Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.6-Exponentials-of-inverse-tangent

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [385]. This is test number [152].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (385)	% 0.00 (0)
Mathematica	% 95.58 (368)	% 4.42 (17)
Maple	% 52.73 (203)	% 47.27 (182)
Maxima	% 34.81 (134)	% 65.19 (251)
Fricas	% 74.29 (286)	% 25.71 (99)
Sympy	% 18.44 (71)	% 81.56 (314)
Giac	% 30.39 (117)	% 69.61 (268)
Mupad	% 38.18 (147)	% 61.82 (238)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

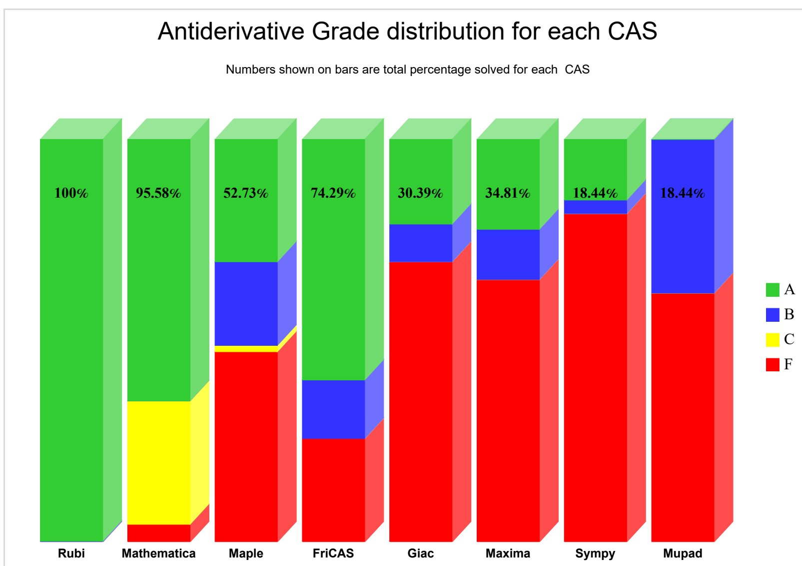
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

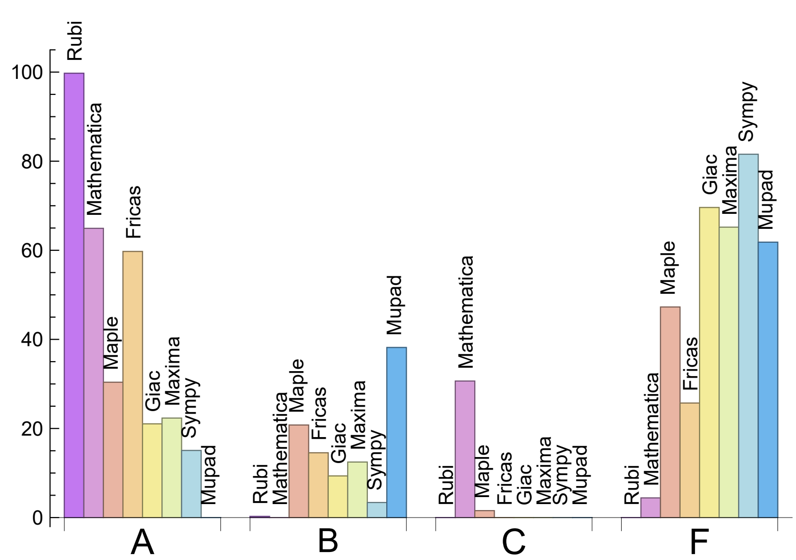
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.74	0.26	0.00	0.00
Mathematica	64.94	0.00	30.65	4.42
Maple	30.39	20.78	1.56	47.27
Maxima	22.34	12.47	0.00	65.19
Fricas	59.74	14.55	0.00	25.71
Sympy	15.06	3.38	0.00	81.56
Giac	21.04	9.35	0.00	69.61
Mupad	0.00	38.18	0.00	61.82

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	17	100.00 %	0.00 %	0.00 %
Maple	182	100.00 %	0.00 %	0.00 %
Maxima	251	96.02 %	0.00 %	3.98 %
Fricas	99	100.00 %	0.00 %	0.00 %
Sympy	314	77.39 %	22.61 %	0.00 %
Giac	268	50.75 %	6.72 %	42.54 %
Mupad	238	99.58 %	0.42 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

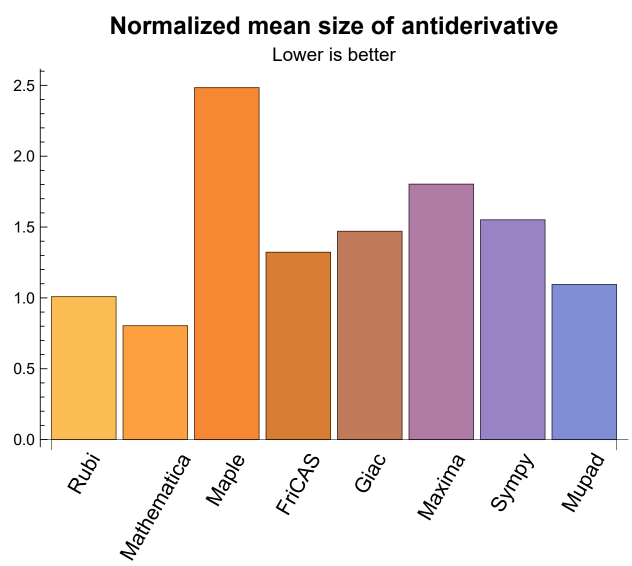
1.3 Performance

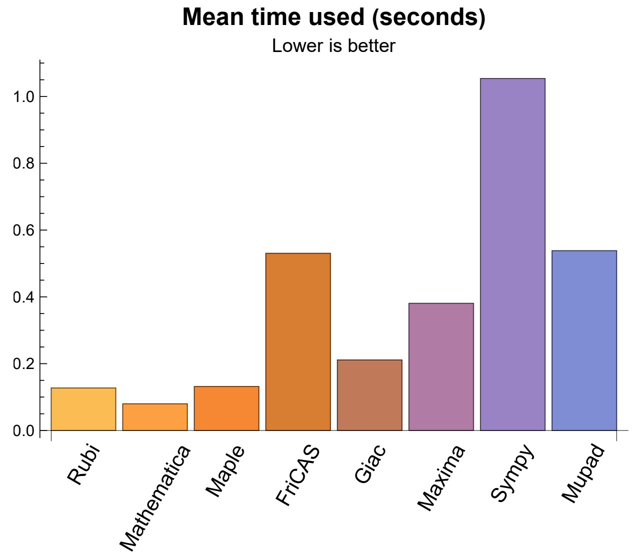
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	142.58	1.01	95.00	1.00
Mathematica	0.08	85.32	0.80	76.50	0.83
Maple	0.13	284.72	2.48	87.00	1.42
Maxima	0.38	222.52	1.80	65.50	1.25
Fricas	0.53	177.37	1.32	120.00	1.00
Sympy	1.05	75.31	1.55	48.00	1.13
Giac	0.21	118.82	1.47	73.00	1.22
Mupad	0.54	68.97	1.09	52.00	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {340, 344, 345, 346}

Mathematica {6, 7, 8, 9, 10, 14, 23, 24, 25, 26, 30, 39, 40, 41, 42, 43, 47, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143, 154, 157, 162, 163, 164, 165, 167, 168, 169, 170, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 202, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 240, 247, 248, 249, 250, 251, 252, 262, 263, 264, 265, 266, 276, 277, 278, 279, 280, 290, 291, 292, 293, 294, 302, 304, 307, 309, 311, 313, 316, 318, 329, 331, 334, 336, 339, 340, 343, 344, 345, 346, 347, 365}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about

2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
```

```
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

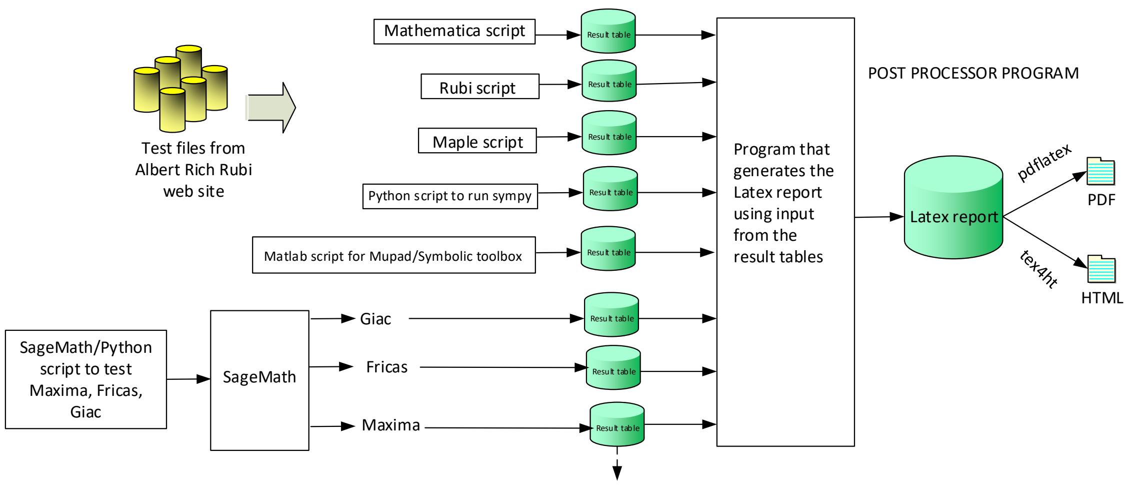
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { 344 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 134, 135, 136, 137, 138, 139, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 248, 249, 250, 251, 252, 263, 264, 265, 266, 277, 278, 279, 280, 291, 292, 293, 294, 302, 311, 343, 347 }

F grade: { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 236, 364, 366, 367, 368, 369, 370 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 140, 141, 165, 166, 202, 203, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 314, 315, 316, 317, 318, 319, 321, 323, 324, 325, 326, 327, 328, 330, 332, 333, 335, 343, 347, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385 }

B grade: { 6, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 307, 309, 311, 313, 320, 322, 329, 331, 334, 336, 379 }

C grade: { 134, 135, 136, 137, 138, 139 }

F grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 166, 191, 192, 193, 198, 199, 200, 201, 202, 203, 211, 248, 263, 277, 291, 301, 303, 304, 306, 307, 308, 315, 316, 317, 318, 322, 323, 325, 326, 333, 334, 335, 343, 372, 383, 384 }

B grade: { 53, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 204, 205, 206, 207, 208, 209, 210, 302, 305, 309, 319, 320, 321, 327, 336, 375, 376, 377, 385 }

C grade: { }

F grade: { 40, 41, 42, 43, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 194, 195, 196, 197, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 310, 311, 312, 313, 314, 324, 328, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 378, 379, 380, 381, 382 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 216, 217, 218, 219, 221, 222, 223, 226, 227, 228, 229, 231, 232, 233, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 334, 335, 336, 343, 347, 372, 373, 374, 377, 378 }

B grade: { 6, 7, 23, 24, 39, 40, 56, 57, 65, 75, 93, 102, 111, 112, 167, 168, 169, 170, 185, 186, 187, 188, 194, 195, 196, 197, 212, 213, 214, 215, 220, 224, 225, 230, 234, 235, 310, 311, 312, 313, 314, 315, 316, 317, 318, 323, 324, 332, 333, 375, 376, 379, 380, 381, 384, 385 }

C grade: { }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 382, 383 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 141, 166, 171, 172, 173, 174, 175, 198, 199, 200, 201, 202, 301, 303, 305, 306, 308, 319, 321, 323, 324, 326, 377, 378 }

B grade: { 136, 137, 176, 177, 178, 179, 203, 204, 205, 206, 375, 376, 379 }

C grade: { }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 304, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 322, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 380, 381, 382, 383, 384, 385 }

2.1.7 Giac

A grade: { 2, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 49, 51, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 189, 190, 191, 192, 193, 194, 195, 202, 207, 208, 209, 210, 248, 263, 277, 291, 301, 302, 303, 305, 306, 308, 309, 311, 313, 316, 318, 319, 321, 322, 323, 324, 325, 326, 331, 334, 343, 377 }

B grade: { 6, 7, 8, 9, 10, 39, 44, 45, 46, 47, 48, 50, 169, 170, 184, 185, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 211, 212, 320, 327, 329, 336, 375, 376, 378, 379 }

C grade: { }

F grade: { 1, 3, 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 186, 187, 188, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 307, 310, 312, 314, 315, 317, 328, 330, 332, 333, 335, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 380, 381, 382, 383, 384, 385 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 198, 199, 200, 201, 202, 203, 204, 205, 206, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 334, 335, 336, 343, 347, 372, 373, 376, 377, 378, 379, 380, 381, 384, 385 }

C grade: { }

F grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 310, 311, 312, 313, 314, 315, 316, 317, 318, 332, 333, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 382, 383 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	64	128	100	67	138	0	98
normalized size	1	1.00	0.57	1.13	0.88	0.59	1.22	0.00	0.87
time (sec)	N/A	0.088	0.056	0.164	0.359	0.461	5.024	0.000	0.098

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	56	109	81	59	119	73	85
normalized size	1	1.00	0.62	1.21	0.90	0.66	1.32	0.81	0.94
time (sec)	N/A	0.064	0.044	0.176	0.317	0.468	4.494	2.008	0.425

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	89	62	51	75	0	71
normalized size	1	1.00	0.61	1.19	0.83	0.68	1.00	0.00	0.95
time (sec)	N/A	0.047	0.034	0.182	0.317	0.427	3.213	0.000	0.411

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	69	42	43	51	54	51
normalized size	1	1.00	0.90	1.64	1.00	1.02	1.21	1.29	1.21
time (sec)	N/A	0.019	0.028	0.168	0.332	0.431	3.122	0.122	0.041

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	25	37	68	41	32
normalized size	1	1.00	0.90	1.66	0.86	1.28	2.34	1.41	1.10
time (sec)	N/A	0.009	0.015	0.104	0.320	0.446	1.253	0.135	0.037

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	29	48	18	58	53	69	32
normalized size	1	1.00	1.16	1.92	0.72	2.32	2.12	2.76	1.28
time (sec)	N/A	0.037	0.015	0.157	0.313	0.425	3.346	0.137	0.037

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	29	66	26	76	33
normalized size	1	1.00	1.24	0.89	0.76	1.74	0.68	2.00	0.87
time (sec)	N/A	0.038	0.030	0.164	0.339	0.454	2.302	0.137	0.037

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	53	48	83	48	155	52
normalized size	1	1.00	0.90	0.84	0.76	1.32	0.76	2.46	0.83
time (sec)	N/A	0.050	0.042	0.161	0.331	1.198	2.911	0.150	0.040

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	75	67	92	75	164	74
normalized size	1	1.00	0.78	0.83	0.74	1.02	0.83	1.82	0.82
time (sec)	N/A	0.069	0.063	0.157	0.329	0.411	3.090	0.158	0.036

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	76	97	86	101	122	240	95
normalized size	1	1.00	0.67	0.86	0.76	0.89	1.08	2.12	0.84
time (sec)	N/A	0.090	0.063	0.162	0.333	0.467	4.301	0.157	0.031

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	55	56	46	42	48	43
normalized size	1	1.00	1.00	1.15	1.17	0.96	0.88	1.00	0.90
time (sec)	N/A	0.036	0.022	0.042	0.429	0.445	0.125	0.114	0.418

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	47	47	37	32	39	36
normalized size	1	1.00	1.00	1.21	1.21	0.95	0.82	1.00	0.92
time (sec)	N/A	0.028	0.015	0.045	0.433	0.464	0.130	0.147	0.421

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	38	38	29	24	30	27
normalized size	1	1.00	1.00	1.31	1.31	1.00	0.83	1.03	0.93
time (sec)	N/A	0.019	0.012	0.043	0.441	0.380	0.104	0.138	0.062

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	30	28	21	14	16	19
normalized size	1	1.00	1.58	1.58	1.47	1.11	0.74	0.84	1.00
time (sec)	N/A	0.009	0.010	0.040	0.441	0.417	0.106	0.119	0.043

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	21	15	17	15	14
normalized size	1	1.00	1.00	1.77	1.62	1.15	1.31	1.15	1.08
time (sec)	N/A	0.019	0.007	0.045	0.453	0.391	0.152	0.124	0.439

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	31	26	32	23	17
normalized size	1	1.00	1.00	1.31	1.19	1.00	1.23	0.88	0.65
time (sec)	N/A	0.024	0.010	0.047	0.442	0.396	0.163	0.132	0.062

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	42	39	42	32	27
normalized size	1	1.00	1.00	1.25	1.17	1.08	1.17	0.89	0.75
time (sec)	N/A	0.027	0.012	0.046	0.430	0.421	0.185	0.115	0.079

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	55	51	47	54	42	34
normalized size	1	1.00	1.00	1.15	1.06	0.98	1.12	0.88	0.71
time (sec)	N/A	0.030	0.014	0.050	0.416	0.405	0.206	0.140	0.072

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	80	143	114	88	0	0	137
normalized size	1	1.00	0.58	1.04	0.83	0.64	0.00	0.00	1.00
time (sec)	N/A	0.621	0.063	0.163	0.316	0.416	0.000	0.000	0.456

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	123	95	81	0	0	114
normalized size	1	1.00	0.62	1.21	0.93	0.79	0.00	0.00	1.12
time (sec)	N/A	0.568	0.056	0.159	0.319	0.410	0.000	0.000	0.063

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	104	76	72	0	0	104
normalized size	1	1.00	0.59	1.13	0.83	0.78	0.00	0.00	1.13
time (sec)	N/A	0.326	0.067	0.156	0.323	0.439	0.000	0.000	0.427

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	81	57	60	0	0	72
normalized size	1	1.00	0.70	1.35	0.95	1.00	0.00	0.00	1.20
time (sec)	N/A	0.045	0.035	0.114	0.312	0.419	0.000	0.000	0.422

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	77	46	100	0	0	73
normalized size	1	1.00	1.08	1.51	0.90	1.96	0.00	0.00	1.43
time (sec)	N/A	0.663	0.044	0.143	0.321	0.447	0.000	0.000	0.428

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	80	60	109	0	0	75
normalized size	1	1.00	0.97	1.27	0.95	1.73	0.00	0.00	1.19
time (sec)	N/A	0.560	0.050	0.148	0.313	0.441	0.000	0.000	0.059

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	79	105	81	131	0	0	99
normalized size	1	1.00	0.86	1.14	0.88	1.42	0.00	0.00	1.08
time (sec)	N/A	0.601	0.082	0.153	0.311	0.483	0.000	0.000	0.426

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	141	100	139	0	0	116
normalized size	1	1.00	0.76	1.21	0.85	1.19	0.00	0.00	0.99
time (sec)	N/A	0.616	0.081	0.150	0.313	0.461	0.000	0.000	0.415

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	70	77	70	56	63	60
normalized size	1	1.00	1.00	1.08	1.18	1.08	0.86	0.97	0.92
time (sec)	N/A	0.045	0.048	0.060	0.416	0.421	0.239	0.121	0.431

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	60	68	62	44	53	51
normalized size	1	1.00	1.00	1.13	1.28	1.17	0.83	1.00	0.96
time (sec)	N/A	0.040	0.035	0.056	0.420	0.851	0.235	0.123	0.059

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	53	60	54	36	45	43
normalized size	1	1.00	1.00	1.18	1.33	1.20	0.80	1.00	0.96
time (sec)	N/A	0.026	0.028	0.056	0.419	0.421	0.190	0.137	0.064

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	42	41	45	43	22	26	32
normalized size	1	1.00	1.35	1.32	1.45	1.39	0.71	0.84	1.03
time (sec)	N/A	0.014	0.021	0.052	0.425	0.522	0.173	0.121	0.427

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	18	10	14	14
normalized size	1	1.00	1.00	0.94	1.38	1.12	0.62	0.88	0.88
time (sec)	N/A	0.022	0.010	0.054	0.423	0.536	0.211	0.132	0.077

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	45	53	60	44	37	37
normalized size	1	1.00	1.00	1.18	1.39	1.58	1.16	0.97	0.97
time (sec)	N/A	0.031	0.026	0.059	0.424	0.419	0.297	0.141	0.450

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	60	69	78	60	47	43
normalized size	1	1.00	1.00	1.15	1.33	1.50	1.15	0.90	0.83
time (sec)	N/A	0.035	0.041	0.056	0.423	0.469	0.347	0.123	0.468

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	68	77	86	71	59	55
normalized size	1	1.00	1.00	1.10	1.24	1.39	1.15	0.95	0.89
time (sec)	N/A	0.039	0.040	0.058	0.419	0.466	0.382	0.137	0.130

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	56	187	76	59	0	0	85
normalized size	1	1.00	0.62	2.08	0.84	0.66	0.00	0.00	0.94
time (sec)	N/A	0.066	0.047	0.190	0.426	0.457	0.000	0.000	0.063

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	168	59	51	0	0	71
normalized size	1	1.00	0.61	2.24	0.79	0.68	0.00	0.00	0.95
time (sec)	N/A	0.047	0.035	0.165	0.437	0.571	0.000	0.000	0.416

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	152	42	43	0	54	51
normalized size	1	1.00	0.90	3.62	1.00	1.02	0.00	1.29	1.21
time (sec)	N/A	0.020	0.028	0.155	0.428	0.544	0.000	0.121	0.404

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	97	25	37	0	42	32
normalized size	1	1.00	0.90	3.34	0.86	1.28	0.00	1.45	1.10
time (sec)	N/A	0.009	0.015	0.102	0.428	0.442	0.000	0.121	0.405

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	29	121	26	58	0	68	32
normalized size	1	1.00	1.16	4.84	1.04	2.32	0.00	2.72	1.28
time (sec)	N/A	0.037	0.015	0.162	0.474	0.464	0.000	0.141	0.038

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	194	0	66	0	0	33
normalized size	1	1.00	1.24	5.11	0.00	1.74	0.00	0.00	0.87
time (sec)	N/A	0.037	0.027	0.168	0.000	0.487	0.000	0.000	0.036

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	219	0	83	0	0	52
normalized size	1	1.00	0.90	3.48	0.00	1.32	0.00	0.00	0.83
time (sec)	N/A	0.051	0.044	0.167	0.000	0.438	0.000	0.000	0.039

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	237	0	92	0	0	74
normalized size	1	1.00	0.78	2.63	0.00	1.02	0.00	0.00	0.82
time (sec)	N/A	0.070	0.054	0.170	0.000	0.446	0.000	0.000	0.035

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	76	259	0	101	0	0	95
normalized size	1	1.00	0.67	2.29	0.00	0.89	0.00	0.00	0.84
time (sec)	N/A	0.089	0.064	0.179	0.000	0.460	0.000	0.000	0.033

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	55	44	46	42	80	43
normalized size	1	1.00	1.00	1.12	0.90	0.94	0.86	1.63	0.88
time (sec)	N/A	0.035	0.021	0.053	0.319	0.406	0.124	0.117	0.056

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	47	35	37	32	68	36
normalized size	1	1.00	1.00	1.18	0.88	0.92	0.80	1.70	0.90
time (sec)	N/A	0.030	0.014	0.053	0.319	0.469	0.128	0.117	0.415

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	38	28	29	24	58	27
normalized size	1	1.00	1.00	1.27	0.93	0.97	0.80	1.93	0.90
time (sec)	N/A	0.021	0.012	0.052	0.330	0.401	0.106	0.129	0.420

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	30	16	21	15	67	19
normalized size	1	1.00	1.50	1.50	0.80	1.05	0.75	3.35	0.95
time (sec)	N/A	0.010	0.011	0.050	0.327	0.439	0.114	0.133	0.409

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	23	12	15	17	47	14
normalized size	1	1.00	1.00	1.64	0.86	1.07	1.21	3.36	1.00
time (sec)	N/A	0.020	0.007	0.055	0.320	0.408	0.151	0.131	0.451

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	34	26	32	37	17
normalized size	1	1.00	1.00	1.26	1.26	0.96	1.19	1.37	0.63
time (sec)	N/A	0.026	0.011	0.066	0.327	0.422	0.156	0.133	0.420

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	45	50	39	42	63	26
normalized size	1	1.00	1.00	1.22	1.35	1.05	1.14	1.70	0.70
time (sec)	N/A	0.027	0.012	0.059	0.332	0.400	0.182	0.133	0.068

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	55	57	47	54	77	33
normalized size	1	1.00	1.00	1.12	1.16	0.96	1.10	1.57	0.67
time (sec)	N/A	0.032	0.015	0.059	0.326	0.422	0.209	0.136	0.430

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	80	296	216	88	0	0	138
normalized size	1	1.00	0.58	2.16	1.58	0.64	0.00	0.00	1.01
time (sec)	N/A	0.624	0.062	0.179	0.455	0.409	0.000	0.000	0.466

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	224	181	81	0	0	115
normalized size	1	1.00	0.62	2.20	1.77	0.79	0.00	0.00	1.13
time (sec)	N/A	0.574	0.056	0.172	0.441	0.411	0.000	0.000	0.071

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	60	226	112	72	0	0	105
normalized size	1	1.00	0.65	2.46	1.22	0.78	0.00	0.00	1.14
time (sec)	N/A	0.324	0.046	0.169	0.454	0.420	0.000	0.000	0.421

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	219	65	60	0	0	73
normalized size	1	1.00	0.70	3.65	1.08	1.00	0.00	0.00	1.22
time (sec)	N/A	0.044	0.036	0.111	0.453	0.487	0.000	0.000	0.416

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	55	257	0	100	0	0	74
normalized size	1	1.00	1.06	4.94	0.00	1.92	0.00	0.00	1.42
time (sec)	N/A	0.572	0.045	0.171	0.000	0.437	0.000	0.000	0.433

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	305	0	109	0	0	76
normalized size	1	1.00	0.95	4.77	0.00	1.70	0.00	0.00	1.19
time (sec)	N/A	0.552	0.053	0.168	0.000	0.409	0.000	0.000	0.422

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	376	0	131	0	0	100
normalized size	1	1.00	0.85	4.04	0.00	1.41	0.00	0.00	1.08
time (sec)	N/A	0.593	0.087	0.175	0.000	0.439	0.000	0.000	0.432

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	89	392	0	139	0	0	117
normalized size	1	1.00	0.75	3.32	0.00	1.18	0.00	0.00	0.99
time (sec)	N/A	0.604	0.082	0.176	0.000	0.556	0.000	0.000	0.066

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	95	416	0	146	0	0	139
normalized size	1	1.00	0.68	2.99	0.00	1.05	0.00	0.00	1.00
time (sec)	N/A	0.662	0.088	0.193	0.000	0.479	0.000	0.000	0.434

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	244	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.041	0.248	0.000	0.611	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	236	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.023	0.168	0.000	0.524	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	209	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.030	0.112	0.000	0.461	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	243	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.040	0.161	0.000	0.432	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	71	0	0	151	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.020	0.162	0.000	0.448	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	175	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.024	0.164	0.000	0.506	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	183	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.028	0.167	0.000	0.435	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	192	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.033	0.162	0.000	0.434	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	111	0	0	199	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.037	0.165	0.000	0.451	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	148	0	0	254	0	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.146	0.178	0.000	0.450	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	247	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.043	0.173	0.000	0.484	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	61	0	0	239	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.020	0.179	0.000	0.446	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	215	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.030	0.116	0.000	0.511	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	243	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.035	0.181	0.000	0.534	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	0	0	157	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.016	0.170	0.000	0.743	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	179	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.020	0.172	0.000	1.188	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	186	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.025	0.173	0.000	0.491	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	195	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.081	0.033	0.168	0.000	0.440	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	96	0	0	251	0	0	-1
normalized size	1	1.00	0.26	0.00	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.046	0.187	0.000	0.559	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	86	0	0	244	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.037	0.197	0.000	0.524	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	72	0	0	236	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.039	0.181	0.000	0.799	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	41	0	0	209	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.041	0.121	0.000	0.424	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	112	0	0	267	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.044	0.179	0.000	0.729	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	87	0	0	152	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.021	0.179	0.000	0.417	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	99	0	0	176	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.025	0.184	0.000	1.007	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	106	0	0	183	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.082	0.031	0.185	0.000	0.728	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	118	0	0	192	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.100	0.037	0.187	0.000	0.519	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	255	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.108	0.188	0.000	0.541	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	247	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.032	0.178	0.000	0.452	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	238	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.018	0.179	0.000	0.497	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	213	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.030	0.119	0.000	0.421	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	243	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.033	0.177	0.000	0.469	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	156	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.016	0.167	0.000	0.458	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	178	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.020	0.215	0.000	0.443	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	92	0	0	186	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.027	0.171	0.000	0.489	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	195	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.081	0.034	0.171	0.000	0.463	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	251	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.115	0.189	0.000	0.432	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	243	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.031	0.191	0.000	0.449	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	236	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.016	0.180	0.000	0.477	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	39	0	0	210	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.041	0.126	0.000	0.508	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	243	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.031	0.178	0.000	0.632	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	152	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.018	0.199	0.000	1.111	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	175	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.035	0.187	0.000	0.418	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	183	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.027	0.188	0.000	0.421	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	191	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.081	0.037	0.199	0.000	0.499	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	100	0	0	306	0	0	-1
normalized size	1	1.00	0.27	0.00	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.045	0.187	0.000	0.426	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	91	0	0	298	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.033	0.181	0.000	0.423	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	63	0	0	292	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.031	0.175	0.000	0.652	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	39	0	0	262	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.050	0.115	0.000	0.580	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	106	0	0	330	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.069	0.171	0.000	0.560	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	69	0	0	213	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.019	0.173	0.000	0.618	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	81	0	0	238	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.020	0.171	0.000	0.636	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	93	0	0	246	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	1.21	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.027	0.170	0.000	0.750	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	99	0	0	252	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.099	0.037	0.167	0.000	0.541	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	73	0	0	208	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.036	0.166	0.000	0.628	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	57	0	0	195	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.024	0.139	0.000	0.475	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	34	0	0	198	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.018	0.085	0.000	0.440	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	90	0	0	339	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.031	0.103	0.000	0.505	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	64	0	0	211	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.016	0.101	0.000	3.169	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	72	0	0	234	0	0	-1
normalized size	1	1.00	0.26	0.00	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.018	0.104	0.000	2.273	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	81	0	0	244	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.021	0.101	0.000	1.957	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	73	0	0	121	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.036	0.063	0.000	1.536	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	54	0	0	116	0	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.022	0.130	0.000	0.624	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	34	0	0	107	0	0	-1
normalized size	1	1.00	0.29	0.00	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.019	0.106	0.000	0.634	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	90	0	0	147	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.027	0.092	0.000	0.507	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	59	0	0	120	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.013	0.065	0.000	0.629	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	69	0	0	138	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.016	0.099	0.000	0.640	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	741	741	83	0	0	435	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.59	0.00	0.00	-0.00
time (sec)	N/A	0.733	0.041	0.206	0.000	0.652	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	63	0	0	428	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.025	0.161	0.000	0.901	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	41	0	0	383	0	0	-1
normalized size	1	1.00	0.06	0.00	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.025	0.105	0.000	0.675	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	859	859	97	0	0	509	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.59	0.00	0.00	-0.00
time (sec)	N/A	0.546	0.036	0.162	0.000	0.632	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	71	0	0	345	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.017	0.151	0.000	0.542	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	84	0	0	381	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.023	0.158	0.000	0.496	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	94	748	0	0	0	0	-1
normalized size	1	1.00	0.82	6.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.045	0.752	0.000	0.589	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	417	0	0	0	0	-1
normalized size	1	1.00	1.16	8.34	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.024	0.574	0.000	0.570	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	175	0	0	136	0	-1
normalized size	1	1.00	0.74	4.49	0.00	0.00	3.49	0.00	-0.03
time (sec)	N/A	0.023	0.007	0.513	0.000	0.634	3.404	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	158	0	0	136	0	-1
normalized size	1	1.00	0.74	4.05	0.00	0.00	3.49	0.00	-0.03
time (sec)	N/A	0.023	0.009	0.420	0.000	0.462	4.063	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	428	0	0	0	0	-1
normalized size	1	1.00	1.16	8.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.022	0.565	0.000	0.428	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	1196	0	0	0	0	-1
normalized size	1	1.00	0.82	10.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.043	0.744	0.000	0.460	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	113	146	0	0	0	0	-1
normalized size	1	1.00	0.71	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.774	0.085	0.428	0.000	0.469	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	71	0	0	95	0	-1
normalized size	1	1.00	1.08	0.90	0.00	0.00	1.20	0.00	-0.01
time (sec)	N/A	0.041	0.037	0.313	0.000	0.464	3.130	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.042	0.449	0.000	0.486	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	113	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.705	0.083	0.476	0.000	0.468	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.309	0.125	0.000	0.493	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.265	0.120	0.000	0.455	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.244	0.123	0.000	0.605	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.268	0.114	0.000	0.451	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.278	0.114	0.000	0.439	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.339	0.115	0.000	0.535	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.216	0.214	0.000	0.454	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.209	0.193	0.000	0.466	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.245	0.114	0.000	0.474	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.249	0.171	0.000	0.447	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	210	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.198	0.135	0.000	0.439	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	116	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.054	0.133	0.000	0.453	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.029	0.132	0.000	0.473	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.034	0.119	0.000	0.447	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.029	0.132	0.000	0.450	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.016	0.132	0.000	0.529	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.034	0.157	0.000	0.600	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	119	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.064	0.161	0.000	0.471	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	217	656	749	175	0	214	-1
normalized size	1	1.00	0.79	2.38	2.71	0.63	0.00	0.78	-0.00
time (sec)	N/A	0.201	0.526	0.200	0.356	0.494	0.000	0.196	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	176	465	529	137	0	163	-1
normalized size	1	1.00	0.88	2.31	2.63	0.68	0.00	0.81	-0.00
time (sec)	N/A	0.192	0.308	0.186	0.352	0.628	0.000	0.868	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	302	351	106	0	117	-1
normalized size	1	1.00	0.79	1.77	2.05	0.62	0.00	0.68	-0.01
time (sec)	N/A	0.125	0.143	0.177	0.348	0.476	0.000	0.195	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	171	209	77	0	76	-1
normalized size	1	1.00	0.98	1.55	1.90	0.70	0.00	0.69	-0.01
time (sec)	N/A	0.075	0.122	0.170	0.331	0.432	0.000	0.188	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	28	69	62	60	36	51	97
normalized size	1	1.00	0.54	1.33	1.19	1.15	0.69	0.98	1.87
time (sec)	N/A	0.034	0.021	0.109	0.329	0.436	3.414	0.482	1.092

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	142	157	233	155	0	112	118
normalized size	1	1.00	1.60	1.76	2.62	1.74	0.00	1.26	1.33
time (sec)	N/A	0.075	0.080	0.165	0.331	0.468	0.000	3.919	1.149

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	120	236	239	227	0	145	218
normalized size	1	1.00	0.92	1.82	1.84	1.75	0.00	1.12	1.68
time (sec)	N/A	0.064	0.087	0.171	0.322	0.496	0.000	0.232	1.676

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	154	405	424	455	0	475	-1
normalized size	1	1.00	0.77	2.01	2.11	2.26	0.00	2.36	-0.00
time (sec)	N/A	0.138	0.136	0.175	0.344	0.450	0.000	0.218	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	247	611	644	700	0	900	-1
normalized size	1	1.00	0.87	2.16	2.28	2.47	0.00	3.18	-0.00
time (sec)	N/A	0.183	0.286	0.181	0.351	0.568	0.000	0.289	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	347	150	105	114	130	201
normalized size	1	1.00	1.00	3.77	1.63	1.14	1.24	1.41	2.18
time (sec)	N/A	0.083	0.078	0.044	0.451	0.478	0.502	0.125	0.585

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	255	118	77	78	88	153
normalized size	1	1.00	1.00	3.54	1.64	1.07	1.08	1.22	2.12
time (sec)	N/A	0.059	0.071	0.044	0.428	0.473	0.380	0.134	0.532

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	176	87	53	49	57	107
normalized size	1	1.00	1.00	3.26	1.61	0.98	0.91	1.06	1.98
time (sec)	N/A	0.048	0.035	0.042	0.420	0.414	0.311	0.135	0.513

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	107	66	35	32	36	60
normalized size	1	1.00	1.00	2.89	1.78	0.95	0.86	0.97	1.62
time (sec)	N/A	0.030	0.023	0.041	0.441	0.502	0.224	0.133	0.129

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	51	46	22	17	17	21
normalized size	1	1.00	1.60	2.55	2.30	1.10	0.85	0.85	1.05
time (sec)	N/A	0.012	0.012	0.040	0.423	0.584	0.156	0.119	0.464

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	149	80	27	107	36	32
normalized size	1	1.00	0.82	3.92	2.11	0.71	2.82	0.95	0.84
time (sec)	N/A	0.034	0.021	0.046	0.415	0.629	0.796	0.121	0.704

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	260	125	40	158	68	98
normalized size	1	1.00	0.71	4.73	2.27	0.73	2.87	1.24	1.78
time (sec)	N/A	0.041	0.029	0.051	0.421	0.502	0.610	0.122	0.637

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	406	188	70	226	98	154
normalized size	1	1.00	0.83	5.34	2.47	0.92	2.97	1.29	2.03
time (sec)	N/A	0.049	0.038	0.052	0.432	0.428	0.876	0.122	0.693

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	560	263	94	286	136	199
normalized size	1	1.00	0.95	6.02	2.83	1.01	3.08	1.46	2.14
time (sec)	N/A	0.056	0.052	0.062	0.440	0.513	1.153	0.123	0.718

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	249	933	3117	263	0	345	-1
normalized size	1	1.00	0.77	2.88	9.62	0.81	0.00	1.06	-0.00
time (sec)	N/A	0.267	0.480	0.196	0.408	0.496	0.000	0.212	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	201	711	2321	217	0	293	-1
normalized size	1	1.00	0.81	2.86	9.32	0.87	0.00	1.18	-0.00
time (sec)	N/A	0.242	0.279	0.186	0.371	0.515	0.000	0.186	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	160	519	1626	174	0	249	-1
normalized size	1	1.00	0.70	2.29	7.16	0.77	0.00	1.10	-0.00
time (sec)	N/A	0.169	0.239	0.182	0.359	0.459	0.000	0.191	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	132	358	1116	137	0	214	-1
normalized size	1	1.00	0.81	2.20	6.85	0.84	0.00	1.31	-0.01
time (sec)	N/A	0.120	0.182	0.175	0.366	0.506	0.000	0.192	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	45	362	740	102	0	183	-1
normalized size	1	1.00	0.48	3.85	7.87	1.09	0.00	1.95	-0.01
time (sec)	N/A	0.042	0.045	0.111	0.344	0.520	0.000	0.211	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	196	818	738	381	0	259	-1
normalized size	1	1.00	1.46	6.10	5.51	2.84	0.00	1.93	-0.01
time (sec)	N/A	0.095	0.800	0.171	0.345	0.512	0.000	0.363	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	145	1358	993	404	0	0	-1
normalized size	1	1.00	0.82	7.72	5.64	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.169	0.174	0.352	0.471	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	194	1955	1541	580	0	0	-1
normalized size	1	1.00	0.73	7.41	5.84	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.243	0.178	0.367	0.559	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	282	2624	2327	853	0	0	-1
normalized size	1	1.00	0.83	7.76	6.88	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.426	0.181	0.377	0.546	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	248	1208	456	175	0	214	-1
normalized size	1	1.00	0.90	4.38	1.65	0.63	0.00	0.78	-0.00
time (sec)	N/A	0.225	0.718	0.225	0.467	0.468	0.000	0.157	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	202	894	308	137	0	163	-1
normalized size	1	1.00	1.00	4.45	1.53	0.68	0.00	0.81	-0.00
time (sec)	N/A	0.194	0.516	0.190	0.440	0.515	0.000	0.148	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	162	605	161	106	0	117	-1
normalized size	1	1.00	0.95	3.54	0.94	0.62	0.00	0.68	-0.01
time (sec)	N/A	0.126	0.299	0.183	0.439	0.447	0.000	0.138	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	131	350	97	77	0	78	-1
normalized size	1	1.00	1.19	3.18	0.88	0.70	0.00	0.71	-0.01
time (sec)	N/A	0.076	0.137	0.175	0.430	0.469	0.000	0.161	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	28	122	35	60	0	52	-1
normalized size	1	1.00	0.54	2.35	0.67	1.15	0.00	1.00	-0.02
time (sec)	N/A	0.034	0.020	0.110	0.425	0.551	0.000	0.128	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	142	283	0	155	0	77	-1
normalized size	1	1.00	1.60	3.18	0.00	1.74	0.00	0.87	-0.01
time (sec)	N/A	0.065	0.077	0.177	0.000	0.471	0.000	0.151	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	602	0	227	0	147	-1
normalized size	1	1.00	0.92	4.63	0.00	1.75	0.00	1.13	-0.01
time (sec)	N/A	0.064	0.083	0.184	0.000	0.489	0.000	0.230	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	154	1146	0	455	0	481	-1
normalized size	1	1.00	0.77	5.70	0.00	2.26	0.00	2.39	-0.00
time (sec)	N/A	0.117	0.149	0.181	0.000	0.522	0.000	0.259	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	282	247	1738	0	700	0	906	-1
normalized size	1	1.00	0.87	6.14	0.00	2.47	0.00	3.20	-0.00
time (sec)	N/A	0.183	0.299	0.201	0.000	0.521	0.000	0.232	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	292	102	105	117	235	165
normalized size	1	1.00	0.96	2.95	1.03	1.06	1.18	2.37	1.67
time (sec)	N/A	0.086	0.074	0.056	0.318	0.462	0.516	0.122	0.166

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	211	74	77	80	173	129
normalized size	1	1.00	1.00	2.74	0.96	1.00	1.04	2.25	1.68
time (sec)	N/A	0.059	0.063	0.056	0.318	0.439	0.486	0.126	0.538

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	143	52	53	53	120	90
normalized size	1	1.00	0.93	2.42	0.88	0.90	0.90	2.03	1.53
time (sec)	N/A	0.048	0.034	0.056	0.320	0.527	0.359	0.138	0.537

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	85	36	35	32	80	51
normalized size	1	1.00	1.00	2.12	0.90	0.88	0.80	2.00	1.28
time (sec)	N/A	0.032	0.023	0.056	0.316	0.442	0.249	0.138	0.503

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	40	19	22	19	38	21
normalized size	1	1.00	1.39	1.74	0.83	0.96	0.83	1.65	0.91
time (sec)	N/A	0.013	0.012	0.054	0.318	0.508	0.216	0.133	0.060

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	74	47	27	102	77	34
normalized size	1	1.00	0.83	1.80	1.15	0.66	2.49	1.88	0.83
time (sec)	N/A	0.036	0.021	0.056	0.330	0.443	0.972	0.135	0.715

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	152	113	40	156	106	100
normalized size	1	1.00	0.68	2.45	1.82	0.65	2.52	1.71	1.61
time (sec)	N/A	0.042	0.027	0.062	0.325	0.453	0.741	0.121	0.654

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	66	246	163	70	226	157	156
normalized size	1	0.98	0.80	2.96	1.96	0.84	2.72	1.89	1.88
time (sec)	N/A	0.052	0.040	0.065	0.340	0.462	0.958	0.136	0.698

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	91	349	222	94	286	202	199
normalized size	1	0.98	0.88	3.36	2.13	0.90	2.75	1.94	1.91
time (sec)	N/A	0.061	0.050	0.062	0.341	0.434	1.402	0.124	0.772

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	299	2058	1368	263	0	353	-1
normalized size	1	1.00	0.92	6.35	4.22	0.81	0.00	1.09	-0.00
time (sec)	N/A	0.281	0.524	0.199	0.478	0.504	0.000	0.196	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	244	1529	979	217	0	303	-1
normalized size	1	1.00	0.98	6.14	3.93	0.87	0.00	1.22	-0.00
time (sec)	N/A	0.246	0.390	0.188	0.465	0.439	0.000	0.181	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	198	1026	624	174	0	257	-1
normalized size	1	1.00	0.86	4.48	2.72	0.76	0.00	1.12	-0.00
time (sec)	N/A	0.168	0.322	0.184	0.442	0.441	0.000	0.179	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	157	676	293	137	0	219	-1
normalized size	1	1.00	0.96	4.15	1.80	0.84	0.00	1.34	-0.01
time (sec)	N/A	0.119	0.335	0.183	0.438	0.478	0.000	0.148	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	45	329	103	102	0	185	-1
normalized size	1	1.00	0.48	3.50	1.10	1.09	0.00	1.97	-0.01
time (sec)	N/A	0.044	0.043	0.115	0.436	0.471	0.000	0.167	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	189	1278	0	381	0	263	-1
normalized size	1	1.00	1.41	9.54	0.00	2.84	0.00	1.96	-0.01
time (sec)	N/A	0.094	0.658	0.145	0.000	0.493	0.000	0.331	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	145	1917	0	404	0	0	-1
normalized size	1	1.00	0.81	10.77	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.174	0.188	0.000	0.461	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	194	3042	0	580	0	0	-1
normalized size	1	1.00	0.73	11.52	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.269	0.198	0.000	0.486	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	275	4390	0	853	0	0	-1
normalized size	1	1.00	0.81	12.95	0.00	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.424	0.200	0.000	0.529	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	554	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.095	0.345	0.000	0.508	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	81	0	0	415	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.048	0.190	0.000	0.497	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	257	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.017	0.115	0.000	0.446	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	124	0	0	414	0	0	-1
normalized size	1	1.00	0.31	0.00	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.102	0.171	0.000	0.493	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	110	0	0	612	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	2.99	0.00	0.00	-0.00
time (sec)	N/A	0.103	0.034	0.168	0.000	0.605	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	561	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.093	0.194	0.000	0.478	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	79	0	0	421	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.047	0.190	0.000	0.463	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	268	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.019	0.125	0.000	0.461	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	122	0	0	690	0	0	-1
normalized size	1	1.00	0.29	0.00	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.096	0.181	0.000	0.514	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	106	0	0	711	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	3.37	0.00	0.00	-0.00
time (sec)	N/A	0.108	0.028	0.181	0.000	0.479	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	99	0	0	561	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.401	0.097	0.186	0.000	0.481	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	421	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.064	0.185	0.000	0.445	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	266	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.023	0.123	0.000	0.478	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	126	0	0	470	0	0	-1
normalized size	1	1.00	0.32	0.00	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.040	0.184	0.000	0.543	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	107	0	0	728	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	3.47	0.00	0.00	-0.00
time (sec)	N/A	0.101	0.032	0.172	0.000	0.469	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	98	0	0	555	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.093	0.198	0.000	0.472	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	415	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.297	0.045	0.190	0.000	0.626	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	43	0	0	258	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.020	0.128	0.000	0.485	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	128	0	0	629	0	0	-1
normalized size	1	1.00	0.30	0.00	0.00	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.043	0.194	0.000	0.540	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	107	0	0	626	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.034	0.198	0.000	0.504	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.942	0.208	0.000	0.506	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	272	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.385	0.118	0.000	0.437	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	160	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.165	0.074	0.000	0.486	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.132	0.070	0.000	0.457	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.033	0.066	0.000	0.433	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	170	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.047	0.110	0.000	0.605	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	125	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.026	0.105	0.000	0.489	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	173	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.081	0.124	0.000	0.459	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.032	0.306	0.000	0.519	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.015	0.273	0.000	0.447	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.013	0.066	0.000	0.469	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.019	0.063	0.000	0.472	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	35	13	12	12	0	12	12
normalized size	1	1.00	2.69	1.00	0.92	0.92	0.00	0.92	0.92
time (sec)	N/A	0.026	0.008	0.038	0.435	0.433	0.000	0.128	0.534

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	39	0	39	0	0	44
normalized size	1	1.00	1.20	0.78	0.00	0.78	0.00	0.00	0.88
time (sec)	N/A	0.050	0.024	0.041	0.000	0.532	0.000	0.000	0.556

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	114	55	0	66	0	0	74
normalized size	1	1.00	1.37	0.66	0.00	0.80	0.00	0.00	0.89
time (sec)	N/A	0.078	0.285	0.041	0.000	0.544	0.000	0.000	0.624

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	71	0	93	0	0	104
normalized size	1	1.00	1.06	0.61	0.00	0.80	0.00	0.00	0.90
time (sec)	N/A	0.110	0.303	0.040	0.000	0.468	0.000	0.000	0.687

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	153	87	0	120	0	0	134
normalized size	1	1.00	1.03	0.58	0.00	0.81	0.00	0.00	0.90
time (sec)	N/A	0.140	0.324	0.042	0.000	0.515	0.000	0.000	0.738

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.024	0.295	0.000	0.506	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.020	0.291	0.000	0.533	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.021	0.290	0.000	0.644	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	0	42	0	0	33
normalized size	1	1.00	1.00	1.06	0.00	1.20	0.00	0.00	0.94
time (sec)	N/A	0.036	0.019	0.040	0.000	0.508	0.000	0.000	0.602

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	54	0	70	0	0	78
normalized size	1	1.00	0.83	0.75	0.00	0.97	0.00	0.00	1.08
time (sec)	N/A	0.073	0.038	0.037	0.000	0.477	0.000	0.000	0.646

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	79	70	0	97	0	0	120
normalized size	1	1.00	0.73	0.65	0.00	0.90	0.00	0.00	1.11
time (sec)	N/A	0.113	0.040	0.037	0.000	0.658	0.000	0.000	0.666

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.029	0.319	0.000	0.677	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.016	0.265	0.000	0.521	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.013	0.065	0.000	0.501	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.024	0.062	0.000	0.552	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	16	15	15	0	15	15
normalized size	1	1.00	1.89	0.89	0.83	0.83	0.00	0.83	0.83
time (sec)	N/A	0.028	0.008	0.036	0.431	0.560	0.000	0.123	0.532

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	40	0	40	0	0	46
normalized size	1	1.00	1.04	0.75	0.00	0.75	0.00	0.00	0.87
time (sec)	N/A	0.057	0.022	0.037	0.000	0.477	0.000	0.000	0.577

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	86	57	0	68	0	0	79
normalized size	1	1.00	0.98	0.65	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.089	0.137	0.041	0.000	0.565	0.000	0.000	0.623

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	122	73	0	95	0	0	111
normalized size	1	1.00	0.99	0.59	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.120	0.273	0.042	0.000	0.483	0.000	0.000	0.666

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.024	0.325	0.000	0.478	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.022	0.290	0.000	0.450	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.020	0.288	0.000	0.501	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	0	44	0	0	35
normalized size	1	1.00	1.00	1.05	0.00	1.19	0.00	0.00	0.95
time (sec)	N/A	0.040	0.018	0.039	0.000	0.475	0.000	0.000	0.608

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	56	0	72	0	0	80
normalized size	1	1.00	0.82	0.74	0.00	0.95	0.00	0.00	1.05
time (sec)	N/A	0.081	0.037	0.037	0.000	0.428	0.000	0.000	0.219

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	81	72	0	99	0	0	122
normalized size	1	1.00	0.71	0.63	0.00	0.87	0.00	0.00	1.07
time (sec)	N/A	0.125	0.040	0.040	0.000	0.421	0.000	0.000	0.694

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.028	0.312	0.000	0.457	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.016	0.267	0.000	0.458	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.014	0.070	0.000	0.532	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.024	0.060	0.000	0.489	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	36	16	23	15	0	15	15
normalized size	1	1.00	2.25	1.00	1.44	0.94	0.00	0.94	0.94
time (sec)	N/A	0.027	0.010	0.039	0.353	0.471	0.000	0.107	0.526

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	41	0	41	0	0	47
normalized size	1	1.00	1.11	0.76	0.00	0.76	0.00	0.00	0.87
time (sec)	N/A	0.054	0.026	0.040	0.000	0.569	0.000	0.000	0.556

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	91	57	0	68	0	0	80
normalized size	1	1.00	1.02	0.64	0.00	0.76	0.00	0.00	0.90
time (sec)	N/A	0.083	0.176	0.039	0.000	0.457	0.000	0.000	0.617

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	127	73	0	95	0	0	112
normalized size	1	1.00	1.02	0.59	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.119	0.389	0.040	0.000	0.456	0.000	0.000	0.678

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.026	0.291	0.000	0.463	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.023	0.290	0.000	0.457	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.026	0.283	0.000	0.439	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	39	0	44	0	0	35
normalized size	1	1.00	0.97	1.03	0.00	1.16	0.00	0.00	0.92
time (sec)	N/A	0.039	0.025	0.039	0.000	0.428	0.000	0.000	0.626

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	56	0	72	0	0	81
normalized size	1	1.00	0.81	0.73	0.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.080	0.050	0.038	0.000	0.418	0.000	0.000	0.650

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	72	0	99	0	0	123
normalized size	1	1.00	0.70	0.63	0.00	0.86	0.00	0.00	1.07
time (sec)	N/A	0.124	0.051	0.038	0.000	0.415	0.000	0.000	0.667

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.027	0.312	0.000	0.473	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.017	0.272	0.000	0.413	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.015	0.066	0.000	0.409	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.023	0.064	0.000	0.448	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	18	23	15	0	15	15
normalized size	1	1.00	1.89	1.00	1.28	0.83	0.00	0.83	0.83
time (sec)	N/A	0.028	0.009	0.037	0.341	0.441	0.000	0.125	0.532

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	42	0	40	0	0	47
normalized size	1	1.00	1.02	0.78	0.00	0.74	0.00	0.00	0.87
time (sec)	N/A	0.058	0.025	0.039	0.000	0.475	0.000	0.000	0.566

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	59	0	68	0	0	79
normalized size	1	1.00	0.96	0.66	0.00	0.76	0.00	0.00	0.89
time (sec)	N/A	0.088	0.175	0.039	0.000	0.443	0.000	0.000	0.621

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	121	75	0	95	0	0	111
normalized size	1	1.00	0.98	0.60	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.121	0.369	0.039	0.000	0.491	0.000	0.000	0.672

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.028	0.313	0.000	0.531	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.021	0.291	0.000	0.436	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.022	0.297	0.000	0.478	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	41	0	44	0	0	35
normalized size	1	1.00	0.97	1.08	0.00	1.16	0.00	0.00	0.92
time (sec)	N/A	0.039	0.024	0.037	0.000	0.429	0.000	0.000	0.178

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	72	0	0	81
normalized size	1	1.00	0.81	0.75	0.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.080	0.052	0.036	0.000	0.507	0.000	0.000	0.633

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	74	0	99	0	0	123
normalized size	1	1.00	0.70	0.64	0.00	0.86	0.00	0.00	1.07
time (sec)	N/A	0.125	0.056	0.039	0.000	0.503	0.000	0.000	0.661

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	45	63	53	37	30	49
normalized size	1	1.00	0.84	0.90	1.26	1.06	0.74	0.60	0.98
time (sec)	N/A	0.043	0.025	0.060	0.428	0.508	0.308	0.147	0.135

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	113	112	86	0	24	92
normalized size	1	1.00	0.66	1.55	1.53	1.18	0.00	0.33	1.26
time (sec)	N/A	0.039	0.014	0.181	0.324	0.433	0.000	0.151	0.530

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	40	44	31	19	25	28
normalized size	1	1.00	1.00	1.33	1.47	1.03	0.63	0.83	0.93
time (sec)	N/A	0.038	0.018	0.056	0.426	0.411	0.163	0.119	0.491

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	63	40	54	0	0	55
normalized size	1	1.00	1.27	1.54	0.98	1.32	0.00	0.00	1.34
time (sec)	N/A	0.035	0.033	0.170	0.325	0.432	0.000	0.000	0.489

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	26	24	15	10	13	15
normalized size	1	1.00	1.00	1.73	1.60	1.00	0.67	0.87	1.00
time (sec)	N/A	0.029	0.006	0.036	0.428	0.431	0.058	0.109	0.474

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	26	12	15	14	13	15
normalized size	1	1.00	1.00	1.62	0.75	0.94	0.88	0.81	0.94
time (sec)	N/A	0.032	0.006	0.033	0.319	0.392	0.054	0.112	0.475

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	56	143	33	54	0	0	56
normalized size	1	1.00	1.37	3.49	0.80	1.32	0.00	0.00	1.37
time (sec)	N/A	0.035	0.066	0.184	0.434	0.483	0.000	0.000	0.486

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	41	31	19	28	29
normalized size	1	1.00	1.00	1.28	1.28	0.97	0.59	0.88	0.91
time (sec)	N/A	0.038	0.016	0.052	0.316	0.511	0.158	0.136	0.493

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	82	262	108	86	0	24	93
normalized size	1	1.00	1.12	3.59	1.48	1.18	0.00	0.33	1.27
time (sec)	N/A	0.038	0.085	0.186	0.423	0.476	0.000	0.151	0.102

