Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.7-Inverse-tangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [153]. This is test number [153].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (153)	% 0.00 (0)
Mathematica	% 100.00 (153)	% 0.00 (0)
Maple	% 86.93 (133)	% 13.07 (20)
Maxima	% 56.21 (86)	% 43.79 (67)
Fricas	% 85.62 (131)	% 14.38 (22)
Sympy	% 32.68 (50)	% 67.32 (103)
Giac	% 39.87 (61)	% 60.13 (92)
Mupad	% 35.95 (55)	% 64.05 (98)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

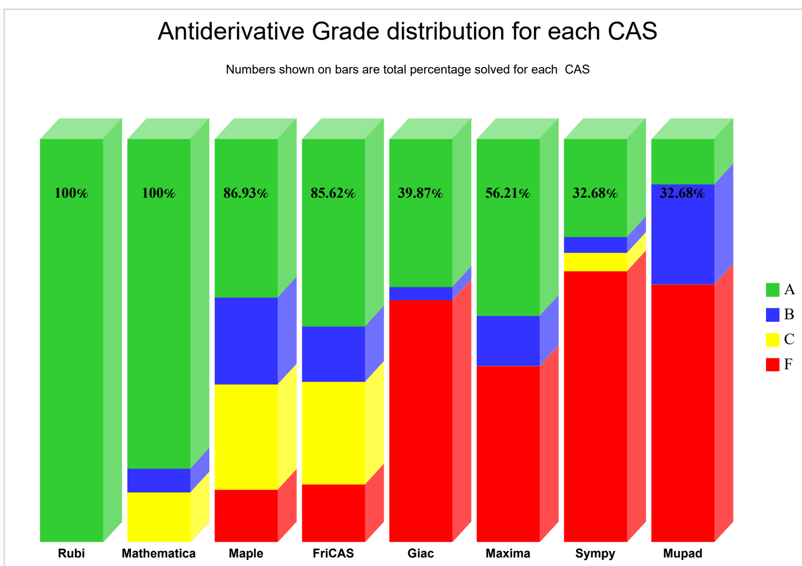
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

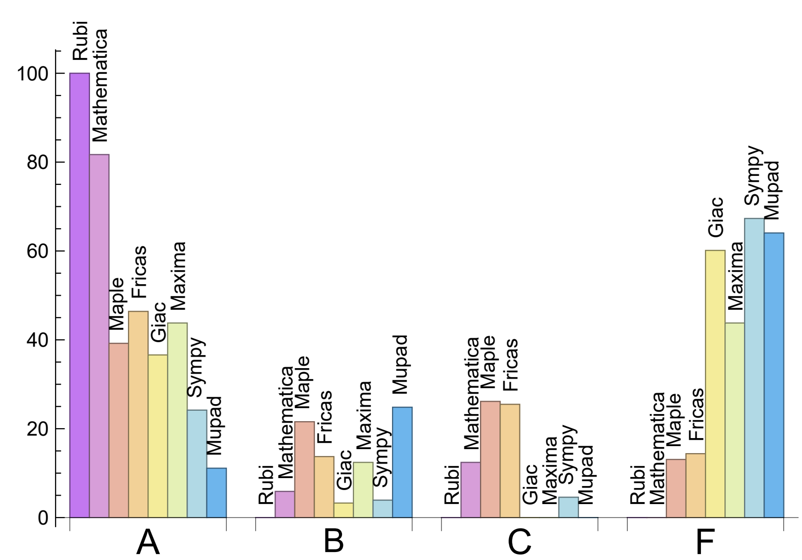
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	81.70	5.88	12.42	0.00
Maple	39.22	21.57	26.14	13.07
Maxima	43.79	12.42	0.00	43.79
Fricas	46.41	13.73	25.49	14.38
Sympy	24.18	3.92	4.58	67.32
Giac	36.60	3.27	0.00	60.13
Mupad	11.11	24.84	0.00	64.05

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	20	100.00 %	0.00 %	0.00 %
Maxima	67	50.75 %	2.99 %	46.27 %
Fricas	22	100.00 %	0.00 %	0.00 %
Sympy	103	33.98 %	42.72 %	23.30 %
Giac	92	90.22 %	6.52 %	3.26 %
Mupad	98	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

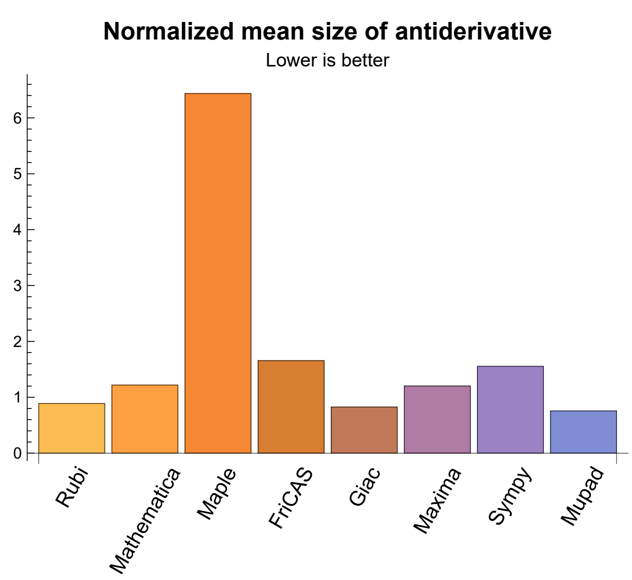
1.3 Performance

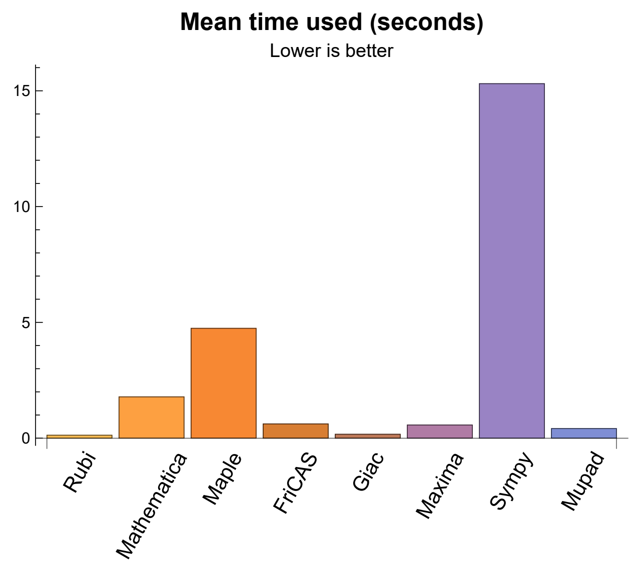
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	111.39	0.89	85.00	1.00
Mathematica	1.78	143.27	1.22	83.00	0.90
Maple	4.74	1100.48	6.44	163.00	2.01
Maxima	0.57	104.80	1.20	51.00	0.94
Fricas	0.62	255.71	1.66	83.00	1.25
Sympy	15.31	87.14	1.55	56.50	0.96
Giac	0.17	48.59	0.83	27.00	0.79
Mupad	0.41	37.20	0.76	21.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {117, 118}

Mathematica {6, 50, 54, 58, 63, 67, 71, 117, 118, 146, 147, 148, 149, 150, 151, 152}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

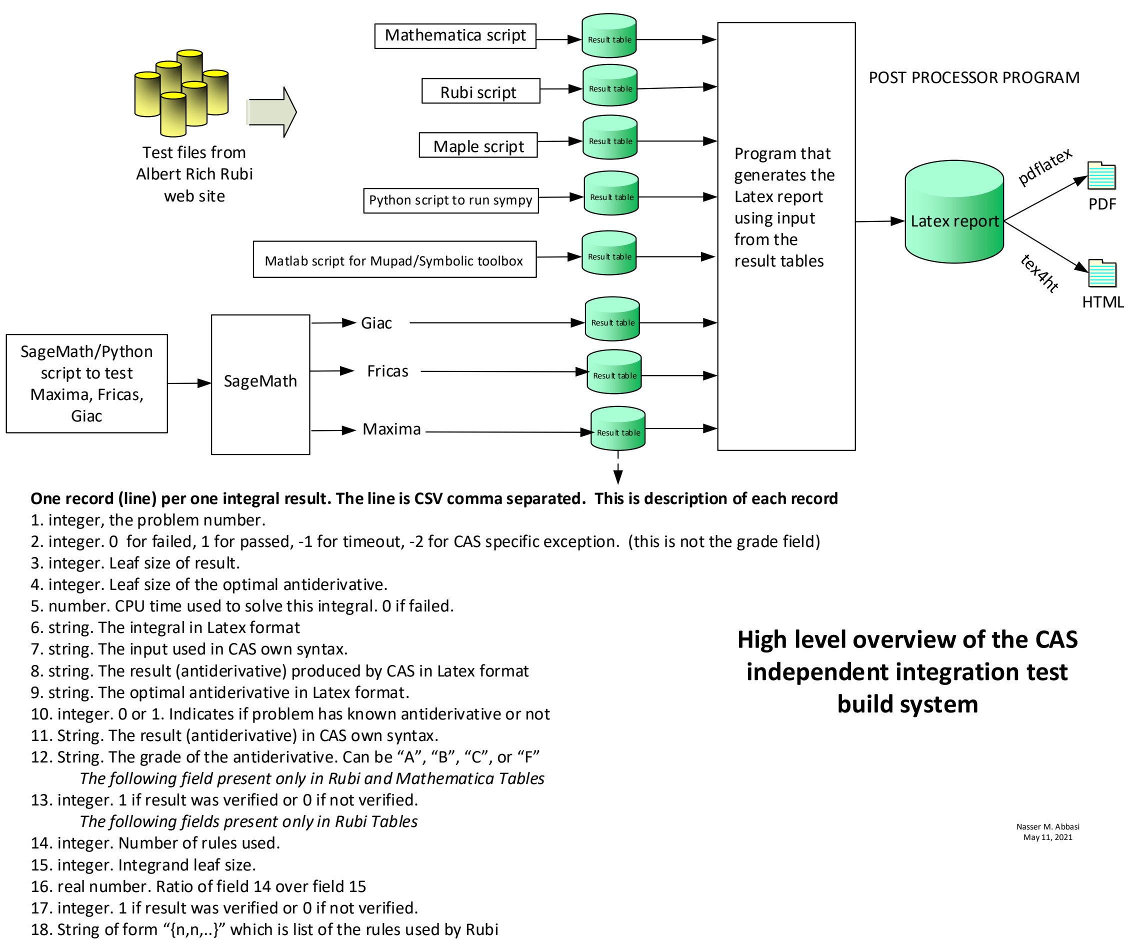
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152 }

B grade: { 50, 54, 58, 63, 67, 71, 75, 76, 93 }

C grade: { 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 121, 148, 149, 150, 151, 153 }

F grade: { }

2.1.3 Maple

A grade: { 1, 3, 4, 5, 7, 8, 9, 10, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 51, 55, 59, 64, 68, 72, 73, 80, 84, 88, 92, 97, 101, 105, 109, 111, 112, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 11, 12, 13, 14, 32, 33, 34, 43, 44, 45, 50, 54, 58, 60, 63, 67, 71, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 114, 115, 117, 118, 130, 131 }

C grade: { 2, 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 147, 148, 149, 150, 151, 152, 153 }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 140, 141, 142, 143, 144, 145, 146 }

2.1.4 Maxima

A grade: { 1, 2, 7, 8, 9, 10, 11, 12, 13, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 60, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 101, 102, 103, 104, 105, 106, 107, 108, 109, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 139, 147, 148, 149, 150, 151, 152 }

B grade: { 14, 38, 39, 50, 52, 53, 54, 56, 57, 58, 63, 65, 66, 67, 69, 70, 71, 110, 113 }

C grade: { }

F grade: { 3, 4, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 48, 49, 51, 55, 59, 61, 62, 64, 68, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 93, 94, 95, 96, 98, 99, 100, 111, 112, 114, 115, 117, 118, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 153 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 67, 68, 71, 72, 80, 84, 88, 92, 97, 101, 105, 109, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 143, 144, 145, 147 }

B grade: { 50, 54, 58, 63, 73, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 134, 148, 149, 150, 151, 152 }

C grade: { 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 111, 112, 114, 115, 117, 118, 153 }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 130, 135, 136, 141, 142, 146 }

2.1.6 Sympy

A grade: { 1, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 60, 80, 119, 122, 123, 124, 128, 129 }

B grade: { 9, 10, 120, 121, 125, 131 }

C grade: { 19, 20, 21, 22, 26, 27, 28 }

F grade: { 2, 6, 18, 23, 24, 25, 29, 32, 33, 34, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 126, 127, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 20, 26, 30, 31, 35, 36, 40, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109, 119, 120, 122, 123, 124, 126, 127, 128, 129, 131, 132, 133, 147, 148, 151, 152 }

B grade: { 7, 8, 9, 10, 125 }

C grade: { }

F grade: { 6, 18, 19, 21, 22, 23, 24, 25, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 153 }

2.1.8 Mupad

A grade: { 31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109 }

B grade: { 1, 2, 14, 30, 38, 39, 40, 43, 44, 45, 47, 60, 110, 113, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 137, 138, 139, 147, 148, 149, 150, 151, 152 }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 41, 42, 46, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 111, 112, 114, 115, 116, 117, 118, 130, 135, 136, 140, 141, 142, 143, 144, 145, 146, 153 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	46	37	45	60	37	230
normalized size	1	1.00	0.88	1.10	0.88	1.07	1.43	0.88	5.48
time (sec)	N/A	0.041	0.018	0.039	0.313	0.444	3.823	0.115	0.686

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	140	40	58	0	40	58
normalized size	1	1.00	0.89	3.11	0.89	1.29	0.00	0.89	1.29
time (sec)	N/A	0.042	0.045	0.294	0.311	0.483	0.000	0.122	0.632

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	86	211	0	76	129	88	-1
normalized size	1	1.00	0.60	1.47	0.00	0.53	0.90	0.61	-0.01
time (sec)	N/A	0.080	0.087	0.050	0.000	0.457	7.248	0.264	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	74	163	0	65	102	75	-1
normalized size	1	1.00	0.64	1.41	0.00	0.56	0.88	0.65	-0.01
time (sec)	N/A	0.042	0.062	0.046	0.000	0.450	2.455	0.247	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	116	0	49	71	62	-1
normalized size	1	1.00	0.67	1.32	0.00	0.56	0.81	0.70	-0.01
time (sec)	N/A	0.029	0.045	0.041	0.000	0.690	0.992	0.246	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	171	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	2.872	0.268	0.000	0.645	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	67	58	44	53	105	-1
normalized size	1	1.00	0.95	1.18	1.02	0.77	0.93	1.84	-0.02
time (sec)	N/A	0.019	0.041	0.040	0.369	0.617	4.595	0.262	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	67	69	68	58	83	198	-1
normalized size	1	1.00	0.79	0.81	0.80	0.68	0.98	2.33	-0.01
time (sec)	N/A	0.028	0.051	0.041	0.334	0.629	5.128	0.257	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	78	117	109	68	352	282	-1
normalized size	1	1.00	0.69	1.04	0.96	0.60	3.12	2.50	-0.01
time (sec)	N/A	0.039	0.067	0.043	0.339	0.839	6.502	0.278	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	89	167	132	80	575	361	-1
normalized size	1	1.00	0.63	1.18	0.94	0.57	4.08	2.56	-0.01
time (sec)	N/A	0.050	0.075	0.046	0.339	0.795	9.155	0.263	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	83	230	167	79	124	137	-1
normalized size	1	1.00	0.67	1.85	1.35	0.64	1.00	1.10	-0.01
time (sec)	N/A	0.067	0.119	0.052	0.338	0.476	11.608	0.246	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	72	180	139	68	97	102	-1
normalized size	1	1.00	0.73	1.82	1.40	0.69	0.98	1.03	-0.01
time (sec)	N/A	0.051	0.101	0.045	0.341	0.492	3.563	0.252	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	132	111	56	70	64	-1
normalized size	1	1.00	0.81	1.78	1.50	0.76	0.95	0.86	-0.01
time (sec)	N/A	0.036	0.091	0.043	0.339	0.442	1.239	0.227	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	84	77	41	39	41	35
normalized size	1	1.00	1.00	1.95	1.79	0.95	0.91	0.95	0.81
time (sec)	N/A	0.010	0.024	0.042	0.335	0.442	1.058	0.182	0.658

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	57	0	148	60	54	-1
normalized size	1	1.00	1.46	0.97	0.00	2.51	1.02	0.92	-0.02
time (sec)	N/A	0.033	0.066	0.039	0.000	0.599	5.543	0.247	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	100	0	198	82	83	-1
normalized size	1	1.00	1.11	1.10	0.00	2.18	0.90	0.91	-0.01
time (sec)	N/A	0.046	0.106	0.042	0.000	0.596	11.583	0.228	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	114	150	0	228	148	108	-1
normalized size	1	1.00	0.96	1.26	0.00	1.92	1.24	0.91	-0.01
time (sec)	N/A	0.060	0.131	0.041	0.000	0.634	19.727	0.248	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	170	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.625	0.236	0.000	0.530	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	158	0	0	0	75	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.088	0.463	0.237	0.000	0.599	150.302	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	0	0	0	75	1	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.49	0.01	-0.01
time (sec)	N/A	0.074	0.310	0.247	0.000	0.481	3.962	0.253	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	0	0	0	71	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.58	0.00	-0.01
time (sec)	N/A	0.061	0.130	0.237	0.000	0.476	5.369	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	0	0	0	78	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.075	0.285	0.237	0.000	0.478	146.125	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	162	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.342	0.233	0.000	0.549	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	171	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.108	0.615	0.237	0.000	0.437	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	139	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.160	0.234	0.000	0.549	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	119	0	0	0	75	1	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.00
time (sec)	N/A	0.161	0.123	0.234	0.000	1.270	13.071	0.264	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	89	0	0	0	71	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.27	0.00	-0.00
time (sec)	N/A	0.139	0.119	0.240	0.000	0.516	8.901	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	121	0	0	0	78	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.26	0.00	-0.00
time (sec)	N/A	0.166	0.154	0.235	0.000	0.537	13.090	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	137	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.136	0.233	0.000	0.547	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	43	42	47	41	43	42
normalized size	1	1.00	1.00	0.86	0.84	0.94	0.82	0.86	0.84
time (sec)	N/A	0.146	0.017	0.046	0.425	0.531	0.888	0.137	0.759

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.101	2.563	0.000	0.608	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	530	1631	0	0	0	0	-1
normalized size	1	1.00	1.23	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	0.205	1.991	0.000	0.607	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	354	837	0	0	0	0	-1
normalized size	1	1.00	1.25	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.135	1.325	0.000	0.457	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	93	263	0	0	0	0	-1
normalized size	1	1.00	0.95	2.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.034	0.796	0.000	0.566	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.101	1.336	0.000	0.495	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.890	1.260	0.000	0.571	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	33	158	0	-1
normalized size	1	1.00	0.92	1.11	1.03	0.89	4.27	0.00	-0.03
time (sec)	N/A	0.024	0.064	0.185	0.340	0.517	2.700	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	81	13	32	0	19
normalized size	1	1.00	0.87	0.87	3.52	0.57	1.39	0.00	0.83
time (sec)	N/A	0.009	0.017	0.177	0.324	0.825	0.360	0.000	0.120

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	57	13	32	0	19
normalized size	1	1.00	0.87	0.87	2.48	0.57	1.39	0.00	0.83
time (sec)	N/A	0.007	0.017	0.177	0.318	0.489	0.199	0.000	0.061

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
normalized size	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.003	0.009	0.038	0.321	0.760	0.143	0.128	0.039

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	42	8	34	0	-1
normalized size	1	1.00	0.90	1.00	2.00	0.38	1.62	0.00	-0.05
time (sec)	N/A	0.031	0.017	0.186	0.419	0.794	0.925	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	56	40	42	160	62	-1
normalized size	1	1.00	0.86	1.56	1.11	1.17	4.44	1.72	-0.03
time (sec)	N/A	0.021	0.054	0.312	0.325	0.551	6.606	0.137	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	65	17	17	26	19	25
normalized size	1	1.00	0.87	2.83	0.74	0.74	1.13	0.83	1.09
time (sec)	N/A	0.008	0.018	0.434	0.322	0.444	0.390	0.130	0.542

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	54	17	17	49	19	25
normalized size	1	1.00	0.87	2.35	0.74	0.74	2.13	0.83	1.09
time (sec)	N/A	0.007	0.016	0.440	0.318	0.394	0.303	0.137	0.072

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	15	15	24	15	21
normalized size	1	1.00	1.12	3.19	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.003	0.009	0.045	0.322	0.674	0.131	0.113	0.071

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	35	14	14	27	15	-1
normalized size	1	1.00	1.00	1.84	0.74	0.74	1.42	0.79	-0.05
time (sec)	N/A	0.033	0.019	0.386	0.310	0.469	4.571	0.113	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
normalized size	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.003	0.001	0.036	0.317	0.589	0.140	0.113	0.002

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	363	8040	0	1973	0	0	-1
normalized size	1	1.00	0.90	19.95	0.00	4.90	0.00	0.00	-0.00
time (sec)	N/A	0.517	1.078	65.269	0.000	0.755	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	272	7648	0	1557	0	0	-1
normalized size	1	1.00	0.89	25.08	0.00	5.10	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.698	6.247	0.000	0.672	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	555	1002	433	1117	0	0	-1
normalized size	1	1.00	2.80	5.06	2.19	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.234	7.922	1.216	0.523	0.696	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.132	5.242	0.719	0.000	0.428	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1532	309	322	0	0	-1
normalized size	1	1.00	0.91	9.95	2.01	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.485	6.712	0.382	0.572	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	218	271	0	0	-1
normalized size	1	1.00	0.89	12.17	1.77	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.358	6.033	0.363	0.492	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	967	1489	448	202	0	0	-1
normalized size	1	1.00	11.38	17.52	5.27	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.125	18.752	0.509	0.471	0.630	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.120	0.735	1.638	0.000	0.635	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	137	1533	310	320	0	0	-1
normalized size	1	1.00	0.88	9.89	2.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.513	6.704	0.384	0.508	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1498	219	269	0	0	-1
normalized size	1	1.00	0.90	12.08	1.77	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.391	5.480	0.369	0.711	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	847	1681	448	200	0	0	-1
normalized size	1	1.00	9.85	19.55	5.21	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.129	17.583	0.560	0.463	0.752	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.136	1.136	1.896	0.000	0.423	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	15	15	24	15	21
normalized size	1	1.00	1.12	3.19	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.003	0.009	0.046	0.319	0.483	0.143	0.126	0.002

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	359	7906	0	1585	0	0	-1
normalized size	1	1.00	0.90	19.81	0.00	3.97	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.006	73.710	0.000	0.780	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	270	7538	0	1285	0	0	-1
normalized size	1	1.00	0.89	24.88	0.00	4.24	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.628	6.507	0.000	0.913	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	1648	1159	532	961	0	0	-1
normalized size	1	1.00	8.32	5.85	2.69	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.250	22.192	1.398	0.550	0.715	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.134	5.129	1.301	0.000	0.482	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1532	309	165	0	0	-1
normalized size	1	1.00	0.88	9.95	2.01	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.383	6.565	0.366	0.419	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	218	143	0	0	-1
normalized size	1	1.00	0.89	12.17	1.77	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.234	5.463	0.344	0.434	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	929	1495	456	111	0	0	-1
normalized size	1	1.00	10.93	17.59	5.36	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.131	20.268	0.688	0.442	0.627	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.121	0.764	1.796	0.000	0.491	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	140	1533	310	166	0	0	-1
normalized size	1	1.00	0.90	9.89	2.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.381	6.618	0.374	0.766	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1498	219	144	0	0	-1
normalized size	1	1.00	0.89	12.08	1.77	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.223	5.573	0.349	0.681	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	872	1753	456	112	0	0	-1
normalized size	1	1.00	10.14	20.38	5.30	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.134	15.673	0.700	0.473	0.547	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.119	0.746	2.062	0.000	0.830	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	64	52	0	58	0	0	-1
normalized size	1	1.00	1.64	1.33	0.00	1.49	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.031	0.379	0.000	0.604	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	105	732	0	93	0	0	-1
normalized size	1	1.00	1.42	9.89	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.026	1.344	0.000	0.451	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	356	758	0	125	0	0	-1
normalized size	1	1.00	3.30	7.02	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.118	0.628	0.000	0.584	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	1448	0	0	-1
normalized size	1	1.00	2.01	24.33	0.00	4.84	0.00	0.00	-0.00
time (sec)	N/A	0.209	5.494	47.255	0.000	0.597	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	994	0	0	-1
normalized size	1	1.00	1.64	23.69	0.00	4.34	0.00	0.00	-0.00
time (sec)	N/A	0.152	2.935	37.576	0.000	0.608	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2414	0	596	0	0	-1
normalized size	1	1.00	1.75	15.18	0.00	3.75	0.00	0.00	-0.01
time (sec)	N/A	0.097	1.942	4.748	0.000	0.640	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	132	178	0	334	0	0	-1
normalized size	1	1.00	1.78	2.41	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.070	0.564	0.000	0.935	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.038	6.652	1.394	0.000	0.580	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	305	6936	0	1335	0	0	-1
normalized size	1	1.00	0.86	19.54	0.00	3.76	0.00	0.00	-0.00
time (sec)	N/A	0.461	5.808	60.801	0.000	0.626	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	229	6586	0	1103	0	0	-1
normalized size	1	1.00	0.86	24.67	0.00	4.13	0.00	0.00	-0.00
time (sec)	N/A	0.373	4.871	6.160	0.000	0.930	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	288	350	0	851	0	0	-1
normalized size	1	1.00	1.66	2.01	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.231	4.520	0.363	0.000	1.138	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.126	8.677	0.829	0.000	0.513	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1555	129	291	0	0	-1
normalized size	1	1.00	0.90	10.95	0.91	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.228	5.757	6.865	1.943	0.726	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1519	106	245	0	0	-1
normalized size	1	1.00	0.90	13.44	0.94	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.195	5.722	5.013	2.044	0.616	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	1381	80	187	0	0	-1
normalized size	1	1.00	0.90	17.48	1.01	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.945	0.579	1.992	0.675	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.112	4.094	1.753	0.000	1.003	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1570	129	291	0	0	-1
normalized size	1	1.00	0.88	10.83	0.89	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.220	5.813	6.790	1.950	0.615	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1534	107	245	0	0	-1
normalized size	1	1.00	0.88	13.22	0.92	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.197	5.677	4.946	1.946	0.521	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	1351	80	187	0	0	-1
normalized size	1	1.00	0.87	16.48	0.98	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.852	0.457	1.948	0.551	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.116	4.104	1.658	0.000	0.641	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	1448	0	0	-1
normalized size	1	1.00	2.01	24.33	0.00	4.84	0.00	0.00	-0.00
time (sec)	N/A	0.209	6.055	49.104	0.000	1.067	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	994	0	0	-1
normalized size	1	1.00	1.64	23.69	0.00	4.34	0.00	0.00	-0.00
time (sec)	N/A	0.153	2.998	37.481	0.000	0.573	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2415	0	596	0	0	-1
normalized size	1	1.00	1.75	15.19	0.00	3.75	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.184	4.685	0.000	0.521	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	132	178	0	334	0	0	-1
normalized size	1	1.00	1.81	2.44	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.085	0.550	0.000	0.475	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.043	5.933	2.477	0.000	0.456	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	299	6864	0	1315	0	0	-1
normalized size	1	1.00	0.85	19.56	0.00	3.75	0.00	0.00	-0.00
time (sec)	N/A	0.465	6.176	60.585	0.000	0.590	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	225	6514	0	1087	0	0	-1
normalized size	1	1.00	0.85	24.58	0.00	4.10	0.00	0.00	-0.00
time (sec)	N/A	0.370	4.763	6.266	0.000	0.645	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	287	350	0	839	0	0	-1
normalized size	1	1.00	1.65	2.01	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.228	4.259	0.542	0.000	0.965	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.121	8.874	1.064	0.000	0.470	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1554	129	291	0	0	-1
normalized size	1	1.00	0.90	10.94	0.91	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.236	1.830	6.835	1.995	0.513	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1518	106	245	0	0	-1
normalized size	1	1.00	0.90	13.43	0.94	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.206	1.793	5.278	2.041	0.556	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	1381	80	187	0	0	-1
normalized size	1	1.00	0.90	17.48	1.01	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.791	0.730	1.999	0.529	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.110	4.133	2.008	0.000	0.456	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1571	129	291	0	0	-1
normalized size	1	1.00	0.88	10.83	0.89	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.231	1.848	7.321	2.006	0.477	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1535	107	245	0	0	-1
normalized size	1	1.00	0.88	13.23	0.92	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.201	1.700	5.235	2.029	0.490	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	1351	80	187	0	0	-1
normalized size	1	1.00	0.87	16.48	0.98	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.795	0.770	1.963	0.513	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.112	4.268	1.903	0.000	0.506	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	59	59	34	40	0	0	21
normalized size	1	1.00	1.90	1.90	1.10	1.29	0.00	0.00	0.68
time (sec)	N/A	0.026	0.048	0.056	0.428	0.465	0.000	0.000	0.688

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	44	0	65	0	0	-1
normalized size	1	1.00	0.79	0.70	0.00	1.03	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.012	0.132	0.000	0.451	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	70	0	87	0	0	-1
normalized size	1	1.00	1.00	0.77	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.009	0.125	0.000	0.481	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	83	106	63	103	0	0	37
normalized size	1	1.00	1.84	2.36	1.40	2.29	0.00	0.00	0.82
time (sec)	N/A	0.028	0.113	0.059	0.440	0.500	0.000	0.000	0.745

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	71	349	0	151	0	0	-1
normalized size	1	1.00	0.78	3.84	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.016	0.229	0.000	0.805	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	407	0	187	0	0	-1
normalized size	1	1.00	1.00	3.06	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.010	0.230	0.000	0.521	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	167	186	189	212	0	0	-1
normalized size	1	1.00	0.85	0.95	0.96	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.196	0.062	0.500	1.032	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	250	236	672	0	304	0	0	-1
normalized size	1	1.08	1.02	2.90	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.100	0.322	0.000	0.455	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	313	334	758	0	378	0	0	-1
normalized size	1	1.04	1.11	2.51	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.016	0.294	0.000	0.812	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	19	28	19	20	20
normalized size	1	1.00	1.00	0.92	0.76	1.12	0.76	0.80	0.80
time (sec)	N/A	0.021	0.019	0.042	0.316	0.554	6.360	0.113	0.089

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	32	31	50	153	32	31
normalized size	1	1.00	0.78	0.71	0.69	1.11	3.40	0.71	0.69
time (sec)	N/A	0.032	0.038	0.046	0.411	0.675	0.516	0.111	0.124

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	54	54	77	223	0	46
normalized size	1	1.00	1.27	0.84	0.84	1.20	3.48	0.00	0.72
time (sec)	N/A	0.067	0.047	0.047	0.405	1.130	0.604	0.000	0.700

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	24	18	26	24	24
normalized size	1	1.00	0.73	0.83	0.80	0.60	0.87	0.80	0.80
time (sec)	N/A	0.011	0.011	0.037	0.413	0.653	0.210	0.125	0.075

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.025	0.033	0.036	0.313	0.501	0.257	0.123	0.225

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	16	15	14	16	15
normalized size	1	1.00	1.00	1.12	0.94	0.88	0.82	0.94	0.88
time (sec)	N/A	0.039	0.048	0.076	0.326	0.577	0.515	0.115	0.135

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	26	53	104	16
normalized size	1	1.00	1.00	0.94	0.89	1.44	2.94	5.78	0.89
time (sec)	N/A	0.151	0.033	0.046	0.565	0.539	0.622	0.143	0.127

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	45	44	40	0	44	72
normalized size	1	1.00	0.85	0.66	0.65	0.59	0.00	0.65	1.06
time (sec)	N/A	0.028	0.064	0.056	0.455	0.567	0.000	0.128	1.653

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	40	39	35	0	39	65
normalized size	1	1.00	0.90	0.68	0.66	0.59	0.00	0.66	1.10
time (sec)	N/A	0.026	0.033	0.055	0.463	0.580	0.000	0.151	0.940

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	28	39	34	58
normalized size	1	1.00	0.96	0.70	0.68	0.56	0.78	0.68	1.16
time (sec)	N/A	0.018	0.030	0.053	0.461	0.992	161.339	0.152	0.850

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	28	26	22	29	27	40
normalized size	1	1.00	1.05	0.76	0.70	0.59	0.78	0.73	1.08
time (sec)	N/A	0.010	0.428	0.054	0.451	0.428	73.759	0.136	0.894

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	84	194	43	0	0	0	-1
normalized size	1	1.00	2.00	4.62	1.02	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.202	1.402	0.433	0.750	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	57	29	28	537	28	44
normalized size	1	1.00	0.98	1.39	0.71	0.68	13.10	0.68	1.07
time (sec)	N/A	0.022	0.034	0.057	0.451	0.614	68.660	0.148	1.409

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	35	0	34	49
normalized size	1	1.00	0.96	0.70	0.68	0.70	0.00	0.68	0.98
time (sec)	N/A	0.024	0.031	0.073	0.452	0.567	0.000	0.145	1.359

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	40	39	40	0	39	56
normalized size	1	1.00	0.86	0.68	0.66	0.68	0.00	0.66	0.95
time (sec)	N/A	0.025	0.044	0.077	0.453	0.501	0.000	0.148	0.939

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	73	0	126	0	0	57
normalized size	1	1.00	1.00	1.16	0.00	2.00	0.00	0.00	0.90
time (sec)	N/A	0.107	0.063	0.517	0.000	0.785	0.000	0.000	0.727

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	0	0	0	0	-1
normalized size	1	1.00	1.00	1.22	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.031	0.433	0.000	0.527	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	0	0	0	0	-1
normalized size	1	1.00	1.00	1.22	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.022	0.433	0.000	0.954	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	71	0	83	0	0	49
normalized size	1	1.00	1.00	1.29	0.00	1.51	0.00	0.00	0.89
time (sec)	N/A	0.108	0.056	1.785	0.000	0.551	0.000	0.000	0.599

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	71	29	82	0	0	51
normalized size	1	1.00	1.00	1.25	0.51	1.44	0.00	0.00	0.89
time (sec)	N/A	0.099	0.027	0.425	0.559	0.444	0.000	0.000	0.602

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	72	29	82	0	0	51
normalized size	1	1.00	1.00	1.22	0.49	1.39	0.00	0.00	0.86
time (sec)	N/A	0.101	0.027	0.434	0.615	0.543	0.000	0.000	0.614

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	0	132	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.238	1.113	0.000	0.680	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.133	0.444	0.000	0.840	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.062	0.448	0.000	0.717	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	83	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	1.30	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.109	0.448	0.000	0.667	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	0	0	82	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	1.24	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.100	0.445	0.000	0.819	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	83	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.096	0.446	0.000	0.614	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	166	0	0	0	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	3.034	6.751	0.000	0.544	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	61	1299	48	75	0	65	66
normalized size	1	1.00	1.27	27.06	1.00	1.56	0.00	1.35	1.38
time (sec)	N/A	0.075	0.115	1.345	0.432	0.701	0.000	2.064	0.769

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1375	131	221	0	154	133
normalized size	1	1.00	1.42	13.35	1.27	2.15	0.00	1.50	1.29
time (sec)	N/A	0.155	0.157	2.615	0.430	0.881	0.000	0.914	0.314

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	431	0	0	164
normalized size	1	1.00	0.49	7.53	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.182	0.126	2.616	0.422	0.506	0.000	0.000	1.416

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	431	0	0	164
normalized size	1	1.00	0.49	7.53	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.179	0.124	2.070	0.426	0.683	0.000	0.000	1.656

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	842	169	276	0	154	135
normalized size	1	1.00	1.41	8.17	1.64	2.68	0.00	1.50	1.31
time (sec)	N/A	0.145	0.160	1.876	0.452	0.647	0.000	0.137	0.819

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	885	47	131	0	66	67
normalized size	1	1.00	1.21	18.83	1.00	2.79	0.00	1.40	1.43
time (sec)	N/A	0.079	0.116	0.980	0.417	0.607	0.000	0.126	0.697

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	116	896	0	246	0	0	-1
normalized size	1	1.00	0.71	5.50	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.572	0.299	0.701	0.000	0.616	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [73] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	12	0.333
2	A	4	4	1.00	14	0.286
3	A	6	4	1.00	25	0.160
4	A	5	4	1.00	25	0.160
5	A	4	4	1.00	23	0.174
6	A	8	8	1.00	25	0.320
7	A	2	2	1.00	25	0.080
8	A	3	3	1.00	25	0.120
9	A	4	3	1.00	25	0.120
10	A	5	3	1.00	25	0.120
11	A	4	3	1.00	25	0.120
12	A	4	3	1.00	25	0.120
13	A	4	3	1.00	25	0.120
14	A	2	2	1.00	21	0.095
15	A	4	4	1.00	25	0.160
16	A	5	5	1.00	25	0.200
17	A	6	5	1.00	25	0.200
18	A	6	4	1.00	27	0.148
19	A	5	4	1.00	27	0.148
20	A	4	4	1.00	27	0.148
21	A	3	3	1.00	27	0.111
22	A	4	4	1.00	27	0.148
23	A	5	4	1.00	27	0.148
24	A	6	4	1.00	27	0.148
25	A	7	6	1.00	27	0.222
26	A	6	6	1.00	27	0.222
27	A	5	5	1.00	27	0.185
28	A	6	6	1.00	27	0.222
29	A	7	6	1.00	27	0.222
30	A	8	7	1.00	11	0.636
31	A	0	0	0.00	0	0.000
32	A	9	7	1.00	40	0.175
33	A	7	6	1.00	40	0.150
34	A	4	4	1.00	38	0.105
35	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	0	0	0.00	0	0.000
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	11	0.182
39	A	2	2	1.00	9	0.222
40	A	2	2	1.00	7	0.286
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	9	0.222
45	A	2	2	1.00	7	0.286
46	A	2	2	1.00	11	0.182
47	A	2	2	1.00	7	0.286
48	A	11	6	1.00	15	0.400
49	A	9	5	1.00	13	0.385
50	A	7	4	1.00	11	0.364
51	A	0	0	0.00	0	0.000
52	A	7	7	1.00	21	0.333
53	A	6	6	1.00	19	0.316
54	A	5	5	1.00	17	0.294
55	A	0	0	0.00	0	0.000
56	A	7	7	1.00	21	0.333
57	A	6	6	1.00	19	0.316
58	A	5	5	1.00	17	0.294
59	A	0	0	0.00	0	0.000
60	A	2	2	1.00	7	0.286
61	A	11	6	1.00	15	0.400
62	A	9	5	1.00	13	0.385
63	A	7	4	1.00	11	0.364
64	A	0	0	0.00	0	0.000
65	A	7	7	1.00	21	0.333
66	A	6	6	1.00	19	0.316
67	A	5	5	1.00	17	0.294
68	A	0	0	0.00	0	0.000
69	A	7	7	1.00	21	0.333
70	A	6	6	1.00	19	0.316
71	A	5	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	0	0	0.00	0	0.000
73	A	6	4	1.00	3	1.333
74	A	8	5	1.00	5	1.000
75	A	10	6	1.00	7	0.857
76	A	12	6	1.00	15	0.400
77	A	10	6	1.00	15	0.400
78	A	8	5	1.00	13	0.385
79	A	6	4	1.00	7	0.571
80	A	0	0	0.00	0	0.000
81	A	11	6	1.00	15	0.400
82	A	9	5	1.00	13	0.385
83	A	7	4	1.00	11	0.364
84	A	0	0	0.00	0	0.000
85	A	7	7	1.00	19	0.368
86	A	6	6	1.00	17	0.353
87	A	5	5	1.00	15	0.333
88	A	0	0	0.00	0	0.000
89	A	7	7	1.00	22	0.318
90	A	6	6	1.00	20	0.300
91	A	5	5	1.00	18	0.278
92	A	0	0	0.00	0	0.000
93	A	12	6	1.00	15	0.400
94	A	10	6	1.00	15	0.400
95	A	8	5	1.00	13	0.385
96	A	6	4	1.00	7	0.571
97	A	0	0	0.00	0	0.000
98	A	11	6	1.00	15	0.400
99	A	9	5	1.00	13	0.385
100	A	7	4	1.00	11	0.364
101	A	0	0	0.00	0	0.000
102	A	7	7	1.00	19	0.368
103	A	6	6	1.00	17	0.353
104	A	5	5	1.00	15	0.333
105	A	0	0	0.00	0	0.000
106	A	7	7	1.00	22	0.318
107	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	5	5	1.00	18	0.278
109	A	0	0	0.00	0	0.000
110	A	4	3	1.00	4	0.750
111	A	7	4	1.00	6	0.667
112	A	9	5	1.00	8	0.625
113	A	4	3	1.00	8	0.375
114	A	7	4	1.00	10	0.400
115	A	9	5	1.00	12	0.417
116	A	6	6	1.00	12	0.500
117	A	9	5	1.08	14	0.357
118	A	11	6	1.04	16	0.375
119	A	5	6	1.00	10	0.600
120	A	5	4	1.00	8	0.500
121	A	9	8	1.00	14	0.571
122	A	4	4	1.00	8	0.500
123	A	1	1	1.00	14	0.071
124	A	1	1	1.00	19	0.053
125	A	5	3	1.00	26	0.115
126	A	9	6	1.00	21	0.286
127	A	8	6	1.00	21	0.286
128	A	7	6	1.00	19	0.316
129	A	6	6	1.00	18	0.333
130	A	6	5	1.00	21	0.238
131	A	6	6	1.00	21	0.286
132	A	7	6	1.00	21	0.286
133	A	8	6	1.00	21	0.286
134	A	2	2	1.00	39	0.051
135	A	2	2	1.00	39	0.051
136	A	2	2	1.00	37	0.054
137	A	2	2	1.00	39	0.051
138	A	2	2	1.00	39	0.051
139	A	2	2	1.00	39	0.051
140	A	2	2	1.00	40	0.050
141	A	2	2	1.00	40	0.050
142	A	2	2	1.00	38	0.053
143	A	2	2	1.00	40	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	2	2	1.00	40	0.050
145	A	2	2	1.00	40	0.050
146	A	9	7	1.00	24	0.292
147	A	5	5	1.00	20	0.250
148	A	8	7	1.00	20	0.350
149	A	13	10	1.00	20	0.500
150	A	13	10	1.00	20	0.500
151	A	8	7	1.00	20	0.350
152	A	5	5	1.00	20	0.250
153	A	13	9	1.00	24	0.375

Chapter 3

Listing of integrals

3.1 $\int x^3 \tan^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b}-\frac{\log\left(\left(a+bx^4\right)^2+1\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arctan(b*x^4+a)/b-1/8*ln(1+(b*x^4+a)^2)/b
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5039, 4846, 260}

$$\frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b}-\frac{\log\left(\left(a+bx^4\right)^2+1\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTan[a + b*x^4])/(4*b) - Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^3 \tan^{-1}\left(a+bx^4\right) dx = \frac{1}{4}\operatorname{Subst}\left(\int \tan^{-1}(a+bx)\,dx, x, x^4\right)$$
$$= \frac{\operatorname{Subst}\left(\int \tan^{-1}(x)\,dx, x, a+bx^4\right)}{4b}$$
$$= \frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}$$
$$= \frac{\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\log\left(1+\left(a+bx^4\right)^2\right)}{8b}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.88

$$-\frac{\log\left(\left(a+bx^4\right)^2+1\right)-2\left(a+bx^4\right)\tan^{-1}\left(a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] -1/8*(-2*(a + b*x^4)*ArcTan[a + b*x^4] + Log[1 + (a + b*x^4)^2])/b
```

fricas [A] time = 0.44, size = 45, normalized size = 1.07

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/
b
```

giac [A] time = 0.12, size = 37, normalized size = 0.88

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

maple [A] time = 0.04, size = 46, normalized size = 1.10

$$\frac{\arctan\left(b\,x^4+a\right)x^4}{4}+\frac{\arctan\left(b\,x^4+a\right)a}{4b}-\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x^4+a),x)
```

```
[Out] 1/4*arctan(b*x^4+a)*x^4+1/4/b*arctan(b*x^4+a)*a-1/8*ln(1+(b*x^4+a)^2)/b
```

maxima [A] time = 0.31, size = 37, normalized size = 0.88

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

mupad [B] time = 0.69, size = 230, normalized size = 5.48

$$\frac{x^4\,\mathrm{atan}\left(b\,x^4+a\right)}{4}-\frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}+\frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{}{a^6+3\,a^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a + b*x^4),x)
```

```
[Out] (x^4*atan(a + b*x^4))/4 - log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (a*ata
n(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^5)
/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*a^2
 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*b*x^
4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))/(4*b
)
```

sympy [A] time = 3.82, size = 60, normalized size = 1.43

$$\begin{cases} \frac{a\,\mathrm{atan}\left(a+bx^4\right)}{4b}+\frac{x^4\,\mathrm{atan}\left(a+bx^4\right)}{4}-\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for } b\neq 0 \\ \frac{x^4\,\mathrm{atan}\,(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x**4+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x**4)/(4*b) + x**4*atan(a + b*x**4)/4 - log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*atan(a)/4, True))
```

3.2 $\int x^{-1+n} \tan^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=45

$$\frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\log\left((a+bx^n)^2+1\right)}{2bn}$$

[Out] (a+b*x^n)*arctan(a+b*x^n)/b/n-1/2*ln(1+(a+b*x^n)^2)/b/n

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5039, 4846, 260}

$$\frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\log\left((a+bx^n)^2+1\right)}{2bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*ArcTan[a + b*x^n],x]

[Out] ((a + b*x^n)*ArcTan[a + b*x^n])/(b*n) - Log[1 + (a + b*x^n)^2]/(2*b*n)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5039

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\tan^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \tan^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn}-\frac{\log\left(1+(a+bx^n)^2\right)}{2bn}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.89

$$-\frac{\log \left((a+bx^n)^2+1\right)-2 (a+bx^n) \tan^{-1}(a+bx^n)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

```
[Out] -1/2*(-2*(a + b*x^n)*ArcTan[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(b*n)
```

fricas [A] time = 0.48, size = 58, normalized size = 1.29

$$\frac{2\, bx^n \arctan(bx^n+a)+2\, a \arctan(bx^n+a)-\log\left(b^2x^{2\,n}+2\,abx^n+a^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arctan(b*x^n + a) + 2*a*arctan(b*x^n + a) - log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

giac [A] time = 0.12, size = 40, normalized size = 0.89

$$\frac{2\,(bx^n+a) \arctan(bx^n+a)-\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

maple [C] time = 0.29, size = 140, normalized size = 3.11

$$-\frac{ix^n \ln(1+i(a+b\,x^n))}{2n}+\frac{ix^n \ln(1-i(a+b\,x^n))}{2n}-\frac{\ln\left(\frac{i+a}{b}+x^n\right)}{2bn}-\frac{\ln\left(x^n-\frac{i-a}{b}\right)}{2bn}+\frac{i\ln\left(\frac{i+a}{b}+x^n\right)a}{2bn}-\frac{i\ln\left(x^n-\frac{i-a}{b}\right)a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arctan(a+b*x^n),x)
```

```
[Out] -1/2*I/n*x^n*ln(1+I*(a+b*x^n))+1/2*I/n*x^n*ln(1-I*(a+b*x^n))-1/2/b/n*ln((I+
a)/b+x^n)-1/2/b/n*ln(x^n-(I-a)/b)+1/2*I/b/n*ln((I+a)/b+x^n)*a-1/2*I/b/n*ln(
x^n-(I-a)/b)*a
```

maxima [A] time = 0.31, size = 40, normalized size = 0.89

$$\frac{2\,(bx^n+a) \arctan(bx^n+a)-\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

mupad [B] time = 0.63, size = 58, normalized size = 1.29

$$\frac{x^n \operatorname{atan}(a+b\,x^n)}{n}-\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)-2\,a \operatorname{atan}(a+b\,x^n)}{2\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*atan(a + b*x^n),x)
```

```
[Out] (x^n*atan(a + b*x^n))/n - (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1) - 2*a*ata
n(a + b*x^n))/(2*b*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atan(a+b*x**n),x)
```

```
[Out] Timed out
```

3.3 $\int x^5 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=144

$$\frac{5d^3\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}+\frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}}+\frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}$$

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+5/96*d^3*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(7/2)+5/96*d^2*x*(e*x^2+d)^(1/2)/(-e)^(5/2)+5/14
4*d*x^3*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/36*x^5*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5151, 321, 217, 206}

$$\frac{5d^3\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}+\frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}}+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}+\frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (5*d^2*x*Sqrt[d + e*x^2])/(96*(-e)^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*
(-e)^(3/2)) + (x^5*Sqrt[d + e*x^2])/(36*Sqrt[-e]) + (x^6*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/6 + (5*d^3*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(96*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \frac{x^6}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(5d)\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{36\sqrt{-e}}$$

$$= \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(5d^2\right)\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{48(-e)^{3/2}}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(5d^3\right)\ \cdots}{96\ \cdots}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(5d^3\right)\ \cdots}{\cdots}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{5d^3\tan\cdots}{96\cdots}$$

Mathematica [A] time = 0.09, size = 86, normalized size = 0.60

$$\frac{3\left(5d^3+16e^3x^6\right)\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\sqrt{-e}\,x\sqrt{d+ex^2}\left(-15d^2+10dex^2-8e^2x^4\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 3*(5*d^3 +
 16*e^3*x^6)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(288*e^3)
```

fricas [A] time = 0.46, size = 76, normalized size = 0.53

$$-\frac{\left(8\,e^2x^5-10\,dex^3+15\,d^2x\right)\sqrt{ex^2+d}\,\sqrt{-e}\,-3\left(16\,e^3x^6+5\,d^3\right)\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{288\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/288*((8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e) - 3*(1
6*e^3*x^6 + 5*d^3)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^3
```

giac [A] time = 0.26, size = 88, normalized size = 0.61

$$\frac{1}{6}\,x^6\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{5}{96}\,d^3\arcsin\left(\frac{xe}{\sqrt{-de}}\right)e^{(-3)}-\frac{1}{288}\left(2\left(4\,x^2e^{(-1)}-5\,de^{(-2)}\right)x^2+15\,d^2e^{(-3)}\right)\sqrt{-x^2e^2-de}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^6*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 5/96*d^3*arcsin(x*e/sqrt(-d*e)
)*e^(-3) - 1/288*(2*(4*x^2*e^(-1) - 5*d*e^(-2))*x^2 + 15*d^2*e^(-3))*sqrt(-
x^2*e^2 - d*e)*x
```

maple [A] time = 0.05, size = 211, normalized size = 1.47

$$\frac{x^{6}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{6}+\frac{\sqrt{-e}\;x^{7}\sqrt{e\,x^{2}+d}}{48d}-\frac{7\sqrt{-e}\;x^{5}\sqrt{e\,x^{2}+d}}{288e}+\frac{35\sqrt{-e}\;d\,x^{3}\sqrt{e\,x^{2}+d}}{1152e^{2}}-\frac{5\sqrt{-e}\;d^{2}x\sqrt{e\,x^{2}+d}}{128e^{3}}+\frac{5\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/48*(-e)^(1/2)/d*x^7*(e*x^2+d
)^(1/2)-7/288*(-e)^(1/2)/e*x^5*(e*x^2+d)^(1/2)+35/1152*(-e)^(1/2)/e^2*d*x^3
*(e*x^2+d)^(1/2)-5/128*(-e)^(1/2)/e^3*d^2*x*(e*x^2+d)^(1/2)+5/96*(-e)^(1/2)
/e^(7/2)*d^3*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/48*(-e)^(1/2)/d*x^5*(e*x^2+d)^
(3/2)/e+5/288*(-e)^(1/2)/e^2*x^3*(e*x^2+d)^(3/2)-5/384*(-e)^(1/2)*d/e^3*x*(
e*x^2+d)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5}\,\mathrm{atan}\left(\frac{\sqrt{-e}\;x}{\sqrt{e\,x^{2}+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 7.25, size = 129, normalized size = 0.90

$$\begin{cases} \frac{5id^{3}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^{2}}}\right)}{96e^{3}}-\frac{5id^{2}x\sqrt{d+ex^{2}}}{96e^{\frac{5}{2}}}+\frac{5idx^{3}\sqrt{d+ex^{2}}}{144e^{\frac{3}{2}}}+\frac{ix^{6}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^{2}}}\right)}{6}-\frac{ix^{5}\sqrt{d+ex^{2}}}{36\sqrt{e}} & \text{for}\: e\neq 0\\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*I*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*I*d**2*
x*sqrt(d + e*x**2)/(96*e**(5/2)) + 5*I*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2
)) + I*x**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - I*x**5*sqrt(d + e*x**2)/(
36*sqrt(e)), Ne(e, 0)), (0, True))
```

3.4 $\int x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=116

$$-\frac{3d^2\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}+\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}$$

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-3/32*d^2*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(5/2)+3/32*d*x*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/16*x
^3*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5151, 321, 217, 206}

$$-\frac{3d^2\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*(-e)^(3/2)) + (x^3*Sqrt[d + e*x^2])/(16*Sqrt[-e
]) + (x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (3*d^2*Sqrt[-e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\int \frac{x^4}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d)\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{16\sqrt{-e}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(3d^2\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{32(-e)^{3/2}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,d\right.}{32(-e)^{3/2}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+e}}\right.}{32e^{5/2}}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.64

$$\frac{\left(8e^2x^4-3d^2\right)\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\sqrt{-e}\,x\sqrt{d+ex^2}\left(3d-2ex^2\right)}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + (-3*d^2 + 8*e^2*x^4)*ArcTan[(
Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(32*e^2)
```

fricas [A] time = 0.45, size = 65, normalized size = 0.56

$$-\frac{\left(2\,ex^3-3\,dx\right)\sqrt{ex^2+d}\,\sqrt{-e}-\left(8\,e^2x^4-3\,d^2\right)\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{32\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*((2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(-e) - (8*e^2*x^4 - 3*d^2)*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^2
```

giac [A] time = 0.25, size = 75, normalized size = 0.65

$$\frac{1}{4}\,x^4\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)+\frac{3}{32}\,d^2\arcsin\left(\frac{xe}{\sqrt{-de}}\right)e^{(-2)}-\frac{1}{32}\,\sqrt{-x^2e^2-de}\left(2\,x^2e^{(-1)}-3\,de^{(-2)}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 3/32*d^2*arcsin(x*e/sqrt(-d*e)
)*e^(-2) - 1/32*sqrt(-x^2*e^2 - d*e)*(2*x^2*e^(-1) - 3*d*e^(-2))*x
```

maple [A] time = 0.05, size = 163, normalized size = 1.41

$$\frac{x^4\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4}+\frac{\sqrt{-e}\;x^5\sqrt{e\,x^2+d}}{24d}-\frac{5\sqrt{-e}\;x^3\sqrt{e\,x^2+d}}{96e}+\frac{\sqrt{-e}\;dx\sqrt{e\,x^2+d}}{16e^2}-\frac{3\sqrt{-e}\;d^2\ln\left(x\sqrt{e}+\sqrt{e\,x^2+\right.}{32e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/24*(-e)^(1/2)/d*x^5*(e*x^2+d
)^(1/2)-5/96*(-e)^(1/2)/e*x^3*(e*x^2+d)^(1/2)+1/16*(-e)^(1/2)/e^2*d*x*(e*x^
2+d)^(1/2)-3/32*(-e)^(1/2)/e^(5/2)*d^2*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/24*(
-e)^(1/2)/d*x^3*(e*x^2+d)^(3/2)/e+1/32*(-e)^(1/2)/e^2*x*(e*x^2+d)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\, x^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [**A**] time = 2.46, size = 102, normalized size = 0.88

$$\begin{cases} -\frac{3 i d^2 \operatorname{atanh}\left(\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right)}{32 e^2} + \frac{3 i d x \sqrt{d+e x^2}}{32 e^{\frac{3}{2}}} + \frac{i x^4 \operatorname{atanh}\left(\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right)}{4} - \frac{i x^3 \sqrt{d+e x^2}}{16 \sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*I*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*I*d*x*
sqrt(d + e*x**2)/(32*e**(3/2)) + I*x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4
 - I*x**3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