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	69	84	0	366	0	0	-1
normalized size	1	1.00	0.53	0.64	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.041	0.197	0.000	0.602	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	71	800	0	187	0	139	-1
normalized size	1	1.00	0.74	8.33	0.00	1.95	0.00	1.45	-0.01
time (sec)	N/A	0.085	0.025	0.242	0.000	0.613	0.000	0.283	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	55	61	0	360	0	0	-1
normalized size	1	1.00	0.65	0.73	0.00	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.031	0.159	0.000	0.571	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	204	0	152	0	72	-1
normalized size	1	1.00	1.44	3.24	0.00	2.41	0.00	1.14	-0.02
time (sec)	N/A	0.060	0.039	0.181	0.000	0.434	0.000	0.173	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	53	0	255	0	0	-1
normalized size	1	1.00	1.00	1.26	0.00	6.07	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.016	0.099	0.000	0.479	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	42	15	255	0	0	-1
normalized size	1	1.00	1.00	0.98	0.35	5.93	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.017	0.047	0.336	0.539	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	117	87	40	152	0	70	-1
normalized size	1	1.00	1.86	1.38	0.63	2.41	0.00	1.11	-0.02
time (sec)	N/A	0.061	0.065	0.218	0.422	0.547	0.000	0.205	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	60	66	35	360	0	0	-1
normalized size	1	1.00	0.70	0.77	0.41	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.033	0.155	0.336	0.621	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	132	136	76	187	0	137	-1
normalized size	1	1.00	1.38	1.42	0.79	1.95	0.00	1.43	-0.01
time (sec)	N/A	0.078	0.096	0.192	0.422	0.506	0.000	0.250	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	20	59	35	37	19	21
normalized size	1	1.00	0.69	0.57	1.69	1.00	1.06	0.54	0.60
time (sec)	N/A	0.038	0.019	0.057	0.428	0.422	0.311	0.115	0.097

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	47	269	95	76	0	111	41
normalized size	1	1.00	0.70	4.01	1.42	1.13	0.00	1.66	0.61
time (sec)	N/A	0.039	0.017	0.178	0.333	0.435	0.000	0.175	0.515

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	35	22	20	13	24
normalized size	1	1.00	0.95	0.79	1.84	1.16	1.05	0.68	1.26
time (sec)	N/A	0.032	0.022	0.056	0.425	0.381	0.190	0.140	0.500

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	104	45	51	0	66	33
normalized size	1	1.00	0.72	1.55	0.67	0.76	0.00	0.99	0.49
time (sec)	N/A	0.039	0.014	0.178	0.322	0.473	0.000	0.139	0.496

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	38	28	49	34	41	23
normalized size	1	1.00	0.75	1.36	1.00	1.75	1.21	1.46	0.82
time (sec)	N/A	0.042	0.018	0.040	0.426	0.452	0.210	0.137	0.482

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	25	0	49	34	44	25
normalized size	1	1.00	0.72	0.86	0.00	1.69	1.17	1.52	0.86
time (sec)	N/A	0.041	0.017	0.097	0.000	0.562	0.204	0.133	0.482

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	93	59	51	0	66	31
normalized size	1	1.00	0.72	1.39	0.88	0.76	0.00	0.99	0.46
time (sec)	N/A	0.040	0.014	0.175	0.415	0.504	0.000	0.158	0.060

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	16	13	22	22	14	24
normalized size	1	1.00	0.95	0.84	0.68	1.16	1.16	0.74	1.26
time (sec)	N/A	0.032	0.021	0.034	0.319	0.368	0.172	0.111	0.051

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	47	92	100	76	0	111	40
normalized size	1	1.00	0.70	1.37	1.49	1.13	0.00	1.66	0.60
time (sec)	N/A	0.039	0.017	0.175	0.320	0.459	0.000	0.195	0.516

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	56	48	0	101	0	0	48
normalized size	1	1.00	0.59	0.51	0.00	1.06	0.00	0.00	0.51
time (sec)	N/A	0.080	0.033	0.165	0.000	0.494	0.000	0.000	1.620

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	77	940	0	67	0	135	46
normalized size	1	1.00	1.12	13.62	0.00	0.97	0.00	1.96	0.67
time (sec)	N/A	0.065	0.038	0.193	0.000	0.516	0.000	0.158	0.979

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	44	0	71	0	0	41
normalized size	1	1.00	1.16	0.90	0.00	1.45	0.00	0.00	0.84
time (sec)	N/A	0.069	0.041	0.096	0.000	0.480	0.000	0.000	1.245

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	398	0	47	0	76	32
normalized size	1	1.00	1.44	7.37	0.00	0.87	0.00	1.41	0.59
time (sec)	N/A	0.055	0.032	0.182	0.000	0.555	0.000	0.180	0.652

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	51	54	0	317	0	0	-1
normalized size	1	1.00	0.58	0.61	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.035	0.182	0.000	0.479	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	86	52	317	0	0	-1
normalized size	1	1.00	0.67	0.97	0.58	3.56	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.041	0.197	0.332	0.477	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	137	59	47	0	75	32
normalized size	1	1.00	1.44	2.54	1.09	0.87	0.00	1.39	0.59
time (sec)	N/A	0.055	0.037	0.181	0.327	0.456	0.000	0.254	0.652

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	45	29	71	0	0	49
normalized size	1	1.00	1.16	0.92	0.59	1.45	0.00	0.00	1.00
time (sec)	N/A	0.072	0.048	0.041	0.335	0.540	0.000	0.000	1.065

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	77	307	120	67	0	134	45
normalized size	1	1.00	1.12	4.45	1.74	0.97	0.00	1.94	0.65
time (sec)	N/A	0.066	0.046	0.185	0.332	0.720	0.000	0.166	0.996

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.024	0.270	0.000	0.606	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	88	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.018	0.067	0.000	0.478	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.019	0.066	0.000	0.410	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	206	141	0	0	0	0	0	-1
normalized size	1	1.57	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.129	0.306	0.000	0.529	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	121	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.128	0.286	0.000	0.473	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	109	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.064	0.315	0.000	0.429	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	42	18	17	17	0	17	17
normalized size	1	1.00	2.33	1.00	0.94	0.94	0.00	0.94	0.94
time (sec)	N/A	0.030	0.008	0.036	0.426	0.423	0.000	0.122	0.634

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	132	120	0	0	0	0	0	-1
normalized size	1	2.03	1.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.042	0.295	0.000	0.455	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	180	142	0	0	0	0	0	-1
normalized size	1	2.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.056	0.298	0.000	0.462	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	242	174	0	0	0	0	0	-1
normalized size	1	1.92	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.081	0.293	0.000	0.485	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	165	166	0	298	0	0	281
normalized size	1	1.00	0.91	0.92	0.00	1.65	0.00	0.00	1.55
time (sec)	N/A	0.175	0.521	0.041	0.000	0.455	0.000	0.000	0.869

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.116	0.300	0.000	0.455	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.057	0.287	0.000	0.542	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.039	0.291	0.000	0.442	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	217	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.275	0.287	0.000	0.577	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	214	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.218	0.295	0.000	0.451	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	248	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.324	0.296	0.000	0.516	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	206	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.185	0.289	0.000	0.487	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	187	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.147	0.286	0.000	0.428	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.029	0.034	0.000	0.578	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	120	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.047	0.291	0.000	0.442	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	142	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.069	0.289	0.000	0.446	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	159	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.084	0.295	0.000	0.457	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.069	0.289	0.000	0.453	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.050	0.293	0.000	0.451	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.048	0.292	0.000	0.477	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.057	0.288	0.000	0.450	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.521	0.189	0.000	0.509	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	96	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.171	0.346	0.000	0.489	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.495	0.344	0.000	0.488	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.794	0.357	0.000	0.481	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.372	0.315	0.000	0.483	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.483	0.312	0.000	0.488	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.629	0.312	0.000	0.505	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.036	0.294	0.000	0.430	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	76	42	0	0	54
normalized size	1	1.00	0.74	0.77	1.43	0.79	0.00	0.00	1.02
time (sec)	N/A	0.065	0.032	0.095	0.361	0.629	0.000	0.000	0.658

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	0	44	0	0	54
normalized size	1	1.00	0.74	0.77	0.00	0.83	0.00	0.00	1.02
time (sec)	N/A	0.061	0.028	0.086	0.000	0.444	0.000	0.000	0.591

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	62	0	78	0	0	-1
normalized size	1	1.00	0.92	1.03	0.00	1.30	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.026	0.142	0.000	0.469	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	292	379	437	318	-1
normalized size	1	1.00	0.95	0.89	7.68	9.97	11.50	8.37	-0.03
time (sec)	N/A	0.079	1.227	0.370	0.539	1.341	6.496	0.137	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	35	155	171	192	151	160
normalized size	1	1.00	0.95	0.92	4.08	4.50	5.05	3.97	4.21
time (sec)	N/A	0.078	0.235	0.129	0.433	0.410	1.765	0.145	3.581

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	62	49	53	48	47
normalized size	1	1.00	0.95	0.89	1.63	1.29	1.39	1.26	1.24
time (sec)	N/A	0.077	0.038	0.062	0.417	0.537	0.435	0.118	0.640

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	62	0	49	53	93	65
normalized size	1	1.00	0.95	1.63	0.00	1.29	1.39	2.45	1.71
time (sec)	N/A	0.077	0.036	0.101	0.000	0.522	0.404	0.131	0.588

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	218	0	171	192	151	159
normalized size	1	1.00	0.95	5.74	0.00	4.50	5.05	3.97	4.18
time (sec)	N/A	0.078	0.229	0.112	0.000	0.514	1.493	0.145	3.400

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	58	0	497	0	0	46
normalized size	1	1.00	0.97	0.89	0.00	7.65	0.00	0.00	0.71
time (sec)	N/A	0.201	0.568	0.175	0.000	0.611	0.000	0.000	2.210

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	0	193	0	0	48
normalized size	1	1.00	1.00	0.89	0.00	2.97	0.00	0.00	0.74
time (sec)	N/A	0.204	0.109	0.151	0.000	0.493	0.000	0.000	1.598

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	74	87	0	0	0	0	-1
normalized size	1	1.00	0.52	0.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.059	0.174	0.000	0.651	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	86	55	0	0	0	-1
normalized size	1	1.00	0.52	0.60	0.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.064	0.176	0.337	0.592	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	93	193	0	0	57
normalized size	1	1.00	1.00	0.89	1.43	2.97	0.00	0.00	0.88
time (sec)	N/A	0.208	0.118	0.154	0.356	0.497	0.000	0.000	1.589

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	58	274	497	0	0	47
normalized size	1	1.00	0.97	0.89	4.22	7.65	0.00	0.00	0.72
time (sec)	N/A	0.209	0.601	0.164	0.450	0.659	0.000	0.000	3.089

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [131] had the largest ratio of [1.250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	14	0.286
2	A	5	4	1.00	14	0.286
3	A	7	5	1.00	14	0.357
4	A	3	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	6	6	1.00	14	0.429
7	A	5	5	1.00	14	0.357
8	A	6	6	1.00	14	0.429
9	A	7	6	1.00	14	0.429
10	A	8	6	1.00	14	0.429
11	A	3	2	1.00	14	0.143
12	A	3	2	1.00	14	0.143
13	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	3	2	1.00	10	0.200
15	A	3	2	1.00	14	0.143
16	A	3	2	1.00	14	0.143
17	A	3	2	1.00	14	0.143
18	A	3	2	1.00	14	0.143
19	A	14	11	1.00	14	0.786
20	A	10	9	1.00	14	0.643
21	A	9	7	1.00	12	0.583
22	A	5	5	1.00	10	0.500
23	A	8	7	1.00	14	0.500
24	A	8	7	1.00	14	0.500
25	A	12	8	1.00	14	0.571
26	A	14	9	1.00	14	0.643
27	A	3	2	1.00	14	0.143
28	A	3	2	1.00	14	0.143
29	A	3	2	1.00	12	0.167
30	A	3	2	1.00	10	0.200
31	A	3	2	1.00	14	0.143
32	A	3	2	1.00	14	0.143
33	A	3	2	1.00	14	0.143
34	A	3	2	1.00	14	0.143
35	A	5	4	1.00	14	0.286
36	A	7	5	1.00	14	0.357
37	A	3	3	1.00	12	0.250
38	A	3	3	1.00	10	0.300
39	A	6	6	1.00	14	0.429
40	A	5	5	1.00	14	0.357
41	A	6	6	1.00	14	0.429
42	A	7	6	1.00	14	0.429
43	A	8	6	1.00	14	0.429
44	A	3	2	1.00	14	0.143
45	A	3	2	1.00	14	0.143
46	A	3	2	1.00	12	0.167
47	A	3	2	1.00	10	0.200
48	A	3	2	1.00	14	0.143
49	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	3	2	1.00	14	0.143
51	A	3	2	1.00	14	0.143
52	A	14	11	1.00	14	0.786
53	A	10	9	1.00	14	0.643
54	A	9	7	1.00	12	0.583
55	A	5	5	1.00	10	0.500
56	A	8	7	1.00	14	0.500
57	A	8	7	1.00	14	0.500
58	A	12	8	1.00	14	0.571
59	A	14	9	1.00	14	0.643
60	A	19	9	1.00	14	0.643
61	A	15	12	1.00	16	0.750
62	A	14	11	1.00	14	0.786
63	A	13	10	1.00	12	0.833
64	A	17	14	1.00	16	0.875
65	A	6	6	1.00	16	0.375
66	A	7	7	1.00	16	0.438
67	A	9	8	1.00	16	0.500
68	A	10	8	1.00	16	0.500
69	A	11	8	1.00	16	0.500
70	A	15	12	1.00	16	0.750
71	A	15	12	1.00	16	0.750
72	A	14	11	1.00	14	0.786
73	A	13	10	1.00	12	0.833
74	A	17	14	1.00	16	0.875
75	A	6	6	1.00	16	0.375
76	A	7	7	1.00	16	0.438
77	A	9	8	1.00	16	0.500
78	A	10	8	1.00	16	0.500
79	A	16	13	1.00	16	0.812
80	A	16	12	1.00	16	0.750
81	A	15	11	1.00	14	0.786
82	A	14	11	1.00	12	0.917
83	A	19	16	1.00	16	1.000
84	A	7	6	1.00	16	0.375
85	A	8	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	10	9	1.00	16	0.562
87	A	11	9	1.00	16	0.562
88	A	15	12	1.00	16	0.750
89	A	15	12	1.00	16	0.750
90	A	14	11	1.00	14	0.786
91	A	13	10	1.00	12	0.833
92	A	17	14	1.00	16	0.875
93	A	6	6	1.00	16	0.375
94	A	7	7	1.00	16	0.438
95	A	9	8	1.00	16	0.500
96	A	10	8	1.00	16	0.500
97	A	15	12	1.00	16	0.750
98	A	15	12	1.00	16	0.750
99	A	14	11	1.00	14	0.786
100	A	13	10	1.00	12	0.833
101	A	17	14	1.00	16	0.875
102	A	6	6	1.00	16	0.375
103	A	7	7	1.00	16	0.438
104	A	9	8	1.00	16	0.500
105	A	10	8	1.00	16	0.500
106	A	16	13	1.00	16	0.812
107	A	16	12	1.00	16	0.750
108	A	15	11	1.00	14	0.786
109	A	14	11	1.00	12	0.917
110	A	19	16	1.00	16	1.000
111	A	7	6	1.00	16	0.375
112	A	8	7	1.00	16	0.438
113	A	10	9	1.00	16	0.562
114	A	11	9	1.00	16	0.562
115	A	16	12	1.00	14	0.857
116	A	15	11	1.00	12	0.917
117	A	14	10	1.00	10	1.000
118	A	25	13	1.00	14	0.929
119	A	13	9	1.00	14	0.643
120	A	14	10	1.00	14	0.714
121	A	16	11	1.00	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	5	5	1.00	14	0.357
123	A	4	4	1.00	12	0.333
124	A	3	3	1.00	10	0.300
125	A	4	4	1.00	14	0.286
126	A	3	3	1.00	14	0.214
127	A	4	4	1.00	14	0.286
128	A	27	13	1.00	16	0.812
129	A	26	12	1.00	14	0.857
130	A	25	11	1.00	12	0.917
131	A	39	20	1.00	16	1.250
132	A	16	13	1.00	16	0.812
133	A	17	14	1.00	16	0.875
134	A	4	4	1.00	14	0.286
135	A	4	4	1.00	14	0.286
136	A	3	3	1.00	14	0.214
137	A	3	3	1.00	14	0.214
138	A	4	4	1.00	14	0.286
139	A	4	4	1.00	14	0.286
140	A	9	5	1.00	14	0.357
141	A	4	3	1.00	14	0.214
142	A	4	3	1.00	14	0.214
143	A	9	5	1.00	14	0.357
144	A	2	2	1.00	16	0.125
145	A	2	2	1.00	16	0.125
146	A	2	2	1.00	16	0.125
147	A	2	2	1.00	16	0.125
148	A	2	2	1.00	16	0.125
149	A	2	2	1.00	16	0.125
150	A	2	2	1.00	12	0.167
151	A	2	2	1.00	12	0.167
152	A	2	2	1.00	16	0.125
153	A	2	2	1.00	15	0.133
154	A	4	4	1.00	15	0.267
155	A	4	4	1.00	15	0.267
156	A	3	3	1.00	13	0.231
157	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	4	4	1.00	15	0.267
159	A	2	2	1.00	15	0.133
160	A	3	3	1.00	15	0.200
161	A	5	5	1.00	15	0.333
162	A	8	8	1.00	16	0.500
163	A	7	7	1.00	16	0.438
164	A	7	7	1.00	16	0.438
165	A	6	6	1.00	14	0.429
166	A	5	5	1.00	12	0.417
167	A	7	7	1.00	16	0.438
168	A	4	4	1.00	16	0.250
169	A	5	5	1.00	16	0.312
170	A	7	6	1.00	16	0.375
171	A	3	2	1.00	16	0.125
172	A	3	2	1.00	16	0.125
173	A	3	2	1.00	16	0.125
174	A	3	2	1.00	14	0.143
175	A	3	2	1.00	12	0.167
176	A	3	2	1.00	16	0.125
177	A	3	2	1.00	16	0.125
178	A	3	2	1.00	16	0.125
179	A	3	2	1.00	16	0.125
180	A	9	8	1.00	16	0.500
181	A	8	8	1.00	16	0.500
182	A	8	7	1.00	16	0.438
183	A	7	6	1.00	14	0.429
184	A	6	6	1.00	12	0.500
185	A	8	8	1.00	16	0.500
186	A	5	4	1.00	16	0.250
187	A	6	5	1.00	16	0.312
188	A	8	7	1.00	16	0.438
189	A	8	8	1.00	16	0.500
190	A	7	7	1.00	16	0.438
191	A	7	7	1.00	16	0.438
192	A	6	6	1.00	14	0.429
193	A	5	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	7	7	1.00	16	0.438
195	A	4	4	1.00	16	0.250
196	A	5	5	1.00	16	0.312
197	A	7	6	1.00	16	0.375
198	A	3	2	1.00	16	0.125
199	A	3	2	1.00	16	0.125
200	A	3	2	1.00	16	0.125
201	A	3	2	1.00	14	0.143
202	A	3	2	1.00	12	0.167
203	A	3	2	1.00	16	0.125
204	A	3	2	1.00	16	0.125
205	A	3	2	0.98	16	0.125
206	A	3	2	0.98	16	0.125
207	A	9	8	1.00	16	0.500
208	A	8	8	1.00	16	0.500
209	A	8	7	1.00	16	0.438
210	A	7	6	1.00	14	0.429
211	A	6	6	1.00	12	0.500
212	A	8	8	1.00	16	0.500
213	A	5	4	1.00	16	0.250
214	A	6	5	1.00	16	0.312
215	A	8	7	1.00	16	0.438
216	A	15	12	1.00	18	0.667
217	A	14	11	1.00	16	0.688
218	A	13	10	1.00	14	0.714
219	A	15	12	1.00	18	0.667
220	A	6	6	1.00	18	0.333
221	A	15	12	1.00	18	0.667
222	A	14	11	1.00	16	0.688
223	A	13	10	1.00	14	0.714
224	A	17	14	1.00	18	0.778
225	A	6	6	1.00	18	0.333
226	A	15	12	1.00	18	0.667
227	A	14	11	1.00	16	0.688
228	A	13	10	1.00	14	0.714
229	A	14	11	1.00	18	0.611

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
230	A	5	5	1.00	18	0.278
231	A	15	12	1.00	18	0.667
232	A	14	11	1.00	16	0.688
233	A	13	10	1.00	14	0.714
234	A	17	14	1.00	18	0.778
235	A	6	6	1.00	18	0.333
236	A	4	3	1.00	14	0.214
237	A	4	4	1.00	14	0.286
238	A	4	4	1.00	14	0.286
239	A	3	3	1.00	12	0.250
240	A	2	2	1.00	10	0.200
241	A	4	4	1.00	14	0.286
242	A	2	2	1.00	14	0.143
243	A	3	3	1.00	14	0.214
244	A	3	3	1.00	19	0.158
245	A	2	2	1.00	19	0.105
246	A	2	2	1.00	17	0.118
247	A	2	2	1.00	6	0.333
248	A	1	1	1.00	19	0.053
249	A	2	2	1.00	19	0.105
250	A	3	2	1.00	19	0.105
251	A	4	2	1.00	19	0.105
252	A	5	2	1.00	19	0.105
253	A	3	3	1.00	21	0.143
254	A	3	3	1.00	21	0.143
255	A	3	3	1.00	21	0.143
256	A	1	1	1.00	21	0.048
257	A	2	2	1.00	21	0.095
258	A	3	2	1.00	21	0.095
259	A	3	3	1.00	21	0.143
260	A	2	2	1.00	21	0.095
261	A	2	2	1.00	19	0.105
262	A	2	2	1.00	8	0.250
263	A	1	1	1.00	21	0.048
264	A	2	2	1.00	21	0.095
265	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
266	A	4	2	1.00	21	0.095
267	A	3	3	1.00	23	0.130
268	A	3	3	1.00	23	0.130
269	A	3	3	1.00	23	0.130
270	A	1	1	1.00	23	0.043
271	A	2	2	1.00	23	0.087
272	A	3	2	1.00	23	0.087
273	A	3	3	1.00	21	0.143
274	A	2	2	1.00	21	0.095
275	A	2	2	1.00	19	0.105
276	A	2	2	1.00	8	0.250
277	A	1	1	1.00	21	0.048
278	A	2	2	1.00	21	0.095
279	A	3	2	1.00	21	0.095
280	A	4	2	1.00	21	0.095
281	A	3	3	1.00	23	0.130
282	A	3	3	1.00	23	0.130
283	A	3	3	1.00	23	0.130
284	A	1	1	1.00	23	0.043
285	A	2	2	1.00	23	0.087
286	A	3	2	1.00	23	0.087
287	A	3	3	1.00	21	0.143
288	A	2	2	1.00	21	0.095
289	A	2	2	1.00	19	0.105
290	A	2	2	1.00	8	0.250
291	A	1	1	1.00	21	0.048
292	A	2	2	1.00	21	0.095
293	A	3	2	1.00	21	0.095
294	A	4	2	1.00	21	0.095
295	A	3	3	1.00	23	0.130
296	A	3	3	1.00	23	0.130
297	A	3	3	1.00	23	0.130
298	A	1	1	1.00	23	0.043
299	A	2	2	1.00	23	0.087
300	A	3	2	1.00	23	0.087
301	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
302	A	5	4	1.00	24	0.167
303	A	3	2	1.00	24	0.083
304	A	4	4	1.00	24	0.167
305	A	2	2	1.00	24	0.083
306	A	2	2	1.00	24	0.083
307	A	4	4	1.00	24	0.167
308	A	3	2	1.00	24	0.083
309	A	5	4	1.00	24	0.167
310	A	4	3	1.00	25	0.120
311	A	5	5	1.00	25	0.200
312	A	4	3	1.00	25	0.120
313	A	4	4	1.00	25	0.160
314	A	3	3	1.00	25	0.120
315	A	3	3	1.00	25	0.120
316	A	4	4	1.00	25	0.160
317	A	4	3	1.00	25	0.120
318	A	5	5	1.00	25	0.200
319	A	3	2	1.00	24	0.083
320	A	3	3	1.00	24	0.125
321	A	2	2	1.00	24	0.083
322	A	3	3	1.00	24	0.125
323	A	4	3	1.00	24	0.125
324	A	4	3	1.00	24	0.125
325	A	3	3	1.00	24	0.125
326	A	2	2	1.00	24	0.083
327	A	3	3	1.00	24	0.125
328	A	4	3	1.00	25	0.120
329	A	3	3	1.00	25	0.120
330	A	3	3	1.00	25	0.120
331	A	3	3	1.00	25	0.120
332	A	5	4	1.00	25	0.160
333	A	5	4	1.00	25	0.160
334	A	3	3	1.00	25	0.120
335	A	3	3	1.00	25	0.120
336	A	3	3	1.00	25	0.120
337	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	2	2	1.00	19	0.105
339	A	2	2	1.00	8	0.250
340	A	4	4	1.57	24	0.167
341	A	4	4	1.00	24	0.167
342	A	3	3	1.00	22	0.136
343	A	1	1	1.00	21	0.048
344	B	3	3	2.03	24	0.125
345	A	5	5	2.00	24	0.208
346	A	6	6	1.92	24	0.250
347	A	4	2	1.00	21	0.095
348	A	3	3	1.00	23	0.130
349	A	3	3	1.00	23	0.130
350	A	3	3	1.00	23	0.130
351	A	5	5	1.00	26	0.192
352	A	5	5	1.00	26	0.192
353	A	5	5	1.00	26	0.192
354	A	5	5	1.00	26	0.192
355	A	4	4	1.00	24	0.167
356	A	3	3	1.00	23	0.130
357	A	3	3	1.00	26	0.115
358	A	4	4	1.00	26	0.154
359	A	6	6	1.00	26	0.231
360	A	3	3	1.00	23	0.130
361	A	3	3	1.00	23	0.130
362	A	3	3	1.00	23	0.130
363	A	3	3	1.00	23	0.130
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	24	0.083
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	3	3	1.00	26	0.115
369	A	3	3	1.00	26	0.115
370	A	3	3	1.00	26	0.115
371	A	3	3	1.00	21	0.143
372	A	3	3	1.00	24	0.125
373	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	1	1	1.00	35	0.029
375	A	2	2	1.00	26	0.077
376	A	2	2	1.00	26	0.077
377	A	2	2	1.00	26	0.077
378	A	2	2	1.00	26	0.077
379	A	2	2	1.00	26	0.077
380	A	3	3	1.00	28	0.107
381	A	3	3	1.00	28	0.107
382	A	4	3	1.00	28	0.107
383	A	4	3	1.00	28	0.107
384	A	3	3	1.00	28	0.107
385	A	3	3	1.00	28	0.107

Chapter 3

Listing of integrals

3.1 $\int e^{i\tan^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=113

$$\frac{3\sinh^{-1}(ax)}{8a^5}+\frac{ix^4\sqrt{a^2x^2+1}}{5a}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}+\frac{(-45ax+64i)\sqrt{a^2x^2+1}}{120a^5}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}$$

```
[Out] 3/8*arcsinh(a*x)/a^5-4/15*I*x^2*(a^2*x^2+1)^(1/2)/a^3+1/4*x^3*(a^2*x^2+1)^(
1/2)/a^2+1/5*I*x^4*(a^2*x^2+1)^(1/2)/a+1/120*(64*I-45*a*x)*(a^2*x^2+1)^(1/2
)/a^5
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}+\frac{(-45ax+64i)\sqrt{a^2x^2+1}}{120a^5}+\frac{3\sinh^{-1}(ax)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^4,x]
```

```
[Out] (((-4*I)/15)*x^2*Sqrt[1 + a^2*x^2])/a^3 + (x^3*Sqrt[1 + a^2*x^2])/(4*a^2) +
 ((I/5)*x^4*Sqrt[1 + a^2*x^2])/a + ((64*I - 45*a*x)*Sqrt[1 + a^2*x^2])/(120
*a^5) + (3*ArcSinh[a*x])/(8*a^5)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{i\tan^{-1}(ax)}x^4\,dx &= \int \frac{x^4(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x^3(-4ia+5a^2x)}{\sqrt{1+a^2x^2}}\,dx}{5a^2} \\
&= \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x^2(-15a^2-16ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{20a^4} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x(32ia^3-45a^4x)}{\sqrt{1+a^2x^2}}\,dx}{60a^6} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5} + \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,d}{8a^4} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5} + \frac{3\sinh^{-1}(ax)}{8a^5}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.57

$$
\frac{45\sinh^{-1}(ax) + \sqrt{a^2x^2+1}\,\left(24ia^4x^4 + 30a^3x^3 - 32ia^2x^2 - 45ax + 64i\right)}{120a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^4,x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(64*I - 45*a*x - (32*I)*a^2*x^2 + 30*a^3*x^3 + (24*I)*a^
4*x^4) + 45*ArcSinh[a*x])/(120*a^5)
```

fricas [A] time = 0.46, size = 67, normalized size = 0.59

$$
\frac{\left(24i\,a^4x^4 + 30\,a^3x^3 - 32i\,a^2x^2 - 45\,ax + 64i\right)\sqrt{a^2x^2+1} - 45\,\log\left(-ax + \sqrt{a^2x^2+1}\right)}{120\,a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/120*((24*I*a^4*x^4 + 30*a^3*x^3 - 32*I*a^2*x^2 - 45*a*x + 64*I)*sqrt(a^2*
x^2 + 1) - 45*log(-a*x + sqrt(a^2*x^2 + 1)))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 128, normalized size = 1.13

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}+\frac{8i\sqrt{a^2x^2+1}}{15a^5}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3x\sqrt{a^2x^2+1}}{8a^4}+\frac{3\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^4\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x)
```

```
[Out] 1/5*I*x^4*(a^2*x^2+1)^(1/2)/a-4/15*I*x^2*(a^2*x^2+1)^(1/2)/a^3+8/15*I/a^5*(
a^2*x^2+1)^(1/2)+1/4*x^3*(a^2*x^2+1)^(1/2)/a^2-3/8/a^4*x*(a^2*x^2+1)^(1/2)+
3/8/a^4*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.36, size = 100, normalized size = 0.88

$$\frac{i\sqrt{a^2x^2+1}\,x^4}{5\,a}+\frac{\sqrt{a^2x^2+1}\,x^3}{4\,a^2}-\frac{4i\,\sqrt{a^2x^2+1}\,x^2}{15\,a^3}-\frac{3\,\sqrt{a^2x^2+1}\,x}{8\,a^4}+\frac{3\,\operatorname{arsinh}\left(ax\right)}{8\,a^5}+\frac{8i\,\sqrt{a^2x^2+1}}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*I*sqrt(a^2*x^2 + 1)*x^4/a + 1/4*sqrt(a^2*x^2 + 1)*x^3/a^2 - 4/15*I*sqrt
(a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(a^2*x^2 + 1)*x/a^4 + 3/8*arcsinh(a*x)/a^5
+ 8/15*I*sqrt(a^2*x^2 + 1)/a^5
```

mupad [B] time = 0.10, size = 98, normalized size = 0.87

$$\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x^3\,{\left(a^2\right)}^{3/2}}{4\,a^4}-\frac{3\,x\,\sqrt{a^2}}{8\,a^4}+\frac{a\,8{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}-\frac{a^3\,x^2\,4{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}+\frac{a^5\,x^4\,1{}\mathrm{i}}{5\,{\left(a^2\right)}^{5/2}}\right)}{\sqrt{a^2}}+\frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{8\,a^4\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a*8i)/(15*(a^2)^(5/2)) - (a^3*x^2*4i)/(15*(a^2)^(5/2
)) + (x^3*(a^2)^(3/2))/(4*a^4) + (a^5*x^4*1i)/(5*(a^2)^(5/2)) - (3*x*(a^2)^
(1/2))/(8*a^4)))/(a^2)^(1/2) + (3*asinh(x*(a^2)^(1/2)))/(8*a^4*(a^2)^(1/2))
```

sympy [A] time = 5.02, size = 138, normalized size = 1.22

$$ia\left(\begin{cases}\frac{x^4\sqrt{a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{a^2x^2+1}}{15a^4}+\frac{8\sqrt{a^2x^2+1}}{15a^6} & \text{for}\: a \neq 0 \\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{x^5}{4\sqrt{a^2x^2+1}}-\frac{x^3}{8a^2\sqrt{a^2x^2+1}}-\frac{3x}{8a^4\sqrt{a^2x^2+1}}+\frac{3\operatorname{asinh}}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**4,x)
```

```
[Out] I*a*Piecewise((x**4*sqrt(a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(a**2*x**2 +
1)/(15*a**4) + 8*sqrt(a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True))
+ x**5/(4*sqrt(a**2*x**2 + 1)) - x**3/(8*a**2*sqrt(a**2*x**2 + 1)) - 3*x/(8
*a**4*sqrt(a**2*x**2 + 1)) + 3*asinh(a*x)/(8*a**5)
```

3.2 $\int e^{i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=90

$$\frac{3i\sinh^{-1}(ax)}{8a^4}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}+\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{(16+9iax)\sqrt{a^2x^2+1}}{24a^4}$$

```
[Out] 3/8*I*arcsinh(a*x)/a^4+1/3*x^2*(a^2*x^2+1)^(1/2)/a^2+1/4*I*x^3*(a^2*x^2+1)^
(1/2)/a-1/24*(16+9*I*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16+9iax)\sqrt{a^2x^2+1}}{24a^4}+\frac{3i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^3,x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 +
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) + (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x^2(-3ia+4a^2x)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x(-8a^2-9ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3}$$
$$= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{3i\sinh^{-1}(ax)}{8a^4}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.62

$$\frac{\sqrt{a^2x^2+1}\left(6ia^3x^3+8a^2x^2-9iax-16\right)+9i\sinh^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^3,x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-16 - (9*I)*a*x + 8*a^2*x^2 + (6*I)*a^3*x^3) + (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

fricas [A] time = 0.47, size = 59, normalized size = 0.66

$$\frac{\left(6i\,a^3x^3+8\,a^2x^2-9i\,ax-16\right)\sqrt{a^2x^2+1}-9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*((6*I*a^3*x^3 + 8*a^2*x^2 - 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) - 9*I*log(
-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

giac [A] time = 2.01, size = 73, normalized size = 0.81

$$\frac{1}{24}\sqrt{a^2x^2+1}\left(\left(2\left(\frac{3\,ix}{a}+\frac{4}{a^2}\right)x-\frac{9\,i}{a^3}\right)x-\frac{16}{a^4}\right)-\frac{3\,i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*x^2 + 1)*((2*(3*i*x/a + 4/a^2)*x - 9*i/a^3)*x - 16/a^4) - 3/8
*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^3*abs(a))
```

maple [A] time = 0.18, size = 109, normalized size = 1.21

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{3ix\sqrt{a^2x^2+1}}{8a^3}+\frac{3i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x)
```

```
[Out] 1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-3/8*I/a^3*x*(a^2*x^2+1)^(1/2)+3/8*I/a^3*ln(x*
a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+1/3*x^2*(a^2*x^2+1)^(1/2)/a^
2-2/3*(a^2*x^2+1)^(1/2)/a^4
```

maxima [A] time = 0.32, size = 81, normalized size = 0.90

$$\frac{i\sqrt{a^2x^2+1}\,x^3}{4\,a}+\frac{\sqrt{a^2x^2+1}\,x^2}{3\,a^2}-\frac{3i\sqrt{a^2x^2+1}\,x}{8\,a^3}+\frac{3i\,\operatorname{arsinh}(ax)}{8\,a^4}-\frac{2\sqrt{a^2x^2+1}}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(a^2*x^2 + 1)*x^3/a + 1/3*sqrt(a^2*x^2 + 1)*x^2/a^2 - 3/8*I*sqrt(
a^2*x^2 + 1)*x/a^3 + 3/8*I*arcsinh(a*x)/a^4 - 2/3*sqrt(a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.43, size = 85, normalized size = 0.94

$$\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)3\mathrm{i}}{8\,a^3\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\left(\frac{2}{3\left(a^2\right)^{3/2}}-\frac{a^2\,x^2}{3\left(a^2\right)^{3/2}}-\frac{x^3\left(a^2\right)^{3/2}\,1\mathrm{i}}{4\,a^3}+\frac{x\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] (asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/(3*
(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) - (x^3*(a^2)^(3/2)*1i)/(4*a^3) + (
x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

sympy [A] time = 4.49, size = 119, normalized size = 1.32

$$\frac{iax^5}{4\sqrt{a^2x^2+1}}+\begin{cases}\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}-\frac{ix^3}{8a\sqrt{a^2x^2+1}}-\frac{3ix}{8a^3\sqrt{a^2x^2+1}}+\frac{3i\operatorname{asinh}(ax)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**3,x)
```

```
[Out] I*a*x**5/(4*sqrt(a**2*x**2 + 1)) + Piecewise((x**2*sqrt(a**2*x**2 + 1)/(3*a
**2) - 2*sqrt(a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - I*x**3/
(8*a*sqrt(a**2*x**2 + 1)) - 3*I*x/(8*a**3*sqrt(a**2*x**2 + 1)) + 3*I*asinh(
a*x)/(8*a**4)
```

3.3 $\int e^{i \tan^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=75

$$-\frac{\sinh^{-1}(ax)}{2a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}+\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}-\frac{i\sqrt{a^2x^2+1}}{a^3}$$

```
[Out] 1/3*I*(a^2*x^2+1)^(3/2)/a^3-1/2*arcsinh(a*x)/a^3-I*(a^2*x^2+1)^(1/2)/a^3+1/
2*x*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 797, 641, 195, 215}

$$\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/3)*(1 +
a^2*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1+iax)\sqrt{1+a^2x^2}\,dx}{a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.03, size = 46, normalized size = 0.61

$$
\frac{-3\sinh^{-1}(ax) + \left(2ia^2x^2 + 3ax - 4i\right)\sqrt{a^2x^2+1}}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-4*I + 3*a*x + (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

fricas [**A**] time = 0.43, size = 51, normalized size = 0.68

$$
\frac{\sqrt{a^2x^2+1}\left(2i\,a^2x^2 + 3\,ax - 4i\right) + 3\,\log\left(-ax + \sqrt{a^2x^2+1}\right)}{6\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 + 3*a*x - 4*I) + 3*log(-a*x + sqrt(a^2*
x^2 + 1)))/a^3
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.18, size = 89, normalized size = 1.19

$$
\frac{ix^2\sqrt{a^2x^2+1}}{3a} - \frac{2i\sqrt{a^2x^2+1}}{3a^3} + \frac{x\sqrt{a^2x^2+1}}{2a^2} - \frac{\ln\left(\frac{x\,a^2}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x)
```

```
[Out] 1/3*I/a*x^2*(a^2*x^2+1)^(1/2)-2/3*I/a^3*(a^2*x^2+1)^(1/2)+1/2*x*(a^2*x^2+1)
^(1/2)/a^2-1/2/a^2*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 62, normalized size = 0.83

$$\frac{i\,\sqrt{a^2x^2+1}\,x^2}{3\,a}+\frac{\sqrt{a^2x^2+1}\,x}{2\,a^2}-\frac{\operatorname{arsinh}\left(ax\right)}{2\,a^3}-\frac{2i\,\sqrt{a^2x^2+1}}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*sqrt(a^2*x^2 + 1)*x^2/a + 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/2*arcsinh(a
*x)/a^3 - 2/3*I*sqrt(a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.41, size = 71, normalized size = 0.95

$$\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}-\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}+\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a^3*x^2*1i)/(3*(a^2)^(3/2)) - (a*2i)/(3*(a^2)^(3/2))
 + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a^2
)^(1/2))
```

sympy [A] time = 3.21, size = 75, normalized size = 1.00

$$ia\left(\begin{cases}\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4} & \text{for}\ a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\operatorname{asinh}\left(ax\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2,x)
```

```
[Out] I*a*Piecewise((x**2*sqrt(a**2*x**2 + 1)/(3*a**2) - 2*sqrt(a**2*x**2 + 1)/(3
*a**4), Ne(a, 0)), (x**4/4, True)) + x*sqrt(a**2*x**2 + 1)/(2*a**2) - asinh
(a*x)/(2*a**3)
```

3.4 $\int e^{i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=42

$$\frac{(2+iax)\sqrt{a^2x^2+1}}{2a^2}-\frac{i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] -1/2*I*arcsinh(a*x)/a^2+1/2*(2+I*a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5060, 780, 215}

$$\frac{(2+iax)\sqrt{a^2x^2+1}}{2a^2}-\frac{i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x,x]
```

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/2)*ArcSinh[a*x])/a^2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{i\tan^{-1}(ax)}x\,dx&=\int\frac{x(1+iax)}{\sqrt{1+a^2x^2}}\,dx\\&=\frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2}-\frac{i\int\frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\&=\frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2}-\frac{i\sinh^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.90

$$\frac{(2+iax)\sqrt{a^2x^2+1}-i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])*x,x]
```

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2] - I*ArcSinh[a*x])/(2*a^2)
```

fricas [A] time = 0.43, size = 43, normalized size = 1.02

$$\frac{\sqrt{a^2x^2+1}\,(i\,ax+2)+i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(I*a*x + 2) + I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

giac [A] time = 0.12, size = 54, normalized size = 1.29

$$\frac{1}{2}\,\sqrt{a^2x^2+1}\left(\frac{ix}{a}+\frac{2}{a^2}\right)+\frac{i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*(i*x/a + 2/a^2) + 1/2*i*log(-x*abs(a) + sqrt(a^2*x^2
+ 1))/(a*abs(a))
```

maple [A] time = 0.17, size = 69, normalized size = 1.64

$$\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x)
```

```
[Out] 1/2*I/a*x*(a^2*x^2+1)^(1/2)-1/2*I/a*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))
/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)/a^2
```

maxima [A] time = 0.33, size = 42, normalized size = 1.00

$$\frac{i\,\sqrt{a^2x^2+1}\,x}{2\,a}-\frac{i\,\mathrm{arsinh}\,(ax)}{2\,a^2}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(a^2*x^2 + 1)*x/a - 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a^
2
```

mupad [B] time = 0.04, size = 51, normalized size = 1.21

$$\frac{\left(\frac{1}{\sqrt{a^2}}+\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\,\sqrt{a^2\,x^2+1}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((1/(a^2)^(1/2) + (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) - (asinh(x*
(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

sympy [A] time = 3.12, size = 51, normalized size = 1.21

$$\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ \frac{\sqrt{a^2x^2+1}}{a^2} & \text{otherwise} \end{cases} + \frac{ix\sqrt{a^2x^2+1}}{2a} - \frac{i\operatorname{asinh}(ax)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x,x)
```

```
[Out] Piecewise((x**2/2, Eq(a**2, 0)), (sqrt(a**2*x**2 + 1)/a**2, True)) + I*x*sq
rt(a**2*x**2 + 1)/(2*a) - I*asinh(a*x)/(2*a**2)
```

3.5 $\int e^{i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh^{-1}(ax)}{a}+\frac{i\sqrt{a^2x^2+1}}{a}$$

```
[Out] arcsinh(a*x)/a+I*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5059, 641, 215}

$$\frac{\sinh^{-1}(ax)}{a}+\frac{i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x]),x]
[Out] (I*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{i\tan^{-1}(ax)}\,dx &= \int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{i\sqrt{1+a^2x^2}}{a}+\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{i\sqrt{1+a^2x^2}}{a}+\frac{\sinh^{-1}(ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{\sinh^{-1}(ax)+i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x]),x]
[Out] (I*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

fricas [A] time = 0.45, size = 37, normalized size = 1.28

$$\frac{i\sqrt{a^2x^2+1} - \log\left(-ax + \sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

giac [A] time = 0.13, size = 41, normalized size = 1.41

$$\frac{\sqrt{a^2x^2+1}\,i}{a} - \frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(a^2*x^2 + 1)*i/a - log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

maple [A] time = 0.10, size = 48, normalized size = 1.66

$$\frac{\ln\left(\frac{x a^2}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} + \frac{i\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2),x)
[Out] ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+I*(a^2*x^2+1)^(1/2)/a
```

maxima [A] time = 0.32, size = 25, normalized size = 0.86

$$\frac{\operatorname{arsinh}(ax)}{a} + \frac{i\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)/a + I*sqrt(a^2*x^2 + 1)/a
```

mupad [B] time = 0.04, size = 32, normalized size = 1.10

$$\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{\sqrt{a^2\,x^2+1}\;1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2),x)
[Out] ((a^2*x^2 + 1)^(1/2)*1i)/a + asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

sympy [A] time = 1.25, size = 68, normalized size = 2.34

$$ia\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ \frac{\sqrt{a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right) + \begin{cases} \sqrt{-\frac{1}{a^2}}\,\operatorname{asin}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0 \\ \sqrt{\frac{1}{a^2}}\,\operatorname{asinh}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] I*a*Piecewise((x**2/2, Eq(a**2, 0)), (sqrt(a**2*x**2 + 1)/a**2, True)) + Pi
ecewise((sqrt(-1/a**2)*asin(x*sqrt(-a**2)), a**2 < 0), (sqrt(a**(-2))*asinh
(x*sqrt(a**2)), a**2 > 0))
```

3.6 $\int \frac{e^{i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=25

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

```
[Out] I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 844, 215, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x,x]
```

```
[Out] I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1+iax}{x\sqrt{1+a^2x^2}}\,dx\\ &= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\ &= i\sinh^{-1}(ax) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)\\ &= i\sinh^{-1}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2}\\ &= i\sinh^{-1}(ax) - \tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.16

$$-\log\left(\sqrt{a^2x^2+1}+1\right) + i\sinh^{-1}(ax) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x,x]
```

```
[Out] I*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.43, size = 58, normalized size = 2.32

$$-\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - i\log\left(-ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

giac [B] time = 0.14, size = 69, normalized size = 2.76

$$-\frac{ai\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} - \log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + \log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -a*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a
^2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

maple [B] time = 0.16, size = 48, normalized size = 1.92

$$\frac{ia\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] I*a*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-arctanh(1/(a^2*x^2+
1)^(1/2))
```

maxima [A] time = 0.31, size = 18, normalized size = 0.72

$$i \operatorname{arsinh}(ax) - \operatorname{arsinh}\left(\frac{1}{a|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] I*arcsinh(a*x) - arcsinh(1/(a*abs(x)))
```

mupad [B] time = 0.04, size = 32, normalized size = 1.28

$$-\operatorname{atanh}\left(\sqrt{a^2 x^2 + 1}\right) + \frac{a \operatorname{asinh}\left(x \sqrt{a^2}\right) 1\mathrm{i}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

sympy [A] time = 3.35, size = 53, normalized size = 2.12

$$ia\left(\begin{cases} \sqrt{-\frac{1}{a^2}} \operatorname{asin}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0 \\ \sqrt{\frac{1}{a^2}} \operatorname{asinh}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0 \end{cases}\right) - \operatorname{asinh}\left(\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] I*a*Piecewise((sqrt(-1/a**2)*asin(x*sqrt(-a**2)), a**2 < 0), (sqrt(a**(-2))
*asinh(x*sqrt(a**2)), a**2 > 0)) - asinh(1/(a*x))
```

3.7 $\int \frac{e^{i \tan^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

[Out] -I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 807, 266, 63, 208}

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Int[E^(I*ArcTan[a*x])/x^2,x]

[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*ArcTanh[Sqrt[1 + a^2*x^2]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+iax}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + (ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{i\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.24

$$
-\frac{\sqrt{a^2x^2+1}}{x} - ia\log\left(\sqrt{a^2x^2+1}+1\right) + ia\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*Log[x] - I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.45, size = 66, normalized size = 1.74

$$
\frac{-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*a*x*log(-a*x + sqrt(a^2*x^2 +
 1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

giac [B] time = 0.14, size = 76, normalized size = 2.00

$$
-ai\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+ai\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)+\frac{2|a|}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -a*i*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + a*i*log(abs(-x*abs(a) +
sqrt(a^2*x^2 + 1) - 1)) + 2*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

maple [A] time = 0.16, size = 34, normalized size = 0.89

$$
-ia\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{\sqrt{a^2x^2+1}}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -I*a*arctanh(1/(a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x
```

maxima [A] time = 0.34, size = 29, normalized size = 0.76

$$-i\,a\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{\sqrt{a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -I*a*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x
```

mupad [B] time = 0.04, size = 33, normalized size = 0.87

$$-\frac{\sqrt{a^2\,x^2+1}}{x}-a\,\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] - a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

sympy [A] time = 2.30, size = 26, normalized size = 0.68

$$-a\sqrt{1+\frac{1}{a^2x^2}}-ia\,\operatorname{asinh}\left(\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(1 + 1/(a**2*x**2)) - I*a*asinh(1/(a*x))
```

3.8 $\int \frac{e^{i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=63

$$-\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 1/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2-I*a*(a^2*x^2+1
)^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$-\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+iax}{x^3\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\ &= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{ia\sqrt{1+a^2x^2}}{x}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.90

$$\frac{1}{2}\left(\frac{(-1-2iax)\sqrt{a^2x^2+1}}{x^2}+a^2\log\left(\sqrt{a^2x^2+1}+1\right)+a^2(-\log(x))\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^3,x]
```

```
[Out] (((-1 - (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

fricas [A] time = 1.20, size = 83, normalized size = 1.32

$$\frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-2i\,a^2x^2+\sqrt{a^2x^2+1}\,(-2i\,ax-1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) - 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(-2*I*a*x - 1))/x^2
```

giac [B] time = 0.15, size = 155, normalized size = 2.46

$$\frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)-\frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)+\frac{\left(x|a|-\sqrt{a^2x^2+1}\right)^3a^2+2\left(x|a|-\sqrt{a^2\right.}}{\left(\left(x|a|-\sqrt{\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^2*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 + 1) - 1)) + ((x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^2 + 2*
(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a*i*abs(a) + (x*abs(a) - sqrt(a^2*x^2 + 1)
)*a^2 - 2*a*i*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^2
```

maple [A] time = 0.16, size = 53, normalized size = 0.84

$$-\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -I*a*(a^2*x^2+1)^(1/2)/x-1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x
^2+1)^(1/2))
```

maxima [A] time = 0.33, size = 48, normalized size = 0.76

$$\frac{1}{2}\,a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{i\,\sqrt{a^2x^2+1}\,a}{x}-\frac{\sqrt{a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arcsinh(1/(a*abs(x))) - I*sqrt(a^2*x^2 + 1)*a/x - 1/2*sqrt(a^2*x^2
+ 1)/x^2
```

mupad [B] time = 0.04, size = 52, normalized size = 0.83

$$\frac{a^2\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}-\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a*(a^2*
x^2 + 1)^(1/2)*1i)/x
```

sympy [A] time = 2.91, size = 48, normalized size = 0.76

$$-ia^2\sqrt{1+\frac{1}{a^2x^2}}+\frac{a^2\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] -I*a**2*sqrt(1 + 1/(a**2*x**2)) + a**2*asinh(1/(a*x))/2 - a*sqrt(1 + 1/(a**
2*x**2))/(2*x)
```

3.9 $\int \frac{e^{i \tan^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=90

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x} - \frac{ia\sqrt{a^2x^2+1}}{2x^2} - \frac{\sqrt{a^2x^2+1}}{3x^3} + \frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3-1/2*I*a*(a^2
*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x} - \frac{ia\sqrt{a^2x^2+1}}{2x^2} - \frac{\sqrt{a^2x^2+1}}{3x^3} + \frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) + (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1+iax}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{1}{3}\int \frac{-3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{1}{6}\int \frac{-4a^2-3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{4}\left(ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}+\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.78

$$\frac{1}{6}\left(-3ia^3\log(x)+\frac{\sqrt{a^2x^2+1}\left(4a^2x^2-3iax-2\right)}{x^3}+3ia^3\log\left(\sqrt{a^2x^2+1}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2 - (3*I)*a*x + 4*a^2*x^2))/x^3 - (3*I)*a^3*Log[x] +
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

fricas [A] time = 0.41, size = 92, normalized size = 1.02

$$\frac{3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2-3i\,ax-2\right)\sqrt{a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*I*a^3*x^3*log(-a*x +
 sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 - 3*I*a*x - 2)*sqrt(a^2*x^
2 + 1))/x^3
```

giac [B] time = 0.16, size = 164, normalized size = 1.82

$$\frac{1}{2}\,a^{3}i\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}+1}+1\right|\right)-\frac{1}{2}\,a^{3}i\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}+1}-1\right|\right)+\frac{3\left(x|a|-\sqrt{a^{2}x^{2}+1}\right)^{5}a^{3}i-3\left(x|a|-\right.}{3\left(\left(x|\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*a^3*i*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^3*i*log(abs(-
x*abs(a) + sqrt(a^2*x^2 + 1) - 1)) + 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^
5*a^3*i - 3*(x*abs(a) - sqrt(a^2*x^2 + 1))*a^3*i + 12*(x*abs(a) - sqrt(a^2*
x^2 + 1))^2*a^2*abs(a) - 4*a^2*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 -
1)^3
```

maple [A] time = 0.16, size = 75, normalized size = 0.83

$$-\frac{\sqrt{a^{2}x^{2}+1}}{3x^{3}}+\frac{2a^{2}\sqrt{a^{2}x^{2}+1}}{3x}+ia\left(-\frac{\sqrt{a^{2}x^{2}+1}}{2x^{2}}+\frac{a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/3*(a^2*x^2+1)^(1/2)/x^3+2/3*a^2*(a^2*x^2+1)^(1/2)/x+I*a*(-1/2*(a^2*x^2+1
)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.33, size = 67, normalized size = 0.74

$$\frac{1}{2}i\,a^{3}\operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{2\sqrt{a^{2}x^{2}+1}\,a^{2}}{3\,x}-\frac{i\sqrt{a^{2}x^{2}+1}\,a}{2\,x^{2}}-\frac{\sqrt{a^{2}x^{2}+1}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*I*a^3*arcsinh(1/(a*abs(x))) + 2/3*sqrt(a^2*x^2 + 1)*a^2/x - 1/2*I*sqrt(
a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(a^2*x^2 + 1)/x^3
```

mupad [B] time = 0.04, size = 74, normalized size = 0.82

$$\frac{a^{3}\operatorname{atan}\left(\sqrt{a^{2}\,x^{2}+1}\;1\mathrm{i}\right)}{2}-\frac{\sqrt{a^{2}\,x^{2}+1}}{3\,x^{3}}+\frac{2\,a^{2}\,\sqrt{a^{2}\,x^{2}+1}}{3\,x}-\frac{a\,\sqrt{a^{2}\,x^{2}+1}\;1\mathrm{i}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(x^4*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a*(a^
2*x^2 + 1)^(1/2)*1i)/(2*x^2) + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

sympy [A] time = 3.09, size = 75, normalized size = 0.83

$$\frac{2a^{3}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3}+\frac{ia^{3}\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{ia^{2}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{2x}-\frac{a\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] 2*a**3*sqrt(1 + 1/(a**2*x**2))/3 + I*a**3*asinh(1/(a*x))/2 - I*a**2*sqrt(1
+ 1/(a**2*x**2))/(2*x) - a*sqrt(1 + 1/(a**2*x**2))/(3*x**2)
```

3.10 $\int \frac{e^{i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=113

$$\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+\frac{2ia^3\sqrt{a^2x^2+1}}{3x}$$

```
[Out] -3/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4-1/3*I*a*(a^2*
x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+1)^(1/2)/x^2+2/3*I*a^3*(a^2*x^2+1)^(1/2)/
x
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/x^5,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) - ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) + (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{x^5}\,dx &= \int \frac{1+iax}{x^5\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{-4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{-9a^2-8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a\cdots}}\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\cdots\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.67

$$\frac{1}{24}\left(9a^4\log(x) - 9a^4\log\left(\sqrt{a^2x^2+1}+1\right) + \frac{\sqrt{a^2x^2+1}\left(16ia^3x^3+9a^2x^2-8iax-6\right)}{x^4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])/x^5,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-6 - (8*I)*a*x + 9*a^2*x^2 + (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

fricas [A] time = 0.47, size = 101, normalized size = 0.89

$$-\frac{9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - 9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}-1\right) - 16i\,a^4x^4 - \left(16i\,a^3x^3+9\,a^2x^2-8i\,ax-\cdots\right.}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 16*I*a^4*x^4 - (16*I*a^3*x^3 + 9*a^2*x^2 - 8*I*a*x
- 6)*sqrt(a^2*x^2 + 1))/x^4
```

giac [B] time = 0.16, size = 240, normalized size = 2.12

$$-\frac{3}{8}a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)+\frac{3}{8}a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)-\frac{9\left(x|a|-\sqrt{a^2x^2+1}\right)^7a^4-33\left(x|a|-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/8*a^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + 3/8*a^4*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/12*(9*(x*abs(a) - sqrt(a^2*x^2 + 1))^7*
a^4 - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^5*a^4 - 48*(x*abs(a) - sqrt(a^2*x^2
 + 1))^4*a^3*i*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^4 + 64*(x*abs
(a) - sqrt(a^2*x^2 + 1))^2*a^3*i*abs(a) + 9*(x*abs(a) - sqrt(a^2*x^2 + 1))*
a^4 - 16*a^3*i*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^4
```

maple [A] time = 0.16, size = 97, normalized size = 0.86

$$ia\left(-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{2a^2\sqrt{a^2x^2+1}}{3x}\right)-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3a^2\left(-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] I*a*(-1/3*(a^2*x^2+1)^(1/2)/x^3+2/3*a^2*(a^2*x^2+1)^(1/2)/x)-1/4*(a^2*x^2+1
)^(1/2)/x^4-3/4*a^2*(-1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+
1)^(1/2)))
```

maxima [A] time = 0.33, size = 86, normalized size = 0.76

$$-\frac{3}{8}a^4\operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{2i\sqrt{a^2x^2+1}\,a^3}{3\,x}+\frac{3\sqrt{a^2x^2+1}\,a^2}{8\,x^2}-\frac{i\sqrt{a^2x^2+1}\,a}{3\,x^3}-\frac{\sqrt{a^2x^2+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*a^4*arcsinh(1/(a*abs(x))) + 2/3*I*sqrt(a^2*x^2 + 1)*a^3/x + 3/8*sqrt(a
^2*x^2 + 1)*a^2/x^2 - 1/3*I*sqrt(a^2*x^2 + 1)*a/x^3 - 1/4*sqrt(a^2*x^2 + 1)
/x^4
```

mupad [B] time = 0.03, size = 95, normalized size = 0.84

$$\frac{a^4\operatorname{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,3\mathrm{i}}{8}-\frac{\sqrt{a^2\,x^2+1}}{4\,x^4}-\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{3\,x^3}+\frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2}+\frac{a^3\,\sqrt{a^2\,x^2+1}\,2\mathrm{i}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(x^5*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) - (a*
(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) + (a^
3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

sympy [A] time = 4.30, size = 122, normalized size = 1.08

$$\frac{2ia^4\sqrt{1+\frac{1}{a^2x^2}}}{3}-\frac{3a^4\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^3}{8x\sqrt{1+\frac{1}{a^2x^2}}}-\frac{ia^2\sqrt{1+\frac{1}{a^2x^2}}}{3x^2}+\frac{a}{8x^3\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^5\sqrt{1+\frac{1}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] 2*I*a**4*sqrt(1 + 1/(a**2*x**2))/3 - 3*a**4*asinh(1/(a*x))/8 + 3*a**3/(8*x*
sqrt(1 + 1/(a**2*x**2))) - I*a**2*sqrt(1 + 1/(a**2*x**2))/(3*x**2) + a/(8*x
**3*sqrt(1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(1 + 1/(a**2*x**2)))
```