3.5 $\int x \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=88

$$\frac{d\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}+\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*d*arctanh(x*e^(1/2)/(e*x^2
+d)^(1/2))*(-e)^(1/2)/e^(3/2)+1/4*x*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5151, 321, 217, 206}

$$\frac{d\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}+\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(4*Sqrt[-e]) + (x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\int\frac{x^2}{\sqrt{d+ex^2}}dx$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{d\int\frac{1}{\sqrt{d+ex^2}}dx}{4\sqrt{-e}}$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{d\,\text{Subst}\left(\int\frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e}}$$
$$= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e^2}}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.67

$$\frac{\left(d+2ex^2\right)\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \sqrt{-e}\,x\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-(Sqrt[-e]*x*Sqrt[d + e*x^2]) + (d + 2*e*x^2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/(4*e)
```

fricas [A] time = 0.69, size = 49, normalized size = 0.56

$$-\frac{\sqrt{ex^2+d}\,\sqrt{-e}\,x - \left(2\,ex^2+d\right)\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(e*x^2 + d)*sqrt(-e)*x - (2*e*x^2 + d)*arctan(sqrt(-e)*x/sqrt(e*x
^2 + d)))/e
```

giac [A] time = 0.25, size = 62, normalized size = 0.70

$$\frac{1}{2}\,x^2\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right) - \frac{1}{4}\,d\arcsin\left(\frac{xe}{\sqrt{-de}}\right)e^{(-1)} - \frac{1}{4}\,\sqrt{-x^2e^2-de}\,xe^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/4*d*arcsin(x*e/sqrt(-d*e))*e
^(-1) - 1/4*sqrt(-x^2*e^2 - d*e)*x*e^(-1)
```

maple [A] time = 0.04, size = 116, normalized size = 1.32

$$\frac{x^2\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2} + \frac{\sqrt{-e}\;x^3\sqrt{e\,x^2+d}}{8d} - \frac{\sqrt{-e}\;x\sqrt{e\,x^2+d}}{8e} + \frac{\sqrt{-e}\;d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{4e^{\frac{3}{2}}} - \frac{\sqrt{-e}\;x\left(e\,x^2+d\right)^{\frac{3}{2}}}{8de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/8*(-e)^(1/2)/d*x^3*(e*x^2+d)
^(1/2)-1/8*(-e)^(1/2)/e*x*(e*x^2+d)^(1/2)+1/4*(-e)^(1/2)/e^(3/2)*d*ln(x*e^(
1/2)+(e*x^2+d)^(1/2))-1/8*(-e)^(1/2)/d*x*(e*x^2+d)^(3/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.99, size = 71, normalized size = 0.81

$$\begin{cases} \frac{id\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{ix^2\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2} - \frac{ix\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((I*d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + I*x**2*atanh(sqrt(
e)*x/sqrt(d + e*x**2))/2 - I*x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0,
 True))
```

3.6 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\mathrm{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\,\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}}+\log(x)\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}}}{}$$

```
[Out] arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2
*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(
1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1
/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln
(x)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+1/2*polylo
g(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)
^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5149, 2327, 2325, 5659, 3716, 2190, 2279, 2391}

$$\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\,\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}}+\log(x)\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{d}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -(Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*S
qrt[e]*Sqrt[d + e*x^2]) + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sq
rt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*Sqr
t[d + e*x^2]) - (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/S
qrt[d]]*Log[x])/(Sqrt[e]*Sqrt[d + e*x^2]) + ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]*Log[x] + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2327

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e*x^2)/d]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e*x^2)/d], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5149

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Simp
[ArcTan[(c*x)/Sqrt[a + b*x^2]]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b*x
^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{-e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}} \cdots\right.}{\sqrt{e}\sqrt{d} \cdots}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}} \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \tan^{-1}\left(\cdots\right.$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\cdots}{\sqrt{d}}\right)}\cdots\right.}{\sqrt{e}\sqrt{d+ex^2}} \cdots$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\cdots}{\sqrt{d}}\right)}\cdots\right.}{\sqrt{e}\sqrt{d+ex^2}} \cdots$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\cdots}{\sqrt{d}}\right)}\cdots\right.}{\sqrt{e}\sqrt{d+ex^2}} \cdots$$

Mathematica [A] time = 2.87, size = 171, normalized size = 0.59

$$\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(-\text{Li}_2\left(e^{-2\sinh^{-1}\left(\sqrt{\frac{e}{d}}x\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{ex^2}{d}+1}+x\sqrt{\frac{e}{d}}\right) + \sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)^2 + 2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)\log \cdots\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[-e]*Sqrt[1 + (e*x^2)/d]
*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqr
t[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2,
E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

```
[Out] Integral(atan(x*sqrt(-e)/sqrt(d + e*x**2))/x, x)
```

3.7 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*(e*x^2+d)^(1/2
)/d/x
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5151, 264}

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/
(2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}+\frac{1}{2}\sqrt{-e}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx\\ &= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-e}\,x\sqrt{d+ex^2}+d\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[-e]*x*Sqrt[d + e*x^2] + d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/
(d*x^2)
```

fricas [A] time = 0.62, size = 44, normalized size = 0.77

$$-\frac{\sqrt{ex^2+d}\,\sqrt{-e}\,x+d\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(e*x^2 + d)*sqrt(-e)*x + d*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(d
*x^2)
```

giac [B] time = 0.26, size = 105, normalized size = 1.84

$$\frac{xe^3}{4\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)d}-\frac{\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)e^{(-1)}}{4\,dx}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*x*e^3/((sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d) - 1/4*(sqrt(-x^2*e^2
- d*e)*e - sqrt(-d*e)*e)*e^(-1)/(d*x) - 1/2*arctan(x*sqrt(-e)/sqrt(x^2*e +
d))/x^2
```

maple [A] time = 0.04, size = 67, normalized size = 1.18

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d^2x}+\frac{\sqrt{-e}\,ex\sqrt{e\,x^2+d}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x)
```

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)/d^2/x*(e*x^2+d
)^(3/2)+1/2*(-e)^(1/2)/d^2*e*x*(e*x^2+d)^(1/2)
```

maxima [A] time = 0.37, size = 58, normalized size = 1.02

$$-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{2\,x^2}-\frac{\sqrt{-e}\,ex^2+d\sqrt{-e}}{2\,\sqrt{ex^2+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(sqrt(-e)*e*x^2 + d*sqrt(
-e))/(sqrt(e*x^2 + d)*d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

sympy [**A**] time = 4.60, size = 53, normalized size = 0.93

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(2*x**2) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(2*d)
```

3.8 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

Optimal. Leaf size=85

$$-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}$$

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4-1/6*(-e)^(3/2)*(e*x^2+d)^(1/2
)/d^2/x-1/12*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5151, 271, 264}

$$-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(12*d*x^3) - ((-e)^(3/2)*Sqrt[d + e*x^2])/(6*d^
2*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{1}{4}\sqrt{-e}\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx\\
&= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{(-e)^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{6d}\\
&= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.79

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\left(2ex^2-d\right)-3d^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTan[(Sqrt[-e]*x)/Sqrt
[d + e*x^2]])/(12*d^2*x^4)
```

fricas [A] time = 0.63, size = 58, normalized size = 0.68

$$-\frac{3\,d^2\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\left(2\,ex^3-dx\right)\sqrt{ex^2+d}\,\sqrt{-e}}{12\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*e*x^3 - d*x)*sqrt(e*x^
2 + d)*sqrt(-e))/(d^2*x^4)
```

giac [B] time = 0.26, size = 198, normalized size = 2.33

$$-\frac{x^3\left(\frac{9\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^2e^{(-2)}}{x^2}+e^2\right)e^6}{96\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^3d^2}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{4\,x^4}+\frac{\left(\frac{9\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)d^4e^6}{x}+\frac{\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^3d^4e^2}{x^3}\right)e^{(-6)}}{96\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/96*x^3*(9*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^2*e^(-2)/x^2 + e^2)*e^
6/((sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^2) - 1/4*arctan(x*sqrt(-e)/s
qrt(x^2*e + d))/x^4 + 1/96*(9*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^4*e
^6/x + (sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^4*e^2/x^3)*e^(-6)/d^6
```

maple [A] time = 0.04, size = 69, normalized size = 0.81

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{\sqrt{-e}\;e\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{-e}\;\left(e\,x^2+d\right)^{\frac{3}{2}}}{12d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x)
```

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*(-e)^(1/2)*e/d^2/x*(e*x^2
+d)^(1/2)-1/12*(-e)^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.33, size = 68, normalized size = 0.80

$$\frac{\sqrt{ex^2+d}\,\sqrt{-e}\,e}{4\,d^2x}-\frac{\left(ex^2+d\right)^{\frac{3}{2}}\sqrt{-e}}{12\,d^2x^3}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*sqrt(-e)*e/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(-e)/(d
^2*x^3) - 1/4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

sympy [A] time = 5.13, size = 83, normalized size = 0.98

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4x^4} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{12dx^2} + \frac{e^{\frac{3}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*x**4) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(12*d*x**2) + e**(3/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(6*d**2)
```

3.9 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=113

$$-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}$$

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-2/45*(-e)^(3/2)*(e*x^2+d)^(1/
2)/d^2/x^3-4/45*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^5
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{5151, 271, 264\}$

$$-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(30*d*x^5) - (2*(-e)^(3/2)*Sqrt[d + e*x^2])/(45
*d^2*x^3) - (4*(-e)^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]]/(6*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{1}{6}\sqrt{-e}\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(2(-e)^{3/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(4(-e)^{5/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{45d^2}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.69

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\left(-3d^2+4dex^2-8e^2x^4\right)-15d^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

fricas [A] time = 0.84, size = 68, normalized size = 0.60

$$-\frac{15\,d^3\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)+\left(8\,e^2x^5-4\,dex^3+3\,d^2x\right)\sqrt{ex^2+d}\,\sqrt{-e}}{90\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(15*d^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*e^2*x^5 - 4*d*e*x^3 +
 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^3*x^6)
```

giac [B] time = 0.28, size = 282, normalized size = 2.50

$$\frac{x^5\left(\frac{25\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^2e^{(-1)}}{x^2}+\frac{150\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^4e^{(-5)}}{x^4}+3\,e^3\right)e^{10}}{2880\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^5d^3}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{6\,x^6}-\frac{\left(\frac{150\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)d^1}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] 1/2880*x^5*(25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^2*e^(-1)/x^2 + 150*(
sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^4*e^(-5)/x^4 + 3*e^3)*e^10/((sqrt(-x
^2*e^2 - d*e)*e - sqrt(-d*e)*e)^5*d^3) - 1/6*arctan(x*sqrt(-e)/sqrt(x^2*e +
 d))/x^6 - 1/2880*(150*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^12*e^16/x
+ 25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^3*d^12*e^12/x^3 + 3*(sqrt(-x^2
*e^2 - d*e)*e - sqrt(-d*e)*e)^5*d^12*e^8/x^5)*e^(-15)/d^15
```

maple [A] time = 0.04, size = 117, normalized size = 1.04

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}+\frac{\sqrt{-e}\,e\sqrt{e\,x^2+d}}{18d^2x^3}-\frac{\sqrt{-e}\,e^2\sqrt{e\,x^2+d}}{9d^3x}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{30d^2x^5}+\frac{\sqrt{-e}\,e\left(e\,x^2+d\right)^{\frac{3}{2}}}{45d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x)
```

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6+1/18*(-e)^(1/2)*e/d^2/x^3*(e*
x^2+d)^(1/2)-1/9*(-e)^(1/2)*e^2/d^3/x*(e*x^2+d)^(1/2)-1/30*(-e)^(1/2)/d^2/x
^5*(e*x^2+d)^(3/2)+1/45*(-e)^(1/2)/d^3*e/x^3*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.34, size = 109, normalized size = 0.96

$$-\frac{\left(2\,e^2x^4+dex^2-d^2\right)\sqrt{-e}\,e}{18\,\sqrt{ex^2+d}\,d^3x^3}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{6\,x^6}+\frac{\left(2\,e^2x^4-dex^2-3\,d^2\right)\sqrt{ex^2+d}\,\sqrt{-e}}{90\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*sqrt(-e)*e/(sqrt(e*x^2 + d)*d^3*x^3) - 1/
6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^
2)*sqrt(e*x^2 + d)*sqrt(-e)/(d^3*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

sympy [B] time = 6.50, size = 352, normalized size = 3.12

$$-\frac{d^4e^{\frac{9}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8}-\frac{d^3e^{\frac{11}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8}-\frac{d^2e^{\frac{13}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8}-\frac{2d}{15d^5e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] -d**4*e**(9/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e
**5*x**6 + 30*d**3*e**6*x**8) - d**3*e**(11/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2
) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x**6 + 45*d**3*e**6*x**8) - d**2*e
**(13/2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e*
*5*x**6 + 30*d**3*e**6*x**8) - 2*d*e**(15/2)*x**6*sqrt(-e)*sqrt(d/(e*x**2)
+ 1)/(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8) - 4*e**(17
/2)*x**8*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x*
*6 + 45*d**3*e**6*x**8) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(6*x**6)
```

3.10 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal. Leaf size=141

$$-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7}$$

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-3/140*(-e)^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^5-1/35*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3-2/35*(-e)^(7/2)*(e*x^2+
d)^(1/2)/d^4/x-1/56*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

Rubi [A] time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5151, 271, 264}

$$-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(56*d*x^7) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(14
0*d^2*x^5) - ((-e)^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) - (2*(-e)^(7/2)*Sqrt
[d + e*x^2])/(35*d^4*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{-e}\int \frac{1}{x^8\sqrt{d+ex^2}}dx$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}dx}{28d}$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{5/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{35d^2}$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(2(-e)^{7/2}\right) \int}{3}$$
$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\tan^{-1}\left(\frac{}{\sqrt{}}\right)}{8x^8}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.63

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\left(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6\right)-35d^4\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^
6) - 35*d^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

fricas [A] time = 0.79, size = 80, normalized size = 0.57

$$-\frac{35\,d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(16\,e^3x^7-8\,de^2x^5+6\,d^2ex^3-5\,d^3x\right)\sqrt{ex^2+d}\,\sqrt{-e}}{280\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(35*d^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*e^3*x^7 - 8*d*e^2*x
^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^4*x^8)
```

giac [B] time = 0.26, size = 361, normalized size = 2.56

$$-\frac{x^7\left(\frac{245\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^4e^{(-4)}}{x^4}+\frac{1225\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^6e^{(-8)}}{x^6}+\frac{49\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^2}{x^2}+5\,e^4\right)e^{14}}{35840\left(\sqrt{-x^2e^2-de}\,e-\sqrt{-de}\,e\right)^7d^4}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{8\,x^8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] -1/35840*x^7*(245*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^4*e^(-4)/x^4 + 12
25*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^6*e^(-8)/x^6 + 49*(sqrt(-x^2*e^2
 - d*e)*e - sqrt(-d*e)*e)^2/x^2 + 5*e^4)*e^14/((sqrt(-x^2*e^2 - d*e)*e - sq
rt(-d*e)*e)^7*d^4) - 1/8*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^8 + 1/35840*(
1225*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)*d^24*e^30/x + 245*(sqrt(-x^2*e
^2 - d*e)*e - sqrt(-d*e)*e)^3*d^24*e^26/x^3 + 49*(sqrt(-x^2*e^2 - d*e)*e -
```

```
sqrt(-d*e)*e)^5*d^24*e^22/x^5 + 5*(sqrt(-x^2*e^2 - d*e)*e - sqrt(-d*e)*e)^7
*d^24*e^18/x^7)*e^(-28)/d^28
```

maple [A] time = 0.05, size = 167, normalized size = 1.18

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}+\frac{\sqrt{-e}\;e\sqrt{e\,x^2+d}}{40d^2x^5}-\frac{\sqrt{-e}\;e^2\sqrt{e\,x^2+d}}{30d^3x^3}+\frac{\sqrt{-e}\;e^3\sqrt{e\,x^2+d}}{15d^4x}-\frac{\sqrt{-e}\;\left(e\,x^2+d\right)^{\frac{3}{2}}}{56d^2x^7}+\frac{\sqrt{-e}\;e\left(e\,x^2+\right.}{70d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x)
```

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8+1/40*(-e)^(1/2)*e/d^2/x^5*(e*
x^2+d)^(1/2)-1/30*(-e)^(1/2)*e^2/d^3/x^3*(e*x^2+d)^(1/2)+1/15*(-e)^(1/2)*e^
3/d^4/x*(e*x^2+d)^(1/2)-1/56*(-e)^(1/2)/d^2/x^7*(e*x^2+d)^(3/2)+1/70*(-e)^(
1/2)/d^3*e/x^5*(e*x^2+d)^(3/2)-1/105*(-e)^(1/2)/d^4*e^2/x^3*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.34, size = 132, normalized size = 0.94

$$\frac{\left(8\,e^3x^6+4\,de^2x^4-d^2ex^2+3\,d^3\right)\sqrt{-e}\,e}{120\,\sqrt{ex^2+d}\,d^4x^5}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{8\,x^8}-\frac{\left(8\,e^3x^6-4\,de^2x^4+3\,d^2ex^2+15\,d^3\right)\sqrt{ex^2+d}\,\sqrt{-e}}{840\,d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] 1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*sqrt(-e)*e/(sqrt(e*x^2
+ d)*d^4*x^5) - 1/8*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x
^6 - 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(-e)/(d^4*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

sympy [B] time = 9.15, size = 575, normalized size = 4.08

$$-\frac{5d^6e^{\frac{19}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}}-\frac{9d^5e^{\frac{21}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] -5*d**6*e**(19/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d
**6*e**10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 9*d**5*e**(
21/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**
10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 5*d**4*e**(23/2)*x
**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8
 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 5*d**3*e**(25/2)*x**6*sqr
t(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8 + 840*
d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 15*d**2*e**(27/2)*x**8*sqrt(-e)*
sqrt(d/(e*x**2) + 1)/(140*d**7*e**9*x**6 + 420*d**6*e**10*x**8 + 420*d**5*e
**11*x**10 + 140*d**4*e**12*x**12) + 5*d*e**(29/2)*x**10*sqrt(-e)*sqrt(d/(e
```

```
*x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10
 + 35*d**4*e**12*x**12) + 2*e**(31/2)*x**12*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(
35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e*
*12*x**12) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(8*x**8)
```

3.11 $\int x^6 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=124

$$\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}}+\frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/7*d^2*(e*x^2+d)^(3/2)/(-e)^(7/2)+3/35*d*(e*x^2+d)^(5/2)/(-e)^(7/2)-1/49*
(e*x^2+d)^(7/2)/(-e)^(7/2)+1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/7
*d^3*(e*x^2+d)^(1/2)/(-e)^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5151, 266, 43}

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}}+\frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*(-e)^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*(-e)^(7/2
)) + (3*d*(d + e*x^2)^(5/2))/(35*(-e)^(7/2)) - (d + e*x^2)^(7/2)/(49*(-e)^(
7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^6 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{-e}\int \frac{x^7}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{d+ex}}\,dx, x, x^2\right)$$
$$= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int\left(-\frac{d^3}{e^3\sqrt{d+ex}} + \frac{3d^2\sqrt{d+ex}}{e^3} - \frac{3d(d+}{e^3}\right.\right.$$
$$= \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2\left(d+ex^2\right)^{3/2}}{7(-e)^{7/2}} + \frac{3d\left(d+ex^2\right)^{5/2}}{35(-e)^{7/2}} - \frac{\left(d+ex^2\right)^{7/2}}{49(-e)^{7/2}} + \frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-}}{\sqrt{d+}}\right.$$

Mathematica [A] time = 0.12, size = 83, normalized size = 0.67

$$\frac{\sqrt{d+ex^2}\left(16d^3-8d^2ex^2+6de^2x^4-5e^3x^6\right)}{245(-e)^{7/2}} + \frac{1}{7}x^7\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*(-e
)^(7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

fricas [A] time = 0.48, size = 79, normalized size = 0.64

$$\frac{35\,e^4x^7\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right) - \left(5\,e^3x^6 - 6\,de^2x^4 + 8\,d^2ex^2 - 16\,d^3\right)\sqrt{ex^2+d}\,\sqrt{-e}}{245\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*e^4*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (5*e^3*x^6 - 6*d*e^2
*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(-e))/e^4
```

giac [A] time = 0.25, size = 137, normalized size = 1.10

$$\frac{1}{7}x^7\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right) + \frac{1}{7}\sqrt{-x^2e^2-de}\,d^3e^{(-4)} + \frac{1}{245}\left(35\left(-x^2e^2-de\right)^{\frac{3}{2}}d^2e^2 + 21\left(x^2e^2+de\right)^2\sqrt{-x^2e^2-de}\,de - 5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/7*x^7*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 1/7*sqrt(-x^2*e^2 - d*e)*d^3*e
^(-4) + 1/245*(35*(-x^2*e^2 - d*e)^(3/2)*d^2*e^2 + 21*(x^2*e^2 + d*e)^2*sqr
t(-x^2*e^2 - d*e)*d*e - 5*(x^2*e^2 + d*e)^3*sqrt(-x^2*e^2 - d*e))*e^(-7)
```

maple [B] time = 0.05, size = 230, normalized size = 1.85

$$\frac{x^7\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{7} + \frac{\sqrt{-e}\,x^8\sqrt{e\,x^2+d}}{63d} - \frac{8\sqrt{-e}\,x^6\sqrt{e\,x^2+d}}{441e} + \frac{16\sqrt{-e}\,d\,x^4\sqrt{e\,x^2+d}}{735e^2} - \frac{64\sqrt{-e}\,d^2x^2\sqrt{e\,x^2+d}}{2205e^3} + \frac{128}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/63*(-e)^(1/2)/d*x^8*(e*x^2+d
)^(1/2)-8/441*(-e)^(1/2)/e*x^6*(e*x^2+d)^(1/2)+16/735*(-e)^(1/2)/e^2*d*x^4*
(e*x^2+d)^(1/2)-64/2205*(-e)^(1/2)/e^3*d^2*x^2*(e*x^2+d)^(1/2)+128/2205*(-e
)^(1/2)/e^4*d^3*(e*x^2+d)^(1/2)-1/63*(-e)^(1/2)/d*x^6*(e*x^2+d)^(3/2)/e+2/1
47*(-e)^(1/2)/e^2*x^4*(e*x^2+d)^(3/2)-8/735*(-e)^(1/2)*d/e^3*x^2*(e*x^2+d)^
(3/2)+16/2205*(-e)^(1/2)*d^2/e^4*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.34, size = 167, normalized size = 1.35

$$\frac{1}{7}\,x^7\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\frac{\left(35\left(ex^2+d\right)^{\frac{9}{2}}-135\left(ex^2+d\right)^{\frac{7}{2}}d+189\left(ex^2+d\right)^{\frac{5}{2}}d^2-105\left(ex^2+d\right)^{\frac{3}{2}}d^3\right)\sqrt{-e}}{2205\,de^4}+\frac{\left(35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2) -
 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3/2
)*d^3)*sqrt(-e)/(d*e^4) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7
/2)*d + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*
x^2 + d)*d^4)*sqrt(-e)/(d*e^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 11.61, size = 124, normalized size = 1.00

$$\begin{cases} \frac{16id^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}}-\frac{8id^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}}+\frac{6idx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}}+\frac{ix^7\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7}-\frac{ix^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*I*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*I*d**2*x**2*sqrt(d
 + e*x**2)/(245*e**(5/2)) + 6*I*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + I*
x**7*atanh(sqrt(e)*x/sqrt(d + e*x**2))/7 - I*x**6*sqrt(d + e*x**2)/(49*sqrt
(e)), Ne(e, 0)), (0, True))
```

3.12 $\int x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=99

$$\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}+\frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}}-\frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

[Out] -2/15*d*(e*x^2+d)^(3/2)/(-e)^(5/2)+1/25*(e*x^2+d)^(5/2)/(-e)^(5/2)+1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*d^2*(e*x^2+d)^(1/2)/(-e)^(5/2)

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5151, 266, 43}

$$\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}+\frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}}-\frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]

[Out] (d^2*Sqrt[d + e*x^2])/(5*(-e)^(5/2)) - (2*d*(d + e*x^2)^(3/2))/(15*(-e)^(5/2)) + (d + e*x^2)^(5/2)/(25*(-e)^(5/2)) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{-e}\int\frac{x^5}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int\left(\frac{d^2}{e^2\sqrt{d+ex}}-\frac{2d\sqrt{d+ex}}{e^2}+\frac{(d+ex)^{3/}}{e^2}\right.\right.\\
&= \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}-\frac{2d\left(d+ex^2\right)^{3/2}}{15(-e)^{5/2}}+\frac{\left(d+ex^2\right)^{5/2}}{25(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.73

$$\frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{75(-e)^{5/2}}+\frac{1}{5}x^5\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*(-e)^(5/2)) + (x^5*Ar
cTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5
```

fricas [A] time = 0.49, size = 68, normalized size = 0.69

$$\frac{15\,e^3x^5\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\left(3\,e^2x^4-4\,dex^2+8\,d^2\right)\sqrt{ex^2+d}\,\sqrt{-e}}{75\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*e^3*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e^2*x^4 - 4*d*e*x^
2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(-e))/e^3
```

giac [A] time = 0.25, size = 102, normalized size = 1.03

$$\frac{1}{5}\,x^5\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{1}{5}\,\sqrt{-x^2e^2-de}\,d^2e^{(-3)}-\frac{1}{75}\left(10\left(-x^2e^2-de\right)^{\frac{3}{2}}de+3\left(x^2e^2+de\right)^2\sqrt{-x^2e^2-de}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/5*sqrt(-x^2*e^2 - d*e)*d^2*e
^(-3) - 1/75*(10*(-x^2*e^2 - d*e)^(3/2)*d*e + 3*(x^2*e^2 + d*e)^2*sqrt(-x^2
*e^2 - d*e))*e^(-5)
```

maple [B] time = 0.04, size = 180, normalized size = 1.82

$$\frac{x^5\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{5}+\frac{\sqrt{-e}\,x^6\sqrt{e\,x^2+d}}{35d}-\frac{6\sqrt{-e}\,x^4\sqrt{e\,x^2+d}}{175e}+\frac{8\sqrt{-e}\,d\,x^2\sqrt{e\,x^2+d}}{175e^2}-\frac{16\sqrt{-e}\,d^2\sqrt{e\,x^2+d}}{175e^3}-\frac{\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/35*(-e)^(1/2)/d*x^6*(e*x^2+d
)^(1/2)-6/175*(-e)^(1/2)/e*x^4*(e*x^2+d)^(1/2)+8/175*(-e)^(1/2)/e^2*d*x^2*(
e*x^2+d)^(1/2)-16/175*(-e)^(1/2)/e^3*d^2*(e*x^2+d)^(1/2)-1/35*(-e)^(1/2)/d*
x^4*(e*x^2+d)^(3/2)/e+4/175*(-e)^(1/2)/e^2*x^2*(e*x^2+d)^(3/2)-8/525*(-e)^(
1/2)*d/e^3*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.34, size = 139, normalized size = 1.40

$$\frac{1}{5}\,x^5\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\frac{\left(15\left(ex^2+d\right)^{\frac{7}{2}}-42\left(ex^2+d\right)^{\frac{5}{2}}d+35\left(ex^2+d\right)^{\frac{3}{2}}d^2\right)\sqrt{-e}}{525\,de^3}+\frac{\left(5\left(ex^2+d\right)^{\frac{7}{2}}-21\left(ex^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)*sqrt(-e)/(d*e^3) + 1/175
*(5*(e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 -
 35*sqrt(e*x^2 + d)*d^3)*sqrt(-e)/(d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\,x^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 3.56, size = 97, normalized size = 0.98

$$\begin{cases} -\frac{8id^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}} + \frac{4idx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}} + \frac{ix^5 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5} - \frac{ix^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*I*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*I*d*x**2*sqrt(d + e
*x**2)/(75*e**(3/2)) + I*x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - I*x**4*
sqrt(d + e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

3.13 $\int x^2 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=74

$$-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/9*(e*x^2+d)^(3/2)/(-e)^(3/2)+1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)
)+1/3*d*(e*x^2+d)^(1/2)/(-e)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5151, 266, 43}

$$-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*(-e)^(3/2)) - (d + e*x^2)^(3/2)/(9*(-e)^(3/2)) + (x^
3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{-e}\int\frac{x^3}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int\left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)dx,x,x^2\right)\\
&= \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{1}{3}x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 0.81

$$\frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}}+3x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 3*x^3*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]])/9
```

fricas [A] time = 0.44, size = 56, normalized size = 0.76

$$\frac{3\,e^2x^3\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\sqrt{ex^2+d}\left(ex^2-2\,d\right)\sqrt{-e}}{9\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*e^2*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*(e*x^2
- 2*d)*sqrt(-e))/e^2
```

giac [A] time = 0.23, size = 64, normalized size = 0.86

$$\frac{1}{3}\,x^3\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)+\frac{1}{3}\,\sqrt{-x^2e^2-de}\,de^{(-2)}+\frac{1}{9}\left(-x^2e^2-de\right)^{\frac{3}{2}}e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) + 1/3*sqrt(-x^2*e^2 - d*e)*d*e^(
-2) + 1/9*(-x^2*e^2 - d*e)^(3/2)*e^(-3)
```

maple [B] time = 0.04, size = 132, normalized size = 1.78

$$\frac{x^3\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{3}+\frac{\sqrt{-e}\;x^4\sqrt{e\,x^2+d}}{15d}-\frac{4\sqrt{-e}\;x^2\sqrt{e\,x^2+d}}{45e}+\frac{8\sqrt{-e}\;d\sqrt{e\,x^2+d}}{45e^2}-\frac{\sqrt{-e}\;x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}{15de}+\frac{2\sqrt{-e}\;\left(e\,x\right.}{45e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/15*(-e)^(1/2)/d*x^4*(e*x^2+d
)^(1/2)-4/45*(-e)^(1/2)/e*x^2*(e*x^2+d)^(1/2)+8/45*(-e)^(1/2)/e^2*d*(e*x^2+
d)^(1/2)-1/15*(-e)^(1/2)/d*x^2*(e*x^2+d)^(3/2)/e+2/45*(-e)^(1/2)/e^2*(e*x^2
+d)^(3/2)
```

maxima [A] time = 0.34, size = 111, normalized size = 1.50

$$\frac{1}{3}\,x^3\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\frac{\left(3\left(ex^2+d\right)^{\frac{5}{2}}-5\left(ex^2+d\right)^{\frac{3}{2}}d\right)\sqrt{-e}}{45\,de^2}+\frac{\left(3\left(ex^2+d\right)^{\frac{5}{2}}-10\left(ex^2+d\right)^{\frac{3}{2}}d+15\sqrt{ex^2+d}\,d^2\right)}{45\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5*
(e*x^2 + d)^(3/2)*d)*sqrt(-e)/(d*e^2) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x
^2 + d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)*sqrt(-e)/(d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\, x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 1.24, size = 70, normalized size = 0.95

$$\begin{cases} \frac{2id\sqrt{d+ex^2}}{9e^{\frac{3}{2}}} + \frac{ix^3 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3} - \frac{ix^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*I*d*sqrt(d + e*x**2)/(9*e**(3/2)) + I*x**3*atanh(sqrt(e)*x/sqr
t(d + e*x**2))/3 - I*x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True
))
```

3.14 $\int \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=43

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5147, 261}

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5147

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*Ar
cTan[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx &= x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \sqrt{-e}\int \frac{x}{\sqrt{d+ex^2}}\,dx\\
&= \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

fricas [A] time = 0.44, size = 41, normalized size = 0.95

$$\frac{ex\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right) - \sqrt{ex^2+d}\,\sqrt{-e}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
[Out] (e*x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*sqrt(-e))/e
```

giac [A] time = 0.18, size = 41, normalized size = 0.95

$$x \arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2 e + d}}\right) - \sqrt{-x^2 e^2 - de}\, e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] x*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - sqrt(-x^2*e^2 - d*e)*e^(-1)
```

maple [B] time = 0.04, size = 84, normalized size = 1.95

$$x \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2 + d}}\right) + \frac{\sqrt{-e}\,x^2\sqrt{e\,x^2 + d}}{3d} - \frac{2\sqrt{-e}\,\sqrt{e\,x^2 + d}}{3e} - \frac{\sqrt{-e}\,\left(e\,x^2 + d\right)^{\frac{3}{2}}}{3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/3*(-e)^(1/2)/d*x^2*(e*x^2+d)^(1/2)
-2/3*(-e)^(1/2)/e*(e*x^2+d)^(1/2)-1/3*(-e)^(1/2)/d/e*(e*x^2+d)^(3/2)
```

maxima [B] time = 0.34, size = 77, normalized size = 1.79

$$x \arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2 + d}}\right) - \frac{\left(ex^2 + d\right)^{\frac{3}{2}}\sqrt{-e}}{3\,de} + \frac{\left(\left(ex^2 + d\right)^{\frac{3}{2}} - 3\,\sqrt{ex^2 + d}\,d\right)\sqrt{-e}}{3\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)*sqrt(-e)/(d*e)
 + 1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)*sqrt(-e)/(d*e)
```

mupad [B] time = 0.66, size = 35, normalized size = 0.81

$$\frac{\sqrt{e\,x^2 + d}}{\sqrt{-e}} + x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2 + d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] (d + e*x^2)^(1/2)/(-e)^(1/2) + x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))
```

sympy [A] time = 1.06, size = 39, normalized size = 0.91

$$\begin{cases} ix\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{i\sqrt{d+ex^2}}{\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Piecewise((I*x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - I*sqrt(d + e*x**2)/sqrt(
e), Ne(e, 0)), (0, True))
```

3.15 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*(-
e)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{5151, 266, 63, 208\}$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x} + \sqrt{-e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x} + \frac{1}{2}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{\sqrt{-e}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 1.46

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x} + \frac{i\sqrt{e}\log\left(-\frac{2\sqrt{-e}\sqrt{d+ex^2}}{ex} + \frac{2i\sqrt{d}}{\sqrt{e}\,x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) + (I*Sqrt[e]*Log[((2*I)*Sqrt[d])/
(Sqrt[e]*x) - (2*Sqrt[-e]*Sqrt[d + e*x^2])/(e*x)])/Sqrt[d]
```

fricas [A] time = 0.60, size = 148, normalized size = 2.51

$$\left[\frac{x\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}\,d\sqrt{-e}\sqrt{-\frac{e}{d}}+2de}{x^2}\right) - 2\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{2x}, -\frac{x\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}\,d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right) + \arctan\left(\frac{\sqrt{-}}{\sqrt{ex^{}}}\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-e/d)
+ 2*d*e)/x^2) - 2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/x, -(x*sqrt(e/d)*arct
an(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + arctan(sqrt(-e)*
x/sqrt(e*x^2 + d)))/x]
```

giac [A] time = 0.25, size = 54, normalized size = 0.92

$$-\frac{\arctan\left(\frac{\sqrt{-x^2e^2-de}\,e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}} - \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(1/2)/sqrt(d) - arctan(x*s
qrt(-e)/sqrt(x^2*e + d))/x
```

maple [A] time = 0.04, size = 57, normalized size = 0.97

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{-e}\;\ln\left(\frac{2d+2\sqrt{d}\;\sqrt{e\,x^{2}+d}}{x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x)
```

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-(-e)^(1/2)/d^(1/2)*ln((2*d+2*d^(1/2
)*(e*x^2+d)^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

sympy [A] time = 5.54, size = 60, normalized size = 1.02

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{x}+\frac{\sqrt{-e}\,\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{d}}\,\sqrt{d+ex^{2}}}\right)}{d\sqrt{-\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/x + sqrt(-e)*atan(1/(sqrt(-1/d)*sqrt(d +
 e*x**2)))/(d*sqrt(-1/d))
```

3.16 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

Optimal. Leaf size=91

$$-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3-1/6*(-e)^(3/2)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/d^(3/2)-1/6*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5151, 266, 51, 63, 208}

$$-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]/(3*x^3) - ((-e)^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
```

```
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{-e}\int\frac{1}{x^3\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{-e}\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{(-e)^{3/2}\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{12d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{-e}\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{6d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 101, normalized size = 1.11

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) + (e^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-e]
)/(Sqrt[e]*Sqrt[d + e*x^2])])/(6*d^(3/2)) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]/(3*x^3)
```

fricas [A] time = 0.60, size = 198, normalized size = 2.18

$$\left[\frac{ex^3\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}\,d\sqrt{-e}\sqrt{-\frac{e}{d}}+2de}{x^2}\right)-2\sqrt{ex^2+d}\sqrt{-e}\,x-4\,d\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{12\,dx^3},\ \frac{ex^3\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}\,d\sqrt{-}}{e^2x^2+d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(e*x^3*sqrt(-e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-
e/d) + 2*d*e)/x^2) - 2*sqrt(e*x^2 + d)*sqrt(-e)*x - 4*d*arctan(sqrt(-e)*x/s
qrt(e*x^2 + d)))/(d*x^3), 1/6*(e*x^3*sqrt(e/d)*arctan(sqrt(e*x^2 + d)*d*sqr
t(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) - sqrt(e*x^2 + d)*sqrt(-e)*x - 2*d*arctan(
sqrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^3)]
```

giac [A] time = 0.23, size = 83, normalized size = 0.91

$$\frac{1}{6}\left(\frac{\arctan\left(\frac{\sqrt{-x^2e^2-de}\,e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{d^{\frac{3}{2}}}-\frac{\sqrt{-x^2e^2-de}\,e}{dx^2}\right)e^{(-1)}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(5/2)/d^(3/2) - sqrt(-
x^2*e^2 - d*e)*e/(d*x^2))*e^(-1) - 1/3*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x
^3
```

maple [A] time = 0.04, size = 100, normalized size = 1.10

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{3x^3}+\frac{\sqrt{-e}\;e\ln\left(\frac{2d+2\sqrt{d}\;\sqrt{e\,x^2+d}}{x}\right)}{6d^{\frac{3}{2}}}-\frac{\sqrt{-e}\;\left(e\,x^2+d\right)^{\frac{3}{2}}}{6d^2x^2}+\frac{\sqrt{-e}\;e\sqrt{e\,x^2+d}}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x)
```

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*(-e)^(1/2)*e/d^(3/2)*ln((
2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/6*(-e)^(1/2)/d^2/x^2*(e*x^2+d)^(3/2)+1/
6*(-e)^(1/2)/d^2*e*(e*x^2+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\;x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

sympy [A] time = 11.58, size = 82, normalized size = 0.90

$$-\frac{\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\,\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{6dx}+\frac{e\sqrt{-e}\;\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\,x}\right)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**3) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(6*d*x) + e*sqrt(-e)*asinh(sqrt(d)/(sqrt(e)*x))/(6*d**(3/2))
```

3.17 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

Optimal. Leaf size=119

$$-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4}$$

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*(-e)^(5/2)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))/d^(5/2)-3/40*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*(-
e)^(1/2)*(e*x^2+d)^(1/2)/d/x^4
```

Rubi [A] time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5151, 266, 51, 63, 208}

$$-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -(Sqrt[-e]*Sqrt[d + e*x^2])/(20*d*x^4) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(40
*d^2*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*(-e)^(5/2)*Ar
cTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{5}\sqrt{-e}\int\frac{1}{x^5\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int\frac{1}{x^3\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)}{40d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3(-e)^{5/2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\right.}{80d^2}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,d\right.}{40d^2}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 114, normalized size = 0.96

$$-\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{40d^{5/2}}+\sqrt{-e}\left(\frac{3e}{40d^2x^2}-\frac{1}{20dx^4}\right)\sqrt{d+ex^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] Sqrt[-e]*(-1/20*1/(d*x^4) + (3*e)/(40*d^2*x^2))*Sqrt[d + e*x^2] - (3*e^(5/2
)*ArcTan[(Sqrt[d]*Sqrt[-e])/(Sqrt[e]*Sqrt[d + e*x^2])])/(40*d^(5/2)) - ArcT
an[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5)
```

fricas [A] time = 0.63, size = 228, normalized size = 1.92

$$\left[\frac{3\,e^2x^5\sqrt{-\frac{e}{d}}\,\log\left(-\frac{e^2x^2+2\,\sqrt{ex^2+d}\,d\sqrt{-e}\,\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-16\,d^2\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)+2\left(3\,ex^3-2\,dx\right)\sqrt{ex^2+d}\,\sqrt{-e}}{80\,d^2x^5},-\frac{3\,e^2\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(3*e^2*x^5*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sq
rt(-e/d) + 2*d*e)/x^2) - 16*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3*e
*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^2*x^5), -1/40*(3*e^2*x^5*sqrt(e/
d)*arctan(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + 8*d^2*arc
```

```
tan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e
))/(d^2*x^5)]
```

giac [A] time = 0.25, size = 108, normalized size = 0.91

$$-\frac{1}{40}\left(\frac{3\arctan\left(\frac{\sqrt{-x^2e^2-de}\,e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{d^{\frac{5}{2}}}+\frac{\left(5\sqrt{-x^2e^2-de}\,de^5+3\left(-x^2e^2-de\right)^{\frac{3}{2}}e^4\right)e^{(-4)}}{d^2x^4}\right)e^{(-1)}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] -1/40*(3*arctan(sqrt(-x^2*e^2 - d*e)*e^(-1/2)/sqrt(d))*e^(7/2)/d^(5/2) + (5
*sqrt(-x^2*e^2 - d*e)*d*e^5 + 3*(-x^2*e^2 - d*e)^(3/2)*e^4)*e^(-4)/(d^2*x^4
))*e^(-1) - 1/5*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^5
```

maple [A] time = 0.04, size = 150, normalized size = 1.26

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{20d^2x^4}+\frac{\sqrt{-e}\,e\left(e\,x^2+d\right)^{\frac{3}{2}}}{40d^3x^2}-\frac{3\sqrt{-e}\,e^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{40d^{\frac{5}{2}}}-\frac{\sqrt{-e}\,e^2\sqrt{e\,x^2+d}}{40d^3}+\sqrt{-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x)
```

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-1/20*(-e)^(1/2)/d^2/x^4*(e*x^
2+d)^(3/2)+1/40*(-e)^(1/2)/d^3*e/x^2*(e*x^2+d)^(3/2)-3/40*(-e)^(1/2)/d^(5/2
)*e^2*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/40*(-e)^(1/2)/d^3*e^2*(e*x^2+
d)^(1/2)+1/10*(-e)^(1/2)*e/d^2/x^2*(e*x^2+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

sympy [A] time = 19.73, size = 148, normalized size = 1.24

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\sqrt{-e}}{20\sqrt{e}\,x^5\sqrt{\frac{d}{ex^2}+1}}+\frac{\sqrt{e}\,\sqrt{-e}}{40dx^3\sqrt{\frac{d}{ex^2}+1}}+\frac{3e^{\frac{3}{2}}\sqrt{-e}}{40d^2x\sqrt{\frac{d}{ex^2}+1}}-\frac{3e^2\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\,x}\right)}{40d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**5) - sqrt(-e)/(20*sqrt(e)*x**5*sqr
t(d/(e*x**2) + 1)) + sqrt(e)*sqrt(-e)/(40*d*x**3*sqrt(d/(e*x**2) + 1)) + 3*
e**(3/2)*sqrt(-e)/(40*d**2*x*sqrt(d/(e*x**2) + 1)) - 3*e**2*sqrt(-e)*asinh(
sqrt(d)/(sqrt(e)*x))/(40*d**(5/2))
```

3.18 $\int x^{9/2} \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=211

$$\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx^{5/2}\sqrt{d+}}{847(-e)^{3/2}}$$

```
[Out] 2/11*x^(11/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+
d)^(1/2)/(-e)^(3/2)+4/121*x^(9/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)+60/847*d^2*x^(
1/2)*(e*x^2+d)^(1/2)/(-e)^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(13/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx}{84}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*(-e)^(5/2)) + (36*d*x^(5/2)*Sqrt[d +
e*x^2])/(847*(-e)^(3/2)) + (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[-e]) + (2*
x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*Sqrt[-e]*(
Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*
ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(13/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{11}\left(2\sqrt{-e}\right)\int \frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(18d)\int \frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx}{121\sqrt{-e}}$$
$$= \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(90d^2\right)\int \frac{}{\sqrt{}}}{847(-e}$$
$$= \frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{d}}\right.$$
$$= \frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{d}}\right.$$
$$= \frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{d}}\right.$$

Mathematica [C] time = 0.62, size = 170, normalized size = 0.81

$$-\frac{60id^3x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{847(-e)^{5/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}} + \frac{4\sqrt{x}\,\sqrt{d+ex^2}\left(15d^2-9dex^2+7e^2x^4\right)}{847(-e)^{5/2}} + \frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/(847*(-e)^(5/2
)) + (2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (((60*I)/847)*d
^3*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt
[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*(-e)^(5/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^{\frac{9}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

3.19 $\int x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=181

$$-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}$$

```
[Out] 2/7*x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/49*x^(5/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/(-e)^(3/2)-10/147*d^(7/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-
e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4
)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{2}{7}x$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*(-e)^(3/2)) + (4*x^(5/2)*Sqrt[d + e*x^2
])/(49*Sqrt[-e]) + (2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (10
*d^(7/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]
*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(9/4)*Sq
rt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\left(2\sqrt{-e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx$$

$$=\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{49\sqrt{-e}}$$

$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(10d^2\right)\int\frac{1}{\sqrt{x}\sqrt{d+e}}}{147(-e)^{3/2}}$$

$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(20d^2\right)\text{Subst}\left(\int\right.}{147}$$

$$=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.46, size = 158, normalized size = 0.87

$$\frac{2}{147}\sqrt{x}\left(\frac{2\left(5d-3ex^2\right)\sqrt{d+ex^2}}{(-e)^{3/2}}+21x^3\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)-\frac{20id^2x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{147(-e)^{3/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 21*x^3*ArcTan[
(Sqrt[-e]*x)/Sqrt[d + e*x^2]]))/147 - (((20*I)/147)*d^2*Sqrt[1 + d/(e*x^2)]
*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sq
rt[d])/Sqrt[e]]*(-e)^(3/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [C] time = 150.30, size = 75, normalized size = 0.41

$$\frac{2x^{\frac{7}{2}}\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^{\frac{9}{2}}\sqrt{-e}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{7\sqrt{d}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(7/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**(9/2)*sqrt(-e)*gamma(9/
4)*hyper((1/2, 9/4), (13/4,), e*x**2*exp_polar(I*pi)/d)/(7*sqrt(d)*gamma(13
/4))
```

3.20 $\int \sqrt{x} \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=153

$$\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 2/3*x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/9*x^(1/2)*(e*x^2+d)^(1/2
)/(-e)^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)
+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 321, 329, 220}

$$\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)]
, 1/2])/(9*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
```

```
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\left(2\sqrt{-e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx\\ &= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(2d)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{9\sqrt{-e}}\\ &= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(4d)\,\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{9\sqrt{-e}}\\ &= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F}{9\sqrt{-e}\sqrt[4]{e}\sqrt{d+e}}\end{aligned}$$

Mathematica [C] time = 0.31, size = 147, normalized size = 0.96

$$\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}-\frac{4idx\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{9\sqrt{-e}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 - (((4*I)/9)*d*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[-
e]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x}\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

giac [A] time = 0.25, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sqrt{x}\, \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\, \mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^{2}+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [C] time = 3.96, size = 75, normalized size = 0.49

$$\frac{2x^{\frac{3}{2}}\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{3} - \frac{x^{\frac{5}{2}}\sqrt{-e}\,\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{ex^{2}e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(3/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/3 - x**(5/2)*sqrt(-e)*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(9/4
))
```

3.21 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

```
[Out] -2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*(cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(si
n(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(
1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/e^(1/4)/(e*x^2+d)^(1/
2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5151, 329, 220}

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^
(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{-e}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,dx$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.13, size = 115, normalized size = 0.94

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{-e}\,x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[-e]*Sqrt[1
+ d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

sympy [C] time = 5.37, size = 71, normalized size = 0.58

$$-\frac{2\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}+\frac{\sqrt{x}\sqrt{-e}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/sqrt(x) + sqrt(x)*sqrt(-e)*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(5/4))
```

3.22 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2\sqrt{-e}\,e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

```
[Out] -2/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(3/2)-2/15*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1
/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x
^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^
(1/2)+x*e^(1/2))^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 325, 329, 220}

$$-\frac{2\sqrt{-e}\,e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[
d + e*x^2]])/(5*x^(5/2)) - (2*Sqrt[-e]*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(
d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^
(1/4)], 1/2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
```

```
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{1}{5}\left(2\sqrt{-e}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx\\ &= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{\left(2(-e)^{3/2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,dx}{15d}\\ &= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{\left(4(-e)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{15d}\\ &= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{2(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\right.}{15d^{5/4}\sqrt[4]{e}\,\sqrt{d+ex^2}}\end{aligned}$$

Mathematica [C] time = 0.29, size = 150, normalized size = 0.96

$$-\frac{2\left(2\sqrt{-e}\,x\sqrt{d+ex^2}+3d\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}}+\frac{4i(-e)^{3/2}x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{15d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[-e]*x*Sqrt[d + e*x^2] + 3*d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]))/(15*d*x^(5/2)) + (((4*I)/15)*(-e)^(3/2)*Sqrt[1 + d/(e*x^2)]*x*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d*Sqrt[(I*Sqrt[d])/Sqr
t[e]]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

sympy [C] time = 146.13, size = 78, normalized size = 0.50

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^{\frac{5}{2}}}+\frac{\sqrt{-e}\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\,x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**(5/2)) + sqrt(-e)*gamma(-3/4)*hy
per((-3/4, 1/2), (1/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*x**(3/2)*gamm
a(1/4))
```

3.23 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=186

$$\frac{10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{9x^{9/}}$$

```
[Out] -2/9*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)-20/189*(-e)^(3/2)*(e*x^2+
d)^(1/2)/d^2/x^(3/2)-4/63*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(7/
4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2
))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d
^(9/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5151, 325, 329, 220}

$$\frac{10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) - (20*(-e)^(3/2)*Sqrt[d + e*x^
2])/(189*d^2*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)
) + (10*Sqrt[-e]*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(
9/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{1}{9}\left(2\sqrt{-e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{189d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(20(-e)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+e}}\right.}{189d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)}{189d}$$

Mathematica [C] time = 0.34, size = 162, normalized size = 0.87

$$\frac{4\sqrt{-e}x\sqrt{d+ex^2}\left(5ex^2-3d\right)-42d^2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}} + \frac{20i(-e)^{5/2}x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{189d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (((20*I)/189)*(-e)^(5/2)*Sqrt[1 + d/
(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d^
2*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="frica
s")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{11}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{30\sqrt{-e}\,e^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{-e}\,\sqrt{}}{143d}$$

```
[Out] -2/13*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)-36/1001*(-e)^(3/2)*(e*x
^2+d)^(1/2)/d^2/x^(7/2)-60/1001*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/14
3*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(11/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5151, 325, 329, 220}

$$-\frac{30\sqrt{-e}\,e^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) - (36*(-e)^(3/2)*Sqrt[d + e*
x^2])/(1001*d^2*x^(7/2)) - (60*(-e)^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2
)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*Sqrt[-e]*
e^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d +
e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{-e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(18(-e)^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(90(-e)^{5/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}}{1001d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + -$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + -$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + -$$

Mathematica [C] time = 0.61, size = 171, normalized size = 0.79

$$\frac{2\left(\frac{30i(-e)^{7/2}x^{15/2}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^3\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}} - \frac{2\sqrt{-e}\sqrt{d+ex^2}\left(7d^2x-9dex^3+15e^2x^5\right)}{d^3} - 77\tan^{-1}\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[-e]*Sqrt[d + e*x^2]*(7*d^2*x - 9*d*e*x^3 + 15*e^2*x^5))/d^3 -
77*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + ((30*I)*(-e)^(7/2)*Sqrt[1 + d/(e*
x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(d^3*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="frica
s")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

3.25 $\int x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=326

$$-\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-}\right.}{135e^{11/4}\sqrt{d+ex^2}}$$

```
[Out] 2/9*x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)
^(1/2)/(-e)^(3/2)+4/81*x^(7/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)-28/135*d^2*(-e)^(
1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(5/2)/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e
)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4
)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)
^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4
)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/
(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 321, 329, 305, 220, 1196}

$$-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*(-e)^(3/2)) + (4*x^(7/2)*Sqrt[d + e*x^2
])/(81*Sqrt[-e]) - (28*d^2*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^(5/2)*(
Sqrt[d] + Sqrt[e]*x)) + (2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9
+ (28*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sq
rt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(11
/4)*Sqrt[d + e*x^2]) - (14*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(135*e^(11/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{9}\left(2\sqrt{-e}\right)\int\frac{x^{9/2}}{\sqrt{d+ex^2}}\,dx$$

$$=\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx}{81\sqrt{-e}}$$

$$=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(14d^2\right)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}}{135(-e)^{3/2}}$$

$$=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(28d^2\right)\text{Subst}\left(\int\right.}{135}$$

$$=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(28d^{5/2}\sqrt{-e}\right)\text{Su}}{}$$

$$=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{\cdot}}{\sqrt{d\cdot}}\right.$$