3.11 $\int e^{2i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=48

$$-\frac{2\log(ax+i)}{a^4}-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[Out] -2*I*x/a^3+x^2/a^2+2/3*I*x^3/a-1/4*x^4-2*ln(I+a*x)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{2\log(ax+i)}{a^4}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+iax)}{1-iax}\,dx\\
&= \int \left(-\frac{2i}{a^3}+\frac{2x}{a^2}+\frac{2ix^2}{a}-x^3-\frac{2}{a^3(i+ax)}\right)dx\\
&= -\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{2\log(ax+i)}{a^4}-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

fricas [A] time = 0.44, size = 46, normalized size = 0.96

$$-\frac{3\,a^4x^4 - 8i\,a^3x^3 - 12\,a^2x^2 + 24i\,ax + 24\,\log\left(\frac{ax+i}{a}\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x + 24*log((a*x + I)/a
))/a^4
```

giac [A] time = 0.11, size = 48, normalized size = 1.00

$$-\frac{3\,a^4x^4 - 8\,a^3ix^3 - 12\,a^2x^2 + 24\,aix}{12\,a^4} - \frac{2\,\log\left(ax+i\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*a^3*i*x^3 - 12*a^2*x^2 + 24*a*i*x)/a^4 - 2*log(a*x + i
)/a^4
```

maple [A] time = 0.04, size = 55, normalized size = 1.15

$$-\frac{x^4}{4} + \frac{2ix^3}{3a} + \frac{x^2}{a^2} - \frac{2ix}{a^3} - \frac{\ln\left(a^2x^2+1\right)}{a^4} + \frac{2i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^3,x)
```

```
[Out] -1/4*x^4+2/3*I*x^3/a+x^2/a^2-2*I*x/a^3-1/a^4*ln(a^2*x^2+1)+2*I/a^4*arctan(a
*x)
```

maxima [A] time = 0.43, size = 56, normalized size = 1.17

$$-\frac{3\,a^3x^4 - 8i\,a^2x^3 - 12\,ax^2 + 24i\,x}{12\,a^3} + \frac{2i\,\arctan\left(ax\right)}{a^4} - \frac{\log\left(a^2x^2+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 - 8*I*a^2*x^3 - 12*a*x^2 + 24*I*x)/a^3 + 2*I*arctan(a*x)/a
^4 - log(a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.42, size = 43, normalized size = 0.90

$$\frac{x^2}{a^2} - \frac{x^4}{4} - \frac{2\,\ln\left(x + \frac{1\mathrm{i}}{a}\right)}{a^4} - \frac{x\,2\mathrm{i}}{a^3} + \frac{x^3\,2\mathrm{i}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

```
[Out] (x^3*2i)/(3*a) - (x*2i)/a^3 - x^4/4 - (2*log(x + 1i/a))/a^4 + x^2/a^2
```

sympy [A] time = 0.13, size = 42, normalized size = 0.88

$$-\frac{x^4}{4} + \frac{2ix^3}{3a} + \frac{x^2}{a^2} - \frac{2ix}{a^3} - \frac{2\log\left(iax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**3,x)
```

```
[Out] -x**4/4 + 2*I*x**3/(3*a) + x**2/a**2 - 2*I*x/a**3 - 2*log(I*a*x - 1)/a**4
```

3.12 $\int e^{2i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=39

$$-\frac{2i\log(ax+i)}{a^3}+\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[Out] 2*x/a^2+I*x^2/a-1/3*x^3-2*I*ln(I+a*x)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2x}{a^2}-\frac{2i\log(ax+i)}{a^3}+\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1+iax)}{1-iax}\,dx\\&=\int\left(\frac{2}{a^2}+\frac{2ix}{a}-x^2-\frac{2i}{a^2(i+ax)}\right)dx\\&=\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$-\frac{2i\log(ax+i)}{a^3}+\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

fricas [A] time = 0.46, size = 37, normalized size = 0.95

$$-\frac{a^3x^3-3i\,a^2x^2-6\,ax+6i\,\log\left(\frac{ax+i}{a}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="fricas")
[Out] -1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x + 6*I*log((a*x + I)/a))/a^3
```

giac [A] time = 0.15, size = 39, normalized size = 1.00

$$-\frac{2 i \log(ax+i)}{a^3} - \frac{a^3x^3 - 3a^2ix^2 - 6ax}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="giac")
[Out] -2*i*log(a*x + i)/a^3 - 1/3*(a^3*x^3 - 3*a^2*i*x^2 - 6*a*x)/a^3
```

maple [A] time = 0.04, size = 47, normalized size = 1.21

$$-\frac{x^3}{3} + \frac{ix^2}{a} + \frac{2x}{a^2} - \frac{i\ln\left(a^2x^2+1\right)}{a^3} - \frac{2\arctan(ax)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2,x)
[Out] -1/3*x^3+I*x^2/a+2*x/a^2-I/a^3*ln(a^2*x^2+1)-2/a^3*arctan(a*x)
```

maxima [A] time = 0.43, size = 47, normalized size = 1.21

$$-\frac{a^2x^3 - 3i\,ax^2 - 6x}{3a^2} - \frac{2\arctan(ax)}{a^3} - \frac{i\log\left(a^2x^2+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="maxima")
[Out] -1/3*(a^2*x^3 - 3*I*a*x^2 - 6*x)/a^2 - 2*arctan(a*x)/a^3 - I*log(a^2*x^2 +
1)/a^3
```

mupad [B] time = 0.42, size = 36, normalized size = 0.92

$$\frac{2x}{a^2} - \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)2\mathrm{i}}{a^3} - \frac{x^3}{3} + \frac{x^2\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
[Out] (2*x)/a^2 - (log(x + 1i/a)*2i)/a^3 - x^3/3 + (x^2*1i)/a
```

sympy [A] time = 0.13, size = 32, normalized size = 0.82

$$-\frac{x^3}{3} + \frac{ix^2}{a} + \frac{2x}{a^2} - \frac{2i\log(iax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2,x)
[Out] -x**3/3 + I*x**2/a + 2*x/a**2 - 2*I*log(I*a*x - 1)/a**3
```

3.13 $\int e^{2 i \tan^{-1}(a x)} x \, dx$

Optimal. Leaf size=29

$$\frac{2 \log(a x+i)}{a^2}+\frac{2 i x}{a}-\frac{x^2}{2}$$

[Out] 2*I*x/a-1/2*x^2+2*ln(I+a*x)/a^2

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$\frac{2 \log(a x+i)}{a^2}+\frac{2 i x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[E^((2*I)*ArcTan[a*x])*x,x]

[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2 i \tan^{-1}(a x)} x \, dx &= \int \frac{x(1+i a x)}{1-i a x} \, dx \\
&= \int \left(\frac{2 i}{a}-x+\frac{2}{a(i+a x)}\right) dx \\
&= \frac{2 i x}{a}-\frac{x^2}{2}+\frac{2 \log(i+a x)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2 \log(a x+i)}{a^2}+\frac{2 i x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^((2*I)*ArcTan[a*x])*x,x]

[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2

fricas [A] time = 0.38, size = 29, normalized size = 1.00

$$-\frac{a^2 x^2 - 4 i \, a x - 4 \, \log\left(\frac{a x+i}{a}\right)}{2 \, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 - 4*I*a*x - 4*log((a*x + I)/a))/a^2
```

giac [A] time = 0.14, size = 30, normalized size = 1.03

$$-\frac{a^2x^2 - 4\,aix}{2\,a^2} + \frac{2\,\log\left(ax + i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="giac")
[Out] -1/2*(a^2*x^2 - 4*a*i*x)/a^2 + 2*log(a*x + i)/a^2
```

maple [A] time = 0.04, size = 38, normalized size = 1.31

$$-\frac{x^2}{2} + \frac{2ix}{a} + \frac{\ln\left(a^2x^2 + 1\right)}{a^2} - \frac{2i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x,x)
[Out] -1/2*x^2+2*I*x/a+1/a^2*ln(a^2*x^2+1)-2*I/a^2*arctan(a*x)
```

maxima [A] time = 0.44, size = 38, normalized size = 1.31

$$-\frac{ax^2 - 4i\,x}{2\,a} - \frac{2i\,\arctan\left(ax\right)}{a^2} + \frac{\log\left(a^2x^2 + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="maxima")
[Out] -1/2*(a*x^2 - 4*I*x)/a - 2*I*arctan(a*x)/a^2 + log(a^2*x^2 + 1)/a^2
```

mupad [B] time = 0.06, size = 27, normalized size = 0.93

$$\frac{2\,\ln\left(x + \frac{1\mathrm{i}}{a}\right)}{a^2} - \frac{x^2}{2} + \frac{x\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
[Out] (2*log(x + 1i/a))/a^2 + (x*2i)/a - x^2/2
```

sympy [A] time = 0.10, size = 24, normalized size = 0.83

$$-\frac{x^2}{2} + \frac{2ix}{a} + \frac{2\log\left(iax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x,x)
[Out] -x**2/2 + 2*I*x/a + 2*log(I*a*x - 1)/a**2
```

3.14 $\int e^{2i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$-x+\frac{2i\log(ax+i)}{a}$$

[Out] -x+2*I*ln(I+a*x)/a

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5061, 43}

$$-x+\frac{2i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^((2*I)*ArcTan[a*x]),x]

[Out] -x + ((2*I)*Log[I + a*x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}\,dx&=\int\frac{1+iax}{1-iax}\,dx\\&=\int\left(-1+\frac{2i}{i+ax}\right)dx\\&=-x+\frac{2i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.58

$$\frac{i\log\left(a^2x^2+1\right)}{a}+\frac{2\tan^{-1}(ax)}{a}-x$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^((2*I)*ArcTan[a*x]),x]

[Out] -x + (2*ArcTan[a*x])/a + (I*Log[1 + a^2*x^2])/a

fricas [A] time = 0.42, size = 21, normalized size = 1.11

$$-\frac{ax-2i\log\left(\frac{ax+i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x - 2*I*log((a*x + I)/a))/a
```

giac [A] time = 0.12, size = 16, normalized size = 0.84

$$-x+\frac{2\,i\log\left(ax+i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -x + 2*i*log(a*x + i)/a
```

maple [A] time = 0.04, size = 30, normalized size = 1.58

$$-x+\frac{i\ln\left(a^{2}x^{2}+1\right)}{a}+\frac{2\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1),x)
```

```
[Out] -x+I/a*ln(a^2*x^2+1)+2*arctan(a*x)/a
```

maxima [A] time = 0.44, size = 28, normalized size = 1.47

$$-x+\frac{2\arctan\left(ax\right)}{a}+\frac{i\log\left(a^{2}x^{2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x + 2*arctan(a*x)/a + I*log(a^2*x^2 + 1)/a
```

mupad [B] time = 0.04, size = 19, normalized size = 1.00

$$-x+\frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1),x)
```

```
[Out] (log(x + 1i/a)*2i)/a - x
```

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$-x+\frac{2i\log\left(iax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1),x)
```

```
[Out] -x + 2*I*log(I*a*x - 1)/a
```

3.15 $\int \frac{e^{2i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$\log(x) - 2\log(ax + i)$$

[Out] ln(x)-2*ln(I+a*x)

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 72}

$$\log(x) - 2\log(ax + i)$$

Antiderivative was successfully verified.

[In] Int[E^((2*I)*ArcTan[a*x])/x,x]

[Out] Log[x] - 2*Log[I + a*x]

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1+iax}{x(1-iax)}\,dx\\ &= \int \left(\frac{1}{x} - \frac{2a}{i+ax}\right)dx\\ &= \log(x) - 2\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\log(x) - 2\log(ax + i)$$

Antiderivative was successfully verified.

[In] Integrate[E^((2*I)*ArcTan[a*x])/x,x]

[Out] Log[x] - 2*Log[I + a*x]

fricas [A] time = 0.39, size = 15, normalized size = 1.15

$$\log(x) - 2\log\left(\frac{ax+i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="fricas")

```
[Out] log(x) - 2*log((a*x + I)/a)
```

giac [A] time = 0.12, size = 15, normalized size = 1.15

$$2\,i^2 \log\left(ax + i\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="giac")
[Out] 2*i^2*log(a*x + i) + log(abs(x))
```

maple [A] time = 0.04, size = 23, normalized size = 1.77

$$\ln(x) + 2i \arctan\left(ax\right) - \ln\left(a^2x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x,x)
[Out] ln(x)+2*I*arctan(a*x)-ln(a^2*x^2+1)
```

maxima [A] time = 0.45, size = 21, normalized size = 1.62

$$2i\ \arctan\left(ax\right) - \log\left(a^2x^2 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="maxima")
[Out] 2*I*arctan(a*x) - log(a^2*x^2 + 1) + log(x)
```

mupad [B] time = 0.44, size = 14, normalized size = 1.08

$$\ln(x) - 2\ \ln\left(x + \frac{1\mathrm{i}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(x*(a^2*x^2 + 1)),x)
[Out] log(x) - 2*log(x + 1i/a)
```

sympy [A] time = 0.15, size = 17, normalized size = 1.31

$$\log\left(3ax\right) - 2\log\left(3ax + 3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x,x)
[Out] log(3*a*x) - 2*log(3*a*x + 3*I)
```

3.16 $\int \frac{e^{2i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=26

$$2ia\log(x) - 2ia\log(ax+i) - \frac{1}{x}$$

```
[Out] -1/x+2*I*a*ln(x)-2*I*a*ln(I+a*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$2ia\log(x) - 2ia\log(ax+i) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+iax}{x^2(1-iax)}\,dx \\
&= \int \left(\frac{1}{x^2} + \frac{2ia}{x} - \frac{2ia^2}{i+ax}\right)dx \\
&= -\frac{1}{x} + 2ia\log(x) - 2ia\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$2ia\log(x) - 2ia\log(ax+i) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

fricas [A] time = 0.40, size = 26, normalized size = 1.00

$$\frac{2i\,ax\log(x) - 2i\,ax\log\left(\frac{ax+i}{a}\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] (2*I*a*x*log(x) - 2*I*a*x*log((a*x + I)/a) - 1)/x
```

giac [A] time = 0.13, size = 23, normalized size = 0.88

$$-2\,ai\log\left(ax+i\right) + 2\,ai\log\left(|x|\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*a*i*log(a*x + i) + 2*a*i*log(abs(x)) - 1/x
```

maple [A] time = 0.05, size = 34, normalized size = 1.31

$$-\frac{1}{x} + 2ia\ln(x) - ia\ln\left(a^2x^2+1\right) - 2a\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^2,x)
[Out] -1/x+2*I*a*ln(x)-I*a*ln(a^2*x^2+1)-2*a*arctan(a*x)
```

maxima [A] time = 0.44, size = 31, normalized size = 1.19

$$-2\,a\arctan\left(ax\right) - i\,a\log\left(a^2x^2+1\right) + 2i\,a\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="maxima")
[Out] -2*a*arctan(a*x) - I*a*log(a^2*x^2 + 1) + 2*I*a*log(x) - 1/x
```

mupad [B] time = 0.06, size = 17, normalized size = 0.65

$$-4\,a\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(x^2*(a^2*x^2 + 1)),x)
[Out] - 4*a*atan(2*a*x + 1i) - 1/x
```

sympy [A] time = 0.16, size = 32, normalized size = 1.23

$$-2a\left(-i\log\left(4a^2x\right) + i\log\left(4a^2x + 4ia\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**2,x)
[Out] -2*a*(-I*log(4*a**2*x) + I*log(4*a**2*x + 4*I*a)) - 1/x
```

3.17 $\int \frac{e^{2i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=36

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2-2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I+a*x)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+iax}{x^3(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{2ia}{x^2}-\frac{2a^2}{x}+\frac{2a^3}{i+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

fricas [A] time = 0.42, size = 39, normalized size = 1.08

$$-\frac{4\,a^2x^2\log(x) - 4\,a^2x^2\log\left(\frac{ax+i}{a}\right) + 4i\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x + I)/a) + 4*I*a*x + 1)/x^2
```

giac [A] time = 0.12, size = 32, normalized size = 0.89

$$2\,a^2\log\left(ax+i\right) - 2\,a^2\log\left(|x|\right) - \frac{4\,aix+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(a*x + i) - 2*a^2*log(abs(x)) - 1/2*(4*a*i*x + 1)/x^2
```

maple [A] time = 0.05, size = 45, normalized size = 1.25

$$-\frac{1}{2x^2} - \frac{2ia}{x} - 2a^2\ln(x) - 2ia^2\arctan\left(ax\right) + a^2\ln\left(a^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2-2*I*a/x-2*a^2*ln(x)-2*I*a^2*arctan(a*x)+a^2*ln(a^2*x^2+1)
```

maxima [A] time = 0.43, size = 42, normalized size = 1.17

$$-2i\,a^2\arctan\left(ax\right) + a^2\log\left(a^2x^2+1\right) - 2\,a^2\log(x) - \frac{4i\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -2*I*a^2*arctan(a*x) + a^2*log(a^2*x^2 + 1) - 2*a^2*log(x) - 1/2*(4*I*a*x +
 1)/x^2
```

mupad [B] time = 0.08, size = 27, normalized size = 0.75

$$-a^2\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right)\,4\mathrm{i} - \frac{\frac{1}{2}+a\,x\,2\mathrm{i}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(x^3*(a^2*x^2 + 1)),x)
```

```
[Out] - a^2*atan(2*a*x + 1i)*4i - (a*x*2i + 1/2)/x^2
```

sympy [A] time = 0.19, size = 42, normalized size = 1.17

$$-2a^2\left(\log\left(4a^3x\right) - \log\left(4a^3x + 4ia^2\right)\right) - \frac{4iax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**3,x)
```

```
[Out] -2*a**2*(log(4*a**3*x) - log(4*a**3*x + 4*I*a**2)) - (4*I*a*x + 1)/(2*x**2)
```

3.18 $\int \frac{e^{2i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=48

$$-2ia^3\log(x)+2ia^3\log(ax+i)+\frac{2a^2}{x}-\frac{ia}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3-I*a/x^2+2*a^2/x-2*I*a^3*ln(x)+2*I*a^3*ln(I+a*x)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{2a^2}{x}-2ia^3\log(x)+2ia^3\log(ax+i)-\frac{ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1+iax}{x^4(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{2ia}{x^3}-\frac{2a^2}{x^2}-\frac{2ia^3}{x}+\frac{2ia^4}{i+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{ia}{x^2}+\frac{2a^2}{x}-2ia^3\log(x)+2ia^3\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-2ia^3\log(x)+2ia^3\log(ax+i)+\frac{2a^2}{x}-\frac{ia}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

fricas [A] time = 0.41, size = 47, normalized size = 0.98

$$\frac{-6i\,a^3x^3\log(x)+6i\,a^3x^3\log\left(\frac{ax+i}{a}\right)+6\,a^2x^2-3i\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*a^3*x^3*log(x) + 6*I*a^3*x^3*log((a*x + I)/a) + 6*a^2*x^2 - 3*I*a
*x - 1)/x^3
```

giac [A] time = 0.14, size = 42, normalized size = 0.88

$$2\,a^3i\log\left(ax+i\right)-2\,a^3i\log\left(|x|\right)+\frac{6\,a^2x^2-3\,aix-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*i*log(a*x + i) - 2*a^3*i*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*i*x - 1)/
x^3
```

maple [A] time = 0.05, size = 55, normalized size = 1.15

$$-\frac{1}{3x^3}-2ia^3\ln(x)-\frac{ia}{x^2}+\frac{2a^2}{x}+ia^3\ln\left(a^2x^2+1\right)+2a^3\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3-2*I*a^3*ln(x)-I*a/x^2+2*a^2/x+I*a^3*ln(a^2*x^2+1)+2*a^3*arctan(a*x
)
```

maxima [A] time = 0.42, size = 51, normalized size = 1.06

$$2\,a^3\arctan\left(ax\right)+i\,a^3\log\left(a^2x^2+1\right)-2i\,a^3\log(x)+\frac{6\,a^2x^2-3i\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*arctan(a*x) + I*a^3*log(a^2*x^2 + 1) - 2*I*a^3*log(x) + 1/3*(6*a^2*x^
2 - 3*I*a*x - 1)/x^3
```

mupad [B] time = 0.07, size = 34, normalized size = 0.71

$$4\,a^3\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right)-\frac{-2\,a^2\,x^2+a\,x\,1\mathrm{i}+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(x^4*(a^2*x^2 + 1)),x)
```

```
[Out] 4*a^3*atan(2*a*x + 1i) - (a*x*1i - 2*a^2*x^2 + 1/3)/x^3
```

sympy [A] time = 0.21, size = 54, normalized size = 1.12

$$-2a^3\left(i\log\left(4a^4x\right)-i\log\left(4a^4x+4ia^3\right)\right)-\frac{-6a^2x^2+3iax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(I*log(4*a**4*x) - I*log(4*a**4*x + 4*I*a**3)) - (-6*a**2*x**2 + 3*
I*a*x + 1)/(3*x**3)
```

3.19 $\int e^{3i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=137

$$-\frac{51i\sinh^{-1}(ax)}{8a^4}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{9i(-3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1+iax)^3}{a^4\sqrt{a^2x^2+1}}$$

```
[Out] -51/8*I*arcsinh(a*x)/a^4+(1+I*a*x)^3/a^4/(a^2*x^2+1)^(1/2)+27/4*(a^2*x^2+1)
^(1/2)/a^4-x^2*(a^2*x^2+1)^(1/2)/a^2-1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-9/8*I*(2
*I-3*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.62, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 215}

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(-3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1+iax)^3}{a^4\sqrt{a^2x^2+1}}-\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] (1 + I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I - 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^3}{a}-x^4\right)}{(1-iax)^2}\,dx\right) \\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx \\
&= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx \\
&= \int \frac{x^3(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1+iax)^2\left(\frac{3i}{a^3}-\frac{x}{a^2}-\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{\frac{12i}{a}-28x-27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{36ia-108a^2x-81ia^3x^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int -\frac{9ia(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{(3i)\int \frac{(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^3} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{(3i)\int \frac{-17a^2-18}{\sqrt{1+a^2}}}{8a^5} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} \\
&= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.58

$$
\sqrt{a^2x^2+1}\left(\frac{4i}{a^4(ax+i)}+\frac{6}{a^4}+\frac{19ix}{8a^3}-\frac{x^2}{a^2}-\frac{ix^3}{4a}\right)-\frac{51i\sinh^{-1}(ax)}{8a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 + (((19*I)/8)*x)/a^3 - x^2/a^2 - ((I/4)*x^3)/a + (
4*I)/(a^4*(I + a*x))) - (((51*I)/8)*ArcSinh[a*x])/a^4
```

fricas [A] time = 0.42, size = 88, normalized size = 0.64

$$\frac{32i\,ax-51\,(-i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\left(-2i\,a^4x^4-6\,a^3x^3+11i\,a^2x^2+29\,ax+80i\right)\sqrt{a^2x^2+1}-32}{8\,a^5x+8i\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] (32*I*a*x - 51*(-I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (-2*I*a^4*x^4 -
 6*a^3*x^3 + 11*I*a^2*x^2 + 29*a*x + 80*I)*sqrt(a^2*x^2 + 1) - 32)/(8*a^5*x
 + 8*I*a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 143, normalized size = 1.04

$$-\frac{ia\,x^5}{4\sqrt{a^2x^2+1}}+\frac{17ix^3}{8a\sqrt{a^2x^2+1}}+\frac{51ix}{8a^3\sqrt{a^2x^2+1}}-\frac{51i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}-\frac{x^4}{\sqrt{a^2x^2+1}}+\frac{5x^2}{a^2\sqrt{a^2x^2+1}}+\frac{10}{a^4\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x)
```

```
[Out] -1/4*I*a*x^5/(a^2*x^2+1)^(1/2)+17/8*I/a*x^3/(a^2*x^2+1)^(1/2)+51/8*I/a^3*x/
(a^2*x^2+1)^(1/2)-51/8*I/a^3*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^
(1/2)-x^4/(a^2*x^2+1)^(1/2)+5*x^2/a^2/(a^2*x^2+1)^(1/2)+10/a^4/(a^2*x^2+1)^
(1/2)
```

maxima [A] time = 0.32, size = 114, normalized size = 0.83

$$-\frac{i\,ax^5}{4\,\sqrt{a^2x^2+1}}-\frac{x^4}{\sqrt{a^2x^2+1}}+\frac{17i\,x^3}{8\,\sqrt{a^2x^2+1}\,a}+\frac{5\,x^2}{\sqrt{a^2x^2+1}\,a^2}+\frac{51i\,x}{8\,\sqrt{a^2x^2+1}\,a^3}-\frac{51i\,\operatorname{arsinh}(ax)}{8\,a^4}+\frac{10}{\sqrt{a^2x^2+1}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*a*x^5/sqrt(a^2*x^2 + 1) - x^4/sqrt(a^2*x^2 + 1) + 17/8*I*x^3/(sqrt(a
^2*x^2 + 1)*a) + 5*x^2/(sqrt(a^2*x^2 + 1)*a^2) + 51/8*I*x/(sqrt(a^2*x^2 + 1
)*a^3) - 51/8*I*arcsinh(a*x)/a^4 + 10/(sqrt(a^2*x^2 + 1)*a^4)
```

mupad [B] time = 0.46, size = 137, normalized size = 1.00

$$\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{{\left(a^2\right)}^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}-\frac{x^3\,{\left(a^2\right)}^{3/2}\,1{}\mathrm{i}}{4\,a^3}+\frac{x\,\sqrt{a^2}\,19{}\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51{}\mathrm{i}}{8\,a^3\,\sqrt{a^2}}+\frac{\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{a^3\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/2
))/a^2 - (x^3*(a^2)^(3/2)*1i)/(4*a^3) + (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^2)
^(1/2) - (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)^(1
/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx+\int \frac{a^3x^6}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**3,x)
```

```
[Out] -I*(Integral(I*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1
)), x) + Integral(a**3*x**6/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(-3*I*a**2*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(
a**2*x**2 + 1)), x))
```

3.20 $\int e^{3i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=102

$$\frac{11\sinh^{-1}(ax)}{2a^3}+\frac{i(1+iax)^3}{a^3\sqrt{a^2x^2+1}}+\frac{i(3+iax)^2\sqrt{a^2x^2+1}}{3a^3}+\frac{(-3ax+28i)\sqrt{a^2x^2+1}}{6a^3}$$

```
[Out] 11/2*arcsinh(a*x)/a^3+I*(1+I*a*x)^3/a^3/(a^2*x^2+1)^(1/2)+1/6*(28*I-3*a*x)*
(a^2*x^2+1)^(1/2)/a^3+1/3*I*(3+I*a*x)^2*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.57, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1654, 780, 215}

$$\frac{i(1+iax)^3}{a^3\sqrt{a^2x^2+1}}+\frac{i(3+iax)^2\sqrt{a^2x^2+1}}{3a^3}+\frac{(-3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (I*(1 + I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) + ((28*I - 3*a*x)*Sqrt[1 + a^2*x^
2])/(6*a^3) + ((I/3)*(3 + I*a*x)^2*Sqrt[1 + a^2*x^2])/a^3 + (11*ArcSinh[a*x
])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^2}{a}-x^3\right)}{(1-iax)^2}\,dx\right) \\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx \\
&= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx \\
&= \int \frac{x^2(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} - \int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(1+iax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(-5-3iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\sinh^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.62

$$
\frac{33\sinh^{-1}(ax)+\frac{\sqrt{a^2x^2+1}\left(-2ia^3x^3-7a^2x^2+19iax-52\right)}{ax+i}}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 + (19*I)*a*x - 7*a^2*x^2 - (2*I)*a^3*x^3))/(I + a*
x) + 33*ArcSinh[a*x])/(6*a^3)
```

fricas [A] time = 0.41, size = 81, normalized size = 0.79

$$
-\frac{24\,ax+(33\,ax+33i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(-2i\,a^3x^3-7\,a^2x^2+19i\,ax-52\right)\sqrt{a^2x^2+1}+24i}{6\,a^4x+6i\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -(24*a*x + (33*a*x + 33*I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (-2*I*a^3*x^3 -
7*a^2*x^2 + 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) + 24*I)/(6*a^4*x + 6*I*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
undef
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.16, size = 123, normalized size = 1.21

$$-\frac{i a\,x^{4}}{3 \sqrt{a^{2} x^{2}+1}}+\frac{13 i x^{2}}{3 a \sqrt{a^{2} x^{2}+1}}+\frac{26 i}{3 a^{3} \sqrt{a^{2} x^{2}+1}}-\frac{3 x^{3}}{2 \sqrt{a^{2} x^{2}+1}}-\frac{11 x}{2 a^{2} \sqrt{a^{2} x^{2}+1}}+\frac{11 \ln \left(\frac{x\,a^{2}}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}+1}\right)}{2 a^{2} \sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x)
```

```
[Out] -1/3*I*a*x^4/(a^2*x^2+1)^(1/2)+13/3*I/a*x^2/(a^2*x^2+1)^(1/2)+26/3*I/a^3/(a
^2*x^2+1)^(1/2)-3/2*x^3/(a^2*x^2+1)^(1/2)-11/2*x/a^2/(a^2*x^2+1)^(1/2)+11/2
/a^2*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 95, normalized size = 0.93

$$-\frac{i\,a x^{4}}{3\,\sqrt{a^{2} x^{2}+1}}-\frac{3\,x^{3}}{2\,\sqrt{a^{2} x^{2}+1}}+\frac{13 i\,x^{2}}{3\,\sqrt{a^{2} x^{2}+1}\,a}-\frac{11\,x}{2\,\sqrt{a^{2} x^{2}+1}\,a^{2}}+\frac{11\,\operatorname{arsinh}\left(a x\right)}{2\,a^{3}}+\frac{26 i}{3\,\sqrt{a^{2} x^{2}+1}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*I*a*x^4/sqrt(a^2*x^2 + 1) - 3/2*x^3/sqrt(a^2*x^2 + 1) + 13/3*I*x^2/(sq
rt(a^2*x^2 + 1)*a) - 11/2*x/(sqrt(a^2*x^2 + 1)*a^2) + 11/2*arcsinh(a*x)/a^3
 + 26/3*I/(sqrt(a^2*x^2 + 1)*a^3)
```

mupad [B] time = 0.06, size = 114, normalized size = 1.12

$$\frac{11\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,x\,\sqrt{a^2}}{2\,a^2}-\frac{a\,14{}\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}+\frac{a^3\,x^2\,1{}\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{4\,\sqrt{a^2\,x^2+1}}{a^2\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a^3*
x^2*1i)/(3*(a^2)^(3/2)) - (a*14i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*a^
2)))/(a^2)^(1/2) - (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a + x*(a^
2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i x^{2}}{a^{2} x^{2} \sqrt{a^{2} x^{2}+1}+\sqrt{a^{2} x^{2}+1}}\, dx+\int \left(-\frac{3 a x^{3}}{a^{2} x^{2} \sqrt{a^{2} x^{2}+1}+\sqrt{a^{2} x^{2}+1}}\right) dx+\int \frac{a^{3} x^{5}}{a^{2} x^{2} \sqrt{a^{2} x^{2}+1}+\sqrt{a^{2} x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2,x)
```

```
[Out] -I*(Integral(I*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1
)), x) + Integral(a**3*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(-3*I*a**2*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(
a**2*x**2 + 1)), x))
```

3.21 $\int e^{3i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{a^2(1-iax)^3}-\frac{3\left(a^2x^2+1\right)^{3/2}}{2a^2(1-iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}+\frac{9i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] -3/2*(a^2*x^2+1)^(3/2)/a^2/(1-I*a*x)-(a^2*x^2+1)^(5/2)/a^2/(1-I*a*x)^3+9/2*
I*arcsinh(a*x)/a^2-9/2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.33, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5060, 1633, 1593, 12, 793, 665, 215}

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{a^2(1-iax)^3}-\frac{3\left(a^2x^2+1\right)^{3/2}}{2a^2(1-iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}+\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x,x]
```

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 - I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 - I*a*x)^3) + (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= -\left((ia)\int \frac{\left(\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)\\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)\\
&= a^2\int \frac{x\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx\\
&= \int \frac{x\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx\\
&= -\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(3i)\int \frac{\left(1+a^2x^2\right)^{3/2}}{(1-iax)^2}\,dx}{a}\\
&= -\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1-iax}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1-iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1-iax)^3}+\frac{9i\sinh^{-1}(ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.59

$$
-\frac{i\left(-9\sinh^{-1}(ax)+\frac{\sqrt{a^2x^2+1}\left(a^2x^2-5iax+14\right)}{ax+i}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-1/2*I)*((Sqrt[1 + a^2*x^2]*(14 - (5*I)*a*x + a^2*x^2))/(I + a*x) - 9*Arc
Sinh[a*x]))/a^2
```

fricas [A] time = 0.44, size = 72, normalized size = 0.78

$$
\frac{-8i\,ax-9\,(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}\left(-i\,a^2x^2-5\,ax-14i\right)+8}{2\,a^3x+2i\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] (-8*I*a*x - 9*(I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)
*(-I*a^2*x^2 - 5*a*x - 14*I) + 8)/(2*a^3*x + 2*I*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.16, size = 104, normalized size = 1.13

$$-\frac{ia\,x^3}{2\sqrt{a^2x^2+1}}-\frac{9ix}{2a\sqrt{a^2x^2+1}}+\frac{9i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}-\frac{3x^2}{\sqrt{a^2x^2+1}}-\frac{7}{a^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x)
```

```
[Out] -1/2*I*a*x^3/(a^2*x^2+1)^(1/2)-9/2*I/a*x/(a^2*x^2+1)^(1/2)+9/2*I/a*ln(x*a^2
/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-3*x^2/(a^2*x^2+1)^(1/2)-7/a^2/(
a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.32, size = 76, normalized size = 0.83

$$-\frac{i\,ax^3}{2\,\sqrt{a^2x^2+1}}-\frac{3\,x^2}{\sqrt{a^2x^2+1}}-\frac{9i\,x}{2\,\sqrt{a^2x^2+1}\,a}+\frac{9i\,\operatorname{arsinh}(ax)}{2\,a^2}-\frac{7}{\sqrt{a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*I*a*x^3/sqrt(a^2*x^2 + 1) - 3*x^2/sqrt(a^2*x^2 + 1) - 9/2*I*x/(sqrt(a^
2*x^2 + 1)*a) + 9/2*I*arcsinh(a*x)/a^2 - 7/(sqrt(a^2*x^2 + 1)*a^2)
```

mupad [B] time = 0.43, size = 104, normalized size = 1.13

$$-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,\sqrt{a^2}}{a^2}+\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^2}}+\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((3*(a^2
)^(1/2))/a^2 + (x*(a^2)^(1/2)*1i)/(2*a)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2
)*4i)/(a*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int\frac{ix}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx+\int\frac{a^3x^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x,x)
```

```
[Out] -I*(Integral(I*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x)
+ Integral(-3*a*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) + Integral(a**3*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
1)), x) + Integral(-3*I*a**2*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**
2*x**2 + 1)), x))
```

3.22 $\int e^{3i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$-\frac{2i(1+iax)^2}{a\sqrt{a^2x^2+1}}-\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

```
[Out] -3*arcsinh(a*x)/a-2*I*(1+I*a*x)^2/a/(a^2*x^2+1)^(1/2)-3*I*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5059, 853, 669, 641, 215}

$$-\frac{2i(1+iax)^2}{a\sqrt{a^2x^2+1}}-\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*(1 + I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2])/a
- (3*ArcSinh[a*x])/a
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{3i\tan^{-1}(ax)}\,dx &= \int \frac{(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\ &= \int \frac{(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\ &= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx \\ &= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\ &= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 42, normalized size = 0.70

$$-\frac{3\sinh^{-1}(ax)}{a} + \frac{\sqrt{a^2x^2+1}\left(\frac{4}{ax+i}-i\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x]),x]
[Out] (Sqrt[1 + a^2*x^2]*(-I + 4/(I + a*x)))/a - (3*ArcSinh[a*x])/a
```

fricas [**A**] time = 0.42, size = 60, normalized size = 1.00

$$\frac{4\,ax + (3\,ax + 3i)\log\left(-ax + \sqrt{a^2x^2+1}\right) + \sqrt{a^2x^2+1}\,(-i\,ax + 5) + 4i}{a^2x + i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] (4*a*x + (3*a*x + 3*I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(-
I*a*x + 5) + 4*I)/(a^2*x + I*a)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] undef
```

maple [**A**] time = 0.11, size = 81, normalized size = 1.35

$$\frac{4x}{\sqrt{a^2x^2+1}} - \frac{ia\,x^2}{\sqrt{a^2x^2+1}} - \frac{5i}{a\sqrt{a^2x^2+1}} - \frac{3\ln\left(\frac{x\,a^2}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x)
[Out] 4*x/(a^2*x^2+1)^(1/2)-I*a*x^2/(a^2*x^2+1)^(1/2)-5*I/a/(a^2*x^2+1)^(1/2)-3*l
n(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.31, size = 57, normalized size = 0.95

$$-\frac{i\,ax^2}{\sqrt{a^2x^2+1}}+\frac{4\,x}{\sqrt{a^2x^2+1}}-\frac{3\,\operatorname{arsinh}(ax)}{a}-\frac{5i}{\sqrt{a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*a*x^2/sqrt(a^2*x^2 + 1) + 4*x/sqrt(a^2*x^2 + 1) - 3*arcsinh(a*x)/a - 5*I
/(sqrt(a^2*x^2 + 1)*a)
```

mupad [B] time = 0.42, size = 72, normalized size = 1.20

$$-\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}-\frac{3\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}}+\frac{4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
- (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int\frac{i}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx+\int\frac{a^3x^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) +
Integral(-3*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) +
 Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*I*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**
2 + 1)), x))
```

3.23 $\int \frac{e^{3i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=51

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

```
[Out] -I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))+4*I*(a^2*x^2+1)^(1/2)/(I+a*x)
```

Rubi [A] time = 0.66, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5060, 6742, 215, 266, 63, 208, 651}

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x,x]

[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1+iax)^2}{x(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(-\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right)-(4a)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2} \\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\sinh^{-1}(ax)-\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.08

$$\frac{4i\sqrt{a^2x^2+1}}{ax+i}-\log\left(\sqrt{a^2x^2+1}+1\right)-i\sinh^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x,x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] + Log[x] - Log[1 + Sqr
t[1 + a^2*x^2]]
```

fricas [B] time = 0.45, size = 100, normalized size = 1.96

$$\frac{4i\,ax-(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}-\right.}{ax+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*I*a*x - (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (I*a*x - 1)*log(-a
*x + sqrt(a^2*x^2 + 1)) + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 4*I
*sqrt(a^2*x^2 + 1) - 4)/(a*x + I)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.14, size = 77, normalized size = 1.51

$$\frac{4iax}{\sqrt{a^2x^2+1}} - \frac{ia\ln\left(\frac{xa^2}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} + \frac{4}{\sqrt{a^2x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 4*I*a*x/(a^2*x^2+1)^(1/2)-I*a*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)
^(1/2)+4/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.32, size = 46, normalized size = 0.90

$$\frac{4i\,ax}{\sqrt{a^2x^2+1}} + \frac{4}{\sqrt{a^2x^2+1}} - i\,\operatorname{arsinh}(ax) - \operatorname{arsinh}\left(\frac{1}{a|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*I*a*x/sqrt(a^2*x^2 + 1) + 4/sqrt(a^2*x^2 + 1) - I*arcsinh(a*x) - arcsinh(
1/(a*abs(x)))
```

mupad [B] time = 0.43, size = 73, normalized size = 1.43

$$-\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right) - \frac{a\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}} + \frac{a\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2} + \frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(x*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2
)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x^3\sqrt{a^2x^2+1} + x\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ax}{a^2x^3\sqrt{a^2x^2+1} + x\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^3\sqrt{a^2x^2+1} + x\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] -I*(Integral(I/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1)), x)
+ Integral(-3*a*x/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1)),
x) + Integral(a**3*x**3/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 +
 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(
a**2*x**2 + 1)), x))
```

3.24 $\int \frac{e^{3i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{4a\sqrt{a^2x^2+1}}{ax+i}-\frac{\sqrt{a^2x^2+1}}{x}-3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -3*I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x-4*a*(a^2*x^2+1)^(1/2)
/(I+a*x)
```

Rubi [A] time = 0.56, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5060, 6742, 264, 266, 63, 208, 651}

$$-\frac{4a\sqrt{a^2x^2+1}}{ax+i}-\frac{\sqrt{a^2x^2+1}}{x}-3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - (4*a*Sqrt[1 + a^2*x^2])/(I + a*x) - (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
```

```
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^2}{x^2(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}}+\frac{3ia}{x\sqrt{1+a^2x^2}}-\frac{4ia^2}{(i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= (3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx-\left(4ia^2\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{x}-\frac{4a\sqrt{1+a^2x^2}}{i+ax}+\frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{x}-\frac{4a\sqrt{1+a^2x^2}}{i+ax}+\frac{(3i)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a}\\
&= -\frac{\sqrt{1+a^2x^2}}{x}-\frac{4a\sqrt{1+a^2x^2}}{i+ax}-3ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.97

$$\sqrt{a^2x^2+1}\left(-\frac{1}{x}-\frac{4a}{ax+i}\right)-3ia\log\left(\sqrt{a^2x^2+1}+1\right)+3ia\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(I + a*x)) + (3*I)*a*Log[x] - (3*I)*a*Lo
g[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.44, size = 109, normalized size = 1.73

$$-\frac{5a^2x^2+5iax+3\left(ia^2x^2-ax\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3\left(-ia^2x^2+ax\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+\cdots}{ax^2+ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 + 5*I*a*x + 3*(I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
1) + 3*(-I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x + I))/(a*x^2 + I*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.15, size = 80, normalized size = 1.27

$$\frac{ia}{\sqrt{a^2x^2+1}}-\frac{5a^2x}{\sqrt{a^2x^2+1}}+3ia\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)-\frac{1}{x\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] I*a/(a^2*x^2+1)^(1/2)-5*a^2*x/(a^2*x^2+1)^(1/2)+3*I*a*(1/(a^2*x^2+1)^(1/2)-
arctanh(1/(a^2*x^2+1)^(1/2)))-1/x/(a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.31, size = 60, normalized size = 0.95

$$-\frac{5\,a^2x}{\sqrt{a^2x^2+1}}-3i\,a\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{4i\,a}{\sqrt{a^2x^2+1}}-\frac{1}{\sqrt{a^2x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -5*a^2*x/sqrt(a^2*x^2 + 1) - 3*I*a*arcsinh(1/(a*abs(x))) + 4*I*a/sqrt(a^2*x
^2 + 1) - 1/(sqrt(a^2*x^2 + 1)*x)
```

mupad [B] time = 0.06, size = 75, normalized size = 1.19

$$-a\,\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,3\mathrm{i}-\frac{\sqrt{a^2\,x^2+1}}{x}-\frac{4\,a^2\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(x^2*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] - a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x - (4*a^2*(a^2*x^2
 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int\frac{i}{a^2x^4\sqrt{a^2x^2+1}+x^2\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax}{a^2x^4\sqrt{a^2x^2+1}+x^2\sqrt{a^2x^2+1}}\right)dx+\int\frac{a^3x^3}{a^2x^4\sqrt{a^2x^2+1}+x^2\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] -I*(Integral(I/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**4*sqrt(a**2*x**2 + 1)
+ x**2*sqrt(a**2*x**2 + 1)), x))
```

3.25 $\int \frac{e^{3i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=92

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 9/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2-3*I*a*(a^2*x^2
+1)^(1/2)/x-4*I*a^2*(a^2*x^2+1)^(1/2)/(I+a*x)
```

Rubi [A] time = 0.60, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5060, 6742, 266, 51, 63, 208, 264, 651}

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) - ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I + a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^2}{x^3(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}}+\frac{3ia}{x^2\sqrt{1+a^2x^2}}-\frac{4a^2}{x\sqrt{1+a^2x^2}}+\frac{4a^3}{(i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= (3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-\left(4a^2\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx+\left(4a^3\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{}{x^3\sqrt{}} \\
&= -\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)-\left(2a^2\right)\operatorname{Subst}\left(\int \frac{}{x\sqrt{}}\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}-4\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)-\frac{1}{4} \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+4a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{}{-\frac{1}{a^2}}\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.86

$$\sqrt{a^2x^2+1}\left(-\frac{4ia^2}{ax+i}-\frac{3ia}{x}-\frac{1}{2x^2}\right)+\frac{9}{2}a^2\log\left(\sqrt{a^2x^2+1}+1\right)-\frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 - ((3*I)*a)/x - ((4*I)*a^2)/(I + a*x)) - (9*a
^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

fricas [A] time = 0.48, size = 131, normalized size = 1.42

$$\frac{-14i\,a^3x^3+14\,a^2x^2+\left(9\,a^3x^3+9i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-\left(9\,a^3x^3+9i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}-\right.}{2\,ax^3+2i\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] (-14*I*a^3*x^3 + 14*a^2*x^2 + (9*a^3*x^3 + 9*I*a^2*x^2)*log(-a*x + sqrt(a^2
*x^2 + 1) + 1) - (9*a^3*x^3 + 9*I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1
) + sqrt(a^2*x^2 + 1)*(-14*I*a^2*x^2 + 5*a*x - I))/(2*a*x^3 + 2*I*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.15, size = 105, normalized size = 1.14

$$-\frac{ia^3x}{\sqrt{a^2x^2+1}}-\frac{9a^2\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}+3ia\left(-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{2a^2x}{\sqrt{a^2x^2+1}}\right)-\frac{1}{2x^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -I*a^3*x/(a^2*x^2+1)^(1/2)-9/2*a^2*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+
1)^(1/2)))+3*I*a*(-1/x/(a^2*x^2+1)^(1/2)-2*a^2*x/(a^2*x^2+1)^(1/2))-1/2/x^2
/(a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.31, size = 81, normalized size = 0.88

$$-\frac{7i\,a^3x}{\sqrt{a^2x^2+1}}+\frac{9}{2}\,a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{9\,a^2}{2\,\sqrt{a^2x^2+1}}-\frac{3i\,a}{\sqrt{a^2x^2+1}\,x}-\frac{1}{2\,\sqrt{a^2x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7*I*a^3*x/sqrt(a^2*x^2 + 1) + 9/2*a^2*arcsinh(1/(a*abs(x))) - 9/2*a^2/sqrt
(a^2*x^2 + 1) - 3*I*a/(sqrt(a^2*x^2 + 1)*x) - 1/2/(sqrt(a^2*x^2 + 1)*x^2)
```

mupad [B] time = 0.43, size = 99, normalized size = 1.08

$$-\frac{a^2\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}-\frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x}-\frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(x^3*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] - (a^2*atan((a^2*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (
a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1
i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int\frac{i}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\right)dx+\int\frac{a^3x^3}{a^2x^5\sqrt{a^2x^2+1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] -I*(Integral(I/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**5*sqrt(a**2*x**2 + 1)
+ x**3*sqrt(a**2*x**2 + 1)), x))
```

3.26 $\int \frac{e^{3 i \tan^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=117

$$\frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}+\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 11/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3-3/2*I*a*(a^
2*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2+1)^(1/2)/x+4*a^3*(a^2*x^2+1)^(1/2)/(I+
a*x)
```

Rubi [A] time = 0.62, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) - (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 + a^2*x^2])/(I + a*x) + ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+iax)^2}{x^4(1-iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}} + \frac{3ia}{x^3\sqrt{1+a^2x^2}} - \frac{4a^2}{x^2\sqrt{1+a^2x^2}} - \frac{4ia^3}{x\sqrt{1+a^2x^2}} + \frac{4ia^4}{(i+ax)\sqrt{1+a^2x^2}}\right)dx$$

$$= (3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx - \left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx - \left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx + \left(4ia^4\right)\int \frac{}{(i}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{4a^2\sqrt{1+a^2x^2}}{x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + \frac{1}{2}(3ia)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) - \frac{1}{3}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} - (4ia)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + 4ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} + \frac{4a^3\sqrt{1+a^2x^2}}{i+ax} + \frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right.$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.76

$$\frac{1}{6}\left(-33ia^3\log(x) + 33ia^3\log\left(\sqrt{a^2x^2+1}+1\right) + \frac{\sqrt{a^2x^2+1}\left(52a^3x^3+19ia^2x^2+7ax-2i\right)}{x^3(ax+i)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2*I + 7*a*x + (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(I +
 a*x)) - (33*I)*a^3*Log[x] + (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

fricas [A] time = 0.46, size = 139, normalized size = 1.19

$$\frac{52\,a^4x^4+52i\,a^3x^3-33\left(-i\,a^4x^4+a^3x^3\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-33\left(i\,a^4x^4-a^3x^3\right)\log\left(-ax+\sqrt{a^2x^2+1}\right.}{6\,ax^4+6i\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] (52*a^4*x^4 + 52*I*a^3*x^3 - 33*(-I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(a^2*
x^2 + 1) + 1) - 33*(I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
+ (52*a^3*x^3 + 19*I*a^2*x^2 + 7*a*x - 2*I)*sqrt(a^2*x^2 + 1))/(6*a*x^4 + 6
*I*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.15, size = 141, normalized size = 1.21

$$-\frac{1}{3x^3\sqrt{a^2x^2+1}}-\frac{13a^2\left(-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{2a^2x}{\sqrt{a^2x^2+1}}\right)}{3}-ia^3\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)+3ia\left(-\frac{1}{2x^2\sqrt{a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/3/x^3/(a^2*x^2+1)^(1/2)-13/3*a^2*(-1/x/(a^2*x^2+1)^(1/2)-2*a^2*x/(a^2*x^
2+1)^(1/2))-I*a^3*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2)))+3*I*a*
(-1/2/x^2/(a^2*x^2+1)^(1/2)-3/2*a^2*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2
+1)^(1/2))))
```

maxima [A] time = 0.31, size = 100, normalized size = 0.85

$$\frac{26\,a^4x}{3\,\sqrt{a^2x^2+1}}+\frac{11}{2}i\,a^3\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{11i\,a^3}{2\,\sqrt{a^2x^2+1}}+\frac{13\,a^2}{3\,\sqrt{a^2x^2+1}\,x}-\frac{3i\,a}{2\,\sqrt{a^2x^2+1}\,x^2}-\frac{1}{3\,\sqrt{a^2x^2+1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 26/3*a^4*x/sqrt(a^2*x^2 + 1) + 11/2*I*a^3*arcsinh(1/(a*abs(x))) - 11/2*I*a^
3/sqrt(a^2*x^2 + 1) + 13/3*a^2/(sqrt(a^2*x^2 + 1)*x) - 3/2*I*a/(sqrt(a^2*x^
2 + 1)*x^2) - 1/3/(sqrt(a^2*x^2 + 1)*x^3)
```

mupad [B] time = 0.41, size = 116, normalized size = 0.99

$$\frac{11\,a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{3\,x^3}-\frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2}+\frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x}+\frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(x^4*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] (11*a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a*
(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) + (4*a
^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x^6\sqrt{a^2x^2+1}+x^4\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax}{a^2x^6\sqrt{a^2x^2+1}+x^4\sqrt{a^2x^2+1}}\right)dx+\int \frac{a^3x^3}{a^2x^6\sqrt{a^2x^2+1}+x^4\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] -I*(Integral(I/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**6*sqrt(a**2*x**2 + 1)
+ x**4*sqrt(a**2*x**2 + 1)), x))
```

3.27 $\int e^{4i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=65

$$\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}+\frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

```
[Out] 12*I*x/a^3-4*x^2/a^2-4/3*I*x^3/a+1/4*x^4+4*I/a^4/(I+a*x)+16*ln(I+a*x)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I + 
a*x)) + (16*Log[I + a*x])/a^4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}x^3\,dx&=\int\frac{x^3(1+iax)^2}{(1-iax)^2}\,dx\\&=\int\left(\frac{12i}{a^3}-\frac{8x}{a^2}-\frac{4ix^2}{a}+x^3-\frac{4i}{a^3(i+ax)^2}+\frac{16}{a^3(i+ax)}\right)dx\\&=\frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.00

$$\frac{4i}{a^4(ax+i)}+\frac{16\log(ax+i)}{a^4}+\frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I + 
a*x)) + (16*Log[I + a*x])/a^4
```

fricas [A]   time = 0.42, size = 70, normalized size = 1.08

$$\frac{3\,a^5x^5 - 13i\,a^4x^4 - 32\,a^3x^3 + 96i\,a^2x^2 - 144\,ax + (192\,ax + 192i)\log\left(\frac{ax+i}{a}\right) + 48i}{12\,a^5x + 12i\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] (3*a^5*x^5 - 13*I*a^4*x^4 - 32*a^3*x^3 + 96*I*a^2*x^2 - 144*a*x + (192*a*x
+ 192*I)*log((a*x + I)/a) + 48*I)/(12*a^5*x + 12*I*a^4)
```

giac [A]   time = 0.12, size = 63, normalized size = 0.97

$$\frac{16\,\log\left(ax+i\right)}{a^4} + \frac{4\,i}{(ax+i)a^4} + \frac{3\,a^8x^4 - 16\,a^7ix^3 - 48\,a^6x^2 + 144\,a^5ix}{12\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 16*log(a*x + i)/a^4 + 4*i/((a*x + i)*a^4) + 1/12*(3*a^8*x^4 - 16*a^7*i*x^3
- 48*a^6*x^2 + 144*a^5*i*x)/a^8
```

maple [A]   time = 0.06, size = 70, normalized size = 1.08

$$\frac{x^4}{4} - \frac{4ix^3}{3a} - \frac{4x^2}{a^2} + \frac{12ix}{a^3} + \frac{4i}{a^4\,(ax+i)} + \frac{8\ln\left(a^2x^2+1\right)}{a^4} - \frac{16i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x)
```

```
[Out] 1/4*x^4-4/3*I*x^3/a-4*x^2/a^2+12*I*x/a^3+4*I/a^4/(I+a*x)+8/a^4*ln(a^2*x^2+1
)-16*I/a^4*arctan(a*x)
```

maxima [A]   time = 0.42, size = 77, normalized size = 1.18

$$-\frac{4\,(-i\,ax-1)}{a^6x^2+a^4} + \frac{3\,a^3x^4 - 16i\,a^2x^3 - 48\,ax^2 + 144i\,x}{12\,a^3} - \frac{16i\,\arctan\left(ax\right)}{a^4} + \frac{8\,\log\left(a^2x^2+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(a^6*x^2 + a^4) + 1/12*(3*a^3*x^4 - 16*I*a^2*x^3 - 48*a*x^2
 + 144*I*x)/a^3 - 16*I*arctan(a*x)/a^4 + 8*log(a^2*x^2 + 1)/a^4
```

mupad [B]   time = 0.43, size = 60, normalized size = 0.92

$$\frac{16\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^4} + \frac{x^4}{4} - \frac{4\,x^2}{a^2} + \frac{4\mathrm{i}}{a^5\,\left(x+\frac{1\mathrm{i}}{a}\right)} + \frac{x\,12\mathrm{i}}{a^3} - \frac{x^3\,4\mathrm{i}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] 4i/(a^5*(x + 1i/a)) + (16*log(x + 1i/a))/a^4 + (x*12i)/a^3 + x^4/4 - (x^3*4
i)/(3*a) - (4*x^2)/a^2
```

sympy [**A**] time = 0.24, size = 56, normalized size = 0.86

$$\frac{x^4}{4} + \frac{4i}{a^5x + ia^4} - \frac{4ix^3}{3a} - \frac{4x^2}{a^2} + \frac{12ix}{a^3} + \frac{16\log{(ax + i)}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**3,x)
```

```
[Out] x**4/4 + 4*I/(a**5*x + I*a**4) - 4*I*x**3/(3*a) - 4*x**2/a**2 + 12*I*x/a**3
 + 16*log(a*x + I)/a**4
```

3.28 $\int e^{4i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=53

$$-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