Mathematica [C] time = 0.16, size = 139, normalized size = 0.43

$$\frac{2x^{3/2}\left(2\sqrt{-e}\left(7d^2+2dex^2-5e^2x^4\right)-14d^2\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)+45e^2x^3\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)}{405e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(2*Sqrt[-e]*(7*d^2 + 2*d*e*x^2 - 5*e^2*x^4) + 45*e^2*x^3*Sqrt[d
+ e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[-e]*Sqrt[1 + (e
*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^2*Sqrt[d +
 e*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{7}{2}} \arctan\left(\frac{\sqrt{-e}\, x}{\sqrt{ex^2 + d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\, \mathrm{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\,x^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Timed out
```

3.26 $\int x^{3/2} \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=296

$$\frac{6d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{25e^{7/4}\sqrt{d+ex^2}}$$

```
[Out] 2/5*x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/25*x^(3/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+12/25*d*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)/(d^(1/2)+x
*e^(1/2))-12/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+
x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2
)
```

Rubi [A] time = 0.16, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5151, 321, 329, 305, 220, 1196}

$$\frac{6d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{25e^{7/4}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[-e]) + (12*d*Sqrt[-e]*Sqrt[x]*Sqrt[d +
 e*x^2])/(25*e^(3/2)*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/5 - (12*d^(5/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
+ e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2]) + (6*d^(5/4)*Sqrt[-e]*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\left(2\sqrt{-e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{25\sqrt{-e}}\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(12d)\,\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{25\sqrt{-e}}\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\left(12d^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{25\sqrt{-e^2}}\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}-\frac{12d\sqrt{x}\sqrt{d+ex^2}}{25\sqrt{-e^2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{12d^{5/4}\left(\sqrt{d}\right.}{}\end{aligned}$$

Mathematica [C] time = 0.12, size = 119, normalized size = 0.40

$$\frac{2x^{3/2}\left(2d\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)-2\sqrt{-e}\left(d+ex^2\right)+5ex\sqrt{d+ex^2}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)}{25e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*Sqrt[-e]*(d + e*x^2) + 5*e*x*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e
]*x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[
1/2, 3/4, 7/4, -((e*x^2)/d)]))/(25*e*Sqrt[d + e*x^2])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] integral(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

giac [A] time = 0.26, size = 1, normalized size = 0.00

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [C] time = 13.07, size = 75, normalized size = 0.25

$$\frac{2x^{\frac{5}{2}}\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^{\frac{7}{2}}\sqrt{-e}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(5/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**(7/2)*sqrt(-e)*gamma(7/
4)*hyper((1/2, 7/4), (11/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*gamma(11
/4))
```

3.27 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{e^{3/4}\sqrt{d+ex^2}}$$

```
[Out] 2*x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-4*(-e)^(1/2)*x^(1/2)*(e*x^2+
d)^(1/2)/e^(1/2)/(d^(1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2
*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2
))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(3/4)/(e*x^2+d)^(1/2)-2*d^(1/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^
(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5151, 329, 305, 220, 1196}

$$-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{e^{3/4}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) + 2*S
qrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*Sqrt[-e]*(Sqrt[d]
+ Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*Sqr
t[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \left(2\sqrt{-e}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx \\
&= 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right) \\
&= 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(4\sqrt{d}\,\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{e}} + \frac{\left(4\sqrt{d}\,\sqrt{-e}\right)\text{Subst}\left(\cdots\right.}{\cdots} \\
&= -\frac{4\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)} + 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) + \frac{4\sqrt[4]{d}\,\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\cdots}{e^{3/4}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 89, normalized size = 0.34

$$2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{-e}\,x^{3/2}\sqrt{\frac{ex^2}{d}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[-e]*x^(3/2)*Sqrt[1
 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e
*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

sympy [C] time = 8.90, size = 71, normalized size = 0.27

$$2\sqrt{x}\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right) - \frac{x^{\frac{3}{2}}\sqrt{-e}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - x**(3/2)*sqrt(-e)*gamma(3/4)*
hyper((1/2, 3/4), (7/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(7/4))
```

3.28 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{2\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{3d^{3/4}\sqrt{d+ex^2}}$$

```
[Out] -2/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^(1/2)+4/3*(-e^2)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/
2))-4/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arct
an(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)
)),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2
))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)+2/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 325, 329, 305, 220, 1196}

$$\frac{2\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{3d^{3/4}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*Sqrt[-e^2]*Sqrt[x]*Sqrt[d
 + e*x^2])/(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x
^2]])/(3*x^(3/2)) - (4*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x
^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2]) + (2*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*S
qrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{-e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(2(-e)^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{d}\sqrt{e}} + \frac{(4(-}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{4(-e)^{3/2}\sqrt{x}\sqrt{d+ex^2}}{3d\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}{3}$$

Mathematica [C] time = 0.15, size = 121, normalized size = 0.41

$$-\frac{2\left(2(-e)^{3/2}x^3\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)+6\sqrt{-e}\,x\left(d+ex^2\right)+3d\sqrt{d+ex^2}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)}{9dx^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-2*(6*Sqrt[-e]*x*(d + e*x^2) + 3*d*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]] + 2*(-e)^(3/2)*x^3*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((e*x^2)/d)]))/(9*d*x^(3/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

sympy [C] time = 13.09, size = 78, normalized size = 0.26

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^{\frac{3}{2}}}+\frac{\sqrt{-e}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\,\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**(3/2)) + sqrt(-e)*gamma(-1/4)*hy
per((-1/4, 1/2), (3/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*sqrt(x)*gamma
(3/4))
```

3.29 $\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{}}{}\right.\right.}{35d^{7/4}\sqrt{d+ex^2}}$$

```
[Out] -2/7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(5/2)-12/35*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^(3/2)
*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(5/4)*(
cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(
-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/
4)/(e*x^2+d)^(1/2)-6/35*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^
(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(
d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5151, 325, 329, 305, 220, 1196}

$$-\frac{12e^{3/2}\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) - (12*(-e)^(3/2)*Sqrt[d + e*x^
2])/(35*d^2*Sqrt[x]) - (12*Sqrt[-e]*e^(3/2)*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^
2*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7
/2)) + (12*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d
^(7/4)*Sqrt[d + e*x^2]) - (6*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
 + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(
1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5151

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d*x)^(m + 1)*ArcTan[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{-e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{\left(6(-e)^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6(-e)^{5/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{35d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12(-e)^{5/2}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{d}} \right.}{35d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12\sqrt{-e}\,e^{3/2}\right)\mathrm{Subst}\left(\int \right.}{35d^{3/2}}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12\sqrt{-e}\,e^{3/2}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

Mathematica [C] time = 0.14, size = 137, normalized size = 0.41

$$\frac{4\sqrt{-e}\,x\left(-d^2+2dex^2+3e^2x^4\right)-10d^2\sqrt{d+ex^2}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-4(-e)^{5/2}x^5\sqrt{\frac{ex^2}{d}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[-e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*(-e)^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2]
)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas
")
```

```
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\tan^{-1}(1+x+x^2)}{x^2} \, dx$

Optimal. Leaf size=50

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] 1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Rubi [A] time = 0.15, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5205, 6742, 260, 634, 617, 204, 628}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
```

```
!FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(1+x+x^2\right)}{x^2}\,dx = -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \frac{1+2x}{x\left(2+2x+3x^2+2x^3+x^4\right)}\,dx$$
$$= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \left(\frac{1}{2x}-\frac{x}{1+x^2}+\frac{2+x}{2\left(2+2x+x^2\right)}\right)dx$$
$$= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\int \frac{2+x}{2+2x+x^2}\,dx-\int \frac{x}{1+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\int \frac{2+2x}{2+2x+x^2}\,dx+\frac{1}{2}\int \frac{}{2+2}$$
$$= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)-\frac{1}{2}\text{Subst}\Bigg(\int$$
$$= \frac{1}{2}\tan^{-1}(1+x)-\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x\right.$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{\tan^{-1}\left(x^2+x+1\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

fricas [A] time = 0.53, size = 47, normalized size = 0.94

$$\frac{2\,x\arctan\left(x+1\right)+x\log\left(x^2+2\,x+2\right)-2\,x\log\left(x^2+1\right)+2\,x\log(x)-4\,\arctan\left(x^2+x+1\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*arctan(x + 1) + x*log(x^2 + 2*x + 2) - 2*x*log(x^2 + 1) + 2*x*log(
x) - 4*arctan(x^2 + x + 1))/x
```

giac [A] time = 0.14, size = 43, normalized size = 0.86

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\,\arctan\left(x+1\right)+\frac{1}{4}\,\log\left(x^2+2\,x+2\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="giac")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(abs(x))
```

maple [A] time = 0.05, size = 43, normalized size = 0.86

$$\frac{\arctan(x+1)}{2}-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{\ln(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{\ln\left(x^2+2x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^2+x+1)/x^2,x)
```

```
[Out] 1/2*arctan(x+1)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

maxima [A] time = 0.43, size = 42, normalized size = 0.84

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\arctan(x+1)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(x)
```

mupad [B] time = 0.76, size = 42, normalized size = 0.84

$$\frac{\operatorname{atan}(x+1)}{2}+\frac{\ln\left(x^2+2x+2\right)}{4}-\frac{\ln\left(x^2+1\right)}{2}+\frac{\ln(x)}{2}-\frac{\operatorname{atan}\left(x^2+x+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x + x^2 + 1)/x^2,x)
```

```
[Out] atan(x + 1)/2 + log(2*x + x^2 + 2)/4 - log(x^2 + 1)/2 + log(x)/2 - atan(x +
 x^2 + 1)/x
```

sympy [A] time = 0.89, size = 41, normalized size = 0.82

$$\frac{\log(x)}{2}-\frac{\log\left(x^2+1\right)}{2}+\frac{\log\left(x^2+2x+2\right)}{4}+\frac{\operatorname{atan}(x+1)}{2}-\frac{\operatorname{atan}\left(x^2+x+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**2+x+1)/x**2,x)
```

```
[Out] log(x)/2 - log(x**2 + 1)/2 + log(x**2 + 2*x + 2)/4 + atan(x + 1)/2 - atan(x
**2 + x + 1)/x
```

3.31 $\int \frac{\left(a+b \tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

maple [A] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate((b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{n}}{c^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

3.32 $\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=431

$$\frac{3b^2\text{Li}_3\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{Li}_3\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{Li}_2\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+\cdots}\right.}{}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arctan((-
c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(
1/2)))/c+3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(
1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2)))*polylog(3,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*I*b^3
*polylog(4,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,-1
+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] time = 0.48, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 4850, 4988, 4884, 4994, 4998, 6610}

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1
 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (3*b^2*(a + b*ArcTan[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c
*x])])/(2*c) - (3*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3
, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/(2*c) - (((3*I)/4)*b^3*Pol
yLog[4, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*Po
lyLog[4, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
```

```
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)\,\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{1\cdots}\right.}{\cdots}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)\,\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{1+x\cdots}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\cdots}{2c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\cdots}{2c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\cdots}{2c}$$

Mathematica [A] time = 0.20, size = 530, normalized size = 1.23

$$-\frac{6b^2\text{Li}_3\left(-\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right) - 6b^2\text{Li}_3\left(\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right) + 6ib\text{Li}_2\left(-\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(8*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - (2*I)/(I
- Sqrt[1 - c*x]/Sqrt[1 + c*x])] + (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])^2*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x]
- I*Sqrt[1 + c*x]))] - (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^
2*PolyLog[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 +
c*x])] + 6*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sq
rt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] - 6*b^2*
(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + I*S
qrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x])] - (3*I)*b^3*PolyLog[4, -((
Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] + (3*I
)*b^3*PolyLog[4, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[
1 + c*x])])/c
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^3\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3\,ab^2\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3\,a^2b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^3}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo rithm="fricas")

[Out] integral(-(b^3*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctan(sqrt (-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a^3)/(c^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo rithm="giac")

[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)

maple [B] time = 1.99, size = 1631, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)

[Out] 3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-3/2*I*a^2*b/c*dilog((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2*I*a^2*b/c*dilog(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/2*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+6*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)+3/2*a*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-6*a*b^2/c*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3/4*I*b^3/c*polylog(4,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-6*I*b^3/c*polylog(4,(1+I*(-c*

```
x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*I*b^3/c*polylog(4,-
(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\frac{15}{2}\left(b^3\log(cx+1)-b^3\log(-cx+1)\right)\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)^3-\frac{45}{8}\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x
 + 1)) - 64*c*integrate(1/128*(112*b^3*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1
))^3 + 384*a*b^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - 3*(b^3*log(2)^2
*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log(
-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)
))/(c^2*x^2 - 1), x))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=283

$$\frac{ib\mathrm{Li}_2\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\mathrm{Li}_2\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(\right.}{c}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2
*b^2*polylog(3,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*polylog(3,
-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] time = 0.30, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6681, 4850, 4988, 4884, 4994, 6610}

$$\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (I*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c - (I*b*(a
 + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])])/c + (b^2*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/S
qrt[1 + c*x])])/(2*c) - (b^2*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[
1 + c*x])])/(2*c)
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\,\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))}{1+}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\,\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))}{1+x}\right.}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{L}}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{L}}{c}$$

Mathematica [A] time = 0.13, size = 354, normalized size = 1.25

$$-\frac{2ib\text{Li}_2\left(-\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-2ib\text{Li}_2\left(\frac{\sqrt{1-cx}+i\sqrt{cx+1}}{\sqrt{1-cx}-i\sqrt{cx+1}}\right)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+4\tanh^{-1}\left(1\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(4*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - (2*I)/(I
- Sqrt[1 - c*x]/Sqrt[1 + c*x])] + (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] -
I*Sqrt[1 + c*x]))] - (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Po
lyLog[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]
```

```
)] + b^2*PolyLog[3, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*
Sqrt[1 + c*x]))] - b^2*PolyLog[3, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1
 - c*x] - I*Sqrt[1 + c*x])])/c
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^2\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctan(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

maple [B] time = 1.32, size = 837, normalized size = 2.96

$$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}+\frac{b^2\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(\frac{\left(1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{\frac{-cx+1}{cx+1}+1}+1\right)}{c}-\frac{ib^2\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{i}{\cdots}\right.}{\cdots}\right.}{c}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-
I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)
/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*b^2/c*polylog(3,-(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-b^2/c*arctan((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)
^(1/2))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x
+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c*polylog(3,(1+I
*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^2/c*arctan((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x
+1)/(c*x+1)+1)^(1/2))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylo
g(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c
*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-
2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/
2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-I*a*b/c
*dilog((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+I*a*b/c
*dilog(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)-\frac{b^2\log(2)^2\log\left(cx+1\right)-b^2\log(2)^2\log\left(-cx+1\right)-4\left(b^2\log\left(cx+1\right)-b^2\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
ctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 - 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(-c*x + 1)/sqrt(c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) + 64*b^2*integr
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))*
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(-c*x +
 1)/sqrt(c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(-c*x + 1)/sqrt(c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}-\frac{ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*I*b*polylog(2,-I*(-c*x+1)^(1/2)/(
c*x+1)^(1/2))/c+1/2*I*b*polylog(2,I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {206, 6681, 4848, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 - c*x])/Sqrt[1 + c*x]])/c + ((I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1
+ c*x]])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx\right.}{2c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 0.95

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\frac{1}{2}ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 - c
*x])/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/
c)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

maple [B] time = 0.80, size = 263, normalized size = 2.68

$$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{\left(1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{\frac{-cx+1}{cx+1}+1}\right)}{c}+\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\frac{\left(1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{\frac{-cx+1}{cx+1}+1}+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+b/c*arct
an((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((
-c*x+1)/(c*x+1)+1)+1)-1/2*I*b/c*dilog((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/
((-c*x+1)/(c*x+1)+1)+1)+1/2*I*b/c*dilog(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2/((-c*x+1)/(c*x+1)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(\left(\log(cx+1)-\log(-cx+1)\right)\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)-c\int\frac{e^{\left(\frac{1}{2}\log(cx+1)+\frac{1}{2}\log\right.}}{}\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) - 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{ac^2x^2+(bc^2x^2-b)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^{2}x^{2}-1\right)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

maple [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(c^{2}x^{2}-1\right)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ac^{2}x^{2}-a+bc^{2}x^{2}\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.36 $\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+(b^2c^2x^2-b^2)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\,(abc^2x^2-ab)\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctan(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

maple [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\left(b^2c^2\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)+abc^2\right)\sqrt{cx+1}\,\sqrt{-cx+1}\int\frac{x}{\left(abc^2x^2-ab+\left(b^2c^2x^2-b^2\right)\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)\right)\sqrt{cx+1}\,\sqrt{}}\right.}{\left(b^2c\arctan\left(\sqrt{-cx+1},\sqrt{cx+1}\right)+abc\right)\sqrt{cx+1}\,\sqrt{-cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] 2*(2*(b^2*c^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c^2)*sqrt(c*x +
1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)
*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
+ 1)/((b^2*c*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c)*sqrt(c*x + 1)*
sqrt(-c*x + 1))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\mathrm{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\mathrm{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atan(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*at
an(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
))**2), x)
```

3.37 $\int x^m \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\tan^{-1}(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctan(tan(b*x+a))/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tan^{-1}(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Tan[a + b*x]],x]
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\tan^{-1}(\tan(a+bx))\,dx &= \frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\tan^{-1}(\tan(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Tan[a + b*x]],x]
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTan[Tan[a + b*x]]))/(1 + m))
```

fricas [A] time = 0.52, size = 33, normalized size = 0.89

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.18, size = 41, normalized size = 1.11

$$\frac{b\,x^2 \mathrm{e}^{m\ln(x)}}{2+m}+\frac{(\arctan(\tan(bx+a))-bx)\,x\,\mathrm{e}^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(tan(b*x+a)),x)
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctan(tan(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

maxima [A] time = 0.34, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\arctan(\tan(bx+a))}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctan(tan(b*x + a))/(m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\,\mathrm{atan}\left(\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(tan(a + b*x)),x)
[Out] int(x^m*atan(tan(a + b*x)), x)
```

sympy [A] time = 2.70, size = 158, normalized size = 4.27

$$\begin{cases} b\log(x)-\frac{\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor}{x} & \text{for } m=-2\\ -bx\log(x)+bx+\left(\operatorname{atan}(\tan(a+bx))+2\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x) & \text{for } m=-1\\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2}+\frac{2xx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(tan(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/
x, Eq(m, -2)), (-b*x*log(x) + b*x + (atan(tan(a + b*x)) + 2*pi*floor((a + b
*x - pi/2)/pi))*log(x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x*
*m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m + 2) +
2*x*x**m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m +
 2), True))
```

3.38 $\int x^2 \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{bx^4}{12}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Tan[a + b*x]],x]
[Out] -(b*x^4)/12 + (x^3*ArcTan[Tan[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\tan(a+bx))\,dx&=\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{1}{3}b\int x^3\,dx\\&=-\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Tan[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcTan[Tan[a + b*x]]))
```

fricas [A] time = 0.82, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.18, size = 20, normalized size = 0.87

$$-\frac{b\,x^4}{12}+\frac{x^3\arctan\left(\tan\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(tan(b*x+a)),x)
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

maxima [B] time = 0.32, size = 81, normalized size = 3.52

$$\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan(\tan(bx+a))}{b^2}-\frac{(bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2}{b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan(tan(b*x +
a))/b^2 - ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2)/b^2)/b
```

mupad [B] time = 0.12, size = 19, normalized size = 0.83

$$\frac{x^3\,\mathrm{atan}\left(\tan\left(a+b\,x\right)\right)}{3}-\frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(tan(a + b*x)),x)
[Out] (x^3*atan(tan(a + b*x)))/3 - (b*x^4)/12
```

sympy [A] time = 0.36, size = 32, normalized size = 1.39

$$-\frac{bx^4}{12}+\frac{x^3\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(tan(b*x+a)),x)
[Out] -b*x**4/12 + x**3*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/3
```

3.39 $\int x \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2 \tan^{-1}(\tan(a+bx)) - \frac{bx^3}{6}$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5171, 30}

$$\frac{1}{2}x^2 \tan^{-1}(\tan(a+bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[Tan[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}\int x \tan^{-1}(\tan(a+bx))\,dx &= \frac{1}{2}x^2 \tan^{-1}(\tan(a+bx)) - \frac{1}{2}b\int x^2\,dx\\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx - 3\tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/6*(x^2*(b*x - 3*ArcTan[Tan[a + b*x]]))
```

fricas [A] time = 0.49, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="fricas")
```

[Out] 1/3*b*x^3 + 1/2*a*x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.18, size = 20, normalized size = 0.87

$$-\frac{b\,x^3}{6}+\frac{x^2 \arctan\left(\tan\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(tan(b*x+a)),x)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

maxima [B] time = 0.32, size = 57, normalized size = 2.48

$$\frac{\frac{3\left((bx+a)^2-2\,(bx+a)a\right)\arctan(\tan(bx+a))}{b}-\frac{(bx+a)^3-3\,(bx+a)^2 a}{b}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*((b*x + a)^2 - 2*(b*x + a)*a)*arctan(tan(b*x + a))/b - ((b*x + a)^3
- 3*(b*x + a)^2*a)/b)/b
```

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{atan}\left(\tan\left(a+b\,x\right)\right)}{2}-\frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(tan(a + b*x)),x)
```

```
[Out] (x^2*atan(tan(a + b*x)))/2 - (b*x^3)/6
```

sympy [A] time = 0.20, size = 32, normalized size = 1.39

$$-\frac{bx^3}{6}+\frac{x^2\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(tan(b*x+a)),x)
```

```
[Out] -b*x**3/6 + x**2*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/2
```

3.40 $\int \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]],x]
```

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x\tan^{-1}(\tan(a+bx)) - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

fricas [A] time = 0.76, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.13, size = 26, normalized size = 1.62

$$\frac{1}{2}\,bx^2 - \pi x \left\lfloor \frac{bx + a}{\pi} + \frac{1}{2} \right\rfloor + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

maple [A] time = 0.04, size = 15, normalized size = 0.94

$$\frac{\arctan\left(\tan\left(bx + a\right)\right)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x)
```

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

maxima [A] time = 0.32, size = 12, normalized size = 0.75

$$\frac{(bx + a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x + a)^2/b
```

mupad [B] time = 0.04, size = 16, normalized size = 1.00

$$x\,\mathrm{atan}\left(\tan\left(a + b\,x\right)\right) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x)),x)
```

```
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

sympy [A] time = 0.14, size = 42, normalized size = 2.62

$$\begin{cases} \frac{\left(\mathrm{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor \frac{a+bx-\frac{\pi}{2}}{\pi} \right\rfloor\right)^2}{2b} & \text{for}\ b \neq 0 \\ x\left(\mathrm{atan}\left(\tan\left(a\right)\right) + \pi\left\lfloor \frac{a-\frac{\pi}{2}}{\pi} \right\rfloor\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
```

```
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

3.41 $\int \frac{\tan^{-1}(\tan(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tan^{-1}(\tan(a+bx))\right)$$

```
[Out] b*x-(b*x-arctan(tan(b*x+a)))*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tan^{-1}(\tan(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\tan(a+bx))}{x} \, dx &= bx - \left(bx - \tan^{-1}(\tan(a+bx))\right) \int \frac{1}{x} \, dx \\ &= bx - \left(bx - \tan^{-1}(\tan(a+bx))\right) \log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.90

$$\log(x)\left(\tan^{-1}(\tan(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcTan[Tan[a + b*x]])*Log[x]
```

fricas [A] time = 0.79, size = 8, normalized size = 0.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.19, size = 21, normalized size = 1.00

$$\ln(x)\arctan(\tan(bx+a)) - \ln(x)xb + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a))/x,x)
```

```
[Out] ln(x)*arctan(tan(b*x+a))-ln(x)*x*b+b*x
```

maxima [A] time = 0.42, size = 42, normalized size = 2.00

$$\frac{b\arctan(\tan(bx+a))\log(bx) + \left(bx - (bx+a)\log(bx) + a\log(bx) + a\right)b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] (b*arctan(tan(b*x + a))*log(b*x) + (b*x - (b*x + a)*log(b*x) + a*log(b*x) +
 a)*b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atan}(\tan(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x))/x,x)
```

```
[Out] int(atan(tan(a + b*x))/x, x)
```

sympy [A] time = 0.93, size = 34, normalized size = 1.62

$$-bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a))/x,x)
```

```
[Out] -b*x*log(x) + b*x + (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))*lo
g(x)
```

3.42 $\int x^m \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}\tan^{-1}(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*(1/2*Pi-arccot(cot(b*x+a)))/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tan^{-1}(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Cot[a + b*x]],x]
[Out] (b*x^(2 + m))/(2 + 3*m + m^2) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\tan^{-1}(\cot(a+bx))\,dx&=\frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}+\frac{b\int x^{1+m}\,dx}{1+m}\\&=\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 0.86

$$\frac{x^{m+1}\left((m+2)\tan^{-1}(\cot(a+bx))+bx\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Cot[a + b*x]],x]
[Out] (x^(1 + m)*(b*x + (2 + m)*ArcTan[Cot[a + b*x]]))/((1 + m)*(2 + m))
```

fricas [A] time = 0.55, size = 42, normalized size = 1.17

$$-\frac{\left(2\,(bm+b)x^2-(\pi(m+2)-2\,am-4\,a)x\right)x^m}{2\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*m + b)*x^2 - (pi*(m + 2) - 2*a*m - 4*a)*x)*x^m/(m^2 + 3*m + 2)
```

giac [A] time = 0.14, size = 62, normalized size = 1.72

$$-\frac{2\,bmx^2x^m - \pi mxx^m + 2\,amxx^m + 2\,bx^2x^m - 2\,\pi xx^m + 4\,axx^m}{2\left(m^2 + 3\,m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*m*x^2*x^m - pi*m*x*x^m + 2*a*m*x*x^m + 2*b*x^2*x^m - 2*pi*x*x^m +
 4*a*x*x^m)/(m^2 + 3*m + 2)
```

maple [A] time = 0.31, size = 56, normalized size = 1.56

$$\frac{\pi\,x^{1+m}}{2+2m} - \frac{b\,x^2\mathrm{e}^{m\ln(x)}}{2+m} - \frac{(\mathrm{arccot}\,(\cot\,(bx+a)) - bx)\,x\,\mathrm{e}^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/2*Pi*x^(1+m)/(1+m)-b/(2+m)*x^2*exp(m*ln(x))-(arccot(cot(b*x+a))-b*x)/(1+m
)*x*exp(m*ln(x))
```

maxima [A] time = 0.32, size = 40, normalized size = 1.11

$$-\frac{bx^{m+2}}{m+2} + \frac{\pi x^{m+1}}{2\,(m+1)} - \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -b*x^(m + 2)/(m + 2) + 1/2*pi*x^(m + 1)/(m + 1) - a*x^(m + 1)/(m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\left(\frac{\Pi}{2} - \mathrm{acot}\,(\cot\,(a+b\,x))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(Pi/2 - acot(cot(a + b*x))),x)
```

```
[Out] int(x^m*(Pi/2 - acot(cot(a + b*x))), x)
```

sympy [A] time = 6.61, size = 160, normalized size = 4.44

$$\begin{cases} -b\log(x) + \frac{\mathrm{acot}\,(\cot\,(a+bx))}{x} - \frac{\pi}{2x} & \text{for}\ m = -2 \\ bx\log(x) - bx - \log(x)\,\mathrm{acot}\,(\cot\,(a+bx)) + \frac{\pi\log(x)}{2} & \text{for}\ m = -1 \\ \frac{2bx^2x^m}{2m^2+6m+4} - \frac{2mxx^m\,\mathrm{acot}\,(\cot\,(a+bx))}{2m^2+6m+4} + \frac{\pi mxx^m}{2m^2+6m+4} - \frac{4xx^m\,\mathrm{acot}\,(\cot\,(a+bx))}{2m^2+6m+4} + \frac{2\pi xx^m}{2m^2+6m+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Piecewise((-b*log(x) + acot(cot(a + b*x))/x - pi/(2*x), Eq(m, -2)), (b*x*lo
g(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2, Eq(m, -1)), (2*b*x**2
*x**m/(2*m**2 + 6*m + 4) - 2*m*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6*m + 4)
 + pi*m*x*x**m/(2*m**2 + 6*m + 4) - 4*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6
*m + 4) + 2*pi*x*x**m/(2*m**2 + 6*m + 4), True))
```

3.43 $\int x^2 \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{bx^4}{12}$$

```
[Out] 1/12*b*x^4+1/3*x^3*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Cot[a + b*x]],x]
[Out] (b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\cot(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{1}{3}b\int x^3\,dx\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$\frac{1}{12}x^3\left(4\tan^{-1}(\cot(a+bx))+bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Cot[a + b*x]],x]
[Out] (x^3*(b*x + 4*ArcTan[Cot[a + b*x]]))/12
```

fricas [A] time = 0.44, size = 17, normalized size = 0.74

$$-\frac{1}{4}bx^4+\frac{1}{6}(\pi-2a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

giac [A] time = 0.13, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,bx^4+\frac{1}{6}\,\pi x^3-\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/4*b*x^4 + 1/6*pi*x^3 - 1/3*a*x^3
```

maple [B] time = 0.43, size = 65, normalized size = 2.83

$$\frac{\pi\,x^3}{6}-\frac{x^3\mathrm{arccot}\left(\cot\left(bx+a\right)\right)}{3}-\frac{-\frac{(bx+a)^4}{4}+a\left(bx+a\right)^3-\frac{3(bx+a)^2a^2}{2}+\left(bx+a\right)a^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/6*Pi*x^3-1/3*x^3*arccot(cot(b*x+a))-1/3/b^3*(-1/4*(b*x+a)^4+a*(b*x+a)^3-3
/2*(b*x+a)^2*a^2+(b*x+a)*a^3)
```

maxima [A] time = 0.32, size = 17, normalized size = 0.74

$$-\frac{1}{4}\,bx^4+\frac{1}{6}\,(\pi-2\,a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

mupad [B] time = 0.54, size = 25, normalized size = 1.09

$$\frac{\Pi\,x^3}{6}+\frac{b\,x^4}{12}-\frac{x^3\,\mathrm{acot}\left(\cot\left(a+b\,x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi/2 - acot(cot(a + b*x))),x)
```

```
[Out] (Pi*x^3)/6 + (b*x^4)/12 - (x^3*acot(cot(a + b*x)))/3
```

sympy [A] time = 0.39, size = 26, normalized size = 1.13

$$\frac{bx^4}{12}-\frac{x^3\,\mathrm{acot}\left(\cot\left(a+bx\right)\right)}{3}+\frac{\pi x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] b*x**4/12 - x**3*acot(cot(a + b*x))/3 + pi*x**3/6
```

3.44 $\int x \tan^{-1}(\cot(a + bx))\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2 \tan^{-1}(\cot(a + bx)) + \frac{bx^3}{6}$$

```
[Out] 1/6*b*x^3+1/2*x^2*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5173, 30}

$$\frac{1}{2}x^2 \tan^{-1}(\cot(a + bx)) + \frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Cot[a + b*x]],x]
[Out] (b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}\int x \tan^{-1}(\cot(a + bx))\, dx &= \frac{1}{2}x^2 \tan^{-1}(\cot(a + bx)) + \frac{1}{2}b \int x^2\, dx\\ &= \frac{bx^3}{6} + \frac{1}{2}x^2 \tan^{-1}(\cot(a + bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$\frac{1}{6}x^2 \left(3 \tan^{-1}(\cot(a + bx)) + bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Cot[a + b*x]],x]
[Out] (x^2*(b*x + 3*ArcTan[Cot[a + b*x]]))/6
```

fricas [A] time = 0.39, size = 17, normalized size = 0.74

$$-\frac{1}{3}\, bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

giac [A] time = 0.14, size = 19, normalized size = 0.83

$$-\frac{1}{3}\,bx^3 + \frac{1}{4}\,\pi x^2 - \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/3*b*x^3 + 1/4*pi*x^2 - 1/2*a*x^2
```

maple [B] time = 0.44, size = 54, normalized size = 2.35

$$\frac{\pi\,x^2}{4} - \frac{x^2\mathrm{arccot}\left(\cot\left(bx+a\right)\right)}{2} - \frac{-\frac{(bx+a)^3}{3} + \left(bx+a\right)^2 a - a^2\left(bx+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/2*Pi-arccot(cot(b*x+a))),x)
```

```
[Out] 1/4*Pi*x^2-1/2*x^2*arccot(cot(b*x+a))-1/2/b^2*(-1/3*(b*x+a)^3+(b*x+a)^2*a-a
^2*(b*x+a))
```

maxima [A] time = 0.32, size = 17, normalized size = 0.74

$$-\frac{1}{3}\,bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

mupad [B] time = 0.07, size = 25, normalized size = 1.09

$$\frac{\Pi\,x^2}{4} + \frac{b\,x^3}{6} - \frac{x^2\,\mathrm{acot}\left(\cot\left(a+b\,x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi/2 - acot(cot(a + b*x))),x)
```

```
[Out] (Pi*x^2)/4 + (b*x^3)/6 - (x^2*acot(cot(a + b*x)))/2
```

sympy [A] time = 0.30, size = 49, normalized size = 2.13

$$\begin{cases} \frac{\pi x^2}{4} - \frac{x\,\mathrm{acot}^2\left(\cot\left(a+bx\right)\right)}{2b} + \frac{\mathrm{acot}^3\left(\cot\left(a+bx\right)\right)}{6b^2} & \text{for } b \neq 0 \\ \frac{x^2\left(-\,\mathrm{acot}\left(\cot\left(a\right)\right)+\frac{\pi}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Piecewise((pi*x**2/4 - x*acot(cot(a + b*x))**2/(2*b) + acot(cot(a + b*x))**
3/(6*b**2), Ne(b, 0)), (x**2*(-acot(cot(a)) + pi/2)/2, True))
```

3.45 $\int \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[Cot[a + b*x]],x]

[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x\tan^{-1}(\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[Cot[a + b*x]],x]

[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]

fricas [A] time = 0.67, size = 15, normalized size = 0.94

$$-\frac{1}{2}bx^2+\frac{1}{2}(\pi-2a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")

[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x

giac [A] time = 0.11, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

maple [B] time = 0.04, size = 51, normalized size = 3.19

$$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2} - \operatorname{arccot}\left(\cot\left(bx+a\right)\right)\right)\operatorname{arccot}\left(\cot\left(bx+a\right)\right) - \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^2}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x)
```

```
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

maxima [A] time = 0.32, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

mupad [B] time = 0.07, size = 21, normalized size = 1.31

$$\frac{\Pi\,x}{2} - x\operatorname{acot}\left(\cot\left(a+b\,x\right)\right) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
```

```
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

sympy [A] time = 0.13, size = 24, normalized size = 1.50

$$\frac{\pi x}{2} - \begin{cases} \frac{\operatorname{acot}^2\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\ b \neq 0 \\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

3.46 $\int \frac{\tan^{-1}(\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

```
[Out] -b*x+(b*x+1/2*Pi-arccot(cot(b*x+a)))*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]]/x,x]
```

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\cot(a+bx))}{x}\,dx &= -bx-\left(-bx-\tan^{-1}(\cot(a+bx))\right)\int \frac{1}{x}\,dx\\ &= -bx+\left(bx+\tan^{-1}(\cot(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\log(x)\left(\tan^{-1}(\cot(a+bx))+bx\right)-bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]]/x,x]
```

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

fricas [A] time = 0.47, size = 14, normalized size = 0.74

$$-bx+\frac{1}{2}\,(\pi-2\,a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

giac [A] time = 0.11, size = 15, normalized size = 0.79

$$-bx + \frac{1}{2}(\pi - 2a)\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="giac")
[Out] -b*x + 1/2*(pi - 2*a)*log(abs(x))
```

maple [A] time = 0.39, size = 35, normalized size = 1.84

$$\frac{\pi\ln(x)}{2} - bx - a\ln(x) - \ln(x)(\text{arccot}(\cot(bx+a)) - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/2*Pi-arccot(cot(b*x+a)))/x,x)
[Out] 1/2*Pi*ln(x)-b*x-a*ln(x)-ln(x)*(arccot(cot(b*x+a))-b*x-a)
```

maxima [A] time = 0.31, size = 14, normalized size = 0.74

$$-bx + \frac{1}{2}(\pi - 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="maxima")
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\frac{\Pi}{2} - \text{acot}(\cot(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi/2 - acot(cot(a + b*x)))/x,x)
[Out] int((Pi/2 - acot(cot(a + b*x)))/x, x)
```

sympy [A] time = 4.57, size = 27, normalized size = 1.42

$$bx\log(x) - bx - \log(x)\,\text{acot}(\cot(a + bx)) + \frac{\pi\log(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-acot(cot(b*x+a)))/x,x)
[Out] b*x*log(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2
```

3.47 $\int \tan^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tan^{-1}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]],x]
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.12

$$x\tan^{-1}(\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

fricas [A] time = 0.59, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.11, size = 26, normalized size = 1.62

$$\frac{1}{2}\,bx^2 - \pi x\left\lfloor\frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

maple [A] time = 0.04, size = 15, normalized size = 0.94

$$\frac{\arctan\left(\tan\left(bx+a\right)\right)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x)
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

maxima [A] time = 0.32, size = 12, normalized size = 0.75

$$\frac{\left(bx+a\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(b*x + a)^2/b
```

mupad [B] time = 0.00, size = 16, normalized size = 1.00

$$x\,\mathrm{atan}\left(\tan\left(a+b\,x\right)\right) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x)),x)
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

sympy [A] time = 0.14, size = 42, normalized size = 2.62

$$\begin{cases} \frac{\left(\mathrm{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for}\ b \neq 0 \\ x\left(\mathrm{atan}\left(\tan\left(a\right)\right) + \pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

3.48 $\int x^2 \tan^{-1}(c + d \tan(a + bx))\, dx$

Optimal. Leaf size=403

$$-\frac{\text{Li}_4\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}+\frac{\text{Li}_4\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-\frac{(ic+d\ldots}{i\ldots}\right)}{4b}$$

```
[Out] 1/3*x^3*arctan(c+d*tan(b*x+a))+1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x^2*polylog(2,-
(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/4*I*x*polylog(3,-(1+I*c+d)*
exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2-1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2*I*a+
2*I*b*x)/(c+I*(1+d)))/b^2-1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*
c-d))/b^3+1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b^3
```

Rubi [A] time = 0.52, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5175, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}+\frac{\text{PolyLog}\left(4,-\ldots\right)}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 - PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 5175

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 - I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 + I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x^3}{1-ic+d+(1-ic-d)\text{}}$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}ix^3$$

Mathematica [A] time = 1.08, size = 363, normalized size = 0.90

$$\frac{1}{3}x^3\tan^{-1}(d\tan(a+bx)+c)+\frac{4ib^3x^3\log\left(1+\frac{(c-i(d+1))e^{2i(a+bx)}}{c+i(d-1)}\right)-4ib^3x^3\log\left(1+\frac{(c-id+i)e^{2i(a+bx)}}{c+i(d+1)}\right)+6b^2x^2\text{Li}_2\left(-\frac{(c-i}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 + ((c - I*(1 + d)
)*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - (4*I)*b^3*x^3*Log[1 + ((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] + 6*b^2*x^2*PolyLog[2, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - 6*b^2*x^2*PolyLog[2, -(
((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + (6*I)*b*x*PolyLog[3
, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - (6*I)*b*x*Po
lyLog[3, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] - 3*PolyLo
g[4, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + 3*PolyLog
[4, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))])/(24*b^3)
```

fricas [C] time = 0.76, size = 1973, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*tan(b*x + a) + c) + 6*b^2*x^2*dilog((2*(I*c*d - d
^2 + d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*
I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^
2 - 2*d + 1) + 1) - 6*b^2*x^2*dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2
*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)
/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*
x^2*dilog((2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2
 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - 6*b^2*x^2*dilog((2*(-I*c*d - d^2
 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*
tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 +
 2*d + 1) + 1) + 4*I*a^3*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*
d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2 + 1))
 - 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I
*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log
(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d +
 I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2
 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x +
a) + d + 1)/(tan(b*x + a)^2 + 1)) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 +
2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 +
2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 -
c^2 + 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 6*I*b*x*pol
ylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (
2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d
- d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d
^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log(-(2*(I*c*d - d^2 + d)*tan(b*x
 + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a)
 + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) +
 (4*I*b^3*x^3 + 4*I*a^3)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 -
 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2
 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (4*I*b^3*x^3 + 4
*I*a^3)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*
c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)
*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log(-(2*
(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*
I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2
+ c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x +
 a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a
) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*po
```

```
lylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 +
(-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)
*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 + 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 +
2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 +
2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2
+ d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tan(b*x + a) + c), x)
```

maple [C] time = 65.27, size = 8040, normalized size = 19.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}x^3\arctan\left(c\cos\left(2\,bx+2\,a\right)+\left(d+1\right)\sin\left(2\,bx+2\,a\right)+c,\left(d+1\right)\cos\left(2\,bx+2\,a\right)-c\sin\left(2\,bx+2\,a\right)-d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/6*x^3*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b
*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*co
s(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c
^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(
4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*
a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4
+ d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2
*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c
^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*
d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2
*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(c+d\,\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*tan(a + b*x)),x)
[Out] int(x^2*atan(c + d*tan(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tan(b*x+a)),x)
[Out] Timed out
```

3.49 $\int x \tan^{-1}(c + d \tan(a + bx)) \, dx$

Optimal. Leaf size=305

$$\frac{i\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1\right.$$

```
[Out] 1/2*x^2*arctan(c+d*tan(b*x+a))+1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x*polylog(2,-(c+I
*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/8*I*polylog(3,-(1+I*c+d)*exp(2*
I*a+2*I*b*x)/(1+I*c-d))/b^2-1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)
/(c+I*(1+d)))/b^2
```

Rubi [A] time = 0.41, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5175, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5175

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 - I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 + I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{2}(b(1-ic-d))\int\frac{e^{2ia+2ibx}x^2}{1-ic+d+(1-ic-d)e^{2ia+}}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{4}ix^2\log($$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{4}ix^2\log($$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{4}ix^2\log($$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{4}ix^2\log($$

Mathematica [A] time = 0.70, size = 272, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(d\tan(a+bx)+c)+\frac{i\left(2b^2x^2\log\left(1+\frac{(c-i(d+1))e^{2i(a+bx)}}{c+i(d-1)}\right)-2b^2x^2\log\left(1+\frac{(c-id+i)e^{2i(a+bx)}}{c+i(d+1)}\right)-2ibx\mathrm{Li}_2\left(-\frac{(c-i(d+1)}{c+i(}\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((c - I*(1 +
 d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - 2*b^2*x^2*Log[1 + ((I + c - I
*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] - (2*I)*b*x*PolyLog[2, -(((c - I*
(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + (2*I)*b*x*PolyLog[2, -((
(I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + PolyLog[3, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - PolyLog[3, -(((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))]))/b^2
```

fricas [C] time = 0.67, size = 1557, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*tan(b*x + a) + c) + 2*b*x*dilog((2*(I*c*d - d^2 +
d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*ta
n(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2
*d + 1) + 1) - 2*b*x*dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 - 2*
I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog((2*
(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*
I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilog((2*(-I*c*d - d^2 - d)*tan(b*x + a
)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 2*
d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) -
2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^
2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((
I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)
*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) - 2*I*a^2*log(((I*c*d - d^2 +
d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a)
- d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((I*c*d - d^2 - d)*tan(b*x + a
)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(
b*x + a)^2 + 1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log(-(2*(I*c*d - d^2 + d)*tan(b
*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x +
a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
 + (2*I*b^2*x^2 - 2*I*a^2)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2
 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c
^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (2*I*b^2*x^2 -
 2*I*a^2)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*
I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log(-(
2*(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d -
2*I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x
 + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2 + 2*I)*tan(b*x +
 a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*
polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2
+ (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d -
d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 + (2*I*c^2 - 4*c*d - 2*I*d^2
+ 2*I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
- 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
+ 2*I*c*d + d^2 + (-2*I*c^2 - 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 1)/((c^
2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan(d \tan(bx+a)+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tan(b*x + a) + c), x)
```

maple [C] time = 6.25, size = 7648, normalized size = 25.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2 \arctan \left(c\cos \left(2\,bx + 2\,a\right) + \left(d + 1\right)\sin \left(2\,bx + 2\,a\right) + c, \left(d + 1\right)\cos \left(2\,bx + 2\,a\right) - c\sin \left(2\,bx + 2\,a\right) - d + 1\right) + \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/4*x^2*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*x +
 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*
a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) -
 (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*
b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 +
 d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*
x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2
 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 +
d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^
4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a)
- 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 -
 d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d^3
+ (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))
*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}\left(c+d\,\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*tan(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

3.50 $\int \tan^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=198

$$\frac{\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2}}{c+i(d+1}\right.$$

```
[Out] x*arctan(c+d*tan(b*x+a))+1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d
))-1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*polylog(2,-
(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*polylog(2,-(c+I*(1-d))*exp(2*
I*a+2*I*b*x)/(c+I*(1+d)))/b
```

Rubi [A] time = 0.23, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5167, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{2}ix\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] - (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] + PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) - PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5167

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[(x*E^(2*I*a + 2*I*b
*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x] - Dist[b*(1
+ I*c + d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I
*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c + d\tan(a + bx))\,dx = x\tan^{-1}(c + d\tan(a + bx)) + (b(1 - ic - d))\int \frac{e^{2ia+2ibx}x}{1 - ic + d + (1 - ic - d)e^{2ia+2ibx}}\,dx$$

$$= x\tan^{-1}(c + d\tan(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{2}ix\log\left(1 + \frac{(c}{}\right.$$

$$= x\tan^{-1}(c + d\tan(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{2}ix\log\left(1 + \frac{(c}{}\right.$$

$$= x\tan^{-1}(c + d\tan(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{2}ix\log\left(1 + \frac{(c}{}\right.$$

Mathematica [B] time = 7.92, size = 555, normalized size = 2.80

$$x\tan^{-1}(d\tan(a+bx)+c)+\frac{x\left(-i\sqrt{-d^2}\left(\text{Li}_2\left(\frac{d^2(1-i\tan(a+bx))}{d^2+icd-i\sqrt{-d^2}}\right)+\log(1-i\tan(a+bx))\log\left(\frac{d^2(-\tan(a+bx))-cd+\sqrt{-d^2}}{-cd+id^2+\sqrt{-d^2}}\right)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (x*(-4*a*d*ArcTan[c + d*Tan[a + b*x]] - I*Sq
rt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))/
(I*c*d + d^2 - I*Sqrt[-d^2])]) + I*Sqrt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[
(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[-d^2])] + PolyLog
[2, (d^2*(1 - I*Tan[a + b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])]) + I*Sqrt[-d^2
]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d +
 I*d^2 - Sqrt[-d^2])] + PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/((-I)*c*d + d
^2 + I*Sqrt[-d^2])]) - I*Sqrt[-d^2]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqr
t[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(
1 + I*Tan[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))])))/(2*d*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

fricas [B] time = 0.70, size = 1117, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*tan(b*x + a) + c) + (-2*I*b*x - 2*I*a)*log(-(2*(I*c*d -
 d^2 + d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 -
2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 +
d^2 - 2*d + 1)) + (2*I*b*x + 2*I*a)*log(-(2*(I*c*d - d^2 - d)*tan(b*x + a)^
2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d
 - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + (2*I*
b*x + 2*I*a)*log(-(2*(-I*c*d - d^2 + d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d -
(2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) + 2*d - 2)/((c^2 + d^2 - 2*d
 + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + (-2*I*b*x - 2*I*a)*log(-(2*(
-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I
*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 +
 c^2 + d^2 + 2*d + 1)) + 2*I*a*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 - c^2
+ I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x + a)^2
 + 1)) - 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2
```

```
 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 2*I*a*l
og(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d
 + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d - d^2
 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x +
a) + d + 1)/(tan(b*x + a)^2 + 1)) + dilog((2*(I*c*d - d^2 + d)*tan(b*x + a)
^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) + 2*
d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) -
dilog((2*(I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 - 2*I*c*d - (-2*I*c^2 + 4
*c*d + 2*I*d^2 - 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan(b*
x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + dilog((2*(-I*c*d - d^2 + d)*tan(b*x
+ a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2 + 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) +
 2*d - 2)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
 - dilog((2*(-I*c*d - d^2 - d)*tan(b*x + a)^2 - 2*c^2 + 2*I*c*d - (2*I*c^2
+ 4*c*d - 2*I*d^2 + 2*I)*tan(b*x + a) - 2*d - 2)/((c^2 + d^2 + 2*d + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tan(b*x + a) + c), x)
```

maple [B] time = 1.22, size = 1002, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctan(c+d*tan(b*x+a))+1/2*I/b*arctan((c+d*tan(b*x+a
))/d-c/d)*ln(1-(-I*d-I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a
))/d-c/d)^2+1)/(-I*d+I-c))+1/2/b*arctan((c+d*tan(b*x+a))/d-c/d)^2+1/4/b*pol
ylog(2,(-I*d-I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d
)^2+1)/(-I*d+I-c))+1/2*d/b/(I+I*d+c)*ln(1-(-I*d+I+c)*(1+I*((c+d*tan(b*x+a))
/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a
))/d-c/d)+1/2/b/(I+I*d+c)*ln(1-(-I*d+I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/
(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a))/d-c/d)-1
/2*I/b/(I+I*d+c)*ln(1-(-I*d+I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*ta
n(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan((c+d*tan(b*x+a))/d-c/d)*c-1/2*I*d/
b/(I+I*d+c)*arctan((c+d*tan(b*x+a))/d-c/d)^2-1/4*I*d/b/(I+I*d+c)*polylog(2,
(-I*d+I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/
(-I*d-I-c))-1/2*I/b/(I+I*d+c)*arctan((c+d*tan(b*x+a))/d-c/d)^2-1/2/b/(I+I*d
+c)*arctan((c+d*tan(b*x+a))/d-c/d)^2*c-1/4*I/b/(I+I*d+c)*polylog(2,(-I*d+I+
c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-
c))-1/4/b/(I+I*d+c)*polylog(2,(-I*d+I+c)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(
((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c
```

maxima [B] time = 0.52, size = 433, normalized size = 2.19

$$d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d} - \frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1}, \frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right) - 4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+a)}{c^2+d^2-2\,d+1}, \frac{cd\tan(bx+a)+c^2\cdots}{c^2+d^2-2\,d+\cdots}\right.\cdots}{\cdots}\right.\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*arc
tan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x + a
) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2 -
 d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/(
c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2
*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*d
 + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*tan
(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) + 8*(b*x + a)*arct
an(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(c+d\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(atan(c + d*tan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(c+d\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a)),x)
```

```
[Out] Integral(atan(c + d*tan(a + b*x)), x)
```

3.51 $\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(d\tan(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*tan(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

maple [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*tan(b*x+a))/x,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}\left(c+d\,\tan\left(a+b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tan(a + b*x))/x,x)
```

```
[Out] int(atan(c + d*tan(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(c + d \tan{\left(a + bx \right)} \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atan(c + d*tan(a + b*x))/x, x)
```

3.52 $\int x^2 \tan^{-1}(c + (1 + ic) \tan(a + bx))\,dx$

Optimal. Leaf size=154

$$\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+b$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(c+(1+I*c)*tan(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(3,
I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] time = 0.24, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5171, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyLog
[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(c+(1+ic)\tan(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{3}(ib)\int\frac{x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,d\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{i(1+ic)+c+ce}\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\end{aligned}$$

Mathematica [A] time = 0.48, size = 140, normalized size = 0.91

$$\frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{4ib^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)-6b^2x^2\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{c}\right)+6ibx\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{c}\right)+3\text{Li}_4\left(\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
^((2*I)*(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.57, size = 322, normalized size = 2.09

$$-\frac{b^4x^4 - 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 - 2*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2
*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - (-2*I*b^3*x^3 - 2*
I*a^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - (-2*I*b^3*x^3 - 2*I*a^3)*
log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e
^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

maple [C] time = 6.71, size = 1532, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*e
xp(I*(b*x+a))*(-I*c)^(1/2))*x-1/12*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*e
xp(2*I*(b*x+a)))+1/6*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))
^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)
/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*
I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*
(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3
*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*
(c-I)/(exp(2*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(ex
p(2*I*(b*x+a))+1))^3+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)-1/12*x^3*Pi*csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-1/12
*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*
exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/12*b*x^4+1/8*polylog(4,I*c*e
xp(2*I*(b*x+a)))/b^3-1/3*I*x^3*ln(exp(I*(b*x+a)))-1/6*I*x^3*ln(c-I)+1/12*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(ex
p(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a)
)*c+I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*x*a^2+1
/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3+1/6*Pi*x^3+1/12*x^3*Pi*csgn(I/(exp(
2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I/(e
xp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1
/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-
I)/(exp(2*I*(b*x+a))+1))^3+1/4/b^3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2-1/2/
b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/12*x^3*Pi*csgn((exp(2*I*(b
```

```
*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-
I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^
2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp
(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*
c+I)-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I
*exp(I*(b*x+a))*(-I*c)^(1/2))-1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2+1
/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi*
csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(
c-I)/(exp(2*I*(b*x+a))+1))-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2
))-1/4*x^2*polylog(2,I*c*exp(2*I*(b*x+a)))/b-1/12*x^3*Pi*csgn(I*exp(2*I*(b*
x+a)))^3+1/12*x^3*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/1
2*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2
```

maxima [B] time = 0.38, size = 309, normalized size = 2.01

$$\frac{\frac{\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\arctan((ic+1)\tan(bx+a)+c)}{b^2}-\frac{3\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+\left(-8i(bx+a)^3+18i(bx+a)^2a-18i(bx+a)\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c + 1)*tan
(b*x + a) + c)/b^2 - 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x +
 a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18
*I*(b*x + a)*a - 9*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*
b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(
2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b^2
*(12*c - 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.53 $\int x\tan^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=123

$$-\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{bx^3}{6}$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(c+(1+I*c)*tan(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5171, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])
/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
```

```
1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{i(1+ic)+c+ce^{2}}$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-$$