```
[Out] -8*x/a^2-2*I*x^2/a+1/3*x^3-4/a^3/(I+a*x)+12*I*ln(I+a*x)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{8x}{a^2}-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(-\frac{8}{a^2}-\frac{4ix}{a}+x^2+\frac{4}{a^2(i+ax)^2}+\frac{12i}{a^2(i+ax)}\right)dx\\ &= -\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.00

$$-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

fricas [A] time = 0.85, size = 62, normalized size = 1.17

$$\frac{a^4x^4 - 5i\,a^3x^3 - 18\,a^2x^2 - 24i\,ax - 36\,(-i\,ax + 1)\log\left(\frac{ax+i}{a}\right) - 12}{3\,a^4x + 3i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] (a^4*x^4 - 5*I*a^3*x^3 - 18*a^2*x^2 - 24*I*a*x - 36*(-I*a*x + 1)*log((a*x +
 I)/a) - 12)/(3*a^4*x + 3*I*a^3)
```

giac [A] time = 0.12, size = 53, normalized size = 1.00

$$\frac{12\,i\log(ax+i)}{a^3} - \frac{4}{(ax+i)a^3} + \frac{a^6x^3 - 6\,a^5ix^2 - 24\,a^4x}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 12*i*log(a*x + i)/a^3 - 4/((a*x + i)*a^3) + 1/3*(a^6*x^3 - 6*a^5*i*x^2 - 24
*a^4*x)/a^6
```

maple [A] time = 0.06, size = 60, normalized size = 1.13

$$\frac{x^3}{3} - \frac{2ix^2}{a} - \frac{8x}{a^2} - \frac{4}{a^3\,(ax+i)} + \frac{6i\ln\left(a^2x^2+1\right)}{a^3} + \frac{12\arctan(ax)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x)
```

```
[Out] 1/3*x^3-2*I*x^2/a-8*x/a^2-4/a^3/(I+a*x)+6*I/a^3*ln(a^2*x^2+1)+12/a^3*arctan
(a*x)
```

maxima [A] time = 0.42, size = 68, normalized size = 1.28

$$-\frac{8\,ax - 8i}{2\left(a^5x^2 + a^3\right)} + \frac{a^2x^3 - 6i\,ax^2 - 24\,x}{3\,a^2} + \frac{12\,\arctan(ax)}{a^3} + \frac{6i\,\log\left(a^2x^2+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*x - 8*I)/(a^5*x^2 + a^3) + 1/3*(a^2*x^3 - 6*I*a*x^2 - 24*x)/a^2 +
 12*arctan(a*x)/a^3 + 6*I*log(a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.06, size = 51, normalized size = 0.96

$$\frac{x^3}{3} + \frac{\ln\left(x + \frac{1\mathrm{i}}{a}\right)\,12\mathrm{i}}{a^3} - \frac{8\,x}{a^2} - \frac{4}{a^4\left(x + \frac{1\mathrm{i}}{a}\right)} - \frac{x^2\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] (log(x + 1i/a)*12i)/a^3 - 4/(a^4*(x + 1i/a)) - (8*x)/a^2 + x^3/3 - (x^2*2i)
/a
```

sympy [A] time = 0.24, size = 44, normalized size = 0.83

$$\frac{x^3}{3} - \frac{4}{a^4x + ia^3} - \frac{2ix^2}{a} - \frac{8x}{a^2} + \frac{12i\log(ax + i)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2,x)
```

```
[Out] x**3/3 - 4/(a**4*x + I*a**3) - 2*I*x**2/a - 8*x/a**2 + 12*I*log(a*x + I)/a*
*3
```

3.29 $\int e^{4i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=45

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

```
[Out] -4*I*x/a+1/2*x^2-4*I/a^2/(I+a*x)-8*ln(I+a*x)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(-\frac{4i}{a}+x+\frac{4i}{a(i+ax)^2}-\frac{8}{a(i+ax)}\right)dx\\ &= -\frac{4ix}{a}+\frac{x^2}{2}-\frac{4i}{a^2(i+ax)}-\frac{8\log(i+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

fricas [A] time = 0.42, size = 54, normalized size = 1.20

$$\frac{a^3x^3 - 7i\,a^2x^2 + 8\,ax - (16\,ax + 16i)\log\left(\frac{ax+i}{a}\right) - 8i}{2\,a^3x + 2i\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 7*I*a^2*x^2 + 8*a*x - (16*a*x + 16*I)*log((a*x + I)/a) - 8*I)/(2
*a^3*x + 2*I*a^2)
```

giac [A] time = 0.14, size = 45, normalized size = 1.00

$$-\frac{8\,\log\left(ax + i\right)}{a^2} - \frac{4\,i}{(ax + i)a^2} + \frac{a^4x^2 - 8\,a^3ix}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="giac")
```

```
[Out] -8*log(a*x + i)/a^2 - 4*i/((a*x + i)*a^2) + 1/2*(a^4*x^2 - 8*a^3*i*x)/a^4
```

maple [A] time = 0.06, size = 53, normalized size = 1.18

$$\frac{x^2}{2} - \frac{4ix}{a} - \frac{4i}{a^2\,(ax + i)} - \frac{4\ln\left(a^2x^2 + 1\right)}{a^2} + \frac{8i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x,x)
```

```
[Out] 1/2*x^2-4*I*x/a-4*I/a^2/(I+a*x)-4/a^2*ln(a^2*x^2+1)+8*I/a^2*arctan(a*x)
```

maxima [A] time = 0.42, size = 60, normalized size = 1.33

$$-\frac{4\,(i\,ax + 1)}{a^4x^2 + a^2} + \frac{ax^2 - 8i\,x}{2\,a} + \frac{8i\,\arctan\left(ax\right)}{a^2} - \frac{4\,\log\left(a^2x^2 + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="maxima")
```

```
[Out] -4*(I*a*x + 1)/(a^4*x^2 + a^2) + 1/2*(a*x^2 - 8*I*x)/a + 8*I*arctan(a*x)/a^
2 - 4*log(a^2*x^2 + 1)/a^2
```

mupad [B] time = 0.06, size = 43, normalized size = 0.96

$$\frac{x^2}{2} - \frac{8\,\ln\left(x + \frac{1\mathrm{i}}{a}\right)}{a^2} - \frac{4\mathrm{i}}{a^3\,\left(x + \frac{1\mathrm{i}}{a}\right)} - \frac{x\,4\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] x^2/2 - (8*log(x + 1i/a))/a^2 - (x*4i)/a - 4i/(a^3*(x + 1i/a))
```

sympy [A] time = 0.19, size = 36, normalized size = 0.80

$$\frac{x^2}{2} - \frac{4i}{a^3x + ia^2} - \frac{4ix}{a} - \frac{8\log\left(ax + i\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x,x)
```

```
[Out] x**2/2 - 4*I/(a**3*x + I*a**2) - 4*I*x/a - 8*log(a*x + I)/a**2
```

3.30 $\int e^{4i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=31

$$\frac{4}{a(ax+i)}-\frac{4i\log(ax+i)}{a}+x$$

```
[Out] x+4/a/(I+a*x)-4*I*ln(I+a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5061, 43}

$$\frac{4}{a(ax+i)}-\frac{4i\log(ax+i)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x]),x]
[Out] x + 4/(a*(I + a*x)) - ((4*I)*Log[I + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{4i\tan^{-1}(ax)}\,dx &= \int \frac{(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(1-\frac{4}{(i+ax)^2}-\frac{4i}{i+ax}\right)dx\\ &= x+\frac{4}{a(i+ax)}-\frac{4i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.35

$$-\frac{2i\log\left(a^2x^2+1\right)}{a}+\frac{4}{a(ax+i)}-\frac{4\tan^{-1}(ax)}{a}+x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x]),x]
[Out] x + 4/(a*(I + a*x)) - (4*ArcTan[a*x])/a - ((2*I)*Log[1 + a^2*x^2])/a
```

fricas [A] time = 0.52, size = 43, normalized size = 1.39

$$\frac{a^2x^2+i\,ax-4\,(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+4}{a^2x+i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="fricas")
[Out] (a^2*x^2 + I*a*x - 4*(I*a*x - 1)*log((a*x + I)/a) + 4)/(a^2*x + I*a)
```

giac [A] time = 0.12, size = 26, normalized size = 0.84

$$x - \frac{4 i \log(ax + i)}{a} + \frac{4}{(ax + i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="giac")
[Out] x - 4*i*log(a*x + i)/a + 4/((a*x + i)*a)
```

maple [A] time = 0.05, size = 41, normalized size = 1.32

$$x + \frac{4}{a(ax + i)} - \frac{2i \ln\left(a^2x^2 + 1\right)}{a} - \frac{4 \arctan(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2,x)
[Out] x+4/a/(I+a*x)-2*I/a*ln(a^2*x^2+1)-4*arctan(a*x)/a
```

maxima [A] time = 0.43, size = 45, normalized size = 1.45

$$x + \frac{8\,ax - 8i}{2\left(a^3x^2 + a\right)} - \frac{4\ \arctan(ax)}{a} - \frac{2i\ \log\left(a^2x^2 + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] x + 1/2*(8*a*x - 8*I)/(a^3*x^2 + a) - 4*arctan(a*x)/a - 2*I*log(a^2*x^2 + 1
)/a
```

mupad [B] time = 0.43, size = 32, normalized size = 1.03

$$x + \frac{4}{a^2\left(x + \frac{1\mathrm{i}}{a}\right)} - \frac{\ln\left(x + \frac{1\mathrm{i}}{a}\right)\,4\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^2,x)
[Out] x + 4/(a^2*(x + 1i/a)) - (log(x + 1i/a)*4i)/a
```

sympy [A] time = 0.17, size = 22, normalized size = 0.71

$$x + \frac{4}{a^2x + ia} - \frac{4i \log(ax + i)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2,x)
[Out] x + 4/(a**2*x + I*a) - 4*I*log(a*x + I)/a
```

3.31 $\int \frac{e^{4i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=16

$$\log(x) + \frac{4i}{ax+i}$$

```
[Out] 4*I/(I+a*x)+ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5062, 88}

$$\log(x) + \frac{4i}{ax+i}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x,x]
```

```
[Out] (4*I)/(I + a*x) + Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1+iax)^2}{x(1-iax)^2}\,dx\\
&= \int \left(\frac{1}{x} - \frac{4ia}{(i+ax)^2}\right)dx\\
&= \frac{4i}{i+ax} + \log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\log(x) + \frac{4i}{ax+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x,x]
```

```
[Out] (4*I)/(I + a*x) + Log[x]
```

fricas [A] time = 0.54, size = 18, normalized size = 1.12

$$\frac{(ax+i)\log(x) + 4i}{ax+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="fricas")
[Out] ((a*x + I)*log(x) + 4*I)/(a*x + I)
```

giac [A] time = 0.13, size = 14, normalized size = 0.88

$$\frac{4\,i}{ax+i}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="giac")
[Out] 4*i/(a*x + i) + log(abs(x))
```

maple [A] time = 0.05, size = 15, normalized size = 0.94

$$\frac{4i}{ax+i}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x,x)
[Out] 4*I/(I+a*x)+ln(x)
```

maxima [A] time = 0.42, size = 22, normalized size = 1.38

$$-\frac{4\,(-i\,ax-1)}{a^2x^2+1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="maxima")
[Out] -4*(-I*a*x - 1)/(a^2*x^2 + 1) + log(x)
```

mupad [B] time = 0.08, size = 14, normalized size = 0.88

$$\ln(x)+\frac{4\mathrm{i}}{a\,x+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(x*(a^2*x^2 + 1)^2),x)
[Out] log(x) + 4i/(a*x + 1i)
```

sympy [A] time = 0.21, size = 10, normalized size = 0.62

$$\log\left(x\right)+\frac{4i}{ax+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x,x)
[Out] log(x) + 4*I/(a*x + I)
```

3.32 $\int \frac{e^{4i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

```
[Out] -1/x-4*a/(I+a*x)+4*I*a*ln(x)-4*I*a*ln(I+a*x)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^2}{x^2(1-iax)^2}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{4ia}{x}+\frac{4a^2}{(i+ax)^2}-\frac{4ia^2}{i+ax}\right)dx\\ &= -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

fricas [A] time = 0.42, size = 60, normalized size = 1.58

$$-\frac{5\,ax+4\left(-i\,a^2x^2+ax\right)\log(x)+4\left(i\,a^2x^2-ax\right)\log\left(\frac{ax+i}{a}\right)+i}{ax^2+i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(-I*a^2*x^2 + a*x)*log(x) + 4*(I*a^2*x^2 - a*x)*log((a*x + I)/a
) + I)/(a*x^2 + I*x)
```

giac [A] time = 0.14, size = 37, normalized size = 0.97

$$-4\,ai\log\left(ax+i\right)+4\,ai\log\left(|x|\right)-\frac{5\,ax+i}{ax^2+ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*i*log(a*x + i) + 4*a*i*log(abs(x)) - (5*a*x + i)/(a*x^2 + i*x)
```

maple [A] time = 0.06, size = 45, normalized size = 1.18

$$-\frac{1}{x}+4ia\ln(x)-\frac{4a}{ax+i}-2ia\ln\left(a^2x^2+1\right)-4a\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x)
```

```
[Out] -1/x+4*I*a*ln(x)-4*a/(I+a*x)-2*I*a*ln(a^2*x^2+1)-4*a*arctan(a*x)
```

maxima [A] time = 0.42, size = 53, normalized size = 1.39

$$-4\,a\arctan\left(ax\right)-2i\,a\log\left(a^2x^2+1\right)+4i\,a\log(x)-\frac{10\,a^2x^2-8i\,ax+2}{2\left(a^2x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*arctan(a*x) - 2*I*a*log(a^2*x^2 + 1) + 4*I*a*log(x) - 1/2*(10*a^2*x^2
- 8*I*a*x + 2)/(a^2*x^3 + x)
```

mupad [B] time = 0.45, size = 37, normalized size = 0.97

$$-8\,a\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right)-\frac{5\,x+\frac{1\mathrm{i}}{a}}{x^2+\frac{x\,1\mathrm{i}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(x^2*(a^2*x^2 + 1)^2),x)
```

```
[Out] - 8*a*atan(2*a*x + 1i) - (5*x + 1i/a)/((x*1i)/a + x^2)
```

sympy [A] time = 0.30, size = 44, normalized size = 1.16

$$4a\left(i\log\left(8a^2x\right)-i\log\left(8a^2x+8ia\right)\right)+\frac{5ax+i}{-ax^2-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**2,x)
```

```
[Out] 4*a*(I*log(8*a**2*x) - I*log(8*a**2*x + 8*I*a)) + (5*a*x + I)/(-a*x**2 - I*
x)
```

3.33 $\int \frac{e^{4i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=52

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

```
[Out] -1/2/x^2-4*I*a/x-4*I*a^2/(I+a*x)-8*a^2*ln(x)+8*a^2*ln(I+a*x)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5062, 88}

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^2}{x^3(1-iax)^2}\,dx\\
&= \int \left(\frac{1}{x^3} + \frac{4ia}{x^2} - \frac{8a^2}{x} + \frac{4ia^3}{(i+ax)^2} + \frac{8a^3}{i+ax}\right)dx\\
&= -\frac{1}{2x^2} - \frac{4ia}{x} - \frac{4ia^2}{i+ax} - 8a^2\log(x) + 8a^2\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.00

$$-\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

fricas [**A**] time = 0.47, size = 78, normalized size = 1.50

$$\frac{-16i\,a^2x^2+7\,ax-\left(16\,a^3x^3+16i\,a^2x^2\right)\log(x)+\left(16\,a^3x^3+16i\,a^2x^2\right)\log\left(\frac{ax+i}{a}\right)-i}{2\,ax^3+2i\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] (-16*I*a^2*x^2 + 7*a*x - (16*a^3*x^3 + 16*I*a^2*x^2)*log(x) + (16*a^3*x^3 +
 16*I*a^2*x^2)*log((a*x + I)/a) - I)/(2*a*x^3 + 2*I*x^2)
```

giac [**A**] time = 0.12, size = 47, normalized size = 0.90

$$8\,a^2\log\left(ax+i\right)-8\,a^2\log\left(|x|\right)-\frac{16\,a^2ix^2-7\,ax+i}{2\,(ax+i)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(a*x + i) - 8*a^2*log(abs(x)) - 1/2*(16*a^2*i*x^2 - 7*a*x + i)/((a
*x + i)*x^2)
```

maple [**A**] time = 0.06, size = 60, normalized size = 1.15

$$-\frac{1}{2x^2}-\frac{4ia}{x}-8a^2\ln(x)-\frac{4ia^2}{ax+i}+4a^2\ln\left(a^2x^2+1\right)-8ia^2\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-4*I*a/x-8*a^2*ln(x)-4*I*a^2/(I+a*x)+4*a^2*ln(a^2*x^2+1)-8*I*a^2*ar
ctan(a*x)
```

maxima [**A**] time = 0.42, size = 69, normalized size = 1.33

$$-8i\,a^2\arctan\left(ax\right)+4\,a^2\log\left(a^2x^2+1\right)-8\,a^2\log(x)+\frac{-16i\,a^3x^3-9\,a^2x^2-8i\,ax-1}{2\left(a^2x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*I*a^2*arctan(a*x) + 4*a^2*log(a^2*x^2 + 1) - 8*a^2*log(x) + 1/2*(-16*I*a
^3*x^3 - 9*a^2*x^2 - 8*I*a*x - 1)/(a^2*x^4 + x^2)
```

mupad [**B**] time = 0.47, size = 43, normalized size = 0.83

$$-a^2\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right)\,16\mathrm{i}+\frac{8\,a^2\,x^2+\frac{a\,x\,7\mathrm{i}}{2}+\frac{1}{2}}{x^2\,\left(-1+a\,x\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(x^3*(a^2*x^2 + 1)^2),x)
```

```
[Out] ((a*x*7i)/2 + 8*a^2*x^2 + 1/2)/(x^2*(a*x*1i - 1)) - a^2*atan(2*a*x + 1i)*16
i
```

sympy [**A**] time = 0.35, size = 60, normalized size = 1.15

$$8a^2\left(-\log\left(16a^3x\right)+\log\left(16a^3x+16ia^2\right)\right)+\frac{16ia^2x^2-7ax+i}{-2ax^3-2ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**3,x)
```

```
[Out] 8*a**2*(-log(16*a**3*x) + log(16*a**3*x + 16*I*a**2)) + (16*I*a**2*x**2 - 7
*a*x + I)/(-2*a*x**3 - 2*I*x**2)
```

3.34 $\int \frac{e^{4i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=62

$$\frac{4a^3}{ax+i} - 12ia^3\log(x) + 12ia^3\log(ax+i) + \frac{8a^2}{x} - \frac{2ia}{x^2} - \frac{1}{3x^3}$$

```
[Out] -1/3/x^3-2*I*a/x^2+8*a^2/x+4*a^3/(I+a*x)-12*I*a^3*ln(x)+12*I*a^3*ln(I+a*x)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 88}

$$\frac{4a^3}{ax+i} + \frac{8a^2}{x} - 12ia^3\log(x) + 12ia^3\log(ax+i) - \frac{2ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1+iax)^2}{x^4(1-iax)^2}\,dx \\
&= \int \left(\frac{1}{x^4} + \frac{4ia}{x^3} - \frac{8a^2}{x^2} - \frac{12ia^3}{x} - \frac{4a^4}{(i+ax)^2} + \frac{12ia^4}{i+ax}\right)dx \\
&= -\frac{1}{3x^3} - \frac{2ia}{x^2} + \frac{8a^2}{x} + \frac{4a^3}{i+ax} - 12ia^3\log(x) + 12ia^3\log(i+ax)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.00

$$\frac{4a^3}{ax+i} - 12ia^3\log(x) + 12ia^3\log(ax+i) + \frac{8a^2}{x} - \frac{2ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

fricas [A] time = 0.47, size = 86, normalized size = 1.39

$$\frac{36\,a^3x^3+18i\,a^2x^2+5\,ax-36\left(i\,a^4x^4-a^3x^3\right)\log(x)-36\left(-i\,a^4x^4+a^3x^3\right)\log\left(\frac{ax+i}{a}\right)-i}{3\,ax^4+3i\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] (36*a^3*x^3 + 18*I*a^2*x^2 + 5*a*x - 36*(I*a^4*x^4 - a^3*x^3)*log(x) - 36*(
-I*a^4*x^4 + a^3*x^3)*log((a*x + I)/a) - I)/(3*a*x^4 + 3*I*x^3)
```

giac [A] time = 0.14, size = 59, normalized size = 0.95

$$12\,a^3i\log\left(ax+i\right)-12\,a^3i\log\left(|x|\right)+\frac{36\,a^3x^3+18\,a^2ix^2+5\,ax-i}{3\,(ax+i)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*i*log(a*x + i) - 12*a^3*i*log(abs(x)) + 1/3*(36*a^3*x^3 + 18*a^2*i*x
^2 + 5*a*x - i)/((a*x + i)*x^3)
```

maple [A] time = 0.06, size = 68, normalized size = 1.10

$$-\frac{1}{3x^3}-12ia^3\ln(x)-\frac{2ia}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{ax+i}+6ia^3\ln\left(a^2x^2+1\right)+12a^3\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-12*I*a^3*ln(x)-2*I*a/x^2+8*a^2/x+4*a^3/(I+a*x)+6*I*a^3*ln(a^2*x^2+
1)+12*a^3*arctan(a*x)
```

maxima [A] time = 0.42, size = 77, normalized size = 1.24

$$12\,a^3\arctan\left(ax\right)+6i\,a^3\log\left(a^2x^2+1\right)-12i\,a^3\log(x)+\frac{72\,a^4x^4-36i\,a^3x^3+46\,a^2x^2-12i\,ax-2}{6\left(a^2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] 12*a^3*arctan(a*x) + 6*I*a^3*log(a^2*x^2 + 1) - 12*I*a^3*log(x) + 1/6*(72*a
^4*x^4 - 36*I*a^3*x^3 + 46*a^2*x^2 - 12*I*a*x - 2)/(a^2*x^5 + x^3)
```

mupad [B] time = 0.13, size = 55, normalized size = 0.89

$$24\,a^3\,\mathrm{atan}\left(2\,a\,x+1\mathrm{i}\right)+\frac{\frac{5\,x}{3}+12\,a^2\,x^3+a\,x^2\,6\mathrm{i}-\frac{1\mathrm{i}}{3\,a}}{x^4+\frac{x^3\,1\mathrm{i}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(x^4*(a^2*x^2 + 1)^2),x)
```

```
[Out] 24*a^3*atan(2*a*x + 1i) + ((5*x)/3 + a*x^2*6i - 1i/(3*a) + 12*a^2*x^3)/(x^4
 + (x^3*1i)/a)
```

sympy [**A**] time = 0.38, size = 71, normalized size = 1.15

$$12a^3\left(-i\log\left(24a^4x\right)+i\log\left(24a^4x+24ia^3\right)\right)+\frac{-36a^3x^3-18ia^2x^2-5ax+i}{-3ax^4-3ix^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**4,x)
```

```
[Out] 12*a**3*(-I*log(24*a**4*x) + I*log(24*a**4*x + 24*I*a**3)) + (-36*a**3*x**3
 - 18*I*a**2*x**2 - 5*a*x + I)/(-3*a*x**4 - 3*I*x**3)
```

3.35 $\int e^{-i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=90

$$-\frac{3i\sinh^{-1}(ax)}{8a^4}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{(16-9iax)\sqrt{a^2x^2+1}}{24a^4}$$

```
[Out] -3/8*I*arcsinh(a*x)/a^4+1/3*x^2*(a^2*x^2+1)^(1/2)/a^2-1/4*I*x^3*(a^2*x^2+1)
^(1/2)/a-1/24*(16-9*I*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5060, 833, 780, 215}

$$-\frac{ix^3\sqrt{a^2x^2+1}}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{(16-9iax)\sqrt{a^2x^2+1}}{24a^4}-\frac{3i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 -
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) - (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1-iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{ix^3\sqrt{1+a^2x^2}}{4a}+\frac{\int \frac{x^2(3ia+4a^2x)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}+\frac{\int \frac{x(-8a^2+9ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4}-\frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4}-\frac{3i\sinh^{-1}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.62

$$\frac{\sqrt{a^2x^2+1}\left(-6ia^3x^3+8a^2x^2+9iax-16\right)-9i\sinh^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(I*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-16 + (9*I)*a*x + 8*a^2*x^2 - (6*I)*a^3*x^3) - (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

fricas [A] time = 0.46, size = 59, normalized size = 0.66

$$\frac{\left(-6i\,a^3x^3+8\,a^2x^2+9i\,ax-16\right)\sqrt{a^2x^2+1}+9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((-6*I*a^3*x^3 + 8*a^2*x^2 + 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) + 9*I*log
(-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.19, size = 187, normalized size = 2.08

$$-\frac{ix\left(a^2x^2+1\right)^{\frac{3}{2}}}{4a^3}+\frac{5ix\sqrt{a^2x^2+1}}{8a^3}+\frac{5i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}+\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3a^4}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^4}-\frac{i\ln\Big(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*I/a^3*x*(a^2*x^2+1)^(3/2)+5/8*I/a^3*x*(a^2*x^2+1)^(1/2)+5/8*I/a^3*ln(x
*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+1/3/a^4*(a^2*x^2+1)^(3/2)-1
/a^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)-I/a^3*ln((I*a+(x-I/a)*a^2)/(a^2)^(
1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 76, normalized size = 0.84

$$-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3}+\frac{5i\,\sqrt{a^2x^2+1}\,x}{8\,a^3}+\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^4}-\frac{3i\,\operatorname{arsinh}\left(ax\right)}{8\,a^4}-\frac{\sqrt{a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*I*(a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*I*sqrt(a^2*x^2 + 1)*x/a^3 + 1/3*(a^2
*x^2 + 1)^(3/2)/a^4 - 3/8*I*arcsinh(a*x)/a^4 - sqrt(a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.06, size = 85, normalized size = 0.94

$$-\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,3\mathrm{i}}{8\,a^3\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{2}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^2\,x^2}{3\,{\left(a^2\right)}^{3/2}}+\frac{x^3\,{\left(a^2\right)}^{3/2}\,1\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] - (asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/(
3*(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) + (x^3*(a^2)^(3/2)*1i)/(4*a^3) -
 (x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{x^3\sqrt{a^2x^2+1}}{ax-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -I*Integral(x**3*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

3.36 $\int e^{-i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=75

$$-\frac{\sinh^{-1}(ax)}{2a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{i\sqrt{a^2x^2+1}}{a^3}$$

```
[Out] -1/3*I*(a^2*x^2+1)^(3/2)/a^3-1/2*arcsinh(a*x)/a^3+I*(a^2*x^2+1)^(1/2)/a^3+1
/2*x*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 797, 641, 195, 215}

$$-\frac{i\left(a^2x^2+1\right)^{3/2}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}+\frac{i\sqrt{a^2x^2+1}}{a^3}-\frac{\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/3)*(1 + a^2
*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1-iax)\sqrt{1+a^2x^2}\,dx}{a^2}$$
$$= \frac{i\sqrt{1+a^2x^2}}{a^3} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2}$$
$$= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}$$
$$= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i\left(1+a^2x^2\right)^{3/2}}{3a^3} - \frac{\sinh^{-1}(ax)}{2a^3}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.61

$$\frac{-3\sinh^{-1}(ax) + \left(-2ia^2x^2 + 3ax + 4i\right)\sqrt{a^2x^2+1}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((4*I + 3*a*x - (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

fricas [A] time = 0.57, size = 51, normalized size = 0.68

$$\frac{\sqrt{a^2x^2+1}\left(-2i\,a^2x^2 + 3\,ax + 4i\right) + 3\,\log\left(-ax + \sqrt{a^2x^2+1}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(-2*I*a^2*x^2 + 3*a*x + 4*I) + 3*log(-a*x + sqrt(a^2
*x^2 + 1)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.16, size = 168, normalized size = 2.24

$$-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}+\frac{\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}+\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)}}{a^3}-\frac{\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2 \cdots}\right)}{a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*I*(a^2*x^2+1)^(3/2)/a^3+1/2*x*(a^2*x^2+1)^(1/2)/a^2+1/2/a^2*ln(x*a^2/(
a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+I/a^3*((x-I/a)^2*a^2+2*I*a*(x-I/a
))^(1/2)-1/a^2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a
))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.44, size = 59, normalized size = 0.79

$$\frac{\sqrt{a^2x^2+1}\,x}{2\,a^2}-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}-\frac{\operatorname{arsinh}(ax)}{2\,a^3}+\frac{i\,\sqrt{a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 - 1/2*arcsinh(a
*x)/a^3 + I*sqrt(a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.42, size = 71, normalized size = 0.95

$$\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}+\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a*2i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2))
 + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a^2
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{x^2\sqrt{a^2x^2+1}}{ax-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

3.37 $\int e^{-i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a^2x^2+1}\,(2-iax)}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] 1/2*I*arcsinh(a*x)/a^2+1/2*(2-I*a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5060, 780, 215}

$$\frac{\sqrt{a^2x^2+1}\,(2-iax)}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/2)*ArcSinh[a*x])/a^2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1-iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\sinh^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.90

$$\frac{\sqrt{a^2x^2+1}\,(2-iax)+i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2] + I*ArcSinh[a*x])/(2*a^2)
```

fricas [A] time = 0.54, size = 43, normalized size = 1.02

$$\frac{\sqrt{a^2x^2+1}\,(-i\,ax+2)-i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(-I*a*x + 2) - I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

giac [A] time = 0.12, size = 54, normalized size = 1.29

$$-\frac{1}{2}\sqrt{a^2x^2+1}\left(\frac{ix}{a}-\frac{2}{a^2}\right)-\frac{i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 + 1)*(i*x/a - 2/a^2) - 1/2*i*log(-x*abs(a) + sqrt(a^2*x^2
 + 1))/(a*abs(a))
```

maple [B] time = 0.16, size = 152, normalized size = 3.62

$$-\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{i\ln\left(\frac{xa^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}+\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2}+\frac{i\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*I/a*x*(a^2*x^2+1)^(1/2)-1/2*I/a*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)
)/(a^2)^(1/2)+1/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+I/a*ln((I*a+(x-I/a)
*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 42, normalized size = 1.00

$$-\frac{i\sqrt{a^2x^2+1}\,x}{2\,a}+\frac{i\,\mathrm{arsinh}\,(ax)}{2\,a^2}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(a^2*x^2 + 1)*x/a + 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a
^2
```

mupad [B] time = 0.40, size = 51, normalized size = 1.21

$$\frac{\left(\frac{1}{\sqrt{a^2}}-\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\sqrt{a^2\,x^2+1}+\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] ((1/(a^2)^(1/2) - (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) + (asinh(x*
(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{x\sqrt{a^2x^2+1}}{ax-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -I*Integral(x*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

3.38 $\int e^{-i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh^{-1}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

```
[Out] arcsinh(a*x)/a-I*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5059, 641, 215}

$$\frac{\sinh^{-1}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-i\tan^{-1}(ax)}\,dx&=\int\frac{1-iax}{\sqrt{1+a^2x^2}}\,dx\\&=-\frac{i\sqrt{1+a^2x^2}}{a}+\int\frac{1}{\sqrt{1+a^2x^2}}\,dx\\&=-\frac{i\sqrt{1+a^2x^2}}{a}+\frac{\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{\sinh^{-1}(ax)-i\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

fricas [A] time = 0.44, size = 37, normalized size = 1.28

$$\frac{-i\sqrt{a^2x^2+1}-\log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

giac [A] time = 0.12, size = 42, normalized size = 1.45

$$-\frac{\sqrt{a^2x^2+1}\,i}{a}-\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(a^2*x^2 + 1)*i/a - log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

maple [B] time = 0.10, size = 97, normalized size = 3.34

$$-\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a}+\frac{\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
[Out] -I/a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+(
(x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 25, normalized size = 0.86

$$\frac{\operatorname{arsinh}(ax)}{a}-\frac{i\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)/a - I*sqrt(a^2*x^2 + 1)/a
```

mupad [B] time = 0.40, size = 32, normalized size = 1.10

$$\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1),x)
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^2x^2+1}}{ax-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

3.39 $\int \frac{e^{-i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=25

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

```
[Out] -I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 844, 215, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{1-iax}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right) + \int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -i\sinh^{-1}(ax) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -i\sinh^{-1}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2} \\
&= -i\sinh^{-1}(ax) - \tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.16

$$-\log\left(\sqrt{a^2x^2+1}+1\right) - i\sinh^{-1}(ax) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.46, size = 58, normalized size = 2.32

$$-\log\left(-ax+\sqrt{a^2x^2+1}+1\right) + i\log\left(-ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

giac [B] time = 0.14, size = 68, normalized size = 2.72

$$\frac{ai\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} - \log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + \log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*i*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a^
2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

maple [B] time = 0.16, size = 121, normalized size = 4.84

$$\sqrt{a^2x^2+1} - \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) - \sqrt{\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)} - \frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}} + \sqrt{\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right.}\right.}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))-((x-I/a)^2*a^2+2*I*a*(x-I/a)
)^(1/2)-I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^
(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.47, size = 26, normalized size = 1.04

$$-i\,a\left(\frac{\operatorname{arsinh}(ax)}{a}-\frac{i\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -I*a*(arcsinh(a*x)/a - I*arcsinh(1/(a*abs(x)))/a)
```

mupad [B] time = 0.04, size = 32, normalized size = 1.28

$$-\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)-\frac{a\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(x*(a*x*1i + 1)),x)
```

```
[Out] - atanh((a^2*x^2 + 1)^(1/2)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^2x^2+1}}{ax^2-ix}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**2 - I*x), x)
```

3.40 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 807, 266, 63, 208}

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(ax)}}{x^2}\,dx = \int \frac{1-iax}{x^2\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1+a^2x^2}}{x} - (ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{1}{2}(ia)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a}$$
$$= -\frac{\sqrt{1+a^2x^2}}{x} + ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.24

$$-\frac{\sqrt{a^2x^2+1}}{x} + ia\log\left(\sqrt{a^2x^2+1}+1\right) - ia\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*Log[x] + I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.49, size = 66, normalized size = 1.74

$$\frac{i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right) - ax - \sqrt{a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*a*x*log(-a*x + sqrt(a^2*x^2 +
1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.17, size = 194, normalized size = 5.11

$$ia\,\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) - ia\sqrt{a^2x^2+1} - \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{x} + a^2x\sqrt{a^2x^2+1} + \frac{a^2\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} + ia\sqrt{\left(x-\frac{i}{a}\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] I*a*arctanh(1/(a^2*x^2+1)^(1/2))-I*a*(a^2*x^2+1)^(1/2)-1/x*(a^2*x^2+1)^(3/2
)+a^2*x*(a^2*x^2+1)^(1/2)+a^2*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)
^(1/2)+I*a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)-a^2*ln((I*a+(x-I/a)*a^2)/(a^
2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^2), x)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.87

$$-\frac{\sqrt{a^2\,x^2+1}}{x}+a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^2*(a*x*1i + 1)),x)
```

```
[Out] a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2x^2+1}}{ax^3-ix^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**3 - I*x**2), x)
```

3.41 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=63

$$\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 1/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2+I*a*(a^2*x^2+1
)^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-iax}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{1}{2}\int \frac{2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.90

$$\frac{1}{2}\left(\frac{(-1+2iax)\sqrt{a^2x^2+1}}{x^2} + a^2\log\left(\sqrt{a^2x^2+1}+1\right) + a^2(-\log(x))\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^3),x]
```

```
[Out] (((-1 + (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

fricas [A] time = 0.44, size = 83, normalized size = 1.32

$$\frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right) + 2i\,a^2x^2 + \sqrt{a^2x^2+1}\,(2i\,ax-1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(2*I*a*x - 1))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 219, normalized size = 3.48

$$\frac{a^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}-\frac{a^2\sqrt{a^2x^2+1}}{2}+\frac{ia\left(a^2x^2+1\right)^{\frac{3}{2}}}{x}-ia^3x\sqrt{a^2x^2+1}-\frac{ia^3\ln\left(\frac{x a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+a^2\sqrt{\left(x-\frac{i}{a}\right)^2 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-1/2*a^2*(a^2*x^2+1)^(1/2)+I*a/x*(a^2*x
^2+1)^(3/2)-I*a^3*x*(a^2*x^2+1)^(1/2)-I*a^3*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1
)^(1/2))/(a^2)^(1/2)+a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+I*a^3*ln((I*a+
(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)-1
/2/x^2*(a^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^3), x)
```

mupad [B] time = 0.04, size = 52, normalized size = 0.83

$$\frac{a^2\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}+\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^3*(a*x*1i + 1)),x)
```

```
[Out] (a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) + (a*(a^2*
x^2 + 1)^(1/2)*1i)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^2x^2+1}}{ax^4-ix^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**4 - I*x**3), x)
```

3.42 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=90

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}+\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3+1/2*I*a*(a^
2*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$\frac{2a^2\sqrt{a^2x^2+1}}{3x}+\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) - (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-iax}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{1}{3}\int \frac{3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{1}{6}\int \frac{-4a^2+3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{4}\left(ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.78

$$\frac{1}{6}\left(3ia^3\log(x) + \frac{\sqrt{a^2x^2+1}\left(4a^2x^2+3iax-2\right)}{x^3} - 3ia^3\log\left(\sqrt{a^2x^2+1}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2 + (3*I)*a*x + 4*a^2*x^2))/x^3 + (3*I)*a^3*Log[x] -
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

fricas [A] time = 0.45, size = 92, normalized size = 1.02

$$\frac{-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2+3i\,ax-2\right)\sqrt{a^2x^2+}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(-3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + 3*I*a^3*x^3*log(-a*x
+ sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 + 3*I*a*x - 2)*sqrt(a^2*x
^2 + 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.17, size = 237, normalized size = 2.63

$$-\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3x^{3}}-\frac{ia^{3}\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{2}+\frac{ia^{3}\sqrt{a^{2}x^{2}+1}}{2}+\frac{a^{2}\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{x}-a^{4}x\sqrt{a^{2}x^{2}+1}-\frac{a^{4}\ln\left(\frac{x\,a^{2}}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/3/x^3*(a^2*x^2+1)^(3/2)-1/2*I*a^3*arctanh(1/(a^2*x^2+1)^(1/2))+1/2*I*a^3
*(a^2*x^2+1)^(1/2)+a^2/x*(a^2*x^2+1)^(3/2)-a^4*x*(a^2*x^2+1)^(1/2)-a^4*ln(x
*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-I*a^3*((x-I/a)^2*a^2+2*I*a*
(x-I/a))^(1/2)+a^4*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x
-I/a))^(1/2))/(a^2)^(1/2)+1/2*I*a/x^2*(a^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}+1}}{(i\,ax+1)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^4), x)
```

mupad [B] time = 0.04, size = 74, normalized size = 0.82

$$\frac{2\,a^{2}\,\sqrt{a^{2}\,x^{2}+1}}{3\,x}-\frac{\sqrt{a^{2}\,x^{2}+1}}{3\,x^{3}}-\frac{a^{3}\,\mathrm{atan}\left(\sqrt{a^{2}\,x^{2}+1}\;1\mathrm{i}\right)}{2}+\frac{a\,\sqrt{a^{2}\,x^{2}+1}\;1\mathrm{i}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^4*(a*x*1i + 1)),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*1i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a^3*ata
n((a^2*x^2 + 1)^(1/2)*1i))/2 + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^{2}x^{2}+1}}{ax^{5}-ix^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**5 - I*x**4), x)
```

3.43 $\int \frac{e^{-i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=113

$$\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{2ia^3\sqrt{a^2x^2+1}}{3x}$$

```
[Out] -3/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4+1/3*I*a*(a^2*
x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+1)^(1/2)/x^2-2/3*I*a^3*(a^2*x^2+1)^(1/2)/
x
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5060, 835, 807, 266, 63, 208}

$$-\frac{2ia^3\sqrt{a^2x^2+1}}{3x}+\frac{3a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{3x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) - (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{x^5}\,dx &= \int \frac{1-iax}{x^5\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{-9a^2+8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{-16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1+a^2x^2}} \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\mathrm{Subst}\left(\int \frac{}{x\sqrt{1}}\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}}\right. \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1+a^2x^2}\right.
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.67

$$\frac{1}{24}\left(9a^4\log(x) - 9a^4\log\left(\sqrt{a^2x^2+1}+1\right) + \frac{\sqrt{a^2x^2+1}\left(-16ia^3x^3+9a^2x^2+8iax-6\right)}{x^4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-6 + (8*I)*a*x + 9*a^2*x^2 - (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

fricas [A] time = 0.46, size = 101, normalized size = 0.89

$$-\frac{9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - 9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}-1\right) + 16i\,a^4x^4 - \left(-16i\,a^3x^3+9\,a^2x^2+8i\right.}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) + 16*I*a^4*x^4 - (-16*I*a^3*x^3 + 9*a^2*x^2 + 8*I*a*x
 - 6)*sqrt(a^2*x^2 + 1))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 259, normalized size = 2.29

$$\frac{ia\left(a^2x^2+1\right)^{\frac{3}{2}}}{3x^3}-\frac{3a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{8}+\frac{3a^4\sqrt{a^2x^2+1}}{8}-\frac{ia^3\left(a^2x^2+1\right)^{\frac{3}{2}}}{x}+ia^5x\sqrt{a^2x^2+1}+\frac{ia^5\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+}\right.}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] 1/3*I*a/x^3*(a^2*x^2+1)^(3/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))+3/8*a^4*
(a^2*x^2+1)^(1/2)-I*a^3/x*(a^2*x^2+1)^(3/2)+I*a^5*x*(a^2*x^2+1)^(1/2)+I*a^5
*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-a^4*((x-I/a)^2*a^2+2*I
*a*(x-I/a))^(1/2)-I*a^5*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I
*a*(x-I/a))^(1/2))/(a^2)^(1/2)-1/4/x^4*(a^2*x^2+1)^(3/2)+5/8*a^2/x^2*(a^2*x
^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^5), x)
```

mupad [B] time = 0.03, size = 95, normalized size = 0.84

$$\frac{a^4\operatorname{atan}\left(\sqrt{a^2\,x^2+1}\;1\mathrm{i}\right)\,3\mathrm{i}}{8}-\frac{\sqrt{a^2\,x^2+1}}{4\,x^4}+\frac{a\,\sqrt{a^2\,x^2+1}\;1\mathrm{i}}{3\,x^3}+\frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2}-\frac{a^3\,\sqrt{a^2\,x^2+1}\;2\mathrm{i}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^5*(a*x*1i + 1)),x)
```

```
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (a*
(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (a^
3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2x^2+1}}{ax^6-ix^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**6 - I*x**5), x)
```

3.44 $\int e^{-2i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=49

$$-\frac{2\log(-ax+i)}{a^4}+\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[Out] 2*I*x/a^3+x^2/a^2-2/3*I*x^3/a-1/4*x^4-2*ln(I-a*x)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log(-ax+i)}{a^4}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((2*I)*ArcTan[a*x]),x]
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^3\,dx&=\int\frac{x^3(1-iax)}{1+iax}\,dx\\&=\int\left(\frac{2i}{a^3}+\frac{2x}{a^2}-\frac{2ix^2}{a}-x^3-\frac{2}{a^3(-i+ax)}\right)dx\\&=\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$-\frac{2\log(-ax+i)}{a^4}+\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a*x]),x]
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

fricas [A] time = 0.41, size = 46, normalized size = 0.94

$$-\frac{3\,a^4x^4+8i\,a^3x^3-12\,a^2x^2-24i\,ax+24\,\log\left(\frac{ax-i}{a}\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*I*a^3*x^3 - 12*a^2*x^2 - 24*I*a*x + 24*log((a*x - I)/a
))/a^4
```

giac [B] time = 0.12, size = 80, normalized size = 1.63

$$-\frac{(aix+1)^4\left(\frac{20\,i^2}{aix+1}-\frac{84\,i^4}{(aix+1)^3}-\frac{54\,i^2}{(aix+1)^2}+3\right)}{12\,a^4i^4}+\frac{2\,\log\left(\frac{1}{\sqrt{a^2x^2+1}\,|a|}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*(a*i*x + 1)^4*(20*i^2/(a*i*x + 1) - 84*i^4/(a*i*x + 1)^3 - 54*i^2/(a*
i*x + 1)^2 + 3)/(a^4*i^4) + 2*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^4
```

maple [A] time = 0.05, size = 55, normalized size = 1.12

$$-\frac{x^4}{4}-\frac{2ix^3}{3a}+\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{\ln\left(a^2x^2+1\right)}{a^4}-\frac{2i\arctan\left(ax\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -1/4*x^4-2/3*I*x^3/a+x^2/a^2+2*I*x/a^3-1/a^4*ln(a^2*x^2+1)-2*I/a^4*arctan(a
*x)
```

maxima [A] time = 0.32, size = 44, normalized size = 0.90

$$-\frac{i\left(-3i\,a^3x^4+8\,a^2x^3+12i\,ax^2-24\,x\right)}{12\,a^3}-\frac{2\,\log\left(i\,ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/12*I*(-3*I*a^3*x^4 + 8*a^2*x^3 + 12*I*a*x^2 - 24*x)/a^3 - 2*log(I*a*x +
1)/a^4
```

mupad [B] time = 0.06, size = 43, normalized size = 0.88

$$\frac{x^2}{a^2}-\frac{x^4}{4}-\frac{2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)}{a^4}+\frac{x\,2\mathrm{i}}{a^3}-\frac{x^3\,2\mathrm{i}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

```
[Out] (x*2i)/a^3 - (2*log(x - 1i/a))/a^4 - x^4/4 - (x^3*2i)/(3*a) + x^2/a^2
```

sympy [A] time = 0.12, size = 42, normalized size = 0.86

$$-\frac{x^4}{4}-\frac{2ix^3}{3a}+\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{2\log\left(iax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**4/4 - 2*I*x**3/(3*a) + x**2/a**2 + 2*I*x/a**3 - 2*log(I*a*x + 1)/a**4
```

3.45 $\int e^{-2i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=40

$$\frac{2i\log(-ax+i)}{a^3}+\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[Out] 2*x/a^2-I*x^2/a-1/3*x^3+2*I*ln(I-a*x)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{5062, 77\}$

$$\frac{2x}{a^2}+\frac{2i\log(-ax+i)}{a^3}-\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1-iax)}{1+iax}\,dx\\&=\int\left(\frac{2}{a^2}-\frac{2ix}{a}-x^2+\frac{2i}{a^2(-i+ax)}\right)dx\\&=\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2i\log(-ax+i)}{a^3}+\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

fricas [A] time = 0.47, size = 37, normalized size = 0.92

$$-\frac{a^3x^3+3i\,a^2x^2-6\,ax-6i\,\log\left(\frac{ax-i}{a}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 + 3*I*a^2*x^2 - 6*a*x - 6*I*log((a*x - I)/a))/a^3
```

giac [B] time = 0.12, size = 68, normalized size = 1.70

$$-\frac{2\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}\,|a|}\right)}{a^3}-\frac{(aix+1)^3\left(\frac{6\,i^2}{aix+1}-\frac{15\,i^2}{(aix+1)^2}+1\right)}{3\,a^3i^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - 1/3*(a*i*x + 1)^3*(6*i^2/(a*i*
x + 1) - 15*i^2/(a*i*x + 1)^2 + 1)/(a^3*i^3)
```

maple [A] time = 0.05, size = 47, normalized size = 1.18

$$-\frac{x^3}{3}-\frac{ix^2}{a}+\frac{2x}{a^2}+\frac{i\ln\left(a^2x^2+1\right)}{a^3}-\frac{2\arctan\left(ax\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -1/3*x^3-I*x^2/a+2*x/a^2+I/a^3*ln(a^2*x^2+1)-2/a^3*arctan(a*x)
```

maxima [A] time = 0.32, size = 35, normalized size = 0.88

$$-\frac{a^2x^3+3i\,ax^2-6\,x}{3\,a^2}+\frac{2i\,\log\left(i\,ax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*x^3 + 3*I*a*x^2 - 6*x)/a^2 + 2*I*log(I*a*x + 1)/a^3
```

mupad [B] time = 0.41, size = 36, normalized size = 0.90

$$\frac{\ln\left(x-\frac{1\mathrm{i}}{a}\right)2\mathrm{i}}{a^3}+\frac{2\,x}{a^2}-\frac{x^3}{3}-\frac{x^2\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

```
[Out] (log(x - 1i/a)*2i)/a^3 + (2*x)/a^2 - x^3/3 - (x^2*1i)/a
```

sympy [A] time = 0.13, size = 32, normalized size = 0.80

$$-\frac{x^3}{3}-\frac{ix^2}{a}+\frac{2x}{a^2}+\frac{2i\log\left(iax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**3/3 - I*x**2/a + 2*x/a**2 + 2*I*log(I*a*x + 1)/a**3
```

3.46 $\int e^{-2i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=30

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

```
[Out] -2*I*x/a-1/2*x^2+2*ln(I-a*x)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 77}

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1-iax)}{1+iax}\,dx\\
&= \int \left(-\frac{2i}{a}-x+\frac{2}{a(-i+ax)}\right)dx\\
&= -\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

fricas [A] time = 0.40, size = 29, normalized size = 0.97

$$-\frac{a^2x^2+4i\,ax-4\,\log\left(\frac{ax-i}{a}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 + 4*I*a*x - 4*log((a*x - I)/a))/a^2
```

giac [B] time = 0.13, size = 58, normalized size = 1.93

$$\frac{i\left(\frac{4\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}\,|a|}\right)}{a}+\frac{(aix+1)^2\left(i-\frac{6\,i}{aix+1}\right)}{ai^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] 1/2*i*(4*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a + (a*i*x + 1)^2*(i - 6*i/(a*
i*x + 1))/(a*i^2))/a
```

maple [A] time = 0.05, size = 38, normalized size = 1.27

$$-\frac{x^2}{2}-\frac{2ix}{a}+\frac{\ln\left(a^2x^2+1\right)}{a^2}+\frac{2i\arctan\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)^2*(a^2*x^2+1),x)
[Out] -1/2*x^2-2*I*x/a+1/a^2*ln(a^2*x^2+1)+2*I/a^2*arctan(a*x)
```

maxima [A] time = 0.33, size = 28, normalized size = 0.93

$$\frac{i\left(i\,ax^2-4\,x\right)}{2\,a}+\frac{2\,\log\left(i\,ax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] 1/2*I*(I*a*x^2 - 4*x)/a + 2*log(I*a*x + 1)/a^2
```

mupad [B] time = 0.42, size = 27, normalized size = 0.90

$$\frac{2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)}{a^2}-\frac{x^2}{2}-\frac{x\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
[Out] (2*log(x - 1i/a))/a^2 - (x*2i)/a - x^2/2
```

sympy [A] time = 0.11, size = 24, normalized size = 0.80

$$-\frac{x^2}{2}-\frac{2ix}{a}+\frac{2\log\left(iax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)**2*(a**2*x**2+1),x)
[Out] -x**2/2 - 2*I*x/a + 2*log(I*a*x + 1)/a**2
```

3.47 $\int e^{-2i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=20

$$-x-\frac{2i\log(-ax+i)}{a}$$

```
[Out] -x-2*I*ln(I-a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5061, 43}

$$-x-\frac{2i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-2*I)*ArcTan[a*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2i\tan^{-1}(ax)}\,dx &= \int \frac{1-iax}{1+iax}\,dx\\
&= \int \left(-1-\frac{2i}{-i+ax}\right)dx\\
&= -x-\frac{2i\log(i-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.50

$$-\frac{i\log\left(a^2x^2+1\right)}{a}+\frac{2\tan^{-1}(ax)}{a}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-2*I)*ArcTan[a*x]),x]
```

```
[Out] -x + (2*ArcTan[a*x])/a - (I*Log[1 + a^2*x^2])/a
```

fricas [A] time = 0.44, size = 21, normalized size = 1.05

$$-\frac{ax+2i\log\left(\frac{ax-i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x + 2*I*log((a*x - I)/a))/a
```

giac [B] time = 0.13, size = 67, normalized size = 3.35

$$a^2\left(\frac{(aix+1)i}{a^3}+\frac{2\,i\log\left(\frac{1}{\sqrt{a^2x^2+1}\,|a|}\right)}{a^3}-\frac{i}{(aix+1)a^3}\right)+\frac{i}{(aix+1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] a^2*((a*i*x + 1)*i/a^3 + 2*i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - i/((a*
i*x + 1)*a^3)) + i/((a*i*x + 1)*a)
```

maple [A] time = 0.05, size = 30, normalized size = 1.50

$$-x-\frac{i\ln\left(a^2x^2+1\right)}{a}+\frac{2\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1),x)
```

```
[Out] -x-I/a*ln(a^2*x^2+1)+2*arctan(a*x)/a
```

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$-x-\frac{2i\,\log\left(i\,ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x - 2*I*log(I*a*x + 1)/a
```

mupad [B] time = 0.41, size = 19, normalized size = 0.95

$$-x-\frac{\ln\left(x-\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(a*x*1i + 1)^2,x)
```

```
[Out] - x - (log(x - 1i/a)*2i)/a
```

sympy [A] time = 0.11, size = 15, normalized size = 0.75

$$-x-\frac{2i\log\left(iax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x - 2*I*log(I*a*x + 1)/a
```

3.48 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x} dx$

Optimal. Leaf size=14

$$\log(x) - 2\log(-ax + i)$$

[Out] ln(x)-2*ln(I-a*x)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 72}

$$\log(x) - 2\log(-ax + i)$$

Antiderivative was successfully verified.