Mathematica [A] time = 0.36, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

fricas [C] time = 0.49, size = 271, normalized size = 2.20

$$-\frac{2\,b^3x^3-3i\,b^2x^2\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+2\,a^3+6\,bx\text{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)+6\,bx\text{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*s
qrt(4*I*c))/c) - (-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*
a) + 1) - (-3*I*b^2*x^2 + 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1
```

```
) + 6*I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*s
qrt(4*I*c)*e^(I*b*x + I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

maple [C] time = 6.03, size = 1497, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/2*I/b*ln(1-I*exp(2*I*(b*x+a))*c)*x*a-1/4*I*x^2*ln(1-I*exp(2*I*(b*x+a))*c
)+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1-I*exp(I*(b
*x+a))*(-I*c)^(1/2))*x+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b
*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*c
sgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(exp(2*
I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3-1/4/b^2*polylog(2,I*c*exp(2*I*(b*x
+a)))*a+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(
exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I
*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*
(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*
Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))+1))^2+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c+I)+1/4*Pi*x^2-1/2*I*x^2*ln(exp
(I*(b*x+a)))-1/4*I*x^2*ln(c-I)-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn
(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(I*(c-I)/(exp(2
*I*(b*x+a))+1))^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a)
)+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I
/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn
(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*exp(I*(b*x
+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*e
xp(2*I*(b*x+a)))^2+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^2
*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))
*c+I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2
*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+
1))^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x
+a))*c+I)/(exp(2*I*(b*x+a))+1))^3-1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2-
1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*
(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/4*x*polyl
og(2,I*c*exp(2*I*(b*x+a)))/b-1/8*I*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2-1/6*
b*x^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+
a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi
*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))
*c+I)/(exp(2*I*(b*x+a))+1))
```

maxima [B] time = 0.36, size = 218, normalized size = 1.77

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan\left((i\,c+1)\tan(bx+a)+c\right)}{b}-\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(c\cos\ldots}{\ldots}}{2\,b\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c + 1)*tan(b*x + a) + c)/b - 2
*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2*
I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c
*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
+ 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3
, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(12*c - 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(c + \tan(a + b\,x)\,(1 + c\,1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.54 $\int \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=85

$$-\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{bx^2}{2}$$

```
[Out] -1/2*b*x^2+x*arctan(c+(1+I*c)*tan(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5163, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5163

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c+(1+ic)\tan(a+bx)) - (ib)\int \frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) + (bc)\int \frac{e^{2ia+2ibx}x}{i(1+ic)+c+ce^{2ia+2ibx}}$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{1}{2}i\int$$

Subs

$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) +$$
$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{\mathrm{Li}_2(}{}$$

Mathematica [B] time = 18.75, size = 967, normalized size = 11.38

$$((c+i)\cos(a+bx)+(ic+1)\sin(a+bx))\left(\log(i\tan(bx)+1)\tan(bx)\cos^2(a)+2bx-i\log\left(1-\frac{\sec(bx)((c-i)\cos(a)+i(}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] + (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[
b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*
x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((
1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*
Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (
Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a])*(Cos[
a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*C
os[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]
- I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] +
I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1
 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)
*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((I + c)*Cos[a +
 b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*
Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a +
b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]]*Tan[b*x] + Cos[a]
^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (
Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
 (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Si
n[a])*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(
I + Tan[b*x]))*(-I + Tan[a + b*x])*(1 - I*c + (-I + c)*Tan[a + b*x]))
```

fricas [B] time = 0.63, size = 202, normalized size = 2.38

$$-\frac{b^2x^2 - i\,bx\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 - (-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - (-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) -
 (-I*b*x - I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqr
t(4*I*c)*e^(I*b*x + I*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c), x)
```

maple [B] time = 0.51, size = 1489, normalized size = 17.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/4*I/(1+I*c)/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln(1/2*(I+c+(1+I*c)*tan(
b*x+a))/c)*c^2+1/4*I/(1+I*c)/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln((c+(1+I
*c)*tan(b*x+a)-I)/(-2*I+2*c))*c^2-2*I/(1+I*c)/b*arctan(c+(1+I*c)*tan(b*x+a)
)/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c+2*I/(1+I*c)/b*arctan(c+(1+I*c)*ta
n(b*x+a))/(2*I-2*c)*ln(c+(1+I*c)*tan(b*x+a)-I)*c+1/4*I/(1+I*c)/b/(I-c)*ln(-
1/2*I*(I+c+(1+I*c)*tan(b*x+a)))*ln(c+(1+I*c)*tan(b*x+a)-I)*c^2-1/4*I/(1+I*c
)/b/(I-c)*dilog((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c))-1/4*I/(1+I*c)/b/(I-c)*
dilog(-1/2*I*(I+c+(1+I*c)*tan(b*x+a)))+1/8*I/(1+I*c)/b/(I-c)*ln(c+(1+I*c)*t
an(b*x+a)-I)^2-1/4/(1+I*c)/b/(I-c)*ln(c+(1+I*c)*tan(b*x+a)-I)^2*c-1/(1+I*c)
/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)+1/(1+
I*c)/b*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(c+(1+I*c)*tan(b*x+a)-I)-1/
2/(1+I*c)/b/(I-c)*dilog(1/2*(I+c+(1+I*c)*tan(b*x+a))/c)*c+1/2/(1+I*c)/b/(I-
c)*dilog((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c))*c+1/2/(1+I*c)/b/(I-c)*dilog(-
1/2*I*(I+c+(1+I*c)*tan(b*x+a)))*c+1/4*I/(1+I*c)/b/(I-c)*dilog(1/2*(I+c+(1+I
*c)*tan(b*x+a))/c)-1/2/(1+I*c)/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln(1/2*(
I+c+(1+I*c)*tan(b*x+a))/c)*c+1/2/(1+I*c)/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I
)*ln((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c))*c+1/(1+I*c)/b*arctan(c+(1+I*c)*ta
n(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c^2-1/4*I/(1+I*c)/b/(I-c)*l
n(-1/2*I*(I+c+(1+I*c)*tan(b*x+a)))*ln(c+(1+I*c)*tan(b*x+a)-I)+1/4*I/(1+I*c)
/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln(1/2*(I+c+(1+I*c)*tan(b*x+a))/c)-1/4
*I/(1+I*c)/b/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln((c+(1+I*c)*tan(b*x+a)-I)/
(-2*I+2*c))+1/4*I/(1+I*c)/b/(I-c)*dilog((c+(1+I*c)*tan(b*x+a)-I)/(-2*I+2*c)
)*c^2+1/4*I/(1+I*c)/b/(I-c)*dilog(-1/2*I*(I+c+(1+I*c)*tan(b*x+a)))*c^2-1/8*
I/(1+I*c)/b/(I-c)*ln(c+(1+I*c)*tan(b*x+a)-I)^2*c^2-1/(1+I*c)/b*arctan(c+(1+
I*c)*tan(b*x+a))/(2*I-2*c)*ln(c+(1+I*c)*tan(b*x+a)-I)*c^2+1/2/(1+I*c)/b/(I-
c)*ln(-1/2*I*(I+c+(1+I*c)*tan(b*x+a)))*ln(c+(1+I*c)*tan(b*x+a)-I)*c-1/4*I/(
1+I*c)/b/(I-c)*dilog(1/2*(I+c+(1+I*c)*tan(b*x+a))/c)*c^2
```

maxima [B] time = 0.47, size = 448, normalized size = 5.27

$$-\frac{(i\,c+1)\left(\frac{4i\,(bx+a)\log\left(\frac{2i\,c^2-2\,(c^2-2i\,c-1)\tan(bx+a)+4\,c-2i}{2i\,c^2-2\,(c^2-2i\,c-1)\tan(bx+a)+2i}\right)}{i\,c+1}-\frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+(c^2-2i\,c-1)\tan(bx+a)-2\,c+i\right)-\log\left(-i\,c^2+(c^2-2i\,c-1)\tan(bx\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log((2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x +
 a) + 4*c - 2*I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c +
 1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I
) - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1
)*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*lo
g(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x
+ a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*
c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a)
- 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*
dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arctan((I*c + 1)*
tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log((2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan
(b*x + a) + 4*c - 2*I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I)))
/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.55 $\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c)/x, x)
```

maple [A] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(c+\left(ic+1\right) \tan \left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is c-1 zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1))/x,x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.56 $\int x^2 \tan^{-1}(c + (-1 + ic)\tan(a + bx))\, dx$

Optimal. Leaf size=155

$$-\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan^{-1}(c-(1-ic)\text{ta}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctan(c-(1-I*c)*tan(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*I*
a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] time = 0.24, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5171, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}+\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(-1+ic)+c+ce^{2ia+2ib}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x}{i(-1+ic)+c+c}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -$$

Mathematica [A] time = 0.51, size = 137, normalized size = 0.88

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3} + \frac{6ix\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} - \frac{6x^2\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + 4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) + 8x^3\tan^{-1}(c+i(c+i)\text{t}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x*
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(a
 + b*x)))])/b^3)/24
```

fricas [C] time = 0.51, size = 320, normalized size = 2.06

$$b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + (2*I*b^3*x^3 +
 2*I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (2*I*b^3*x^3 + 2*I*a^
3)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I
*c)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^
3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

maple [C] time = 6.70, size = 1533, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b^
3*a^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a)
)*(I*c)^(1/2))-1/3*I/b^3*ln(1+I*c*exp(2*I*(b*x+a)))*a^3-1/6*I/b^3*a^3*ln(-e
xp(2*I*(b*x+a))*c+I)+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/4*x^2*polylog
(2,-I*exp(2*I*(b*x+a))*c)/b+1/12*b*x^4-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I
+c)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*x*a^2+1/2*
I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(I*(
b*x+a))*(I*c)^(1/2))*x-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp
(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))+1/6
*Pi*x^3+1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2/b^3*a^2*dilog
(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)*a
^2+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/12*x^3
*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi*csgn((e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*
x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1)
*(I+c))^3+1/6*I*x^3*ln(1+I*c*exp(2*I*(b*x+a)))-1/6*I*x^3*ln(exp(2*I*(b*x+a)
)*c-I)+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn
((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*
(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))+1))-1/12*x^3*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+
1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))+1))^2-1/8*polylog(4,-I*exp(2*I*(b*x+a))*c)/b^3-1/12*x^3*Pi*cs
```

```
gn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))+1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))
*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2-1/12*x^3*Pi*csgn(I*
(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(e
xp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(
2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*x*polylog(3,-I*exp(2*I*(b*x
+a))*c)/b^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I
+c)/(exp(2*I*(b*x+a))+1))^2+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/6*I*x^3*ln(I+c)+
1/12*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*x^3*Pi*cs
gn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*x^3*Pi*csgn(I/(exp(2*I
*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))+1/12*x^3*Pi*
csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*
(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))
```

maxima [B] time = 0.38, size = 310, normalized size = 2.00

$$\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b^2}+\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c - 1)*tan
(b*x + a) + c)/b^2 + 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x +
 a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*arc
tan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18
*I*(b*x + a)*a - 9*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
- 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2
*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*e
^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(
b^2*(12*c + 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.57 $\int x\tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=124

$$\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{bx^3}{6}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctan(c-(1-I*c)*tan(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*I*a
+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,-I*c*
exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.21, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5171, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5171

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tan[a + b*x]])/(f*(m +
```

```
1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{i(-1+ic)+c+ce^{2ia+2ibx}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{i(-1+ic)+c+ce$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) +$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) +$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) +$$

Mathematica [A] time = 0.39, size = 111, normalized size = 0.90

$$\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{c}\right)+\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+i(c+i)\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + I*(I + c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

fricas [C] time = 0.71, size = 269, normalized size = 2.17

$$\frac{2b^3x^3+3ib^2x^2\log\left(-\frac{(c+i)e^{(2ibx+2ia)}}{ce^{(2ibx+2ia)}-i}\right)+2a^3+6bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)+6bx\text{Li}_2\left(-\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)+3ia^2\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) + (3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x +
I*a) + 1) + (3*I*b^2*x^2 - 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) +
```

```
 1) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/
2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan \left((i\,c - 1) \tan \left(bx + a\right) + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

maple [C] time = 5.48, size = 1498, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I*x^2*ln(exp(I*(b*x+a
)))+1/4*I*x^2*ln(I+c)-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))+1/2*I/b*ln(1+I*
c*exp(2*I*(b*x+a)))*x*a-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/4*
I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))-1/4*I*x^2*ln(exp(2*I*(b*x+a))*c-I)-1/8*x^2
*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a
))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/4*Pi*x^2+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a)
)*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x
+a)))-1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/8*x^2*
Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(I*
exp(2*I*(b*x+a)))^3-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I
*(b*x+a))+1)*(I+c))^2-1/8*x^2*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*
(I+c))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/
(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(
I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I
*(b*x+a))+1)*(I+c))^3-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*x^
2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+
a))*(I+c)/(exp(2*I*(b*x+a))+1))+1/6*b*x^3+1/4*x*polylog(2,-I*exp(2*I*(b*x+a
))*c)/b+1/8*I*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2-1/8*x^2*Pi*csgn(I/(exp(2
*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I
)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I
*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/8
*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn
(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1)
)^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)
/(exp(2*I*(b*x+a))+1))^2+1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2+1/4*I/b^2
*a^2*ln(-exp(2*I*(b*x+a))*c+I)-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1
/2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^2*a*dilog(1-I*ex
p(I*(b*x+a))*(I*c)^(1/2))+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I
*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*P
i*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(I*exp
(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/4/b^2*polylog(2,-I*exp(2*I*(b
*x+a))*c)*a+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(e
xp(2*I*(b*x+a))+1)*(I+c))
```

maxima [B] time = 0.37, size = 219, normalized size = 1.77

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b}+\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(c\cos}{}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c - 1)*tan(b*x + a) + c)/b + 2
*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x + 2
*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), -
c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
 + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(
3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(12*c + 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c + \tan\left(a + b\,x\right)\,\left(-1 + c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.58 $\int \tan^{-1}(c + (-1 + ic)\tan(a + bx))\,dx$

Optimal. Leaf size=86

$$\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b} + \frac{1}{2}ix\log\left(1 + ice^{2ia+2ibx}\right) + x\tan^{-1}(c - (1 - ic)\tan(a + bx)) + \frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+x*arctan(c-(1-I*c)*tan(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*x
))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5163, 2184, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{1}{2}ix\log\left(1 + ice^{2ia+2ibx}\right) + x\tan^{-1}(c - (1 - ic)\tan(a + bx)) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5163

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c-(1-ic)\tan(a+bx)) - (ib)\int \frac{x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c-(1-ic)\tan(a+bx)) - (bc)\int \frac{e^{2ia+2ibx}x}{i(-1+ic)+c+ce^{2ia+2ib}$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{1}{2}i\int$$

Subs

$$= \frac{bx^2}{2} + x\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) -$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) + \frac{\mathrm{Li}_2(}{}$$

Mathematica [**B**] time = 17.58, size = 847, normalized size = 9.85

$$x\tan^{-1}(c+i(c+i)\tan(a+bx)) + \frac{ix\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\right.}{((c-i)\cos(a+bx)+i(c+i)\sin(a+bx))\left(-\frac{\log\left(\frac{1}{2}\sec(bx)(\cos(a)+i\sin(a))((ic+1)\cos(a+bx)-(c\right.}{\tan(bx)-i}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a +
b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*Log
[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec
[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/
(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + S
in[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*x + I*Log
[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
+ b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*
Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + ((1 + I*c)*(
Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1 - I*c)*Cos[a
+ b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 - (Sec[b*x]*((I
 + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Ta
n[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]*Tan[b*x] - (Log
[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*((I + c
)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]
) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a + b*x] + I*(I + c)*S
in[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

fricas [**B**] time = 0.75, size = 200, normalized size = 2.33

$$\frac{b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (
```

```
I*b*x + I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*
e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c), x)
```

maple [B] time = 0.56, size = 1681, normalized size = 19.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] -1/4*I/(-1+I*c)/b/(I+c)*dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))*c^2+1/4*I/(
-1+I*c)/b/(I+c)*dilog((-c-(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))*c^2+1/2/(-1+I*
c)/b/(I+c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c
)*c-1/4*I/(-1+I*c)/b/(I+c)*ln(I+c+(-1+I*c)*tan(b*x+a))*ln(-1/2*I*(-c-(-1+I*
c)*tan(b*x+a)+I))+1/4*I/(-1+I*c)/b/(I+c)*ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)
))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))-1/2/(-1+I*c)/b/(I+c)*ln(c-(-1+I*c)
*tan(b*x+a)+I)*ln((-c-(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))*c+1/4/(-1+I*c)/b/(
I+c)*ln(I+c+(-1+I*c)*tan(b*x+a))^2*c+1/(-1+I*c)/b*arctan(c+(-1+I*c)*tan(b*x
+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a))*c^2-1/(-1+I*c)/b*arctan(c+(-1+I*
c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c^2-1/2/(-1+I*c)/b/(I+
c)*ln(I+c+(-1+I*c)*tan(b*x+a))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))*c+1/2/
(-1+I*c)/b/(I+c)*ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))*ln(-1/2*I*(-c-(-1+I*c
)*tan(b*x+a)+I))*c-1/4*I/(-1+I*c)/b/(I+c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*ln((-
c-(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))+1/4*I/(-1+I*c)/b/(I+c)*ln(c-(-1+I*c)*t
an(b*x+a)+I)*ln(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)-1/4*I/(-1+I*c)/b/(I+c)*d
ilog(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)*c^2-1/8*I/(-1+I*c)/b/(I+c)*ln(I+c+(
-1+I*c)*tan(b*x+a))^2*c^2-1/2/(-1+I*c)/b/(I+c)*dilog((-c-(-1+I*c)*tan(b*x+a
)-I)/(-2*I-2*c))*c+1/2/(-1+I*c)/b/(I+c)*dilog(-1/2*(-c-(-1+I*c)*tan(b*x+a)+
I)/c)*c-1/(-1+I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c
)*tan(b*x+a))+1/(-1+I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1
+I*c)*tan(b*x+a)+I)-1/4*I/(-1+I*c)/b/(I+c)*dilog((-c-(-1+I*c)*tan(b*x+a)-I)
/(-2*I-2*c))+1/4*I/(-1+I*c)/b/(I+c)*dilog(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c
)+1/8*I/(-1+I*c)/b/(I+c)*ln(I+c+(-1+I*c)*tan(b*x+a))^2+1/4*I/(-1+I*c)/b/(I+
c)*dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))+1/2/(-1+I*c)/b/(I+c)*dilog(-1/2*
I*(I+c+(-1+I*c)*tan(b*x+a)))*c+2*I/(-1+I*c)/b*arctan(c+(-1+I*c)*tan(b*x+a))
/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a))*c-2*I/(-1+I*c)/b*arctan(c+(-1+I*c)*t
an(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c+1/4*I/(-1+I*c)/b/(I+c)*l
n(I+c+(-1+I*c)*tan(b*x+a))*ln(-1/2*I*(-c-(-1+I*c)*tan(b*x+a)+I))*c^2-1/4*I/
(-1+I*c)/b/(I+c)*ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))*ln(-1/2*I*(-c-(-1+I*c
)*tan(b*x+a)+I))*c^2+1/4*I/(-1+I*c)/b/(I+c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(
(-c-(-1+I*c)*tan(b*x+a)-I)/(-2*I-2*c))*c^2-1/4*I/(-1+I*c)/b/(I+c)*ln(c-(-1+
I*c)*tan(b*x+a)+I)*ln(-1/2*(-c-(-1+I*c)*tan(b*x+a)+I)/c)*c^2
```

maxima [B] time = 0.46, size = 448, normalized size = 5.21

$$-\frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(\frac{2i\,c^2-2\left(c^2+2i\,c-1\right)\tan(bx+a)+2i}{2i\,c^2-2\left(c^2+2i\,c-1\right)\tan(bx+a)-4\,c-2i}\right)}{i\,c-1}+\frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(bx+a)+2\,c+i\right)-\log\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(b\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log((2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x +
 a) + 2*I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c -
 1) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I
) - log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1
)*tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log
(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x +
 a) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c
 + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a)
+ 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*
dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arctan((I*c - 1)*
tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log((2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan
(b*x + a) + 2*I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))
/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.59 $\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} \, dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} \, dx = \int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} \, dx$$

Mathematica [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i \log\left(-\frac{(c+i)e^{(2i bx+2i a)}}{ce^{(2i bx+2i a)}-i}\right)}{2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((i c-1)\tan\left(bx+a\right)+c\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c)/x, x)
```

maple [**A**] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c + (ic - 1)\tan\left(bx + a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is c-1 zero or nonzero?
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c + \tan\left(a + b\,x\right)\,\left(-1 + c\,1\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1))/x,x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.60 $\int \tan^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$-\frac{\tan^{-1}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]],x]
[Out] -ArcTan[Cot[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x\tan^{-1}(\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

fricas [A] time = 0.48, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

giac [**A**] time = 0.13, size = 15, normalized size = 0.94

$$-\frac{1}{2}bx^2 + \frac{1}{2}\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

maple [**B**] time = 0.05, size = 51, normalized size = 3.19

$$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2} - \operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a)) - \frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^2}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x)
```

```
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

maxima [**A**] time = 0.32, size = 15, normalized size = 0.94

$$-\frac{1}{2}bx^2 + \frac{1}{2}\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

mupad [**B**] time = 0.00, size = 21, normalized size = 1.31

$$\frac{\Pi\,x}{2} - x\operatorname{acot}(\cot(a+b\,x)) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
```

```
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

sympy [**A**] time = 0.14, size = 24, normalized size = 1.50

$$\frac{\pi x}{2} - \begin{cases} \frac{\operatorname{acot}^2(\cot(a+bx))}{2b} & \text{for } b \neq 0 \\ x\operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

3.61 $\int x^2 \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=399

$$-\frac{\text{Li}_4\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}+\frac{\text{Li}_4\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}+\frac{ix\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(\frac{(ic-d+1)e^{2ia}}{ic+d+1}\right.}{4b}$$

```
[Out] 1/3*x^3*arctan(c+d*cot(b*x+a))+1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x^2*polylog(2,(c
+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/4*I*x*polylog(3,(1+I*c-d)*exp
(2*I*a+2*I*b*x)/(1+I*c+d))/b^2-1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*
b*x)/(c+I*(1-d)))/b^2-1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))
/b^3+1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

Rubi [A] time = 0.51, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5177, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}+\frac{\text{PolyLog}\left(4,\frac{(c+i\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 - Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) + P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 5177

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 + I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 - I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}(c+d\cot(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{3}(b(1+ic-d))\int \frac{e^{2ia+2ibx}x^3}{1+ic+d+(-1-ic+d)e^{2}} \\
&= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log \\
&= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log \\
&= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log \\
&= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log \\
&= \frac{1}{3}x^3\tan^{-1}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log
\end{aligned}
$$

Mathematica [A] time = 1.01, size = 359, normalized size = 0.90

$$
\frac{1}{3}x^3\tan^{-1}(d\cot(a+bx)+c)+\frac{4ib^3x^3\log\left(1-\frac{(c+i(d-1))e^{2i(a+bx)}}{c-i(d+1)}\right)-4ib^3x^3\log\left(1-\frac{(c+i(d+1))e^{2i(a+bx)}}{c-id+i}\right)+6b^2x^2\mathrm{Li}_2\left(\frac{(c+i(d-1}{c-i(}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 - ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (4*I)*b^3*x^3*Log[1 - ((c + I*(1
 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + 6*b^2*x^2*PolyLog[2, ((c + I*(
-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 6*b^2*x^2*PolyLog[2, ((c +
 I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + (6*I)*b*x*PolyLog[3, ((c
+ I*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (6*I)*b*x*PolyLog[3,
((c + I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - 3*PolyLog[4, ((c + I
*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + 3*PolyLog[4, ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)])/(24*b^3)
```

fricas [C] time = 0.78, size = 1585, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*cot(b*x + a) + c) + 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
 - 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 4*I*a^3
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) + (4*I*b^3*x^3 + 4*I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*
cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log((c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) + (-4*I*b^3*x^3 - 4*I*a^3)*log((c
^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I
*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + (4*I*b^3*x^3
 + 4*I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (
I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1
)) - 3*polylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*
c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((
c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin
(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*
a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))
/b^3
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left(d \cot\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*cot(b*x + a) + c), x)
```

maple **[C]** time = 73.71, size = 7906, normalized size = 19.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\,x^3 \arctan\left(-c\cos\left(2\,bx + 2\,a\right) + (d+1)\sin\left(2\,bx + 2\,a\right) + c, (d+1)\cos\left(2\,bx + 2\,a\right) + c\sin\left(2\,bx + 2\,a\right) + d - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*x^3*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/6*x^3*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2
*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*
x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x +
2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*co
s(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x +
2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(
c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^
4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x +
 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*
(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(
c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x +
 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(c + d\cot\left(a + b\,x\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*cot(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*cot(a + b*x)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

3.62 $\int x \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=303

$$\frac{i\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-\frac{(ic-d}{ic}\right.$$

```
[Out] 1/2*x^2*arctan(c+d*cot(b*x+a))+1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x*polylog(2,(c+I*(
1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/8*I*polylog(3,(1+I*c-d)*exp(2*I*a
+2*I*b*x)/(1+I*c+d))/b^2-1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+
I*(1-d)))/b^2
```

Rubi [A] time = 0.40, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5177, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+i(d+1}{c+i}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5177

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] + (Dist[(b*(1 + I*c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2
*I*a + 2*I*b*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x]
- Dist[(b*(1 - I*c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I
*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}(b(1+ic-d))\int \frac{e^{2ia+2ibx}x^2}{1+ic+d+(-1-ic+d)$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

Mathematica [A] time = 0.63, size = 270, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(d\cot(a+bx)+c)+\frac{i\left(2b^2x^2\log\left(1-\frac{(c+i(d-1))e^{2i(a+bx)}}{c-i(d+1)}\right)-2b^2x^2\log\left(1-\frac{(c+i(d+1))e^{2i(a+bx)}}{c-id+i}\right)-2ibx\mathrm{Li}_2\left(\frac{(c+i(\ldots}{c\ldots}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - ((c + I*(-1
+ d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 2*b^2*x^2*Log[1 - ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - (2*I)*b*x*PolyLog[2, ((c + I*(-
1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (2*I)*b*x*PolyLog[2, ((c +
I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + PolyLog[3, ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - PolyLog[3, ((c + I*(1 + d))*E^((
2*I)*(a + b*x)))/(I + c - I*d)]))/b^2
```

fricas [C] time = 0.91, size = 1285, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*cot(b*x + a) + c) + 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log(-1/2*c
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + (2*I*b^2*x^2 - 2*I*a^2)*log((c
^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I
*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) + (-2*I*b^2*x^
2 + 2*I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) +
(I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d +
1)) + (-2*I*b^2*x^2 + 2*I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/
(c^2 + d^2 - 2*d + 1)) + (2*I*b^2*x^2 - 2*I*a^2)*log((c^2 + d^2 - (c^2 - 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^
2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^
2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2
*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1))
- I*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*
d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^
2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(
2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan \left(d \cot \left(bx+a\right)+c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*cot(b*x + a) + c), x)
```

maple [C] time = 6.51, size = 7538, normalized size = 24.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,x^2 \arctan\left(-c\cos\left(2\,bx+2\,a\right)+\left(d+1\right)\sin\left(2\,bx+2\,a\right)+c,\left(d+1\right)\cos\left(2\,bx+2\,a\right)+c\sin\left(2\,bx+2\,a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^2*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/4*x^2*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a)
 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atan}(c + d \cot(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*cot(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*cot(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

3.63 $\int \tan^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=198

$$\frac{\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

```
[Out] x*arctan(c+d*cot(b*x+a))+1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d
))-1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*polylog(2,(
1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*polylog(2,(c+I*(1+d))*exp(2*I*
a+2*I*b*x)/(c+I*(1-d)))/b
```

Rubi [A] time = 0.25, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5169, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] - (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] + PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) - PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5169

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] + (Dist[b*(1 + I*c - d), Int[(x*E^(2*I*a + 2*I*b
*x))/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x)), x], x] - Dist[b*(1
- I*c + d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - I*c - d - (1 - I*c + d)*E^(2*I
*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\cot(a+bx))\,dx = x\tan^{-1}(c+d\cot(a+bx)) + (b(1+ic-d))\int \frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)e^{2ia+2i}}$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

Mathematica [B] time = 22.19, size = 1648, normalized size = 8.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

fricas [B] time = 0.71, size = 961, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*cot(b*x + a) + c) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) + (2*I*b*x + 2*I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 +
d^2 + 2*d + 1)) + (-2*I*b*x - 2*I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d
+ 1)/(c^2 + d^2 + 2*d + 1)) + (-2*I*b*x - 2*I*a)*log((c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + (2*I*b*x + 2*I*a)*log((c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + dilog(-(c^2 + d^2 - (
c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*si
n(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 -
(c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*si
n(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 -
(c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*s
in(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - dilog(-(c^2 + d^2 -
 (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*s
in(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c), x)
```

maple [B] time = 1.40, size = 1159, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b*arctan(c+d*cot(b*x+a))*Pi+1/b*arctan(c+d*cot(b*x+a))*arccot(cot(b*x+
a))+1/b*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan((c+d*cot(b*x+a))/d-c/d)
-1/2*I*d/b*ln(1-(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d
*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)
/(1+I*c+d)-1/2*I/b*ln(1-(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/(
(d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d
-c/d)+c)/(1+I*c+d)-1/2*I/b/(-I*d-I+c)*ln(1-(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x
+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*
((c+d*cot(b*x+a))/d-c/d)+c)*c-1/2*d/b*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^
```

```
2/(1+I*c+d)-1/4*d/b*polylog(2,(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c
))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)-1/2/b*arctan
(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)-1/2/b/(-I*d-I+c)*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)^2*c-1/4/b*polylog(2,(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x+
a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)-
1/4/b/(-I*d-I+c)*polylog(2,(-I*d+I+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^
2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c+1/2*I/b*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)*ln(1-(I+I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2
/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))+1/2/b*arctan(d*((c+d*cot(
b*x+a))/d-c/d)+c)^2+1/4/b*polylog(2,(I+I*d+c)*(1+I*(d*((c+d*cot(b*x+a))/d-c
/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))
```

maxima [B] time = 0.55, size = 532, normalized size = 2.69

$$-\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d}-\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+\left(c^2+d+1\right)\tan(bx+a)}{c^2+d^2+2\,d+1},-\frac{cd\,t\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
 a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*ta
n(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d
+ 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan((
c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a)
)/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d
 + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) -
 log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2
+ 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) -
 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)
*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*
d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))
/d) - 8*(b*x + a)*arctan(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c
^2 + 1)*tan(b*x + a))/d))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+d\cot\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*cot(a + b*x)),x)
```

```
[Out] int(atan(c + d*cot(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(d\cot(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*cot(b*x+a))/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 5.13, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Cot[a + b*x]]/x, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(d\cot\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*cot(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c)/x, x)
```

maple [A] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*cot(b*x+a))/x,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}\left(c+d\,\mathrm{cot}\left(a+b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*cot(a + b*x))/x,x)
```

```
[Out] int(atan(c + d*cot(a + b*x))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.65 $\int x^2 \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=154

$$-\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctan(-c-(1-I*c)*cot(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] time = 0.26, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5173, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}+\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{-i(1-ic)+c-ce^{2ia+2i}$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}}{-i(1-ic)+c-}$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)$$

$$= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)$$

Mathematica [A] time = 0.38, size = 136, normalized size = 0.88

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}+\frac{6ix\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2}-\frac{6x^2\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{c}\right)}{b}+4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+8x^3\tan^{-1}(c+($$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

fricas [C] time = 0.42, size = 165, normalized size = 1.07

$$\frac{2\,b^4x^4 + 4i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) + 6i\,bx\mathrm{polylog}\left(3, i\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 4*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*I*a
^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3, I*c*e^(2*I*b*x +
 2*I*a)) + (4*I*b^3*x^3 + 4*I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*po
lylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

maple [C] time = 6.56, size = 1532, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*
Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn(I*exp(I*(b*x+a)))^
2*csgn(I*exp(2*I*(b*x+a)))-1/6*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a)))^2-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^
2+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-1/12*x^3*
Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))
-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/2*I/b^2*ln(1-I*exp(2*I*(b*x
+a))*c)*x*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2*I/b^2
*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a)
)*c)-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I/(exp(2*I*(b*x+a))-1)
)*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+1/12*b*x^4-1/6*I*x^3*
ln(exp(2*I*(b*x+a))*c+I)+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))-1))^3+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1
))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(
exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+1/6*Pi*x^3+1/12*x^3*Pi*csgn(I
*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+1/1
2*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I
*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*(exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))-1))-1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3+1/2*I/b^3*a^3*ln(1-I*exp
(I*(b*x+a))*(-I*c)^(1/2))-1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*
a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+
a)))/b^2-1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c
)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2
-1/4/b^3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2+1/2/b^3*a^2*dilog(1-I*exp(I*(b
*x+a))*(-I*c)^(1/2))+1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/1
```

```
2*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+1/3*I*x^3*ln(e
xp(I*(b*x+a)))+1/6*I*x^3*ln(I+c)-1/12*x^3*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(ex
p(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/
(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(
exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/8*polyl
og(4,I*c*exp(2*I*(b*x+a)))/b^3+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))+1/4*x
^2*polylog(2,I*c*exp(2*I*(b*x+a)))/b
```

maxima [B] time = 0.37, size = 309, normalized size = 2.01

$$\frac{\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b^2}+\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c + 1)*co
t(b*x + a) + c)/b^2 + 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 1
8*I*(b*x + a)*a - 9*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
- 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2
*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^
(2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b^
2*(12*c + 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.66 $\int x \tan^{-1}(c + (1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=123

$$\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{bx^3}{6}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctan(-c-(1-I*c)*cot(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.22, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5173, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{b}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
```

```
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{-i(1-ic)+c-ce^{2ia+2ib}}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{-i(1-ic)+c-c}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) -$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) +$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) +$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) +$$

Mathematica [A] time = 0.23, size = 110, normalized size = 0.89

$$\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

fricas [C] time = 0.43, size = 143, normalized size = 1.16

$$\frac{4\,b^3x^3+6i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+4\,a^3+6\,bx\text{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+\left(6i\,b^2x^2-6i\,a^2\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 6*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^2*l
og((c*e^(2*I*b*x + 2*I*a) + I)/c) + (6*I*b^2*x^2 - 6*I*a^2)*log(-I*c*e^(2*I
*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

maple [C] time = 5.46, size = 1497, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/4/b^2*polylog(2,I*c*exp(2*I*(b*x+a)))*a+1/2*I*x^2*ln(exp(I*(b*x+a)))+1/4*
I*x^2*ln(I+c)+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1)
)*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*exp
(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(
2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))
-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(
I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-1
/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/
(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I/(e
xp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+1/8
*x^2*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b
*x+a))-1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a
))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1/2/b^2*a*dilog(
1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^
(1/2))+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+1/8*x
^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I
*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+1/4*Pi*x^2+1/8*x^2*Pi*csgn(I*exp(I*(b*
x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*
exp(2*I*(b*x+a)))^2-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))^2+1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/4*I*x^2*ln(exp(2*
I*(b*x+a))*c+I)+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/4*I/b^2*a^2*ln(exp(
2*I*(b*x+a))*c+I)-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b
^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/8*I*polylog(3,I*c*exp(2*I*(b*x
+a)))/b^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*cs
gn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn(I*(exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))-1))+1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2+1/2*I/b*ln(1-I*exp(2*I*
(b*x+a))*c)*x*a-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/b*a*l
n(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/4*I*x^2*ln(1-I*exp(2*I*(b*x+a))*c)-1
/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/6*b*x^3+1/8
*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/4*x*polylog
(2,I*c*exp(2*I*(b*x+a)))/b
```

maxima [B] time = 0.34, size = 218, normalized size = 1.77

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b}+\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(c\cos(2}{}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c + 1)*cot(b*x + a) + c)/b +
2*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
```

```
*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
 + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(
3, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(12*c + 12*I)))/b
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(c - \cot(a + b x)\ (-1 + c\,1\mathrm{i}))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.67 $\int \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=85

$$\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2-x*arctan(-c-(1-I*c)*cot(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5165, 2184, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5165

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c+(1-ic)\cot(a+bx)) - (ib)\int \frac{x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + (bc)\int \frac{e^{2ia+2ibx}x}{-i(1-ic)+c-ce^{2ia+2i}}$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{1}{2}i\int$$

Sub

$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) -$$
$$= \frac{bx^2}{2} + x\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{\mathrm{Li}_2}{}$$

Mathematica [B] time = 20.27, size = 929, normalized size = 10.93

$$x\tan^{-1}(c+(1-ic)\cot(a+bx)) - \frac{}{(\cot(a+bx)+i)(ic+(c+i)\cot(a+bx)+1)\left(i\log(i\tan(bx)+1)\tan(bx)\cos^2(a\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I +
c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] -
(I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*Sin[b*x]
)*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*Cot[a + b
*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(C
os[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a
] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I + c)*Cos[a +
 b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x
]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]) + 2*b*x*
Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a
+ b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-1 - I*c)
*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - I*
Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + I*
Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[b*x]*((1 - I*c)*Cos[a +
 b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) - (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x])))
```

fricas [A] time = 0.63, size = 111, normalized size = 1.31

$$\frac{2\,b^2x^2 + 2i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) - 2\,a^2 + (2i\,bx+2i\,a)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) + \mathrm{Li}_2\left(\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 2*I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2
*I*a) + I)) - 2*a^2 + (2*I*b*x + 2*I*a)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) -
 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x + 2*I*a)))
/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

maple [B] time = 0.69, size = 1495, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] -1/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(I+c+cot(b*x+a)*(-1
+I*c))+1/8*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)^2-1/4*I/b/(-1+I*c
)/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))+1/4*I/b/(-1+I*c)/(I+c)*
dilog(-1/2*(I+cot(b*x+a)*(-1+I*c)-c)/c)-1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*I
*(I+cot(b*x+a)*(-1+I*c)-c))-1/2/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(I+cot(b*x+a)
*(-1+I*c)-c))*c+1/4/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)^2*c-1/2/b/
(-1+I*c)/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*c+1/2/b/(-1+I*c)
/(I+c)*dilog(-1/2*(I+cot(b*x+a)*(-1+I*c)-c)/c)*c+1/b/(-1+I*c)*arctan(cot(b*
x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)-c-I)+1/b/(-1+I*c)*arctan(
cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(I+c+cot(b*x+a)*(-1+I*c))*c^2-1/2/b/(-1+
I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)*ln(-1/2*I*(I+cot(b*x+a)*(-1+I*c)-c))
*c-1/2/b/(-1+I*c)/(I+c)*ln(I+c+cot(b*x+a)*(-1+I*c))*ln((cot(b*x+a)*(-1+I*c)
-c-I)/(-2*I-2*c))*c+2*I/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*
ln(I+c+cot(b*x+a)*(-1+I*c))*c+1/4*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)
-c-I)*ln(-1/2*I*(I+cot(b*x+a)*(-1+I*c)-c))*c^2+1/4*I/b/(-1+I*c)/(I+c)*ln(I+
c+cot(b*x+a)*(-1+I*c))*ln((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*c^2-1/4*I/b
/(-1+I*c)/(I+c)*ln(I+c+cot(b*x+a)*(-1+I*c))*ln(-1/2*(I+cot(b*x+a)*(-1+I*c)-
c)/c)*c^2-2*I/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x
+a)*(-1+I*c)-c-I)*c+1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(I+cot(b*x+a)*(-1+I
*c)-c))*c^2-1/8*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I)^2*c^2+1/4*I/
b/(-1+I*c)/(I+c)*dilog((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))*c^2+1/2/b/(-1+
I*c)/(I+c)*ln(I+c+cot(b*x+a)*(-1+I*c))*ln(-1/2*(I+cot(b*x+a)*(-1+I*c)-c)/c)
*c-1/b/(-1+I*c)*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I
*c)-c-I)*c^2+1/4*I/b/(-1+I*c)/(I+c)*ln(I+c+cot(b*x+a)*(-1+I*c))*ln(-1/2*(I+
cot(b*x+a)*(-1+I*c)-c)/c)-1/4*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)-c-I
)*ln(-1/2*I*(I+cot(b*x+a)*(-1+I*c)-c))-1/4*I/b/(-1+I*c)/(I+c)*ln(I+c+cot(b*
x+a)*(-1+I*c))*ln((cot(b*x+a)*(-1+I*c)-c-I)/(-2*I-2*c))-1/4*I/b/(-1+I*c)/(I
+c)*dilog(-1/2*(I+cot(b*x+a)*(-1+I*c)-c)/c)*c^2
```

maxima [B] time = 0.44, size = 456, normalized size = 5.36

$$-\frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(\frac{-2i\,c^2+2\left(c^2+1\right)\tan(bx+a)+4\,c+2i}{-2i\,c^2+2\left(c^2+1\right)\tan(bx+a)-2i}\right)}{i\,c-1} - \frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+\left(c^2+1\right)\tan(bx+a)+2\,c+i\right)-\log\left(-i\,c^2+\left(c^2+1\right)\tan(bx+a)-i\right)\right)-2i\log\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) + 4
*c + 2*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I))/(I*c - 1) - I*(4*(b*
x + a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I) - log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I
)*log(-1/2*((I*c + 1)*tan(b*x + a) + c + I)/c + 1) + 2*I*log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) + 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(1/2*(c - I)*ta
n(b*x + a) - 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)
^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*l
og(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) -
 2*I*dilog(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c
 + 1)*tan(b*x + a) + c + I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c
- 1)) - 8*(b*x + a)*arctan(c + (-I*c + 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*
log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) + 4*c + 2*I)/(-2*I*c^2 + 2*(c^2 +
1)*tan(b*x + a) - 2*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.68 $\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) + I)
)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c)/x, x)
```

maple [A] time = 1.80, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-c-(-ic+1)\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is c-1 zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1))/x,x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.69 $\int x^2 \tan^{-1}(c + (-1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=155

$$\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+$$

```
[Out] -1/12*b*x^4-1/3*x^3*arctan(-c+(1+I*c)*cot(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
I*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(
3,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5173, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*
b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + P
olyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{-i(-1-ic)+c-ce^{2ia}\cdots}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia+\cdots}}{-i(-1-ic)+\cdots}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2i\cdots}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2i\cdots}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2i\cdots}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2i\cdots}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2i\cdots}\right.$$

Mathematica [A] time = 0.38, size = 140, normalized size = 0.90

$$\frac{1}{3}x^3\tan^{-1}(c+(-1-ic)\cot(a+bx)) - \frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{c}\right) + 3\text{Li}_4\left(\cdots\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6
*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*
(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.77, size = 166, normalized size = 1.07

$$-\frac{2\,b^4x^4-4i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+6i\,bxp}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 4*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*
I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -I*c*e^(2*I*b
*x + 2*I*a)) - (-4*I*b^3*x^3 - 4*I*a^3)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) -
3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

maple [C] time = 6.62, size = 1533, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] -1/12*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/6*x^3*Pi*c
sgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*polylog(4,-I*exp(2*I*(
b*x+a))*c)/b^3-1/6*I*x^3*ln(1+I*c*exp(2*I*(b*x+a)))-1/12*x^3*Pi*csgn(I*exp(
2*I*(b*x+a)))^3-1/12*b*x^4+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))
-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2
*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csg
n(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2
-1/12*x^3*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2
*I*(b*x+a))-1))-1/3*I*x^3*ln(exp(I*(b*x+a)))-1/6*I*x^3*ln(c-I)-1/12*x^3*Pi*
csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/2*I/b^2*a^2*ln(1-I*
exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/
2))+1/3*I/b^3*ln(1+I*c*exp(2*I*(b*x+a)))*a^3+1/6*I/b^3*a^3*ln(-exp(2*I*(b*x
+a))*c+I)-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*x*polylog(
3,-I*exp(2*I*(b*x+a))*c)/b^2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))-1))^3+1/6*Pi*x^3-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))-1))^2+1/2*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^2*a^
2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/4*x^2*polylog(2,-I*exp(2*I*(b*x+a)
)*c)/b+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+
a))-1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/12*x^3
*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)
^(1/2))-1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/4/b^3*polylog(2
,-I*exp(2*I*(b*x+a))*c)*a^2+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*
I*(b*x+a))-1))^3-1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*
```

```
Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a)
)-1))^3-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))
-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/12*x^3*Pi*csgn(I
*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)-1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csg
n((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*(c-I))*
csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2
```

maxima [B] time = 0.37, size = 310, normalized size = 2.00

$$\frac{\frac{\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b^2}-\frac{3\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c - 1)*co
t(b*x + a) + c)/b^2 - 3*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 1
8*I*(b*x + a)*a - 9*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(
2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*
e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/
(b^2*(12*c - 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.70 $\int x \tan^{-1}(c + (-1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=124

$$-\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{bx^3}{6}$$

```
[Out] -1/6*b*x^3-1/2*x^2*arctan(-c+(1+I*c)*cot(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I*
c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5173, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x
)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5173

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Cot[a + b*x]])/(f*(m +
```

```
 1)), x] - Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{-i(-1-ic)+c-ce^{2ia+}}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia+2i}}{-i(-1-ic)+}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ib}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ib}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ib}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ib}\right.$$

Mathematica [A] time = 0.22, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\tan^{-1}(c+(-1-ic)\cot(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{c}\right)+\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

fricas [C] time = 0.68, size = 144, normalized size = 1.16

$$-\frac{4b^3x^3-6ib^2x^2\log\left(-\frac{\left(ce^{(2ibx+2ia)}-i\right)e^{(-2ibx-2ia)}}{c-i}\right)+4a^3+6bx\text{Li}_2\left(-ice^{(2ibx+2ia)}\right)+6ia^2\log\left(\frac{ce^{(2ibx+2ia)}-i}{c}\right)-\left(-6i\right.}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 6*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^
2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) - (-6*I*b^2*x^2 + 6*I*a^2)*log(I*c*e^(
2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-x\arctan\left(-(-ic-1)\cot(bx+a)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

maple [C] time = 5.57, size = 1498, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*I/b*a*ln(1-I*exp(I*(b*x+
a))*(I*c)^(1/2))*x-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a
))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn(I*
exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)
)*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I/(
exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+
1/8*x^2*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*cs
gn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*
csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1)
)^2-1/8*I*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(
b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))-1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*cs
gn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/4*Pi*x^2-1/2*I*x^2*ln(e
xp(I*(b*x+a)))-1/4*I*x^2*ln(c-I)+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-
1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I
*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(
exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b
*x+a))-1))^3-1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csg
n(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn((exp(2*I*(
b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*cs
gn(I*exp(2*I*(b*x+a)))+1/4*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*
x+a)))^2+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c-I)-1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x
+a)))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x
+a))*c-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(
2*I*(b*x+a))-1))-1/2*I/b*ln(1+I*c*exp(2*I*(b*x+a)))*x*a+1/2*I/b^2*a^2*ln(1+
I*exp(I*(b*x+a))*(I*c)^(1/2))-1/6*b*x^3-1/4*x*polylog(2,-I*exp(2*I*(b*x+a))
*c)/b+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2/b^2*a*dilog(1-I*e
xp(I*(b*x+a))*(I*c)^(1/2))-1/4/b^2*polylog(2,-I*exp(2*I*(b*x+a))*c)*a-1/8*x
^2*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x
+a))-1))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))
-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn((e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I/b^2*ln(1+I*c*exp(2*I*(b*
x+a)))*a^2-1/4*I/b^2*a^2*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^2*a^2*ln(1-I*exp
(I*(b*x+a))*(I*c)^(1/2))
```

maxima [B] time = 0.35, size = 219, normalized size = 1.77

$$\frac{\frac{\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b}-\frac{2\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(c\cos(2\,\cdots}{\cdots}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c - 1)*cot(b*x + a) + c)/b -
2*(-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
```

```
2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
-c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*
x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog
(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(12*c - 12*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c - \cot\left(a + b\,x\right)\,\left(1 + c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(x*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(-1-I*c)*cot(b*x+a)),x)
[Out] Exception raised: CoercionFailed
```

3.71 $\int \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=86

$$-\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{bx^2}{2}$$

```
[Out] -1/2*b*x^2-x*arctan(-c+(1+I*c)*cot(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b
*x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5165, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
(2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5165

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c-(1+ic)\cot(a+bx))-(ib)\int \frac{x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-(bc)\int \frac{e^{2ia+2ibx}x}{-i(-1-ic)+c-ce^{2}}$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-$$

Mathematica [**B**] time = 15.67, size = 872, normalized size = 10.14

$$x\tan^{-1}(c+(-ic-1)\cot(a+bx))+\frac{ix\csc(a+bx)\left(2bx\log(2\cos(bx\right.}{(\cot(a+bx)+i)((c-i)\cos(a+bx)+i(c+i)\sin(a+bx))\left(\frac{\log\left(\frac{1}{2}\sec(bx)(\cos(a)+i\sin\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos[
b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I +
c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a +
b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] +
I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] -
I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a]
 + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x]
 + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[
a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a
])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]]*((I + c)*
Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*
Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[
a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + (Log[(Sec[b*x
]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*S
ec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (Sec[b*x]*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x
] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]]*Tan[b*x] + (I*Lo
g[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*
((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + T
an[b*x])))
```

fricas [**A**] time = 0.55, size = 112, normalized size = 1.30

$$-\frac{2\,b^2x^2-2i\,bx\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)-2\,a^2-(-2i\,bx-2i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}-}{c}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2 - 2*I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*
I*a)/(c - I)) - 2*a^2 - (-2*I*b*x - 2*I*a)*log(I*c*e^(2*I*b*x + 2*I*a) + 1)
 - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x + 2*I*a
)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

maple [B] time = 0.70, size = 1753, normalized size = 20.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] 1/4*I/b/(1+I*c)/(I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln(1/2*(-(1+I*c)*cot(b*x+
a)+c+I)/c)-1/4*I/b/(1+I*c)/(I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln((-(1+I*c)*c
ot(b*x+a)+c-I)/(-2*I+2*c))+1/4*I/b/(1+I*c)/(I-c)*ln(-1/2*I*(I+(1+I*c)*cot(b
*x+a)-c))*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))-1/4*I/b/(1+I*c)/(I-c)*ln(I+(
1+I*c)*cot(b*x+a)-c)*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))-1/4*I/b/(1+I*c)/(
I-c)*dilog(-1/2*I*(I+(1+I*c)*cot(b*x+a)-c))*c^2-1/8*I/b/(1+I*c)/(I-c)*ln(I+
(1+I*c)*cot(b*x+a)-c)^2*c^2-1/4*I/b/(1+I*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b
*x+a)+c+I)/c)*c^2+1/4*I/b/(1+I*c)/(I-c)*dilog((-(1+I*c)*cot(b*x+a)+c-I)/(-2
*I+2*c))*c^2+1/2/b/(1+I*c)/(I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln((-(1+I*c)*c
ot(b*x+a)+c-I)/(-2*I+2*c))*c+1/2/b/(1+I*c)/(I-c)*ln(I+(1+I*c)*cot(b*x+a)-c)
*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*c-1/2/b/(1+I*c)/(I-c)*ln(-1/2*I*(I+(1
+I*c)*cot(b*x+a)-c))*ln(-1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*c-1/2/b/(1+I*c)/(
I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*c+1/b/
(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x+a)-c)*
c^2-1/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x
+a)-c+I)*c^2-1/4*I/b/(1+I*c)/(I-c)*ln(-1/2*I*(I+(1+I*c)*cot(b*x+a)-c))*ln(-
1/2*I*(-(1+I*c)*cot(b*x+a)+c+I))*c^2-1/4*I/b/(1+I*c)/(I-c)*ln(-(1+I*c)*cot(
b*x+a)-c+I)*ln(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*c^2+1/4*I/b/(1+I*c)/(I-c)*l
n(-(1+I*c)*cot(b*x+a)-c+I)*ln((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c))*c^2-2*I
/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x+a)-
c)*c+2*I/b/(1+I*c)*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(
b*x+a)-c+I)*c+1/4*I/b/(1+I*c)/(I-c)*ln(I+(1+I*c)*cot(b*x+a)-c)*ln(-1/2*I*(-
(1+I*c)*cot(b*x+a)+c+I))*c^2-1/2/b/(1+I*c)/(I-c)*dilog(-1/2*I*(I+(1+I*c)*co
t(b*x+a)-c))*c-1/4/b/(1+I*c)/(I-c)*ln(I+(1+I*c)*cot(b*x+a)-c)^2*c-1/2/b/(1+
I*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)*c+1/2/b/(1+I*c)/(I-c)*dil
og((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c))*c-1/b/(1+I*c)*arctan(-c+(1+I*c)*co
t(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x+a)-c)+1/b/(1+I*c)*arctan(-c+(1+I*c
)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)+1/4*I/b/(1+I*c)/(I-c)*d
ilog(-1/2*I*(I+(1+I*c)*cot(b*x+a)-c))+1/8*I/b/(1+I*c)/(I-c)*ln(I+(1+I*c)*co
t(b*x+a)-c)^2+1/4*I/b/(1+I*c)/(I-c)*dilog(1/2*(-(1+I*c)*cot(b*x+a)+c+I)/c)-
1/4*I/b/(1+I*c)/(I-c)*dilog((-(1+I*c)*cot(b*x+a)+c-I)/(-2*I+2*c))
```

maxima [B] time = 0.47, size = 456, normalized size = 5.30

$$-\frac{(i\,c+1)\left(\frac{4i\,(bx+a)\log\left(\frac{-2i\,c^2+2\,(c^2+1)\tan(bx+a)-2i}{-2i\,c^2+2\,(c^2+1)\tan(bx+a)-4\,c+2i}\right)}{i\,c+1}+\frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+(c^2+1)\tan(bx+a)-2\,c+i\right)-\log\left(-i\,c^2+(c^2+1)\tan(bx+a)-i\right)\right)-2i\,\log\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2
*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 4*c + 2*I))/(I*c + 1) + I*(4*(b*
x + a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I) - log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I
)*log(-1/2*((I*c - 1)*tan(b*x + a) + c - I)/c + 1) + 2*I*log(-I*c^2 + (c^2
+ 1)*tan(b*x + a) - 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(-1/2*(c + I)*t
an(b*x + a) + 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I
)^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*
log(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1)
- 2*I*dilog(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c
 - 1)*tan(b*x + a) + c - I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c
+ 1)) - 8*(b*x + a)*arctan(c + (-I*c - 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*
log((-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I)/(-2*I*c^2 + 2*(c^2 + 1)*tan
(b*x + a) - 4*c + 2*I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(c - \cot\left(a + b\,x\right)\,\left(1 + c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.72 $\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]

fricas [A] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\,\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c)/x, x)
```

maple [**A**] time = 2.06, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-c-(-ic-1)\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is c-1 zero or nonzero?
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1))/x,x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1))/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.73 $\int \tan^{-1}(\sinh(x))\,dx$

Optimal. Leaf size=39

$$i\text{Li}_2\left(-ie^x\right)-i\text{Li}_2\left(ie^x\right)-2x\tan^{-1}\left(e^x\right)+x\tan^{-1}(\sinh(x))$$

```
[Out] -2*x*arctan(exp(x))+x*arctan(sinh(x))+I*polylog(2,-I*exp(x))-I*polylog(2,I*
exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {5203, 4180, 2279, 2391}

$$i\text{PolyLog}\left(2,-ie^x\right)-i\text{PolyLog}\left(2,ie^x\right)-2x\tan^{-1}\left(e^x\right)+x\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sinh[x]],x]
```

```
[Out] -2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2
, I*E^x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}(\sinh(x))\,dx &= x\tan^{-1}(\sinh(x))-\int x\text{sech}(x)\,dx\\
&= -2x\tan^{-1}\left(e^x\right)+x\tan^{-1}(\sinh(x))+i\int \log\left(1-ie^x\right)\,dx-i\int \log\left(1+ie^x\right)\,dx\\
&= -2x\tan^{-1}\left(e^x\right)+x\tan^{-1}(\sinh(x))+i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^x\right)-i\,\text{Subst}\left(\int \frac{\log(1}{x}\right.\\
&= -2x\tan^{-1}\left(e^x\right)+x\tan^{-1}(\sinh(x))+i\text{Li}_2\left(-ie^x\right)-i\text{Li}_2\left(ie^x\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.64

$$x\tan^{-1}(\sinh(x)) + i\left(\text{Li}_2\left(-ie^{-x}\right) - \text{Li}_2\left(ie^{-x}\right) + x\left(\log\left(1 - ie^{-x}\right) - \log\left(1 + ie^{-x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sinh[x]],x]
```

```
[Out] x*ArcTan[Sinh[x]] + I*(x*(Log[1 - I/E^x] - Log[1 + I/E^x]) + PolyLog[2, (-I
)/E^x] - PolyLog[2, I/E^x])
```

fricas [B] time = 0.60, size = 58, normalized size = 1.49

$$x\arctan(\sinh(x)) + i\,x\log(i\cosh(x) + i\sinh(x) + 1) - i\,x\log(-i\cosh(x) - i\sinh(x) + 1) - i\,\text{Li}_2(i\cosh(x) + i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(sinh(x)) + I*x*log(I*cosh(x) + I*sinh(x) + 1) - I*x*log(-I*cosh(x)
 - I*sinh(x) + 1) - I*dilog(I*cosh(x) + I*sinh(x)) + I*dilog(-I*cosh(x) - I
*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(sinh(x)), x)
```

maple [A] time = 0.38, size = 52, normalized size = 1.33

$$x\arctan(\sinh(x)) - ix\left(\ln\left(1 - ie^{x}\right) - \ln\left(1 + ie^{x}\right)\right) + i\,\text{dilog}\left(1 + ie^{x}\right) - i\,\text{dilog}\left(1 - ie^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(sinh(x)),x)
```

```
[Out] x*arctan(sinh(x))-I*x*(ln(1-I*exp(x))-ln(1+I*exp(x)))+I*dilog(1+I*exp(x))-I
*dilog(1-I*exp(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - 2\int \frac{xe^{x}}{e^{(2x)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{atan}(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(sinh(x)),x)
```

```
[Out] int(atan(sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(sinh(x)),x)
```

```
[Out] Integral(atan(sinh(x)), x)
```

3.74 $\int x \tan^{-1}(\sinh(x)) \, dx$

Optimal. Leaf size=74

$$ix\text{Li}_2\left(-ie^x\right) - ix\text{Li}_2\left(ie^x\right) - i\text{Li}_3\left(-ie^x\right) + i\text{Li}_3\left(ie^x\right) + x^2\left(-\tan^{-1}\left(e^x\right)\right) + \frac{1}{2}x^2\tan^{-1}(\sinh(x))$$

```
[Out] -x^2*arctan(exp(x))+1/2*x^2*arctan(sinh(x))+I*x*polylog(2,-I*exp(x))-I*x*po
lylog(2,I*exp(x))-I*polylog(3,-I*exp(x))+I*polylog(3,I*exp(x))
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5205, 4180, 2531, 2282, 6589}

$$ix\text{PolyLog}\left(2,-ie^x\right)-ix\text{PolyLog}\left(2,ie^x\right)-i\text{PolyLog}\left(3,-ie^x\right)+i\text{PolyLog}\left(3,ie^x\right)+x^2\left(-\tan^{-1}\left(e^x\right)\right)+\frac{1}{2}x^2\tan^{-1}(\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sinh[x]],x]
```

```
[Out] -(x^2*ArcTan[E^x]) + (x^2*ArcTan[Sinh[x]])/2 + I*x*PolyLog[2, (-I)*E^x] - I
*x*PolyLog[2, I*E^x] - I*PolyLog[3, (-I)*E^x] + I*PolyLog[3, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(\sinh(x))\,dx = \frac{1}{2}x^2\tan^{-1}(\sinh(x)) - \frac{1}{2}\int x^2\text{sech}(x)\,dx$$
$$= -x^2\tan^{-1}(e^x) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + i\int x\log(1-ie^x)\,dx - i\int x\log(1+ie^x)\,dx$$
$$= -x^2\tan^{-1}(e^x) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2(-ie^x) - ix\text{Li}_2(ie^x) - i\int \text{Li}_2(-ie^x)\,dx + i\int$$
$$= -x^2\tan^{-1}(e^x) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2(-ie^x) - ix\text{Li}_2(ie^x) - i\,\text{Subst}\left(\int \frac{\text{Li}_2(-ix)}{x}\,d\right.$$
$$= -x^2\tan^{-1}(e^x) + \frac{1}{2}x^2\tan^{-1}(\sinh(x)) + ix\text{Li}_2(-ie^x) - ix\text{Li}_2(ie^x) - i\text{Li}_3(-ie^x) + i\text{Li}_3(ie^x)$$

Mathematica [A] time = 0.03, size = 105, normalized size = 1.42

$$\frac{1}{2}x^2\tan^{-1}(\sinh(x)) - \frac{1}{2}i\left(-2x\left(\text{Li}_2(-ie^{-x}) - \text{Li}_2(ie^{-x})\right) - 2\left(\text{Li}_3(-ie^{-x}) - \text{Li}_3(ie^{-x})\right) - \left(x^2\left(\log(1-ie^{-x}) - \log(1+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sinh[x]],x]
```

```
[Out] (x^2*ArcTan[Sinh[x]])/2 - (I/2)*(-(x^2*(Log[1 - I/E^x] - Log[1 + I/E^x])) -
 2*x*(PolyLog[2, (-I)/E^x] - PolyLog[2, I/E^x]) - 2*(PolyLog[3, (-I)/E^x] -
 PolyLog[3, I/E^x]))
```

fricas [C] time = 0.45, size = 93, normalized size = 1.26

$$\frac{1}{2}x^2\arctan(\sinh(x)) + \frac{1}{2}ix^2\log(i\cosh(x) + i\sinh(x) + 1) - \frac{1}{2}ix^2\log(-i\cosh(x) - i\sinh(x) + 1) - ix\text{Li}_2(i\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(sinh(x)) + 1/2*I*x^2*log(I*cosh(x) + I*sinh(x) + 1) - 1/2*I*
x^2*log(-I*cosh(x) - I*sinh(x) + 1) - I*x*dilog(I*cosh(x) + I*sinh(x)) + I*
x*dilog(-I*cosh(x) - I*sinh(x)) + I*polylog(3, I*cosh(x) + I*sinh(x)) - I*p
olylog(3, -I*cosh(x) - I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\arctan(\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(sinh(x)), x)
```

maple [C] time = 1.34, size = 732, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(sinh(x)),x)
```

```
[Out] I*x*polylog(2,-I*exp(x))-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)^3+1/8*Pi*x
^2*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)+1/8*Pi*x^2*csgn(
I*exp(-x)*(exp(x)+I)^2)^3-1/2*I*x^2*ln(1-I*exp(x))-1/2*I*x^2*ln(exp(x)-I)+1
/2*I*x^2*ln(1+I*exp(x))+1/2*I*x^2*ln(exp(x)+I)-1/8*Pi*x^2*csgn(I*(exp(x)-I)
^2)^3+1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)^3-1/8*Pi*x^2*csgn(I*(exp(x)-I)^2)*csg
n(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)+1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)
-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2-1/8*Pi*x^2*csgn(exp(-x)*(exp(x)+I)^2)^2
-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2+1/8*P
i*x^2*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2+1/8*Pi*x^2*csgn(I
*(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)-1/4*Pi*x^2*csgn(I*(exp(x)+I))*csgn(I*(e
xp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)^2)^
2-1/8*Pi*x^2*csgn(I*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)+1/4*Pi*x^2*csgn(I*(e
xp(x)-I))*csgn(I*(exp(x)-I)^2)^2+1/8*Pi*x^2*csgn(I*exp(-x))*csgn(I*exp(-x)*
(exp(x)-I)^2)^2-1/8*Pi*x^2*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-1
/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)+I*polylog
(3,I*exp(x))+1/4*Pi*x^2-I*polylog(3,-I*exp(x))+1/8*Pi*x^2*csgn(I*(exp(x)+I)
^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)-I*x*polylog(2,I*exp(x))+1/
8*Pi*x^2*csgn(exp(-x)*(exp(x)+I)^2)^3+1/8*Pi*x^2*csgn(exp(-x)*(exp(x)-I)^2)
^3-1/8*Pi*x^2*csgn(exp(-x)*(exp(x)-I)^2)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)-\int\frac{x^2e^x}{e^{(2\,x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - integrate(x^2*e^x/(e^(2*x) + 1),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}\left(\sinh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(sinh(x)),x)
```

```
[Out] int(x*atan(sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atan}\left(\sinh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(sinh(x)),x)
```

```
[Out] Integral(x*atan(sinh(x)), x)
```

3.75 $\int x^2 \tan^{-1}(\sinh(x))\, dx$

Optimal. Leaf size=108

$$ix^2\text{Li}_2\left(-ie^x\right)-ix^2\text{Li}_2\left(ie^x\right)-2ix\text{Li}_3\left(-ie^x\right)+2ix\text{Li}_3\left(ie^x\right)+2i\text{Li}_4\left(-ie^x\right)-2i\text{Li}_4\left(ie^x\right)-\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))$$

```
[Out] -2/3*x^3*arctan(exp(x))+1/3*x^3*arctan(sinh(x))+I*x^2*polylog(2,-I*exp(x))-
I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*x*polylog(3,I*exp(
x))+2*I*polylog(4,-I*exp(x))-2*I*polylog(4,I*exp(x))
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5205, 4180, 2531, 6609, 2282, 6589}

$$ix^2\text{PolyLog}\left(2,-ie^x\right)-ix^2\text{PolyLog}\left(2,ie^x\right)-2ix\text{PolyLog}\left(3,-ie^x\right)+2ix\text{PolyLog}\left(3,ie^x\right)+2i\text{PolyLog}\left(4,-ie^x\right)-2i\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (-2*x^3*ArcTan[E^x])/3 + (x^3*ArcTan[Sinh[x]])/3 + I*x^2*PolyLog[2, (-I)*E^
x] - I*x^2*PolyLog[2, I*E^x] - (2*I)*x*PolyLog[3, (-I)*E^x] + (2*I)*x*PolyL
og[3, I*E^x] + (2*I)*PolyLog[4, (-I)*E^x] - (2*I)*PolyLog[4, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\sinh(x))\,dx &= \frac{1}{3}x^3\tan^{-1}(\sinh(x))-\frac{1}{3}\int x^3\operatorname{sech}(x)\,dx\\ &= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+i\int x^2\log\left(1-ie^x\right)\,dx-i\int x^2\log\left(1+ie^x\right)\\ &= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\operatorname{Li}_2\left(-ie^x\right)-ix^2\operatorname{Li}_2\left(ie^x\right)-2i\int x\operatorname{Li}_2\left(-ie\right.\\ &= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\operatorname{Li}_2\left(-ie^x\right)-ix^2\operatorname{Li}_2\left(ie^x\right)-2ix\operatorname{Li}_3\left(-ie^x\right)+\\ &= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\operatorname{Li}_2\left(-ie^x\right)-ix^2\operatorname{Li}_2\left(ie^x\right)-2ix\operatorname{Li}_3\left(-ie^x\right)+\\ &= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right)+\frac{1}{3}x^3\tan^{-1}(\sinh(x))+ix^2\operatorname{Li}_2\left(-ie^x\right)-ix^2\operatorname{Li}_2\left(ie^x\right)-2ix\operatorname{Li}_3\left(-ie^x\right)+\end{aligned}$$

Mathematica [B] time = 0.12, size = 356, normalized size = 3.30

$$\frac{1}{192}i\left(192x^2\operatorname{Li}_2\left(-ie^x\right)+192i\pi x\operatorname{Li}_2\left(ie^x\right)+384x\operatorname{Li}_3\left(-ie^{-x}\right)-384x\operatorname{Li}_3\left(-ie^x\right)-48(\pi-2ix)^2\operatorname{Li}_2\left(-ie^{-x}\right)-48\pi^2\operatorname{L}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (I/192)*(7*Pi^4 + (8*I)*Pi^3*x + 24*Pi^2*x^2 - (32*I)*Pi*x^3 - 16*x^4 - (64
*I)*x^3*ArcTan[Sinh[x]] + (8*I)*Pi^3*Log[1 + I/E^x] + 48*Pi^2*x*Log[1 + I/E
^x] - (96*I)*Pi*x^2*Log[1 + I/E^x] - 64*x^3*Log[1 + I/E^x] - 48*Pi^2*x*Log[
1 - I*E^x] + (96*I)*Pi*x^2*Log[1 - I*E^x] - (8*I)*Pi^3*Log[1 + I*E^x] + 64*
x^3*Log[1 + I*E^x] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*x)/4]] - 48*(Pi - (2*I)
*x)^2*PolyLog[2, (-I)/E^x] + 192*x^2*PolyLog[2, (-I)*E^x] - 48*Pi^2*PolyLog
[2, I*E^x] + (192*I)*Pi*x*PolyLog[2, I*E^x] + (192*I)*Pi*PolyLog[3, (-I)/E^
x] + 384*x*PolyLog[3, (-I)/E^x] - 384*x*PolyLog[3, (-I)*E^x] - (192*I)*Pi*P
olyLog[3, I*E^x] + 384*PolyLog[4, (-I)/E^x] + 384*PolyLog[4, (-I)*E^x])
```

fricas [C] time = 0.58, size = 125, normalized size = 1.16

$$\frac{1}{3}x^3\arctan\left(\sinh(x)\right)+\frac{1}{3}ix^3\log\left(i\cosh(x)+i\sinh(x)+1\right)-\frac{1}{3}ix^3\log\left(-i\cosh(x)-i\sinh(x)+1\right)-ix^2\operatorname{Li}_2\left(i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(sinh(x)) + 1/3*I*x^3*log(I*cosh(x) + I*sinh(x) + 1) - 1/3*I*
x^3*log(-I*cosh(x) - I*sinh(x) + 1) - I*x^2*dilog(I*cosh(x) + I*sinh(x)) +
I*x^2*dilog(-I*cosh(x) - I*sinh(x)) + 2*I*x*polylog(3, I*cosh(x) + I*sinh(x
```

```
)) - 2*I*x*polylog(3, -I*cosh(x) - I*sinh(x)) - 2*I*polylog(4, I*cosh(x) +
I*sinh(x)) + 2*I*polylog(4, -I*cosh(x) - I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan(\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(sinh(x)), x)
```

maple [C] time = 0.63, size = 758, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(sinh(x)),x)
```

```
[Out] 1/3*I*x^3*ln(1+I*exp(x))-1/3*I*x^3*ln(exp(x)-I)+1/3*I*x^3*ln(exp(x)+I)-1/3*
I*x^3*ln(1-I*exp(x))-1/12*Pi*x^3*csgn(exp(-x)*(exp(x)+I)^2)^2+1/12*Pi*x^3*c
sgn(exp(-x)*(exp(x)+I)^2)^3+1/12*Pi*x^3*csgn(exp(-x)*(exp(x)-I)^2)^3-1/12*P
i*x^3*csgn(exp(-x)*(exp(x)-I)^2)^2+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)+I)^2)
^3+I*x^2*polylog(2,-I*exp(x))+2*I*x*polylog(3,I*exp(x))+2*I*polylog(4,-I*ex
p(x))+1/12*Pi*x^3*csgn(I*(exp(x)+I)^2)^3-1/12*Pi*x^3*csgn(I*(exp(x)-I)^2)^3
+1/12*Pi*x^3*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2+1/12*Pi*x^
3*csgn(I*(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)-1/6*Pi*x^3*csgn(I*(exp(x)+I))*c
sgn(I*(exp(x)+I)^2)^2+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)
*(exp(x)-I)^2)^2-1/12*Pi*x^3*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)
^2)^2+1/12*Pi*x^3*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2-1/12*Pi*x^
3*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)-1/12*Pi*x^3*csgn(
I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x
)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*(exp(x)-I))^2*csgn(
I*(exp(x)-I)^2)+1/6*Pi*x^3*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-I)^2)^2-1/12*P
i*x^3*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)+1/1
2*Pi*x^3*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)+
1/6*Pi*x^3-2*I*polylog(4,I*exp(x))-I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(
3,-I*exp(x))+1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+
I)^2)-1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)-I)^2)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\, x^3 \arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - 2\int \frac{x^3 e^x}{3\left(e^{(2\,x)} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(1/3*x^3*e^x/(e^(2*x)
 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}(\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(sinh(x)),x)
```

```
[Out] int(x^2*atan(sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atan}(\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(sinh(x)),x)
[Out] Integral(x**2*atan(sinh(x)), x)
```

3.76 $\int (e+fx)^3 \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=299

$$-\frac{3if^3\text{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}+\frac{3if^3\text{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2\text{L}}{8b}$$

```
[Out] -1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,I
*exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8*I
*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4,-I
*exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-3/16
*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b*x+2*
a))/b^4
```

Rubi [A] time = 0.21, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5183, 4180, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Tanh[a +
 b*x]])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^
2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[
3, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*
a + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/
b^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f
^3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3\tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} - \frac{b\int (e+fx)^4\operatorname{sech}(2a+2bx)\,dx}{4f}$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{1}{2}i\int (e+$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3}{}$$

$$= -\frac{(e+fx)^4\tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3}{}$$

Mathematica [B] time = 5.49, size = 600, normalized size = 2.01

$$\frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\tan^{-1}(\tanh(a+bx)) - \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)-8b^4e^3x\log\left(1+ie^{2(a+bx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Tanh[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

fricas [C] time = 0.60, size = 1448, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a)/cos
h(b*x + a)) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4
*I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-4*I*b^
3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-1/2
*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e
*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
 a) + sinh(b*x + a))) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^
2*f*x + 4*I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
 + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x
- 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*
e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2
*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2
 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*
a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f
^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*
e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f +
 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)
*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 +
6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(
b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3
*b*e*f^2 + I*a^4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)
) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(
4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b
*e*f^2)*polylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (12*I
*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 12*I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a))) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 1
2*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
 (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x - 12*I*b^2*e^2*f)*polylog(3, 1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*
f^2*x - 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*
x + a))))/b^4
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 47.26, size = 7275, normalized size = 24.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(tanh(b*x+a)),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(f^3x^4 + 4\,ef^2x^3 + 6\,e^2fx^2 + 4\,e^3x\right)\arctan\left(\frac{e^{(2\,bx+2\,a)} - 1}{e^{(2\,bx+2\,a)} + 1}\right) - \int \frac{\left(bf^3x^4e^{(2\,a)} + 4\,bef^2x^3e^{(2\,a)} + 6\,be^2fx^2e^{(2\,a)} + \cdots\right.}{2\left(e^{(4\,bx+4\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="maxima")
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}\left(\tanh\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^3,x)
[Out] int(atan(tanh(a + b*x))*(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e + fx\right)^{3} \operatorname{atan}{\left(\tanh{\left(a + bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(tanh(b*x+a)),x)
[Out] Integral((e + f*x)**3*atan(tanh(a + b*x)), x)
```

3.77 $\int (e+fx)^2 \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=229

$$\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}+\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+2b}\right.}{4b}$$

```
[Out] -1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,I
*exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f
*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*x+2
*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5183, 4180, 2531, 6609, 2282, 6589}

$$-\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Tanh[a +
 b*x]])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*Poly
Log[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a
+ 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*
f^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
```

```
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} - \frac{b\int (e+fx)^3 \text{sech}(2a+2bx)\,dx}{3f}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{1}{2}i\int (e+$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2}{}$$

$$= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2}{}$$

Mathematica [A] time = 2.94, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tan^{-1}(\tanh(a+bx)) - \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)+12b^3e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Tanh[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

fricas [C] time = 0.61, size = 994, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-3*I*b^2
*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (-3*I*b^2*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(
-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (3*I*b^2*f^2*x^2 + 6*I*b^
2*e*f*x + 3*I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (3*I*b^2*f^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(
cosh(b*x + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b
^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I
*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3
*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 +
 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(-
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f
+ I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*
b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(-
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f
 - I*a^3*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (6*I
*b*f^2*x + 6*I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, -1/2*
sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 37.58, size = 5425, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(f^2x^3 + 3\,efx^2 + 3\,e^2x\right)\arctan\left(\frac{e^{(2\,bx+2\,a)} - 1}{e^{(2\,bx+2\,a)} + 1}\right) - \int \frac{2\left(bf^2x^3e^{(2\,a)} + 3\,befx^2e^{(2\,a)} + 3\,be^2xe^{(2\,a)}\right)e^{(2\,bx)}}{3\left(e^{(4\,bx+4\,a)} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}\left(\tanh\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atan(tanh(a + b*x))*(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e + f x\right)^{2} \operatorname{atan}{\left(\tanh{\left(a + b x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atan(tanh(a + b*x)), x)
```

3.78 $\int (e+fx)\tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=159

$$-\frac{if\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}+\frac{if\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}-\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}+$$

```
[Out] -1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*exp
(2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3,I
*exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5183, 4180, 2531, 2282, 6589}

$$-\frac{if\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}+\frac{if\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2a}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Tanh[a +
 b*x]])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/
4)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^
(2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5183

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Tanh[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} - \frac{b\int (e+fx)^2\text{sech}(2a+2bx)\,dx}{2f}$$
$$= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{1}{2}i\int (e+$$
$$= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{L}}{}$$
$$= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{L}}{}$$
$$= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+fx)\text{L}}{}$$

Mathematica [A] time = 1.94, size = 278, normalized size = 1.75

$$-\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\text{Li}_2\left(-ie^{2(a+bx)}\right)+2bx\text{Li}_2\left(ie^{2(a+bx)}\right)+\text{Li}_3\left(-ie^{2(a+}\right.\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Tanh[a + b*x]] + (f*x^2*ArcTan[Tanh[a + b*x]])/2 - (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) - ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

fricas [C] time = 0.64, size = 596, normalized size = 3.75

$$2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)+\left(-2i\,bfx-2i\,be\right)\text{Li}_2\left(\frac{1}{2}\sqrt{4i}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+\left(-2i\,bf\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-2
*I*b*f*x - 2*I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
 (2*I*b*f*x + 2*I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (2*I*b*f*x + 2*I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
 a))) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*
a*b*e + I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) +
```

```
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I
*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*a*b
*e - I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a
*b*e - I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2
*I*a*b*e + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
 (-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I
*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polyl
og(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, -1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 4.75, size = 2414, normalized size = 15.18

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(tanh(b*x+a)),x)
```

```
[Out] -1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(
exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*I*ln(exp(2*b*x+2*a)-I)*f*x^2-1/
2*I*ln(exp(2*b*x+2*a)-I)*e*x+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2
+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+
1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(
exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/
b^2+1/8*Pi*f*x^2+1/4*Pi*e*x+1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)+1/8*Pi*x^
2*f*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/4*Pi*x*e*csgn(I*(
exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I
)/(exp(2*b*x+2*a)+1))^3-1/2*I*f/b^2*a*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))-1
/2*I*f/b^2*a*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*(b*x+a)*ln((
(-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)+exp
(b*x+a))/(-I)^(1/2))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(
I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp
(2*b*x+2*a)+1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*cs
gn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*f*csgn(I*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)+1))^2+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(e
xp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x
+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/2*I/b*a*e*ln(exp(2*b*x+2*a)+I)-1/4*I/b^2*f
*a^2*ln(exp(2*b*x+2*a)+I)-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/4*Pi*x*e*csgn
(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*
Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*
```

```
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2
*a)+I)-1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/4*P
i*x*e*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn((
1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*f*csgn((1+I)*(exp(
2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-1/8
*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*cs
gn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/2*I*e/b*dilog(1+exp(b*x
+a)*(-1)^(3/4))+1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e*dilog(((
-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a))/(
-I)^(1/2))+1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b^
2*f*a*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(1+exp(
b*x+a)*(-1)^(3/4))+1/2*I*e/b*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f/b^
2*(b*x+a)^2*ln(1+I*exp(2*b*x+2*a))+1/4*I*f/b^2*(b*x+a)*polylog(2,-I*exp(2*b
*x+2*a))-1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*dilog(1
-exp(b*x+a)*(-1)^(3/4))-1/4*I*f/b^2*(b*x+a)^2*ln(1-I*exp(2*b*x+2*a))-1/4*I*
f/b^2*(b*x+a)*polylog(2,I*exp(2*b*x+2*a))-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)-
exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(
1/2))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x
^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*f*csgn((1-I
)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a
)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^
2+1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(fx^2+2\,ex\right)\arctan\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)-\int\frac{\left(bfx^2e^{(2\,a)}+2\,bexe^{(2\,a)}\right)e^{(2\,bx)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(\tanh\left(a+b\,x\right)\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x),x)
```

```
[Out] int(atan(tanh(a + b*x))*(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e+fx\right)\operatorname{atan}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(tanh(a + b*x)), x)
```

3.79 $\int \tan^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=74

$$\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}-x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))$$

```
[Out] -x*arctan(exp(2*b*x+2*a))+x*arctan(tanh(b*x+a))+1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5179, 4180, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}-x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -(x*ArcTan[E^(2*a + 2*b*x)]) + x*ArcTan[Tanh[a + b*x]] + ((I/4)*PolyLog[2,
(-I)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5179

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Tanh[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}(\tanh(a+bx))\,dx &= x\tan^{-1}(\tanh(a+bx))-b\int x\text{sech}(2a+2bx)\,dx\\
&= -x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))+\frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)dx-\frac{1}{2}i\int \log\\
&= -x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))+\frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}-\frac{i\,\text{S}}{}\\
&= -x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\tanh(a+bx))+\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 132, normalized size = 1.78

$$x\tan^{-1}(\tanh(a+bx))-\frac{-2i\left(\text{Li}_2\left(-ie^{2(a+bx)}\right)-\text{Li}_2\left(ie^{2(a+bx)}\right)\right)-\left((-4ia-4ibx+\pi)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+\right.\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[Tanh[a + b*x]] - (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

fricas [B] time = 0.93, size = 334, normalized size = 4.51

$$\frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{4i}\,(\cosh\,(bx+a)+\sinh\,(bx+a))+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(tanh(b*x + a)), x)
```

maple [B] time = 0.56, size = 178, normalized size = 2.41

$$\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\arctan\left(\tanh\left(bx+a\right)\right)}{b}-\frac{i\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\left(\tanh^2(bx+a)\right)}\right)}{2b}+\frac{i\operatorname{arctanh}\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a)),x)
```

```
[Out] 1/b*arctanh(tanh(b*x+a))*arctan(tanh(b*x+a))-1/2*I/b*arctanh(tanh(b*x+a))*l
n(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))+1/2*I/b*arctanh(tanh(b*x+a))*ln(
1+I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))+1/4*I/b*dilog(1+I*(tanh(b*x+a)+1)^
2/(1-tanh(b*x+a)^2))-1/4*I/b*dilog(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\arctan\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)-2\,b\int\frac{xe^{(2\,bx+2\,a)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(\tanh\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x)),x)
```

```
[Out] int(atan(tanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a)),x)
```

```
[Out] Integral(atan(tanh(a + b*x)), x)
```

3.80 $\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx} \, dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(\tanh(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctan(tanh(b*x+a))/(f*x+e),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx} \, dx = \int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Mathematica [A] time = 6.65, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(tanh(b*x + a))/(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}\left(\tanh\left(a+b\,x\right)\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))/(e + f*x),x)
```

```
[Out] int(atan(tanh(a + b*x))/(e + f*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\tanh\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(atan(tanh(a + b*x))/(e + f*x), x)
```

3.81 $\int x^2 \tan^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=355

$$\frac{i\text{Li}_4\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{Li}_4\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\frac{(-c-d+i)e^{2}}{-c+d+}\right.}{4b}$$

```
[Out] 1/3*x^3*arctan(c+d*tanh(b*x+a))+1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/
(I+c-d))/b-1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*p
olylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,-(I-c-d)*exp(
2*b*x+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

Rubi [A] time = 0.46, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5199, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b - ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b - ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 + ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 + ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
- ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5199

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] + (Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(I*
b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^3}{i+c-d+(i+c+d)e}$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\log$$

Mathematica [A] time = 5.81, size = 305, normalized size = 0.86

$$\frac{1}{3}x^3\tan^{-1}(d\tanh(a+bx)+c) + \frac{i\left(4b^3x^3\log\left(1+\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right) - 4b^3x^3\log\left(1+\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right) + 6b^2x^2\mathrm{Li}_2\left(-\frac{(c+d-i)e^{2}}{c-d-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(-I + c - d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
 + b*x)))/(I + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x
```

```
)))/(-I + c - d))] - 6*b^2*x^2*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(
I + c - d))] - 6*b*x*PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c -
d))] + 6*b*x*PolyLog[3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))] + 3*P
olyLog[4, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] - 3*PolyLog[4, -(
((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^3
```

fricas [C] time = 0.63, size = 1335, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
3*I*b^2*x^2*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-1/2*sqrt(-(4*c^
2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*I*b^2*x^2*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-1/2*sqr
t(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2
+ 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4
*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2
 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) +
 I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) + (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d +
4)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
 + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*s
qrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylo
g(3, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -1/2*sqrt(-(4*c^2 - 4*d^2 +
8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*
b*x*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1
))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -1/2*sqrt(-(4*c^2
- 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
))) + (I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3
)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-(4*c^2
 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog
(4, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) + 6*I*polylog(4, -1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d
 + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polyl
og(4, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I
*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left(d \tanh\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tanh(b*x + a) + c), x)
```

maple [C] time = 60.80, size = 6936, normalized size = 19.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3\arctan\left(\frac{\left(ce^{(2\,a)}+de^{(2\,a)}\right)e^{(2\,bx)}+c-d}{e^{(2\,bx+2\,a)}+1}\right)-4\,bd\int\frac{x^3e^{(2\,bx+2\,a)}}{3\left(c^2-2\,cd+d^2+\left(c^2e^{(4\,a)}+2\,cde^{(4\,a)}+d^2e^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a)
 - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(c+d\tanh\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*tanh(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

3.82 $\int x\tan^{-1}(c+d\tanh(a+bx))\,dx$

Optimal. Leaf size=267

$$-\frac{i\mathrm{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\mathrm{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\mathrm{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\mathrm{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+\frac{(-\right.$$

```
[Out] 1/2*x^2*arctan(c+d*tanh(b*x+a))+1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,-(I-c-
d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c
-d))/b-1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3
,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] time = 0.37, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5199, 2190, 2531, 2282, 6589}

$$-\frac{i\mathrm{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\mathrm{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\mathrm{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\mathrm{PolyLog}\left(2,\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b - ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b - ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
+ ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5199

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
```

```
 + 1)), x] + (Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(I*
b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^2}{i+c-d+(i+c+d)e^{2}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log$$

Mathematica [A] time = 4.87, size = 229, normalized size = 0.86

$$\frac{1}{2}x^2\tan^{-1}(d\tanh(a+bx)+c)+\frac{i\left(2b^2x^2\log\left(1+\frac{(c+d-i)e^{2(a+bx)}}{c-d-i}\right)-2b^2x^2\log\left(1+\frac{(c+d+i)e^{2(a+bx)}}{c-d+i}\right)+2bx\mathrm{Li}_2\left(-\frac{(c+d-i)e^{2(a+}}{c-d-i}\right.\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(-I + c - d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(I + c - d)] + 2*b*x*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x)))/(
-I + c - d))] - 2*b*x*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d
))] - PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] + PolyLog[
3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^2
```

fricas [C] time = 0.93, size = 1103, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
2*I*b*x*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-1/2*sqrt(-(4*c^2 - 4*d^
```

```
2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) -
2*I*b*x*dilog(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-1/2*sqrt(-(4*c^2 - 4*d^
2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) +
I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4
)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
+ a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sq
rt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^
2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a
) + (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d
+ d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 +
 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*
d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(1/2*sq
rt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(
-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) - 2*I*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d
 + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polyl
og(3, -1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, 1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I
*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*pol
ylog(3, -1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left(d \tanh\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tanh(b*x + a) + c), x)
```

maple [C] time = 6.16, size = 6586, normalized size = 24.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\, x^2 \arctan\left(\frac{\left(ce^{(2\,a)} + de^{(2\,a)}\right)e^{(2\,bx)} + c - d}{e^{(2\,bx+2\,a)} + 1}\right) - 2\,bd \int \frac{x^2 e^{(2\,bx+2\,a)}}{c^2 - 2\,cd + d^2 + \left(c^2 e^{(4\,a)} + 2\,cde^{(4\,a)} + d^2 e^{(4\,a)} + e^{(4\,a)}\right)e^{(4\,bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d
^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atan}\left(c + d \tanh\left(a + b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*tanh(a + b*x)), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

3.83 $\int \tan^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=174

$$\frac{i\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right.$$

```
[Out] x*arctan(c+d*tanh(b*x+a))+1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,-(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b-1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5191, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1+\frac{(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b - ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5191

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[(x*E^(2*a + 2*b*x
))/(I - c + d + (I - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[I*b*(I + c + d)
, Int[(x*E^(2*a + 2*b*x))/(I + c - d + (I + c + d)*E^(2*a + 2*b*x)), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c + d\tanh(a + bx))\,dx = x\tan^{-1}(c + d\tanh(a + bx)) + (b(1 - i(c + d)))\int \frac{e^{2a+2bx}x}{i + c - d + (i + c + d)e^{2a+2bx}}$$

$$= x\tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{2}ix\log\left(1 + \right.$$

$$= x\tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{2}ix\log\left(1 + \right.$$

$$= x\tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}ix\log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{2}ix\log\left(1 + \right.$$

Mathematica [A] time = 4.52, size = 288, normalized size = 1.66

$$\frac{d\text{Li}_2\left(-\frac{(c^2+2dc+d^2+1)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - d\text{Li}_2\left(\frac{(c^2+2dc+d^2+1)e^{2(a+bx)}}{-c^2+d^2+2\sqrt{-d^2}-1}\right) - 2d(a+bx)\log\left(\frac{2((c+d)^2+1)e^{2(a+bx)}}{2c^2-2d^2-4\sqrt{-d^2}+2}+1\right) + 2d(a+bx)\log\left(\right.}{4b\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(1
+ (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*(a
+ b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 -
 d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

fricas [B] time = 1.14, size = 851, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2
*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt(-(4*c
^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 -
 d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)
)) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2
 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt(-(4*c^2 - 4*d^2 - 8*I*d
+ 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(1/2*sqrt(-(4*c^2 - 4*d^2
 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (I*b*x + I*a)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqr
t(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d +
4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(
1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(-(4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
```

```
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt(-(4
*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) - I*dilog(-1/2*sqrt(-(4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tanh(b*x + a) + c), x)
```

maple [B] time = 0.36, size = 350, normalized size = 2.01

$$\frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)+d\right)}{2b}-\frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b}-\frac{i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arctan(c+d*tanh(b*x+a))*ln(d*tanh(b*x+a)+d)-1/2/b*arctan(c+d*tanh(b*x
+a))*ln(d*tanh(b*x+a)-d)-1/4*I/b*ln(d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)+I-c
)/(I-c-d))+1/4*I/b*ln(d*tanh(b*x+a)-d)*ln((I+d*tanh(b*x+a)+c)/(I+c+d))-1/4*
I/b*dilog((-d*tanh(b*x+a)+I-c)/(I-c-d))+1/4*I/b*dilog((I+d*tanh(b*x+a)+c)/(
I+c+d))+1/4*I/b*ln(d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)+I-c)/(I-c+d))-1/4*I/
b*ln(d*tanh(b*x+a)+d)*ln((I+d*tanh(b*x+a)+c)/(I+c-d))+1/4*I/b*dilog((-d*tan
h(b*x+a)+I-c)/(I-c+d))-1/4*I/b*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,bd\int \frac{xe^{(2\,bx+2\,a)}}{c^2-2\,cd+d^2+\left(c^2e^{(4\,a)}+2\,cde^{(4\,a)}+d^2e^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}+2\,\left(c^2e^{(2\,a)}-d^2e^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x
) + c - d)/(e^(2*b*x + 2*a) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+d\,\mathrm{tanh}\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tanh(a + b*x)),x)
```

```
[Out] int(atan(c + d*tanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}{\left(c+d\tanh{\left(a+bx\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a)),x)
[Out] Integral(atan(c + d*tanh(a + b*x)), x)
```

3.84 $\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(d\tanh(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 8.68, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*tanh(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*tanh(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}\left(c+d\tanh\left(a+b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tanh(a + b*x))/x,x)
```

```
[Out] int(atan(c + d*tanh(a + b*x))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.85 $\int x^2 \tan^{-1}(c + (i + c) \tanh(a + bx))\, dx$

Optimal. Leaf size=142

$$\frac{i\text{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c+(c+i)\tanh(a$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*tanh(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5195, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1
 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
- ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-I)
*c*E^(2*a + 2*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{3}b\int \frac{x^3}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{-i+ce^{2a+2b}}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.$$

Mathematica [A] time = 5.76, size = 128, normalized size = 0.90

$$\frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-6b^2x^2\text{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right)-6bx\text{Li}_3\left(\frac{ie^{-2(a+bx)}}{c}\right)-3\text{Li}_4\left(\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}+\frac{1}{3}x^3\tan^{-1}(c+(c+i)\tanh($$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*Po
lyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^3
```

fricas [C] time = 0.73, size = 291, normalized size = 2.05

$$-i\,b^4x^4+2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+6i\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,b^2x^2\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)+i\,a^4-2i\,a^3\,\text{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) + (2*I*b^3*x^3 + 2*I*a^3)*log(1/2*sqrt(
-4*I*c)*e^(b*x + a) + 1) + (2*I*b^3*x^3 + 2*I*a^3)*log(-1/2*sqrt(-4*I*c)*e^
(b*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polyl
og(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*tanh(b*x + a) + c), x)
```

maple [C] time = 6.86, size = 1555, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] 1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x
+2*a))/b+1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c-2*I)-1/4*I*x*polylog(3
,-I*c*exp(2*b*x+2*a))/b^2+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*
b*x+2*a)+1))^3-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1)
)^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/12*
Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1
/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(
2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*exp
(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1
/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/4*I/b^3*c/(I+c
)*a^4-1/12*I*b*c/(I+c)*x^4-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*a^2+1/2*I/b
^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*
c)^(1/2))*x+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1
-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))+
1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/4/b^3/(I+c)*a^4+1/12*b/(I
+c)*x^4+1/6*Pi*x^3-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*Pi*x^3*csgn((
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*cs
gn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-
2*I)/(exp(2*b*x+2*a)+1))-1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a^3-1/4*I/b^3*p
olylog(2,-I*c*exp(2*b*x+2*a))*a^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)+1))-1/3/b^3*a^3/(I+c)*ln(exp(b*x+a))+1/3/b^2/(I+c)*
x*a^3+1/3*I/b^3*c*a^3/(I+c)*ln(exp(b*x+a))-1/3*I/b^2*c/(I+c)*x*a^3+1/12*Pi*
x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2
*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*e
```

```
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*
a)+1))
```

maxima [A] time = 1.94, size = 129, normalized size = 0.91

$$\frac{1}{3}\,x^3\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4}-\frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx}{-2\,b^4(-i\,c+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.86 $\int x \tan^{-1}(c + (i + c)\tanh(a + bx))\,dx$

Optimal. Leaf size=113

$$-\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{6}ibx^3$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*tanh(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2
```

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5195, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1
+ I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b - (
(I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
```

```
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{2}b\int \frac{x^2}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}(ibc)\int \frac{e^{2a+2bx}x^2}{-i+ce^{2a+2bx}}\,d$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)$$
$$= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)$$

Mathematica [A] time = 5.72, size = 102, normalized size = 0.90

$$\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right)-\text{Li}_3\left(\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I/
(c*E^(2*(a + b*x)))]))/b^2
```

fricas [C] time = 0.62, size = 245, normalized size = 2.17

$$-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)+3i\,a^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) + (3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (3*I*b^
2*x^2 - 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/
2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a))
)/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan \left((c+i) \tanh (bx+a)+c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*tanh(b*x + a) + c), x)
```

maple [C] time = 5.01, size = 1519, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] 1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))
+1/2/b^2*a^2/(I+c)*ln(exp(b*x+a))-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/8
*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))
*x*a-1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(b*x+a)
*(I*c)^(1/2))*x-1/6*I*b*c*x^3/(I+c)+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))
^3-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/4*I/b^2
*a^2*ln(-exp(2*b*x+2*a)*c+I)+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(
2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))
^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2
*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1)
)^2+1/2*I/b*c/(I+c)*x*a^2-1/2*I/b^2*c*a^2/(I+c)*ln(exp(b*x+a))-1/3/b^2/(I+c
)*a^3+1/6*b*x^3/(I+c)+1/3*I/b^2*c/(I+c)*a^3+1/4*Pi*x^2+1/4*I/b^2*ln(1+I*c*e
xp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a-1/2*I/b^2*a^2
*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2)
)-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b
*x+a)*(I*c)^(1/2))+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)+1))-1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/4*I*x^2*ln(2
*exp(2*b*x+2*a)*c-2*I)-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(
2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-1/2/
b/(I+c)*x*a^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)+1))^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3
```

maxima [A] time = 2.04, size = 106, normalized size = 0.94

$$\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\arctan\left((c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arctan((c + I)*tanh(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c + \tanh\left(a + b\,x\right)\,\left(c + 1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.87 $\int \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*tanh(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5187, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Tanh[a + b*x]] + (I/2)*x*Log[1 + I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5187

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = x\tan^{-1}(c+(i+c)\tanh(a+bx)) - b\int \frac{x}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{1}{2}i$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\mathrm{Su}}{}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\mathrm{Li}}{}$$

Mathematica [A] time = 1.95, size = 71, normalized size = 0.90

$$\frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b} + x\tan^{-1}(c+(c+i)\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

fricas [B] time = 0.67, size = 187, normalized size = 2.37

$$\frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
 I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
 I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
 a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c), x)
```

maple [B] time = 0.58, size = 1381, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] -1/2/(I+c)^2/b*ln(I+c+(I+c)*tanh(b*x+a))*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)+I)
)*c+1/2/(I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*ln(-1/2*I*(-c-(I+c)*ta
nh(b*x+a)+I))*c+1/2/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*t
anh(b*x+a)+I)/c)*c+1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+
c)*tanh(b*x+a)+I)-1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I
+c)*tanh(b*x+a))-1/2/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*tanh(
b*x+a)-I)/(-2*I-2*c))*c-1/4*I/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*tanh(b*x+a)+I)
/c)*c^2+1/4*I/(I+c)^2/b*dilog((-c-(I+c)*tanh(b*x+a)-I)/(-2*I-2*c))*c^2-1/4*
I/(I+c)^2/b*ln(I+c+(I+c)*tanh(b*x+a))*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)+I))+1
/4*I/(I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*ln(-1/2*I*(-c-(I+c)*tanh(
b*x+a)+I))+1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*tanh
(b*x+a)+I)/c)-1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*tanh(b
*x+a)-I)/(-2*I-2*c))-1/8*I/(I+c)^2/b*ln(I+c+(I+c)*tanh(b*x+a))^2*c^2-1/4*I/
(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*c^2-2*I/(I+c)/b*arctan(c+(I
+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c+2*I/(I+c)/b*arctan(c
+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c-1/(I+c)/b*arctan(
c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2+1/4*I/(I+c)^2/
b*ln(I+c+(I+c)*tanh(b*x+a))*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)+I))*c^2-1/4*I/(
I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*ln(-1/2*I*(-c-(I+c)*tanh(b*x+a)
+I))*c^2+1/(I+c)/b*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(
b*x+a))*c^2-1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(-c-(I+c)*tan
h(b*x+a)+I)/c)*c^2+1/4*I/(I+c)^2/b*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-c-(I+c)*t
anh(b*x+a)-I)/(-2*I-2*c))*c^2+1/4/(I+c)^2/b*ln(I+c+(I+c)*tanh(b*x+a))^2*c+1
/2/(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*c+1/2/(I+c)^2/b*dilog(-1
/2*(-c-(I+c)*tanh(b*x+a)+I)/c)*c-1/2/(I+c)^2/b*dilog((-c-(I+c)*tanh(b*x+a)-
I)/(-2*I-2*c))*c+1/4*I/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*tanh(b*x+a)+I)/c)-1/4
*I/(I+c)^2/b*dilog((-c-(I+c)*tanh(b*x+a)-I)/(-2*I-2*c))+1/8*I/(I+c)^2/b*ln(
I+c+(I+c)*tanh(b*x+a))^2+1/4*I/(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a
)))
```

maxima [A] time = 1.99, size = 80, normalized size = 1.01

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*tanh(b*x + a
) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.88 $\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\tan^{-1}(c+(c+i)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}-i}\right)}{2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

[Out] integrate(arctan((c + I)*tanh(b*x + a) + c)/x, x)

maple [A] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+\left(i+c\right)\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)

[Out] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$i\,bx - \frac{1}{4}\left(4\,\pi - 4i\,a - 2\,\arctan(c) - i\,\log\left(c^2+1\right)\right)\log(x) + \frac{1}{2}\int \frac{\arctan\left(ce^{(2\,bx+2\,a)}\right)}{x}\,dx - \frac{1}{4}i\int \frac{\log\left(c^2e^{(4\,bx+4\,a)}+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")

[Out] I*b*x - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*log(x) + 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x + 4*a) + 1)/x, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(c + tanh(a + b*x)*(c + 1i))/x,x)

[Out] int(atan(c + tanh(a + b*x)*(c + 1i))/x, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(c+(I+c)*tanh(b*x+a))/x,x)

[Out] Timed out

3.89 $\int x^2 \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=145

$$-\frac{i\text{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c-(-c+i)\tanh(a+bx)$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*tanh(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*ex
p(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.22, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5195, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a
+ 2*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}b\int \frac{x^3}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{i+ce^{2a+2bx}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2b}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2b}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2b}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2b}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2b}\right.$$

Mathematica [A] time = 5.81, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\tan^{-1}(c+(c-i)\tanh(a+bx)) - \frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\mathrm{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\mathrm{Li}_3\left(-\frac{ie^{-2(a+bx)}}{c}\right) - 3\mathrm{Li}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Tanh[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*
x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x
)))]))/b^3
```

fricas [C] time = 0.61, size = 291, normalized size = 2.01

$$\frac{i b^4x^4 + 2i b^3x^3\log\left(-\frac{\left(ce^{(2bx+2a)}+i\right)e^{(-2bx-2a)}}{c-i}\right) - 6i b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) - 6i b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) - i a^4 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) + (-2*I*b^3*x^3 - 2*I*a^3)*log(1/2*sqrt(4*I*c
)*e^(b*x + a) + 1) + (-2*I*b^3*x^3 - 2*I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x +
 a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -
1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*tanh(b*x + a) + c), x)
```

maple [C] time = 6.79, size = 1570, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] 1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a)
)+1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))^2+1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x
+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/
b+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)+1))^3+1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)+1))^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))
^3-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*exp
(b*x+a)*(-I*c)^(1/2))*x-1/4*I/b^3*c/(I-c)*a^4-1/12*I*b*c/(I-c)*x^4+1/3/b^3*
a^3/(I-c)*ln(exp(b*x+a))-1/3/b^2/(I-c)*x*a^3-1/6*Pi*x^3-1/12*Pi*x^3*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/4/b^3/(I-c)
*a^4-1/12*b/(I-c)*x^4+1/3*I/b^3*c*a^3/(I-c)*ln(exp(b*x+a))+1/12*Pi*x^3*csgn
(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a
)+1))^2-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3
*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)
*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)+1))^2+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*x*a^2-1/12*P
i*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*c
sgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^
3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-1/3*I/b^2*c/(I-c)*x*a^3+1/12*Pi*x^3*
csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/12*
Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(e
xp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-1/8*I*p
```

```
olylog(4,I*c*exp(2*b*x+2*a))/b^3+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/6*
I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/3*I/b^3*ln(1-I*c*exp(2*b*
x+2*a))*a^3+1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2-1/2*I/b^3*a^3*ln(1-
I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/
2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*exp(
b*x+a)*(-I*c)^(1/2))
```

maxima [A] time = 1.95, size = 129, normalized size = 0.89

$$\frac{1}{3}x^3\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3x^4}{4ic+4}-\frac{4b^3x^3\log\left(-ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(ice^{(2bx+2a)}\right)-6}{-2b^4(-ic-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.90 $\int x \tan^{-1}(c - (i - c)\tanh(a + bx))\,dx$

Optimal. Leaf size=116

$$\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{6}ibx^3$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*tanh(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2*b
*x+2*a))/b^2
```

Rubi [A] time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5195, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{6}i$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5195

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Tanh[a + b*x]])/(f*(m
```

```
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}b\int \frac{x^2}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}(ibc)\int \frac{e^{2a+2bx}x^2}{i+ce^{2a+2bx}}\,d$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)$$

Mathematica [A] time = 5.68, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\tan^{-1}(c+(c-i)\tanh(a+bx)) - \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right)-\text{Li}_3\left(-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Tanh[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3
, (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

fricas [C] time = 0.52, size = 245, normalized size = 2.11

$$2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right)-6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
(-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-3*I*b^2*x
^2 + 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sq
rt(4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*tanh(b*x + a) + c), x)
```

maple [C] time = 4.95, size = 1534, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] 1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))+
1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*a^2+1/2*I/b^2*ln(1+I*exp(b*x+a)*(
-I*c)^(1/2))*a^2-1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2
,I*c*exp(2*b*x+2*a))*a+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I
/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2
*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/
2*I/b*c/(I-c)*x*a^2-1/2*I/b^2*c*a^2/(I-c)*ln(exp(b*x+a))-1/2/b^2*a^2/(I-c)*
ln(exp(b*x+a))+1/2/b/(I-c)*x*a^2+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+
2*I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)+1))^3+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1
))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/4*Pi*x^2-1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c
+I)+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)+1))^3+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)+1))^3-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a
)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*
csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/4*I*x^2*ln(-2*
exp(2*b*x+2*a)*c-2*I)-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1
/3/b^2/(I-c)*a^3-1/6*b*x^3/(I-c)+1/2*I/b*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x*
a+1/2*I/b*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x*a+1/3*I/b^2*c/(I-c)*a^3-1/6*I*b
*c*x^3/(I-c)-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x*a+1/8*Pi*x^2*csgn(I*(2*exp(
2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)+1))^2
```

maxima [A] time = 1.95, size = 107, normalized size = 0.92

$$-\left(\frac{2\,x^{3}}{3 i\,c+3}-\frac{2\,b^{2}x^{2}\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_{2}\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_{3}(i\,ce^{(2\,bx+2\,a)})}{-2\,b^{3}(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^{2}\arctan\left((c-i)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arctan((c - I)*tanh(b*x + a) + c)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c + \tanh\left(a + b\,x\right)\,\left(c - \mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.91 $\int \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{2}ibx^2$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*tanh(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5187, 2184, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\tanh(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Tanh[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5187

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = x\tan^{-1}(c-(i-c)\tanh(a+bx)) - b\int \frac{x}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{1}{2}i$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\mathrm{S}}{}$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\mathrm{L}}{}$$

Mathematica [A] time = 1.85, size = 71, normalized size = 0.87

$$x\tan^{-1}(c+(c-i)\tanh(a+bx)) - \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - \mathrm{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

fricas [B] time = 0.55, size = 187, normalized size = 2.28

$$\frac{i\,b^2x^2 + i\,bx\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right) - i\,a^2 + (-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}+1\right) + (-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{4}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I
*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a))
)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c), x)
```

maple [B] time = 0.46, size = 1351, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] -1/2/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)*c+1/2/b/(c-I)/(I-c)
*dilog(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*c+1/b/(c-I)*arctan((c-I)*tanh(b*
x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)+c-I)-1/4*I/b/(c-I)/(I-c)*dilog(((c-I
)*tanh(b*x+a)+c-I)/(-2*I+2*c))+1/8*I/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-I
)^2+1/4*I/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)-1/b/(c-I)*arct
an((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)+1/2/b/(c-I)/(I-
c)*dilog(-1/2*I*(I+(c-I)*tanh(b*x+a)+c))*c-1/4/b/(c-I)/(I-c)*ln((c-I)*tanh(
b*x+a)+c-I)^2*c-1/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*(I+(c-I)*tanh(b*x+a)+c))-1
/4*I/b/(c-I)/(I-c)*ln(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)*ln((c-I)*tanh(b*x+a)-c
+I)*c^2+1/4*I/b/(c-I)/(I-c)*ln(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)
*tanh(b*x+a)-c+I)*c^2+1/4*I/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-I)*ln(-1/2
*I*(I+(c-I)*tanh(b*x+a)+c))*c^2-1/8*I/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-
I)^2*c^2-1/4*I/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)*c^2+1/4*I
/b/(c-I)/(I-c)*dilog(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*c^2-1/4*I/b/(c-I)/
(I-c)*ln((c-I)*tanh(b*x+a)+c-I)*ln(-1/2*I*(I+(c-I)*tanh(b*x+a)+c))+1/4*I/b/
(c-I)/(I-c)*ln(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)*ln((c-I)*tanh(b*x+a)-c+I)-1/4
*I/b/(c-I)/(I-c)*ln(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)*tanh(b*x+a
)-c+I)+1/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*(I+(c-I)*tanh(b*x+a)+c))*c^2+1/b/(c
-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c^2-1/b
/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)+c-I)*c^2-
1/2/b/(c-I)/(I-c)*ln(1/2*(I+(c-I)*tanh(b*x+a)+c)/c)*ln((c-I)*tanh(b*x+a)-c+
I)*c+1/2/b/(c-I)/(I-c)*ln(((c-I)*tanh(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)*tanh
(b*x+a)-c+I)*c+1/2/b/(c-I)/(I-c)*ln((c-I)*tanh(b*x+a)+c-I)*ln(-1/2*I*(I+(c-
I)*tanh(b*x+a)+c))*c-2*I/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((
c-I)*tanh(b*x+a)-c+I)*c+2*I/b/(c-I)*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*l
n((c-I)*tanh(b*x+a)+c-I)*c
```

maxima [A] time = 1.95, size = 80, normalized size = 0.98

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*tanh(b*x +
a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.92 $\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\tan^{-1}(c-(-c+i)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{i\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c)/x, x)
```

maple [A] time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(c-(i-c)\tanh \left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx-\frac{1}{2}\,\pi\log(x)-\frac{1}{4}\left(2\,\pi+4i\,a-2\,\arctan(c)+i\,\log\left(c^2+1\right)\right)\log(x)+\frac{1}{2}\int\frac{\arctan\left(ce^{(2\,bx+2\,a)}\right)}{x}\,dx+\frac{1}{4}i\int\frac{\log\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*
log(x) + 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(lo
g(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c - 1i))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.93 $\int (e+fx)^3 \tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=299

$$\frac{3if^3\text{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}-\frac{3if^3\text{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}-\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2}{8}$$

```
[Out] 1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+3/16*
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A] time = 0.21, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5185, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Coth[a +
b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} + \frac{b\int (e+fx)^4 \text{sech}(2a+2bx)\,dx}{4f}$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{1}{2}i\int (e+fx)^3$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(\right.}{4b}$$

Mathematica [B] time = 6.05, size = 600, normalized size = 2.01

$$\frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\tan^{-1}(\coth(a+bx)) + \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)-8b^4e^3x\log\left(1+ie^{2(a+bx)}\right)+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Coth[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

fricas [C] time = 1.07, size = 1448, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*
f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*p
olylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x + a)/si
nh(b*x + a)) + (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4
*I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (4*I*b^3
*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e
*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
 a) + sinh(b*x + a))) + (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e
^2*f*x - 4*I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x
+ 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e
*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*
e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2
 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*
a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*
f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3
*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f
 - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(
b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f
^3)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3
- 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cos
h(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^
3*b*e*f^2 - I*a^4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a
)) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (24*I*b*f^3*x + 24*I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*b*e*f^2)*polylog(
4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-24*I*b*f^3*x - 24*I*
b*e*f^2)*polylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-12
*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x - 12*I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I
)*(cosh(b*x + a) + sinh(b*x + a))) + (-12*I*b^2*f^3*x^2 - 24*I*b^2*e*f^2*x
- 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e*f^2*x + 12*I*b^2*e^2*f)*polylog(3, 1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (12*I*b^2*f^3*x^2 + 24*I*b^2*e
*f^2*x + 12*I*b^2*e^2*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))))/b^4
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 49.10, size = 7275, normalized size = 24.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(coth(b*x+a)),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2+4\,e^3x\right)\arctan\left(e^{(2\,bx+2\,a)}+1,e^{(2\,bx+2\,a)}-1\right)+\int\frac{\left(bf^3x^4e^{(2\,a)}+4\,bef^2x^3e^{(2\,a)}+6\,be^2f\right.}{2\left(e^{(4\,bx+4\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2*x
^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x
 + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}\left(\mathrm{coth}\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x)^3,x)
[Out] int(atan(coth(a + b*x))*(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)^{3}\operatorname{atan}{\left(\coth{\left(a+bx\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(coth(b*x+a)),x)
[Out] Integral((e + f*x)**3*atan(coth(a + b*x)), x)
```