[In] Int[1/(E^((2*I)*ArcTan[a*x])*x),x]

[Out] Log[x] - 2*Log[I - a*x]

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{x} dx &= \int \frac{1 - iax}{x(1 + iax)} dx \\
&= \int \left(\frac{1}{x} - \frac{2a}{-i + ax}\right) dx \\
&= \log(x) - 2\log(i - ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\log(x) - 2\log(-ax + i)$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x),x]

[Out] Log[x] - 2*Log[I - a*x]

fricas [A] time = 0.41, size = 15, normalized size = 1.07

$$\log(x) - 2 \log\left(\frac{ax - i}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="fricas")

```
[Out] log(x) - 2*log((a*x - I)/a)
```

giac [**B**] time = 0.13, size = 47, normalized size = 3.36

$$-ai\left(\frac{i\log\left(-i+\frac{i}{aix+1}\right)}{a}+\frac{i\log\left(\frac{1}{\sqrt{a^2x^2+1}\,|a|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="giac")
[Out] -a*i*(i*log(-i + i/(a*i*x + 1))/a + i*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a)
```

maple [**A**] time = 0.06, size = 23, normalized size = 1.64

$$\ln(x)-\ln\left(a^2x^2+1\right)-2i\arctan\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x)
[Out] ln(x)-ln(a^2*x^2+1)-2*I*arctan(a*x)
```

maxima [**A**] time = 0.32, size = 12, normalized size = 0.86

$$-2\log\left(i\,ax+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="maxima")
[Out] -2*log(I*a*x + 1) + log(x)
```

mupad [**B**] time = 0.45, size = 14, normalized size = 1.00

$$\ln(x)-2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(x*(a*x*1i + 1)^2),x)
[Out] log(x) - 2*log(x - 1i/a)
```

sympy [**A**] time = 0.15, size = 17, normalized size = 1.21

$$\log\left(3ax\right)-2\log\left(3ax-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x,x)
[Out] log(3*a*x) - 2*log(3*a*x - 3*I)
```

3.49 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=27

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

```
[Out] -1/x-2*I*a*ln(x)+2*I*a*ln(I-a*x)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-iax}{x^2(1+iax)}\,dx\\ &= \int \left(\frac{1}{x^2}-\frac{2ia}{x}+\frac{2ia^2}{-i+ax}\right)dx\\ &= -\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

fricas [A] time = 0.42, size = 26, normalized size = 0.96

$$\frac{-2i\,ax\log(x) + 2i\,ax\log\left(\frac{ax-i}{a}\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] (-2*I*a*x*log(x) + 2*I*a*x*log((a*x - I)/a) - 1)/x
```

giac [A] time = 0.13, size = 37, normalized size = 1.37

$$-2\,ai\log\left(-i + \frac{i}{aix+1}\right) + \frac{a}{i - \frac{i}{aix+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*a*i*log(-i + i/(a*i*x + 1)) + a/(i - i/(a*i*x + 1))
```

maple [A] time = 0.07, size = 34, normalized size = 1.26

$$-\frac{1}{x} - 2ia\ln(x) - 2a\arctan(ax) + ia\ln\left(a^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x)
[Out] -1/x-2*I*a*ln(x)-2*a*arctan(a*x)+I*a*ln(a^2*x^2+1)
```

maxima [A] time = 0.33, size = 34, normalized size = 1.26

$$2i\,a\log(i\,ax+1) - 2i\,a\log(x) - \frac{ax-i}{ax^2-i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="maxima")
[Out] 2*I*a*log(I*a*x + 1) - 2*I*a*log(x) - (a*x - I)/(a*x^2 - I*x)
```

mupad [B] time = 0.42, size = 17, normalized size = 0.63

$$-4\,a\,\mathrm{atan}\left(2\,a\,x - \mathrm{i}\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(x^2*(a*x*1i + 1)^2),x)
[Out] - 4*a*atan(2*a*x - 1i) - 1/x
```

sympy [A] time = 0.16, size = 32, normalized size = 1.19

$$-2a\left(i\log\left(4a^2x\right) - i\log\left(4a^2x - 4ia\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**2,x)
[Out] -2*a*(I*log(4*a**2*x) - I*log(4*a**2*x - 4*I*a)) - 1/x
```

3.50 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=37

$$-2a^2\log(x)+2a^2\log(-ax+i)+\frac{2ia}{x}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I-a*x)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5062, 77}

$$-2a^2\log(x)+2a^2\log(-ax+i)+\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/(2*x^2) + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-iax}{x^3(1+iax)}\,dx\\
&= \int \left(\frac{1}{x^3}-\frac{2ia}{x^2}-\frac{2a^2}{x}+\frac{2a^3}{-i+ax}\right)dx\\
&= -\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i-ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$-2a^2\log(x)+2a^2\log(-ax+i)+\frac{2ia}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/2*1/x^2 + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

fricas [**A**] time = 0.40, size = 39, normalized size = 1.05

$$-\frac{4\,a^2x^2\log(x) - 4\,a^2x^2\log\left(\frac{ax-i}{a}\right) - 4i\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x - I)/a) - 4*I*a*x + 1)/x^2
```

giac [**B**] time = 0.13, size = 63, normalized size = 1.70

$$2\,a^2i^2\log\left(\frac{i^2}{aix+1}+1\right)+\frac{\frac{6\,a^2i^2}{aix+1}+5\,a^2}{2\left(i-\frac{i}{aix+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*i^2*log(i^2/(a*i*x + 1) + 1) + 1/2*(6*a^2*i^2/(a*i*x + 1) + 5*a^2)/(i
 - i/(a*i*x + 1))^2
```

maple [**A**] time = 0.06, size = 45, normalized size = 1.22

$$-\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\ln(x)+2ia^2\arctan(ax)+a^2\ln\left(a^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*I*a^2*arctan(a*x)+a^2*ln(a^2*x^2+1)
```

maxima [**A**] time = 0.33, size = 50, normalized size = 1.35

$$2\,a^2\log(i\,ax+1) - 2\,a^2\log(x) - \frac{4\,a^2x^2 - 3i\,ax + 1}{2i\,ax^3 + 2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] 2*a^2*log(I*a*x + 1) - 2*a^2*log(x) - (4*a^2*x^2 - 3*I*a*x + 1)/(2*I*a*x^3
+ 2*x^2)
```

mupad [**B**] time = 0.07, size = 26, normalized size = 0.70

$$a^2\,\mathrm{atan}\left(2\,a\,x-\mathrm{i}\right)\,4\mathrm{i}+\frac{-\frac{1}{2}+a\,x\,2\mathrm{i}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(x^3*(a*x*1i + 1)^2),x)
```

```
[Out] a^2*atan(2*a*x - 1i)*4i + (a*x*2i - 1/2)/x^2
```

sympy [**A**] time = 0.18, size = 42, normalized size = 1.14

$$-2a^2\left(\log\left(4a^3x\right)-\log\left(4a^3x-4ia^2\right)\right)-\frac{-4iax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**3,x)
```

```
[Out] -2*a**2*(log(4*a**3*x) - log(4*a**3*x - 4*I*a**2)) - (-4*I*a*x + 1)/(2*x**2
)
```

3.51 $\int \frac{e^{-2i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=49

$$2ia^3\log(x) - 2ia^3\log(-ax+i) + \frac{2a^2}{x} + \frac{ia}{x^2} - \frac{1}{3x^3}$$

```
[Out] -1/3/x^3+I*a/x^2+2*a^2/x+2*I*a^3*ln(x)-2*I*a^3*ln(I-a*x)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{5062, 77\}$

$$\frac{2a^2}{x} + 2ia^3\log(x) - 2ia^3\log(-ax+i) + \frac{ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{1-iax}{x^4(1+iax)}\,dx\\
&= \int \left(\frac{1}{x^4} - \frac{2ia}{x^3} - \frac{2a^2}{x^2} + \frac{2ia^3}{x} - \frac{2ia^4}{-i+ax}\right)dx\\
&= -\frac{1}{3x^3} + \frac{ia}{x^2} + \frac{2a^2}{x} + 2ia^3\log(x) - 2ia^3\log(i-ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$2ia^3\log(x) - 2ia^3\log(-ax+i) + \frac{2a^2}{x} + \frac{ia}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*1/x^3 + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

fricas [A] time = 0.42, size = 47, normalized size = 0.96

$$\frac{6i\,a^3x^3\log(x) - 6i\,a^3x^3\log\left(\frac{ax-i}{a}\right) + 6\,a^2x^2 + 3i\,ax - 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*I*a^3*x^3*log(x) - 6*I*a^3*x^3*log((a*x - I)/a) + 6*a^2*x^2 + 3*I*a*
x - 1)/x^3
```

giac [A] time = 0.14, size = 77, normalized size = 1.57

$$2\,a^3i\log\left(-i + \frac{i}{aix+1}\right) + \frac{\frac{24\,a^3i^2}{aix+1} + 10\,a^3 - \frac{15\,a^3i^2}{(aix+1)^2}}{3\left(i - \frac{i}{aix+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*i*log(-i + i/(a*i*x + 1)) + 1/3*(24*a^3*i^2/(a*i*x + 1) + 10*a^3 - 15
*a^3*i^2/(a*i*x + 1)^2)/(i - i/(a*i*x + 1))^3
```

maple [A] time = 0.06, size = 55, normalized size = 1.12

$$-\frac{1}{3x^3} + \frac{ia}{x^2} + 2ia^3\ln(x) + \frac{2a^2}{x} + 2a^3\arctan(ax) - ia^3\ln\left(a^2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3+I*a/x^2+2*I*a^3*ln(x)+2*a^2/x+2*a^3*arctan(a*x)-I*a^3*ln(a^2*x^2+1
)
```

maxima [A] time = 0.33, size = 57, normalized size = 1.16

$$-2i\,a^3\log(i\,ax+1) + 2i\,a^3\log(x) + \frac{6i\,a^3x^3 + 3\,a^2x^2 + 2i\,ax - 1}{3i\,ax^4 + 3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*I*a^3*log(I*a*x + 1) + 2*I*a^3*log(x) + (6*I*a^3*x^3 + 3*a^2*x^2 + 2*I*a
*x - 1)/(3*I*a*x^4 + 3*x^3)
```

mupad [B] time = 0.43, size = 33, normalized size = 0.67

$$4\,a^3\,\mathrm{atan}(2\,a\,x - \mathrm{i}) + \frac{2\,a^2\,x^2 + a\,x\,1\mathrm{i} - \frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(x^4*(a*x*1i + 1)^2),x)
```

```
[Out] 4*a^3*atan(2*a*x - 1i) + (a*x*1i + 2*a^2*x^2 - 1/3)/x^3
```

sympy [A] time = 0.21, size = 54, normalized size = 1.10

$$-2a^3\left(-i\log\left(4a^4x\right)+i\log\left(4a^4x-4ia^3\right)\right)-\frac{-6a^2x^2-3iax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(-I*log(4*a**4*x) + I*log(4*a**4*x - 4*I*a**3)) - (-6*a**2*x**2 - 3
*I*a*x + 1)/(3*x**3)
```

3.52 $\int e^{-3i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=137

$$\frac{51i\sinh^{-1}(ax)}{8a^4}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}+\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{9i(3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1-iax)^3}{a^4\sqrt{a^2x^2+1}}$$

```
[Out] 51/8*I*arcsinh(a*x)/a^4+(1-I*a*x)^3/a^4/(a^2*x^2+1)^(1/2)+27/4*(a^2*x^2+1)^
(1/2)/a^4-x^2*(a^2*x^2+1)^(1/2)/a^2+1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-9/8*I*(2*
I+3*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.62, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 215}

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{9i(3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1-iax)^3}{a^4\sqrt{a^2x^2+1}}+\frac{51i\sinh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (1 - I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I + 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3i\tan^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^3}{a}-x^4\right)}{(1+iax)^2}\,dx \\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx \\
&= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx \\
&= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx \\
&= \int \frac{x^3(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1-iax)^2\left(-\frac{3i}{a^3}-\frac{x}{a^2}+\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-\frac{12i}{a}-28x+27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-36ia-108a^2x+81ia^3x^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{9ia(2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(3i)\int \frac{(2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^3} \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{(3i)\int \frac{-17a^2+18ia\cdots}{\sqrt{1+a^2x^2}}}{8a^5} \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \cdots \\
&= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \cdots
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.58

$$
\frac{51i\sinh^{-1}(ax)}{8a^4} + \sqrt{a^2x^2+1}\left(-\frac{4i}{a^4(ax-i)} + \frac{6}{a^4} - \frac{19ix}{8a^3} - \frac{x^2}{a^2} + \frac{ix^3}{4a}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 - (((19*I)/8)*x)/a^3 - x^2/a^2 + ((I/4)*x^3)/a - (
4*I)/(a^4*(-I + a*x))) + (((51*I)/8)*ArcSinh[a*x])/a^4
```

fricas [A] time = 0.41, size = 88, normalized size = 0.64

$$\frac{-32i\,ax-51\,(i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\left(2i\,a^4x^4-6\,a^3x^3-11i\,a^2x^2+29\,ax-80i\right)\sqrt{a^2x^2+1}-32}{8\,a^5x-8i\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (-32*I*a*x - 51*(I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (2*I*a^4*x^4 -
6*a^3*x^3 - 11*I*a^2*x^2 + 29*a*x - 80*I)*sqrt(a^2*x^2 + 1) - 32)/(8*a^5*x
- 8*I*a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.18, size = 296, normalized size = 2.16

$$\frac{ix\left(a^2x^2+1\right)^{\frac{3}{2}}}{4a^3}+\frac{3ix\sqrt{a^2x^2+1}}{8a^3}+\frac{3i\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}+\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^7\left(x-\frac{i}{a}\right)^3}-\frac{5\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\right.}{a^6\left(x-\frac{i}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*I/a^3*x*(a^2*x^2+1)^(3/2)+3/8*I/a^3*x*(a^2*x^2+1)^(1/2)+3/8*I/a^3*ln(x*
a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+I/a^7/(x-I/a)^3*((x-I/a)^2*a
^2+2*I*a*(x-I/a))^(5/2)-5/a^6/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)
+4/a^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+6*I/a^3*((x-I/a)^2*a^2+2*I*a*(x-
I/a))^(1/2)*x+6*I/a^3*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a
*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.45, size = 216, normalized size = 1.58

$$\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^6x^2-2i\,a^5x-a^4}+\frac{3\left(a^2x^2+1\right)^{\frac{3}{2}}}{2i\,a^5x+2\,a^4}+\frac{6\sqrt{a^2x^2+1}}{i\,a^5x+a^4}+\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3}+\frac{3i\sqrt{a^2x^2+1}\,x}{8\,a^3}-\frac{3i\sqrt{-a^2x^2+4i\,ax+3}\,x}{2\,a^3}-\frac{(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 + 1)^(3/2)/(a^6*x^2 - 2*I*a^5*x - a^4) + 3*(a^2*x^2 + 1)^(3/2)/(2*
I*a^5*x + 2*a^4) + 6*sqrt(a^2*x^2 + 1)/(I*a^5*x + a^4) + 1/4*I*(a^2*x^2 + 1
)^(3/2)*x/a^3 + 3/8*I*sqrt(a^2*x^2 + 1)*x/a^3 - 3/2*I*sqrt(-a^2*x^2 + 4*I*a
*x + 3)*x/a^3 - (a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(I*a*x + 2)/a^4 + 63/
8*I*arcsinh(a*x)/a^4 + 9/2*sqrt(a^2*x^2 + 1)/a^4 - 3*sqrt(-a^2*x^2 + 4*I*a*
x + 3)/a^4
```

mupad [B] time = 0.47, size = 138, normalized size = 1.01

$$\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{\left(a^2\right)^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}+\frac{x^3\,\left(a^2\right)^{3/2}\,1\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,19\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}+\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51\mathrm{i}}{8\,a^3\,\sqrt{a^2}}+\frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a^3\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/2
))/a^2 + (x^3*(a^2)^(3/2)*1i)/(4*a^3) - (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^2)
^(1/2) + (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)^(1
/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int \frac{a^2x^5\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**5*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 -
 3*a*x + I), x))
```

3.53 $\int e^{-3i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=102

$$\frac{11\sinh^{-1}(ax)}{2a^3}-\frac{i(1-iax)^3}{a^3\sqrt{a^2x^2+1}}-\frac{i(3-iax)^2\sqrt{a^2x^2+1}}{3a^3}-\frac{(3ax+28i)\sqrt{a^2x^2+1}}{6a^3}$$

```
[Out] 11/2*arcsinh(a*x)/a^3-I*(1-I*a*x)^3/a^3/(a^2*x^2+1)^(1/2)-1/3*I*(3-I*a*x)^2
*(a^2*x^2+1)^(1/2)/a^3-1/6*(28*I+3*a*x)*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.57, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 1633, 1593, 12, 852, 1635, 1654, 780, 215}

$$-\frac{i(1-iax)^3}{a^3\sqrt{a^2x^2+1}}-\frac{i(3-iax)^2\sqrt{a^2x^2+1}}{3a^3}-\frac{(3ax+28i)\sqrt{a^2x^2+1}}{6a^3}+\frac{11\sinh^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) - ((I/3)*(3 - I*a*x)^2*Sqrt[1
+ a^2*x^2])/a^3 - ((28*I + 3*a*x)*Sqrt[1 + a^2*x^2])/(6*a^3) + (11*ArcSinh[
a*x])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3i\tan^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^2}{a}-x^3\right)}{(1+iax)^2}\,dx \\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx \\
&= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx \\
&= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx \\
&= \int \frac{x^2(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(1-iax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(-5+3iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\sinh^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.06, size = 63, normalized size = 0.62

$$
\frac{33\sinh^{-1}(ax)+\frac{\sqrt{a^2x^2+1}\left(2ia^3x^3-7a^2x^2-19iax-52\right)}{ax-i}}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 - (19*I)*a*x - 7*a^2*x^2 + (2*I)*a^3*x^3))/(-I + a
*x) + 33*ArcSinh[a*x])/(6*a^3)
```

fricas [**A**] time = 0.41, size = 81, normalized size = 0.79

$$
-\frac{24\,ax+(33\,ax-33i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(2i\,a^3x^3-7\,a^2x^2-19i\,ax-52\right)\sqrt{a^2x^2+1}-24i}{6\,a^4x-6i\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(24*a*x + (33*a*x - 33*I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (2*I*a^3*x^3 - 7
*a^2*x^2 - 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) - 24*I)/(6*a^4*x - 6*I*a^3)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
undef
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 224, normalized size = 2.20

$$\frac{4i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^5\left(x-\frac{i}{a}\right)^2}-\frac{11i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3a^3}+\frac{11\sqrt{\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)}\,x}{2a^2}+\frac{11\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)}{\sqrt{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*I/a^5/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-11/3*I/a^3*((x-I/a)^2
*a^2+2*I*a*(x-I/a))^(3/2)+11/2/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x+11
/2/a^2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)
)/(a^2)^(1/2)+1/a^6/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)
```

maxima [B] time = 0.44, size = 181, normalized size = 1.77

$$-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^5x^2-2i\,a^4x-a^3}-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{i\,a^4x+a^3}-\frac{6i\sqrt{a^2x^2+1}}{i\,a^4x+a^3}-\frac{\sqrt{-a^2x^2+4i\,ax+3}\,x}{2\,a^2}+\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}+\frac{\arcsin\left(i\,ax+2\right)}{2\,a^3}+\frac{6\,\text{ars}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(a^2*x^2 + 1)^(3/2)/(a^5*x^2 - 2*I*a^4*x - a^3) - I*(a^2*x^2 + 1)^(3/2)/
(I*a^4*x + a^3) - 6*I*sqrt(a^2*x^2 + 1)/(I*a^4*x + a^3) - 1/2*sqrt(-a^2*x^2
 + 4*I*a*x + 3)*x/a^2 + 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(I*a*x +
2)/a^3 + 6*arcsinh(a*x)/a^3 - 3*I*sqrt(a^2*x^2 + 1)/a^3 + I*sqrt(-a^2*x^2 +
 4*I*a*x + 3)/a^3
```

mupad [B] time = 0.07, size = 115, normalized size = 1.13

$$\frac{11\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,x\,\sqrt{a^2}}{2\,a^2}+\frac{a\,14\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}+\frac{4\,\sqrt{a^2\,x^2+1}}{a^2\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] (11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a*14
i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*a^
2)))/(a^2)^(1/2) + (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a - x*(a^
2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\frac{x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^4\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**4*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 -
 3*a*x + I), x))
```

3.54 $\int e^{-3i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{a^2(1+iax)^3}-\frac{3\left(a^2x^2+1\right)^{3/2}}{2a^2(1+iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

```
[Out] -3/2*(a^2*x^2+1)^(3/2)/a^2/(1+I*a*x)-(a^2*x^2+1)^(5/2)/a^2/(1+I*a*x)^3-9/2*
I*arcsinh(a*x)/a^2-9/2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.32, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5060, 1633, 1593, 12, 793, 665, 215}

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{a^2(1+iax)^3}-\frac{3\left(a^2x^2+1\right)^{3/2}}{2a^2(1+iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 + I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 + I*a*x)^3) - (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-3i\tan^{-1}(ax)}x\,dx &= \int \frac{x(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= (ia)\int \frac{\left(-\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx\\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx\\
&= a^2\int \frac{x\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx\\
&= \int \frac{x\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx\\
&= -\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(3i)\int \frac{\left(1+a^2x^2\right)^{3/2}}{(1+iax)^2}\,dx}{a}\\
&= -\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1+iax}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3\left(1+a^2x^2\right)^{3/2}}{2a^2(1+iax)}-\frac{\left(1+a^2x^2\right)^{5/2}}{a^2(1+iax)^3}-\frac{9i\sinh^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.65

$$\sqrt{a^2x^2+1}\left(\frac{4i}{a^2(ax-i)}-\frac{3}{a^2}+\frac{ix}{2a}\right)-\frac{9i\sinh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-3/a^2 + ((I/2)*x)/a + (4*I)/(a^2*(-I + a*x))) - (((9*I)
/2)*ArcSinh[a*x])/a^2
```

fricas [A] time = 0.42, size = 72, normalized size = 0.78

$$\frac{8i\,ax-9\,(-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}\left(i\,a^2x^2-5\,ax+14i\right)+8}{2\,a^3x-2i\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (8*I*a*x - 9*(-I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)
*(I*a^2*x^2 - 5*a*x + 14*I) + 8)/(2*a^3*x - 2*I*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 226, normalized size = 2.46

$$\frac{3\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^{4}\left(x-\frac{i}{a}\right)^{2}}-\frac{3\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{a^{2}}-\frac{9i\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)}\,x}{2a}-\frac{9i\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^{2}}{\sqrt{a^{2}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 3/a^4/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-3/a^2*((x-I/a)^2*a^2+2*
I*a*(x-I/a))^(3/2)-9/2*I/a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x-9/2*I/a*ln
((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(
1/2)-I/a^5/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)
```

maxima [A] time = 0.45, size = 112, normalized size = 1.22

$$-\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{a^{4}x^{2}-2i\,a^{3}x-a^{2}}-\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{2i\,a^{3}x+2\,a^{2}}-\frac{6\sqrt{a^{2}x^{2}+1}}{i\,a^{3}x+a^{2}}-\frac{9i\,\operatorname{arsinh}\left(ax\right)}{2\,a^{2}}-\frac{3\sqrt{a^{2}x^{2}+1}}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 + 1)^(3/2)/(a^4*x^2 - 2*I*a^3*x - a^2) - (a^2*x^2 + 1)^(3/2)/(2*I
*a^3*x + 2*a^2) - 6*sqrt(a^2*x^2 + 1)/(I*a^3*x + a^2) - 9/2*I*arcsinh(a*x)/
a^2 - 3/2*sqrt(a^2*x^2 + 1)/a^2
```

mupad [B] time = 0.42, size = 105, normalized size = 1.14

$$-\frac{\sqrt{a^{2}\,x^{2}+1}\,\left(\frac{3\,\sqrt{a^{2}}}{a^{2}}-\frac{x\,\sqrt{a^{2}}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^{2}}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^{2}}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^{2}}}-\frac{\sqrt{a^{2}\,x^{2}+1}\,4\mathrm{i}}{a\,\left(-x\,\sqrt{a^{2}}+\frac{\sqrt{a^{2}}\,1\mathrm{i}}{a}\right)\,\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] - ((a^2*x^2 + 1)^(1/2)*((3*(a^2)^(1/2))/a^2 - (x*(a^2)^(1/2)*1i)/(2*a)))/(a
^2)^(1/2) - (asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1
/2)*4i)/(a*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{x\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int \frac{a^2x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I),
x) + Integral(a**2*x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*
a*x + I), x))
```

3.55 $\int e^{-3i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{2i(1-iax)^2}{a\sqrt{a^2x^2+1}}+\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

```
[Out] -3*arcsinh(a*x)/a+2*I*(1-I*a*x)^2/a/(a^2*x^2+1)^(1/2)+3*I*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5059, 853, 669, 641, 215}

$$\frac{2i(1-iax)^2}{a\sqrt{a^2x^2+1}}+\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*I)*ArcTan[a*x]),x]
```

```
[Out] ((2*I)*(1 - I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2])/a -
 (3*ArcSinh[a*x])/a
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 5059

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3i\tan^{-1}(ax)}\,dx &= \int \frac{(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\ &= \int \frac{(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\ &= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.70

$$-\frac{3\sinh^{-1}(ax)}{a} + \frac{\sqrt{a^2x^2+1}\left(\frac{4}{ax-i}+i\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-3*I)*ArcTan[a*x]),x]
[Out] (Sqrt[1 + a^2*x^2]*(I + 4/(-I + a*x)))/a - (3*ArcSinh[a*x])/a
```

fricas [A] time = 0.49, size = 60, normalized size = 1.00

$$\frac{4\,ax + (3\,ax - 3i)\log\left(-ax + \sqrt{a^2x^2+1}\right) + \sqrt{a^2x^2+1}\,(i\,ax+5) - 4i}{a^2x - i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] (4*a*x + (3*a*x - 3*I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(I
*a*x + 5) - 4*I)/(a^2*x - I*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] undef
```

maple [B] time = 0.11, size = 219, normalized size = 3.65

$$-\frac{\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^4\left(x-\frac{i}{a}\right)^3} - \frac{2i\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x-\frac{i}{a}\right)^2} + \frac{2i\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{a} - 3\sqrt{\left(x-\frac{i}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a^4/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-2*I/a^3/(x-I/a)^2*((x-
I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+2*I/a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)-3
*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x-3*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+(
(x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.45, size = 65, normalized size = 1.08

$$\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^3x^2-2i\,a^2x-a}-\frac{3\ \mathrm{arsinh}\,(ax)}{a}+\frac{6i\,\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(a^2*x^2 + 1)^(3/2)/(a^3*x^2 - 2*I*a^2*x - a) - 3*arcsinh(a*x)/a + 6*I*sq
rt(a^2*x^2 + 1)/(I*a^2*x + a)
```

mupad [B] time = 0.42, size = 73, normalized size = 1.22

$$\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}}-\frac{4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^3,x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*1i)/a - (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - (4*(a^2
*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\frac{\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I), x)
 + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*
x + I), x))
```

3.56 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=52

$$\frac{4i\sqrt{a^2x^2+1}}{-ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

```
[Out] I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))+4*I*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] time = 0.57, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5060, 6742, 215, 266, 63, 208, 651}

$$\frac{4i\sqrt{a^2x^2+1}}{-ax+i}-\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+i\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I - a*x) + I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1-iax)^2}{x(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx-(4a)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2}\\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\sinh^{-1}(ax)-\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.06

$$
-\frac{4i\sqrt{a^2x^2+1}}{ax-i}-\log\left(\sqrt{a^2x^2+1}+1\right)+i\sinh^{-1}(ax)+\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

```
[Out] ((-4*I)*Sqrt[1 + a^2*x^2])/(-I + a*x) + I*ArcSinh[a*x] + Log[x] - Log[1 + S
qrt[1 + a^2*x^2]]
```

fricas [B] time = 0.44, size = 100, normalized size = 1.92

$$
\frac{-4i\,ax-(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax-i)\log\left(-ax+\sqrt{a^2x^2+\cdots}\right.}{ax-i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (-4*I*a*x - (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (-I*a*x - 1)*log(
-a*x + sqrt(a^2*x^2 + 1)) + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) - 4
*I*sqrt(a^2*x^2 + 1) - 4)/(a*x - I)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
undef
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 257, normalized size = 4.94

$$\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3}+\sqrt{a^{2}x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)+\frac{i\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^{3}\left(x-\frac{i}{a}\right)^{3}}-\frac{\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a^{2}\left(x-\frac{i}{a}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 1/3*(a^2*x^2+1)^(3/2)+(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))+I/a^3/
(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-1/a^2/(x-I/a)^2*((x-I/a)^2*a^
2+2*I*a*(x-I/a))^(5/2)+2/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+I*a*((x-I/a)
^2*a^2+2*I*a*(x-I/a))^(1/2)*x+I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)
^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x), x)
```

mupad [B] time = 0.43, size = 74, normalized size = 1.42

$$-\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)+\frac{a\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,1{}\mathrm{i}}{\sqrt{a^2}}+\frac{a\,\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(x*(a*x*1i + 1)^3),x)
```

```
[Out] (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2)) + (a*(
a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^{2}x^{2}+1}}{a^{3}x^{4}-3ia^{2}x^{3}-3ax^{2}+ix}\,dx+\int \frac{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}}{a^{3}x^{4}-3ia^{2}x^{3}-3ax^{2}+ix}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3 - 3*a*x**2 + I*x
), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3 -
 3*a*x**2 + I*x), x))
```

3.57 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=64

$$\frac{4a\sqrt{a^2x^2+1}}{-ax+i}-\frac{\sqrt{a^2x^2+1}}{x}+3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 3*I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x+4*a*(a^2*x^2+1)^(1/2)/
(I-a*x)
```

Rubi [A] time = 0.55, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5060, 6742, 264, 266, 63, 208, 651}

$$\frac{4a\sqrt{a^2x^2+1}}{-ax+i}-\frac{\sqrt{a^2x^2+1}}{x}+3ia\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + (4*a*Sqrt[1 + a^2*x^2])/(I - a*x) + (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
```

```
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^2}{x^2(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}} - \frac{3ia}{x\sqrt{1+a^2x^2}} + \frac{4ia^2}{(-i+ax)\sqrt{1+a^2x^2}}\right) dx \\
&= -\left((3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\right) + \left(4ia^2\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} + 3ia\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.95

$$\sqrt{a^2x^2+1}\left(-\frac{1}{x}-\frac{4a}{ax-i}\right)+3ia\log\left(\sqrt{a^2x^2+1}+1\right)-3ia\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(-I + a*x)) - (3*I)*a*Log[x] + (3*I)*a*L
og[1 + Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.41, size = 109, normalized size = 1.70

$$-\frac{5\,a^2x^2-5i\,ax+3\left(-i\,a^2x^2-ax\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3\left(i\,a^2x^2+ax\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+}{ax^2-i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 - 5*I*a*x + 3*(-I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
 1) + 3*(I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x - I))/(a*x^2 - I*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 305, normalized size = 4.77

$$ia\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}+3ia\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)-ia\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}-\frac{\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}}{x}+a^{2}x\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}+\frac{3a^{2}x\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] I*a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+3*I*a*arctanh(1/(a^2*x^2+1)^(1/2))-
I*a*(a^2*x^2+1)^(3/2)-1/x*(a^2*x^2+1)^(5/2)+a^2*x*(a^2*x^2+1)^(3/2)+3/2*a^2
*x*(a^2*x^2+1)^(1/2)+3/2*a^2*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^
(1/2)-3*I*a*(a^2*x^2+1)^(1/2)-3/2*a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x
-3/2*a^2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/
2))/(a^2)^(1/2)+1/a^2/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^{3}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^2), x)
```

mupad [B] time = 0.42, size = 76, normalized size = 1.19

$$a\operatorname{atanh}\left(\sqrt{a^{2}\,x^{2}+1}\right)\,3\mathrm{i}-\frac{\sqrt{a^{2}\,x^{2}+1}}{x}+\frac{4\,a^{2}\,\sqrt{a^{2}\,x^{2}+1}}{\left(-x\,\sqrt{a^{2}}+\frac{\sqrt{a^{2}}\,1\mathrm{i}}{a}\right)\,\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^2*(a*x*1i + 1)^3),x)
```

```
[Out] a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x + (4*a^2*(a^2*x^2 +
 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\frac{\sqrt{a^{2}x^{2}+1}}{a^{3}x^{5}-3ia^{2}x^{4}-3ax^{3}+ix^{2}}\,dx+\int\frac{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}}{a^{3}x^{5}-3ia^{2}x^{4}-3ax^{3}+ix^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x**4 - 3*a*x**3 + I*x
**2), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x**
4 - 3*a*x**3 + I*x**2), x))
```

3.58 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=93

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i}+\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] 9/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2+3*I*a*(a^2*x^2
+1)^(1/2)/x-4*I*a^2*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] time = 0.59, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5060, 6742, 266, 51, 63, 208, 264, 651}

$$-\frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i}+\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*x^2) + ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I - a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-iax)^2}{x^3(1+iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}} - \frac{3ia}{x^2\sqrt{1+a^2x^2}} - \frac{4a^2}{x\sqrt{1+a^2x^2}} + \frac{4a^3}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx$$

$$= -\left((3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx + \left(4a^3\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx +$$

$$= \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) - \left(2a^2\right)\operatorname{Subst}\left(\int \ldots\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} - 4\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) -$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + 4a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right) - \frac{1}{2}\operatorname{Subst}\left(\int \ldots\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.85

$$\sqrt{a^2x^2+1}\left(\frac{4ia^2}{ax-i} + \frac{3ia}{x} - \frac{1}{2x^2}\right) + \frac{9}{2}a^2\log\left(\sqrt{a^2x^2+1}+1\right) - \frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 + ((3*I)*a)/x + ((4*I)*a^2)/(-I + a*x)) - (9*
a^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

fricas [A] time = 0.44, size = 131, normalized size = 1.41

$$\frac{14i\,a^3x^3 + 14\,a^2x^2 + \left(9\,a^3x^3 - 9i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - \left(9\,a^3x^3 - 9i\,a^2x^2\right)\log\left(-ax+\sqrt{a^2x^2+1}-1\ldots\right.}{2\,ax^3 - 2i\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] (14*I*a^3*x^3 + 14*a^2*x^2 + (9*a^3*x^3 - 9*I*a^2*x^2)*log(-a*x + sqrt(a^2*
x^2 + 1) + 1) - (9*a^3*x^3 - 9*I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
 + sqrt(a^2*x^2 + 1)*(14*I*a^2*x^2 + 5*a*x + I))/(2*a*x^3 - 2*I*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 376, normalized size = 4.04

$$-\frac{3a^2\left(a^2x^2+1\right)^{\frac{3}{2}}}{2}-\frac{9a^2\sqrt{a^2x^2+1}}{2}+\frac{9a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-\frac{9ia^3x\sqrt{a^2x^2+1}}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -3/2*a^2*(a^2*x^2+1)^(3/2)-9/2*a^2*(a^2*x^2+1)^(1/2)+9/2*a^2*arctanh(1/(a^2
*x^2+1)^(1/2))-I/a/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-9/2*I*a^3*
x*(a^2*x^2+1)^(1/2)-9/2*I*a^3*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)
^(1/2)-3*I*a^3*x*(a^2*x^2+1)^(3/2)+3*a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2
)+3*I*a/x*(a^2*x^2+1)^(5/2)+9/2*I*a^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x
-1/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+9/2*I*a^3*ln((I*a+(x-I/a)*
a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)-1/2/x^2*(
a^2*x^2+1)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^3), x)
```

mupad [B] time = 0.43, size = 100, normalized size = 1.08

$$-\frac{a^2\operatorname{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}+\frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x}-\frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^3*(a*x*1i + 1)^3),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a^2*atan((a^2
*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)
/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^6-3ia^2x^5-3ax^4+ix^3}\,dx+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^6-3ia^2x^5-3ax^4+ix^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x**5 - 3*a*x**4 + I*x
**3), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x**
5 - 3*a*x**4 + I*x**3), x))
```

3.59 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=118

$$\frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}-\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

```
[Out] -11/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3+3/2*I*a*(a
^2*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2+1)^(1/2)/x-4*a^3*(a^2*x^2+1)^(1/2)/(I
-a*x)
```

Rubi [A] time = 0.60, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 271, 264, 266, 51, 63, 208, 651}

$$-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}+\frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*x^3) + (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 + a^2*x^2])/(I - a*x) - ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-iax)^2}{x^4(1+iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}} - \frac{3ia}{x^3\sqrt{1+a^2x^2}} - \frac{4a^2}{x^2\sqrt{1+a^2x^2}} + \frac{4ia^3}{x\sqrt{1+a^2x^2}} - \frac{4ia^4}{(-i+ax)\sqrt{1+a^2x^2}}\right) dx$$

$$= -\left((3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx + \left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx - \left(4ia^4\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{4a^2\sqrt{1+a^2x^2}}{x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{2}(3ia)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) -$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} + (4ia)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - 4ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - \frac{11}{2}ia^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.75

$$\frac{1}{6}\left(33ia^3\log(x) - 33ia^3\log\left(\sqrt{a^2x^2+1}+1\right) + \frac{\sqrt{a^2x^2+1}\left(52a^3x^3 - 19ia^2x^2 + 7ax + 2i\right)}{x^3(ax-i)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 7*a*x - (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(-I +
 a*x)) + (33*I)*a^3*Log[x] - (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

fricas [A] time = 0.56, size = 139, normalized size = 1.18

$$\frac{52\,a^4x^4 - 52i\,a^3x^3 - 33\left(i\,a^4x^4 + a^3x^3\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 33\left(-i\,a^4x^4 - a^3x^3\right)\log\left(-ax + \sqrt{a^2x^2+1}\right.}{6\,ax^4 - 6i\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] (52*a^4*x^4 - 52*I*a^3*x^3 - 33*(I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(a^2*x
^2 + 1) + 1) - 33*(-I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
+ (52*a^3*x^3 - 19*I*a^2*x^2 + 7*a*x + 2*I)*sqrt(a^2*x^2 + 1))/(6*a*x^4 - 6
*I*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 392, normalized size = 3.32

$$-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{3x^3}+\frac{16a^2\left(a^2x^2+1\right)^{\frac{5}{2}}}{3x}-\frac{16a^4x\left(a^2x^2+1\right)^{\frac{3}{2}}}{3}-8a^4x\sqrt{a^2x^2+1}-\frac{8a^4\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{2ia\left(\left(x-\frac{i}{a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/3/x^3*(a^2*x^2+1)^(5/2)+16/3*a^2/x*(a^2*x^2+1)^(5/2)-16/3*a^4*x*(a^2*x^2
+1)^(3/2)-8*a^4*x*(a^2*x^2+1)^(1/2)-8*a^4*ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^
(1/2))/(a^2)^(1/2)+2*I*a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-11/2
*I*a^3*arctanh(1/(a^2*x^2+1)^(1/2))+8*a^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/
2)*x+8*a^4*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(
1/2))/(a^2)^(1/2)+11/6*I*a^3*(a^2*x^2+1)^(3/2)+3/2*I*a/x^2*(a^2*x^2+1)^(5/2
)+11/2*I*a^3*(a^2*x^2+1)^(1/2)-1/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5
/2)-16/3*I*a^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^4), x)
```

mupad [B] time = 0.07, size = 117, normalized size = 0.99

$$\frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x}-\frac{\sqrt{a^2\,x^2+1}}{3\,x^3}+\frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2}-\frac{11\,a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2}-\frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^4*(a*x*1i + 1)^3),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (11*a^3*
atan((a^2*x^2 + 1)^(1/2)*1i))/2 + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) - (4*a
^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^7-3ia^2x^6-3ax^5+ix^4}\,dx+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^7-3ia^2x^6-3ax^5+ix^4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x**6 - 3*a*x**5 + I*x
**4), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x**
6 - 3*a*x**5 + I*x**4), x))
```

3.60 $\int \frac{e^{-3i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=139

$$\frac{19a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{ia\sqrt{a^2x^2+1}}{x^3}+\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{6ia^3\sqrt{a^2x^2+1}}{x}$$

```
[Out] -51/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4+I*a*(a^2*x^2
+1)^(1/2)/x^3+19/8*a^2*(a^2*x^2+1)^(1/2)/x^2-6*I*a^3*(a^2*x^2+1)^(1/2)/x+4*
I*a^4*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] time = 0.66, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5060, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{6ia^3\sqrt{a^2x^2+1}}{x}+\frac{19a^2\sqrt{a^2x^2+1}}{8x^2}+\frac{ia\sqrt{a^2x^2+1}}{x^3}-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(4*x^4) + (I*a*Sqrt[1 + a^2*x^2])/x^3 + (19*a^2*Sqrt[1 +
 a^2*x^2])/(8*x^2) - ((6*I)*a^3*Sqrt[1 + a^2*x^2])/x + ((4*I)*a^4*Sqrt[1 +
a^2*x^2])/(I - a*x) - (51*a^4*ArcTanh[Sqrt[1 + a^2*x^2]])/8
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-iax)^2}{x^5(1+iax)\sqrt{1+a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^5\sqrt{1+a^2x^2}} - \frac{3ia}{x^4\sqrt{1+a^2x^2}} - \frac{4a^2}{x^3\sqrt{1+a^2x^2}} + \frac{4ia^3}{x^2\sqrt{1+a^2x^2}} + \frac{4a^4}{x\sqrt{1+a^2x^2}} - \frac{4}{(-i+ax)\cdots}\right.$$

$$= -\left((3ia)\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx\right) - \left(4a^2\right)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx + \left(4ia^3\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx + \left(4a^4\cdots\right.$$

$$= \frac{ia\sqrt{1+a^2x^2}}{x^3} - \frac{4ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right) - \left(2a\cdots\right.$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{2a^2\sqrt{1+a^2x^2}}{x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - \frac{1}{8}\left(3a^2\right)\cdots$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - 4a^4\tan\cdots$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - 6a^4\tan\cdots$$

$$= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} + \frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} - \frac{51}{8}a^4\tan\cdots$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.68

$$\frac{1}{8}\left(51a^4\log(x) - 51a^4\log\left(\sqrt{a^2x^2+1}+1\right) + \frac{\sqrt{a^2x^2+1}\left(-80ia^4x^4 - 29a^3x^3 - 11ia^2x^2 + 6ax + 2i\right)}{x^4(ax-i)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 6*a*x - (11*I)*a^2*x^2 - 29*a^3*x^3 - (80*I)*a^4
*x^4))/(x^4*(-I + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 + a^2*x^2]]
)/8
```

fricas [A] time = 0.48, size = 146, normalized size = 1.05

$$\frac{-80i\,a^5x^5-80\,a^4x^4-51\left(a^5x^5-i\,a^4x^4\right)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+51\left(a^5x^5-i\,a^4x^4\right)\log\left(-ax+\sqrt{a^2x^2+1}\right.}{8\,ax^5-8i\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] (-80*I*a^5*x^5 - 80*a^4*x^4 - 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt(a^2*
x^2 + 1) + 1) + 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
+ (-80*I*a^4*x^4 - 29*a^3*x^3 - 11*I*a^2*x^2 + 6*a*x + 2*I)*sqrt(a^2*x^2 +
1))/(8*a*x^5 - 8*I*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.19, size = 416, normalized size = 2.99

$$\frac{3a^2\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{\left(x-\frac{i}{a}\right)^2}+\frac{51a^4\sqrt{a^2x^2+1}}{8}-8a^4\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}+\frac{12ia^5\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2}\right.}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] 3*a^2/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+51/8*a^4*(a^2*x^2+1)^(1
/2)-8*a^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+12*I*a^5*ln(x*a^2/(a^2)^(1/2)
+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+12*I*a^5*x*(a^2*x^2+1)^(1/2)+8*I*a^5*x*(a^2
*x^2+1)^(3/2)-8*I*a^3/x*(a^2*x^2+1)^(5/2)+17/8*a^4*(a^2*x^2+1)^(3/2)+I*a/(x
-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-12*I*a^5*ln((I*a+(x-I/a)*a^2)/(
a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)+I*a/x^3*(a^2*x^
2+1)^(5/2)-51/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))-1/4/x^4*(a^2*x^2+1)^(5/2)-
12*I*a^5*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)*x+23/8*a^2/x^2*(a^2*x^2+1)^(5/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{(i\,ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^5), x)
```

mupad [B] time = 0.43, size = 139, normalized size = 1.00

$$\frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,51\mathrm{i}}{8}-\frac{\sqrt{a^2\,x^2+1}}{4\,x^4}+\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x^3}+\frac{19\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2}-\frac{a^3\,\sqrt{a^2\,x^2+1}\,6\mathrm{i}}{x}+\frac{a^5\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^5*(a*x*1i + 1)^3),x)
```

```
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*51i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (a
*(a^2*x^2 + 1)^(1/2)*1i)/x^3 + (19*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (a^3*
(a^2*x^2 + 1)^(1/2)*6i)/x + (a^5*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)
/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\frac{\sqrt{a^2x^2+1}}{a^3x^8-3ia^2x^7-3ax^6+ix^5}\,dx+\int\frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^8-3ia^2x^7-3ax^6+ix^5}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x**7 - 3*a*x**6 + I*x
**5), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x**
7 - 3*a*x**6 + I*x**5), x))
```

3.61 $\int e^{\frac{1}{2} i \tan^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=339

$$-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}-\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{16\sqrt{2}\,a^3}$$

```
[Out] -3/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3-1/12*I*(1-I*a*x)^(3/4)*(1+I*a*x)
^(5/4)/a^3+1/3*x*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^2+3/16*I*arctan(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/16*I*arctan(1+(1-I*a*x)^(1
/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+3/32*I*ln(1+
(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^
3*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}-\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-3*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 - ((I/12)*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(5/4))/a^3 + (x*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(3*a
^2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])
/(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*
a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((3
*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$

$$= \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{2}\right)\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{3a^2}$$

$$= -\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{}{\sqrt[4]{1-i}}}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\text{Subs}}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\text{Subs}}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{(3i)\,\text{Subs}}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\,\text{Subs}}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3i\log\left(1\right.}{}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{3i\tan^{-1}}{}$$

Mathematica [C] time = 0.04, size = 82, normalized size = 0.24

$$\frac{(1-iax)^{3/4}\left(\sqrt[4]{1+iax}\left(4ia^2x^2+5ax-i\right)-6i\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*((1 + I*a*x)^(1/4)*(-I + 5*a*x + (4*I)*a^2*x^2) - (6*I)*
2^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (1 - I*a*x)/2]))/(12*a^3)
```

fricas [A] time = 0.61, size = 244, normalized size = 0.72

$$-\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\,\log\left(\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\,\log\left(-\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-\frac{9i}{64\,a^6}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*I*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*I*a^3*sqrt(9/6
4*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-9/64*I/a^6)*
log(8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12
*a^3*sqrt(-9/64*I/a^6)*log(-8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 2*I*a^2*x^2 - a*x - 11*I)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
[Out] Integral(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.62 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\ldots \tan$$

```
[Out] 1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)
/a^2-1/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)-1/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\ldots \tan$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(3/4)*(1 + I*a
*x)^(5/4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x\,dx=\int\frac{x\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$
$$=\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\int\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\int\frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx}{8a}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\text{Subst}\left(\int\frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-iax}\right)}{2a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}+\frac{\text{Subst}\left(\int\frac{1+}{1+}\right.}{}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2}+\frac{\text{Subst}\left(\right.}{}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1}}{\sqrt{1}}\right.}{8}$$
$$=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+iax}}\right.}{4\sqrt{2}a^2}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{3/4}\left(2\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right)+3(1+iax)^{5/4}\right)}{6a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(3*(1 + I*a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(6*a^2)
```

fricas [A] time = 0.52, size = 236, normalized size = 0.80

$$-\frac{2a^2\sqrt{\frac{i}{16a^4}}\log\left(4a^2\sqrt{\frac{i}{16a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2a^2\sqrt{\frac{i}{16a^4}}\log\left(-4a^2\sqrt{\frac{i}{16a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2a^2\sqrt{-\frac{i}{16a^4}}\log\left(4\right.}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*a^2*sqrt(1/16*I/a^4) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1/16*I/a^4)*log(4*a^2*sq
rt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I
```

```
/a^4)*log(-4*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 (2*a^2*x^2 - I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [**F**] time = 0.17, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x,x)
```

```
[Out] Integral(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.63 $\int e^{\frac{1}{2} i \tan^{-1}(ax)} \, dx$

Optimal. Leaf size=268

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

```
[Out] I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-1/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a*2^(1/2)+1/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4))/a*2^(1/2)+1/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a
*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)-1/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+
I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2
]*a) - ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{2} i \tan^{-1}(ax)}\,dx &= \int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a} \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{i\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{i\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{2a} \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-i}}{\sqrt{1+i}}\right.}{2\sqrt{}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.15

$$
-\frac{8ie^{\frac{5}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};-e^{2i\tan^{-1}(ax)}\right)}{5a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a*x])*Hypergeometric2F1[5/4, 2, 9/4, -E^((2
*I)*ArcTan[a*x])])/a
```

fricas [A] time = 0.46, size = 209, normalized size = 0.78

$$
\frac{a\sqrt{\frac{i}{a^2}}\log\left(ia\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{i}{a^2}}\log\left(-ia\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + a\sqrt{-\frac{i}{a^2}}\log\left(ia\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)) - a*sqrt(I/a^2)*log(-I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + a*sqrt(-I/a^2)*log(I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) - a*sqrt(-I/a^2)*log(-I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) + (2*a*x + 2*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueDone
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

3.64 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+iax}}\right.$$

```
[Out] -2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))-1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)
/(1+I*a*x)^(1/2))*2^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+
(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(
1+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2
^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+iax}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\,dx$$
$$= (ia)\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$
$$= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\,\text{Subst}\Big($$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{Su}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\text{Subs}}{}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\Big(1+}{}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{t}$$

Mathematica [C] time = 0.04, size = 97, normalized size = 0.36

$$-\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}\,(1+iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right)+2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)\right)}{3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] (-2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4
, 7/4, (1 - I*a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x
)]))/(3*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.43, size = 243, normalized size = 0.91

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^{}}}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/2
*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/2
*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - lo
g(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x, x)
```

3.65 $\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=92

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] - I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + \frac{1}{2}(ia)\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + (2ia)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - (ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right. \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 71, normalized size = 0.77

$$
-\frac{i(1-iax)^{3/4}\left(2ax\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+3ax-3i\right)}{3x(1+iax)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-1/3*I)*(1 - I*a*x)^(3/4)*(-3*I + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1,
 7/4, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

fricas [B] time = 0.45, size = 151, normalized size = 1.64

$$
\frac{-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-}{2\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) - I) + I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-I*a*x +
 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**2, x)
```

3.66 $\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=132

$$\frac{1}{4}a^2 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[Out] -1/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5
/4)/x^2+1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$\frac{1}{4}a^2 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}-\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-I/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(3/4)*(1 +
I*a*x)^(5/4))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
 + (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{\sqrt[4]{1+iax}}{x^3\sqrt[4]{1-iax}}\,dx\\
&= -\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} + \frac{1}{4}(ia)\int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} - \frac{1}{8}a^2\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) +\\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} + \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.
\end{aligned}$$

Mathematica [C] time = 0.02, size = 81, normalized size = 0.61

$$\frac{(1-iax)^{3/4}\left(2a^2x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+9a^2x^2-15iax-6\right)}{12x^2(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-6 - (15*I)*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric2
F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(12*x^2*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.51, size = 175, normalized size = 1.33

$$\frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - (6*a^2*x^2 + 2*I*a*x + 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**3, x)
```

3.67 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-ia}{}$$

```
[Out] -1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3-5/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^2+11/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3/8*I*a^3*arctan((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))+3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))
```

Rubi [A] time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{(1-ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) - (((5*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1+iax}}{x^4\sqrt[4]{1-iax}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{1}{3}\int \frac{\frac{5ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} - \frac{1}{6}\int \frac{\frac{11a^2}{4}+\frac{5}{2}ia^3x}{x^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{6}\int -\frac{\ldots}{8x\sqrt[4]{1\ldots}}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{1}{16}\left(3ia^3\right)\int \ldots$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{1}{4}\left(3ia^3\right)\mathrm{Subs}\ldots$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{8}\left(3ia^3\right)\mathrm{Subs}\ldots$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{3}{8}ia^3\tan^{-1}\left(\ldots\right.$$

Mathematica [C] time = 0.03, size = 93, normalized size = 0.55

$$\frac{(1-iax)^{3/4}\left(6ia^3x^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+11ia^3x^3+21a^2x^2-18iax-8\right)}{24x^3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-8 - (18*I)*a*x + 21*a^2*x^2 + (11*I)*a^3*x^3 + (6*I)*a
^3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I
*a*x)^(3/4))
```

fricas [A] time = 0.43, size = 183, normalized size = 1.08

$$\frac{9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a}}{a}}\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)) - I) - 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - 1) + (-22*I*a^3*x^3 + 2*a^2*x^2 - 4*I*a*x - 16)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**4, x)
```

3.68 $\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x^5} \, dx$

Optimal. Leaf size=202

$$-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}+\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}-\frac{(\cdots}{}$$

```
[Out] -1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4-7/24*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^3+29/96*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2+83/192*I*a^3*(1-I*
a*x)^(3/4)*(1+I*a*x)^(1/4)/x-11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}+\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) - (((7*I)/24)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^3 + (29*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(96*x^2) + (((83*I)/192)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x^5} \, dx = \int \frac{\sqrt[4]{1+iax}}{x^5 \sqrt[4]{1-iax}} \, dx$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} + \frac{1}{4} \int \frac{\frac{7ia}{2} - 3a^2 x}{x^4 \sqrt[4]{1-iax} (1+iax)^{3/4}} \, dx$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} - \frac{1}{12} \int \frac{\frac{29a^2}{4} + 7ia^3 x}{x^3 \sqrt[4]{1-iax} (1+iax)^{3/4}} \, dx$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{1}{24} \int \frac{-}{x^2 \sqrt[4]{1-}}$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax}{19}$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax}{19}$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax}{19}$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax}{19}$$

$$= -\frac{(1-iax)^{3/4} \sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4} \sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4} \sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax}{19}$$

Mathematica [C] time = 0.03, size = 99, normalized size = 0.49

$$-\frac{(1-iax)^{3/4}\left(22a^4x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+83a^4x^4-141ia^3x^3-114a^2x^2+104iax+48\right)}{192x^4(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -1/192*((1 - I*a*x)^(3/4)*(48 + (104*I)*a*x - 114*a^2*x^2 - (141*I)*a^3*x^3
 + 83*a^4*x^4 + 22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*
x)]))/(x^4*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.43, size = 192, normalized size = 0.95

$$-\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a^4x^4\log}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - (166*a^4*x^4 + 50*I*a^3*x^3 + 4*a^2*x^2 - 16*I*a*x - 96)
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**5, x)
```

3.69 $\int \frac{e^{\frac{1}{2} i \tan^{-1}(ax)}}{x^6} \, dx$

Optimal. Leaf size=240

$$-\frac{31}{128} i a^5 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{31}{128} i a^5 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{611 a^4 (1-iax)^{3/4} \sqrt[4]{1+iax}}{1920 x}+\frac{269 i a^3 (1-iax)^{3/4} \sqrt[4]{1+i a}}{960 x^2}$$

```
[Out] -1/5*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^5-9/40*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^4+11/48*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+269/960*I*a^3*(1-I
*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-611/1920*a^4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4
)/x-31/128*I*a^5*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-31/128*I*a^5*arcta
nh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.10, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{269 i a^3 (1-iax)^{3/4} \sqrt[4]{1+iax}}{960 x^2}+\frac{11 a^2 (1-iax)^{3/4} \sqrt[4]{1+iax}}{48 x^3}-\frac{611 a^4 (1-iax)^{3/4} \sqrt[4]{1+iax}}{1920 x}-\frac{31}{128} i a^5 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^6,x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(5*x^5) - (((9*I)/40)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^4 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(48*x^3) + (((269*I)/960)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 - (
611*a^4*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(1920*x) - ((31*I)/128)*a^5*Ar
cTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((31*I)/128)*a^5*ArcTanh[(1 + I
*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(ax)}}{x^6}\,dx = \int \frac{\sqrt[4]{1+iax}}{x^6\sqrt[4]{1-iax}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} + \frac{1}{5}\int \frac{\frac{9ia}{2}-4a^2x}{x^5\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} - \frac{1}{20}\int \frac{\frac{55a^2}{4}+\frac{27}{2}ia^3x}{x^4\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{1}{60}\int \frac{}{x^3\sqrt[4]{}}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$
$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-}{}$$

Mathematica [C] time = 0.04, size = 111, normalized size = 0.46

$$\frac{(1-iax)^{3/4}\left(-310ia^5x^5\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)-611ia^5x^5-1149a^4x^4+978ia^3x^3+872a^2x^2-816iax-384\right)}{1920x^5(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^6,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-384 - (816*I)*a*x + 872*a^2*x^2 + (978*I)*a^3*x^3 - 11
49*a^4*x^4 - (611*I)*a^5*x^5 - (310*I)*a^5*x^5*Hypergeometric2F1[3/4, 1, 7/
4, (I + a*x)/(I - a*x)]))/(1920*x^5*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.45, size = 199, normalized size = 0.83

$$\frac{-465i\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+465i\,a^5x^5}{3840\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/3840*(-465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 465*a
^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 465*a^5*x^5*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - 1) + (1222*I*a^5*x^5 - 146*a^4*x^4 + 196*I*a^3*x^3 + 16*
a^2*x^2 - 96*I*a*x - 768)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**6, x)
```

3.70 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=337

$$-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4}-\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}\,a^4}-\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\cdots\right.}{128\sqrt{2}\cdots}$$

```
[Out] -41/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4+1/4*x^2*(1-I*a*x)^(1/4)*(1+I*a*x
)^(7/4)/a^2-1/32*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)*(11+4*I*a*x)/a^4+123/128*a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-123/128*arctan
(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1
/2)-123/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4}-\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\cdots\right.}{128\sqrt{2}\,a^4}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (-41*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(1/4)
*(1 + I*a*x)^(7/4))/(4*a^2) - ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4)*(11 + (4
*I)*a*x))/(32*a^4) + (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$

$$= \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} + \frac{\int \frac{x(1+iax)^{3/4}\left(-2-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{4a^2}$$

$$= \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{(41i)\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{(12}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123}{}$$

$$= -\frac{41\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123}{}$$

Mathematica [C] time = 0.15, size = 148, normalized size = 0.44

$$\frac{\sqrt[4]{1-iax}\left(ia^3x^3(1+iax)^{3/4} + a^2x^2(1+iax)^{3/4} - 24\,2^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right) + 8\,2^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax\right.\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(a^2*x^2*(1 + I*a*x)^(3/4) + I*a^3*x^3*(1 + I*a*x)^(3/4)
 - 24*2^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - I*a*x)/2] + 8*2^(3/4)
```

```
*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - I*a*x)/2] + 2*2^(3/4)*Hypergeometri
c2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(4*a^4)
```

fricas [A] time = 0.45, size = 254, normalized size = 0.75

$$\frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\,\log\left(\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\,\log\left(-\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log
(-64/123*I*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(-15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 32*a^4*sqrt(-15129/4096*I/a^8)*lo
g(-64/123*I*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + (16*I*a^3*x^3 + 24*a^2*x^2 - 30*I*a*x - 63)*sqrt(a^2*x^2 + 1)*sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(\frac{1 + a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
[Out] Timed out
```

3.71 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$-\frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3}-\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+iax}}\right.}{16\sqrt{2}\,a^3}$$

```
[Out] -17/24*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3-1/4*I*(1-I*a*x)^(1/4)*(1+I*a*x
)^(7/4)/a^3+1/3*x*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)/a^2+17/16*I*arctan(1-(1-I
*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/16*I*arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)+17/32*I*ln(1-(1-I*a*x)^(1/4)*2^
(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-17/32*I*
ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2
))/a^3*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2}-\frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3}-\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}-\frac{17i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-17*I)/24)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - ((I/4)*(1 - I*a*x)
^(1/4)*(1 + I*a*x)^(7/4))/a^3 + (x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(3*
a^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) -
(((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)
^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$

$$= \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} + \frac{\int \frac{(1+iax)^{3/4}\left(-1-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{3a^2}$$

$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{24a^2}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{}{(1-iax)^{}}}{16}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}}{}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}}{}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}}{}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} - \frac{(17i)\,\text{Subst}}{}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} + \frac{17i\log\left(1 -\right.}{}$$

$$= -\frac{17i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{3a^2} + \frac{17i\tan^{-1}\left(\right.}{8}$$

Mathematica [C] time = 0.04, size = 82, normalized size = 0.24

$$\frac{\sqrt[4]{1-iax}\left((1+iax)^{3/4}\left(4ia^2x^2+7ax-3i\right)-34i2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(3/4)*(-3*I + 7*a*x + (4*I)*a^2*x^2) - (34*
I)*2^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(12*a^3)
```

fricas [A] time = 0.48, size = 247, normalized size = 0.73

$$-\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\,\log\left(\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\,\log\left(-\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{-\frac{289i}{64\,a^6}}\,\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*a^3*sqrt(289/64*I/a^6) + sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*a^3*sqrt
(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-289/64
*I/a^6)*log(8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) + 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(8*I*a^2*x^2 + 14*a*x -
 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
[Out] Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.72 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9}{}$$