3.94 $\int (e+fx)^2 \tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=229

$$-\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie\right.}{4b}$$

```
[Out] 1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5185, 4180, 2531, 6609, 2282, 6589}

$$\frac{if(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{if^2\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{PolyLog}\left(\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Coth[a +
b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
```

```
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} + \frac{b\int (e+fx)^3 \operatorname{sech}(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} - \frac{1}{2}i\int (e+fx)^2$$
$$= \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}_2\left(\right.}{4b}$$
$$= \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}_2\left(\right.}{4b}$$

Mathematica [A] time = 3.00, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tan^{-1}(\coth(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)+12b^3efx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Coth[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

fricas [C] time = 0.57, size = 994, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) + (3*I*b^2*f
^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a))) + (3*I*b^2*f^2*x^2 + 6*I*b^2*e*f*x + 3*I*b^2*e^2)*dilog(-1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-3*I*b^2*f^2*x^2 - 6*I*b^2*
e*f*x - 3*I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ (-3*I*b^2*f^2*x^2 - 6*I*b^2*e*f*x - 3*I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(c
osh(b*x + a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3
*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^
3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I
*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 -
3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f
- I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a
*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) +
2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(-
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f
+ I*a^3*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-6*I
*b*f^2*x - 6*I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) + (-6*I*b*f^2*x - 6*I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a)
+ sinh(b*x + a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a))) + (6*I*b*f^2*x + 6*I*b*e*f)*polylog(3, -1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 37.48, size = 5425, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(coth(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\arctan\left(e^{(2\,bx+2\,a)}+1,e^{(2\,bx+2\,a)}-1\right)+\int\frac{2\left(bf^2x^3e^{(2\,a)}+3\,befx^2e^{(2\,a)}+3\,be^2xe^{(2\,a)}\right)e^{(2}}{3\left(e^{(4\,bx+4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b*e
^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}\left(\coth\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atan(coth(a + b*x))*(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)^{2}\operatorname{atan}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atan(coth(a + b*x)), x)
```

3.95 $\int (e+fx)\tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=159

$$\frac{if\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}-\frac{if\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}$$

```
[Out] 1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5185, 4180, 2531, 2282, 6589}

$$\frac{if\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{if\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Coth[a +
b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5185

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTan[Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} + \frac{b\int (e+fx)^2\text{sech}(2a+2bx)\,dx}{2f}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{1}{2}i\int (e+fx)\,\text{lo}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-\right.}{4b}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-\right.}{4b}$$
$$= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-\right.}{4b}$$

Mathematica [A] time = 2.18, size = 278, normalized size = 1.75

$$\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\text{Li}_2\left(-ie^{2(a+bx)}\right)+2bx\text{Li}_2\left(ie^{2(a+bx)}\right)+\text{Li}_3\left(-ie^{2(a+bx)}\right)\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Coth[a + b*x]] + (f*x^2*ArcTan[Coth[a + b*x]])/2 + (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) + ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

fricas [C] time = 0.52, size = 596, normalized size = 3.75

$$2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)+\left(2i\,bfx+2i\,be\right)\text{Li}_2\left(\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)+\left(2i\,bfx+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) + (2*I*b
*f*x + 2*I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (2*I
*b*f*x + 2*I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (
-2*I*b*f*x - 2*I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
 + (-2*I*b*f*x - 2*I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
 a))) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*
b*e - I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
```

```
I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I
*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I*a*
b*e + I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I
*a*b*e + I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (
2*I*a*b*e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a))
+ (2*I*a*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I
*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polyl
og(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, -1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 4.68, size = 2415, normalized size = 15.19

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(coth(b*x+a)),x)
```

```
[Out] -1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2
*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(
2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))+1/4*Pi*x*e
*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)-1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp
(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*f*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+
1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(
2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2
+1/8*Pi*f*x^2+1/4*Pi*e*x-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/2*I*e/b*a*ln
(-exp(2*b*x+2*a)+I)-1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*dilo
g(1-exp(b*x+a)*(-1)^(3/4))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/
2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*l
n(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)+exp(
b*x+a))/(-I)^(1/2))+1/4*I*f/b^2*(b*x+a)^2*ln(1-I*exp(2*b*x+2*a))+1/4*I*f/b^
2*(b*x+a)*polylog(2,I*exp(2*b*x+2*a))-1/4*I*f/b^2*(b*x+a)*polylog(2,-I*exp(
2*b*x+2*a))+1/2*I*(-1/2*f*x^2-e*x)*ln(exp(2*b*x+2*a)+I)-1/4*I*f/b^2*a^2*ln(
-exp(2*b*x+2*a)+I)-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))
*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I/(exp
(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+
1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*c
sgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b
*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*cs
gn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(
exp(2*b*x+2*a)-1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+ex
p(b*x+a))/(-I)^(1/2))-1/4*I*f/b^2*(b*x+a)^2*ln(1+I*exp(2*b*x+2*a))-1/2*I*e/
b*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*(b*x+a)*ln(1-exp(b*x+a)*(-1
```

```
)^(3/4))+1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1
-exp(b*x+a)*(-1)^(3/4))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1
/2))+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2
*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a
)-1))+1/2*I*f/b^2*a*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*(b*x+
a)*ln(1-exp(b*x+a)*(-1)^(3/4))-1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)+I)/(ex
p(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)-1))^2+1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+1/4
*I*ln(exp(2*b*x+2*a)-I)*f*x^2+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x-1/2*I*f/b^2*a*
(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*(b*x+a)*ln(((-
I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)-I)
/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-1/8
*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn
((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn((1+I)*(ex
p(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/
(exp(2*b*x+2*a)-1))^3-1/2*I/b*a*e*ln(exp(2*b*x+2*a)+I)+1/4*I/b^2*f*a^2*ln(e
xp(2*b*x+2*a)+I)+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))
*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I/(exp
(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*I*f*pol
ylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)-1))^3+1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^
3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(fx^2+2\,ex\right)\arctan\left(e^{(2\,bx+2\,a)}+1,e^{(2\,bx+2\,a)}-1\right)+\int\frac{\left(bfx^2e^{(2\,a)}+2\,bexe^{(2\,a)}\right)e^{(2\,bx)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(\coth\left(a+b\,x\right)\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x),x)
```

```
[Out] int(atan(coth(a + b*x))*(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)\operatorname{atan}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(coth(a + b*x)), x)
```

3.96 $\int \tan^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=73

$$-\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))$$

```
[Out] x*arctan(exp(2*b*x+2*a))+x*arctan(coth(b*x+a))-1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5181, 4180, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}+x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[E^(2*a + 2*b*x)] + x*ArcTan[Coth[a + b*x]] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5181

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Coth[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \tan^{-1}(\coth(a+bx))\,dx = x\tan^{-1}(\coth(a+bx))+b\int x\text{sech}(2a+2bx)\,dx$$

$$= x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))-\frac{1}{2}i\int\log\left(1-ie^{2a+2bx}\right)dx+\frac{1}{2}i\int\text{log}$$

$$= x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))-\frac{i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}+$$

$$= x\tan^{-1}\left(e^{2a+2bx}\right)+x\tan^{-1}(\coth(a+bx))-\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A] time = 0.08, size = 132, normalized size = 1.81

$$x\tan^{-1}(\coth(a+bx))+\frac{-2i\left(\text{Li}_2\left(-ie^{2(a+bx)}\right)-\text{Li}_2\left(ie^{2(a+bx)}\right)\right)-\left((-4ia-4ibx+\pi)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[Coth[a + b*x]] + (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

fricas [B] time = 0.47, size = 334, normalized size = 4.58

$$\frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i}\,(\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(coth(b*x + a)), x)
```

maple [B] time = 0.55, size = 178, normalized size = 2.44

$$\frac{\operatorname{arctanh}\left(\coth\left(bx+a\right)\right)\arctan\left(\coth\left(bx+a\right)\right)}{b}-\frac{i\operatorname{arctanh}\left(\coth\left(bx+a\right)\right)\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\left(\coth^2(bx+a)\right)}\right)}{2b}+\frac{i\operatorname{arctanh}\left(\coth\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a)),x)
```

```
[Out] 1/b*arctanh(coth(b*x+a))*arctan(coth(b*x+a))-1/2*I/b*arctanh(coth(b*x+a))*l
n(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))+1/2*I/b*arctanh(coth(b*x+a))*ln(
1+I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))+1/4*I/b*dilog(1+I*(coth(b*x+a)+1)^
2/(1-coth(b*x+a)^2))-1/4*I/b*dilog(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \arctan\left(e^{(2\,bx+2\,a)}+1, e^{(2\,bx+2\,a)}-1\right) + 2\,b\int \frac{xe^{(2\,bx+2\,a)}}{e^{(4\,bx+4\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*
b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(\coth\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x)),x)
```

```
[Out] int(atan(coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a)),x)
```

```
[Out] Integral(atan(coth(a + b*x)), x)
```

3.97 $\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(\coth(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctan(coth(b*x+a))/(f*x+e),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[Coth[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 5.93, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(coth(b*x + a))/(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctan(coth(b*x+a))/(f*x+e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}\left(\coth\left(a+b\,x\right)\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))/(e + f*x),x)
```

```
[Out] int(atan(coth(a + b*x))/(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a))/(f*x+e),x)
```

```
[Out] Timed out
```

3.98 $\int x^2 \tan^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=351

$$\frac{i\text{Li}_4\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{Li}_4\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}-\frac{ix\text{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix^2\text{I}}{}$$

```
[Out] 1/3*x^3*arctan(c+d*coth(b*x+a))+1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I
+c-d))/b-1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*poly
log(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^3
```

Rubi [A] time = 0.46, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5201, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a}}{c-d+}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b - ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b - ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 + ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 - ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5201

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(I
*b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^3}{i+c-d+(-i-c-\cdots}$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\cdots$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\cdots$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\cdots$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\cdots$$

$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3\cdots$$

Mathematica [A] time = 6.18, size = 299, normalized size = 0.85

$$\frac{1}{3}x^3\tan^{-1}(d\coth(a+bx)+c)+\frac{i\left(4b^3x^3\log\left(1+\frac{(c+d-i)e^{2(a+bx)}}{-c+d+i}\right)-4b^3x^3\log\left(1+\frac{(c+d+i)e^{2(a+bx)}}{-c+d-i}\right)+6b^2x^2\mathrm{Li}_2\left(\frac{(c+d-i)e\cdots}{c-d\cdots}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(I - c + d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(-I - c + d)] + 6*b^2*x^2*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x))
)/(-I + c - d)] - 6*b^2*x^2*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c
```

```
 - d)] - 6*b*x*PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + 6*
b*x*PolyLog[3, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] + 3*PolyLog[4, ((
-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] - 3*PolyLog[4, ((I + c + d)*E^(2
*(a + b*x)))/(I + c - d)]))/b^3
```

fricas [C] time = 0.59, size = 1315, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
3*I*b^2*x^2*dilog(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-1/2*sqrt((4*c^2
- 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 3*I*b^2*x^2*dilog(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-1/2*sqrt((
4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b
*x + a))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2
+ 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1
)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*
I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*si
nh(b*x + a) + (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2
 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*
c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, 1/2
*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*I*b*x*polylog(3, -1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylo
g(3, 1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -1/2*sqrt((4*c^2 - 4*d^2 - 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (I*b^3
*x^3 + I*a^3)*log(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(-1/2*sq
rt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*
d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*
b^3*x^3 - I*a^3)*log(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d
^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog(4, 1/2*sqrt((4*
c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*I*polylog(4, -1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, 1/2*sqrt((4*
c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) - 6*I*polylog(4, -1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left(d \coth\left(bx + a\right) + c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*coth(b*x + a) + c), x)
```

maple [C] time = 60.58, size = 6864, normalized size = 19.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3\arctan\left(\left(ce^{(2a)}+de^{(2a)}\right)e^{(2bx)}-c+d,e^{(2bx+2a)}-1\right)+4bd\int\frac{}{3\left(c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}\left(c+d\coth\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*coth(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

3.99 $\int x \tan^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=265

$$-\frac{i\text{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-\frac{(-c-d+i)}{-c+d}\right.$$

```
[Out] 1/2*x^2*arctan(c+d*coth(b*x+a))+1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,(I-c-d
)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d
))/b-1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3,(I
+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] time = 0.37, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5201, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b - ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b -
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5201

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
```

```
 + 1)), x] + (-Dist[(I*b*(I - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(I
*b*(I + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx)) - \frac{1}{2}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^2}{i+c-d+(-i-c-$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

$$= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\,\mathrm{lo}$$

Mathematica [A] time = 4.76, size = 225, normalized size = 0.85

$$\frac{1}{2}x^2\tan^{-1}(d\coth(a+bx)+c)+\frac{i\left(2b^2x^2\log\left(1+\frac{(c+d-i)e^{2(a+bx)}}{-c+d+i}\right)-2b^2x^2\log\left(1+\frac{(c+d+i)e^{2(a+bx)}}{-c-d-i}\right)+2bx\mathrm{Li}_2\left(\frac{(c+d-i)e^{2(a}}{c-d-i}\right.\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(I - c + d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a +
 b*x)))/(-I - c + d)] + 2*b*x*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x)))/(-I
 + c - d)] - 2*b*x*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] -
PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + PolyLog[3, ((I +
c + d)*E^(2*(a + b*x)))/(I + c - d)]))/b^2
```

fricas [C] time = 0.65, size = 1087, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
2*I*b*x*dilog(1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-1/2*sqrt((4*c^2 - 4*d^2
```

```
+ 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*
I*b*x*dilog(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-1/2*sqrt((4*c^2 - 4*d^2 -
8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^
2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a)
+ 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt((4
*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*
c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c
^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d
+ d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(1/2*sqrt((4*c^
2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2
+ I*a^2)*log(1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt((
4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b
*x + a)) + 1) - 2*I*polylog(3, 1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -1/2*sq
rt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a))) + 2*I*polylog(3, 1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -1/2*sq
rt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + si
nh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left(d \coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*coth(b*x + a) + c), x)
```

maple [C] time = 6.27, size = 6514, normalized size = 24.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 \arctan\left(\left(ce^{(2a)}+de^{(2a)}\right)e^{(2bx)}-c+d, e^{(2bx+2a)}-1\right)+2bd\int \frac{x^2e^{(2}}{c^2-2cd+d^2+\left(c^2e^{(4a)}+2cde^{(4a)}+d^2e^{(4a)}+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atan}\left(c + d \coth\left(a + b x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*coth(a + b*x)),x)
[Out] int(x*atan(c + d*coth(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*coth(b*x+a)),x)
[Out] Timed out
```

3.100 $\int \tan^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=174

$$\frac{i\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)+x\tan$$

```
[Out] x*arctan(c+d*coth(b*x+a))+1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,(I-c-d)*exp(2*b*x+
2*a)/(I-c+d))/b-1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5193, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+d+}{c-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b - ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5193

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[(x*E^(2*a + 2*b*
x))/(I - c + d - (I - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[I*b*(I + c + d
), Int[(x*E^(2*a + 2*b*x))/(I + c - d - (I + c + d)*E^(2*a + 2*b*x)), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\coth(a+bx))\,dx = x\tan^{-1}(c+d\coth(a+bx)) - (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(-i-c-d)e^{2a+}}$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1-\right.$$

Mathematica [A] time = 4.26, size = 287, normalized size = 1.65

$$\frac{d\text{Li}_2\left(\frac{\left(c^2+2dc+d^2+1\right)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - d\text{Li}_2\left(-\frac{\left(c^2+2dc+d^2+1\right)e^{2(a+bx)}}{-c^2+d^2+2\sqrt{-d^2}-1}\right) + 2d(a+bx)\log\left(1-\frac{\left((c+d)^2+1\right)e^{2(a+bx)}}{c^2-d^2+2\sqrt{-d^2}+1}\right) - 2d(a+bx)\log}{4b\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1 +
(c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b*x)
*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])]
 + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 +
2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

fricas [B] time = 0.97, size = 839, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2
- 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*
(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - (c^2 - d^2 - 2*I*d + 1)*sqrt((4*c^2
 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d +
d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + (c^2 - d
^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1)))
+ I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
1)*sinh(b*x + a) - (c^2 - d^2 + 2*I*d + 1)*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)
/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(1/2*sqrt((4*c^2 - 4*d^2 + 8*
I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I
*b*x + I*a)*log(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt((4*c
^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(1/2*sqrt
((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh
(b*x + a))) + I*dilog(-1/2*sqrt((4*c^2 - 4*d^2 + 8*I*d + 4)/(c^2 - 2*c*d +
```

```
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt((4*c^2 - 4*d^
2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) -
I*dilog(-1/2*sqrt((4*c^2 - 4*d^2 - 8*I*d + 4)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*coth(b*x + a) + c), x)
```

maple [B] time = 0.54, size = 350, normalized size = 2.01

$$-\frac{\arctan\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right)}{2b}+\frac{\arctan\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)+d\right)}{2b}-\frac{i\ln(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a)),x)
```

```
[Out] -1/2/b*arctan(c+d*coth(b*x+a))*ln(d*coth(b*x+a)-d)+1/2/b*arctan(c+d*coth(b*
x+a))*ln(d*coth(b*x+a)+d)-1/4*I/b*ln(d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)+I-
c)/(I-c-d))+1/4*I/b*ln(d*coth(b*x+a)-d)*ln((I+d*coth(b*x+a)+c)/(I+c+d))-1/4
*I/b*dilog((-d*coth(b*x+a)+I-c)/(I-c-d))+1/4*I/b*dilog((I+d*coth(b*x+a)+c)/
(I+c+d))+1/4*I/b*ln(d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)+I-c)/(I-c+d))-1/4*I
/b*ln(d*coth(b*x+a)+d)*ln((I+d*coth(b*x+a)+c)/(I+c-d))+1/4*I/b*dilog((-d*co
th(b*x+a)+I-c)/(I-c+d))-1/4*I/b*dilog((I+d*coth(b*x+a)+c)/(I+c-d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\,bd\int\frac{xe^{(2\,bx+2\,a)}}{c^2-2\,cd+d^2+\left(c^2e^{(4\,a)}+2\,cde^{(4\,a)}+d^2e^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}-2\left(c^2e^{(2\,a)}-d^2e^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}\,dx+x\,\mathrm{arc}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x)
 - c + d, e^(2*b*x + 2*a) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+d\coth\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*coth(a + b*x)),x)
```

```
[Out] int(atan(c + d*coth(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tan^{-1}(d\coth(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*coth(b*x+a))/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 8.87, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*coth(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

maple [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a))/x,x)
[Out] int(arctan(c+d*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atan}\left(c+d\coth\left(a+b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*coth(a + b*x))/x,x)
[Out] int(atan(c + d*coth(a + b*x))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a))/x,x)
[Out] Timed out
```

3.102 $\int x^2 \tan^{-1}(c + (i + c)\coth(a + bx))\, dx$

Optimal. Leaf size=142

$$\frac{i\text{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c+(c+i)\coth(a+bx)$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*coth(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c*e
xp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5197, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/12)*b*x^4 + (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1
 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - (
(I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2*a
 + 2*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(c+(i+c)\coth(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{3}b\int\frac{x^3}{-i-ce^{2a+2bx}}\,dx\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{-i-ce^{2a+2bx}}\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right.\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right.\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right.\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right.\\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right.\end{aligned}$$

Mathematica [A] time = 1.83, size = 128, normalized size = 0.90

$$\frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6b^2x^2\mathrm{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right)-6bx\mathrm{Li}_3\left(-\frac{ie^{-2(a+bx)}}{c}\right)-3\mathrm{Li}_4\left(-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}+\frac{1}{3}x^3\tan^{-1}(c+(c+i)\mathrm{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*x
*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x)
))]))/b^3
```

fricas [C] time = 0.51, size = 291, normalized size = 2.05

$$\frac{-ib^4x^4+2ib^3x^3\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right)+6ib^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right)+6ib^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right)+ia^4-2ia^3\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) + (2*I*b^3*x^3 + 2*I*a^3)*log(1/2*sqrt(4*I*c)
*e^(b*x + a) + 1) + (2*I*b^3*x^3 + 2*I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a
) + 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/
2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left((c+i)\coth(bx+a)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*coth(b*x + a) + c), x)
```

maple [C] time = 6.84, size = 1554, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] -1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*cs
gn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b^2*ln(1-I*
c*exp(2*b*x+2*a))*x*a^2+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*p
olylog(3,I*c*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3-1/
12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/12*Pi*x^3
*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi
*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/4*I/b^3*polylog(
2,I*c*exp(2*b*x+2*a))*a^2-1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/2*I/b^3*
a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^
(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^2*dilog
(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)-1/4*I/b^3*c/(I+c)*a^4-1/12*I*b*c/(I+c)*x^4+1/4/b^3/(I+c)*a^4+1/12*b/
(I+c)*x^4+1/6*Pi*x^3-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c+2*I)+1/12*Pi*x^3*csgn(
I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)
-1))^2-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*
x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/1
2*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))^2-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1)
)^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)-1))-1/3/b^3*a^3/(I+c)*ln(exp(b*x+a))+1/3/b^2/(I+c)*x*a^3+1/12*Pi*x^3*cs
gn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*
x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/12
*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b
```

```
*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/3*I/b^3*c*a^3/(I+c)*ln(exp(b*x+a))-1
/3*I/b^2*c/(I+c)*x*a^3+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/12
*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b
*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/6*I/b^3*a^3*ln(exp(2*b*x+2
*a)*c+I)
```

maxima [A] time = 2.00, size = 129, normalized size = 0.91

$$\frac{1}{3}x^3\arctan\left((c+i)\coth\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4}-\frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx}{-2\,b^4(-i\,c+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*atan(c + coth(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.103 $\int x \tan^{-1}(c + (i + c)\coth(a + bx))\, dx$

Optimal. Leaf size=113

$$-\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{6}ibx^3$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*coth(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(2*
b*x+2*a))/b^2
```

Rubi [A] time = 0.21, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5197, 2184, 2190, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\coth(a+bx))-$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/6)*b*x^3 + (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1
- I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((I/
8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
```

```
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(c+(i+c)\coth(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{-i-ce^{2a+2bx}}\,dx\\ &= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{-i-ce^{2a+2bx}}\,d\\ &= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\ &= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\ &= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\ &= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\end{aligned}$$

Mathematica [A] time = 1.79, size = 102, normalized size = 0.90

$$\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right)-\text{Li}_3\left(-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}+\frac{1}{2}x^2\tan^{-1}(c+(c+i)\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

fricas [C] time = 0.56, size = 245, normalized size = 2.17

$$-2ib^3x^3+3ib^2x^2\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right)-2ia^3+6ibx\text{Li}_2\left(\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right)+6ibx\text{Li}_2\left(-\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right)+3ia^2\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
(3*I*b^2*x^2 - 3*I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (3*I*b^2*x^2
 - 3*I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt
(4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*coth(b*x + a) + c), x)
```

maple [C] time = 5.28, size = 1518, normalized size = 13.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] 1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))+1/2/b^2*a^2/(I+c)*ln(exp(b*x+a))-1/8*Pi*
x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/4*
I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/
6*I*b*c*x^3/(I+c)+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/8*P
i*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/2*I/b^2*a*dilog
(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/4*I/
b^2*polylog(2,I*c*exp(2*b*x+2*a))*a-1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2
))*a^2-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/8*Pi*x^2*csgn(I/(ex
p(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+
2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1)
)*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/2*I/b^2*a^2*ln(1+I*
exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b*c/(I+c)*x*a^2-1/2*I/b^2*c*a^2/(I+c)*ln(exp
(b*x+a))-1/3/b^2/(I+c)*a^3+1/6*b*x^3/(I+c)+1/3*I/b^2*c/(I+c)*a^3+1/4*Pi*x^2
+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/8*Pi
*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/8
*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*
c+2*I)-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1/8
*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^
3-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*ex
p(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)
*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*c
sgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*c
sgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn((2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x
*a-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b*a*ln(1+I*exp(b*x+a)*
(-I*c)^(1/2))*x-1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/2/b/(I+c)*x*a^2
```

maxima [A] time = 2.04, size = 106, normalized size = 0.94

$$\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\arctan\left((c+i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)))
*b*(c + I) + 1/2*x^2*arctan((c + I)*coth(b*x + a) + c)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c + \coth\left(a + b\,x\right)\,\left(c + 1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.104 $\int \tan^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*coth(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5189, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\tan^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-I/2)*b*x^2 + x*ArcTan[c + (I + c)*Coth[a + b*x]] + (I/2)*x*Log[1 - I*c*E^
(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5189

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\coth(a+bx))\,dx = x\tan^{-1}(c+(i+c)\coth(a+bx)) - b\int \frac{x}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\,\mathrm{Su}}{}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.79, size = 71, normalized size = 0.90

$$\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{Li}_2\left(-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}+x\tan^{-1}(c+(c+i)\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

fricas [B] time = 0.53, size = 187, normalized size = 2.37

$$\frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*d
ilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/
b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c), x)
```

maple [B] time = 0.73, size = 1381, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] 1/2/(I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*coth(b*x+a)))*ln(-1/2*I*(-c-(I+c)*coth(b
*x+a)+I))*c-1/2/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln((-c-(I+c)*coth(b*x+a
)-I)/(-2*I-2*c))*c+1/2/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(-c-(I+c
)*coth(b*x+a)+I)/c)*c-1/4*I/(I+c)^2/b*ln(I+c+(I+c)*coth(b*x+a))*ln(-1/2*I*(
-c-(I+c)*coth(b*x+a)+I))+1/4*I/(I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*coth(b*x+a)))
*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))-1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)
+I)*ln((-c-(I+c)*coth(b*x+a)-I)/(-2*I-2*c))+1/4*I/(I+c)^2/b*ln(c-(I+c)*coth
(b*x+a)+I)*ln(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)-1/8*I/(I+c)^2/b*ln(I+c+(I+c)
*coth(b*x+a))^2*c^2-1/4*I/(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a)))*c
^2+1/4*I/(I+c)^2/b*dilog((-c-(I+c)*coth(b*x+a)-I)/(-2*I-2*c))*c^2-1/4*I/(I+
c)^2/b*dilog(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)*c^2+1/(I+c)/b*arctan(c+(I+c)*
coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)-1/(I+c)/b*arctan(c+(I+c)*c
oth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))-1/2/(I+c)^2/b*ln(I+c+(I+c)*
coth(b*x+a))*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*c-2*I/(I+c)/b*arctan(c+(I+
c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c+2*I/(I+c)/b*arctan(c+
(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c-1/4*I/(I+c)^2/b*ln
(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)*c^2-1/(I+c)/b*a
rctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2+1/(I+c)/
b*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2+1/4*I
/(I+c)^2/b*ln(I+c+(I+c)*coth(b*x+a))*ln(-1/2*I*(-c-(I+c)*coth(b*x+a)+I))*c^
2-1/4*I/(I+c)^2/b*ln(-1/2*I*(I+c+(I+c)*coth(b*x+a)))*ln(-1/2*I*(-c-(I+c)*co
th(b*x+a)+I))*c^2+1/4*I/(I+c)^2/b*ln(c-(I+c)*coth(b*x+a)+I)*ln((-c-(I+c)*co
th(b*x+a)-I)/(-2*I-2*c))*c^2+1/4/(I+c)^2/b*ln(I+c+(I+c)*coth(b*x+a))^2*c+1/
2/(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a)))*c-1/2/(I+c)^2/b*dilog((-c
-(I+c)*coth(b*x+a)-I)/(-2*I-2*c))*c+1/2/(I+c)^2/b*dilog(-1/2*(-c-(I+c)*coth
(b*x+a)+I)/c)*c+1/4*I/(I+c)^2/b*dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a)))-1/4*I
/(I+c)^2/b*dilog((-c-(I+c)*coth(b*x+a)-I)/(-2*I-2*c))+1/4*I/(I+c)^2/b*dilog
(-1/2*(-c-(I+c)*coth(b*x+a)+I)/c)+1/8*I/(I+c)^2/b*ln(I+c+(I+c)*coth(b*x+a))
^2
```

maxima [A] time = 2.00, size = 80, normalized size = 1.01

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*coth(b*x + a
) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\coth\left(a+b\,x\right)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.105 $\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\tan^{-1}(c+(c+i)\coth(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arctan(c+(I+c)*coth(b*x+a))/x,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.13, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{i\log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right)}{2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c)/x, x)
```

maple [A] time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$i\,bx+\frac{1}{2}\,\pi\log(x)-\frac{1}{4}\left(4\,\pi-4i\,a-2\,\arctan(c)-i\,\log\left(c^{2}+1\right)\right)\log(x)-\frac{1}{2}\int\frac{\arctan\left(ce^{(2\,bx+2\,a)}\right)}{x}\,dx-\frac{1}{4}i\int\frac{\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x + 1/2*pi*log(x) - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*l
og(x) - 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log
(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{atan}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c + 1i))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.106 $\int x^2 \tan^{-1}(c - (i - c)\coth(a + bx))\,dx$

Optimal. Leaf size=145

$$-\frac{i\text{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\tan^{-1}(c-(-c+i)\coth(a$$

[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*coth(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3

Rubi [A] time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5197, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}-\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \tan^{-1}(c-(i-c)\coth(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.\\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2b}\right.\end{aligned}$$

Mathematica [A] time = 1.85, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\tan^{-1}(c+(c-i)\coth(a+bx)) - \frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{Li}_3\left(\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{Li}_4\left(\frac{i\cdots}{}\right.\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Coth[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 - I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*P
olyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^
3
```

fricas [C] time = 0.48, size = 291, normalized size = 2.01

$$i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right) - 6i\,b^2x^2\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right) - i\,a^{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) + (-2*I*b^3*x^3 - 2*I*a^3)*log(1/2*sqrt
(-4*I*c)*e^(b*x + a) + 1) + (-2*I*b^3*x^3 - 2*I*a^3)*log(-1/2*sqrt(-4*I*c)*
e^(b*x + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*pol
ylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*coth(b*x + a) + c), x)
```

maple [C] time = 7.32, size = 1571, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] 1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)-1))^2-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))+1/12*Pi*x^3*csgn(I*(
2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))^2-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/6*I*x^3*l
n(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))
*a^3+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1/2*I/b^3*ln(1+I*exp(b*x+
a)*(I*c)^(1/2))*a^3-1/4*I/b^3*c/(I-c)*a^4-1/12*I*b*c/(I-c)*x^4-1/2*I/b^3*ln
(1-I*exp(b*x+a)*(I*c)^(1/2))*a^3-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(
1/2))-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/12*Pi*x^3*csgn((-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*x^3*csgn
(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a
)-1))^2+1/3/b^3*a^3/(I-c)*ln(exp(b*x+a))-1/3/b^2/(I-c)*x*a^3-1/6*Pi*x^3+1/1
2*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I
*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x
+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn((2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)
*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c+2*I)-1/4/b^3
/(I-c)*a^4-1/12*b/(I-c)*x^4+1/3*I/b^3*c*a^3/(I-c)*ln(exp(b*x+a))+1/12*Pi*x^
3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/3*I
/b^2*c/(I-c)*x*a^3-1/2*I/b^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x*a^2-1/2*I/b^2
*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x*a^2+1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*
a^2-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*x^3*csgn(I*(-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/6*I/b^3*a^3*ln(
-exp(2*b*x+2*a)*c+I)-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3-1/12*Pi*x^3*c
```

```
sgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(
I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)-1))^2
```

maxima [A] time = 2.01, size = 129, normalized size = 0.89

$$\frac{1}{3}\,x^3 \arctan\left((c-i)\coth\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4}-\frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6}{-2\,b^4(-i\,c-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\coth\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*atan(c + coth(a + b*x)*(c - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.107 $\int x \tan^{-1}(c - (i - c)\coth(a + bx))\, dx$

Optimal. Leaf size=116

$$\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\coth(a+bx))+\frac{1}{6}ibx^3$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*coth(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2
```

Rubi [A] time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5197, 2184, 2190, 2531, 2282, 6589}

$$\frac{i\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\tan^{-1}(c-(-c+i)\coth(a+bx))+$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5197

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTan[c + d*Coth[a + b*x]])/(f*(m
```

```
 + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(c-(i-c)\coth(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}\,dx\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\end{aligned}$$

Mathematica [A] time = 1.70, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\tan^{-1}(c+(c-i)\coth(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right)-\text{Li}_3\left(\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Coth[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I
/(c*E^(2*(a + b*x)))]))/b^2
```

fricas [C] time = 0.49, size = 245, normalized size = 2.11

$$\frac{2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)-6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) + (-3*I*b^2*x^2 + 3*I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-3*I*
b^2*x^2 + 3*I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3,
1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a
)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*coth(b*x + a) + c), x)
```

maple [C] time = 5.24, size = 1535, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] 1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c+2*I)-1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I
)-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/8
*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^3+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
-1))^3+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)
*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*cs
gn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/4*I*x*polylog(2,-I*c*ex
p(2*b*x+2*a))/b+1/2*I/b*c/(I-c)*x*a^2-1/2*I/b^2*c*a^2/(I-c)*ln(exp(b*x+a))-
1/2/b^2*a^2/(I-c)*ln(exp(b*x+a))+1/2/b/(I-c)*x*a^2-1/8*Pi*x^2*csgn(I/(exp(2
*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn((2*
exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1
/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)-1))^2-1/4*Pi*x^2+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+1/2*I/
b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*
c)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1/4*I/b^2*ln(1+I*c*
exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a+1/2*I/b^2*a^
2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))^2-1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*x*a+1/2*I/b*a*ln(1+I*
exp(b*x+a)*(I*c)^(1/2))*x+1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/4*I*
x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/3/b^2/(I-c)*a^3-1/6*b*x^3/(
I-c)+1/3*I/b^2*c/(I-c)*a^3-1/6*I*b*c*x^3/(I-c)-1/8*Pi*x^2*csgn(I*(2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x
+2*a)-1))
```

maxima [A] time = 2.03, size = 107, normalized size = 0.92

$$-\left(\frac{2\,x^{3}}{3i\,c+3}-\frac{2\,b^{2}x^{2}\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^{3}(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^{2}\arctan\left((c-i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)
))*b*(c - I) + 1/2*x^2*arctan((c - I)*coth(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}\left(c + \coth\left(a + b\,x\right)\,\left(c - \mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.108 $\int \tan^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*coth(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5189, 2184, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\tan^{-1}(c-(-c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Coth[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5189

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c-(i-c)\coth(a+bx))\,dx = x\tan^{-1}(c-(i-c)\coth(a+bx)) - b\int \frac{x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{1}{2}i$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{iS}{}$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\mathrm{Li}}{}$$

Mathematica [A] time = 0.80, size = 71, normalized size = 0.87

$$x\tan^{-1}(c+(c-i)\coth(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{Li}_2\left(\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

fricas [B] time = 0.51, size = 187, normalized size = 2.28

$$\frac{ib^2x^2 + ibx\log\left(-\frac{\left(ce^{(2bx+2a)}-i\right)e^{(-2bx-2a)}}{c-i}\right) - ia^2 + (-ibx-ia)\log\left(\frac{1}{2}\sqrt{-4ic}\,e^{(bx+a)}+1\right) + (-ibx-ia)\log\left(-\frac{1}{2}\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
 - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
 + a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c), x)
```

maple [B] time = 0.77, size = 1351, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] -1/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)+
1/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)+c-I)+1
/2/b/(c-I)/(I-c)*dilog(-1/2*I*(I+(c-I)*coth(b*x+a)+c))*c-1/4/b/(c-I)/(I-c)*
ln((c-I)*coth(b*x+a)+c-I)^2*c-1/2/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I)*coth(b*x
+a)+c)/c)*c+1/2/b/(c-I)/(I-c)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*c-1
/4*I/b/(c-I)/(I-c)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))-1/4*I/b/(c-I)/
(I-c)*dilog(-1/2*I*(I+(c-I)*coth(b*x+a)+c))+1/8*I/b/(c-I)/(I-c)*ln((c-I)*co
th(b*x+a)+c-I)^2+1/4*I/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I)*coth(b*x+a)+c)/c)+1
/4*I/b/(c-I)/(I-c)*dilog(-1/2*I*(I+(c-I)*coth(b*x+a)+c))*c^2-1/8*I/b/(c-I)/
(I-c)*ln((c-I)*coth(b*x+a)+c-I)^2*c^2-1/4*I/b/(c-I)/(I-c)*dilog(1/2*(I+(c-I
)*coth(b*x+a)+c)/c)*c^2-1/4*I/b/(c-I)/(I-c)*ln(-1/2*I*(I+(c-I)*coth(b*x+a)+
c))*ln((c-I)*coth(b*x+a)+c-I)+1/4*I/b/(c-I)/(I-c)*ln(1/2*(I+(c-I)*coth(b*x+
a)+c)/c)*ln((c-I)*coth(b*x+a)-c+I)+1/2/b/(c-I)/(I-c)*ln(-1/2*I*(I+(c-I)*cot
h(b*x+a)+c))*ln((c-I)*coth(b*x+a)+c-I)*c-1/2/b/(c-I)/(I-c)*ln(1/2*(I+(c-I)*
coth(b*x+a)+c)/c)*ln((c-I)*coth(b*x+a)-c+I)*c-1/b/(c-I)*arctan((c-I)*coth(b
*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)+c-I)*c^2+1/b/(c-I)*arctan((c-I)*cot
h(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c^2+1/2/b/(c-I)/(I-c)*ln(((
c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)*coth(b*x+a)-c+I)*c+1/4*I/b/(c-I)
/(I-c)*dilog(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*c^2-1/4*I/b/(c-I)/(I-c)*ln
(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)*coth(b*x+a)-c+I)+1/4*I/b/(c-I
)/(I-c)*ln(-1/2*I*(I+(c-I)*coth(b*x+a)+c))*ln((c-I)*coth(b*x+a)+c-I)*c^2-1/
4*I/b/(c-I)/(I-c)*ln(1/2*(I+(c-I)*coth(b*x+a)+c)/c)*ln((c-I)*coth(b*x+a)-c+
I)*c^2+1/4*I/b/(c-I)/(I-c)*ln(((c-I)*coth(b*x+a)+c-I)/(-2*I+2*c))*ln((c-I)*
coth(b*x+a)-c+I)*c^2+2*I/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((
c-I)*coth(b*x+a)+c-I)*c-2*I/b/(c-I)*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*l
n((c-I)*coth(b*x+a)-c+I)*c
```

maxima [A] time = 1.96, size = 80, normalized size = 0.98

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*coth(b*x +
a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\coth\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed
```

3.109 $\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\tan^{-1}(c-(-c+i)\coth(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arctan(c-(I-c)*coth(b*x+a))/x,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.27, size = 0, normalized size = 0.00

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\log\left(-\frac{\left(ce^{(2bx+2a)}-i\right)e^{(-2bx-2a)}}{c-i}\right)}{2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I))/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c)/x, x)
```

maple [A] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx-\frac{1}{4}\left(2\,\pi+4i\,a-2\,\arctan(c)+i\,\log\left(c^2+1\right)\right)\log(x)-\frac{1}{2}\int\frac{\arctan\left(ce^{(2\,bx+2\,a)}\right)}{x}\,dx+\frac{1}{4}i\int\frac{\log\left(c^2e^{(4\,bx+4\,a)}+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*int
egrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x +
 4*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atan}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

3.110 $\int \tan^{-1}(e^x)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}i\text{Li}_2\left(-ie^x\right)-\frac{1}{2}i\text{Li}_2\left(ie^x\right)$$

```
[Out] 1/2*I*polylog(2,-I*exp(x))-1/2*I*polylog(2,I*exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2282, 4848, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}i\text{PolyLog}\left(2,ie^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^x],x]
```

```
[Out] (I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(e^x\right)\,dx &= \text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx,x,e^x\right)\\ &= \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^x\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^x\right)\\ &= \frac{1}{2}i\text{Li}_2\left(-ie^x\right)-\frac{1}{2}i\text{Li}_2\left(ie^x\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.90

$$x\tan^{-1}\left(e^x\right)-\frac{1}{2}i\left(-\text{Li}_2\left(-ie^x\right)+\text{Li}_2\left(ie^x\right)+x\left(\log\left(1-ie^x\right)-\log\left(1+ie^x\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x],x]
```

```
[Out] x*ArcTan[E^x] - (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

fricas [B] time = 0.46, size = 40, normalized size = 1.29

$$x \arctan\left(e^{x}\right) + \frac{1}{2} i x \log\left(i e^{x} + 1\right) - \frac{1}{2} i x \log\left(-i e^{x} + 1\right) - \frac{1}{2} i \operatorname{Li}_2\left(i e^{x}\right) + \frac{1}{2} i \operatorname{Li}_2\left(-i e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(e^x) + 1/2*I*x*log(I*e^x + 1) - 1/2*I*x*log(-I*e^x + 1) - 1/2*I*di
log(I*e^x) + 1/2*I*dilog(-I*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(e^{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^x), x)
```

maple [B] time = 0.06, size = 59, normalized size = 1.90

$$\ln\left(e^{x}\right) \arctan\left(e^{x}\right) + \frac{i \ln\left(e^{x}\right) \ln\left(1 + i e^{x}\right)}{2} - \frac{i \ln\left(e^{x}\right) \ln\left(1 - i e^{x}\right)}{2} + \frac{i \operatorname{dilog}\left(1 + i e^{x}\right)}{2} - \frac{i \operatorname{dilog}\left(1 - i e^{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x)),x)
```

```
[Out] ln(exp(x))*arctan(exp(x))+1/2*I*ln(exp(x))*ln(1+I*exp(x))-1/2*I*ln(exp(x))*
ln(1-I*exp(x))+1/2*I*dilog(1+I*exp(x))-1/2*I*dilog(1-I*exp(x))
```

maxima [B] time = 0.43, size = 34, normalized size = 1.10

$$x \arctan\left(e^{x}\right) - \frac{1}{4} \pi \log\left(e^{(2x)} + 1\right) - \frac{1}{2} i \operatorname{Li}_2\left(i e^{x} + 1\right) + \frac{1}{2} i \operatorname{Li}_2\left(-i e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(e^x) - 1/4*pi*log(e^(2*x) + 1) - 1/2*I*dilog(I*e^x + 1) + 1/2*I*di
log(-I*e^x + 1)
```

mupad [B] time = 0.69, size = 21, normalized size = 0.68

$$\frac{\operatorname{polylog}\left(2, -e^{x}\, 1i\right) 1i}{2} - \frac{\operatorname{polylog}\left(2, e^{x}\, 1i\right) 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(x)),x)
```

```
[Out] (polylog(2, -exp(x)*1i)*1i)/2 - (polylog(2, exp(x)*1i)*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(e^{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x)),x)
```

```
[Out] Integral(atan(exp(x)), x)
```

3.111 $\int x \tan^{-1}(e^x)\, dx$

Optimal. Leaf size=63

$$\frac{1}{2}ix\text{Li}_2\left(-ie^x\right)-\frac{1}{2}ix\text{Li}_2\left(ie^x\right)-\frac{1}{2}i\text{Li}_3\left(-ie^x\right)+\frac{1}{2}i\text{Li}_3\left(ie^x\right)$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5143, 2531, 2282, 6589}

$$\frac{1}{2}ix\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix\text{PolyLog}\left(2,ie^x\right)-\frac{1}{2}i\text{PolyLog}\left(3,-ie^x\right)+\frac{1}{2}i\text{PolyLog}\left(3,ie^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*x*PolyLog[2, (-I)*E^x] - (I/2)*x*PolyLog[2, I*E^x] - (I/2)*PolyLog[3,
 (-I)*E^x] + (I/2)*PolyLog[3, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(e^x)\,dx = \frac{1}{2}i\int x\log(1-ie^x)\,dx - \frac{1}{2}i\int x\log(1+ie^x)\,dx$$
$$= \frac{1}{2}ix\text{Li}_2(-ie^x) - \frac{1}{2}ix\text{Li}_2(ie^x) - \frac{1}{2}i\int \text{Li}_2(-ie^x)\,dx + \frac{1}{2}i\int \text{Li}_2(ie^x)\,dx$$
$$= \frac{1}{2}ix\text{Li}_2(-ie^x) - \frac{1}{2}ix\text{Li}_2(ie^x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\text{Li}_2(-ix)}{x}\,dx, x, e^x\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\text{Li}_2(ix)}{x}\,dx, x, e\right.$$
$$= \frac{1}{2}ix\text{Li}_2(-ie^x) - \frac{1}{2}ix\text{Li}_2(ie^x) - \frac{1}{2}i\text{Li}_3(-ie^x) + \frac{1}{2}i\text{Li}_3(ie^x)$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.79

$$\frac{1}{2}i\left(x\text{Li}_2(-ie^x) - x\text{Li}_2(ie^x) - \text{Li}_3(-ie^x) + \text{Li}_3(ie^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*(x*PolyLog[2, (-I)*E^x] - x*PolyLog[2, I*E^x] - PolyLog[3, (-I)*E^x]
+ PolyLog[3, I*E^x])
```

fricas [C] time = 0.45, size = 65, normalized size = 1.03

$$\frac{1}{2}x^2\arctan(e^x) + \frac{1}{4}ix^2\log(ie^x+1) - \frac{1}{4}ix^2\log(-ie^x+1) - \frac{1}{2}ix\text{Li}_2(ie^x) + \frac{1}{2}ix\text{Li}_2(-ie^x) + \frac{1}{2}i\,\text{polylog}(3, ie^x) - \frac{1}{2}i\,\text{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(e^x) + 1/4*I*x^2*log(I*e^x + 1) - 1/4*I*x^2*log(-I*e^x + 1)
- 1/2*I*x*dilog(I*e^x) + 1/2*I*x*dilog(-I*e^x) + 1/2*I*polylog(3, I*e^x) -
1/2*I*polylog(3, -I*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\arctan(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(e^x), x)
```

maple [A] time = 0.13, size = 44, normalized size = 0.70

$$\frac{ix\,\text{polylog}(2,-ie^x)}{2} - \frac{ix\,\text{polylog}(2,ie^x)}{2} - \frac{i\,\text{polylog}(3,-ie^x)}{2} + \frac{i\,\text{polylog}(3,ie^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(x)),x)
```

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\arctan(e^x) - \int \frac{x^2e^x}{2\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^x) - integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{atan}\left(\mathrm{e}^{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(exp(x)),x)
```

```
[Out] int(x*atan(exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atan}\left(e^{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(x)),x)
```

```
[Out] Integral(x*atan(exp(x)), x)
```

3.112 $\int x^2 \tan^{-1}(e^x)\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}ix^2\text{Li}_2\left(-ie^x\right)-\frac{1}{2}ix^2\text{Li}_2\left(ie^x\right)-ix\text{Li}_3\left(-ie^x\right)+ix\text{Li}_3\left(ie^x\right)+i\text{Li}_4\left(-ie^x\right)-i\text{Li}_4\left(ie^x\right)$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5143, 2531, 6609, 2282, 6589}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix^2\text{PolyLog}\left(2,ie^x\right)-ix\text{PolyLog}\left(3,-ie^x\right)+ix\text{PolyLog}\left(3,ie^x\right)+i\text{PolyLog}\left(4,-ie^x\right)-i\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(e^x)\,dx = \frac{1}{2}i\int x^2 \log(1-ie^x)\,dx - \frac{1}{2}i\int x^2\log(1+ie^x)\,dx$$
$$= \frac{1}{2}ix^2\text{Li}_2(-ie^x) - \frac{1}{2}ix^2\text{Li}_2(ie^x) - i\int x\text{Li}_2(-ie^x)\,dx + i\int x\text{Li}_2(ie^x)\,dx$$
$$= \frac{1}{2}ix^2\text{Li}_2(-ie^x) - \frac{1}{2}ix^2\text{Li}_2(ie^x) - ix\text{Li}_3(-ie^x) + ix\text{Li}_3(ie^x) + i\int \text{Li}_3(-ie^x)\,dx - i\int \text{Li}_3(i$$
$$= \frac{1}{2}ix^2\text{Li}_2(-ie^x) - \frac{1}{2}ix^2\text{Li}_2(ie^x) - ix\text{Li}_3(-ie^x) + ix\text{Li}_3(ie^x) + i\,\text{Subst}\left(\int \frac{\text{Li}_3(-ix)}{x}\,dx, x, e^x$$
$$= \frac{1}{2}ix^2\text{Li}_2(-ie^x) - \frac{1}{2}ix^2\text{Li}_2(ie^x) - ix\text{Li}_3(-ie^x) + ix\text{Li}_3(ie^x) + i\text{Li}_4(-ie^x) - i\text{Li}_4(ie^x)$$

Mathematica [A] time = 0.01, size = 91, normalized size = 1.00

$$\frac{1}{2}ix^2\text{Li}_2(-ie^x) - \frac{1}{2}ix^2\text{Li}_2(ie^x) - ix\text{Li}_3(-ie^x) + ix\text{Li}_3(ie^x) + i\text{Li}_4(-ie^x) - i\text{Li}_4(ie^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^x],x]
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

fricas [C] time = 0.48, size = 87, normalized size = 0.96

$$\frac{1}{3}x^3\arctan(e^x) + \frac{1}{6}i\,x^3\log(i\,e^x+1) - \frac{1}{6}i\,x^3\log(-i\,e^x+1) - \frac{1}{2}i\,x^2\text{Li}_2(i\,e^x) + \frac{1}{2}i\,x^2\text{Li}_2(-i\,e^x) + i\,x\text{polylog}(3, i\,e^x) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="fricas")
[Out] 1/3*x^3*arctan(e^x) + 1/6*I*x^3*log(I*e^x + 1) - 1/6*I*x^3*log(-I*e^x + 1)
- 1/2*I*x^2*dilog(I*e^x) + 1/2*I*x^2*dilog(-I*e^x) + I*x*polylog(3, I*e^x)
- I*x*polylog(3, -I*e^x) - I*polylog(4, I*e^x) + I*polylog(4, -I*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\arctan(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="giac")
[Out] integrate(x^2*arctan(e^x), x)
```

maple [A] time = 0.12, size = 70, normalized size = 0.77

$$\frac{ix^2\,\text{polylog}(2, -ie^x)}{2} - \frac{ix^2\,\text{polylog}(2, ie^x)}{2} - ix\,\text{polylog}(3, -ie^x) + ix\,\text{polylog}(3, ie^x) + i\,\text{polylog}(4, -ie^x) - i\,\text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(x)),x)
```

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3\arctan\left(e^x\right) - \int \frac{x^3e^x}{3\left(e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^x) - integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(exp(x)),x)
```

```
[Out] int(x^2*atan(exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{atan}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(x)),x)
```

```
[Out] Integral(x**2*atan(exp(x)), x)
```

3.113 $\int \tan^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=45

$$\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

```
[Out] 1/2*I*polylog(2,-I*exp(b*x+a))/b-1/2*I*polylog(2,I*exp(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 4848, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*PolyLog[2, I*E^(a + b*x)])/
b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(e^{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{2b}-\frac{i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 83, normalized size = 1.84

$$x\tan^{-1}\left(e^{a+bx}\right)-\frac{i\left(-\text{Li}_2\left(-ie^{a+bx}\right)+\text{Li}_2\left(ie^{a+bx}\right)+bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^(a + b*x)],x]
```

```
[Out] x*ArcTan[E^(a + b*x)] - ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

fricas [B] time = 0.50, size = 103, normalized size = 2.29

$$\frac{2\,bx\arctan\left(e^{(bx+a)}\right)+i\,a\log\left(e^{(bx+a)}+i\right)-i\,a\log\left(e^{(bx+a)}-i\right)+(i\,bx+i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(e^(b*x + a)) + I*a*log(e^(b*x + a) + I) - I*a*log(e^(b*x
+ a) - I) + (I*b*x + I*a)*log(I*e^(b*x + a) + 1) + (-I*b*x - I*a)*log(-I*e^
(b*x + a) + 1) - I*dilog(I*e^(b*x + a)) + I*dilog(-I*e^(b*x + a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(e^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^(b*x + a)), x)
```

maple [B] time = 0.06, size = 106, normalized size = 2.36

$$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\arctan\left(\mathrm{e}^{bx+a}\right)}{b}+\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1-i\mathrm{e}^{bx+a}\right)}{2b}+\frac{i\,\mathrm{dilog}\left(1+i\mathrm{e}^{bx+a}\right)}{2b}-\frac{i\,\mathrm{dilog}\left(1\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(b*x+a)),x)
```

```
[Out] 1/b*ln(exp(b*x+a))*arctan(exp(b*x+a))+1/2*I/b*ln(exp(b*x+a))*ln(1+I*exp(b*x
+a))-1/2*I/b*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))+1/2*I/b*dilog(1+I*exp(b*x+a)
)-1/2*I/b*dilog(1-I*exp(b*x+a))
```

maxima [B] time = 0.44, size = 63, normalized size = 1.40

$$\frac{(bx+a)\arctan\left(e^{(bx+a)}\right)}{b}-\frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctan(e^(b*x + a))/b - 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

mupad [B] time = 0.75, size = 37, normalized size = 0.82

$$-\frac{\mathrm{Li}_2\left(1-\mathrm{e}^{b\,x}\,\mathrm{e}^a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}+\frac{\mathrm{Li}_2\left(1+\mathrm{e}^{b\,x}\,\mathrm{e}^a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(a + b*x)),x)
```

```
[Out] (dilog(exp(b*x)*exp(a)*1i + 1)*1i)/(2*b) - (dilog(1 - exp(b*x)*exp(a)*1i)*1
i)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(e^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(b*x+a)),x)
```

```
[Out] Integral(atan(exp(a + b*x)), x)
```

3.114 $\int x \tan^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=91

$$-\frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{2b^2}+\frac{i\text{Li}_3\left(ie^{a+bx}\right)}{2b^2}+\frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(b*x+a))/b-1/2*I*x*polylog(2,I*exp(b*x+a))/b-1/2*I*
polylog(3,-I*exp(b*x+a))/b^2+1/2*I*polylog(3,I*exp(b*x+a))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5143, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}+\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x*PolyLog[2, I*E^(a + b*x
)])/b - ((I/2)*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + ((I/2)*PolyLog[3, I*E^(a
 + b*x)])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}\left(e^{a+bx}\right)dx = \frac{1}{2}i\int x\log\left(1-ie^{a+bx}\right)dx - \frac{1}{2}i\int x\log\left(1+ie^{a+bx}\right)dx$$
$$= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{i\int \text{Li}_2\left(-ie^{a+bx}\right)dx}{2b} + \frac{i\int \text{Li}_2\left(ie^{a+bx}\right)dx}{2b}$$
$$= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{i\,\text{Subst}\left(\int \frac{\text{Li}_2(-ix)}{x}dx, x, e^{a+bx}\right)}{2b^2} + \frac{i\,\text{Subst}\left(\int \frac{\text{Li}_2(ix)}{x}dx\right.}{2b^2}$$
$$= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{2b^2} + \frac{i\text{Li}_3\left(ie^{a+bx}\right)}{2b^2}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.78

$$\frac{i\left(bx\text{Li}_2\left(-ie^{a+bx}\right) - bx\text{Li}_2\left(ie^{a+bx}\right) - \text{Li}_3\left(-ie^{a+bx}\right) + \text{Li}_3\left(ie^{a+bx}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^(a + b*x)],x]
[Out] ((I/2)*(b*x*PolyLog[2, (-I)*E^(a + b*x)] - b*x*PolyLog[2, I*E^(a + b*x)] -
PolyLog[3, (-I)*E^(a + b*x)] + PolyLog[3, I*E^(a + b*x)]))/b^2
```

fricas [C] time = 0.80, size = 151, normalized size = 1.66

$$\frac{2\,b^2x^2\arctan\left(e^{(bx+a)}\right) - 2i\,bx\text{Li}_2\left(i\,e^{(bx+a)}\right) + 2i\,bx\text{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^2\log\left(e^{(bx+a)}+i\right) + i\,a^2\log\left(e^{(bx+a)}-i\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="fricas")
[Out] 1/4*(2*b^2*x^2*arctan(e^(b*x + a)) - 2*I*b*x*dilog(I*e^(b*x + a)) + 2*I*b*x
*dilog(-I*e^(b*x + a)) - I*a^2*log(e^(b*x + a) + I) + I*a^2*log(e^(b*x + a)
 - I) + (I*b^2*x^2 - I*a^2)*log(I*e^(b*x + a) + 1) + (-I*b^2*x^2 + I*a^2)*l
og(-I*e^(b*x + a) + 1) + 2*I*polylog(3, I*e^(b*x + a)) - 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\arctan\left(e^{(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(e^(b*x + a)), x)
```

maple [B] time = 0.23, size = 349, normalized size = 3.84

$$-\frac{i\ln\left(1-ie^{bx+a}\right)xa}{2b} + \frac{i\ln\left(-i\left(e^{bx+a}+i\right)\right)xa}{2b} - \frac{i\,\text{polylog}\left(3,-ie^{bx+a}\right)}{2b^2} + \frac{i\ln\left(1+ie^{bx+a}\right)xa}{2b} - \frac{ix\,\text{polylog}\left(2,ie^{bx+a}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(b*x+a)),x)
[Out] 1/2*I/b^2*dilog(-I*(exp(b*x+a)+I))*a-1/2*I/b^2*polylog(2,I*exp(b*x+a))*a-1/
2*I/b*ln(1-I*exp(b*x+a))*x*a+1/2*I/b^2*dilog(-I*exp(b*x+a))*a+1/2*I/b*ln(-I
```

```
*(exp(b*x+a)+I))*x*a-1/2*I*x*polylog(2,I*exp(b*x+a))/b+1/2*I/b*ln(1+I*exp(b
*x+a))*x*a-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a))-1/2*I/b^2*ln(-I*(-exp(b*x+a)+I)
)*a^2+1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I*x*polylog(2,-I*exp(b*x+a))
/b+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(-I*exp(b*x+a))*ln(-I*(-
exp(b*x+a)+I))*a+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a))-1/2*I/b*ln(-I*(-exp(b*x+a
)+I))*x*a-1/2*I*polylog(3,-I*exp(b*x+a))/b^2+1/2*I*polylog(3,I*exp(b*x+a))/
b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2 \arctan\left(e^{(bx+a)}\right) - b \int \frac{x^2 e^{(bx+a)}}{2\left(e^{(2\,bx+2\,a)} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(b*x + a)) - b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(exp(a + b*x)),x)
```

```
[Out] int(x*atan(exp(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atan}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x*atan(exp(a)*exp(b*x)), x)
```

3.115 $\int x^2 \tan^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=133

$$\frac{i\text{Li}_4\left(-ie^{a+bx}\right)}{b^3}-\frac{i\text{Li}_4\left(ie^{a+bx}\right)}{b^3}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b-1/2*I*x^2*polylog(2,I*exp(b*x+a))/b-I*
x*polylog(3,-I*exp(b*x+a))/b^2+I*x*polylog(3,I*exp(b*x+a))/b^2+I*polylog(4,
-I*exp(b*x+a))/b^3-I*polylog(4,I*exp(b*x+a))/b^3
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5143, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}+\frac{ix^2\text{PolyLog}(\ldots}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}\left(e^{a+bx}\right) dx = \frac{1}{2}i \int x^2 \log\left(1 - ie^{a+bx}\right) dx - \frac{1}{2}i \int x^2 \log\left(1 + ie^{a+bx}\right) dx$$
$$= \frac{ix^2 \text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix^2 \text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{i \int x \text{Li}_2\left(-ie^{a+bx}\right) dx}{b} + \frac{i \int x \text{Li}_2\left(ie^{a+bx}\right) dx}{b}$$
$$= \frac{ix^2 \text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix^2 \text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{a+bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{a+bx}\right)}{b^2} + \frac{i \int \text{Li}_3\left(-ie^{a+bx}\right)}{b^2}$$
$$= \frac{ix^2 \text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix^2 \text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{a+bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{a+bx}\right)}{b^2} + \frac{i \,\text{Subst}\left(\int \frac{\text{Li}_3(-ix)}{x}\right.}{b^3}$$
$$= \frac{ix^2 \text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix^2 \text{Li}_2\left(ie^{a+bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{a+bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{a+bx}\right)}{b^2} + \frac{i \text{Li}_4\left(-ie^{a+bx}\right)}{b^3} -$$

Mathematica [A] time = 0.01, size = 133, normalized size = 1.00

$$\frac{i \text{Li}_4\left(-ie^{a+bx}\right)}{b^3} - \frac{i \text{Li}_4\left(ie^{a+bx}\right)}{b^3} - \frac{ix \text{Li}_3\left(-ie^{a+bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{a+bx}\right)}{b^2} + \frac{ix^2 \text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{ix^2 \text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

fricas [C] time = 0.52, size = 187, normalized size = 1.41

$$\frac{2\, b^3 x^3 \arctan\left(e^{(bx+a)}\right) - 3i\, b^2 x^2 \text{Li}_2\left(i\, e^{(bx+a)}\right) + 3i\, b^2 x^2 \text{Li}_2\left(-i\, e^{(bx+a)}\right) + i\, a^3 \log\left(e^{(bx+a)} + i\right) - i\, a^3 \log\left(e^{(bx+a)} - i\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(e^(b*x + a)) - 3*I*b^2*x^2*dilog(I*e^(b*x + a)) + 3*I
*b^2*x^2*dilog(-I*e^(b*x + a)) + I*a^3*log(e^(b*x + a) + I) - I*a^3*log(e^(
b*x + a) - I) + 6*I*b*x*polylog(3, I*e^(b*x + a)) - 6*I*b*x*polylog(3, -I*e
^(b*x + a)) + (I*b^3*x^3 + I*a^3)*log(I*e^(b*x + a) + 1) + (-I*b^3*x^3 - I*
a^3)*log(-I*e^(b*x + a) + 1) - 6*I*polylog(4, I*e^(b*x + a)) + 6*I*polylog(
4, -I*e^(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arctan\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(e^(b*x + a)), x)
```

maple [B] time = 0.23, size = 407, normalized size = 3.06

$$-\frac{i\,\mathrm{dilog}\left(-i\left(\mathrm{e}^{bx+a}+i\right)\right)a^{2}}{2b^{3}}-\frac{i\ln\left(-i\left(\mathrm{e}^{bx+a}+i\right)\right)a^{3}}{2b^{3}}+\frac{i\,\mathrm{polylog}\left(2,i\mathrm{e}^{bx+a}\right)a^{2}}{2b^{3}}-\frac{ix\,\mathrm{polylog}\left(3,-i\mathrm{e}^{bx+a}\right)}{b^{2}}+\frac{i\ln\left(-i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(b*x+a)),x)
```

```
[Out] -1/2*I*x^2*polylog(2,I*exp(b*x+a))/b+I*polylog(4,-I*exp(b*x+a))/b^3-1/2*I/b
^3*ln(-I*(exp(b*x+a)+I))*a^3-1/2*I/b^3*polylog(2,-I*exp(b*x+a))*a^2-I*x*pol
ylog(3,-I*exp(b*x+a))/b^2+1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*x*a^2-1/2*I/b^3*
dilog(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a))-1/2*I/b^3*a^3
*ln(1+I*exp(b*x+a))-I*polylog(4,I*exp(b*x+a))/b^3-1/2*I/b^3*ln(-I*exp(b*x+a
))*ln(-I*(-exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2+1/2*I*x^2*
polylog(2,-I*exp(b*x+a))/b-1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2+1/2*I/b^3*pol
ylog(2,I*exp(b*x+a))*a^2-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*x*a^2+I*x*polylog(
3,I*exp(b*x+a))/b^2+1/2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3-1/2*I/b^2*ln(1+I*e
xp(b*x+a))*x*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^{3}\arctan\left(e^{(bx+a)}\right)-b\int\frac{x^{3}e^{(bx+a)}}{3\left(e^{(2\,bx+2\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(b*x + a)) - b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\mathrm{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(exp(a + b*x)),x)
```

```
[Out] int(x^2*atan(exp(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\,\mathrm{atan}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x**2*atan(exp(a)*exp(b*x)), x)
```

3.116 $\int \tan^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=196

$$\frac{i\text{Li}_2\left(1-\frac{2}{1-i\left(bf^{c+dx}+a\right)}\right)}{2d\log(f)}-\frac{i\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)\tan^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+dx}}{(-a+i)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{d\log}$$

```
[Out] -arctan(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arctan(a+b*f^(d*
x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2
,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a)/(
1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

Rubi [A] time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 5047, 4856, 2402, 2315, 2447}

$$\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{2d\log(f)}-\frac{i\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(-a+i)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)\tan^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\text{log}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcTan[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
```

```
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \tan^{-1}\left(a+bf^{c+dx}\right)\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(a+bx)}{x}\,dx,x,f^{c+dx}\right)}{d\log(f)}$$

$$= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx,x,a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right.}{d\log(f)}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right.}{d\log(f)}$$

$$= -\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c+dx}\right)\right)}\right.}{d\log(f)}$$

Mathematica [A] time = 0.20, size = 167, normalized size = 0.85

$$x\tan^{-1}\left(a+bf^{c+dx}\right) - \frac{b\left(\text{Li}_2\left(-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{Li}_2\left(-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)+dx\log(f)\left(\log\left(\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}+1\right)-\log\left(\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}+1\right)\right.\right.}{2\sqrt{-b^2}\,d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcTan[a + b*f^(c + d*x)] - (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

fricas [A] time = 1.03, size = 212, normalized size = 1.08

$$\frac{2\,dx\arctan\left(bf^{dx+c}+a\right)\log(f)+i\,c\log\left(bf^{dx+c}+a+i\right)\log(f)-i\,c\log\left(bf^{dx+c}+a-i\right)\log(f)+(i\,dx+i\,c)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*arctan(b*f^(d*x + c) + a)*log(f) + I*c*log(b*f^(d*x + c) + a + I
)*log(f) - I*c*log(b*f^(d*x + c) + a - I)*log(f) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I*d*x - I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) - I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \arctan\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arctan(b*f^(d*x + c) + a), x)
```

maple [A] time = 0.06, size = 186, normalized size = 0.95

$$\frac{\ln\left(b\,f^{dx+c}\right)\arctan\left(a+b\,f^{dx+c}\right)}{d\ln(f)}+\frac{i\ln\left(b\,f^{dx+c}\right)\ln\left(\frac{-b\,f^{dx+c}-a+i}{i-a}\right)}{2d\ln(f)}-\frac{i\ln\left(b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2d\ln(f)}+\frac{i\,\mathrm{dilog}\left(\frac{-b\,f^{dx+c}-a+i}{i-a}\right.}{2d\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a+b*f^(d*x+c)),x)
```

```
[Out] 1/d/ln(f)*ln(b*f^(d*x+c))*arctan(a+b*f^(d*x+c))+1/2*I/d/ln(f)*ln(b*f^(d*x+c
))*ln((-b*f^(d*x+c)-a+I)/(I-a))-1/2*I/d/ln(f)*ln(b*f^(d*x+c))*ln((I+b*f^(d*
x+c)+a)/(I+a))+1/2*I/d/ln(f)*dilog((-b*f^(d*x+c)-a+I)/(I-a))-1/2*I/d/ln(f)*
dilog((I+b*f^(d*x+c)+a)/(I+a))
```

maxima [A] time = 0.50, size = 189, normalized size = 0.96

$$\frac{(dx+c)\arctan\left(bf^{dx+c}+a\right)}{d}-\frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log(f)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arctan(b*f^(d*x + c) + a)/d - 1/2*(2*(d*x + c)*arctan((b^2*f^(d*x
 + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a*b
*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2*c)
/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b
*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(a+b\,f^{c+d\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*f^(c + d*x)),x)
```

```
[Out] int(atan(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

3.117 $\int x \tan^{-1}\left(a+bf^{c+dx}\right)dx$

Optimal. Leaf size=232

$$\frac{i\text{Li}_3\left(\frac{ibf^{c+dx}}{1-ia}\right)}{2d^2\log^2(f)}-\frac{i\text{Li}_3\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{2d^2\log^2(f)}-\frac{ix\text{Li}_2\left(\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix\text{Li}_2\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{2d\log(f)}-\frac{1}{4}ix^2\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)+\frac{1}{4}ix^2\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)+$$

```
[Out] 1/2*x^2*arctan(a+b*f^(d*x+c))-1/4*I*x^2*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/4*I*x
^2*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d/l
n(f)+1/2*I*x*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+1/2*I*polylog(3,I*b*
f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-1/2*I*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2
```

Rubi [A] time = 0.15, antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5143, 2532, 2531, 2282, 6589}

$$-\frac{i\text{PolyLog}\left(3,\frac{bf^{c+dx}}{-a+i}\right)}{2d^2\log^2(f)}+\frac{i\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)}+\frac{ix\text{PolyLog}\left(2,\frac{bf^{c+dx}}{-a+i}\right)}{2d\log(f)}-\frac{ix\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}+\frac{1}{4}ix^2\log\left(-ia-i\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/4)*x^2*Log[1 - I*a - I*b*f^(c + d*x)] - (I/4)*x^2*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/4)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/4)*x^2*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*
Log[f]) - ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) - ((I
/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + ((I/2)*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{1}{2}i\int x\log\left(1-ia-ibf^{c+dx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ia+ibf^{c+dx}\right)\,dx$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$
$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

Mathematica [A] time = 0.10, size = 236, normalized size = 1.02

$$\frac{i\left(d^2x^2\log^2(f)\log\left(-ia-ibf^{c+dx}+1\right)-d^2x^2\log^2(f)\log\left(ia+ibf^{c+dx}+1\right)-d^2x^2\log^2(f)\log\left(\frac{a+bf^{c+dx}+i}{a+i}\right)+\cdots\right.}{4d^2\,\mathrm{l}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] ((I/4)*(d^2*x^2*Log[f]^2*Log[1 - I*a - I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*
Log[1 + I*a + I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*Log[(I + a + b*f^(c + d*x
))/(I + a)] + d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(-I + a)] + 2*d*x*Lo
g[f]*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 2*d*x*Log[f]*PolyLog[2, -((b*f^(
c + d*x))/(I + a))] - 2*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 2*PolyLog[3,
-((b*f^(c + d*x))/(I + a))]))/(d^2*Log[f]^2)
```

fricas [C] time = 0.45, size = 304, normalized size = 1.31

$$\frac{2\,d^2x^2\arctan\left(bf^{dx+c}+a\right)\log(f)^2-i\,c^2\log\left(bf^{dx+c}+a+i\right)\log(f)^2+i\,c^2\log\left(bf^{dx+c}+a-i\right)\log(f)^2+2i\,dx\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arctan(b*f^(d*x + c) + a)*log(f)^2 - I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 + I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 + 2*I*d*x*di
```

```
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) - 2*I*d*x*di
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (I*d^2*x^2
 - I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I
*d^2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) - 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arctan\left(bf^{dx+c} + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(b*f^(d*x + c) + a), x)
```

maple [B] time = 0.32, size = 672, normalized size = 2.90

$$-\frac{ix^2\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{4}+\frac{ic^2\ln\left(\frac{f^{dx}f^{c}b+i+a}{i+a}\right)}{2d^2}+\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)x^2}{4}-\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia+1}\right)c^2}{4d^2}+\frac{ic\,\mathrm{dilog}\left(\frac{f^{dx}f^{c}b+i+a}{i+a}\right)}{2d^2\ln(f)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a+b*f^(d*x+c)),x)
```

```
[Out] -1/4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))+1/2*I/d^2*c^2*ln((f^(d*x)*f^c*b+I+a)/(I+
a))-1/2*I/d^2/ln(f)*c*dilog((f^(d*x)*f^c*b+a-I)/(-I+a))+1/4*I*ln(1-I*b/(-I*
a-1)*f^(d*x)*f^c)*x^2-1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2-1/2*I/d^2
/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c+1/2*I/d^2/ln(f)*c*dilog((f^(d*x
)*f^c*b+I+a)/(I+a))-1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/4*I/d^2*ln(1-
I*b/(-I*a-1)*f^(d*x)*f^c)*c^2-1/2*I/d*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x*c+1/2
*I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^c)-1/2*I/d*c*ln((f^(d*x)*f^c
*b+a-I)/(-I+a))*x-1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)+1/4*I/d^2*c^2*ln(
I*f^(d*x)*f^c*b+I*a+1)-1/2*I/d^2*c^2*ln((f^(d*x)*f^c*b+a-I)/(-I+a))+1/2*I/d
^2/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c+1/2*I/d*ln(1-I*b/(-I*a-1)*f^
(d*x)*f^c)*x*c+1/2*I/d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/d^
2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/d*c*ln((f^(d*x)*f^c*b+I
+a)/(I+a))*x+1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))-1/2*I/d/ln(f)*polylog(2,I*b/
(1-I*a)*f^(d*x)*f^c)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bdf^{c}\int\frac{f^{dx}x^2}{2\left(b^2f^{2\,dx}f^{2\,c}+2\,abf^{dx}f^{c}+a^2+1\right)}\,dx\log(f)+\frac{1}{2}\,x^2\arctan\left(bf^{dx}f^{c}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(b*f^(d*x)*f^c + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a + b*f^(c + d*x)),x)
```

```
[Out] int(x*atan(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

3.118 $\int x^2 \tan^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=302

$$-\frac{i\text{Li}_4\left(\frac{ibf^{c+dx}}{1-ia}\right)}{d^3\log^3(f)}+\frac{i\text{Li}_4\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{d^3\log^3(f)}+\frac{ix\text{Li}_3\left(\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)}-\frac{ix\text{Li}_3\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{d^2\log^2(f)}-\frac{ix^2\text{Li}_2\left(\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix^2\text{Li}_2\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{2d\log(f)}-\frac{1}{6}ix^3\log\left(1\right.$$

```
[Out] 1/3*x^3*arctan(a+b*f^(d*x+c))-1/6*I*x^3*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/6*I*x
^3*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x^2*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d
/ln(f)+1/2*I*x^2*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+I*x*polylog(3,I*
b*f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-I*x*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2-I*polylog(4,I*b*f^(d*x+c)/(1-I*a))/d^3/ln(f)^3+I*polylog(4,-I*b*f^(
d*x+c)/(1+I*a))/d^3/ln(f)^3
```

Rubi [A] time = 0.20, antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5143, 2532, 2531, 6609, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(3,\frac{bf^{c+dx}}{-a+i}\right)}{d^2\log^2(f)}+\frac{ix\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}+\frac{i\text{PolyLog}\left(4,\frac{bf^{c+dx}}{-a+i}\right)}{d^3\log^3(f)}-\frac{i\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)}+\frac{ix^2\text{PolyLog}\left(2,\right.}{2d\log(f)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/6)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/6)*x^3*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(
d*Log[f]) - ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) -
 (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + (I*x*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) + (I*PolyLog[4, (b*f^(c + d*x)
)/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/(d^
3*Log[f]^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5143

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}\left(a+bf^{c+dx}\right) dx = \frac{1}{2}i\int x^2 \log\left(1-ia-ibf^{c+dx}\right) dx - \frac{1}{2}i\int x^2 \log\left(1+ia+ibf^{c+dx}\right) dx$$

$$= \frac{1}{6}ix^3 \log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3 \log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3 \log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

$$= \frac{1}{6}ix^3 \log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3 \log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3 \log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

$$= \frac{1}{6}ix^3 \log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3 \log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3 \log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

$$= \frac{1}{6}ix^3 \log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3 \log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3 \log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

$$= \frac{1}{6}ix^3 \log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3 \log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3 \log\left(1-\frac{bf^{c+dx}}{i-a}\right.$$

Mathematica [A] time = 0.02, size = 334, normalized size = 1.11

$$\frac{i\text{Li}_4\left(\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)} - \frac{i\text{Li}_4\left(-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)} + \frac{ix\text{Li}_3\left(\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)} - \frac{ix\text{Li}_3\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{d^2\log^2(f)} - \frac{ix^2\text{Li}_2\left(\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)} + \frac{ix^2\text{Li}_2\left(-\frac{ibf^{c+dx}}{ia+1}\right)}{2d\log(f)} + \frac{1}{6}ix^3\log\left(-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] - (I/6)*x^3*Log[1 - (I*b*f^(c + d*x))/(1 - I*a)] + (I/6)*x^3*Log[1
 + (I*b*f^(c + d*x))/(1 + I*a)] - ((I/2)*x^2*PolyLog[2, (I*b*f^(c + d*x))/(
1 - I*a)])/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, ((-I)*b*f^(c + d*x))/(1 + I*a
)])/(d*Log[f]) + (I*x*PolyLog[3, (I*b*f^(c + d*x))/(1 - I*a)])/(d^2*Log[f]^
```

```
2) - (I*x*PolyLog[3, ((-I)*b*f^(c + d*x))/(1 + I*a)])/(d^2*Log[f]^2) + (I*P
olyLog[4, (b*f^(c + d*x))/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(
c + d*x))/(I + a))])/(d^3*Log[f]^3)
```

fricas [C] time = 0.81, size = 378, normalized size = 1.25

$$\frac{2\,d^3x^3\arctan\left(bf^{dx+c}+a\right)\log(f)^3+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log(f)^2-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arctan(b*f^(d*x + c) + a)*log(f)^3 + 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 - I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2
 + 1)) + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) + 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\arctan\left(bf^{dx+c}+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(b*f^(d*x + c) + a), x)
```

maple [B] time = 0.29, size = 758, normalized size = 2.51

$$\frac{ix^3\ln\left(1-i\left(a+b\,f^{dx+c}\right)\right)}{6}+\frac{i\,\mathrm{polylog}\left(2,\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)x^2}{2d\ln(f)}-\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)c^3}{3d^3}+\frac{ic^2\ln\left(\frac{f^{dx}f^{c}b+a-i}{-i+a}\right)x}{2d^2}+\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a+b*f^(d*x+c)),x)
```

```
[Out] -1/2*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/6*I*x^3*ln(1-I*(a+b
*f^(d*x+c)))-1/2*I/d^3/ln(f)*c^2*dilog((f^(d*x)*f^c*b+I+a)/(I+a))+I/d^3/ln(
f)^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)-1/3*I/d^3*ln(1-I*b/(-I*a-1)*f^(d*x
)*f^c)*c^3+1/2*I/d^2*c^2*ln((f^(d*x)*f^c*b+a-I)/(-I+a))*x+1/6*I*ln(1-I*b/(-
I*a-1)*f^(d*x)*f^c)*x^3-1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))-1/2*I/d^3/ln(f)*p
olylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2+1/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c
*b)-1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c^2+1/2*I/d^3*c^3*ln((f^(d*x
)*f^c*b+a-I)/(-I+a))-1/2*I/d^2*c^2*ln((f^(d*x)*f^c*b+I+a)/(I+a))*x+1/2*I/d^
3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+I/d^2/ln(f)^2*polylog(3,I*b/
(1-I*a)*f^(d*x)*f^c)*x+1/2*I/d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x^
2-1/2*I/d^3*c^3*ln((f^(d*x)*f^c*b+I+a)/(I+a))-I/d^2/ln(f)^2*polylog(3,I*b/(
-I*a-1)*f^(d*x)*f^c)*x-I/d^3/ln(f)^3*polylog(4,I*b/(1-I*a)*f^(d*x)*f^c)+1/2
*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x*c^2-1/6*I/d^3*c^3*ln(I*f^(d*x)*f^c*b
+I*a+1)-1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^3+1/3*I/d^3*ln(1-I*b/(1-I*a)*
f^(d*x)*f^c)*c^3+1/2*I/d^3/ln(f)*c^2*dilog((f^(d*x)*f^c*b+a-I)/(-I+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bdf^{c}\int \frac{f^{dx}x^{3}}{3\left(b^{2}f^{2\,dx}f^{2\,c}+2\,abf^{dx}f^{c}+a^{2}+1\right)}\,dx\log(f)+\frac{1}{3}\,x^{3}\arctan\left(bf^{dx}f^{c}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(b*f^(d*x)*f^c + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\,\mathrm{atan}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a + b*f^(c + d*x)),x)
```

```
[Out] int(x^2*atan(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

3.119 $\int e^{-x} \tan^{-1}(e^x)\, dx$

Optimal. Leaf size=25

$$x - \frac{1}{2}\log\left(e^{2x}+1\right) - e^{-x}\tan^{-1}(e^x)$$

```
[Out] x-arctan(exp(x))/exp(x)-1/2*ln(1+exp(2*x))
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2194, 5207, 2282, 36, 29, 31}

$$x - \frac{1}{2}\log\left(e^{2x}+1\right) - e^{-x}\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^x]/E^x,x]
```

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{-x}\tan^{-1}(e^x)\,dx &= -e^{-x}\tan^{-1}(e^x)+\int \frac{1}{1+e^{2x}}\,dx\\ &= -e^{-x}\tan^{-1}(e^x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\ &= -e^{-x}\tan^{-1}(e^x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\ &= x-e^{-x}\tan^{-1}(e^x)-\frac{1}{2}\log\left(1+e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$x-\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x]/E^x,x]
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

fricas [A] time = 0.55, size = 28, normalized size = 1.12

$$\frac{1}{2}\left(2\,xe^{x}-e^{x}\log\left(e^{(2\,x)}+1\right)-2\,\arctan\left(e^{x}\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="fricas")
[Out] 1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) - 2*arctan(e^x))*e^(-x)
```

giac [A] time = 0.11, size = 20, normalized size = 0.80

$$-\arctan\left(e^{x}\right)e^{(-x)}+x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^x)*e^(-x) + x - 1/2*log(e^(2*x) + 1)
```

maple [A] time = 0.04, size = 23, normalized size = 0.92

$$-\arctan\left(\mathrm{e}^{x}\right)\mathrm{e}^{-x}+\ln\left(\mathrm{e}^{x}\right)-\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x))/exp(x),x)
[Out] -arctan(exp(x))/exp(x)+ln(exp(x))-1/2*ln(exp(x)^2+1)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$-\arctan\left(e^{x}\right)e^{(-x)}-\frac{1}{2}\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="maxima")
[Out] -arctan(e^x)*e^(-x) - 1/2*log(e^(-2*x) + 1)
```

mupad [B] time = 0.09, size = 20, normalized size = 0.80

$$x - \frac{\ln\left(e^{2x} + 1\right)}{2} - \operatorname{atan}\left(e^{x}\right) e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(x))*exp(-x),x)
[Out] x - log(exp(2*x) + 1)/2 - atan(exp(x))*exp(-x)
```

sympy [A] time = 6.36, size = 19, normalized size = 0.76

$$x - \frac{\log\left(e^{2x} + 1\right)}{2} - e^{-x} \operatorname{atan}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x))/exp(x),x)
[Out] x - log(exp(2*x) + 1)/2 - exp(-x)*atan(exp(x))
```

3.120 $\int \frac{\tan^{-1}(x)}{(-1+x)^3}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{4}\log(1-x)-\frac{\tan^{-1}(x)}{2(1-x)^2}$$

```
[Out] 1/4/(1-x)-1/2*arctan(x)/(1-x)^2-1/4*ln(1-x)+1/8*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4862, 710, 801, 260}

$$\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{4}\log(1-x)-\frac{\tan^{-1}(x)}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] 1/(4*(1 - x)) - ArcTan[x]/(2*(1 - x)^2) - Log[1 - x]/4 + Log[1 + x^2]/8
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(x)}{(-1+x)^3}\,dx &= -\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{2}\int \frac{1}{(-1+x)^2\left(1+x^2\right)}\,dx\\ &= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{4}\int \frac{-1-x}{(-1+x)\left(1+x^2\right)}\,dx\\ &= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{4}\int \left(\frac{1}{1-x}+\frac{x}{1+x^2}\right)dx\\ &= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{4}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{8}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.78

$$\frac{1}{8}\left(\log\left(x^2+1\right)-\frac{2}{x-1}-2\log(1-x)-\frac{4\tan^{-1}(x)}{(x-1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] (-2/(-1 + x) - (4*ArcTan[x])/(-1 + x)^2 - 2*Log[1 - x] + Log[1 + x^2])/8
```

fricas [A] time = 0.67, size = 50, normalized size = 1.11

$$\frac{\left(x^2-2x+1\right)\log\left(x^2+1\right)-2\left(x^2-2x+1\right)\log\left(x-1\right)-2x-4\arctan(x)+2}{8\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((x^2 - 2*x + 1)*log(x^2 + 1) - 2*(x^2 - 2*x + 1)*log(x - 1) - 2*x - 4*
arctan(x) + 2)/(x^2 - 2*x + 1)
```

giac [A] time = 0.11, size = 32, normalized size = 0.71

$$-\frac{1}{4(x-1)}-\frac{\arctan(x)}{2(x-1)^2}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(abs(x -
 1))
```

maple [A] time = 0.05, size = 32, normalized size = 0.71

$$-\frac{\arctan(x)}{2(x-1)^2}+\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/(x-1)^3,x)
```

```
[Out] -1/2/(x-1)^2*arctan(x)+1/8*ln(x^2+1)-1/4/(x-1)-1/4*ln(x-1)
```

maxima [A] time = 0.41, size = 31, normalized size = 0.69

$$-\frac{1}{4(x-1)}-\frac{\arctan(x)}{2(x-1)^2}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.12, size = 31, normalized size = 0.69

$$\frac{\ln\left(x^2+1\right)}{8}-\frac{\ln(x-1)}{4}-\frac{\frac{x}{4}+\frac{\mathrm{atan}(x)}{2}-\frac{1}{4}}{(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x - 1)^3,x)
```

```
[Out] log(x^2 + 1)/8 - log(x - 1)/4 - (x/4 + atan(x)/2 - 1/4)/(x - 1)^2
```

sympy [B] time = 0.52, size = 153, normalized size = 3.40

$$-\frac{2x^2\log(x-1)}{8x^2-16x+8}+\frac{x^2\log\left(x^2+1\right)}{8x^2-16x+8}+\frac{4x\log(x-1)}{8x^2-16x+8}-\frac{2x\log\left(x^2+1\right)}{8x^2-16x+8}-\frac{2x}{8x^2-16x+8}-\frac{2\log(x-1)}{8x^2-16x+8}+\frac{\log\left(x^2+\right.}{8x^2-16x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/(-1+x)**3,x)
```

```
[Out] -2*x**2*log(x - 1)/(8*x**2 - 16*x + 8) + x**2*log(x**2 + 1)/(8*x**2 - 16*x
+ 8) + 4*x*log(x - 1)/(8*x**2 - 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 - 16*
x + 8) - 2*x/(8*x**2 - 16*x + 8) - 2*log(x - 1)/(8*x**2 - 16*x + 8) + log(x
**2 + 1)/(8*x**2 - 16*x + 8) - 4*atan(x)/(8*x**2 - 16*x + 8) + 2/(8*x**2 -
16*x + 8)
```

3.121 $\int \frac{\tan^{-1}(1+2x)}{(4+3x)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{5}{578}\log\left(2x^2+2x+1\right)-\frac{1}{34(3x+4)}+\frac{5}{289}\log(3x+4)-\frac{\tan^{-1}(2x+1)}{6(3x+4)^2}+\frac{8}{867}\tan^{-1}(2x+1)$$

```
[Out] -1/34/(4+3*x)+8/867*arctan(1+2*x)-1/6*arctan(1+2*x)/(4+3*x)^2+5/289*ln(4+3*
x)-5/578*ln(2*x^2+2*x+1)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5045, 1982, 709, 800, 634, 617, 204, 628}

$$-\frac{5}{578}\log\left(2x^2+2x+1\right)-\frac{1}{34(3x+4)}+\frac{5}{289}\log(3x+4)-\frac{\tan^{-1}(2x+1)}{6(3x+4)^2}+\frac{8}{867}\tan^{-1}(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] -1/(34*(4 + 3*x)) + (8*ArcTan[1 + 2*x])/867 - ArcTan[1 + 2*x]/(6*(4 + 3*x)^
2) + (5*Log[4 + 3*x])/289 - (5*Log[1 + 2*x + 2*x^2])/578
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5045

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcTan[c + d*x])^p)/(f*(m +
 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcTan[c
 + d*x])^(p - 1))/(1 + (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(1+2x)}{(4+3x)^3}\,dx &= -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int \frac{1}{(4+3x)^2\left(1+(1+2x)^2\right)}\,dx\\
&= -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int \frac{1}{(4+3x)^2\left(2+4x+4x^2\right)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int \frac{4-12x}{(4+3x)\left(2+4x+4x^2\right)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int \left(\frac{90}{17(4+3x)}-\frac{2(7+30x)}{17\left(1+2x+2x^2\right)}\right)dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{1}{867}\int \frac{7+30x}{1+2x+2x^2}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\int \frac{2+4x}{1+2x+2x^2}\,dx+\frac{8}{867}\int \frac{}{1
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{34(4+3x)}-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2x+2x^2\right)-\frac{8}{867}\text{Subs}\\
&= -\frac{1}{34(4+3x)}+\frac{8}{867}\tan^{-1}(1+2x)-\frac{\tan^{-1}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2\right.
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 81, normalized size = 1.27

$$\frac{-289\tan^{-1}(2x+1)+(3x+4)((-15+8i)(3x+4)\log((1+i)x+i)-(15+8i)(3x+4)\log(1+(1+i)x)+90x}{1734(3x+4)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] (-289*ArcTan[1 + 2*x] + (4 + 3*x)*(-51 - (15 - 8*I)*(4 + 3*x)*Log[I + (1 +
I)*x] - (15 + 8*I)*(4 + 3*x)*Log[1 + (1 + I)*x] + 120*Log[4 + 3*x] + 90*x*L
og[4 + 3*x]))/(1734*(4 + 3*x)^2)
```

fricas [A] time = 1.13, size = 77, normalized size = 1.20

$$\frac{\left(48\,x^2+128\,x-11\right)\arctan\left(2\,x+1\right)-5\left(9\,x^2+24\,x+16\right)\log\left(2\,x^2+2\,x+1\right)+10\left(9\,x^2+24\,x+16\right)\log\left(3\,x+\right.}{578\left(9\,x^2+24\,x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="fricas")
```

```
[Out] 1/578*((48*x^2 + 128*x - 11)*arctan(2*x + 1) - 5*(9*x^2 + 24*x + 16)*log(2*
x^2 + 2*x + 1) + 10*(9*x^2 + 24*x + 16)*log(3*x + 4) - 51*x - 68)/(9*x^2 +
24*x + 16)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.05, size = 54, normalized size = 0.84

$$-\frac{2\arctan\left(1+2x\right)}{3\left(8+6x\right)^2}-\frac{1}{17\left(8+6x\right)}+\frac{5\ln\left(8+6x\right)}{289}-\frac{5\ln\left(\left(1+2x\right)^2+1\right)}{578}+\frac{8\arctan\left(1+2x\right)}{867}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1+2*x)/(4+3*x)^3,x)
```

```
[Out] -2/3/(8+6*x)^2*arctan(1+2*x)-1/17/(8+6*x)+5/289*ln(8+6*x)-5/578*ln((1+2*x)^
2+1)+8/867*arctan(1+2*x)
```

maxima [A] time = 0.40, size = 54, normalized size = 0.84

$$-\frac{1}{34\left(3\,x+4\right)}-\frac{\arctan\left(2\,x+1\right)}{6\left(3\,x+4\right)^2}+\frac{8}{867}\arctan\left(2\,x+1\right)-\frac{5}{578}\log\left(2\,x^2+2\,x+1\right)+\frac{5}{289}\log\left(3\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1) -
 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(3*x + 4)
```

mupad [B] time = 0.70, size = 46, normalized size = 0.72

$$\frac{5\,\ln\left(x+\frac{4}{3}\right)}{289}-\frac{5\,\ln\left(x^2+x+\frac{1}{2}\right)}{578}+\frac{8\,\mathrm{atan}\left(2\,x+1\right)}{867}-\frac{\frac{3\,x}{34}+\frac{\mathrm{atan}\left(2\,x+1\right)}{6}+\frac{2}{17}}{\left(3\,x+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(2*x + 1)/(3*x + 4)^3,x)
```

```
[Out] (5*log(x + 4/3))/289 - (5*log(x + x^2 + 1/2))/578 + (8*atan(2*x + 1))/867 -
 ((3*x)/34 + atan(2*x + 1)/6 + 2/17)/(3*x + 4)^2
```

sympy [B] time = 0.60, size = 223, normalized size = 3.48

$$\frac{90x^2\log\left(x+\frac{4}{3}\right)}{5202x^2+13872x+9248}-\frac{45x^2\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248}+\frac{48x^2\,\mathrm{atan}\left(2x+1\right)}{5202x^2+13872x+9248}+\frac{240x\log\left(x+\frac{4}{3}\right)}{5202x^2+13872x+9248}-\frac{120}{520}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+2*x)/(4+3*x)**3,x)
```

```
[Out] 90*x**2*log(x + 4/3)/(5202*x**2 + 13872*x + 9248) - 45*x**2*log(2*x**2 + 2*
x + 1)/(5202*x**2 + 13872*x + 9248) + 48*x**2*atan(2*x + 1)/(5202*x**2 + 13
872*x + 9248) + 240*x*log(x + 4/3)/(5202*x**2 + 13872*x + 9248) - 120*x*log
(2*x**2 + 2*x + 1)/(5202*x**2 + 13872*x + 9248) + 128*x*atan(2*x + 1)/(5202
*x**2 + 13872*x + 9248) - 51*x/(5202*x**2 + 13872*x + 9248) + 160*log(x + 4
/3)/(5202*x**2 + 13872*x + 9248) - 80*log(2*x**2 + 2*x + 1)/(5202*x**2 + 13
872*x + 9248) - 11*atan(2*x + 1)/(5202*x**2 + 13872*x + 9248) - 68/(5202*x*
*2 + 13872*x + 9248)
```

3.122 $\int \tan^{-1}\left(\sqrt{1+x}\right) dx$

Optimal. Leaf size=30

$$-\sqrt{x+1}+x\tan^{-1}\left(\sqrt{x+1}\right)+2\tan^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] 2*arctan((1+x)^(1/2))+x*arctan((1+x)^(1/2))-(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 80, 63, 203}

$$-\sqrt{x+1}+x\tan^{-1}\left(\sqrt{x+1}\right)+2\tan^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[1 + x]],x]
```

```
[Out] -Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \tan^{-1}\left(\sqrt{1+x}\right) dx &= x\tan^{-1}\left(\sqrt{1+x}\right)-\int \frac{x}{\sqrt{1+x}\,(4+2x)}\,dx\\
&= -\sqrt{1+x}+x\tan^{-1}\left(\sqrt{1+x}\right)+2\int \frac{1}{\sqrt{1+x}\,(4+2x)}\,dx\\
&= -\sqrt{1+x}+x\tan^{-1}\left(\sqrt{1+x}\right)+4\,\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx,x,\sqrt{1+x}\right)\\
&= -\sqrt{1+x}+2\tan^{-1}\left(\sqrt{1+x}\right)+x\tan^{-1}\left(\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.73

$$(x+2)\tan^{-1}\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[1 + x]],x]
[Out] -Sqrt[1 + x] + (2 + x)*ArcTan[Sqrt[1 + x]]
```

fricas [A] time = 0.65, size = 18, normalized size = 0.60

$$(x+2)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 2)*arctan(sqrt(x + 1)) - sqrt(x + 1)
```

giac [A] time = 0.13, size = 24, normalized size = 0.80

$$(x+1)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}+\arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="giac")
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$(x+1)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}+\arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan((x+1)^(1/2)),x)
[Out] (x+1)*arctan((x+1)^(1/2))-(x+1)^(1/2)+arctan((x+1)^(1/2))
```

maxima [A] time = 0.41, size = 24, normalized size = 0.80

$$(x+1)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}+\arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="maxima")
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

mupad [B] time = 0.08, size = 24, normalized size = 0.80

$$\mathrm{atan}\left(\sqrt{x+1}\right)-\sqrt{x+1}+\mathrm{atan}\left(\sqrt{x+1}\right)(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2)),x)
[Out] atan((x + 1)^(1/2)) - (x + 1)^(1/2) + atan((x + 1)^(1/2))*(x + 1)
```

sympy [**A**] time = 0.21, size = 26, normalized size = 0.87

$$x \operatorname{atan}\left(\sqrt{x+1}\right) - \sqrt{x+1} + 2 \operatorname{atan}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan((1+x)**(1/2)),x)
```

```
[Out] x*atan(sqrt(x + 1)) - sqrt(x + 1) + 2*atan(sqrt(x + 1))
```

3.123 $\int \frac{1}{(1+x^2)(2+\tan^{-1}(x))} dx$

Optimal. Leaf size=5

$$\log\left(\tan^{-1}(x)+2\right)$$

```
[Out] ln(2+arctan(x))
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4882}

$$\log\left(\tan^{-1}(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(2 + ArcTan[x])),x]
[Out] Log[2 + ArcTan[x]]
```

Rule 4882

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)\left(2+\tan^{-1}(x)\right)} dx = \log\left(2+\tan^{-1}(x)\right)$$

Mathematica [A] time = 0.03, size = 5, normalized size = 1.00

$$\log\left(\tan^{-1}(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(2 + ArcTan[x])),x]
[Out] Log[2 + ArcTan[x]]
```

fricas [A] time = 0.50, size = 5, normalized size = 1.00

$$\log\left(\arctan(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="fricas")
[Out] log(arctan(x) + 2)
```

giac [A] time = 0.12, size = 5, normalized size = 1.00

$$\log\left(\arctan(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="giac")
[Out] log(arctan(x) + 2)
```

maple [A] time = 0.04, size = 6, normalized size = 1.20

$$\ln(2 + \arctan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(2+arctan(x)),x)
[Out] ln(2+arctan(x))
```

maxima [A] time = 0.31, size = 5, normalized size = 1.00

$$\log(\arctan(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="maxima")
[Out] log(arctan(x) + 2)
```

mupad [B] time = 0.22, size = 5, normalized size = 1.00

$$\ln(\mathrm{atan}(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(atan(x) + 2)),x)
[Out] log(atan(x) + 2)
```

sympy [A] time = 0.26, size = 5, normalized size = 1.00

$$\log(\mathrm{atan}(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(2+atan(x)),x)
[Out] log(atan(x) + 2)
```

3.124 $\int \frac{1}{(a+ax^2)(b-2b\tan^{-1}(x))}\,dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

```
[Out] -1/2*ln(1-2*arctan(x))/a/b
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4882}

$$-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

```
[Out] -Log[1 - 2*ArcTan[x]]/(2*a*b)
```

Rule 4882

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{\left(a+ax^2\right)\left(b-2b\tan^{-1}(x)\right)}\,dx=-\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

Mathematica [A] time = 0.05, size = 17, normalized size = 1.00

$$-\frac{\log\left(2\tan^{-1}(x)-1\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

```
[Out] -1/2*Log[-1 + 2*ArcTan[x]]/(a*b)
```

fricas [A] time = 0.58, size = 15, normalized size = 0.88

$$-\frac{\log\left(2\arctan(x)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*arctan(x) - 1)/(a*b)
```

giac [A] time = 0.11, size = 16, normalized size = 0.94

$$-\frac{\log\left(|2\arctan(x)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="giac")
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

maple [A] time = 0.08, size = 19, normalized size = 1.12

$$-\frac{\ln\left(2b\arctan(x)-b\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a)/(b-2*b*arctan(x)),x)
[Out] -1/2/a*ln(2*b*arctan(x)-b)/b
```

maxima [A] time = 0.33, size = 16, normalized size = 0.94

$$-\frac{\log\left(|2\arctan(x)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="maxima")
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

mupad [B] time = 0.13, size = 15, normalized size = 0.88

$$-\frac{\ln\left(2\,\mathrm{atan}(x)-1\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x^2)*(b - 2*b*atan(x))),x)
[Out] -log(2*atan(x) - 1)/(2*a*b)
```

sympy [A] time = 0.51, size = 14, normalized size = 0.82

$$-\frac{\log\left(\mathrm{atan}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a)/(b-2*b*atan(x)),x)
[Out] -log(atan(x) - 1/2)/(2*a*b)
```

3.125 $\int \frac{x+x^3+(1+x)^2\tan^{-1}(x)}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

[Out] 1/(1+x)+1/2*arctan(x)^2+ln(1+x)

Rubi [A] time = 0.15, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6725, 43, 4884}

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]

[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^3+(1+x)^2\tan^{-1}(x)}{(1+x)^2\left(1+x^2\right)}\,dx &= \int \left(\frac{x}{(1+x)^2}+\frac{\tan^{-1}(x)}{1+x^2}\right)dx\\
&= \int \frac{x}{(1+x)^2}\,dx+\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)^2+\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx\\
&= \frac{1}{1+x}+\frac{1}{2}\tan^{-1}(x)^2+\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{1}{x+1}+\log(x+1)+\frac{1}{2}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

fricas [A] time = 0.54, size = 26, normalized size = 1.44

$$\frac{(x+1)\arctan(x)^2+2(x+1)\log(x+1)+2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x)^2 + 2*(x + 1)*log(x + 1) + 2)/(x + 1)
```

giac [B] time = 0.14, size = 104, normalized size = 5.78

$$\frac{(x+1)\left(\frac{1}{x+1}-1\right)\arctan\left((x+1)\left(\frac{1}{x+1}-1\right)\right)^2+2(x+1)\left(\frac{1}{x+1}-1\right)\log\left(-(x+1)\left(\frac{1}{x+1}-1\right)+1\right)-\arctan\left((x+1)\left(\frac{1}{x+1}\right.\right.\cdots}{2\left((x+1)\left(\frac{1}{x+1}-1\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*((x + 1)*(1/(x + 1) - 1)*arctan((x + 1)*(1/(x + 1) - 1))^2 + 2*(x + 1)*
(1/(x + 1) - 1)*log(-(x + 1)*(1/(x + 1) - 1) + 1) - arctan((x + 1)*(1/(x +
1) - 1))^2 - 2*log(-(x + 1)*(1/(x + 1) - 1) + 1) - 2)/((x + 1)*(1/(x + 1) -
 1) - 1)
```

maple [A] time = 0.05, size = 17, normalized size = 0.94

$$\frac{1}{x+1}+\frac{\arctan(x)^2}{2}+\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^3+(x+1)^2*arctan(x))/(x+1)^2/(x^2+1),x)
```

```
[Out] 1/(x+1)+1/2*arctan(x)^2+ln(x+1)
```

maxima [A] time = 0.56, size = 16, normalized size = 0.89

$$\frac{1}{2}\arctan(x)^2+\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x)^2 + 1/(x + 1) + log(x + 1)
```

mupad [B] time = 0.13, size = 16, normalized size = 0.89

$$\ln(x+1)+\frac{1}{x+1}+\frac{\mathrm{atan}(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + atan(x)*(x + 1)^2 + x^3)/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] log(x + 1) + 1/(x + 1) + atan(x)^2/2
```

sympy [B] time = 0.62, size = 53, normalized size = 2.94

$$\frac{2x\log(x+1)}{2x+2}+\frac{x\operatorname{atan}^2(x)}{2x+2}+\frac{2\log(x+1)}{2x+2}+\frac{\operatorname{atan}^2(x)}{2x+2}+\frac{2}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**3+(1+x)**2*atan(x))/(1+x)**2/(x**2+1),x)
```

```
[Out] 2*x*log(x + 1)/(2*x + 2) + x*atan(x)**2/(2*x + 2) + 2*log(x + 1)/(2*x + 2)
+ atan(x)**2/(2*x + 2) + 2/(2*x + 2)
```