```
[Out] 3/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)
/a^2-9/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+9/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)+9/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(1/4)*(1 + I
*a*x)^(7/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x\,dx=\int\frac{x(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$
$$=\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}-\frac{(3i)\int\frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{4a}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}-\frac{(9i)\int\frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{9\,\text{Subst}\left(\int\frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-iax}\right)}{2a^2}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{9\,\text{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{9\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}+\frac{9\,\text{Subst}\Big(\int}{}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}+\frac{9\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2}+\frac{9\,\text{Sub}}{}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}-\frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}\,a^2}+\frac{9\log\Big(1+}{}$$
$$=\frac{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2a^2}-\frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}\,a^2}+\frac{9\tan^{-1}\Big(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1-}}}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.21

$$\frac{\sqrt[4]{1-iax}\left(6\,2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right)+(1+iax)^{7/4}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4, 1
/4, 5/4, (1 - I*a*x)/2]))/(2*a^2)
```

fricas [A] time = 0.45, size = 239, normalized size = 0.81

$$-\frac{2a^2\sqrt{\frac{81i}{16a^4}}\log\left(\frac{4}{9}ia^2\sqrt{\frac{81i}{16a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2a^2\sqrt{\frac{81i}{16a^4}}\log\left(-\frac{4}{9}ia^2\sqrt{\frac{81i}{16a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2a^2\sqrt{-\frac{81i}{16a^4}}\log\Big(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*I*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*I*a^2*sqrt(81/
16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-81/16*I/a^4)
*log(4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
2*a^2*sqrt(-81/16*I/a^4)*log(-4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^
```

```
2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a*x + 5)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i\left(ax - i\right)}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x,x)
```

```
[Out] Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.73 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}\,a}$$

```
[Out] I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-3/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a*2^(1/2)+3/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4))/a*2^(1/2)-3/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a
*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)+3/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+
I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sq
rt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sq
rt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}\,dx = \int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{a}$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\right.}{2a}$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a}$$
$$= \frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} - \frac{3i\log\left(1+\right.}{}$$

Mathematica [**C**] time = 0.03, size = 41, normalized size = 0.15

$$-\frac{8ie^{\frac{7}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};-e^{2i\tan^{-1}(ax)}\right)}{7a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a*x])*Hypergeometric2F1[7/4, 2, 11/4, -E^((
2*I)*ArcTan[a*x])])/a
```

fricas [**A**] time = 0.51, size = 215, normalized size = 0.80

$$\frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}a\sqrt{-\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2}}{ax+i}}\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) - a*sqrt(9*I/a^2)*log(-1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(1/3*a*sqrt(-9*I/a^2) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(-1/3*a*sqrt(-9*I/a^2) + sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.74 $\int \frac{e^{\frac{3}{2} i \tan^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=267

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.$$

```
[Out] 2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/
(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(
1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^
(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx$$

$$= (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\,\text{Subst}\Big($$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{Sub}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\Big(1+\frac{\sqrt{1}}{\sqrt{1}}}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\,\text{ta}$$

Mathematica [C] time = 0.03, size = 96, normalized size = 0.36

$$-2\,2^{3/4}\sqrt[4]{1-iax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\;{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{1-iax}{-iax-1}\right)}{\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2
] - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
 I*a*x))])/(1 + I*a*x)^(1/4)
```

fricas [A] time = 0.53, size = 243, normalized size = 0.91

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i} + \sqrt{\frac{i\sqrt{}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 + 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) +
 log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueWarning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWar
ning, need to choose a branch for the root of a polynomial with parameters.
 This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m o
perator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x, x)
```

3.75 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))-3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] - (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} + \frac{1}{2}(3ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} + (6ia)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} - (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + (3ia)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,d\right. \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} + 3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.74

$$
-\frac{i\sqrt[4]{1-iax}\left(6ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+ax-i\right)}{x\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(-I + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
 + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

fricas [B] time = 0.74, size = 157, normalized size = 1.71

$$
\frac{-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + 3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{2\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - I) + 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*s
qrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**2, x)
```

3.76 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}-\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}$$

```
[Out] -3/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7
/4)/x^2-9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}-\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (((-3*I)/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(1/4)*(
1 + I*a*x)^(7/4))/(2*x^2) - (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/
4)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^{3/4}}{x^3(1-iax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}+\frac{1}{4}(3ia)\int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}\,dx\\
&= -\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}-\frac{1}{8}\left(9a^2\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}-\frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}+\frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{7/4}}{2x^2}-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.
\end{aligned}$$

Mathematica [C] time = 0.02, size = 81, normalized size = 0.61

$$\frac{\sqrt[4]{1-iax}\left(18a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+5a^2x^2-7iax-2\right)}{4x^2\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 - (7*I)*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric2
F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

fricas [A] time = 1.19, size = 179, normalized size = 1.36

$$\frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2}}{ax+i}}\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) - 2*sqrt(a^2*x^2 + 1)*(5*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**3, x)
```

3.77 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3}-\frac{7ia\sqrt[4]{1}}{}$$

```
[Out] -1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3-7/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^2+23/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-17/8*I*a^3*arctan((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x}-\frac{17}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) - (((7*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (23*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+iax)^{3/4}}{x^4(1-iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{\frac{7ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{\frac{23a^2}{4}+\frac{7}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{1}{6}\int -\frac{}{8x(1-}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} - \frac{1}{16}\left(17ia^3\right)\int$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} - \frac{1}{4}\left(17ia^3\right)\text{Sul}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{1}{8}\left(17ia^3\right)\text{Sul}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} - \frac{17}{8}ia^3\tan^{-1}\Bigg($$

Mathematica [C] time = 0.02, size = 93, normalized size = 0.55

$$\frac{\sqrt[4]{1-iax}\left(102ia^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+23ia^3x^3+37a^2x^2-22iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 - (22*I)*a*x + 37*a^2*x^2 + (23*I)*a^3*x^3 + (102*I)
*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 +
 I*a*x)^(1/4))
```

fricas [A] time = 0.49, size = 186, normalized size = 1.09

$$\frac{51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-51i\,a^3x^3\log\left(\sqrt{\right.\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x^
3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) - I) - 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - 1) + (46*a^2*x^2 - 28*I*a*x - 16)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**4, x)
```

3.78 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{63ia^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64x}+\frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2}-\cdots$$

```
[Out] -1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4-3/8*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^
(3/4)/x^3+15/32*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2+63/64*I*a^3*(1-I*a*
x)^(1/4)*(1+I*a*x)^(3/4)/x+123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4
))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2}+\frac{63ia^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) - (((3*I)/8)*a*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x^3 + (15*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/
(32*x^2) + (((63*I)/64)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+iax)^{3/4}}{x^5(1-iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{\frac{9ia}{2}-3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4}+9ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{1}{24}\int \frac{-\frac{1}{}}{x^2(1-}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-ia}}{64}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-ia}}{64}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-ia}}{64}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-ia}}{64}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-ia}}{64}$$

Mathematica [C] time = 0.03, size = 99, normalized size = 0.49

$$-\frac{\sqrt[4]{1-iax}\left(246a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+63a^4x^4-93ia^3x^3-54a^2x^2+40iax+16\right)}{64x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -1/64*((1 - I*a*x)^(1/4)*(16 + (40*I)*a*x - 54*a^2*x^2 - (93*I)*a^3*x^3 + 6
3*a^4*x^4 + 246*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]
))/(x^4*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.44, size = 195, normalized size = 0.97

$$-\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-123\,a^4x^4}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) - (126*I*a^3*x^3 + 60*a^2*x^2 - 48*I*a*x - 32)*sqrt(a^
2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
[Out] Timed out
```

3.79 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=373

$$-\frac{i(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}+\frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}\,a^4}-\frac{475\log}{}$$

```
[Out] 475/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4-4*I*x^3*(1+I*a*x)^(5/4)/a/(1-I*a
*x)^(1/4)-17/4*x^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^2-1/96*I*(521*I-452*a*
x)*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^4-475/128*arctan(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/128*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/256*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 97, 153, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}-\frac{i(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}+\frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1}}{\sqrt{1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (475*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) - ((4*I)*x^3*(1 + I*a*x)
^(5/4))/(a*(1 - I*a*x)^(1/4)) - (17*x^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4)
)/(4*a^2) - ((I/96)*(521*I - 452*a*x)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/
a^4 - (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) + (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{x^2\sqrt[4]{1+iax}\left(3+\frac{17iax}{4}\right)}{\sqrt[4]{1-iax}}\,dx}{a}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} + \frac{i\int \frac{x\sqrt[4]{1+iax}\left(-\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \cdots$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)\cdots}{\cdots}$$

Mathematica [C] time = 0.05, size = 96, normalized size = 0.26

$$\frac{380\sqrt[4]{2}\,(1-iax)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right) - \sqrt[4]{1+iax}\,(ax-i)^2\left(6a^2x^2-5iax+59\right)}{24a^4\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(59 - (5*I)*a*x + 6*a^2*x^2)) + 380*2^(1/
4)*(1 - I*a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(24*a^4*(1
 - I*a*x)^(1/4))
```

fricas [A] time = 0.56, size = 251, normalized size = 0.67

$$-\frac{96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\,\log\left(\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}} + \sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right) - 96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\,\log\left(-\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}} + \sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right) + 96\,a^4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/192*(96*a^4*sqrt(225625/4096*I/a^8)*log(64/475*a^4*sqrt(225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(225625/4096*I/a^8)*
log(-64/475*a^4*sqrt(225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + 96*a^4*sqrt(-225625/4096*I/a^8)*log(64/475*a^4*sqrt(-225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(-225625/4096*I/a^8)
*log(-64/475*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) + (48*a^4*x^4 - 136*I*a^3*x^3 - 226*a^2*x^2 + 521*I*a*x - 2467)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3}\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
```

```
[Out] Timed out
```

3.80 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=371

$$\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\sqrt{\ }\right.}{16\sqrt{2}\,a^{\ }}$$

```
[Out] 55/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+11/4*I*(1-I*a*x)^(3/4)*(1+I*a*x)
^(5/4)/a^3+2*I*(1+I*a*x)^(9/4)/a^3/(1-I*a*x)^(1/4)+1/3*I*(1-I*a*x)^(3/4)*(1
+I*a*x)^(9/4)/a^3-55/16*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))
/a^3*2^(1/2)+55/16*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*
2^(1/2)+55/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2
)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-55/32*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*
x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\sqrt{\ }\right.}{16\sqrt{2}\,a^{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((55*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + (((11*I)/4)*(1 - I*a
*x)^(3/4)*(1 + I*a*x)^(5/4))/a^3 + ((2*I)*(1 + I*a*x)^(9/4))/(a^3*(1 - I*a*
x)^(1/4)) + ((I/3)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(9/4))/a^3 - (((55*I)/8)*A
rcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt
[2]*a^3) + (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*
(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((55*I)/16)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} - \frac{(2i)\int \frac{(1+iax)^{5/4}\left(\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{11\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{2a^2}$$

$$= \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{55\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax}{3a^3}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.23

$$\frac{44\sqrt[4]{2}(ax+i)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right) - \sqrt[4]{1+iax}\,(ax-i)^2(ax+7i)}{3a^3\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(7*I + a*x)) + 44*2^(1/4)*(I + a*x)*Hyper
geometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(3*a^3*(1 - I*a*x)^(1/4))
```

fricas [A] time = 0.52, size = 244, normalized size = 0.66

$$12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\,\log\left(\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\,\log\left(-\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-\frac{30}{64}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*a^3*sqrt(3025/64*I/a^6)*log(8/55*I*a^3*sqrt(3025/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(3025/64*I/a^6)*log(-8/55*I*a^
3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(
-3025/64*I/a^6)*log(8/55*I*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I))) - 12*a^3*sqrt(-3025/64*I/a^6)*log(-8/55*I*a^3*sqrt(-3025/64
*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 26*I*a^2*x^2
- 61*a*x - 287*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m
 & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{1 + a x 1\mathrm{i}}{\sqrt{a^2 x^2 + 1}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**2,x)
[Out] Timed out
```

3.81 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=324

$$-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{}}{\sqrt{}}\right.}{}$$

```
[Out] -25/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2-5/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/
4)/a^2-2*(1+I*a*x)^(9/4)/a^2/(1-I*a*x)^(1/4)+25/8*arctan(1-(1-I*a*x)^(1/4)*
2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/8*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/16*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(
1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)+25/16*ln(1+(1-I*a*x)^(1/4
)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{}}{\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (-25*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) - (5*(1 - I*a*x)^(3/4)*(1
 + I*a*x)^(5/4))/(2*a^2) - (2*(1 + I*a*x)^(9/4))/(a^2*(1 - I*a*x)^(1/4)) +
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) - (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$
$$= -\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(5i)\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{a}$$
$$= -\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\right.}{2a^2}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx\right.}{2a^2}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx\right.}{4a^2}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+}\right.}{8a^2}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{8\sqrt{2}\,a^2}$$
$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+i}}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.22

$$\frac{2\left(20i\sqrt[4]{2}\,(ax+i)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right)-3(1+iax)^{9/4}\right)}{3a^2\sqrt[4]{1-iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (2*(-3*(1 + I*a*x)^(9/4) + (20*I)*2^(1/4)*(I + a*x)*Hypergeometric2F1[-5/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(3*a^2*(1 - I*a*x)^(1/4))
```

fricas [A] time = 0.80, size = 236, normalized size = 0.73

$$2\,a^2\sqrt{\frac{625i}{16\,a^4}}\,\log\left(\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{625i}{16\,a^4}}\,\log\left(-\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{625i}{16\,a^4}}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(625/16*I/a^4)*log(4/25*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(625/16*I/a^4)*log(-4/25*a^2*sqrt(625
/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-625/16*I/a^
4)*log(4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- 2*a^2*sqrt(-625/16*I/a^4)*log(-4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 - 9*I*a*x + 43)*sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x,x)
```

```
[Out] Timed out
```

3.82 $\int e^{\frac{5}{2} i \tan^{-1}(ax)}\,dx$

Optimal. Leaf size=299

$$-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i}{}$$

```
[Out] -5*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-4*I*(1+I*a*x)^(5/4)/a/(1-I*a*x)^(1/4
)+5/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/2*I*a
rctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/4*I*ln(1-(1-I*
a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/
2)+5/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*
a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-5*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((4*I)*(1 + I*a*x)^(5/4))/
(a*(1 - I*a*x)^(1/4)) + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (
((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{5}{2}i\tan^{-1}(ax)}\,dx &= \int \frac{(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx \\
&= -\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - 5\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5}{2}\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a} \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(5i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} - \frac{(5i)\,\mathrm{Subst}\left(\right.}{} \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(5i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(5i)\,\mathrm{Su}}{} \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{5i\log\left(1+\right.}{} \\
&= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}a}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 41, normalized size = 0.14

$$
-\frac{8ie^{\frac{9}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(2,\frac{9}{4};\frac{13}{4};-e^{2i\tan^{-1}(ax)}\right)}{9a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/9)*E^(((9*I)/2)*ArcTan[a*x])*Hypergeometric2F1[2, 9/4, 13/4, -E^((
2*I)*ArcTan[a*x])])/a
```

fricas [A] time = 0.42, size = 209, normalized size = 0.70

$$
-\frac{a\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}ia\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}ia\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + a\sqrt{-\frac{25i}{a^2}}\log\left(\frac{1}{5}ia\sqrt{-\frac{25i}{a^2}}+\sqrt{\right.}}{2a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*sqrt(25*I/a^2)*log(1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I))) - a*sqrt(25*I/a^2)*log(-1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-25*I/a^2)*log(1/5*I*a*sqrt(-25*I/a^2) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-25*I/a^2)*log(-1/5*I*a*sqrt(
```

```
-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (2*a*x + 18*I)*sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.83 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=293

$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\right.$$

```
[Out] 8*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5062, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (8*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1+iax)^{5/4}}{x(1-iax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{-\frac{ia}{4}-\frac{a^2x}{4}}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - (ia)\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right.$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\,\text{S}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \text{l}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+ia}}\right.$$

Mathematica [C] time = 0.04, size = 112, normalized size = 0.38

$$\frac{4\left(3\sqrt[4]{2}(1+iax)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-iax)\right) + (-1+iax)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right) + 3iax + 3\right)}{3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (4*(3 + (3*I)*a*x + 3*2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[-1/4, -1/
4, 3/4, (1 - I*a*x)/2] + (-1 + I*a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.73, size = 267, normalized size = 0.91

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x}}{ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
```

```
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 8
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) -
 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)/x, x)
```

3.84 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=121

$$-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 10*I*a*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(5/4)/x/(1-I*a*x)^(1/4)-5*
I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-5*I*a*arctanh((1+I*a*x)^(1/4)/(
1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}+\frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((10*I)*a*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(5/4)/(x*(1 -
I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - (5*I)
*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx \\
&= -\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + \frac{1}{2}(5ia)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx \\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + \frac{1}{2}(5ia)\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + (10ia)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - (5ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (5ia)\,\text{Subst}\left(\int \frac{}{1}\right. \\
&= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - 5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 87, normalized size = 0.72

$$\frac{-3\left(9a^2x^2-8iax+1\right)-10ax(ax+i)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)}{3x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (-3*(1 - (8*I)*a*x + 9*a^2*x^2) - 10*a*x*(I + a*x)*Hypergeometric2F1[3/4, 1
, 7/4, (I + a*x)/(I - a*x)])/(3*x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.42, size = 152, normalized size = 1.26

$$\frac{-5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 5*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 5*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - I) + 5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(
-9*I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

3.85 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}$$

```
[Out] -25/2*a^2*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-5/4*I*a*(1+I*a*x)^(5/4)/x/(1-I*a*
x)^(1/4)-1/2*(1+I*a*x)^(9/4)/x^2/(1-I*a*x)^(1/4)+25/4*a^2*arctan((1+I*a*x)^
(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$-\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (-25*a^2*(1 + I*a*x)^(1/4))/(2*(1 - I*a*x)^(1/4)) - (((5*I)/4)*a*(1 + I*a*x
)^(5/4))/(x*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(9/4)/(2*x^2*(1 - I*a*x)^(1/4)
) + (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1+iax)^{5/4}}{x^3(1-iax)^{5/4}}\,dx\\
&= -\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{1}{4}(5ia)\int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx\\
&= -\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+i}}{\sqrt[4]{1-i}}\right.\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right.\\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\right.
\end{aligned}$$

Mathematica [C] time = 0.03, size = 99, normalized size = 0.61

$$\frac{-129ia^3x^3+50a^2x^2(1-iax)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)-102a^2x^2-33iax-6}{12x^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (-6 - (33*I)*a*x - 102*a^2*x^2 - (129*I)*a^3*x^3 + 50*a^2*x^2*(1 - I*a*x)*H
ypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(12*x^2*(1 - I*a*x)^(1/
4)*(1 + I*a*x)^(3/4))
```

fricas [A] time = 1.01, size = 176, normalized size = 1.08

$$\frac{25\,a^2x^2\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25i\,a^2x^2\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+i\right)-25i\,a^2x^2\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}-i\right)-25\,a^2x^2\log\left(\cdots\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 25*I*a^2*x^2
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 25*I*a^2*x^2*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) - 25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - 1) - (86*a^2*x^2 + 18*I*a*x + 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

3.86 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}-\frac{13ia\sqrt[4]{}}{12x^2\sqrt[4]{}}$$

```
[Out] -287/24*I*a^3*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/3*(1+I*a*x)^(1/4)/x^3/(1-I*
a*x)^(1/4)-13/12*I*a*(1+I*a*x)^(1/4)/x^2/(1-I*a*x)^(1/4)+61/24*a^2*(1+I*a*x
)^(1/4)/x/(1-I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)
)+55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 212, 206, 203}

$$-\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}}+\frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1}}{3x^3\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (((-287*I)/24)*a^3*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(1/4)
/(3*x^3*(1 - I*a*x)^(1/4)) - (((13*I)/12)*a*(1 + I*a*x)^(1/4))/(x^2*(1 - I*
a*x)^(1/4)) + (61*a^2*(1 + I*a*x)^(1/4))/(24*x*(1 - I*a*x)^(1/4)) + ((55*I)
/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((55*I)/8)*a^3*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^4}dx = \int \frac{(1+iax)^{5/4}}{x^4(1-iax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{1}{3}\int \frac{-\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}-13ia^3x}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{6}\int \frac{\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{i\int \frac{165a^4}{16x\sqrt[4]{1-iax}(1+iax)^{3/}}}{3a}$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{16}\left(55ia^3\right)\int \frac{}{x\sqrt[4]{1}}$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{4}\left(55ia^3\right)\text{Subst}\Bigg(\int$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{1}{8}\left(55ia^3\right)\text{Subst}\Bigg(\int$$
$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.03, size = 106, normalized size = 0.52

$$\frac{287a^4x^4+110a^3x^3(ax+i)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)-226ia^3x^3+87a^2x^2-34iax-8}{24x^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (-8 - (34*I)*a*x + 87*a^2*x^2 - (226*I)*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^3
*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(24*x^3*(1
- I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.73, size = 183, normalized size = 0.90

$$\frac{165i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-165i\,a^3x^3\,\mathrm{lo}}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*a^3*
x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 165*a^3*x^3*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I)) - 1) + (-574*I*a^3*x^3 + 122*a^2*x^2 - 52*I*a*x - 16)*sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

3.87 $\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=233

$$\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}-$$

```
[Out] 2467/192*a^4*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/4*(1+I*a*x)^(1/4)/x^4/(1-I*a
*x)^(1/4)-17/24*I*a*(1+I*a*x)^(1/4)/x^3/(1-I*a*x)^(1/4)+113/96*a^2*(1+I*a*x
)^(1/4)/x^2/(1-I*a*x)^(1/4)+521/192*I*a^3*(1+I*a*x)^(1/4)/x/(1-I*a*x)^(1/4)
-475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.10, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] (2467*a^4*(1 + I*a*x)^(1/4))/(192*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(1/4)/(4
*x^4*(1 - I*a*x)^(1/4)) - (((17*I)/24)*a*(1 + I*a*x)^(1/4))/(x^3*(1 - I*a*x
)^(1/4)) + (113*a^2*(1 + I*a*x)^(1/4))/(96*x^2*(1 - I*a*x)^(1/4)) + (((521*
I)/192)*a^3*(1 + I*a*x)^(1/4))/(x*(1 - I*a*x)^(1/4)) - (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}i\tan^{-1}(ax)}}{x^5}dx = \int \frac{(1+iax)^{5/4}}{x^5(1-iax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{1}{4}\int \frac{-\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{1}{12}\int \frac{-\frac{113a^2}{4}-\frac{51}{2}ia^3x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} - \frac{1}{24}\int \frac{\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}dx$$
$$= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}+}{x(1-iax)^{5/}}$$
$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} +$$
$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} +$$
$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} +$$
$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} -$$
$$= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} -$$

Mathematica [C] time = 0.04, size = 118, normalized size = 0.51

$$\frac{2467ia^5x^5 + 950ia^4x^4(ax+i)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right) + 1946a^4x^4 + 747ia^3x^3 + 362a^2x^2 - 184iax - 48}{192x^4\sqrt[4]{1-iax}\,(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] (-48 - (184*I)*a*x + 362*a^2*x^2 + (747*I)*a^3*x^3 + 1946*a^4*x^4 + (2467*I
)*a^5*x^5 + (950*I)*a^4*x^4*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)])/(192*x^4*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.52, size = 192, normalized size = 0.82

$$-\frac{1425\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) + 1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) - 1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) - 1425}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(1425*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*I*
a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 1425*I*a^4*x^4*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*a^4*x^4*log(sqrt(I*sqrt(a^2*
```

```
x^2 + 1)/(a*x + I)) - 1) - (4934*a^4*x^4 + 1042*I*a^3*x^3 + 452*a^2*x^2 - 2
72*I*a*x - 96)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

3.88 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=337

$$-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}\,a^4}+\frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{128}$$

```
[Out] -11/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4+1/4*x^2*(1-I*a*x)^(5/4)*(1+I*a*x
)^(3/4)/a^2-1/96*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)*(25-4*I*a*x)/a^4-11/128*ar
ctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+11/128*arctan(1
+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-11/256*ln(1-(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
+11/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a
*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}-\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ia}}\right.}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (-11*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(5/4)
*(1 + I*a*x)^(3/4))/(4*a^2) - ((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(25 - (4
*I)*a*x))/(96*a^4) - (11*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(64*Sqrt[2]*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (
11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} + \frac{\int \frac{x\left(-2+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{(11i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{64a^3}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax}{96a^4}$$

Mathematica [C] time = 0.11, size = 127, normalized size = 0.38

$$\frac{(1-iax)^{5/4}\left(5a^2x^2(1+iax)^{3/4} + 4\,2^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-iax)\right) - 12\,2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-iax)\right) + 5\,2^{3/4}\,{}_2F_1\right.}{20a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*a^2*x^2*(1 + I*a*x)^(3/4) + 4*2^(3/4)*Hypergeometric2
F1[-7/4, 5/4, 9/4, (1 - I*a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/4,
```

```
 9/4, (1 - I*a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - I*a*
x)/2]))/(20*a^4)
```

fricas [A] time = 0.54, size = 255, normalized size = 0.76

$$-\frac{96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(-\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/192*(96*a^4*sqrt(121/4096*I/a^8)*log(64/11*I*a^4*sqrt(121/4096*I/a^8) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(121/4096*I/a^8)*log(-64/
11*I*a^4*sqrt(121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a
^4*sqrt(-121/4096*I/a^8)*log(64/11*I*a^4*sqrt(-121/4096*I/a^8) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + 96*a^4*sqrt(-121/4096*I/a^8)*log(-64/11*I*a^4*
sqrt(-121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (-48*I*a^3*x
^3 + 56*a^2*x^2 + 58*I*a*x - 83)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**3/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.89 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$\frac{i(1+iax)^{3/4}(1-iax)^{5/4}}{12a^3}+\frac{3i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right.}{16\sqrt{2}\,a^3}$$

```
[Out] 3/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3+1/12*I*(1-I*a*x)^(5/4)*(1+I*a*x)^
(3/4)/a^3+1/3*x*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2+3/16*I*arctan(1-(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/16*I*arctan(1+(1-I*a*x)^(1/
4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)+3/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-3/32*I*ln(1+(
1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3
*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{x(1+iax)^{3/4}(1-iax)^{5/4}}{3a^2}+\frac{i(1+iax)^{3/4}(1-iax)^{5/4}}{12a^3}+\frac{3i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}-3i\,\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (((3*I)/8)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 + ((I/12)*(1 - I*a*x)^(
5/4)*(1 + I*a*x)^(3/4))/a^3 + (x*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(3*a^
2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/
(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*
x)^(1/4)])/(Sqrt[2]*a^3) + (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*
I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{\int \frac{\left(-1+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{3a^2}$$

$$= \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{1}{(1-iax)^{3/4}\cdots}}{16a\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\,\mathrm{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\log\Big(1+\cdots}{\cdots}$$

$$= \frac{3i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\tan^{-1}\Big(1-\cdots}{8\sqrt{\cdots}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.22

$$\frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}(4ax+i)-9i2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-iax)\right)\right)}{60a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4)*(I + 4*a*x) - (9*I)*2^(3/4)*Hyperge
ometric2F1[1/4, 5/4, 9/4, (1 - I*a*x)/2]))/(60*a^3)
```

fricas [A] time = 0.45, size = 247, normalized size = 0.73

$$-\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\,\log\left(\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\,\log\left(-\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{-\frac{9i}{64\,a^6}}\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*a^3*sqrt(9/64*I/
a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-9/64*I/a^6)*log(
8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*s
qrt(-9/64*I/a^6)*log(-8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - sqrt(a^2*x^2 + 1)*(-8*I*a^2*x^2 + 10*a*x + 11*I)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**2/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.90 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+\frac{(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\cdots$$

```
[Out] 1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)
/a^2+1/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-1/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)-1/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+\frac{(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(5/4)*(1 + I*a
*x)^(3/4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{i\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{i\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{\text{Sub}}{}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}\,a^2} - \frac{\log\left(1\right.}{}$$

$$= \frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}\,a^2} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}}{\cdots}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4} - 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-iax)\right)\right)}{10a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 5/
4, 9/4, (1 - I*a*x)/2]))/(10*a^2)
```

fricas [A] time = 0.50, size = 238, normalized size = 0.81

$$\frac{2a^2\sqrt{\frac{i}{16a^4}}\log\left(4ia^2\sqrt{\frac{i}{16a^4}} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - 2a^2\sqrt{\frac{i}{16a^4}}\log\left(-4ia^2\sqrt{\frac{i}{16a^4}} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - 2a^2\sqrt{-\frac{i}{16a^4}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*I*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*I*a^2*sqrt(1/16*I/a^4)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I/a^4)*log(4*I*a
^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1
/16*I/a^4)*log(-4*I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
```

```
 I))) + sqrt(a^2*x^2 + 1)*(-2*I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.91 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$-\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a}+$$

```
[Out] -I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-1/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4))/a*2^(1/2)+1/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x
)^(1/4))/a*2^(1/2)-1/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*
a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)+1/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a}+$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I/2)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((I/2)*Log[1 + Sqrt[1 - I*
a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}\,dx = \int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{2a}$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a} +$$
$$= -\frac{i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{1-ia}}{\sqrt{1+ia}}\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.15

$$-\frac{8ie^{\frac{3}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2i\tan^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-1/2*I)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^((2
*I)*ArcTan[a*x])])/a
```

fricas [A] time = 0.42, size = 213, normalized size = 0.79

$$\frac{a\sqrt{\frac{i}{a^2}}\log\left(a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{i}{a^2}}\log\left(-a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{-\frac{i}{a^2}}\log\left(a\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + a}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - a*sqrt(I/a^2)*log(-a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- a*sqrt(-I/a^2)*log(a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ a*sqrt(-I/a^2)*log(-a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

3.92 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.$$

```
[Out] 2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4))-1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/
(1+I*a*x)^(1/2))*2^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(
1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\,dx$$

$$= -\left((ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\right) + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= 4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) + 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\,\mathrm{Subst}\Bigg(\int$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\,\mathrm{Subs}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\mathrm{Subst}\Big(\int}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\Big(1+\frac{\sqrt{1-i}}{\sqrt{1+i}}}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\tanh$$

Mathematica [C] time = 0.03, size = 96, normalized size = 0.36

$$2\,2^{3/4}\sqrt[4]{1-iax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\;{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{1-iax}{-iax-1}\right)}{\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

```
[Out] 2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2]
 - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
I*a*x))])/(1 + I*a*x)^(1/4)
```

fricas [A] time = 0.47, size = 243, normalized size = 0.91

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I)
+ log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
[Out] Integral(1/(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

3.93 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))+I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x}-ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] + I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} - \frac{1}{2}(ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} - (2ia)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} + (ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x \right. \\
&= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{x} - ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.75

$$
\frac{i\sqrt[4]{1-iax}\left(2ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)-ax+i\right)}{x\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(I - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

fricas [B] time = 0.46, size = 156, normalized size = 1.70

$$
\frac{i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)-2\sqrt{}}{2x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
)) - I) - I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)

[Out] Integral(1/(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

3.94 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2}+\frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}$$

```
[Out] 1/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/
4)/x^2-1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2}+\frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((I/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(5/4)*(1 + I
*a*x)^(3/4))/(2*x^2) - (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
+ (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{1-iax}}{x^3\sqrt[4]{1+iax}}\,dx$$
$$= -\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}(ia)\int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx$$
$$= \frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{8}a^2\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$= \frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$= \frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} + \frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) -$$
$$= \frac{ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

Mathematica [C] time = 0.02, size = 81, normalized size = 0.61

$$\frac{\sqrt[4]{1-iax}\left(2a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)-3a^2x^2+iax-2\right)}{4x^2\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 + I*a*x - 3*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F1[1/
4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.44, size = 178, normalized size = 1.35

$$\frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - 2*sqrt(a^2*x^2 + 1)*(-3*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}}\, x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt{\frac{i\, ax+1}{\sqrt{a^{2}x^{2}+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3} \sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
[Out] Integral(1/(x**3*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

3.95 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x}-\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-i}}{}$$

```
[Out] -1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3+5/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^2+11/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3/8*I*a^3*arctan((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))-3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x}+\frac{3}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-i}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(3*x^3) + (((5*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (11*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1-iax}}{x^4\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{-\frac{5ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{\frac{11a^2}{4}-\frac{5}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{1}{6}\int \frac{}{8x(1-}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{1}{16}\left(3ia^3\right)\int$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{1}{4}\left(3ia^3\right)\mathrm{Sub}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} - \frac{1}{8}\left(3ia^3\right)\mathrm{Sub}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x} + \frac{3}{8}ia^3\tan^{-1}\Big($$

Mathematica [C] time = 0.03, size = 92, normalized size = 0.54

$$\frac{\sqrt[4]{1-iax}\left(-18ia^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+11ia^3x^3+a^2x^2+2iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 + (2*I)*a*x + a^2*x^2 + (11*I)*a^3*x^3 - (18*I)*a^3*
x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*a*
x)^(1/4))
```

fricas [A] time = 0.49, size = 186, normalized size = 1.09

$$\frac{-9i\,a^3x^3\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}-i\right)+9i\,a^3x^3\log\left(\sqrt{\frac{i}{}}\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(-9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) + 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)) - 1) + (22*a^2*x^2 + 20*I*a*x - 16)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

3.96 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=202

$$\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}+\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{\sqrt[4]{1}}{}$$

```
[Out] -1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4+7/24*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^3+29/96*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2-83/192*I*a^3*(1-I*
a*x)^(1/4)*(1+I*a*x)^(3/4)/x+11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 298, 203, 206}

$$\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*x^4) + (((7*I)/24)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^3 + (29*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(96*x^2) - (((83*I)/192)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2} i \tan^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1-iax}}{x^5\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{-\frac{7ia}{2} - 3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} - \frac{1}{12}\int \frac{\frac{29a^2}{4} - 7ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} + \frac{1}{24}\int \frac{\cdots}{x^2(1 \cdots}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-i \cdots}}{1\cdots}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-i \cdots}}{1\cdots}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-i \cdots}}{1\cdots}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-i \cdots}}{1\cdots}$$

$$= -\frac{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-i \cdots}}{1\cdots}$$

Mathematica [C] time = 0.03, size = 99, normalized size = 0.49

$$\frac{\sqrt[4]{1-iax}\left(-66a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+83a^4x^4-25ia^3x^3+2a^2x^2+8iax-48\right)}{192x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-48 + (8*I)*a*x + 2*a^2*x^2 - (25*I)*a^3*x^3 + 83*a^4*x
^4 - 66*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(192*
x^4*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.46, size = 195, normalized size = 0.97

$$-\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a^4x^4\log\left(\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - (-166*I*a^3*x^3 + 116*a^2*x^2 + 112*I*a*x - 96)*sqrt(a^2
*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
[Out] Integral(1/(x**5*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

3.97 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=337

$$-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4}-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}\,a^4}-\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{128}$$

```
[Out] -41/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4+1/4*x^2*(1-I*a*x)^(7/4)*(1+I*a*x
)^(1/4)/a^2-1/32*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)*(11-4*I*a*x)/a^4-123/128*a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/128*arctan
(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1
/2)-123/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 100, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4}-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-41*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) + (x^2*(1 - I*a*x)^(7/4)
*(1 + I*a*x)^(1/4))/(4*a^2) - ((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4)*(11 - (4
*I)*a*x))/(32*a^4) - (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} + \frac{\int \frac{x(1-iax)^{3/4}\left(-2+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} - \frac{(41i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} -$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} +$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} +$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} -$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} +$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} +$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}\,(11-4iax)}{32a^4} -$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.38

$$\frac{(1-iax)^{7/4}\left(7a^2x^2\sqrt[4]{1+iax}+12\sqrt[4]{2}\,{}_2F_1\left(-\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-iax)\right)-20\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-iax)\right)+7\sqrt[4]{2}\,{}_2F_1\left(\right.\right.}{28a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*a^2*x^2*(1 + I*a*x)^(1/4) + 12*2^(1/4)*Hypergeometric
2F1[-5/4, 7/4, 11/4, (1 - I*a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4, 7/
```

```
4, 11/4, (1 - I*a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 -
I*a*x)/2]))/(28*a^4)
```

fricas [A] time = 0.43, size = 251, normalized size = 0.74

$$-\frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\,\log\left(\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\,\log\left(-\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)+32\,a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*a^4*sqrt(15129/4096*I/a^8)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log(
-64/123*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*a^4*sqrt(-15129/4096*I/a^8) + sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(-64
/123*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (
16*a^4*x^4 + 40*I*a^3*x^3 - 54*a^2*x^2 - 93*I*a*x + 63)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.98 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=339

$$\frac{i\sqrt[4]{1+iax}\,(1-iax)^{7/4}}{4a^3}+\frac{17i\sqrt[4]{1+iax}\,(1-iax)^{3/4}}{24a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\right.}{16\sqrt{2}\,a^3}$$

```
[Out] 17/24*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+1/4*I*(1-I*a*x)^(7/4)*(1+I*a*x)
^(1/4)/a^3+1/3*x*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)/a^2+17/16*I*arctan(1-(1-I*
a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/16*I*arctan(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/32*I*ln(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+17/32*I*l
n(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)
)/a^3*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{x\sqrt[4]{1+iax}\,(1-iax)^{7/4}}{3a^2}+\frac{i\sqrt[4]{1+iax}\,(1-iax)^{7/4}}{4a^3}+\frac{17i\sqrt[4]{1+iax}\,(1-iax)^{3/4}}{24a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}\,a^3}+\frac{17i\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((17*I)/24)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + ((I/4)*(1 - I*a*x)^
(7/4)*(1 + I*a*x)^(1/4))/a^3 + (x*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(3*a
^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]
)/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
 I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^
(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{\int \frac{(1-iax)^{3/4}\left(-1+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{3a^2}$$

$$= \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{24a^2}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{1}{\sqrt[4]{1-iax}\,(1+}}{16a^2}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{(17i)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17i\log\left(1+\frac{\sqrt{}}{\sqrt{}}\right.}{16}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{17i\tan^{-1}\left(1-\right.}{8\sqrt{2}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.22

$$\frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}\,(4ax+3i)-17i\sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-iax)\right)\right)}{84a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4)*(3*I + 4*a*x) - (17*I)*2^(1/4)*Hype
rgeometric2F1[3/4, 7/4, 11/4, (1 - I*a*x)/2]))/(84*a^3)
```

fricas [A] time = 0.45, size = 243, normalized size = 0.72

$$-\frac{12\,a^{3}\sqrt{\frac{289i}{64\,a^{6}}}\log\left(\frac{8}{17}i\,a^{3}\sqrt{\frac{289i}{64\,a^{6}}}+\sqrt{\frac{i\sqrt{a^{2}x^{2}+1}}{ax+i}}\right)-12\,a^{3}\sqrt{\frac{289i}{64\,a^{6}}}\log\left(-\frac{8}{17}i\,a^{3}\sqrt{\frac{289i}{64\,a^{6}}}+\sqrt{\frac{i\sqrt{a^{2}x^{2}+1}}{ax+i}}\right)+12\,a^{3}\sqrt{-\frac{28}{64}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*I*a^3*sqrt(289/64*I/a^6) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*I*a^3*
sqrt(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-28
9/64*I/a^6)*log(8/17*I*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*I*a^3*sqrt(-289/64*I/a^6)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (8*a^3*x^3 + 22*I*a^2*x^2 - 37*a*
x - 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.99 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=295

$$\frac{\sqrt[4]{1+iax}\,(1-iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1+iax}\,(1-iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}$$

```
[Out] 3/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)
/a^2+9/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)+9/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{1+iax}\,(1-iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1+iax}\,(1-iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}\,a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(7/4)*(1 + I
*a*x)^(1/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(3i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{4a}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(9i)\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx}{8a}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{9\,\text{Subs}}{}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{9}{}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}\,a^2} + \frac{9\log(}{}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}\,a^2} - \frac{9\tan^{-1}\left(1+\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.21

$$\frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}-3\sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-iax)\right)\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4) - 3*2^(1/4)*Hypergeometric2F1[3/4,
7/4, 11/4, (1 - I*a*x)/2]))/(14*a^2)
```

fricas [A] time = 0.48, size = 236, normalized size = 0.80

$$\frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{81i}{16\,a^4}}\log\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*a^2*sqrt(81/16*I/
a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-81/16*I/a^4)*log(
4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*s
qrt(-81/16*I/a^4)*log(-4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1
```

```
)/(a*x + I))) - (2*a^2*x^2 + 7*I*a*x - 5)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueWarning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWar
ning, need to choose a branch for the root of a polynomial with parameters.
 This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m o
perator + Error: Bad Argument ValueWarning, need to choose a branch for the
 root of a polynomial with parameters. This might be wrong.The choice was d
one assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m
operator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.100 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=268

$$-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

```
[Out] -I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-3/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4))/a*2^(1/2)+3/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x
)^(1/4))/a*2^(1/2)+3/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*
a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)-3/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5061, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}\,dx = \int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(6i)\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(6i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{(3i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{a}$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\right.}{2a}$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a}$$
$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a} + \frac{3i\log\left(1+\right.}{}$$

Mathematica [C] time = 0.04, size = 39, normalized size = 0.15

$$-\frac{8ie^{\frac{1}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-8*I)*E^((I/2)*ArcTan[a*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*ArcT
an[a*x])])/a
```

fricas [A] time = 0.51, size = 210, normalized size = 0.78

$$\frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}ia\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}ia\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}ia\sqrt{-\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+}}{ax+i}}\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(1/3*I*a*sqrt(-9*I/a^2) + sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(-1/3*I*a*sqrt(-9*I/a^2
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a*x + 2*I)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-3/2), x)
```

3.101 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=267

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-$$

```
[Out] -2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)
/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+
(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(
1+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2
^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x}\,dx &= \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx \\
&= -\left((ia)\int \frac{1}{\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx\right)+\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{1-iax}\right)+4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+4\,\text{Subst}\Bigg(\int \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+2\,\text{Sub} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}+\frac{\text{Subst}}{} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1-}}\right.}{} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\tan
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 97, normalized size = 0.36

$$
\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}\,(1+iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-iax)\right)-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)\right)}{3(1+iax)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (1 - I*a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)
]))/(3*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.63, size = 243, normalized size = 0.91

$$
-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x}}{ax}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
[Out] Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

3.102 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))+3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 212, 206, 203}

$$-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] + (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-\frac{1}{2}(3ia)\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}-(6ia)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+(3ia)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+(3ia)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx\right. \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+3ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+3ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.75

$$\frac{i(1-iax)^{3/4}\left(2ax\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)-ax+i\right)}{x(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(3/4)*(I - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

fricas [B] time = 1.11, size = 152, normalized size = 1.65

$$\frac{3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - I) - 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-
I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

3.103 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=132

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[Out] 3/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/
4)/x^2+9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5062, 96, 94, 93, 212, 206, 203}

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] (((3*I)/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(7/4)*(1
 + I*a*x)^(1/4))/(2*x^2) + (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4
)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-iax)^{3/4}}{x^3(1+iax)^{3/4}}\,dx\\ &= -\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}-\frac{1}{4}(3ia)\int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx\\ &= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}-\frac{1}{8}\left(9a^2\right)\int \frac{1}{x\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx\\ &= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}-\frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\cdots}}{\sqrt[4]{1-\cdots}}\right.\\ &= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ia\cdots}}{\sqrt[4]{1-ia\cdots}}\right.\\ &= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2}+\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\tanh^{-1}\left(\cdots\right.\end{aligned}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.61

$$\frac{(1-iax)^{3/4}\left(6a^2x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)-5a^2x^2+3iax-2\right)}{4x^2(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-2 + (3*I)*a*x - 5*a^2*x^2 + 6*a^2*x^2*Hypergeometric2F
1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.42, size = 175, normalized size = 1.33

$$\frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a\cdots}}{a\cdots}}\right.\cdots}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) + (10*a^2*x^2 + 14*I*a*x - 4)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
)/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(\frac{i(ax-i)}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
[Out] Integral(1/(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

3.104 $\int \frac{e^{-\frac{3}{2} i \tan^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=170

$$-\frac{17}{8} i a^3 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8} i a^3 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+7ia(1-i\cdots$$

```
[Out] -1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+7/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^2+23/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-17/8*I*a^3*arctan((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8} i a^3 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8} i a^3 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-(1-ia\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(3*x^3) + (((7*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (23*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^4}\,dx &= \int \frac{(1-iax)^{3/4}}{x^4(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{1}{3}\int \frac{-\frac{7ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{1}{6}\int \frac{\frac{23a^2}{4}-\frac{7}{2}ia^3x}{x^2\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{6}\int \frac{\ }{8x\ \cdots} \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{16}\left(17ia\cdots\right. \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{4}\left(17ia^3\cdots\right. \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{1}{8}\left(17ia^3\cdots\right. \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8}ia^3\tan\cdots
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 93, normalized size = 0.55

$$\frac{(1-iax)^{3/4}\left(-34ia^3x^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+23ia^3x^3+9a^2x^2+6iax-8\right)}{24x^3(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-8 + (6*I)*a*x + 9*a^2*x^2 + (23*I)*a^3*x^3 - (34*I)*a^
3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*
a*x)^(3/4))
```

fricas [A] time = 0.42, size = 183, normalized size = 1.08

$$\frac{-51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+51i\,a^3x^3\log\left(\sqrt{\right.}\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(-51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x
^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) + 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)) - 1) + (-46*I*a^3*x^3 + 74*a^2*x^2 + 44*I*a*x - 16)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(\frac{i \left(a x - i\right)}{\sqrt{a^{2} x^{2} + 1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
[Out] Integral(1/(x**4*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

3.105 $\int \frac{e^{-\frac{3}{2} i \tan^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=202

$$-\frac{123}{64} a^4 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64} a^4 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}+\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{(1}{}$$

```
[Out] -1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4+3/8*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^
(1/4)/x^3+15/32*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-63/64*I*a^3*(1-I*a*
x)^(3/4)*(1+I*a*x)^(1/4)/x-123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4
))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5062, 99, 151, 12, 93, 212, 206, 203}

$$\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}-\frac{123}{64} a^4 \tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64} a^4 \tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3ia(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*x^4) + (((3*I)/8)*a*(1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x^3 + (15*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/
(32*x^2) - (((63*I)/64)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-iax)^{3/4}}{x^5(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{1}{4}\int \frac{-\frac{9ia}{2}-3a^2x}{x^4\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4}-9ia^3x}{x^3\sqrt[4]{1-iax}\,(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} + \frac{1}{24}\int \frac{}{x^2}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-}{}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-}{}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-}{}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-}{}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-}{}$$

Mathematica [C] time = 0.04, size = 99, normalized size = 0.49

$$\frac{(1-iax)^{3/4}\left(-82a^4x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{ax+i}{i-ax}\right)+63a^4x^4-33ia^3x^3+6a^2x^2+8iax-16\right)}{64x^4(1+iax)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-16 + (8*I)*a*x + 6*a^2*x^2 - (33*I)*a^3*x^3 + 63*a^4*x
^4 - 82*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(64*x
^4*(1 + I*a*x)^(3/4))
```

fricas [A] time = 0.50, size = 191, normalized size = 0.95

$$-\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-123\,a^4x^4\log}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) + (126*a^4*x^4 + 186*I*a^3*x^3 - 108*a^2*x^2 - 80*I*a*
x + 32)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Integral(1/(x**5*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

3.106 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=373

$$-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{96a^4}+\frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}\,a^4}-\frac{475\log\left(\cdots\right.}{}$$

```
[Out] 4*I*x^3*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+475/64*(1-I*a*x)^(1/4)*(1+I*a*x)^
(3/4)/a^4-17/4*x^2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-1/96*I*(1-I*a*x)^(5/
4)*(1+I*a*x)^(3/4)*(521*I+452*a*x)/a^4+475/128*arctan(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-475/128*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/256*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 97, 153, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{96a^4}+\frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\cdots\right.}{128\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((4*I)*x^3*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + (475*(1 - I*a*x)^(1/4
)*(1 + I*a*x)^(3/4))/(64*a^4) - (17*x^2*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)
)/(4*a^2) - ((I/96)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(521*I + 452*a*x))/
a^4 + (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) - (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{x^2\sqrt[4]{1-iax}\left(3-\frac{17iax}{4}\right)}{\sqrt[4]{1+iax}}\,dx}{a}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i\int \frac{x\sqrt[4]{1-iax}\left(\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}}{}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 0.27

$$-\frac{\sqrt[4]{1-iax}\,(ax+i)^2\left(3\left(6a^2x^2+5iax+59\right)-95\,2^{3/4}\sqrt[4]{1+iax}\;{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-iax)\right)\right)}{72a^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -1/72*((1 - I*a*x)^(1/4)*(I + a*x)^2*(3*(59 + (5*I)*a*x + 6*a^2*x^2) - 95*2
^(3/4)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))
/(a^4*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.43, size = 306, normalized size = 0.82

$$\left(96\,a^5x-96i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\,\log\left(\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-\left(96\,a^5x-96i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\,\log\left(-\frac{64}{475}i\,a^4\sqrt{\frac{225\ldots}{409\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] ((96*a^5*x - 96*I*a^4)*sqrt(225625/4096*I/a^8)*log(64/475*I*a^4*sqrt(225625
/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (96*a^5*x - 96*I*a^4)
*sqrt(225625/4096*I/a^8)*log(-64/475*I*a^4*sqrt(225625/4096*I/a^8) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - (96*a^5*x - 96*I*a^4)*sqrt(-225625/4096*I/
a^8)*log(64/475*I*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) + (96*a^5*x - 96*I*a^4)*sqrt(-225625/4096*I/a^8)*log(-64/475*I*a
^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (48*I*
a^4*x^4 - 136*a^3*x^3 - 226*I*a^2*x^2 + 521*a*x - 2467*I)*sqrt(a^2*x^2 + 1)
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(192*a^5*x - 192*I*a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

3.107 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=371

$$-\frac{i(1+iax)^{3/4}(1-iax)^{9/4}}{3a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{11i(1+iax)^{3/4}(1-iax)^{5/4}}{4a^3}-\frac{55i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}-\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\cdots\right.}{16\sqrt{2}\,a\cdots}$$

```
[Out] -2*I*(1-I*a*x)^(9/4)/a^3/(1+I*a*x)^(1/4)-55/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(
3/4)/a^3-11/4*I*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^3-1/3*I*(1-I*a*x)^(9/4)*(
1+I*a*x)^(3/4)/a^3-55/16*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)
)/a^3*2^(1/2)+55/16*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3
*2^(1/2)-55/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/
2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+55/32*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a
*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5062, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i(1+iax)^{3/4}(1-iax)^{9/4}}{3a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{11i(1+iax)^{3/4}(1-iax)^{5/4}}{4a^3}-\frac{55i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}-\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\cdots\right.}{16\sqrt{2}\,a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*(1 - I*a*x)^(9/4))/(a^3*(1 + I*a*x)^(1/4)) - (((55*I)/8)*(1 - I*a*x
)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - (((11*I)/4)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^
(3/4))/a^3 - ((I/3)*(1 - I*a*x)^(9/4)*(1 + I*a*x)^(3/4))/a^3 - (((55*I)/8)*
ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) +
(((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a^3) - (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((55*I)/16)*Log[1
+ Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} + \frac{(2i)\int \frac{(1-iax)^{5/4}\left(-\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{11\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{2a^2}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{55\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+ia}{3a^3}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.25

$$-\frac{\sqrt[4]{1-iax}\,(ax+i)^2\left(11i2^{3/4}\sqrt[4]{1+iax}\;{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-iax)\right)+3ax-21i\right)}{9a^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -1/9*((1 - I*a*x)^(1/4)*(I + a*x)^2*(-21*I + 3*a*x + (11*I)*2^(3/4)*(1 + I*
a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))/(a^3*(1 + I*a
*x)^(1/4))
```

fricas [A] time = 0.42, size = 298, normalized size = 0.80

$$\left(12\,a^4x - 12i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\,\log\left(\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}} + \sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right) - \left(12\,a^4x - 12i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\,\log\left(-\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}} + \sqrt{\frac{i\,\sqrt{\ }}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] ((12*a^4*x - 12*I*a^3)*sqrt(3025/64*I/a^6)*log(8/55*a^3*sqrt(3025/64*I/a^6)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (12*a^4*x - 12*I*a^3)*sqrt(3025/6
4*I/a^6)*log(-8/55*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I))) - (12*a^4*x - 12*I*a^3)*sqrt(-3025/64*I/a^6)*log(8/55*a^3*sqrt(-3025
/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (12*a^4*x - 12*I*a^3)*s
qrt(-3025/64*I/a^6)*log(-8/55*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I))) + (8*I*a^3*x^3 - 26*a^2*x^2 - 61*I*a*x - 287)*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(24*a^4*x - 24*I*a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueWarning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWar
ning, need to choose a branch for the root of a polynomial with parameters.
 This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m o
perator + Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

3.108 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x\,dx$

Optimal. Leaf size=324

$$-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{5(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}-\frac{25(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{}}{\sqrt{}}\right.}{}$$

```
[Out] -2*(1-I*a*x)^(9/4)/a^2/(1+I*a*x)^(1/4)-25/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)
/a^2-5/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-25/8*arctan(1-(1-I*a*x)^(1/4)*
2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+25/8*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/16*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(
1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)+25/16*ln(1+(1-I*a*x)^(1/4
)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{5(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}-\frac{25(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{25\log\left(\frac{\sqrt{}}{\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-2*(1 - I*a*x)^(9/4))/(a^2*(1 + I*a*x)^(1/4)) - (25*(1 - I*a*x)^(1/4)*(1 +
 I*a*x)^(3/4))/(4*a^2) - (5*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(2*a^2) -
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) + (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x\,dx = \int \frac{x(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{(5i)\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{a}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+}}}{8a}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\right.}{2a}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{1+x^4}\,d\right.}{2a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,d\right.}{4a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x}\right.}{8a}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right.}{8\sqrt{2}\,a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1+}}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.19

$$\frac{2(1-iax)^{9/4}\left(5\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-iax)\right)-\frac{9}{\sqrt[4]{1+iax}}\right)}{9a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (2*(1 - I*a*x)^(9/4)*(-9/(1 + I*a*x)^(1/4) + 5*2^(3/4)*Hypergeometric2F1[1/
4, 9/4, 13/4, (1 - I*a*x)/2]))/(9*a^2)
```

fricas [A] time = 0.65, size = 292, normalized size = 0.90

$$-\frac{\left(2\,a^3x-2i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\,\log\left(\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-\left(2\,a^3x-2i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\,\log\left(-\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\,\sqrt{a^2x}}{ax+}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -((2*a^3*x - 2*I*a^2)*sqrt(625/16*I/a^4)*log(4/25*I*a^2*sqrt(625/16*I/a^4)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a^3*x - 2*I*a^2)*sqrt(625/16*I/
a^4)*log(-4/25*I*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - (2*a^3*x - 2*I*a^2)*sqrt(-625/16*I/a^4)*log(4/25*I*a^2*sqrt(-625/16*I
/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (2*a^3*x - 2*I*a^2)*sqrt(-62
5/16*I/a^4)*log(-4/25*I*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 - 9*a*x + 43*I)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/(4*a^3*x - 4*I*a^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

3.109 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=299

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i(1+iax)^{3/4}\sqrt[4]{1-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i\tan^{-}}{}$$

```
[Out] 4*I*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+5*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a
+5/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/2*I*ar
ctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)+5/4*I*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2
)-5/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a
*x)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i(1+iax)^{3/4}\sqrt[4]{1-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}\,a}+\frac{5i\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((4*I)*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + ((5*I)*(1 - I*a*x)^(1/4)*
(1 + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(
1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((
(5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}\,dx=\int\frac{(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\int\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{5}{2}\int\frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{(10i)\,\text{Subst}\left(\int\frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-iax}\right)}{a}$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{(10i)\,\text{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{(5i)\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}-\frac{(5i)\,\text{Subst}\left(\int\frac{1\cdots}{1\cdots}\right.}{}$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}-\frac{(5i)\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a}-\frac{(5i)\,\text{Subst}}{}$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}+\frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}\,a}-\frac{5i\log\left(1+\frac{\sqrt{1-ia}}{\sqrt{1+ia}}\right.}{2\sqrt{2}}$$
$$=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}\,(1+iax)^{3/4}}{a}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}\,a}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.13

$$\frac{8ie^{-\frac{1}{2}i\tan^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((8*I)*Hypergeometric2F1[-1/4, 2, 3/4, -E^((2*I)*ArcTan[a*x])])/(a*E^((I/2)
*ArcTan[a*x]))
```

fricas [A] time = 0.58, size = 262, normalized size = 0.88

$$-\frac{\left(a^2x-i\,a\right)\sqrt{\frac{25i}{a^2}}\,\log\left(\frac{1}{5}\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-\left(a^2x-i\,a\right)\sqrt{\frac{25i}{a^2}}\,\log\left(-\frac{1}{5}\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-\left(a^2x-i\,a\right)\sqrt{-\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -((a^2*x - I*a)*sqrt(25*I/a^2)*log(1/5*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(25*I/a^2)*log(-1/5*a*sqrt(25*I/a^2
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(-25*I/a^2)*lo
g(1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (a^2*x - I
*a)*sqrt(-25*I/a^2)*log(-1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(
```

```
a*x + I))) + 2*sqrt(a^2*x^2 + 1)*(-I*a*x - 9)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)))/(2*a^2*x - 2*I*a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueWarning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWar
ning, need to choose a branch for the root of a polynomial with parameters.
 This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m o
perator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-5/2), x)
```