3.126 $\int -x^3 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=68

$$\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{8}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/24*x^(3/2)-1/40*x^(5/2)+1/56*x^(7/2)+1/16*Pi*x^4+1/8*arctan(x^(1/2))-1/8*
x^4*arctan(x^(1/2))-1/8*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^4}{16}+\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{8}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -Sqrt[x]/8 + x^(3/2)/24 - x^(5/2)/40 + x^(7/2)/56 + (Pi*x^4)/16 + ArcTan[Sq
rt[x]]/8 - (x^4*ArcTan[Sqrt[x]])/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -x^3 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x^3 \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi\int x^3\, dx \\
&= \frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{x^{7/2}}{1+x}\, dx \\
&= \frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int \frac{x^{5/2}}{1+x}\, dx \\
&= -\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{x^{3/2}}{1+x}\, dx \\
&= \frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int \frac{\sqrt{x}}{1+x}\, dx \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int \frac{1}{\sqrt{x}(1+x)}\, dx \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\right. \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.85

$$\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{840}\sqrt{x}\left(210x^{7/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)-15x^3+21x^2-35x+105\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[Sqrt[x]]/8 - (Sqrt[x]*(105 - 35*x + 21*x^2 - 15*x^3 + 210*x^(7/2)*Ar
cTan[Sqrt[x] - Sqrt[1 + x]]))/840
```

fricas [A] time = 0.57, size = 40, normalized size = 0.59

$$\frac{1}{4}\left(x^4-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{840}\left(15\,x^3-21\,x^2+35\,x-105\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/840*(15*x^3 - 21*x^2 + 35*x
 - 105)*sqrt(x)
```

giac [A] time = 0.13, size = 44, normalized size = 0.65

$$-\frac{1}{4}\,x^4\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{56}\,x^{\frac{7}{2}}-\frac{1}{40}\,x^{\frac{5}{2}}+\frac{1}{24}\,x^{\frac{3}{2}}-\frac{1}{8}\,\sqrt{x}+\frac{1}{8}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^4*arctan(-sqrt(x + 1) + sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/2
4*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

maple [A] time = 0.06, size = 45, normalized size = 0.66

$$-\frac{x^{4} \arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{4}+\frac{x^{\frac{7}{2}}}{56}-\frac{x^{\frac{5}{2}}}{40}+\frac{x^{\frac{3}{2}}}{24}-\frac{\sqrt{x}}{8}+\frac{\arctan\left(\sqrt{x}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/4*x^4*arctan(x^(1/2)-(x+1)^(1/2))+1/56*x^(7/2)-1/40*x^(5/2)+1/24*x^(3/2)
-1/8*x^(1/2)+1/8*arctan(x^(1/2))
```

maxima [A] time = 0.46, size = 44, normalized size = 0.65

$$\frac{1}{4} x^{4} \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{56} x^{\frac{7}{2}}-\frac{1}{40} x^{\frac{5}{2}}+\frac{1}{24} x^{\frac{3}{2}}-\frac{1}{8} \sqrt{x}+\frac{1}{8} \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(x + 1) - sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/24*
x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

mupad [B] time = 1.65, size = 72, normalized size = 1.06

$$\frac{x^{3/2}}{24}-\frac{\sqrt{x}}{8}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\left(\frac{x^5}{2}+\frac{x^4}{2}\right)}{2\,x+2}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/16 - x^(1/2)/8 + x^(3/2)/24 - x^(5/2)/
40 + x^(7/2)/56 + (atan((x + 1)^(1/2) - x^(1/2))*(x^4/2 + x^5/2))/(2*x + 2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

3.127 $\int -x^2 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx$

Optimal. Leaf size=59

$$\frac{x^{5/2}}{30}-\frac{x^{3/2}}{18}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/18*x^(3/2)+1/30*x^(5/2)+1/12*Pi*x^3-1/6*arctan(x^(1/2))-1/6*x^3*arctan(x
^(1/2))+1/6*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^3}{12}+\frac{x^{5/2}}{30}-\frac{x^{3/2}}{18}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] Sqrt[x]/6 - x^(3/2)/18 + x^(5/2)/30 + (Pi*x^3)/12 - ArcTan[Sqrt[x]]/6 - (x^
3*ArcTan[Sqrt[x]])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -x^2 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi\int x^2\,dx \\
&= \frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int \frac{x^{5/2}}{1+x}\,dx \\
&= \frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int \frac{x^{3/2}}{1+x}\,dx \\
&= -\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int \frac{\sqrt{x}}{1+x}\,dx \\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.90

$$\frac{1}{90}\left(-\sqrt{x}\left(30x^{5/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)-3x^2+5x-15\right)-15\tan^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (-15*ArcTan[Sqrt[x]] - Sqrt[x]*(-15 + 5*x - 3*x^2 + 30*x^(5/2)*ArcTan[Sqrt[
x] - Sqrt[1 + x]]))/90
```

fricas [A] time = 0.58, size = 35, normalized size = 0.59

$$\frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{90}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/90*(3*x^2 - 5*x + 15)*sqrt(
x)
```

giac [A] time = 0.15, size = 39, normalized size = 0.66

$$-\frac{1}{3}\,x^3\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{30}\,x^{\frac{5}{2}}-\frac{1}{18}\,x^{\frac{3}{2}}+\frac{1}{6}\,\sqrt{x}-\frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arctan(-sqrt(x + 1) + sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6
*sqrt(x) - 1/6*arctan(sqrt(x))
```

maple [A] time = 0.06, size = 40, normalized size = 0.68

$$-\frac{x^{3} \arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{3}+\frac{x^{\frac{5}{2}}}{30}-\frac{x^{\frac{3}{2}}}{18}+\frac{\sqrt{x}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/3*x^3*arctan(x^(1/2)-(x+1)^(1/2))+1/30*x^(5/2)-1/18*x^(3/2)+1/6*x^(1/2)-
1/6*arctan(x^(1/2))
```

maxima [A] time = 0.46, size = 39, normalized size = 0.66

$$\frac{1}{3} x^{3} \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{30} x^{\frac{5}{2}}-\frac{1}{18} x^{\frac{3}{2}}+\frac{1}{6} \sqrt{x}-\frac{1}{6} \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x + 1) - sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6*s
qrt(x) - 1/6*arctan(sqrt(x))
```

mupad [B] time = 0.94, size = 65, normalized size = 1.10

$$\frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\left(\frac{2x^4}{3}+\frac{2x^3}{3}\right)}{2x+2}+\frac{\ln\left(\frac{\left(\sqrt{x}-\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/12 + x^(1/2)/6 - x^(3/2)/18 + x^(5/2)/30
 + (atan((x + 1)^(1/2) - x^(1/2))*((2*x^3)/3 + (2*x^4)/3))/(2*x + 2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

3.128 $\int -x \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx$

Optimal. Leaf size=50

$$\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{4}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/12*x^(3/2)+1/8*Pi*x^2+1/4*arctan(x^(1/2))-1/4*x^2*arctan(x^(1/2))-1/4*x^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5159, 30, 5033, 50, 63, 203}

$$\frac{\pi x^2}{8}+\frac{x^{3/2}}{12}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{4}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -Sqrt[x]/4 + x^(3/2)/12 + (Pi*x^2)/8 + ArcTan[Sqrt[x]]/4 - (x^2*ArcTan[Sqrt
[x]])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -x\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)dx &= -\left(\frac{1}{2}\int x\tan^{-1}\left(\sqrt{x}\right)dx\right)+\frac{1}{4}\pi\int x\,dx\\
&= \frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{x^{3/2}}{1+x}dx\\
&= \frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}\int\frac{\sqrt{x}}{1+x}dx\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{1}{\sqrt{x}(1+x)}dx\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x^2}dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.96

$$\frac{1}{12}\left(3\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}\left(6x^{3/2}\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)-x+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (3*ArcTan[Sqrt[x]] - Sqrt[x]*(3 - x + 6*x^(3/2)*ArcTan[Sqrt[x] - Sqrt[1 + x
]]))/12
```

fricas [A] time = 0.99, size = 28, normalized size = 0.56

$$\frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}(x-3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*(x - 3)*sqrt(x)
```

giac [A] time = 0.15, size = 34, normalized size = 0.68

$$-\frac{1}{2}x^2\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2*arctan(-sqrt(x + 1) + sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*
arctan(sqrt(x))
```

maple [A] time = 0.05, size = 35, normalized size = 0.70

$$-\frac{x^2\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(x^(1/2)-(x+1)^(1/2)),x)
```

```
[Out] -1/2*x^2*arctan(x^(1/2)-(x+1)^(1/2))+1/12*x^(3/2)-1/4*x^(1/2)+1/4*arctan(x^
(1/2))
```

maxima [A] time = 0.46, size = 34, normalized size = 0.68

$$\frac{1}{2}x^2\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*ar
ctan(sqrt(x))
```

mupad [B] time = 0.85, size = 58, normalized size = 1.16

$$\frac{x^{3/2}}{12}-\frac{\sqrt{x}}{4}+\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\left(x^3+x^2\right)}{2x+2}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/8 - x^(1/2)/4 + x^(3/2)/12 + (atan((x
+ 1)^(1/2) - x^(1/2))*(x^2 + x^3))/(2*x + 2)
```

sympy [A] time = 161.34, size = 39, normalized size = 0.78

$$\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}-\frac{x^2\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{2}+\frac{\mathrm{atan}\left(\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] x**(3/2)/12 - sqrt(x)/4 - x**2*atan(sqrt(x) - sqrt(x + 1))/2 + atan(sqrt(x)
)/4
```

3.129 $\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=37

$$\frac{\pi x}{4}+\frac{\sqrt{x}}{2}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/4*Pi*x-1/2*arctan(x^(1/2))-1/2*x*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5159, 8, 5027, 50, 63, 203}

$$\frac{\pi x}{4}+\frac{\sqrt{x}}{2}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -\left(\frac{1}{2}\int \tan^{-1}\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int 1\,dx\\
&= \frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.43, size = 39, normalized size = 1.05

$$\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-x\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
[Out] Sqrt[x]/2 - ArcTan[Sqrt[x]]/2 - x*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

fricas [A] time = 0.43, size = 22, normalized size = 0.59

$$(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

giac [A] time = 0.14, size = 27, normalized size = 0.73

$$-x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
[Out] -x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

maple [A] time = 0.05, size = 28, normalized size = 0.76

$$-x\arctan\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2)),x)
[Out] -x*arctan(x^(1/2)-(x+1)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

maxima [A] time = 0.45, size = 26, normalized size = 0.70

$$x\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

mupad [B] time = 0.89, size = 40, normalized size = 1.08

$$x\,\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] x*atan((x + 1)^(1/2) - x^(1/2)) - (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 +
x^(1/2)/2
```

sympy [A] time = 73.76, size = 29, normalized size = 0.78

$$\frac{\sqrt{x}}{2}-x\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right)-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

3.130 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}i\text{Li}_2\left(-i\sqrt{x}\right)+\frac{1}{2}i\text{Li}_2\left(i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

```
[Out] 1/4*Pi*ln(x)-1/2*I*polylog(2,-I*x^(1/2))+1/2*I*polylog(2,I*x^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5159, 29, 5031, 4848, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\text{PolyLog}\left(2,i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] (Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x]
]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x}\,dx$$
$$= \frac{1}{4}\pi\log(x) - \text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}\pi\log(x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \sqrt{x}\right)+\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,d\right.$$
$$= \frac{1}{4}\pi\log(x) - \frac{1}{2}i\text{Li}_2\left(-i\sqrt{x}\right)+\frac{1}{2}i\text{Li}_2\left(i\sqrt{x}\right)$$

Mathematica [A] time = 0.20, size = 84, normalized size = 2.00

$$-\log(x)\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)+\frac{1}{4}i\left(-2\text{Li}_2\left(-i\sqrt{x}\right)+2\text{Li}_2\left(i\sqrt{x}\right)+\left(\log\left(1-i\sqrt{x}\right)-\log\left(1+i\sqrt{x}\right)\right)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] -(ArcTan[Sqrt[x] - Sqrt[1 + x]]*Log[x]) + (I/4)*((Log[1 - I*Sqrt[x]] - Log[
1 + I*Sqrt[x]])*Log[x] - 2*PolyLog[2, (-I)*Sqrt[x]] + 2*PolyLog[2, I*Sqrt[x
]])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x + 1) - sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-sqrt(x + 1) + sqrt(x))/x, x)
```

maple [B] time = 1.40, size = 194, normalized size = 4.62

$$2\arctan\left(\sqrt{x}-\sqrt{x+1}\right)\ln\left(1-\frac{\left(1+i\left(\sqrt{x}-\sqrt{x+1}\right)\right)^4}{\left(\left(\sqrt{x}-\sqrt{x+1}\right)^2+1\right)^2}\right)-2\arctan\left(\sqrt{x}-\sqrt{x+1}\right)\ln\left(1+\frac{\left(1+i\left(\sqrt{x}-\sqrt{x+\right.\right.}}{\left(\left(\sqrt{x}-\sqrt{x+1}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x,x)
```

```
[Out] 2*arctan(x^(1/2)-(x+1)^(1/2))*ln(1-(1+I*(x^(1/2)-(x+1)^(1/2)))^4/((x^(1/2)-
(x+1)^(1/2))^2+1)^2)-2*arctan(x^(1/2)-(x+1)^(1/2))*ln(1+(1+I*(x^(1/2)-(x+1)
```

```
^(1/2)))^4/((x^(1/2)-(x+1)^(1/2))^2+1)^2)+1/2*I*dilog(1+(1+I*(x^(1/2)-(x+1)
^(1/2)))^4/((x^(1/2)-(x+1)^(1/2))^2+1)^2)-1/2*I*dilog(1-(1+I*(x^(1/2)-(x+1)
^(1/2)))^4/((x^(1/2)-(x+1)^(1/2))^2+1)^2)
```

maxima [A] time = 0.43, size = 43, normalized size = 1.02

$$\frac{1}{4}\pi\log\left(x+1\right)+\arctan\left(\sqrt{x+1}-\sqrt{x}\right)\log(x)+\frac{1}{2}i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*log(x + 1) + arctan(sqrt(x + 1) - sqrt(x))*log(x) + 1/2*I*dilog(I*sq
rt(x) + 1) - 1/2*I*dilog(-I*sqrt(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x,x)
```

```
[Out] int(atan((x + 1)^(1/2) - x^(1/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x,x)
```

```
[Out] Timed out
```

3.131 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/4*Pi/x+1/2*arctan(x^(1/2))+1/2*arctan(x^(1/2))/x+1/2/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] -Pi/(4*x) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x]]/(2*x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^2}\,dx\\
&= -\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.98

$$\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
[Out] 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x] - Sqrt[1 + x]]/x
```

fricas [A] time = 0.61, size = 28, normalized size = 0.68

$$-\frac{2\,(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\sqrt{x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="fricas")
[Out] -1/2*(2*(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) - sqrt(x))/x
```

giac [A] time = 0.15, size = 28, normalized size = 0.68

$$\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="giac")
[Out] arctan(-sqrt(x + 1) + sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

maple [B] time = 0.06, size = 57, normalized size = 1.39

$$\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{x+1}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\ln\left(\sqrt{x+1}+1\right)}{4}+\frac{\ln\left(\sqrt{x+1}-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^2,x)
```

```
[Out] arctan(x^(1/2)-(x+1)^(1/2))/x+1/2/x^(1/2)+1/2*arctanh((x+1)^(1/2))+1/2*arct
an(x^(1/2))-1/4*ln((x+1)^(1/2)+1)+1/4*ln((x+1)^(1/2)-1)
```

maxima [A] time = 0.45, size = 29, normalized size = 0.71

$$-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x + 1) - sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

mupad [B] time = 1.41, size = 44, normalized size = 1.07

$$-\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)-\frac{\sqrt{x}}{2}}{x}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^2,x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 - (atan((x + 1)^(1/2) - x^(1/2)) - x
^(1/2)/2)/x
```

sympy [B] time = 68.66, size = 537, normalized size = 13.10

$$-\frac{2x^{\frac{5}{2}}\sqrt{x+1}\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2}+\frac{x^{\frac{5}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2}-\frac{4x^{\frac{3}{2}}\sqrt{x+1}\,\mathrm{atan}\left(\sqrt{x}\right.}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**2,x)
```

```
[Out] -2*x**(5/2)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1
) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + x**(5/2)/(-2*x**(5/2)*sqrt(
x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) - 4*x**(3/2)*sqrt(x + 1)
*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x +
 1) + 2*x**3 + 2*x**2) + x**(3/2)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqr
t(x + 1) + 2*x**3 + 2*x**2) - 2*sqrt(x)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) +
 2*x**3*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*s
qrt(x + 1) + 2*x**3 + 2*x**2) - x**2*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) -
 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + 4*x**2*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) -
 x*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 +
 2*x**2) + 2*x*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**
(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2)
```

3.132 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/8*Pi/x^2+1/12/x^(3/2)-1/4*arctan(x^(1/2))+1/4*arctan(x^(1/2))/x^2-1/4/x^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -Pi/(8*x^2) + 1/(12*x^(3/2)) - 1/(4*Sqrt[x]) - ArcTan[Sqrt[x]]/4 + ArcTan[S
qrt[x]]/(4*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^3}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}+\frac{1}{8}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.96

$$-\frac{3x^2\tan^{-1}\left(\sqrt{x}\right)+(3x-1)\sqrt{x}-6\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -1/12*(Sqrt[x]*(-1 + 3*x) + 3*x^2*ArcTan[Sqrt[x]] - 6*ArcTan[Sqrt[x] - Sqrt
[1 + x]])/x^2
```

fricas [A] time = 0.57, size = 35, normalized size = 0.70

$$\frac{6\left(x^2-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

giac [A] time = 0.15, size = 34, normalized size = 0.68

$$-\frac{3\,x-1}{12\,x^{\frac{3}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*x - 1)/x^(3/2) + 1/2*arctan(-sqrt(x + 1) + sqrt(x))/x^2 - 1/4*arct
an(sqrt(x))
```

maple [A] time = 0.07, size = 35, normalized size = 0.70

$$\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^3,x)
```

```
[Out] 1/2*arctan(x^(1/2)-(x+1)^(1/2))/x^2+1/12/x^(3/2)-1/4/x^(1/2)-1/4*arctan(x^(
1/2))
```

maxima [A] time = 0.45, size = 34, normalized size = 0.68

$$-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x) - 1/2*arctan(sqrt(x + 1) - sqrt(x))/x^2 + 1/12/x^(3/2) - 1/4*a
rctan(sqrt(x))
```

mupad [B] time = 1.36, size = 49, normalized size = 0.98

$$-\frac{\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{12}+\frac{x^{3/2}}{4}}{x^2}+\frac{\ln\left(\frac{\left(\sqrt{x}-\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^3,x)
```

```
[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/8 - (atan((x + 1)^(1/2) - x^(1/2))/2 - x
^(1/2)/12 + x^(3/2)/4)/x^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

3.133 $\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/12*Pi/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6*arctan(x^(1/2))+1/6*arctan(x^(1/
2))/x^3+1/6/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5159, 30, 5033, 51, 63, 203}

$$-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] -Pi/(12*x^3) + 1/(30*x^(5/2)) - 1/(18*x^(3/2)) + 1/(6*Sqrt[x]) + ArcTan[Sqr
t[x]]/6 + ArcTan[Sqrt[x]]/(6*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 5159

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[(Pi*s)/4, Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^4}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^4}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{7/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.86

$$\frac{1}{90}\left(\frac{30\tan^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{x^3}-\frac{-15x^2+5x-3}{x^{5/2}}+15\tan^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] (-((-3 + 5*x - 15*x^2)/x^(5/2)) + 15*ArcTan[Sqrt[x]] + (30*ArcTan[Sqrt[x] -
 Sqrt[1 + x]])/x^3)/90
```

fricas [A] time = 0.50, size = 40, normalized size = 0.68

$$-\frac{30\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\left(15\,x^2-5\,x+3\right)\sqrt{x}}{90\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/90*(30*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) - (15*x^2 - 5*x + 3)*sqrt
(x))/x^3
```

giac [A] time = 0.15, size = 39, normalized size = 0.66

$$\frac{15\,x^2-5\,x+3}{90\,x^{\frac{5}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3}+\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/90*(15*x^2 - 5*x + 3)/x^(5/2) + 1/3*arctan(-sqrt(x + 1) + sqrt(x))/x^3 +
1/6*arctan(sqrt(x))
```

maple [A] time = 0.08, size = 40, normalized size = 0.68

$$\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(x+1)^(1/2))/x^4,x)
```

```
[Out] 1/3*arctan(x^(1/2)-(x+1)^(1/2))/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6/x^(1/2)+1
/6*arctan(x^(1/2))
```

maxima [A] time = 0.45, size = 39, normalized size = 0.66

$$\frac{1}{6\sqrt{x}}-\frac{1}{18\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{3\,x^3}+\frac{1}{30\,x^{\frac{5}{2}}}+\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6/sqrt(x) - 1/18/x^(3/2) - 1/3*arctan(sqrt(x + 1) - sqrt(x))/x^3 + 1/30/x
^(5/2) + 1/6*arctan(sqrt(x))
```

mupad [B] time = 0.94, size = 56, normalized size = 0.95

$$-\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{30}+\frac{x^{3/2}}{18}-\frac{x^{5/2}}{6}}{x^3}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^4,x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/12 - (atan((x + 1)^(1/2) - x^(1/2))/3
- x^(1/2)/30 + x^(3/2)/18 - x^(5/2)/6)/x^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

3.134 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}} \, dx$

Optimal. Leaf size=63

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)*(-c^2*x^2+a)^(1/2)/c/(1+m)/(d-c^2*d*x^
2/a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}} \, dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{a-c^2x^2}} \, dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

fricas [B] time = 0.79, size = 126, normalized size = 2.00

$$-\frac{\sqrt{-c^2x^2+a}\,a\left(-\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)^m\sqrt{-\frac{c^2dx^2-ad}{a}}\,\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}{acdm+acd-\left(c^3dm+c^3d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*(-arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))^m*sqr
t(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))/(a*c*d
*m + a*c*d - (c^3*d*m + c^3*d)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^m}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^m/sqrt(-c^2*d*x^2/a + d), x)
```

maple [A] time = 0.52, size = 73, normalized size = 1.16

$$-\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^{1+m}\left(c^2x^2-a\right)}{(1+m)\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\,\sqrt{-c^2x^2+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)/(1+m)*(c^2*x^2-a)/(-d*(c^2*x^2-a)/a)^
(1/2)/(-c^2*x^2+a)^(1/2)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.73, size = 57, normalized size = 0.90

$$\frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^{m+1}\,\sqrt{a-c^2\,x^2}}{c\,\left(m+1\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^m/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] (atan((c*x)/(a - c^2*x^2)^(1/2))^(m + 1)*(a - c^2*x^2)^(1/2))/(c*(m + 1)*(d
 - (c^2*d*x^2)/a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{m}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**m/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**m/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

3.135 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/3*arctan(c*x/(-c^2*x^2+a)^(1/2))^3*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a\sqrt{-\frac{c^2dx^2-ad}{a}}\,\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)^2}{c^2dx^2-ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x
^2 - a))^2/(c^2*d*x^2 - a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

maple [A] time = 0.43, size = 72, normalized size = 1.22

$$-\frac{\sqrt{-\frac{d(c^2x^2-a)}{a}}\,\sqrt{-c^2x^2+a}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3 a}{3d\left(c^2x^2-a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -1/3*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*arctan(c*x
/(-c^2*x^2+a)^(1/2))^3*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}\left(\frac{cx}{\sqrt{a-c^{2}x^{2}}}\right)}{\sqrt{-d\left(-1+\frac{c^{2}x^{2}}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**2/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

3.136 $\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/2*arctan(c*x/(-c^2*x^2+a)^(1/2))^2*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5157, 5155}

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}{c^2dx^2-ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^
2 - a))/(c^2*d*x^2 - a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

maple [A] time = 0.43, size = 72, normalized size = 1.22

$$-\frac{\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\,\sqrt{-c^2x^2+a}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2 a}{2d\left(c^2x^2-a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -1/2*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*arctan(c*x
/(-c^2*x^2+a)^(1/2))^2*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2),x)
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

3.137 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}dx$

Optimal. Leaf size=55

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] ln(arctan(c*x/(-c^2*x^2+a)^(1/2)))*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5153}

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Rule 5153

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1*Log[ArcTan[(c*x)/Sqrt[a + b*x^2]]])/c, x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

fricas [A] time = 0.55, size = 83, normalized size = 1.51

$$-\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}\,\log\left(2\,\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)}{c^3dx^2-acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)*log(2*arctan(sqrt(-c^2*x^2
 + a)*c*x/(c^2*x^2 - a)))/(c^3*d*x^2 - a*c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

maple [A] time = 1.78, size = 71, normalized size = 1.29

$$-\frac{\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\,\sqrt{-c^2x^2+a}\,\ln\left(\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)\right)a}{d\left(c^2x^2-a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] -(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*ln(arctan(c*x/
(-c^2*x^2+a)^(1/2)))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

mupad [B] time = 0.60, size = 49, normalized size = 0.89

$$\frac{\ln\left(\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\right)\,\sqrt{a-c^2\,x^2}}{c\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] (log(atan((c*x)/(a - c^2*x^2)^(1/2)))*(a - c^2*x^2)^(1/2))/(c*(d - (c^2*d*x
^2)/a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))), x)
```

3.138 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

```
[Out] -(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5157, 5155}

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= -\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.00

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

fricas [A] time = 0.44, size = 82, normalized size = 1.44

$$-\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{\left(c^3dx^2-acd\right)\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*arcta
n(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^2), x)
```

maple [A] time = 0.42, size = 71, normalized size = 1.25

$$\frac{\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\,\sqrt{-c^2x^2+a}\,a}{d\left(c^2x^2-a\right)c\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] (-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*a/arctan(c*x/(-
c^2*x^2+a)^(1/2))
```

maxima [A] time = 0.56, size = 29, normalized size = 0.51

$$-\frac{\sqrt{a}}{c\sqrt{d}\,\arctan\left(cx,\sqrt{-c^2x^2+a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a)))
```

mupad [B] time = 0.60, size = 51, normalized size = 0.89

$$-\frac{\sqrt{a-c^2\,x^2}}{c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(c*atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**2),
x)
```

3.139 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

```
[Out] -1/2*(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5157, 5155}

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -Sqrt[a - c^2*x^2]/(2*c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]^2)
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.00

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2
*x^2]]^2)
```

fricas [A] time = 0.54, size = 82, normalized size = 1.39

$$\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{2\left(c^3dx^2-acd\right)\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*ar
ctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^3), x)
```

maple [A] time = 0.43, size = 72, normalized size = 1.22

$$\frac{\sqrt{-\frac{d\left(c^2x^2-a\right)}{a}}\,\sqrt{-c^2x^2+a}\;a}{2d\left(c^2x^2-a\right)c\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x)
```

```
[Out] 1/2*(-d*(c^2*x^2-a)/a)^(1/2)*(-c^2*x^2+a)^(1/2)/d/(c^2*x^2-a)/c*a/arctan(c*
x/(-c^2*x^2+a)^(1/2))^2
```

maxima [A] time = 0.62, size = 29, normalized size = 0.49

$$-\frac{\sqrt{a}}{2\,c\sqrt{d}\,\arctan\left(cx,\sqrt{-c^2x^2+a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a))^2)
```

mupad [B] time = 0.61, size = 51, normalized size = 0.86

$$-\frac{\sqrt{a-c^2\,x^2}}{2\,c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^3*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(2*c*atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2
)/a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\,\mathrm{atan}^3\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**3/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**3),
x)
```

3.140 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

```
[Out] arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^(1+m)*(-a*e^2/b-e^2*x^2)^(1/2)/e/(1+m)
/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.24, size = 66, normalized size = 0.92

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.68, size = 132, normalized size = 1.83

$$-\frac{\sqrt{bx^2+a}\left(-\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)^m\sqrt{-\frac{be^2x^2+ae^2}{b}}\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{aem+(bem+be)x^2+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))
)^m*sqrt(-(b*e^2*x^2 + a*e^2)/b)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b
*e*x^2 + a*e))/(a*e*m + (b*e*m + b*e)*x^2 + a*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^m/sqrt(b*x^2 + a), x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^{2}}{b}-e^{2}x^{2}}}\right)^{m}}{\sqrt{b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^{2}\,x^{2}-\frac{a\,e^{2}}{b}}}\right)^{m}}{\sqrt{b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2),x)
```

```
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atan}^{m}\left(\frac{ex}{\sqrt{-\frac{ae^{2}}{b}-e^{2}x^{2}}}\right)}{\sqrt{a+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**m/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**m/sqrt(a + b*x**2), x)
```

3.141 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[Out] 1/3*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

Mathematica [**A**] time = 0.13, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^3)
/(3*e*Sqrt[a + b*x^2])
```

fricas [**F**] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2/sqrt(b*
x^2 + a), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2x^2}}\right)^2}{\sqrt{b x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2),x)
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2/sqrt(a + b*x**2), x)
```

3.142 $\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[Out] 1/2*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5157, 5155}

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^2)
/(2*e*Sqrt[a + b*x^2])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))/sqrt(b*x
^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2x^2}}\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))/sqrt(a + b*x**2), x)
```

3.143 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[Out] ln(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2)))*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^2+
a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5157, 5153}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]
]])/(e*Sqrt[a + b*x^2])
```

Rule 5153

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1*Log[ArcTan[(c*x)/Sqrt[a + b*x^2]]])/c, x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.11, size = 58, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*Log[ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]
]])/(e*Sqrt[a + b*x^2])
```

fricas [A] time = 0.67, size = 83, normalized size = 1.30

$$\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}\log\left(2\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)}{bex^2+ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)*log(2*arctan(b*x*sqrt(-(b*e^2*
x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))/(b*e*x^2 + a*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2 x^2}}\right)\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)),x)
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\mathrm{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))), x)
```

3.144 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)}$$

```
[Out] -(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5157, 5155}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^
2)/b) - e^2*x^2]]))
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 0.91

$$\frac{e\sqrt{a+bx^2}}{b\sqrt{-\frac{e^2(a+bx^2)}{b}}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),
x]
```

```
[Out] (e*Sqrt[a + b*x^2])/(b*Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^
2*(a + b*x^2))/b)]])
```

fricas [A] time = 0.82, size = 82, normalized size = 1.24

$$\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{(bex^2+ae)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b*x*sq
rt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2x^2}}\right)^2 \sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\mathrm{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2), x)
```

3.145 $\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}$$

```
[Out] -1/2*(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x
^2+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5157, 5155}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),x]
```

```
[Out] -Sqrt[-((a*e^2)/b) - e^2*x^2]/(2*e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e
^2)/b) - e^2*x^2]]^2)
```

Rule 5155

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[(c*x)/Sqrt[a + b*x^2]]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5157

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[(c
*x)/Sqrt[a + b*x^2]]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 0.91

$$-\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),
x]
```

```
[Out] -1/2*Sqrt[-((e^2*(a + b*x^2))/b)]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((e
^2*(a + b*x^2))/b)]]^2)
```

fricas [A] time = 0.61, size = 83, normalized size = 1.22

$$-\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{2\left(bex^2+ae\right)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b
*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^3), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2x^2}}\right)^3 \sqrt{b x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^3 \sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}^3\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**3), x)
```

3.146 $\int \frac{\tan^{-1}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

Optimal. Leaf size=101

$$\frac{i\text{Li}_2(-ic(a+bx))\log(d(a+bx))}{2b}-\frac{i\text{Li}_2(ic(a+bx))\log(d(a+bx))}{2b}-\frac{i\text{Li}_3(-ic(a+bx))}{2b}+\frac{i\text{Li}_3(ic(a+bx))}{2b}$$

```
[Out] 1/2*I*ln(d*(b*x+a))*polylog(2,-I*c*(b*x+a))/b-1/2*I*ln(d*(b*x+a))*polylog(2
,I*c*(b*x+a))/b-1/2*I*polylog(3,-I*c*(b*x+a))/b+1/2*I*polylog(3,I*c*(b*x+a)
)/b
```

Rubi [A] time = 0.27, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4848, 2391, 5209, 2444, 2433, 2374, 6589}

$$\frac{i\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{PolyLog}(2,ic(a+bx))}{2b}-\frac{i\text{PolyLog}(3,-ic(a+bx))}{2b}+\frac{i\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] ((I/2)*Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)])/b - ((I/2)*Log[d*(a +
 b*x)]*PolyLog[2, I*c*(a + b*x)])/b - ((I/2)*PolyLog[3, (-I)*c*(a + b*x)])/
b + ((I/2)*PolyLog[3, I*c*(a + b*x)])/b
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5209

```
Int[(ArcTan[v_]*Log[w_])/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[I/2, Int[(
Log[1 - I*v]*Log[w])/(a + b*x), x], x] - Dist[I/2, Int[(Log[1 + I*v]*Log[w]
)/(a + b*x), x], x] /; FreeQ[{a, b}, x] && LinearQ[v, x] && LinearQ[w, x] &
& EqQ[Simplify[D[v/(a + b*x), x]], 0] && EqQ[Simplify[D[w/(a + b*x), x]], 0
]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-ic(a+bx))}{a+bx}\,dx - \frac{1}{2}i\int \frac{\log(d(a+bx))\log}{a+b}$$

$$= \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-iac-ibcx)}{a+bx}\,dx - \frac{1}{2}i\int \frac{\log(d(a+bx))\log}{a+b}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{\log(dx)\log\left(\frac{iabc+b(1-iac)}{b}-icx\right)}{x}\,dx, x, a+bx\right)}{2b} - \frac{i\,\text{Subst}\left(\int \frac{\log(dx)\log}{}\right.}{}$$

$$= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))}{2b} - \frac{i}{}$$

$$= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))}{2b} - \frac{i}{}$$

Mathematica [A] time = 3.03, size = 166, normalized size = 1.64

$$\frac{2\log(a+bx)\tan^{-1}(c(a+bx))\log(d(a+bx)) + i(\text{Li}_2(-ic(a+bx))\log(d(a+bx)) - \text{Li}_2(ic(a+bx))\log(d(a+b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] (2*ArcTan[c*(a + b*x)]*Log[a + b*x]*Log[d*(a + b*x)] + I*(Log[a + b*x]*Log[
d*(a + b*x)]*Log[1 + I*a*c + I*b*c*x] - Log[a + b*x]*Log[d*(a + b*x)]*Log[1
 - I*c*(a + b*x)] + Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)] - Log[d*(
a + b*x)]*PolyLog[2, I*c*(a + b*x)] - PolyLog[3, (-I)*c*(a + b*x)] + PolyLo
g[3, I*c*(a + b*x)]))/(2*b)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arctan(bcx+ac)\log(bdx+ad)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*c*x + a*c)*log(b*d*x + a*d)/(b*x + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple **[F]** time = 6.75, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c\left(bx+a\right)\right)\ln\left(d\left(bx+a\right)\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

```
[Out] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(c\,\left(a+b\,x\right)\right)\,\ln\left(d\,\left(a+b\,x\right)\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x),x)
```

```
[Out] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ad+bdx\right)\operatorname{atan}\left(ac+bcx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(a*d + b*d*x)*atan(a*c + b*c*x)/(a + b*x), x)
```

3.147 $\int e^{c(a+bx)}\tan^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=48

$$\frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sinh(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2194, 5207, 2282, 12, 260}

$$\frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sinh[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\sinh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\sinh(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 1.27

$$-\frac{\log\left(e^{2c(a+bx)}+1\right)+e^{c(a+bx)}\tan^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] -((E^(c*(a + b*x))*ArcTan[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] + Log[
1 + E^(2*c*(a + b*x))])/(b*c))
```

fricas [A] time = 0.70, size = 75, normalized size = 1.56

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan(\sinh(bcx+ac))-\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

giac [A] time = 2.06, size = 65, normalized size = 1.35

$$\frac{\left(\arctan\left(\frac{1}{2}e^{(bcx+ac)}-\frac{1}{2}e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(2bcx+2ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

maple [C] time = 1.34, size = 1299, normalized size = 27.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x)
```

```
[Out] 1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(
-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(
b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a)
)-I)^2)*exp(c*(b*x+a))-ln(1+exp(2*c*(b*x+a)))/b/c+I/b/c*exp(c*(b*x+a))*ln(e
xp(c*(b*x+a))+I)+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x
+a))+I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp
(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*
csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(
-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2
)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a)
)*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+
I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c
*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(
c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I
*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(
exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(
b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c
*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*
x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(
c*(b*x+a))+2*a/b+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b
*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x
+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))
*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(ex
p(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-
I)+1/2/b/c*exp(c*(b*x+a))*Pi
```

maxima [A] time = 0.43, size = 48, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

mupad [B] time = 0.77, size = 66, normalized size = 1.38

$$\frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{atan}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}-\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(sinh(a*c + b*c*x)),x)
```

```
[Out] (exp(b*c*x)*exp(a*c)*atan((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))
/2))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.148 $\int e^{c(a+bx)} \tan^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=103

$$-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(cosh(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2194, 5207, 2282, 12, 1247, 632, 31}

$$-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Cosh[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\cosh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\cosh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\sinh(x)}{1+\cosh^2(x)}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x\left(-1+x^2\right)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx,x,e^{2ac+\cdots}\right.}{2bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc} - \cdots$$

Mathematica [C] time = 0.16, size = 146, normalized size = 1.42

$$\frac{\text{RootSum}\left[\#1^4+6\#1^2+1\&,\frac{-7\#1^2\log\left(e^{c(a+bx)}-\#1\right)+7\#1^2ac+7\#1^2bcx-\log\left(e^{c(a+bx)}-\#1\right)+ac+bcx}{3\#1^2+1}\&\right]-4c(a+bx)+2e^{c(a+bx)}\text{ta}\cdots}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(2*E^(c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

fricas [B] time = 0.88, size = 221, normalized size = 2.15

$$\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)\arctan\left(\cosh(bcx+ac)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)\cosh(\cdots}{\cosh(bcx+\cdots}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cos
h(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 +
2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(co
sh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(
b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

giac [A] time = 0.91, size = 154, normalized size = 1.50

$$\frac{\left(\sqrt{2}\,e^{(-ac)}\log\left(-\frac{2\sqrt{2}\,e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}\,e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}+\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}+6\,e\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

maple [C] time = 2.62, size = 1375, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x)
```

```
[Out] 1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))+1/4/b/c*
Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(
I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a)
)+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+
a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a
)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))
*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I
*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x
+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*
x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b
*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(
b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2
*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp
(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))
+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))
*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(ex
p(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c
*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(
b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(
c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp
(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*
x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/2/b/c*exp(c*(b*x+a))*Pi+1/2
/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*2^(1/2)*ln(exp(2*c*
(b*x+a))+(1+2^(1/2))^2)+2*a/b-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+
1-2*I*exp(c*(b*x+a)))-1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*ln
(exp(2*c*(b*x+a))+(1+2^(1/2))^2)
```

maxima [A] time = 0.43, size = 131, normalized size = 1.27

$$\frac{\arctan\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc}-\frac{2\left(bcx+ac\right)}{bc}-\frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) - 2*(b*c*x + a*c)/(b*c) - 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

mupad [B] time = 0.31, size = 133, normalized size = 1.29

$$\frac{\ln\left(-8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}-1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}+1\right)}{2\,b\,c}+\frac{e^{a\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(cosh(a*c + b*c*x)),x)
```

```
[Out] (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^
(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) + 6*2^(1/2)*exp
(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) + (exp(a*c + b*c*x)*atan((exp(b*c*x
)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.149 $\int e^{c(a+bx)} \tan^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=180

$$-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}-\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}\cdot$$

```
[Out] exp(b*c*x+a*c)*arctan(tanh(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2194, 5207, 12, 2249, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}+\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}-\frac{\tan^{-1}\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Tanh[c*(a + b*x)]])/(b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\tanh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}\left(\int \right.}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,e^{ac+bcx}\right)}{2bc} - \frac{\text{Subst}}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+\right.}{}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\tan^{-1}\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+\right.}{bc}$$

Mathematica [C] time = 0.13, size = 89, normalized size = 0.49

$$\frac{\text{RootSum}\left[\#1^4+1\&,\frac{-\log\left(e^{c(a+bx)}-\#1\right)+ac+bcx}{\#1}\&\right]+2e^{c(a+bx)}\tan^{-1}\left(\frac{e^{2c(a+bx)}-1}{e^{2c(a+bx)}+1}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcTan[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

fricas [B] time = 0.51, size = 431, normalized size = 2.39

$$4\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2bcx+2ac)}}\,bc\left(\frac{1}{b^4c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1
/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b
*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*
c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1
/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*c
^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c
))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt(
2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
+ e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*c
```

```
^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b
*c*x + 2*a*c)) + 4*arctan((e^(2*b*c*x + 2*a*c) - 1)/(e^(2*b*c*x + 2*a*c) +
1))*e^(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 2.62, size = 1355, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x)
```

```
[Out] 1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/2*I/b/c*ex
p(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)+1/4*I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)
-1/2*I*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/
2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+
1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b
*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))
))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(
b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+ex
p(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))
))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(
b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a)
)-1/4/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*ln(e
xp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+
1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/
2*I*2^(1/2))*2^(1/2)+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+I)+1/4/b/
c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b
*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c
*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I)
/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a
))))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4
/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c
*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(
2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csg
n((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*
x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*exp(c*(b*x+a))*Pi
```

maxima [A] time = 0.42, size = 167, normalized size = 0.93

$$\frac{\arctan\left(\tanh\left(bcx + ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(bcx+ac)}\right)\right)}{2bc} - \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(bcx+ac)}\right)\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

mupad [B] time = 1.42, size = 164, normalized size = 0.91

$$\frac{4\,e^{ac+bcx}\operatorname{atan}\left(\frac{e^{2bcx}\,e^{2ac}-1}{e^{2bcx}\,e^{2ac}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4i)+e^{bcx}\,e^{ac}\,8i\right)(-1-i)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4i)-e^{bcx}\,e^{ac}\,8i\right)}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(tanh(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(exp
(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(tanh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.150 $\int e^{c(a+bx)} \tan^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=180

$$\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\tan^{-1}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(coth(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2194, 5207, 12, 2249, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\tan^{-1}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c)) + ArcTan[1 + Sqrt[2]*E
^(a*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Coth[c*(a + b*x)]])
/(b*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*
c) - Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)} \tan^{-1}(\coth(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x \tan^{-1}(\coth(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Su}}{}$$
$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(\right.}{}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\tan^{-1}\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\tan^{-1}(\coth(c}{bc}$$

Mathematica [C] time = 0.12, size = 89, normalized size = 0.49

$$\frac{\text{RootSum}\left[\#1^4+1\&, \frac{\log\left(e^{c(a+bx)}-\#1\right)-ac-bcx}{\#1}\&\right]+2e^{c(a+bx)}\tan^{-1}\left(\frac{e^{2c(a+bx)}+1}{e^{2c(a+bx)}-1}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

fricas [B] time = 0.68, size = 431, normalized size = 2.39

$$-\frac{4\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(
1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(
b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4
*c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(
1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*
c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*
c))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt
(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
 + e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*
```

```
c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*
b*c*x + 2*a*c)) - 4*arctan((e^(2*b*c*x + 2*a*c) + 1)/(e^(2*b*c*x + 2*a*c) -
 1))*e^(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 2.07, size = 1355, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x)
```

```
[Out] -1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+
a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*ln(exp(c*(b*x+a))-1/2*2
^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2
^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2
)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*Pi*c
sgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1
))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c
*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(
2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(
2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*c
sgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a)
)-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x
+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x
+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I
)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-
I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*exp(c*(b*x+a))*Pi-1/4/b/c*P
i*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/
4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x
+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(
b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))
^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+
a))-1))^2*exp(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)-1/
4*I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(e
xp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a)
)+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2
)-1/2*I*2^(1/2))*2^(1/2)-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+I)
```

maxima [A] time = 0.43, size = 167, normalized size = 0.93

$$\frac{\arctan\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bcx+ac)}\right)\right)}{2bc}+\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bcx+ac)}\right)\right)}{2bc}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

mupad [B] time = 1.66, size = 164, normalized size = 0.91

$$\frac{4\,e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{e^{2\,b\,c\,x}\,e^{2\,a\,c}+1}{e^{2\,b\,c\,x}\,e^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4\mathrm{i}\right)-e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4\mathrm{i}\right)+e^{b\,c\,x}\,e^{a\,c}}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(coth(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2^(
1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\,\mathrm{atan}\left(\coth\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(coth(a*c + b*c*x)), x)
```

3.151 $\int e^{c(a+bx)} \tan^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sech(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2194, 5207, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sech[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tan^{-1}(\operatorname{sech}(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^x\tan^{-1}(\operatorname{sech}(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{e^x\operatorname{sech}(x)\tanh(x)}{1+\operatorname{sech}^2(x)}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{2\operatorname{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx,x,e^{2ac+\cdots}\right)}{2bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}+\cdots
\end{aligned}$$

Mathematica [C] time = 0.16, size = 145, normalized size = 1.41

$$\frac{\operatorname{RootSum}\left[\#1^4+6\#1^2+1\&,\frac{7\#1^2\log\left(e^{c(a+bx)}-\#1\right)-7\#1^2ac-7\#1^2bcx+\log\left(e^{c(a+bx)}-\#1\right)-ac-bcx}{3\#1^2+1}\&\right]+4c(a+bx)+2e^{c(a+bx)}\tan\cdots}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

fricas [B] time = 0.65, size = 276, normalized size = 2.68

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

giac [A] time = 0.14, size = 154, normalized size = 1.50

$$-\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)-2\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx+4ac)}+6e^{(2bcx+2}\right.\right.}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

maple [C] time = 1.88, size = 842, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))+1/4/b/c
*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x
+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a)))
)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))
)*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*e
xp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x
+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x+a)
)))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a
))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp
(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn
(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b
*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c
*(b*x+a))))^3*exp(c*(b*x+a))-1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(2^(1/2)-1
)^2)+1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)-2*a/b+1/2/b/c*ln(ex
p(2*c*(b*x+a))+(2^(1/2)-1)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)+1/
2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))
```

maxima [A] time = 0.45, size = 169, normalized size = 1.64

$$\frac{\arctan\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc}-\frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2bcx+2ac)}-3}{2\sqrt{2}+e^{(2bcx+2ac)}+3}\right)}{8bc}+\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2bcx-2ac)}-3}{2\sqrt{2}+e^{(-2bcx-2ac)}+3}\right)}{8bc}+\frac{\log\left(e^{(4bcx+4ac)}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
```

```
+ 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) + 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

mupad [B] time = 0.82, size = 135, normalized size = 1.31

$$\frac{e^{a c+b c x} \operatorname{atan}\left(\frac{1}{\frac{e^{b c x} e^{a c}}{2}+\frac{e^{-b c x} e^{-a c}}{2}}\right)}{b c}+\frac{\ln\left(8 e^{2 c (a+b x)}-2\sqrt{2}-6\sqrt{2}\, e^{2 c (a+b x)}\right)\left(\sqrt{2}+1\right)}{2 b c}-\frac{\ln\left(8 e^{2 c (a+b x)}+2\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*atan(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))
/2)))/(b*c) + (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a
+ b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6
*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.152 $\int e^{c(a+bx)} \tan^{-1}(\mathbf{csch}(ac + bcx))\, dx$

Optimal. Leaf size=47

$$\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}+\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(csch(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2194, 5207, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}+\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Csch[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\text{csch}(ac+bcx))\,dx=\frac{\text{Subst}\left(\int e^x\tan^{-1}(\text{csch}(x))\,dx,x,ac+bcx\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx,x,ac+bcx\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{2x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tan^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.12, size = 57, normalized size = 1.21

$$\frac{\log\left(e^{2c(a+bx)}+1\right)+e^{c(a+bx)}\tan^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}-1}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

fricas [B] time = 0.61, size = 131, normalized size = 2.79

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan\left(\frac{2\,(\cosh(bcx+ac)+\sinh(bcx+ac))}{\cosh(bcx+ac)^2+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right)+\log\left(\frac{2\,\text{co}}{\cosh(bcx+}\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

giac [A] time = 0.13, size = 66, normalized size = 1.40

$$\frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

maple [C] time = 0.98, size = 885, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x)
```

```
[Out] -I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I)^2)^3*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(
c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*c
sgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I)^2)*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*cs
gn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1)
)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))
-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*(exp(c
*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(ex
p(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(exp(2*c*(b*x+a))
-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b
/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+
a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*ex
p(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-2*a/b
+ln(1+exp(2*c*(b*x+a)))/b/c+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)
```

maxima [A] time = 0.42, size = 47, normalized size = 1.00

$$\frac{\arctan\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

mupad [B] time = 0.70, size = 67, normalized size = 1.43

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{atan}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*atan(1/((exp(
b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2)))/(b*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(csch(a*c + b*c*x)), x)
```

3.153 $\int \frac{\left(a+b\tan^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=163

$$ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{ibd\text{Li}_2\left(-icx^n\right)}{2n}-\frac{ibd\text{Li}_2\left(icx^n\right)}{2n}+\frac{ibe\text{Li}_2\left(-icx^n\right)\log\left(fx^m\right)}{2n}-\frac{ibe\text{Li}_2\left(icx^n\right)\log\left(fx^m\right)}{2n}-$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*I*b*d*polylog(2,-I*c*x^n)/n+1/2*I*b*e*l
n(f*x^m)*polylog(2,-I*c*x^n)/n-1/2*I*b*d*polylog(2,I*c*x^n)/n-1/2*I*b*e*ln(
f*x^m)*polylog(2,I*c*x^n)/n-1/2*I*b*e*m*polylog(3,-I*c*x^n)/n^2+1/2*I*b*e*m
*polylog(3,I*c*x^n)/n^2
```

Rubi [A] time = 0.57, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2301, 6742, 5031, 4848, 2391, 5007, 5005, 2374, 6589}

$$\frac{ibd\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibd\text{PolyLog}\left(2,icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,-icx^n\right)}{2n}-\frac{ibe\log\left(fx^m\right)\text{PolyLog}\left(2,ic\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])/
n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[2,
 I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*m*
PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5005

```
Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[(Log[d*x^m]*Log[1 - I*c*x^n])/x, x], x] - Dist[I/2, Int[(Log[d*x
^m]*Log[1 + I*c*x^n])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 5007

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTan[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcT
an[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx=\int\left(\frac{d\left(a+b\tan^{-1}(cx^n)\right)}{x}+\frac{e\left(a+b\tan^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\right)dx$$
$$=d\int\frac{a+b\tan^{-1}(cx^n)}{x}\,dx+e\int\frac{\left(a+b\tan^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\,dx$$
$$=(ae)\int\frac{\log\left(fx^m\right)}{x}\,dx+(be)\int\frac{\tan^{-1}(cx^n)\log\left(fx^m\right)}{x}\,dx+\frac{d\,\mathrm{Subst}\,(}{}$$
$$=ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{1}{2}(ibe)\int\frac{\log\left(fx^m\right)\log\left(1-icx^n\right)}{x}\,dx$$
$$=ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{ibd\mathrm{Li}_2\left(-icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\mathrm{Li}_2\left(-i\right.}{2n}$$
$$=ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{ibd\mathrm{Li}_2\left(-icx^n\right)}{2n}+\frac{ibe\log\left(fx^m\right)\mathrm{Li}_2\left(-i\right.}{2n}$$

Mathematica [C] time = 0.30, size = 116, normalized size = 0.71

$$\frac{bcx^n\left(d+e\log\left(fx^m\right)\right){}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n}-\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2}+\frac{1}{2}a\log(x)\left(2d+2e\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2
*x^(2*n))])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(
c^2*x^(2*n))]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log
[f*x^m]))/2
```

fricas [C] time = 0.62, size = 246, normalized size = 1.51

$$\frac{2\,aemn^2\log(x)^2+2i\,bem\mathrm{polylog}\left(3,i\,cx^n\right)-2i\,bem\mathrm{polylog}\left(3,-i\,cx^n\right)+2\left(bemn^2\log(x)^2+2\left(ben^2\log(f)+bd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 + 2*I*b*e*m*polylog(3, I*c*x^n) - 2*I*b*e*m*polyl
og(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*arctan(c*x^n) + (-2*I*b*e*m*n*log(x) - 2*I*b*e*n*log(f) - 2*I*b*d*n)*di
log(I*c*x^n) + (2*I*b*e*m*n*log(x) + 2*I*b*e*n*log(f) + 2*I*b*d*n)*dilog(-I
*c*x^n) + (I*b*e*m*n^2*log(x)^2 + (2*I*b*e*n^2*log(f) + 2*I*b*d*n^2)*log(x)
)*log(I*c*x^n + 1) + (-I*b*e*m*n^2*log(x)^2 + (-2*I*b*e*n^2*log(f) - 2*I*b*
d*n^2)*log(x))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arctan(cx^n) + a)\left(e \log\left(fx^m\right) + d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

maple [C] time = 0.70, size = 896, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] -1/2*I/n*dilog(1-I*c*x^n)*ln(f)*b*e-1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)+1/2
*I/n*dilog(1+I*c*x^n)*ln(f)*b*e+1/4/n*dilog(1+I*c*x^n)*Pi*b*e*csgn(I*x^m)*c
sgn(I*f*x^m)*csgn(I*f)+1/2*I*e*b*ln(1+I*c*x^n)*ln(x)^2*m-1/2*I*e*b*ln(1+I*c
*x^n)*ln(x^m)*ln(x)+1/2*I*e*b*ln(-I*(c*x^n+I))*ln(x)^2*m-1/2*I*e*b*ln(-I*(c
*x^n+I))*ln(x^m)*ln(x)+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f*x^m)^2*csgn(I*f)+1/2
*e*a/m*ln(x^m)^2+1/2*I*b*e*m*polylog(3,I*c*x^n)/n^2+1/4/n*dilog(1-I*c*x^n)*
Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f*x^m)^3-1
/4/n*dilog(1+I*c*x^n)*Pi*b*e*csgn(I*f*x^m)^2*csgn(I*f)-1/2*I*b*e*m*polylog(
3,-I*c*x^n)/n^2-1/2*I*e*b*ln(-I*(-c*x^n+I))*ln(x)^2*m+1/2*I*e*b*ln(-I*(-c*x
^n+I))*ln(x^m)*ln(x)-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*x^m)*csgn(I*f*x^m)*csgn(
I*f)+1/4/n*dilog(1-I*c*x^n)*Pi*b*e*csgn(I*f*x^m)^2*csgn(I*f)-1/4/n*dilog(1+
I*c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*dilog(1+I*c*x^n)*b*d+1/
4/n*dilog(1+I*c*x^n)*Pi*b*e*csgn(I*f*x^m)^3+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*l
n(-I*c*x^n)*m*ln(x)+1/n*ln(x^n)*ln(f)*a*e-1/2*I/n*dilog(1-I*c*x^n)*b*d+1/2*
I*e*b/n*dilog(-I*(c*x^n+I))*m*ln(x)+1/2*I*e*b/n*m*ln(x)*polylog(2,-I*c*x^n)
-1/4/n*dilog(1-I*c*x^n)*Pi*b*e*csgn(I*f*x^m)^3-1/4/n*dilog(1-I*c*x^n)*Pi*b*
e*csgn(I*x^m)*csgn(I*f*x^m)*csgn(I*f)+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*x^m)*cs
gn(I*f*x^m)^2-1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m)+1/2*I*e*b/
n*dilog(-I*c*x^n)*m*ln(x)-1/2*I*e*b*ln(1-I*c*x^n)*ln(x)^2*m+1/2*I*e*b*ln(1-
I*c*x^n)*ln(x^m)*ln(x)-1/2*I*e*b/n*m*ln(x)*polylog(2,I*c*x^n)-1/2*I*e*b/n*d
ilog(-I*(c*x^n+I))*ln(x^m)+1/n*ln(x^n)*a*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae\log\left(fx^m\right)^2}{2\,m}+ad\log(x)-\frac{1}{2}\left(bem\log(x)^2-2\,be\log(x)\log\left(x^m\right)-2\left(be\log(f)+bd\right)\log(x)\right)\arctan\left(cx^n\right)-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(c*x^n) - integrate(-1/2*(b*c*e
*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f) + b*c*d
)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}\,(c\,x^n))\,\left(d + e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

```
[Out] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```