3.110 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=293

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\right.$$

```
[Out] 8*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5062, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\tan^{-1}\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (8*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1-iax)^{5/4}}{x(1+iax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{\frac{ia}{4}-\frac{a^2x}{4}}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i}}{\sqrt[4]{1+i}}\right.$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}}$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} -$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+ia}}\right.$$

Mathematica [C] time = 0.07, size = 106, normalized size = 0.36

$$\frac{\sqrt[4]{1-iax}\left(-20\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right) + 2^{3/4}(1-iax)\sqrt[4]{1+iax}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-iax)\right) + 20\right)}{5\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a
*x)] + 2^(3/4)*(1 - I*a*x)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/
4, (1 - I*a*x)/2]))/(5*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.56, size = 330, normalized size = 1.13

$$\sqrt{4i}\,(ax-i)\log\left(\frac{1}{2}i\sqrt{4i} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \sqrt{4i}\,(ax-i)\log\left(-\frac{1}{2}i\sqrt{4i} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \sqrt{-4i}\,(ax-i)\log\left(\frac{1}{2}i\sqrt{-4i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] (sqrt(4*I)*(a*x - I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - sqrt(4*I)*(a*x - I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/
```

```
(a*x + I))) - sqrt(-4*I)*(a*x - I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + sqrt(-4*I)*(a*x - I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I))) - (2*a*x - 2*I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) + 1) - 2*(-I*a*x - 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I
) - 2*(I*a*x + 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + (2*a*x - 2
*I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 16*I*sqrt(a^2*x^2 + 1)*s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(2*a*x - 2*I)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

3.111 $\int \frac{e^{-\frac{5}{2} i \tan^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=121

$$-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}-\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -10*I*a*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(5/4)/x/(1+I*a*x)^(1/4)-5
*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+5*I*a*arctanh((1+I*a*x)^(1/4)/
(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5062, 94, 93, 298, 203, 206}

$$-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}-\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] ((-10*I)*a*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(5/4)/(x*(1 +
 I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + (5*I
)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx \\
&= -\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - (10ia)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} + (5ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (5ia)\,\text{Subst}\left(\int \right. \\
&= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - 5ia\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.57

$$
\frac{i\sqrt[4]{1-iax}\left(10ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)-9ax+i\right)}{x\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(I - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
+ a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

fricas [B] time = 0.62, size = 213, normalized size = 1.76

$$
-\frac{\sqrt{a^2x^2+1}\,(18\,ax-2i)\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+5\left(-i\,a^2x^2-ax\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)-\left(5\,a^2x^2-5i\,ax\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right.}{2\,ax^2-2i\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(a^2*x^2 + 1)*(18*a*x - 2*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 5*
(-I*a^2*x^2 - a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - (5*a^2*x^
2 - 5*I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + (5*a^2*x^2 - 5*
I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 5*(I*a^2*x^2 + a*x)*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(2*a*x^2 - 2*I*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
[Out] Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

3.112 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}+\frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}}$$

```
[Out] -25/2*a^2*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+5/4*I*a*(1-I*a*x)^(5/4)/x/(1+I*a*
x)^(1/4)-1/2*(1-I*a*x)^(9/4)/x^2/(1+I*a*x)^(1/4)-25/4*a^2*arctan((1+I*a*x)^
(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5062, 96, 94, 93, 298, 203, 206}

$$-\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}+\frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] (-25*a^2*(1 - I*a*x)^(1/4))/(2*(1 + I*a*x)^(1/4)) + (((5*I)/4)*a*(1 - I*a*x
)^(5/4))/(x*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(9/4)/(2*x^2*(1 + I*a*x)^(1/4)
) - (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-iax)^{5/4}}{x^3(1+iax)^{5/4}}\,dx \\
&= -\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{4}(5ia)\int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx \\
&= \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right. \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} + \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right. \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\tanh
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 81, normalized size = 0.50

$$
\frac{\sqrt[4]{1-iax}\left(50a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)-43a^2x^2+9iax-2\right)}{4x^2\sqrt[4]{1+iax}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 + (9*I)*a*x - 43*a^2*x^2 + 50*a^2*x^2*Hypergeometric
2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

fricas [B] time = 0.64, size = 238, normalized size = 1.46

$$\frac{\sqrt{a^2x^2+1}\left(86i\,a^2x^2+18\,ax+4i\right)\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+\left(25\,a^3x^3-25i\,a^2x^2\right)\log\left(\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}+1\right)-25\left(i\,a^3x^3+a^2x^2\right)\mathrm{lo}}{8\,ax^3-8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2*x^2 + 1)*(86*I*a^2*x^2 + 18*a*x + 4*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) + (25*a^3*x^3 - 25*I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) + 1) - 25*(I*a^3*x^3 + a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
) + I) - 25*(-I*a^3*x^3 - a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
- I) - (25*a^3*x^3 - 25*I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
- 1))/(8*a*x^3 - 8*I*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)

[Out] Timed out
```

3.113 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=203

$$\frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}}-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}+\frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}}$$

```
[Out] 287/24*I*a^3*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/3*(1-I*a*x)^(1/4)/x^3/(1+I*a
*x)^(1/4)+13/12*I*a*(1-I*a*x)^(1/4)/x^2/(1+I*a*x)^(1/4)+61/24*a^2*(1-I*a*x)
^(1/4)/x/(1+I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5062, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}}+\frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}}+\frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{55}{8}ia^3\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}}-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] (((287*I)/24)*a^3*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(1/4)/
(3*x^3*(1 + I*a*x)^(1/4)) + (((13*I)/12)*a*(1 - I*a*x)^(1/4))/(x^2*(1 + I*a
*x)^(1/4)) + (61*a^2*(1 - I*a*x)^(1/4))/(24*x*(1 + I*a*x)^(1/4)) + ((55*I)/
8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((55*I)/8)*a^3*ArcTanh
[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^4}dx = \int \frac{(1-iax)^{5/4}}{x^4(1+iax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} - \frac{1}{3}\int \frac{\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}+13ia^3x}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}dx$$
$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{6}\int \frac{-\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{3/4}(1+iax)^{5/4}}dx$$
$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{i\int \frac{165a^4}{16x(1-iax)^{3/4}\sqrt[4]{1+iax}}d}{3a}$$
$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{16}\left(55ia^3\right)\int \frac{}{x(1-ia}$$
$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{4}\left(55ia^3\right)\text{Subst}\left(\int \frac{}{-}\right.$$
$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{8}\left(55ia^3\right)\text{Subst}\left(\int \frac{}{1}\right.$$
$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{55}{8}ia^3\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right.$$

Mathematica [C] time = 0.03, size = 93, normalized size = 0.46

$$\frac{\sqrt[4]{1-iax}\left(-330ia^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+287ia^3x^3+61a^2x^2+26iax-8\right)}{24x^3\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 + (26*I)*a*x + 61*a^2*x^2 + (287*I)*a^3*x^3 - (330*I
)*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1
+ I*a*x)^(1/4))
```

fricas [A] time = 0.75, size = 246, normalized size = 1.21

$$\frac{\left(574\,a^3x^3-122i\,a^2x^2+52\,ax+16i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-165\left(i\,a^4x^4+a^3x^3\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-\left(165\,a^4x\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((574*a^3*x^3 - 122*I*a^2*x^2 + 52*a*x + 16*I)*sqrt(a^2*x^2 + 1)*sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - 165*(I*a^4*x^4 + a^3*x^3)*log(sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)) + 1) - (165*a^4*x^4 - 165*I*a^3*x^3)*log(sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I)) + I) + (165*a^4*x^4 - 165*I*a^3*x^3)*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 165*(-I*a^4*x^4 - a^3*x^3)*log(sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)) - 1))/(48*a*x^4 - 48*I*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{4} \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=233

$$\frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}}+\frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}}-\frac{}{4x}$$

```
[Out] 2467/192*a^4*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/4*(1-I*a*x)^(1/4)/x^4/(1+I*a
*x)^(1/4)+17/24*I*a*(1-I*a*x)^(1/4)/x^3/(1+I*a*x)^(1/4)+113/96*a^2*(1-I*a*x
)^(1/4)/x^2/(1+I*a*x)^(1/4)-521/192*I*a^3*(1-I*a*x)^(1/4)/x/(1+I*a*x)^(1/4)
+475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] time = 0.10, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5062, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}}+\frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}}-\frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{17}{24}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] (2467*a^4*(1 - I*a*x)^(1/4))/(192*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(1/4)/(4
*x^4*(1 + I*a*x)^(1/4)) + (((17*I)/24)*a*(1 - I*a*x)^(1/4))/(x^3*(1 + I*a*x
)^(1/4)) + (113*a^2*(1 - I*a*x)^(1/4))/(96*x^2*(1 + I*a*x)^(1/4)) - (((521*
I)/192)*a^3*(1 - I*a*x)^(1/4))/(x*(1 + I*a*x)^(1/4)) + (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}i\tan^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-iax)^{5/4}}{x^5(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} - \frac{1}{4}\int \frac{\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{1}{12}\int \frac{-\frac{113a^2}{4}+\frac{51}{2}ia^3x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{1}{24}\int \frac{-\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}}{x(1-iax)^{3}}$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} -$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} +$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} +$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} -$$
$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} +$$

Mathematica [C] time = 0.04, size = 99, normalized size = 0.42

$$\frac{\sqrt[4]{1-iax}\left(-2850a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax+i}{i-ax}\right)+2467a^4x^4-521ia^3x^3+226a^2x^2+136iax-48\right)}{192x^4\sqrt[4]{1+iax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-48 + (136*I)*a*x + 226*a^2*x^2 - (521*I)*a^3*x^3 + 246
7*a^4*x^4 - 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)
]))/(192*x^4*(1 + I*a*x)^(1/4))
```

fricas [A] time = 0.54, size = 252, normalized size = 1.08

$$\left(-4934i\,a^4x^4-1042\,a^3x^3-452i\,a^2x^2+272\,ax+96i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}-1425\left(a^5x^5-i\,a^4x^4\right)\log\left(\sqrt{\frac{i\,\sqrt{a}}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] ((-4934*I*a^4*x^4 - 1042*a^3*x^3 - 452*I*a^2*x^2 + 272*a*x + 96*I)*sqrt(a^2
*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1425*(a^5*x^5 - I*a^4*x^4)*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + (1425*I*a^5*x^5 + 1425*a^4*x
^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + (-1425*I*a^5*x^5 - 1425*
```

```
a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 1425*(a^5*x^5 - I*a
^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(384*a*x^5 - 384*I*x^
4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

3.115 $\int e^{\frac{1}{3}i\tan^{-1}(x)}x^2\,dx$

Optimal. Leaf size=319

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}-\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}-\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{19i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{108\sqrt{3}}+\frac{19i\log\left(\cdots\right.}{\cdots}$$

```
[Out] -19/54*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)-1/18*I*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/3
*(1-I*x)^(5/6)*(1+I*x)^(7/6)*x-19/81*I*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))-
19/162*I*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1/2))-19/162*I*arctan(2*(1
-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))-19/324*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3
)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)+19/324*I*ln(1+(1-I*x)^(1/3)/
(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5062, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}-\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}-\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{19i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{108\sqrt{3}}+\frac{19i\log\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((-19*I)/54)*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/18)*(1 - I*x)^(5/6)*(1 +
I*x)^(7/6) + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6)*x)/3 + ((19*I)/162)*ArcTan[Sq
rt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/162)*ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/81)*ArcTan[(1 - I*x)^(1/6)
/(1 + I*x)^(1/6)] - (((19*I)/108)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) -
 (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3] + (((19*I)/108)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{3}i\tan^{-1}(x)}x^2\,dx = \int \frac{\sqrt[6]{1+ix}\,x^2}{\sqrt[6]{1-ix}}\,dx$$

$$= \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{1}{3}\int \frac{\left(-1-\frac{ix}{3}\right)\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$

$$= -\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{54}\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{162}\int \frac{}{\sqrt[6]{1-}}$$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\,\text{Subst}\Bigg($$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\,\text{Subst}\Bigg($$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\,\text{Subst}\Bigg($$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\tan^{-1}\Bigg($$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{81}i\tan^{-1}\Bigg($$

$$= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{19}{162}i\tan^{-1}\Bigg($$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.23

$$\frac{1}{90}(1-ix)^{5/6}\left(5\sqrt[6]{1+ix}\left(6ix^2+7x-i\right)-38i\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}-\frac{ix}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(1/6)*(-I + 7*x + (6*I)*x^2) - (38*I)*2^(1/6)
*Hypergeometric2F1[-1/6, 5/6, 11/6, 1/2 - (I/2)*x]))/90
```

fricas [A] time = 0.63, size = 208, normalized size = 0.65

$$-\frac{19}{324}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)-\frac{19}{324}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)-\frac{19}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] -19/324*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2*I) - 19/324*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) - 1/2*I) - 19/324*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2*I) - 19/324*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) + 1/324*(108*x^3 - 18*I*x^2 - 6*x -
132*I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) - 19/162*log((I*sqrt(x^2 + 1)/(x + I
))^(1/3) + I) + 19/162*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - I)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
```

```
[Out] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x**2,x)
```

```
[Out] Timed out
```

3.116 $\int e^{\frac{1}{3}i\tan^{-1}(x)}x\,dx$

Optimal. Leaf size=278

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}\Big($$

```
[Out] 1/6*(1-I*x)^(5/6)*(1+I*x)^(1/6)+1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/9*arctan(
(1-I*x)^(1/6)/(1+I*x)^(1/6))+1/18*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1
/2))+1/18*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))+1/36*ln(1+(1-I*x)^(
1/3)/(1+I*x)^(1/3)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)-1/36*ln(1+(
1-I*x)^(1/3)/(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5062, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\tan^{-1}\Big($$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/6 + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/2 -
 ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[(1 - I*x)^(1/6)/(1 + I*x
)^(1/6)]/9 + Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(
1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/
3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{3}i\tan^{-1}(x)}x\,dx = \int \frac{\sqrt[6]{1+ix}\,x}{\sqrt[6]{1-ix}}\,dx$$
$$= \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{6}i\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{18}i\int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}}\,dx$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right)$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{9}\,\text{Sul}$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}}\right.$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}} - \frac{\sqrt{3}\sqrt[6]{}}{\sqrt[6]{1}}\right.}{12\sqrt{3}}$$
$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{18}\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{18}\tan^{-1}\left(\sqrt{3}\right.$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.21

$$\frac{1}{10}(1-ix)^{5/6}\left(2\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}-\frac{ix}{2}\right) + 5(1+ix)^{7/6}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5/6
, 11/6, 1/2 - (I/2)*x]))/10
```

fricas [A] time = 0.47, size = 195, normalized size = 0.70

$$-\frac{1}{36}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} + \frac{1}{2}i\right) - \frac{1}{36}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{2}i\right) + \frac{1}{36}\left(\sqrt{3}-i\right)1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2
*I) - 1/36*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
- 1/2*I) + 1/36*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2*I) + 1/36*(sqrt(3) + I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x
+ I))^(1/3) - 1/2*I) + 1/6*(3*x^2 - I*x + 4)*(I*sqrt(x^2 + 1)/(x + I))^(1/3
) - 1/18*I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) + 1/18*I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
[Out] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt[3]{\frac{i (x-i)}{\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x,x)
[Out] Integral(x*(I*(x - I)/sqrt(x**2 + 1))**(1/3), x)
```

3.117 $\int e^{\frac{1}{3}i\tan^{-1}(x)}\,dx$

Optimal. Leaf size=262

$$i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}$$

```
[Out] I*(1-I*x)^(5/6)*(1+I*x)^(1/6)+2/3*I*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))+1/3
*I*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1/2))+1/3*I*arctan(2*(1-I*x)^(1/
6)/(1+I*x)^(1/6)+3^(1/2))+1/6*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3)-(1-I*x)^(1
/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)-1/6*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3)+(
1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5061, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/3)*ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/
6))/(1 + I*x)^(1/6)] + (I/3)*ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)
^(1/6)] + ((2*I)/3)*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] + ((I/2)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3] - ((I/2)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1
 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{3}i\tan^{-1}(x)}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}}\,dx$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + 2i\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right)$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + 2i\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{2}{3}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{2}{3}i\,\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,\right.$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{2}{3}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{6}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{6}$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{2}{3}i\tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}} - \frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right.}{2\sqrt{3}}$$
$$= i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{3}i\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{3}i\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{2}{3}i\tan^{-1}\left(\right.$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.13

$$-\frac{12}{7}ie^{\frac{7}{3}i\tan^{-1}(x)}\,{}_2F_1\left(\frac{7}{6},2;\frac{13}{6};-e^{2i\tan^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x]),x]
```

```
[Out] ((-12*I)/7)*E^(((7*I)/3)*ArcTan[x])*Hypergeometric2F1[7/6, 2, 13/6, -E^((2*
I)*ArcTan[x])]
```

fricas [A] time = 0.44, size = 198, normalized size = 0.76

$$\frac{1}{6}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+\frac{1}{6}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\frac{1}{6}\left(i\sqrt{3}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/
2*I) + 1/6*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/
3) - 1/2*I) + 1/6*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) + 1/2*I) + 1/6*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1
)/(x + I))^(1/3) - 1/2*I) + 1/6*(6*x + 6*I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3)
 + 1/3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - 1/3*log((I*sqrt(x^2 + 1)/
(x + I))^(1/3) - I)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\frac{ix+1}{\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3),x)
[Out] Integral(((I*x + 1)/sqrt(x**2 + 1))**(1/3), x)
```

3.118 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x} dx$

Optimal. Leaf size=430

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.$$

```
[Out] -2*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))-arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)
-3^(1/2))-arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))-2*arctanh((1+I*x)^(
1/6)/(1-I*x)^(1/6))+1/2*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I
*x)^(1/3))-1/2*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3)
)+arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-arctan(1/3*
(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-1/2*ln(1+(1-I*x)^(1/3)/(
1+I*x)^(1/3)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)+1/2*ln(1+(1-I*x)^
(1/3)/(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5062, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x,x]
```

```
[Out] ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ArcTan[Sqrt[3] + (2
*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + Sqrt[3]*ArcTan[(1 - (2*(1 + I*x)^(1/6)
)/(1 - I*x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 -
I*x)^(1/6))/Sqrt[3]] - 2*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] - 2*ArcTan
h[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3)/(1 +
I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2 + (Sqrt[3]*Log[1
 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1
/6)])/2 + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*
x)^(1/3)]/2 - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{3} i \tan^{-1}(x)}}{x}\, dx &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}\, x}\, dx \\
&= i \int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}}\, dx + \int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6} x}\, dx \\
&= -\left(6 \operatorname{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\, dx, x, \sqrt[6]{1-ix}\right)\right) + 6 \operatorname{Subst}\left(\int \frac{1}{-1+x^6}\, dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)\right) - 2 \operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\, dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - 2 \operatorname{Subst}\Bigg(\\
&= -2 \tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{2} \operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\, dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2} \operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\, d\right. \\
&= -2 \tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2 \tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{2} \log\left(1 - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{2} \log\left(1 + \right. \\
&= \sqrt{3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3} \tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - 2 \tan^{-1}\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2 \tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right. \\
&= \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \sqrt{3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3} \tan^{-1}\Bigg(
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.21

$$
-\frac{3(1-ix)^{5/6}\left(\sqrt[6]{2}\,(1+ix)^{5/6}\, {}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}-\frac{ix}{2}\right)+2\, {}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{x+i}{i-x}\right)\right)}{5(1+ix)^{5/6}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x,x]
```

```
[Out] (-3*(1 - I*x)^(5/6)*(2^(1/6)*(1 + I*x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11
/6, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(
5*(1 + I*x)^(5/6))
```

fricas [A] time = 0.50, size = 339, normalized size = 0.79

$$\frac{1}{2}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)+\frac{1}{2}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I
) + 1/2*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1
/2*I) + 1/2*(-I*sqrt(3) - 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2) + 1/2*(-I*sqrt(3) + 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x
 + I))^(1/3) - 1/2) + 1/2*(I*sqrt(3) - 1)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2) + 1/2*(I*sqrt(3) + 1)*log(-1/2*I*sqrt(3) + (I*sq
rt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 1/2*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I) - 1/2*(sqrt(3) + I)*log(-1/2*sqrt(3)
 + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) - log((I*sqrt(x^2 + 1)/(x + I))
^(1/3) + 1) + I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I) + log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x,x)
[Out] int((((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x,x)
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x, x)
```

3.119 $\int \frac{e^{\frac{1}{3} i \tan^{-1}(x)}}{x^2} \, dx$

Optimal. Leaf size=253

$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}+\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{i \text{ ta}\cdots}{\cdots}$$

```
[Out] -(1-I*x)^(5/6)*(1+I*x)^(1/6)/x-2/3*I*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6))+1
/6*I*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))-1/6*I*ln
(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))+1/3*I*arctan(1/
3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-1/3*I*arctan(1/3*(1+2*
(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5062, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}+\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{i \text{ ta}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x) + (I*ArcTan[(1 - (2*(1 + I*x)^(1/6))
/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - (I*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1
 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - ((2*I)/3)*ArcTanh[(1 + I*x)^(1/6)/(1 - I
*x)^(1/6)] + (I/6)*Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3
)/(1 - I*x)^(1/3)] - (I/6)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I
*x)^(1/3)/(1 - I*x)^(1/3)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}\,x^2}\,dx$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{1}{3}i\int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}x}\,dx$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + 2i\,\mathrm{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{2}{3}i\,\mathrm{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1}}{\sqrt[6]{1}}\right.$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\,\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{6}i\,\mathrm{Su}$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{6}i\log\left(1+\right.$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.25

$$-\frac{i(1-ix)^{5/6}\left(2x\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{x+i}{i-x}\right)+5x-5i\right)}{5(1+ix)^{5/6}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^2,x]
```

```
[Out] ((-1/5*I)*(1 - I*x)^(5/6)*(-5*I + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6,
 (I + x)/(I - x)]))/((1 + I*x)^(5/6)*x)
```

fricas [A] time = 3.17, size = 211, normalized size = 0.83

$$\left(\sqrt{3}x-ix\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)+\left(\sqrt{3}x+ix\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}\right)-\left(\sqrt{3}x+ix\right)\log\left(-\frac{1}{2}i\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(3)*x - I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2) + (sqrt(3)*x + I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1
/3) - 1/2) - (sqrt(3)*x + I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I
))^(1/3) + 1/2) - (sqrt(3)*x - I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(
x + I))^(1/3) - 1/2) - 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) + 2*I
*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 6*(-I*x + 1)*(I*sqrt(x^2 + 1)
/(x + I))^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^{2}+1}}\right)^{\frac{1}{3}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^{2}+1}}\right)^{\frac{1}{3}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^{2}+1}}\right)^{1/3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^{2}+1}}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**2,x)
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x**2, x)
```

3.120 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=280

$$-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}-\frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x}-\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)$$

```
[Out] -1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)/x^2-1/6*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x+1/
9*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6))-1/36*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6
)+(1+I*x)^(1/3)/(1-I*x)^(1/3))+1/36*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x
)^(1/3)/(1-I*x)^(1/3))-1/18*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^
(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3
^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5062, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}-\frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x}-\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^3,x]
```

```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/(2*x^2) - ((I/6)*(1 - I*x)^(5/6)*(1 + I*
x)^(1/6))/x - ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*
Sqrt[3]) + ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*Sqr
t[3]) + ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)]/9 - Log[1 - (1 + I*x)^(1/6
)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36 + Log[1 + (1 + I*x)
^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^3}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}\,x^3}\,dx$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} + \frac{1}{6}i\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}\,x^2}\,dx$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{18}\int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}x}\,dx$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{9}\text{Subst}\Bigg($$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\right.$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\log\left(1 - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + \right.$$
$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\tanh^{-}$$

Mathematica [C] time = 0.02, size = 72, normalized size = 0.26

$$\frac{(1-ix)^{5/6}\left(2x^2\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{x+i}{i-x}\right) + 5\left(4x^2-7ix-3\right)\right)}{30(1+ix)^{5/6}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^3,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(-3 - (7*I)*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6, 1
, 11/6, (I + x)/(I - x)]))/(30*(1 + I*x)^(5/6)*x^2)
```

fricas [A] time = 2.27, size = 234, normalized size = 0.84

$$2\,x^2\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right) - 2\,x^2\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right) + \left(i\,\sqrt{3}\,x^2+x^2\right)\log\left(\frac{1}{2}i\,\sqrt{3}+\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right) + \left(i\,\sqrt{3}\,x^2 - x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/36*(2*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 2*x^2*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - 1) + (I*sqrt(3)*x^2 + x^2)*log(1/2*I*sqrt(3) + (I*s
qrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (I*sqrt(3)*x^2 - x^2)*log(1/2*I*sqrt(3
) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (-I*sqrt(3)*x^2 + x^2)*log(-1/
2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (-I*sqrt(3)*x^2 - x^
2)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 6*(4*x^2 +
 I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(1/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**3,x)
[Out] Timed out
```

3.121 $\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^4}\,dx$

Optimal. Leaf size=319

$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2}+\frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x}-\frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{19}{324}i\log\Bigg($$

```
[Out] -1/3*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^3-7/18*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^2
+11/27*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x+19/81*I*arctanh((1+I*x)^(1/6)/(1-I*x)^
(1/6))-19/324*I*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3
))+19/324*I*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))-1
9/162*I*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)+19/16
2*I*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5062, 99, 151, 12, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2}-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}+\frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x}-\frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)+\frac{19}{324}i\log\Bigg($$

Antiderivative was successfully verified.

```
[In] Int[E^((I/3)*ArcTan[x])/x^4,x]
```

```
[Out] -((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(3*x^3) - (((7*I)/18)*(1 - I*x)^(5/6)*(1
 + I*x)^(1/6))/x^2 + (11*(1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(27*x) - (((19*I)
/54)*ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] + (
((19*I)/54)*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt
[3] + ((19*I)/81)*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - ((19*I)/324)*L
og[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)] +
 ((19*I)/324)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}i\tan^{-1}(x)}}{x^4}\,dx = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}\,x^4}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} + \frac{1}{3}\int \frac{\frac{7i}{3}-2x}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}x^3}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} - \frac{1}{6}\int \frac{\frac{22}{9}+\frac{7ix}{3}}{\sqrt[6]{1-ix}\,(1+ix)^{5/6}x^2}\,dx$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{1}{6}\int -\frac{19i}{27\sqrt[6]{1-ix}\,(1+ix)}$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{162}i\int \frac{1}{\sqrt[6]{1-ix}\,(1+ix)^{5}}$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{27}i\,\mathrm{Subst}\left(\int \frac{1}{-1+x^6}\,d\right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) -$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\tanh^{-1}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) -$$
$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19i\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{1}{}$$

Mathematica [C] time = 0.02, size = 81, normalized size = 0.25

$$\frac{(1-ix)^{5/6}\left(38ix^3\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{x+i}{i-x}\right)+5\left(22ix^3+43x^2-39ix-18\right)\right)}{270(1+ix)^{5/6}x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^4,x]
```

```
[Out] ((1 - I*x)^(5/6)*(5*(-18 - (39*I)*x + 43*x^2 + (22*I)*x^3) + (38*I)*x^3*Hyp
ergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(270*(1 + I*x)^(5/6)*x^3)
```

fricas [A] time = 1.96, size = 244, normalized size = 0.76

$$38ix^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right) - 38ix^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right) - \left(19\sqrt{3}x^3-19ix^3\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right) - (19$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/324*(38*I*x^3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 38*I*x^3*log((I*
sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - (19*sqrt(3)*x^3 - 19*I*x^3)*log(1/2*I*s
qrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) - (19*sqrt(3)*x^3 + 19*I*x^
3)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (19*sqrt(3)
*x^3 + 19*I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2
) + (19*sqrt(3)*x^3 - 19*I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) - 1/2) + (-132*I*x^3 + 6*x^2 - 18*I*x - 108)*(I*sqrt(x^2 + 1)/(x
+ I))^(1/3))/x^3
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

maple **[F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1+x\,1{}\mathrm{i}}{\sqrt{x^2+1}}\right)}^{1/3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4,x)
```

```
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**4,x)
```

```
[Out] Timed out
```

3.122 $\int e^{\frac{2}{3}i\tan^{-1}(x)}x^2\,dx$

Optimal. Leaf size=177

$$\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}-\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)+\frac{22i\tan}{}$$

```
[Out] -11/27*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-1/9*I*(1-I*x)^(2/3)*(1+I*x)^(4/3)+1/3*
(1-I*x)^(2/3)*(1+I*x)^(4/3)*x+11/27*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))+11/
81*I*ln(1+I*x)+22/81*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3
^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5062, 90, 80, 50, 60}

$$\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}-\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)+\frac{22i\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((-11*I)/27)*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - (I/9)*(1 - I*x)^(2/3)*(1 + I
*x)^(4/3) + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3)*x)/3 + (((22*I)/27)*ArcTan[1/S
qrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + ((11*I)/
27)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] + ((11*I)/81)*Log[1 + I*x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}i\tan^{-1}(x)}x^2\,dx &= \int \frac{\sqrt[3]{1+ix}\,x^2}{\sqrt[3]{1-ix}}\,dx\\ &= \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{1}{3}\int \frac{\left(-1-\frac{2ix}{3}\right)\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx\\ &= -\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x-\frac{11}{27}\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx\\ &= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x-\frac{22}{81}\int \frac{}{\sqrt[3]{1-i}}\\ &= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{22i\tan^{-1}\left(\frac{1}{\sqrt{}}\right.}{27}\end{aligned}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.41

$$\frac{1}{18}(1-ix)^{2/3}\left(2\sqrt[3]{1+ix}\left(3ix^2+4x-i\right)-11i\sqrt[3]{2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}-\frac{ix}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((1 - I*x)^(2/3)*(2*(1 + I*x)^(1/3)*(-I + 4*x + (3*I)*x^2) - (11*I)*2^(1/3)
*Hypergeometric2F1[-1/3, 2/3, 5/3, 1/2 - (I/2)*x]))/18
```

fricas [A] time = 1.54, size = 121, normalized size = 0.68

$$-\frac{1}{81}\left(11\sqrt{3}+11i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{81}\left(11\sqrt{3}-11i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{81}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")
```

```
[Out] -1/81*(11*sqrt(3) + 11*I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(
3) - 1/2) + 1/81*(11*sqrt(3) - 11*I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) -
1/2*I*sqrt(3) - 1/2) + 1/81*(27*x^3 - 9*I*x^2 - 6*x - 42*I)*(I*sqrt(x^2 + 1
)/(x + I))^(2/3) + 22/81*I*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
[Out] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x**2,x)
[Out] Timed out
```

3.123 $\int e^{\frac{2}{3}i\tan^{-1}(x)}x\,dx$

Optimal. Leaf size=140

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

```
[Out] 1/3*(1-I*x)^(2/3)*(1+I*x)^(1/3)+1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)-1/3*ln(1+(1
-I*x)^(1/3)/(1+I*x)^(1/3))-1/9*ln(1+I*x)-2/9*arctan(1/3*3^(1/2)-2/3*(1-I*x)
^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5062, 80, 50, 60}

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/3 + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/2 -
 (2*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/(3*S
qrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)]/3 - Log[1 + I*x]/9
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{2}{3}i\tan^{-1}(x)}x\,dx=\int\frac{\sqrt[3]{1+ix}\,x}{\sqrt[3]{1-ix}}\,dx$$
$$=\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{1}{3}i\int\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx$$
$$=\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{2}{9}i\int\frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3}}\,dx$$
$$=\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right.$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.39

$$\frac{1}{2}(1-ix)^{2/3}\left(\sqrt[3]{2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}-\frac{ix}{2}\right)+(1+ix)^{4/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(2/3)*((1 + I*x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/
3, 1/2 - (I/2)*x]))/2
```

fricas [A] time = 0.62, size = 116, normalized size = 0.83

$$-\frac{1}{9}\left(i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)-\frac{1}{9}\left(-i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)+\frac{1}{6}\left(3x^2-2ix+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
```

```
[Out] -1/9*(I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
1/2) - 1/9*(-I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqr
t(3) - 1/2) + 1/6*(3*x^2 - 2*I*x + 5)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/9
*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{i x + 1}{\sqrt{x^2 + 1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(\frac{1 + x\,1\mathrm{i}}{\sqrt{x^2 + 1}}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
```

```
[Out] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x,x)
```

```
[Out] Timed out
```

3.124 $\int e^{\frac{2}{3}i\tan^{-1}(x)}\,dx$

Optimal. Leaf size=116

$$i(1-ix)^{2/3}\sqrt[3]{1+ix}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)-\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

```
[Out] I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))-1/3*I*ln(
1+I*x)-2/3*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5061, 50, 60}

$$i(1-ix)^{2/3}\sqrt[3]{1+ix}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)-\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - ((2*I)*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^
(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] - I*Log[1 + (1 - I*x)^(1/3)/(1 +
 I*x)^(1/3)] - (I/3)*Log[1 + I*x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{2}{3}i\tan^{-1}(x)}\,dx = \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx$$
$$= i(1-ix)^{2/3}\sqrt[3]{1+ix} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3}}\,dx$$
$$= i(1-ix)^{2/3}\sqrt[3]{1+ix} - \frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} - i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) - \frac{1}{3}i\log(1+ix)$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.29

$$-\frac{3}{2}ie^{\frac{8}{3}i\tan^{-1}(x)}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};-e^{2i\tan^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x]),x]
```

```
[Out] ((-3*I)/2)*E^(((8*I)/3)*ArcTan[x])*Hypergeometric2F1[4/3, 2, 7/3, -E^((2*I)
*ArcTan[x])]
```

fricas [A] time = 0.63, size = 107, normalized size = 0.92

$$\frac{1}{3}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{1}{2}i\sqrt{3} - \frac{1}{2}\right) - \frac{1}{3}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{1}{2}i\sqrt{3} - \frac{1}{2}\right) + \frac{1}{3}(3x+3i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) - 1/2
) - 1/3*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) -
 1/2) + 1/3*(3*x + 3*I)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/3*I*log((I*sqrt
(x^2 + 1)/(x + I))^(2/3) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3),x)
[Out] Timed out
```

3.125 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=163

$$\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}$$

```
[Out] 3/2*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))+3/2*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1
/2*ln(1+I*x)-1/2*ln(x)+arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3
^(1/2))*3^(1/2)+arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))
*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 105, 60, 91}

$$\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x,x]
```

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] +
 Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))]
+ (3*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)])/2 + (3*Log[(1 - I*x)^(1/3) -
 (1 + I*x)^(1/3)])/2 + Log[1 + I*x]/2 - Log[x]/2
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{2}{3} i \tan^{-1}(x)}}{x} dx &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}\, x} dx \\
&= i \int \frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3}} dx + \int \frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3} x} dx \\
&= \sqrt{3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right) + \sqrt{3} \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right) + \frac{3}{2}\log\left(1 + \frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) + \frac{3}{2}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.55

$$
-\frac{3(1-ix)^{2/3}\left(\sqrt[3]{2}\,(1+ix)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{2}-\frac{ix}{2}\right) + 2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x+i}{i-x}\right)\right)}{4(1+ix)^{2/3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x,x]
```

```
[Out] (-3*(1 - I*x)^(2/3)*(2^(1/3)*(1 + I*x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/
3, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I - x)]))/(4*
(1 + I*x)^(2/3))
```

fricas [A] time = 0.51, size = 147, normalized size = 0.90

$$
\frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}\,(i\,x-1)+x+2i\,\sqrt{x^2+1}\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2\,x+2i}\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}\,(-i\,x+1)+x+2i\,\sqrt{x^2+}}{2\,x+2i}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*log((sqrt(3)*(I*x - 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(
x^2 + 1)/(x + I))^(1/3) + I)/(2*x + 2*I)) + 1/2*(-I*sqrt(3) - 1)*log((sqrt(
3)*(-I*x + 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) + I)/
(2*x + 2*I)) + log(-(x - I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) +
I)/(x + I))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$
\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i(x-i)}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x,x)
```

```
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(2/3)/x, x)
```

3.126 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)+\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

```
[Out] -(1-I*x)^(2/3)*(1+I*x)^(1/3)/x+I*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))-1/3*I*ln(x
)+2/3*I*arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 94, 91}

$$-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)+\frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/x) + ((2*I)*ArcTan[1/Sqrt[3] + (2*(1 -
I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + I*Log[(1 - I*x)^(1/3) - (
1 + I*x)^(1/3)] - (I/3)*Log[x]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^2}\,dx = \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}\,x^2}\,dx$$
$$= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + \frac{2}{3}i\int \frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3}x}\,dx$$
$$= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + \frac{2i\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} + i\log\left(\sqrt[3]{1-ix} - \sqrt[3]{1+ix}\right) - \frac{1}{3}i\log(x)$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.53

$$-\frac{i(1-ix)^{2/3}\left(x\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x+i}{i-x}\right)+x-i\right)}{(1+ix)^{2/3}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

```
[Out] ((-I)*(1 - I*x)^(2/3)*(-I + x + x*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I
 - x)]))/((1 + I*x)^(2/3)*x)
```

fricas [A] time = 0.63, size = 120, normalized size = 1.08

$$\frac{\left(\sqrt{3}\,x - i\,x\right)\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{1}{2}i\,\sqrt{3} + \frac{1}{2}\right) - \left(\sqrt{3}\,x + i\,x\right)\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{1}{2}i\,\sqrt{3} + \frac{1}{2}\right) + 2i\,x\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} -\right.}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(3)*x - I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3)
+ 1/2) - (sqrt(3)*x + I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt
(3) + 1/2) + 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) - 3*(-I*x + 1)*
(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**2,x)
[Out] Timed out
```

3.127 $\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=142

$$-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

```
[Out] -1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)/x^2-1/3*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)/x-1/
3*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+2/3*(1-I
*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 96, 94, 91}

$$-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}-\frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x}-\frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

```
[Out] -((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/(2*x^2) - ((I/3)*(1 - I*x)^(2/3)*(1 + I*
x)^(1/3))/x - (2*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^
(1/3))])/(3*Sqrt[3]) - Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)]/3 + Log[x]/9
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{2}{3}i\tan^{-1}(x)}}{x^3}\,dx &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}\,x^3}\,dx\\
&= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} + \frac{1}{3}i\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}\,x^2}\,dx\\
&= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2}{9}\int \frac{1}{\sqrt[3]{1-ix}\,(1+ix)^{2/3}x}\,dx\\
&= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{\right.
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.49

$$\frac{(1-ix)^{2/3}\left(2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x+i}{i-x}\right)+5x^2-8ix-3\right)}{6(1+ix)^{2/3}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

```
[Out] ((1 - I*x)^(2/3)*(-3 - (8*I)*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5/
3, (I + x)/(I - x)]))/(6*(1 + I*x)^(2/3)*x^2)
```

fricas [A] time = 0.64, size = 138, normalized size = 0.97

$$-\frac{4\,x^2\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-1\right)+2\left(-i\,\sqrt{3}\,x^2-x^2\right)\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\,\sqrt{3}+\frac{1}{2}\right)+2\left(i\,\sqrt{3}\,x^2-x^2\right)\log\left(\left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\right.}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) + 2*(-I*sqrt(3)*x^2 -
 x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) + 1/2) + 2*(I*sqr
t(3)*x^2 - x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) + 1/2)
+ 3*(5*x^2 + 2*I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**3,x)
[Out] Timed out
```

3.128 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=741

$$-\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3}-\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3}-\frac{11i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{256a^3}+\frac{11i\sqrt{2-\sqrt{2}}}{}$$

```
[Out] -11/32*I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^3-1/24*I*(1-I*a*x)^(7/8)*(1+I*a*
x)^(9/8)/a^3+1/3*x*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8)/a^2+11/128*I*arctan((-2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^
(1/2))^(1/2)/a^3-11/128*I*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1
/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^3-11/256*I*ln(1+(1-I*a*x
)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*
(2-2^(1/2))^(1/2)/a^3+11/256*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*
x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^3+11/128*I*
arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2))*(2+2^(1/2))^(1/2)/a^3-11/128*I*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1
/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^3-11/256*I*ln
(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a
*x)^(1/8))*(2+2^(1/2))^(1/2)/a^3+11/256*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a^
3
```

Rubi [A] time = 0.73, antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 90, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3}-\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3}-\frac{11i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1}}{\sqrt[8]{1+iax}}\right.}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-11*I)/32)*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a^3 - ((I/24)*(1 - I*a*x
)^(7/8)*(1 + I*a*x)^(9/8))/a^3 + (x*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(3
*a^2) + (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/a^3 + (((11*I)/128)*S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a^3 - (((11*I)/128)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (
2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/
256)*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[
2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqr
t[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 - (((11*I)/256)*Sqrt[2 + Sq
rt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1
- I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{4} i \tan^{-1}(ax)} x^2 \, dx = \int \frac{x^2 \sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} \, dx$$

$$= \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{4}\right)\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} \, dx}{3a^2}$$

$$= -\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11 \int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} \, dx}{32a^2}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11 \int \frac{}{\sqrt[8]{1}}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\mathrm{Su}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\mathrm{Su}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\mathrm{Su}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{(11i)\,\mathrm{Su}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\,\mathrm{Su}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{\left(11i\sqrt{\frac{1}{2}}\right.}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11i\sqrt{2-}}{}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{11i\sqrt{2-}}{}$$

<u>**Mathematica**</u> **[C]** time = 0.04, size = 83, normalized size = 0.11

$$\frac{(1-iax)^{7/8}\left(7\sqrt[8]{1+iax}\left(8ia^2x^2+9ax-i\right)-66i\sqrt[8]{2}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-iax)\right)\right)}{168a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(1/8)*(-I + 9*a*x + (8*I)*a^2*x^2) - (66*
I)*2^(1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - I*a*x)/2]))/(168*a^3)
```

fricas [A] time = 0.65, size = 435, normalized size = 0.59

$$96i\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}\log\left(\frac{128}{11}\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}+\left(\frac{i\,\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-96\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}\log\left(\frac{128}{11}i\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/96*(96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(14641/2684354
56*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(14641/2
68435456*I/a^12)^(1/4)*log(128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(14641/268435456*I/a^12)^(1/4
)*log(-128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(
a*x + I))^(1/4)) - 96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*
(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 9
6*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(-14641/268435456*I/
a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(-14641/26843
5456*I/a^12)^(1/4)*log(128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(-14641/268435456*I/a^12)^(1/4)*
log(-128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a
*x + I))^(1/4)) - 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*
(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
(32*a^3*x^3 - 4*I*a^2*x^2 - a*x - 37*I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
[Out] Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

3.129 $\int e^{\frac{1}{4} i \tan^{-1}(ax)} x \, dx$

Optimal. Leaf size=689

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}+\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{64a^2}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.}{6}$$

```
[Out] 1/8*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^2+1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8)
/a^2-1/32*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-
2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^2+1/32*arctan((2*(1-I*a*x)^(1/8)/(1+I*a
*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^2+1/64*
ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I
*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^2-1/64*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4
)+(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^2-
1/32*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))*(2+2^(1/2))^(1/2)/a^2+1/32*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(
1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^2+1/64*ln(1+
(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)
^(1/8))*(2+2^(1/2))^(1/2)/a^2-1/64*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-
I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a^2
```

Rubi [A] time = 0.49, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5062, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}+\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{64a^2}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/(8*a^2) + ((1 - I*a*x)^(7/8)*(1 + I*a
*x)^(9/8))/(2*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[
2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
+ I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Log[1 + (
1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8)])/(64*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)
/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/
4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2) - (
Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + S
qrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{4}i\tan^{-1}(ax)}x\,dx = \int \frac{x\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$

$$= \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{8a}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{1}{\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx}{32a}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{4a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}\,a^2} - \text{Sul}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}\,a^2} + \text{Sul}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}\,a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,d\right.}{32a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-i}}{\sqrt[8]{1+iax}}\right.}{64a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \sqrt{2}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.09

$$\frac{(1-iax)^{7/8}\left(2\sqrt[8]{2}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-iax)\right)+7(1+iax)^{9/8}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/8,
 7/8, 15/8, (1 - I*a*x)/2]))/(14*a^2)
```

fricas [A] time = 0.90, size = 428, normalized size = 0.62

$$-\frac{8\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+8i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2}}{ax}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^2*(1/1048576*I/a^8)^(1/4)*log(32*a^2*(1/1048576*I/a^8)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(
32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) -
 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I
*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(1/1048576*I/a^8)^(1/4)*log(-3
2*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*
a^2*(-1/1048576*I/a^8)^(1/4)*log(32*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(
a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*log(32*I*
a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*I
*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(-1/1048576*I/a^8)^(1/4) + (I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32
*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (4
*a^2*x^2 - I*a*x + 5)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x,x)
[Out] Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

3.130 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=674

$$\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}+\frac{i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}-\frac{i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}$$

```
[Out] I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a-1/4*I*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a
*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a+1/4*I*a
rctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/
2))*(2-2^(1/2))^(1/2)/a+1/8*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x
)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a-1/8*I*ln(1+(
1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^
(1/8))*(2-2^(1/2))^(1/2)/a-1/4*I*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)
+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a+1/4*I*arctan((2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^
(1/2))^(1/2)/a+1/8*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(
2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a-1/8*I*ln(1+(1-I*a*x)^
(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2
+2^(1/2))^(1/2)/a
```

Rubi [A] time = 0.43, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5061, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}+\frac{i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}-\frac{i\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a - ((I/4)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a - ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*
a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/4)*Sqrt[2 + Sqrt
[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/S
qrt[2 + Sqrt[2]]])/a + ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] +
 (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/8)*Sq
rt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a - ((I/8)*Sqrt[2 - Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a + ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x
)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8)])/a - ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*
x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{4}i\tan^{-1}(ax)}\,dx &= \int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{1}{4}\int \frac{1}{\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\,\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\,\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-}}{\sqrt[8]{1+}}\right.}{\sqrt{2}a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\,\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\,\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-}}{\sqrt[8]{1+}}\right.}{\sqrt{2}a} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}\,a} + \frac{i\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\right.}{2\sqrt{2\left(2-\sqrt{2}\right.}} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a} + \frac{\left(i\sqrt{\frac{1}{2}\left(3-\right.}\right.}{} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} - \frac{i\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.}{8} \\
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} - \frac{i\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-ia}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right.}{4a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 41, normalized size = 0.06

$$-\frac{16ie^{\frac{9}{4}i\tan^{-1}(ax)}\,{}_2F_1\left(\frac{9}{8},2;\frac{17}{8};-e^{2i\tan^{-1}(ax)}\right)}{9a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x]),x]
```

```
[Out] (((-16*I)/9)*E^(((9*I)/4)*ArcTan[a*x])*Hypergeometric2F1[9/8, 2, 17/8, -E^(
(2*I)*ArcTan[a*x])])/a
```

fricas [A] time = 0.68, size = 383, normalized size = 0.57

$$-i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\,\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\,\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] (-I*a*(1/256*I/a^4)^(1/4)*log(4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1
)/(a*x + I))^(1/4)) + a*(1/256*I/a^4)^(1/4)*log(4*I*a*(1/256*I/a^4)^(1/4) +
 (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(1/256*I/a^4)^(1/4)*log(-4*I*a*
(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*a*(1/256*I
/a^4)^(1/4)*log(-4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4)) - I*a*(-1/256*I/a^4)^(1/4)*log(4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^
2*x^2 + 1)/(a*x + I))^(1/4)) + a*(-1/256*I/a^4)^(1/4)*log(4*I*a*(-1/256*I/a
^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(-1/256*I/a^4)^(1/4)
*log(-4*I*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 I*a*(-1/256*I/a^4)^(1/4)*log(-4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 +
 1)/(a*x + I))^(1/4)) + (a*x + I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueDone
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4),x)
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

3.131 $\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=859

$$-2\tan^{-1}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2+\sqrt{2}}}\right)+\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{2+\cdots}$$

```
[Out] -2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-2*arctanh((1+I*a*x)^(1/8)/(1-I*a
*x)^(1/8))+1/2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(1/8)*2^(1/2)
/(1-I*a*x)^(1/8))*2^(1/2)-1/2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)+(1+I*a*x
)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+arctan(1-(1+I*a*x)^(1/8)*2^(1/2)/(
1-I*a*x)^(1/8))*2^(1/2)-arctan(1+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2
^(1/2)+arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(
1/2))^(1/2))*(2-2^(1/2))^(1/2)-arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2
+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/2*ln(1+(1-I*a*x)^(1
/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2
^(1/2))^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2-2
^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)+arctan((-2*(1-I*a*x)^(1/8)
/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-ar
ctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
))*(2+2^(1/2))^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/
8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-I*a*x)^
(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2
+2^(1/2))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 859, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5062, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-2\tan^{-1}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2+\sqrt{2}}}\right)+\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{2+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(
Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[
2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8))/Sqrt[2 - Sqrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8
))/(1 - I*a*x)^(1/8)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 -
 I*a*x)^(1/8)] - 2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] - (Sqrt[2 -
 Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*
(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 -
I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x
)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sq
rt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + Log[1 - (Sqrt[2]*(1 + I*a*
x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/Sqrt[2]
```

```
- Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)
/(1 - I*a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] && !GtQ[a/b, 0]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[8]{1+iax}}{x\sqrt[8]{1-iax}}\,dx$$
$$= (ia)\int \frac{1}{\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx + \int \frac{1}{x\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx$$
$$= -\left(8\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)\right) + 8\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 8\,\text{Subst}\Big($$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\,\text{Subst}\Big($$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}} + \text{Subs}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\Big(1+}{}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2$$
$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+}}{}\right.$$

Mathematica [C] time = 0.04, size = 97, normalized size = 0.11

$$-\frac{4(1-iax)^{7/8}\left(\sqrt[8]{2}\,(1+iax)^{7/8}\,{}_2F_1\left(\frac{7}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-iax)\right)+2\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{ax+i}{i-ax}\right)\right)}{7(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] (-4*(1 - I*a*x)^(7/8)*(2^(1/8)*(1 + I*a*x)^(7/8)*Hypergeometric2F1[7/8, 7/8
, 15/8, (1 - I*a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (I + a*x)/(I - a
*x)]))/(7*(1 + I*a*x)^(7/8))
```

fricas [A] time = 0.63, size = 509, normalized size = 0.59

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\left(\frac{i\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))
- 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) + 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1
/4)) + I^(1/4)*log(I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*I^(
1/4)*log(I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*I^(1/4)*log
(-I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I^(1/4)*log(-I^(1/4)
 + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (-I)^(1/4)*log((-I)^(1/4) + (I*
sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*(-I)^(1/4)*log(I*(-I)^(1/4) + (I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*(-I)^(1/4)*log(-I*(-I)^(1/4) + (I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (-I)^(1/4)*log(-(-I)^(1/4) + (I*sqrt(a^2
*x^2 + 1)/(a*x + I))^(1/4)) - log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1
) - I*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) + I*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) + log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) -
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x, x)
```

3.132 $\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=328

$$-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}+\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*I*a*arctan((1+I*a*x)^(1/8)/(1-I*a*x)
^(1/8))-1/2*I*a*arctanh((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))+1/4*I*a*arctan(1-(
1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)-1/4*I*a*arctan(1+(1+I*a*x)^
(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+1/8*I*a*ln(1+(1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4)-(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)-1/8*I*a*ln(1+(1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4)+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5062, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}+\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x) - (I/2)*a*ArcTan[(1 + I*a*x)^(1/
8)/(1 - I*a*x)^(1/8)] + ((I/2)*a*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
- I*a*x)^(1/8)])/Sqrt[2] - ((I/2)*a*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/
(1 - I*a*x)^(1/8)])/Sqrt[2] - (I/2)*a*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)
^(1/8)] + ((I/4)*a*Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) +
(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/Sqrt[2] - ((I/4)*a*Log[1 + (Sqrt[2]*(
1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])
/Sqrt[2]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4} i \tan^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + \frac{1}{4}(ia)\int \frac{1}{x\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + (2ia)\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - (ia)\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - (ia)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x,\right.$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}(ia)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{4}(ia)\,\text{Subst}\left(\int\right.$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\tanh^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\log\left(1-\frac{\sqrt{2}\sqrt[8]{1}}{\sqrt[8]{1-i}}\right.}{4\sqrt{}}$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} - \frac{ia\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.02, size = 71, normalized size = 0.22

$$-\frac{i(1-iax)^{7/8}\left(2ax\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{ax+i}{i-ax}\right)+7ax-7i\right)}{7x(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-1/7*I)*(1 - I*a*x)^(7/8)*(-7*I + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1,
 15/8, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(7/8))
```

fricas [A] time = 0.54, size = 345, normalized size = 1.05

$$-i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)+i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + a*x*log((I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x +
I))^(1/4) - I) + I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - 1) + sqr
t(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sqrt(I*a^2))/a)
 - sqrt(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - I*sqrt(I*a^
2))/a) + sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sq
rt(-I*a^2))/a) - sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4
) - I*sqrt(-I*a^2))/a) - 4*(-I*a*x + 1)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**2, x)
```

3.133 $\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=364

$$-\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{a^2\tan^{-1}\left(1-\sqrt{\ldots}\right.}{16\sqrt{2}}$$

```
[Out] -1/8*I*a*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(9
/8)/x^2+1/16*a^2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))+1/16*a^2*arctanh((
1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-1/32*a^2*arctan(1-(1+I*a*x)^(1/8)*2^(1/2)/(
1-I*a*x)^(1/8))*2^(1/2)+1/32*a^2*arctan(1+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)
^(1/8))*2^(1/2)-1/64*a^2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(1/
8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+1/64*a^2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4)+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5062, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}}+\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{a^2\tan^{-1}\left(1-\sqrt{\ldots}\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-I/8)*a*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x - ((1 - I*a*x)^(7/8)*(1 +
I*a*x)^(9/8))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/1
6 - (a^2*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16*Sqr
t[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16
*Sqrt[2]) + (a^2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/16 - (a^2*Lo
g[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1
- I*a*x)^(1/4)])/(32*Sqrt[2]) + (a^2*Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/(32*Sqrt[2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}i\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[8]{1+iax}}{x^3\sqrt[8]{1-iax}}\,dx$$

$$= -\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}(ia)\int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{32}a^2\int \frac{1}{x\sqrt[8]{1-iax}\,(1+iax)^{7/8}}\,dx$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right.$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}a^2\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right.$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\tanh^{-}$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\tanh^{-}$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\tan^{-1}\left(1\right.}{16}$$

Mathematica [C] time = 0.02, size = 84, normalized size = 0.23

$$\frac{(1-iax)^{7/8}\left(2a^2x^2\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{ax+i}{i-ax}\right)+7\left(5a^2x^2-9iax-4\right)\right)}{56x^2(1+iax)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(7*(-4 - (9*I)*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeometr
ic2F1[7/8, 1, 15/8, (I + a*x)/(I - a*x)]))/(56*x^2*(1 + I*a*x)^(7/8))
```

fricas [A] time = 0.50, size = 381, normalized size = 1.05

$$a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)-a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*(a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + I*a^2*x^2*lo
g((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - I*a^2*x^2*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) - a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4) - 1) + sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
 + sqrt(I*a^4))/a^2) - sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))^(1/4) - sqrt(I*a^4))/a^2) + sqrt(-I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2
+ 1)/(a*x + I))^(1/4) + sqrt(-I*a^4))/a^2) - sqrt(-I*a^4)*x^2*log((a^2*(I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - sqrt(-I*a^4))/a^2) - (20*a^2*x^2 + 4*I*
a*x + 16)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**3, x)
```

3.134 $\int e^{6i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=114

$$\frac{2(2m^2+4m+3)x^{m+1}{}_2F_1(1,m+1;m+2;iax)}{m+1}+\frac{4ix^{m+1}\left(a(m^2+3m+3)x+i(m+1)^2\right)}{(m+1)(1-iax)^2}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

```
[Out] -x^(1+m)*(1+I*a*x)^2/(1+m)/(1-I*a*x)^2+4*I*x^(1+m)*(I*(1+m)^2+a*(m^2+3*m+3)
*x)/(1+m)/(1-I*a*x)^2+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*
x)/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 100, 145, 64}

$$\frac{2(2m^2+4m+3)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}+\frac{4ix^{m+1}\left(a(m^2+3m+3)x+i(m+1)^2\right)}{(m+1)(1-iax)^2}-\frac{(1}{(m}$$

Antiderivative was successfully verified.

```
[In] Int[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -((x^(1 + m)*(1 + I*a*x)^2)/((1 + m)*(1 - I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 + a*(3 + 3*m + m^2)*x))/((1 + m)*(1 - I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{6i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)^3}{(1-iax)^3}\,dx$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{i\int \frac{x^m(1+iax)\left(-2ia(1+m)+2a^2(3+m)x\right)}{(1-iax)^3}\,dx}{a(1+m)}$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2+a\left(3+3m+m^2\right)x\right)}{(1+m)(1-iax)^2} + \left(2\left(3+4m+2m^2\right)\right)\int$$

$$= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2+a\left(3+3m+m^2\right)x\right)}{(1+m)(1-iax)^2} + \frac{2\left(3+4m+2m^2\right)x^{1+m}}{1-}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.82

$$\frac{x^{m+1}\left(-a^2x^2+2\left(2m^2+4m+3\right)(ax+i)^2\,{}_2F_1(1,m+1;m+2;iax)+m^2(4-4iax)+4m(2-3iax)-10iax+5\right.}{(m+1)(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(5 - (10*I)*a*x - a^2*x^2 + 4*m*(2 - (3*I)*a*x) + m^2*(4 - (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
I*a*x]))/((1 + m)*(I + a*x)^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^3x^3-3i\,a^2x^2-3\,ax+i\right)x^m}{a^3x^3+3i\,a^2x^2-3\,ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + I)*x^m/(a^3*x^3 + 3*I*a^2*x^2 -
3*a*x - I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^6x^m}{\left(a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

maple [C] time = 0.75, size = 748, normalized size = 6.56

$$\frac{\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(-a^2m^2x^2+2a^2m\,x^2+3a^2x^2-m^2+4m+5\right)}{2(1+m)\left(a^2x^2+1\right)^2}+\frac{4x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(\frac{1}{16}m^3-\frac{3}{16}m^2-\frac{1}{16}m+\frac{3}{16}\right)\Phi\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{4}+\frac{3i\left(a^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x)
```

```
[Out] 1/4*(a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(a^2)^(1/2+1/2*m)*(-a^2*m^2*x^2+2
*a^2*m*x^2+3*a^2*x^2-m^2+4*m+5)/(a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(a^2)^(1/2+1/
2*m)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+3/2*I/
a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+m-2)/(a^2*x^2+1)^2-1/4*x
^m*(a^2)^(1/2*m)*(-2+m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-15/4*(a^2)^(-1/2-1/2*
m)*(1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*(a^2*m*x^2+a^2*x^2+m-1)/(a^2*x^2+1)^2/a^2
-1/4*x^(1+m)*(a^2)^(3/2+1/2*m)*(1+m)*(-1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2
*m))-5*I/a*(a^2)^(-1/2*m)*(-1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+4*a^2*x^2+m+2)
/(a^2*x^2+1)^2+1/4*x^m*(a^2)^(1/2*m)*(2+m)*m*LerchPhi(-a^2*x^2,1,1/2*m))+15
/4*(a^2)^(-1/2-1/2*m)*(-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(a^2*m*x^2+5*a^2*x^2+
m+3)/a^4/(a^2*x^2+1)^2+1/4*x^(1+m)*(a^2)^(5/2+1/2*m)*(m^2+4*m+3)/a^4*LerchP
hi(-a^2*x^2,1,1/2+1/2*m))+3/2*I/a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(8*
a^4*x^4+a^2*m^2*x^2+8*a^2*m*x^2+16*a^2*x^2+m^2+6*m+8)/(a^2*x^2+1)^2/m-1/4*x
^m*(a^2)^(1/2*m)*(m^2+6*m+8)*LerchPhi(-a^2*x^2,1,1/2*m))-1/4*(a^2)^(-1/2-1/
2*m)*(1/2*x^(1+m)*(a^2)^(7/2+1/2*m)*(8*a^4*x^4+a^2*m^2*x^2+10*a^2*m*x^2+25*
a^2*x^2+m^2+8*m+15)/(a^2*x^2+1)^2/(1+m)/a^6-1/4*x^(1+m)*(a^2)^(7/2+1/2*m)*(
m^2+8*m+15)/a^6*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^6 x^m}{\left(a^2x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(1+a\,x\,1\mathrm{i})^6}{\left(a^2\,x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3,x)
```

```
[Out] int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx-\int\frac{15a^2x^2x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx-\int\left(-\frac{15a^4x^4x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**m,x)
```

```
[Out] -Integral(-x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integral(
15*a**2*x**2*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integra
l(-15*a**4*x**4*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Inte
gral(a**6*x**6*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integ
ral(-6*I*a*x*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integra
l(20*I*a**3*x**3*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Int
egral(-6*I*a**5*x**5*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
```

3.135 $\int e^{4i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=50

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;iax)+\frac{4x^{m+1}}{1-iax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(1-I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 89, 80, 64}

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)+\frac{4x^{m+1}}{1-iax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, I*a*x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{4i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)^2}{(1-iax)^2}\,dx$$
$$= \frac{4x^{1+m}}{1-iax} + \frac{\int \frac{x^m\left(-a^2(3+4m)-ia^3x\right)}{1-iax}\,dx}{a^2}$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - (4(1+m))\int \frac{x^m}{1-iax}\,dx$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;iax)$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.16

$$\frac{x^{m+1}(-4(m+1)(ax+i)\,{}_2F_1(1,m+1;m+2;iax)+ax+4im+5i)}{(m+1)(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(5*I + (4*I)*m + a*x - 4*(1 + m)*(I + a*x)*Hypergeometric2F1[1,
1 + m, 2 + m, I*a*x]))/((1 + m)*(I + a*x))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2-2i\,ax-1\right)x^m}{a^2x^2+2i\,ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - 2*I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^4x^m}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

maple [C] time = 0.57, size = 417, normalized size = 8.34

$$\frac{\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}}{2a^2x^2+2}+\frac{2x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^2}{4}+\frac{1}{4}\right)\Phi\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{2}+\frac{2i\left(a^2\right)^{-\frac{m}{2}}\left(\frac{x^m\left(a^2\right)^{\frac{m}{2}}(-m-2)}{(2+m)\left(a^2x^2+1\right)}+\frac{x^m\left(a^2\right)^{\frac{m}{2}}m\Phi\left(-a^2x^2,1,\frac{m}{2}\right)}{2}\right)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x)
```

```
[Out] 1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(1/2+1/2*m)/(2*a^2*x^2+2)+2/(1+m)*x
^(1+m)*(a^2)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+2*I
```

```
/a*(a^2)^(-1/2*m)*(1/(2+m)*x^m*(a^2)^(1/2*m)*(-m-2)/(a^2*x^2+1)+1/2*x^m*(a^
2)^(1/2*m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-3*(a^2)^(-1/2-1/2*m)*(1/(3+m)*x^(1
+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2/(a^2*x^2+1)+1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*
(1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))-2*I/a*(a^2)^(-1/2*m)*(x^m*(a^2)^(
1/2*m)*(2*a^2*x^2+m+2)/(a^2*x^2+1)/m-1/2*x^m*(a^2)^(1/2*m)*(2+m)*LerchPhi(-
a^2*x^2,1,1/2*m))+1/2*(a^2)^(-1/2-1/2*m)*(x^(1+m)*(a^2)^(5/2+1/2*m)*(2*a^2*
x^2+m+3)/(a^2*x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(3+m)/a^4*Lerc
hPhi(-a^2*x^2,1,1/2+1/2*m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^4 x^m}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,(1+a\,x\,1\mathrm{i})^4}{\left(a^2\,x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(ax-i)^4}{\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x - I)**4/(a**2*x**2 + 1)**2, x)
```

3.136 $\int e^{2i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=39

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;iax)}{m+1}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 80, 64}

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*
x])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(ax)}x^m\,dx&=\int\frac{x^m(1+iax)}{1-iax}\,dx\\&=-\frac{x^{1+m}}{1+m}+2\int\frac{x^m}{1-iax}\,dx\\&=-\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;iax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x^{m+1}(-1+2\,{}_2F_1(1,m+1;m+2;iax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x]))/(1 + m)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-i)x^m}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x - I)*x^m/(a*x + I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^2 x^m}{a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

maple [C] time = 0.51, size = 175, normalized size = 4.49

$$\frac{x^{1+m}\left(\frac{1}{2}+\frac{m}{2}\right)\Phi\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}+\frac{i\left(a^2\right)^{-\frac{m}{2}}\left(\frac{2x^m\left(a^2\right)^{\frac{m}{2}}}{m}+\frac{x^m\left(a^2\right)^{\frac{m}{2}}(-m-2)\Phi\left(-a^2x^2,1,\frac{m}{2}\right)}{2+m}\right)}{a}-\frac{\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(a^2\right)^{\frac{3}{2}+\frac{m}{2}}}{(1+m)a^2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^m,x)
```

```
[Out] 1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(-a^2*x^2,1,1/2+1/2*m)+I/a*(a^2)^(-1/2*
m)*(2*x^m*(a^2)^(1/2*m)/m+1/(2+m)*x^m*(a^2)^(1/2*m)*(-m-2)*LerchPhi(-a^2*x^
2,1,1/2*m))-1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(3/2+1/2*m)/(1+m)/a^2+1
/(3+m)*x^(1+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^2 x^m}{a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(1+a\,x\,1\mathrm{i}\right)}^2}{a^2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

```
[Out] int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1), x)
```

sympy [B] time = 3.40, size = 136, normalized size = 3.49

$$\frac{iamx^2x^m\Phi\left(axe^{\frac{5i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{2iax^2x^m\Phi\left(axe^{\frac{5i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{mxx^m\Phi\left(axe^{\frac{5i\pi}{2}},1,m+1\right)\Gamma(m+}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**m,x)
```

```
[Out] I*a*m*x**2*x**m*lerchphi(a*x*exp_polar(5*I*pi/2), 1, m + 2)*gamma(m + 2)/ga
mma(m + 3) + 2*I*a*x**2*x**m*lerchphi(a*x*exp_polar(5*I*pi/2), 1, m + 2)*ga
mma(m + 2)/gamma(m + 3) + m*x*x**m*lerchphi(a*x*exp_polar(5*I*pi/2), 1, m +
 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchphi(a*x*exp_polar(5*I*pi/2), 1,
 m + 1)*gamma(m + 1)/gamma(m + 2)
```

3.137 $\int e^{-2i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=39

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-iax)}{m+1}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5062, 80, 64}

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)
*a*x])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(ax)}x^m\,dx&=\int\frac{x^m(1-iax)}{1+iax}\,dx\\&=-\frac{x^{1+m}}{1+m}+2\int\frac{x^m}{1+iax}\,dx\\&=-\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-iax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x^{m+1}(-1+2\,{}_2F_1(1,m+1;m+2;-iax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((2*I)*ArcTan[a*x]),x]
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x]))/(1 + m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+i)x^m}{ax-i}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-(a*x + I)*x^m/(a*x - I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)x^m}{(i\,ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

maple [C] time = 0.42, size = 158, normalized size = 4.05

$$\frac{i\,(ia)^{-m}\left(\frac{x^m(ia)^m\left(-a^2m\,x^2-iamx-2iax-m^2-3m-2\right)}{(1+m)m(iax+1)}+x^m\,(ia)^m\,(2+m)\,\Phi\left(-iax,1,m\right)\right)}{a}-\frac{i\,(ia)^{-m}\left(\frac{x^m(ia)^m(-1-m)}{(1+m)(iax+1)}+x^m\,(ia)^m\,m\Phi\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^2*(a^2*x^2+1),x)
[Out] I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-a^2*m*x^2-I*a*m*x-m^2-2*I*a*x-3*m-2)/(1+m)/m/
(1+I*a*x)+x^m*(I*a)^m*(2+m)*LerchPhi(-I*a*x,1,m))-I*(I*a)^(-m)/a*(1/(1+m)*x
^m*(I*a)^m*(-1-m)/(1+I*a*x)+x^m*(I*a)^m*m*LerchPhi(-I*a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)x^m}{(i\,ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(a^2\,x^2+1\right)}{(1+a\,x\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
[Out] int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2, x)
```

sympy [B] time = 4.06, size = 136, normalized size = 3.49

$$-\frac{iamx^2x^m\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma\left(m+2\right)}{\Gamma\left(m+3\right)}-\frac{2iax^2x^m\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma\left(m+2\right)}{\Gamma\left(m+3\right)}+\frac{mxx^m\Phi\left(axe^{\frac{3i\pi}{2}},1,m+1\right)\Gamma}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -I*a*m*x**2*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 2)*gamma(m + 2)/g
amma(m + 3) - 2*I*a*x**2*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 2)*g
amma(m + 2)/gamma(m + 3) + m*x*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m
+ 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchphi(a*x*exp_polar(3*I*pi/2), 1
, m + 1)*gamma(m + 1)/gamma(m + 2)
```

3.138 $\int e^{-4i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=50

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(1+I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*x
)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 89, 80, 64}

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 + I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (-I)*a*x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-4i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)^2}{(1+iax)^2}\,dx$$
$$= \frac{4x^{1+m}}{1+iax} + \frac{\int \frac{x^m\left(-a^2(3+4m)+ia^3x\right)}{1+iax}\,dx}{a^2}$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - (4(1+m))\int \frac{x^m}{1+iax}\,dx$$
$$= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;-iax)$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.16

$$\frac{x^{m+1}(-4(m+1)(ax-i)\,{}_2F_1(1,m+1;m+2;-iax)+ax-4im-5i)}{(m+1)(ax-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(-5*I - (4*I)*m + a*x - 4*(1 + m)*(-I + a*x)*Hypergeometric2F1[1
, 1 + m, 2 + m, (-I)*a*x]))/((1 + m)*(-I + a*x))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2i\,ax-1\right)x^m}{a^2x^2-2i\,ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^2 x^m}{(i\,ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

maple [C] time = 0.56, size = 428, normalized size = 8.56

$$-\frac{i\,(ia)^{-m}\left(\frac{x^m(ia)^m\left(6a^4x^4m+6ia^3x^3m+a^2x^2m^4+24ia^3x^3+11a^2x^2m^3-2iax\,m^4+46a^2m^2x^2-21iax\,m^3+90a^2m\,x^2-79iax\,m^2+72a^2x^2-126iamx-m^4\right.}{(1+m)m(iax+1)^3}\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x)
```

```
[Out] -1/6*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(a^2*x^2*m^4+6*a^4*x^4*m+11*a^2*x^2*m^3-72
*I*a*x-21*I*a*x*m^3+46*a^2*m^2*x^2+24*I*a^3*x^3-m^4-2*I*a*x*m^4+90*a^2*m*x^
2+6*I*a^3*x^3*m-10*m^3+72*a^2*x^2-126*I*a*m*x-35*m^2-79*I*a*x*m^2-50*m-24)/
(1+m)/m/(1+I*a*x)^3+x^m*(I*a)^m*(m^3+9*m^2+26*m+24)*LerchPhi(-I*a*x,1,m))+1
```

```
/3*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2-4*a^2*m*x^2+2*I*a*x*m^2-6*a^2
*x^2+7*I*a*m*x+m^2+6*I*a*x+3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*m*(m^2+3*m+2)*Ler
chPhi(-I*a*x,1,m))-1/6*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2+2*a^2*m*x
^2+2*I*a*x*m^2-5*I*a*m*x+m^2-3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*(m^2-3*m+2)*m*L
erchPhi(-I*a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^2 x^m}{(i\,ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\left(a^2\,x^2+1\right)^2}{(1+a\,x\,1\mathrm{i})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4,x)
```

```
[Out] int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a^2x^2+1\right)^2}{(ax-i)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**4*(a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**m*(a**2*x**2 + 1)**2/(a*x - I)**4, x)
```

3.139 $\int e^{-6i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=115

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\,{}_2F_1(1,m+1;m+2;-iax)}{m+1}+\frac{4ix^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(m+1)(1+iax)^2}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax}$$

```
[Out] -x^(1+m)*(1-I*a*x)^2/(1+m)/(1+I*a*x)^2+4*I*x^(1+m)*(I*(1+m)^2-a*(m^2+3*m+3)
*x)/(1+m)/(1+I*a*x)^2+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a
*x)/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5062, 100, 145, 64}

$$\frac{2\left(2m^2+4m+3\right)x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}+\frac{4ix^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right.}{(m+1)(1+iax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] -((x^(1 + m)*(1 - I*a*x)^2)/((1 + m)*(1 + I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 - a*(3 + 3*m + m^2)*x))/((1 + m)*(1 + I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{-6i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)^3}{(1+iax)^3}\,dx$$
$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} - \frac{i\int \frac{x^m(1-iax)\left(2ia(1+m)+2a^2(3+m)x\right)}{(1+iax)^3}\,dx}{a(1+m)}$$
$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2-a\left(3+3m+m^2\right)x\right)}{(1+m)(1+iax)^2} + \left(2\left(3+4m+2m^2\right)\right)\int \frac{}{1}$$
$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}\left(i(1+m)^2-a\left(3+3m+m^2\right)x\right)}{(1+m)(1+iax)^2} + \frac{2\left(3+4m+2m^2\right)x^{1+m}\,{}_2F}{1+}$$

Mathematica [A] time = 0.04, size = 94, normalized size = 0.82

$$\frac{x^{m+1}\left(-a^2x^2+2\left(2m^2+4m+3\right)(ax-i)^2\,{}_2F_1(1,m+1;m+2;-iax)+m^2(4+4iax)+4m(2+3iax)+10iax+5\right)}{(m+1)(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(5 + (10*I)*a*x - a^2*x^2 + 4*m*(2 + (3*I)*a*x) + m^2*(4 + (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(-I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
 (-I)*a*x]))/((1 + m)*(-I + a*x)^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^3x^3+3i\,a^2x^2-3\,ax-i\right)x^m}{a^3x^3-3i\,a^2x^2-3\,ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x - I)*x^m/(a^3*x^3 - 3*I*a^2*x^2 -
3*a*x + I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^3x^m}{\left(i\,ax+1\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

maple [C] time = 0.74, size = 1196, normalized size = 10.40

$$i\,(ia)^{-m}\left(\frac{x^m(ia)^m\left(-a^4x^4m^6-120a^6x^6m-22a^4x^4m^5+4ia^3x^3m^6-120ia^5x^5m-197a^4x^4m^4+87ia^3x^3m^5-720ia^5x^5-932a^4x^4m^3+764ia^3x^3m^4-2556a^4x^4m\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x)
```

```
[Out] 1/120*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-720+764*I*a^3*x^3*m^4+12000*I*a^3*x^3*m
-3095*I*a*x*m^3+11722*a^2*m^2*x^2+6*a^2*x^2*m^6-120*a^6*x^6*m+129*a^2*x^2*m
^5-720*I*a^5*x^5+7200*I*a^3*x^3-3600*I*a*x-197*a^4*x^4*m^4-m^6-175*m^4-4*I*
a*x*m^6-120*I*a^5*x^5*m-932*a^4*x^4*m^3-22*a^4*x^4*m^5+4*I*a^3*x^3*m^6+87*I
*a^3*x^3*m^5+8802*I*a^3*x^3*m^2-720*I*a*x*m^4-8100*I*a*m*x-1764*m+7200*a^2*
x^2+14400*a^2*m*x^2-85*I*a*x*m^5-21*m^5-7076*I*a*x*m^2-2556*a^4*x^4*m^2-420
0*a^4*x^4*m-735*m^3+1112*a^2*x^2*m^4+4911*a^2*x^2*m^3+3483*I*a^3*x^3*m^3-a^
4*x^4*m^6-1624*m^2-3600*a^4*x^4)/(1+m)/m/(1+I*a*x)^5+x^m*(I*a)^m*(m^5+20*m^
4+155*m^3+580*m^2+1044*m+720)*LerchPhi(-I*a*x,1,m))-1/40*I*(I*a)^(-m)/a*(-x
^m*(I*a)^m*(24-312*I*a^3*x^3*m+41*I*a*x*m^3+149*I*a*x*m^2+226*I*a*m*x-239*a
^2*m^2*x^2+a^4*x^4*m^4-240*I*a^3*x^3+120*I*a*x+m^4+11*a^4*x^4*m^3-4*I*a^3*x
^3*m^4-171*I*a^3*x^3*m^2+4*I*a*x*m^4-43*I*a^3*x^3*m^3+50*m-240*a^2*x^2-392*
a^2*m*x^2+46*a^4*x^4*m^2+96*a^4*x^4*m+10*m^3-6*a^2*x^2*m^4-63*a^2*x^2*m^3+3
5*m^2+120*a^4*x^4)/(1+I*a*x)^5+x^m*(I*a)^m*m*(m^4+10*m^3+35*m^2+50*m+24)*Le
rchPhi(-I*a*x,1,m))+1/40*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4+a^4*x^4*
m^3+20*I*a*x-4*a^4*x^4*m^2+18*I*a^3*x^3*m-6*a^2*x^2*m^4-4*a^4*x^4*m+4*I*a*x
*m^4-3*a^2*x^2*m^3+19*I*a^3*x^3*m^2-4*I*a*m*x+31*a^2*m^2*x^2-3*I*a^3*x^3*m^
3+m^4+18*a^2*m*x^2-21*I*a*x*m^2-40*a^2*x^2+I*a*x*m^3-5*m^2-4*I*a^3*x^3*m^4+
4)/(1+I*a*x)^5+x^m*(I*a)^m*(m^2-3*m+2)*m*(m^2+3*m+2)*LerchPhi(-I*a*x,1,m))-
1/120*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4-9*a^4*x^4*m^3+108*I*a^3*x^3
*m+26*a^4*x^4*m^2+4*I*a*x*m^4-6*a^2*x^2*m^4-24*a^4*x^4*m-111*I*a^3*x^3*m^2+
57*a^2*x^2*m^3-154*I*a*m*x+37*I*a^3*x^3*m^3-179*a^2*m^2*x^2+129*I*a*x*m^2+m
^4+188*a^2*m*x^2-39*I*a*x*m^3-10*m^3-4*I*a^3*x^3*m^4+35*m^2-50*m+24)/(1+I*a
*x)^5+x^m*(I*a)^m*(m^4-10*m^3+35*m^2-50*m+24)*m*LerchPhi(-I*a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^3 x^m}{(i\,ax+1)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\left(a^2\,x^2+1\right)^3}{(1+a\,x\,1\mathrm{i})^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6,x)
```

```
[Out] int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2-6iax-1}\,dx-\int \frac{3a^2x^2x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**6*(a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 +
 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(3*a**2*x**2*x**m/(a**6*x**6 - 6
```

```
*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x - 1),
 x) - Integral(3*a**4*x**4*x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 +
 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(a**6*x**6*x**m
/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2*x**2
- 6*I*a*x - 1), x)
```

3.140 $\int e^{3i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=159

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}+\frac{4iax^{m+2}}{}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*x^(2+
m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([
3/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+4*I*a*x^(2+m)*hypergeom([3/2, 1
+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] time = 0.77, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5060, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) + ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx$$
$$= \int \left(-\frac{3x^m}{\sqrt{1+a^2x^2}} - \frac{iax^{1+m}}{\sqrt{1+a^2x^2}} + \frac{4x^m}{(1-iax)\sqrt{1+a^2x^2}}\right)dx$$
$$= -\left(3\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right) + 4\int \frac{x^m}{(1-iax)\sqrt{1+a^2x^2}}\,dx - (ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + 4\int \frac{x^m(1+iax)}{\left(1+a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + 4\int \frac{x^m}{\left(1+a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\ldots\right)}{1+m}$$

Mathematica [C] time = 0.08, size = 113, normalized size = 0.71

$$-\frac{i\sqrt{1-iax}\,\sqrt{ax-i}\,x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-iax,iax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;-iax,iax\right)\right)}{(m+1)\sqrt{1+iax}\,\sqrt{ax+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] ((-I)*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*(AppellF1[1 + m, -1/2, 1/2,
2 + m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, (-I)*a*x, I*a
*x]))/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}\,(-i\,ax-1)x^m}{a^2x^2+2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*(-I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.43, size = 146, normalized size = 0.92

$$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^{2}x^{2}\right)}{1+m}+\frac{3ia\,x^{2+m}\,\mathrm{hypergeom}\left(\left[\frac{3}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^{2}x^{2}\right)}{2+m}-\frac{3a^{2}x^{3+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([3/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+3*I*a/(2+m)*x
^(2+m)*hypergeom([3/2,1+1/2*m],[2+1/2*m],-a^2*x^2)-3*a^2/(3+m)*x^(3+m)*hype
rgeom([3/2,3/2+1/2*m],[5/2+1/2*m],-a^2*x^2)-I*a^3/(4+m)*x^(4+m)*hypergeom([
3/2,2+1/2*m],[3+1/2*m],-a^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^{3}x^{m}}{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^3*x^m/(a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\,(1+a\,x\,1\mathrm{i})^{3}}{\left(a^{2}\,x^{2}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix^{m}}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}+\sqrt{a^{2}x^{2}+1}}\,dx+\int \left(-\frac{3axx^{m}}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}+\sqrt{a^{2}x^{2}+1}}\right)dx+\int \frac{a^{3}x^{3}x^{m}}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}+\sqrt{a^{2}x^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**m,x)
```

```
[Out] -I*(Integral(I*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2*x**m/(a**2*x**2*sqrt(a**2*x**2
+ 1) + sqrt(a**2*x**2 + 1)), x))
```

3.141 $\int e^{i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=79

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5060, 808, 364}

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= (ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx+\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}+\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [C] time = 0.04, size = 85, normalized size = 1.08

$$\frac{i\sqrt{1-iax}\,\sqrt{ax-i}\,x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-iax,iax\right)}{(m+1)\sqrt{1+iax}\,\sqrt{ax+i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a*x])*x^m,x]
```

```
[Out] (I*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*AppellF1[1 + m, -1/2, 1/2, 2 + m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\,\sqrt{a^2x^2+1}\,x^m}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m/(a*x + I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.31, size = 71, normalized size = 0.90

$$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^2x^2\right)}{1+m}+\frac{ia\,x^{2+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^2x^2\right)}{2+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*hypergeom([1/2,1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(1+a\,x\,1\mathrm{i}\right)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 3.13, size = 95, normalized size = 1.20

$$\frac{iax^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**m,x)
```

```
[Out] I*a*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*ex
p_polar(I*pi))/(2*gamma(m/2 + 2)) + x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2
 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(I*pi))/(2*gamma(m/2 + 3/2))
```

3.142 $\int e^{-i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=79

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5060, 808, 364}

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\left((ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx\right)+\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 85, normalized size = 1.08

$$-\frac{i\sqrt{1+iax}\sqrt{ax+i}\,x^{m+1}F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-iax,iax\right)}{(m+1)\sqrt{1-iax}\sqrt{ax-i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((-I)*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*AppellF1[1 + m, 1/2, -1/2, 2
+ m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{i\sqrt{a^2x^2+1}\,x^m}{ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m/(a*x - I), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}\,x^m}{i\,ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{a^2x^2+1}}{iax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}\,x^m}{i\,ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{a^2\,x^2+1}}{1+a\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
[Out] int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{x^m\sqrt{a^2x^2+1}}{ax-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
[Out] -I*Integral(x**m*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

3.143 $\int e^{-3i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=159

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m+2}-\frac{4iax^{m+2}\,{}_2F}{}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+
m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([
3/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-4*I*a*x^(2+m)*hypergeom([3/2, 1
+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] time = 0.70, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5060, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{4x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\text{H}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) - ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 5060

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx$$
$$= \int \left(-\frac{3x^m}{\sqrt{1+a^2x^2}} + \frac{iax^{1+m}}{\sqrt{1+a^2x^2}} + \frac{4x^m}{(1+iax)\sqrt{1+a^2x^2}}\right)dx$$
$$= -\left(3\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right) + 4\int \frac{x^m}{(1+iax)\sqrt{1+a^2x^2}}\,dx + (ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + 4\int \frac{x^m(1-iax}{\left(1+a^2x^2\right)^{3}}$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + 4\int \frac{x^m}{\left(1+a^2x^2\right)^{3}}$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+m} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+}{}\right.}{1+}$$

Mathematica [C] time = 0.08, size = 113, normalized size = 0.71

$$\frac{i\sqrt{1+iax}\,\sqrt{ax+i}\,x^{m+1}\left(F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-iax,iax\right)-2F_1\left(m+1;\frac{3}{2},-\frac{1}{2};m+2;-iax,iax\right)\right)}{(m+1)\sqrt{1-iax}\,\sqrt{ax-i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^((3*I)*ArcTan[a*x]),x]
[Out] (I*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*(AppellF1[1 + m, 1/2, -1/2, 2 +
m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, (-I)*a*x, I*a*x])
)/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,(i\,ax-1)x^m}{a^2x^2-2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(i a x +1\right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
[Out] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} x^{2}+1\right)^{\frac{3}{2}} x^{m}}{\left(i\, a x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^m/(I*a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(a^2\,x^2+1\right)}^{3/2}}{{\left(1+a\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
[Out] int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i \left(\int \frac{x^{m} \sqrt{a^{2} x^{2} + 1}}{a^{3} x^{3} - 3 i a^{2} x^{2} - 3 a x + i}\, dx + \int \frac{a^{2} x^{2} x^{m} \sqrt{a^{2} x^{2} + 1}}{a^{3} x^{3} - 3 i a^{2} x^{2} - 3 a x + i}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
[Out] I*(Integral(x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**2*x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x
**2 - 3*a*x + I), x))
```

3.144 $\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,-5/4,5/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{5}{4},-\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{5}{4},-\frac{5}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int e^{\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-i)x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x - I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x + I), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
```

```
[Out] Timed out
```

3.145 $\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,-3/4,3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{3}{4},-\frac{3}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int e^{\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{i\sqrt{a^2x^2+1}\,x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax+i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x +
 I), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^{2} x^{2}+1}}\right)^{\frac{3}{2}} x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(\frac{i\,ax+1}{\sqrt{a^{2} x^{2}+1}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{m} \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
[Out] Timed out
```

3.146 $\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

[Out] x^(1+m)*AppellF1(1+m,-1/4,1/4,2+m,-I*a*x,I*a*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^((I/2)*ArcTan[a*x])*x^m,x]

[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx&=\int\frac{x^m\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx\\&=\frac{x^{1+m}F_1\left(1+m;\frac{1}{4},-\frac{1}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[E^((I/2)*ArcTan[a*x])*x^m,x]

[Out] Integrate[E^((I/2)*ArcTan[a*x])*x^m, x]

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**m,x)
[Out] Integral(x**m*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.147 $\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/4,-1/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{1}{4},\frac{1}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{-\frac{1}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/E^((I/2)*ArcTan[a*x]), x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{i\sqrt{a^2x^2+1}\,x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x
- I), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{m}}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

3.148 $\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,3/4,-3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{3}{4},\frac{3}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int e^{-\frac{3}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]), x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+i)x^m\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}}{ax-i},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*x + I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x - I), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueDone
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{m}}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**m/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

3.149 $\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,5/4,-5/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{5}{4},\frac{5}{4};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int e^{-\frac{5}{2}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]), x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,(i\,ax-1)x^m\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}}{a^2x^2-2i\,ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))/(a^2*x^2 - 2*I*a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueWarning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m op
erator + Error: Bad Argument ValueWarning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWar
ning, need to choose a branch for the root of a polynomial with parameters.
 This might be wrong.The choice was done assuming 0=[0,0]index.cc index_m o
perator + Error: Bad Argument ValueDone
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Timed out
```

3.150 $\int e^{\frac{2}{3}\tan^{-1}(x)} x^m \, dx$

Optimal. Leaf size=38

$$\frac{x^{m+1} F_1\left(m+1; -\frac{i}{3}, \frac{i}{3}; m+2; ix, -ix\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/3*I,-1/3*I,2+m,-I*x,I*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 133}

$$\frac{x^{m+1} F_1\left(m+1; -\frac{i}{3}, \frac{i}{3}; m+2; ix, -ix\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTan[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/3, I/3, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{2}{3}\tan^{-1}(x)} x^m \, dx &= \int (1-ix)^{\frac{i}{3}} (1+ix)^{-\frac{i}{3}} x^m \, dx \\
&= \frac{x^{1+m} F_1\left(1+m; -\frac{i}{3}, \frac{i}{3}; 2+m; ix, -ix\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int e^{\frac{2}{3}\tan^{-1}(x)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcTan[x])/3)*x^m,x]
```

```
[Out] Integrate[E^((2*ArcTan[x])/3)*x^m, x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m e^{\left(\frac{2}{3}\arctan(x)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(2/3*arctan(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\left(\frac{2}{3} \arctan(x)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*e^(2/3*arctan(x)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{\frac{2 \arctan(x)}{3}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2/3*arctan(x))*x^m,x)
```

```
[Out] int(exp(2/3*arctan(x))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\left(\frac{2}{3} \arctan(x)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(2/3*arctan(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \, \mathrm{e}^{\frac{2 \operatorname{atan}(x)}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp((2*atan(x))/3),x)
```

```
[Out] int(x^m*exp((2*atan(x))/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\frac{2 \operatorname{atan}(x)}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2/3*atan(x))*x**m,x)
```

```
[Out] Integral(x**m*exp(2*atan(x)/3), x)
```

3.151 $\int e^{\frac{1}{3}\tan^{-1}(x)} x^m \, dx$

Optimal. Leaf size=38

$$\frac{x^{m+1}F_1\left(m+1;-\frac{i}{6},\frac{i}{6};m+2;ix,-ix\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/6*I,-1/6*I,2+m,-I*x,I*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{i}{6},\frac{i}{6};m+2;ix,-ix\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTan[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -I/6, I/6, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{3}\tan^{-1}(x)}x^m\,dx &= \int (1-ix)^{\frac{i}{6}}(1+ix)^{-\frac{i}{6}}x^m\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{i}{6},\frac{i}{6};2+m;ix,-ix\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{3}\tan^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTan[x]/3)*x^m,x]
```

```
[Out] Integrate[E^(ArcTan[x]/3)*x^m, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m e^{\left(\frac{1}{3}\arctan(x)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="fricas")
[Out] integral(x^m*e^(1/3*arctan(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="giac")
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/3*arctan(x))*x^m,x)
[Out] int(exp(1/3*arctan(x))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\,e^{\frac{\mathrm{atan}(x)}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(atan(x)/3),x)
[Out] int(x^m*exp(atan(x)/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{\frac{\operatorname{atan}(x)}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/3*atan(x))*x**m,x)
[Out] Integral(x**m*exp(atan(x)/3), x)
```

3.152 $\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{8},-\frac{1}{8};m+2;iax,-iax\right)}{m+1}$$

[Out] x^(1+m)*AppellF1(1+m,-1/8,1/8,2+m,-I*a*x,I*a*x)/(1+m)

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{8},-\frac{1}{8};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^((I/4)*ArcTan[a*x])*x^m,x]

[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, I*a*x, (-I)*a*x])/(1 + m)

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{8},-\frac{1}{8};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{4}i\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[E^((I/4)*ArcTan[a*x])*x^m,x]

[Out] Integrate[E^((I/4)*ArcTan[a*x])*x^m, x]

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming 0=[0,0]index.cc index_m oper
ator + Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming 0=[0,0]index.cc index_m operator + Error: Bad Argument ValueWarni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming 0=[0,0]index.cc index_m ope
rator + Error: Bad Argument ValueDone
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{1}{4}} x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(\frac{i\,ax+1}{\sqrt{a^{2}x^{2}+1}}\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{m}\,{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^{2}\,x^{2}+1}}\right)}^{1/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
```

```
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
```

```
[Out] Timed out
```

3.153 $\int e^{in\tan^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=40

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;iax,-iax\right)}{m+1}$$

[Out] x^(1+m)*AppellF1(1+m,-1/2*n,1/2*n,2+m,-I*a*x,I*a*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5062, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^(I*n*ArcTan[a*x])*x^m,x]

[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, I*a*x, (-I)*a*x])/(1 + m)

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{in\tan^{-1}(ax)}x^m\,dx&=\int x^m(1-iax)^{-n/2}(1+iax)^{n/2}\,dx\\&=\frac{x^{1+m}F_1\left(1+m;\frac{n}{2},-\frac{n}{2};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int e^{in\tan^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(I*n*ArcTan[a*x])*x^m,x]

[Out] Integrate[E^(I*n*ArcTan[a*x])*x^m, x]

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="fricas")
[Out] integral(x^m/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{in\arctan(ax)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^m,x)
[Out] int(exp(I*n*arctan(a*x))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*e^(I*n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atan(a*x)*1i),x)
[Out] int(x^m*exp(n*atan(a*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{in\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**m,x)
[Out] Integral(x**m*exp(I*n*atan(a*x)), x)
```

3.154 $\int e^{in\tan^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=171

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}-\frac{(1+iax)^{\frac{n+2}{2}}\left(2ianx+n^2+6\right)(1-iax)^{1-\frac{n}{2}}}{24a^4}+\frac{x^2}{}$$

```
[Out] 1/4*x^2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2-1/24*(1-I*a*x)^(1-1/2*n
)*(1+I*a*x)^(1+1/2*n)*(6+n^2+2*I*a*n*x)/a^4-1/3*2^(-2+1/2*n)*n*(n^2+8)*(1-I
*a*x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^4/(2
-n)
```

Rubi [A] time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 100, 147, 69}

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}-\frac{(1+iax)^{\frac{n+2}{2}}\left(2ianx+n^2+6\right)(1-iax)^{1-\frac{n}{2}}}{24a^4}+\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^3,x]
```

```
[Out] (x^2*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(4*a^2) - ((1 - I*a*x)^
(1 - n/2)*(1 + I*a*x)^((2 + n)/2)*(6 + n^2 + (2*I)*a*n*x))/(24*a^4) - (2^(-
2 + n/2)*n*(8 + n^2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2,
 2 - n/2, (1 - I*a*x)/2])/(3*a^4*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^3\,dx = \int x^3(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} + \frac{\int x(1-iax)^{-n/2}(1+iax)^{n/2}(-2-ianx)\,dx}{4a^2}$$
$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}\left(6+n^2+2ianx\right)}{24a^4} + \frac{\left(in\left(8+n^2\right)\right)\int(1 \ldots}{\ldots}$$
$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}\left(6+n^2+2ianx\right)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n\left(8+n^2\right)(\ldots}{\ldots}$$

Mathematica [A]   time = 0.20, size = 210, normalized size = 1.23

$$\frac{(ax+i)(1-iax)^{-n/2}\left((n-2)\left(a^2x^2(ax-i)(1+iax)^{n/2}-i2^{\frac{n}{2}+1}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)\right)-i2^{\frac{n}{2}+3}n\,{}_2F_1\left(-\frac{n}{2}\ldots\right.\right.}{4a^4(n-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^3,x]
```

```
[Out] ((I + a*x)*((-I)*2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/2
, (1 - I*a*x)/2] + I*2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n/2, 1 - n
/2, 2 - n/2, (1 - I*a*x)/2] + (-2 + n)*(a^2*x^2*(1 + I*a*x)^(n/2)*(-I + a*x
) - I*2^(1 + n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2
])))/(4*a^4*(-2 + n)*(1 - I*a*x)^(n/2))
```

fricas [F]   time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F]   time = 0.14, size = 0, normalized size = 0.00

$$\int e^{in\arctan(ax)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^3,x)
[Out] int(exp(I*n*arctan(a*x))*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="maxima")
[Out] integrate(x^3*e^(I*n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(n*atan(a*x)*1i),x)
[Out] int(x^3*exp(n*atan(a*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{in\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**3,x)
[Out] Integral(x**3*exp(I*n*atan(a*x)), x)
```

3.155 $\int e^{in\tan^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=159

$$-\frac{i2^{n/2}\left(n^2+2\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6a^3}+\frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1}}{3a^2}$$

```
[Out] -1/6*I*n*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^3+1/3*x*(1-I*a*x)^(1-1/2
*n)*(1+I*a*x)^(1+1/2*n)/a^2-1/3*I*2^(1/2*n)*(n^2+2)*(1-I*a*x)^(1-1/2*n)*hyp
ergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^3/(2-n)
```

Rubi [A] time = 0.09, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 90, 80, 69}

$$-\frac{i2^{n/2}\left(n^2+2\right)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6a^3}+\frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-I/6)*n*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/a^3 + (x*(1 - I*a*
x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*a^2) - ((I/3)*2^(n/2)*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2
])/(a^3*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^2\,dx = \int x^2(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} + \frac{\int(1-iax)^{-n/2}(1+iax)^{n/2}(-1-ianx)\,dx}{3a^2}$$
$$= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{(2+n^2)\int(1-iax)^{-n/2}(1+ia}{6a^2}$$
$$= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{i2^{n/2}(2+n^2)(1-iax)^{1-\frac{n}{2}}\,{}_2F_1}{3a^3(2}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.73

$$\frac{(ax+i)(1-iax)^{-n/2}\left(2^{\frac{n}{2}+1}\left(n^2+2\right)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)+(n-2)(ax-i)(2ax-in)(1+iax)^{n/2}\right)}{6a^3(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2,x]
```

```
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x)*((-I)*n + 2*a*x) + 2^(1 +
 n/2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2])
)/(6*a^3*(-2 + n)*(1 - I*a*x)^(n/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int e^{in\arctan(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^2,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(I*n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atan(a*x)*1i),x)
[Out] int(x^2*exp(n*atan(a*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{in\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**2,x)
[Out] Integral(x**2*exp(I*n*atan(a*x)), x)
```

3.156 $\int e^{in\tan^{-1}(ax)} x\,dx$

Optimal. Leaf size=107

$$\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

```
[Out] 1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2+2^(1/2*n)*n*(1-I*a*x)^(1-1/
2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^2/(2-n)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5062, 80, 69}

$$\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*a^2) + (2^(n/2)*n*(1 - I
*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - I*a*x)/2])/(
a^2*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{in\tan^{-1}(ax)}x\,dx &= \int x(1-iax)^{-n/2}(1+iax)^{n/2}\,dx\\
&= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2}-\frac{(in)\int(1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{2a}\\
&= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2}+\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a^2(2-n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 105, normalized size = 0.98

$$\frac{(ax+i)(1-iax)^{-n/2}\left(i2^{\frac{n}{2}+1}n\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)+(n-2)(ax-i)(1+iax)^{n/2}\right)}{2a^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x) + I*2^(1 + n/2)*n*Hyperge
ometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2]))/(2*a^2*(-2 + n)*(1 -
I*a*x)^(n/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int e^{in\arctan(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(I*n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atan(a*x)*1i),x)
[Out] int(x*exp(n*atan(a*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{in\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x,x)
[Out] Integral(x*exp(I*n*atan(a*x)), x)
```

3.157 $\int e^{in\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=71

$$\frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

```
[Out] I*2^(1+1/2*n)*(1-I*a*x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2
-1/2*I*a*x)/a/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5061, 69}

$$\frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x]),x]
```

```
[Out] (I*2^(1 + n/2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n
/2, (1 - I*a*x)/2])/(a*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{in\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-n/2}(1+iax)^{n/2}\,dx\\ &= \frac{i2^{1+\frac{n}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{a(2-n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.75

$$-\frac{4ie^{i(n+2)\tan^{-1}(ax)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2i\tan^{-1}(ax)}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*n*ArcTan[a*x]),x]
```

```
[Out] ((-4*I)*E^(I*(2 + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^((2*I)*ArcTan[a*x])])/(a*(2 + n))
```

fricas [F]   time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="fricas")
[Out] integral(1/((-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F]   time = 0.12, size = 0, normalized size = 0.00

$$\int e^{in\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x)),x)
[Out] int(exp(I*n*arctan(a*x)),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)*1i),x)
[Out] int(exp(n*atan(a*x)*1i), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int e^{in\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x)),x)
[Out] Integral(exp(I*n*atan(a*x)), x)
```

3.158 $\int \frac{e^{in\tan^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=125

$$\frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{n}$$

```
[Out] 2*(1+I*a*x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(1-I*a*x)/(1+I*a*x))/n/
((1-I*a*x)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2-1/
2*I*a*x)/n/((1-I*a*x)^(1/2*n))
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5062, 105, 69, 131}

$$\frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x,x]
```

```
[Out] (2*(1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - I*a*x)/(1 + I
*a*x)])/(n*(1 - I*a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2,
1 - n/2, (1 - I*a*x)/2])/(n*(1 - I*a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x}\,dx$$
$$= -\left((ia)\int (1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}\,dx\right) + \int \frac{(1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}}{x}\,dx$$
$$= \frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-iax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(\right.}{n}$$

Mathematica [A] time = 0.03, size = 106, normalized size = 0.85

$$\frac{2(1-iax)^{-n/2}\left((1+iax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{ax+i}{i-ax}\right)-2^{n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-iax)\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x,x]
[Out] (2*((1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (I + a*x)/(I - a*x)] - 2^(n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2, (1 - I*a*x)/2]))/(n*(1 - I*a*x)^(n/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="fricas")
[Out] integral(1/(x*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x,x)
[Out] int(exp(I*n*arctan(a*x))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(I*n*arctan(a*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)*1i)/x,x)
```

```
[Out] int(exp(n*atan(a*x)*1i)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x,x)
```

```
[Out] Integral(exp(I*n*atan(a*x))/x, x)
```

3.159 $\int \frac{e^{in\tan^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}$$

```
[Out] -4*I*a*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1
/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5062, 131}

$$-\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-4*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{in\tan^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx\\ &= -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.04

$$-\frac{2ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n}{2}-1}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};-\frac{1-iax}{-iax-1}\right)}{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-2*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^(-1 + n/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, -((1 - I*a*x)/(-1 - I*a*x))])/(1 - n/2)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="fricas")
[Out] integral(1/(x^2*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^2,x)
[Out] int(exp(I*n*arctan(a*x))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atan}(a\,x)\,1i}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)*1i)/x^2,x)
[Out] int(exp(n*atan(a*x)*1i)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**2,x)
[Out] Integral(exp(I*n*atan(a*x))/x**2, x)
```

3.160 $\int \frac{e^{in\tan^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=120

$$\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

```
[Out] -1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^2+2*a^2*n*(1-I*a*x)^(1-1/2*n
)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*x)
)/(2-n)
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5062, 96, 131}

$$\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{2-n}-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*x^2) + (2*a^2*n*(1 - I*
a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^3}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{1}{2}(ian)\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{1+iax}\right)}{2-n}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.95

$$\frac{(ax+i)(1-iax)^{-n/2}(1+iax)^{n/2}\left(4a^2nx^2\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{ax+i}{i-ax}\right)-(n-2)(ax-i)^2\right)}{2(n-2)x^2(ax-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 + I*a*x)^(n/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2) + 4*a^2*n*x^2*Hyperg
eometric2F1[2, 1 - n/2, 2 - n/2, (I + a*x)/(I - a*x)]))/(2*(-2 + n)*x^2*(1
- I*a*x)^(n/2)*(-I + a*x))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x^3\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^3,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)*1i)/x^3,x)
[Out] int(exp(n*atan(a*x)*1i)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**3,x)
[Out] Integral(exp(I*n*atan(a*x))/x**3, x)
```

3.161 $\int \frac{e^{in\tan^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=171

$$\frac{2ia^3\left(n^2+2\right)(1+iax)^{\frac{n-2}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{3(2-n)}-\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3x^3}-\frac{ian(1+iax)^{\frac{n+2}{2}}(1-ia}{6x^2}$$

```
[Out] -1/3*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^3-1/6*I*a*n*(1-I*a*x)^(1-1/2
*n)*(1+I*a*x)^(1+1/2*n)/x^2+2/3*I*a^3*(n^2+2)*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)
^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Rubi [A] time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5062, 129, 151, 12, 131}

$$\frac{2ia^3\left(n^2+2\right)(1+iax)^{\frac{n-2}{2}}(1-iax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-iax}{iax+1}\right)}{3(2-n)}-\frac{ian(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6x^2}-\frac{(1+iax)^{\frac{n+2}{2}}(1-ia}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*x^3) - ((I/6)*a*n*(1 -
I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (((2*I)/3)*a^3*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 5062

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 - I*a
*x)^((I*n)/2))/(1 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\int \frac{e^{in\tan^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^4}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{1}{3}\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}\left(-ian+a^2x\right)}{x^3}\,dx$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} - \frac{1}{6}\int \frac{a^2\left(2+n^2\right)(1-iax)^{-n/2}(1}{x^2}$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} - \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\int \frac{(1-iax)^{-n/2}(}{x^2}$$
$$= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} + \frac{2ia^3\left(2+n^2\right)(1-iax)^{1-\frac{n}{2}}(1+i}{3(}$$

Mathematica [A] time = 0.06, size = 119, normalized size = 0.70

$$-\frac{(ax+i)(1-iax)^{-n/2}(1+iax)^{\frac{n-2}{2}}\left(4a^3\left(n^2+2\right)x^3\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{ax+i}{i-ax}\right)-(n-2)(ax-i)^2(anx-2i)\right)}{6(n-2)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*((1 + I*a*x)^((-2 + n)/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2*(-2*I + a
*n*x)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (I + a*
x)/(I - a*x)]))/((-2 + n)*x^3*(1 - I*a*x)^(n/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^4\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(1/(x^4*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{e^{in \arctan(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))/x^4,x)
[Out] int(exp(I*n*arctan(a*x))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(i\,n \arctan(ax))}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atan}(a\,x)\,1i}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)*1i)/x^4,x)
[Out] int(exp(n*atan(a*x)*1i)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{in \operatorname{atan}{(ax)}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))/x**4,x)
[Out] Integral(exp(I*n*atan(a*x))/x**4, x)
```

3.162 $\int e^{i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+114a+19i\right)}{120b^5}+\frac{\left(8ia^4-16a^3-24ia\right.}{}$$

[Out] 1/8*(3-12*I*a-24*a^2+16*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5-1/20*(I+8*a)*x^2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^3+1/5*x^3*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2+1/120*(1+I*a+I*b*x)^(3/2)*(19*I+114*a-86*I*a^2-96*a^3-2*(13-14*I*a-36*a^2)*b*x)*(1-I*a-I*b*x)^(1/2)/b^5+1/8*(3*I+12*a-24*I*a^2-16*a^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5

Rubi [A] time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5095, 100, 153, 147, 50, 53, 619, 215}

$$\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-2\left(-36a^2-14ia+13\right)bx-96a^3-86ia^2+114a+19i\right)}{120b^5}+\frac{\left(8ia^4-16a^3-24ia\right.}{}$$

Antiderivative was successfully verified.

[In] Int[E^(I*ArcTan[a + b*x])*x^4,x]

[Out] ((3*I + 12*a - (24*I)*a^2 - 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(8*b^5) - ((I + 8*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(20*b^3) + (x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(5*b^2) + (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(19*I + 114*a - (86*I)*a^2 - 96*a^3 - 2*(13 - (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 - (12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x^3\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x^2\sqrt{1+ia+ibx}\left(-3\left(1+a^2\right)-(i+8a)bx\right)}{\sqrt{1-ia-ibx}}dx}{5b^2}$$

$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x\sqrt{1+ia}}{}}{}$$

$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia}}{}$$

$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}}{20b^3}$$

$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}}{20b^3}$$

$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}}{20b^3}$$

$$= \frac{\left(3i+12a-24ia^2-16a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}}{20b^3}$$

Mathematica [A] time = 0.53, size = 217, normalized size = 0.79

$$\frac{\sqrt[4]{-1}\left(8a^4+16ia^3-24a^2-12ia+3\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4b^{11/2}} + \frac{i\sqrt{a^2+2abx+b^2x^2+1}\,\left(24a^4+a^3(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^4,x]
```

```
[Out] ((I/120)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(64 + 24*a^4 + (45*I)*b*x - 32*b
^2*x^2 - (30*I)*b^3*x^3 + 24*b^4*x^4 + a^3*(250*I - 24*b*x) + 2*a^2*(-166 -
 (65*I)*b*x + 12*b^2*x^2) + a*(-275*I + 116*b*x + (70*I)*b^2*x^2 - 24*b^3*x
^3)))/b^5 + ((-1)^(1/4)*(3 - (12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*Sqrt[(
-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]]
)/(4*b^(11/2))
```

fricas [A] time = 0.49, size = 175, normalized size = 0.63

$$\frac{186i\,a^5-1345\,a^4-1730i\,a^3+1320\,a^2-\left(960\,a^4+1920i\,a^3-2880\,a^2-1440i\,a+360\right)\log\left(-bx-a+\sqrt{b^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/960*(186*I*a^5 - 1345*a^4 - 1730*I*a^3 + 1320*a^2 - (960*a^4 + 1920*I*a^3
 - 2880*a^2 - 1440*I*a + 360)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)) + (192*I*b^4*x^4 - 48*(4*I*a - 5)*b^3*x^3 + (192*I*a^2 - 560*a - 256*I
)*b^2*x^2 + 192*I*a^4 - 2000*a^3 + (-192*I*a^3 + 1040*a^2 + 928*I*a - 360)*
b*x - 2656*I*a^2 + 2200*a + 512*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 300*
I*a)/b^5
```

giac [A] time = 0.20, size = 214, normalized size = 0.78

$$\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\left(\frac{4\,ix}{b}-\frac{4\,ab^7i-5\,b^7}{b^9}\right)x+\frac{12\,a^2b^6i-35\,ab^6-16\,b^6i}{b^9}\right)x-\frac{24\,a^3b^5i-130\,a^2b^5-116\,ab^5i}{b^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/120*sqrt((b*x + a)^2 + 1)*((2*(3*(4*i*x/b - (4*a*b^7*i - 5*b^7)/b^9)*x +
(12*a^2*b^6*i - 35*a*b^6 - 16*b^6*i)/b^9)*x - (24*a^3*b^5*i - 130*a^2*b^5 -
 116*a*b^5*i + 45*b^5)/b^9)*x + (24*a^4*b^4*i - 250*a^3*b^4 - 332*a^2*b^4*i
 + 275*a*b^4 + 64*b^4*i)/b^9) - 1/8*(8*a^4 + 16*a^3*i - 24*a^2 - 12*a*i + 3
)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

maple [B] time = 0.20, size = 656, normalized size = 2.38

$$-\frac{7a\,x^2\sqrt{b^2x^2+2abx+a^2+1}}{12b^3}+\frac{13a^2x\sqrt{b^2x^2+2abx+a^2+1}}{12b^4}+\frac{a^4\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b^4\sqrt{b^2}}-\frac{3a^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x)
```

```
[Out] -7/12*a/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/12*a^2/b^4*x*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+a^4/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)-3*a^2/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)-83/30*I/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+29/3
0*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^
(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/5*I/b^2*a*x^3*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)+1/5*I/b^5*a^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/5*I/b^3*a
^2*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-4/15*I/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)+8/15*I/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^4*a^3*x*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)+2*I/b^4*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+
a^2+1)^(1/2))/(b^2)^(1/2)+1/4*x^3/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-25/12*a
^3/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+55/24*a/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+1/5*I/b*x^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/8/b^4*x*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+3/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/(b^2)^(1/2)
```

maxima [B] time = 0.36, size = 749, normalized size = 2.71

$$\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x^4}{5\,b}-\frac{9i\sqrt{b^2x^2+2\,abx+a^2+1}\,ax^3}{20\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,(-i\,a-1)x^3}{4\,b^2}+\frac{21i\sqrt{b^2x^2+2\,a}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^4/b - 9/20*I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a*x^3/b^2 - 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*
x^3/b^2 + 21/20*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x^2/b^3 - 7/12*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x^2/b^3 - 63/8*I*a^5*arcsinh(2*(b
^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35/8*a^4*(I*a + 1)*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 21/8*I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 + 35/24*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a^2*(I*a + 1)*x/b^4 - 1/15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(4*I*a^
2 + 4*I)*x^2/b^3 + 35/4*I*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
```

```
*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 15/4*(a^2 + 1)*a^2*(I*a + 1)*arcsinh(2*(b^2*
x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 63/8*I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^4/b^5 - 35/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*(I*a
+ 1)/b^5 + 161/120*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^4 -
3/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*(I*a + 1)*x/b^4 - 15/8*I*(a
^2 + 1)^2*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5
 - 3/8*(a^2 + 1)^2*(-I*a - 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2))/b^5 - 49/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a^2
/b^5 + 55/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*(I*a + 1)/b^5 +
8/15*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}{\sqrt{{\left(a+b\,x\right)}^2+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i \left(\int \left(- \frac{i x^{4}}{\sqrt{a^{2} + 2 a b x + b^{2} x^{2} + 1}}\right)\, dx + \int \frac{a x^{4}}{\sqrt{a^{2} + 2 a b x + b^{2} x^{2} + 1}}\, dx + \int \frac{b x^{5}}{\sqrt{a^{2} + 2 a b x + b^{2} x^{2} + 1}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**4,x)
```

```
[Out] I*(Integral(-I*x**4/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(a*x
**4/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(b*x**5/sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1), x))
```

3.163 $\int e^{i \tan^{-1}(a+bx)} x^3 \, dx$

Optimal. Leaf size=201

$$-\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-18a^2+2(6a+i)bx-10ia+7\right)}{24b^4}-\frac{\left(8ia^3-12a^2-12ia+3\right)\sqrt{-ia-ibx+1}\,\sqrt{ia\cdots}}{8b^4}$$

```
[Out] 1/8*(3*I+12*a-12*I*a^2-8*a^3)*arcsinh(b*x+a)/b^4+1/4*x^2*(1+I*a+I*b*x)^(3/2
)*(1-I*a-I*b*x)^(1/2)/b^2-1/24*(1+I*a+I*b*x)^(3/2)*(7-10*I*a-18*a^2+2*(I+6*
a)*b*x)*(1-I*a-I*b*x)^(1/2)/b^4-1/8*(3-12*I*a-12*a^2+8*I*a^3)*(1-I*a-I*b*x)
^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
```

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 100, 147, 50, 53, 619, 215}

$$-\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-18a^2+2(6a+i)bx-10ia+7\right)}{24b^4}-\frac{\left(8ia^3-12a^2-12ia+3\right)\sqrt{-ia-ibx+1}\,\sqrt{ia\cdots}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] -((3 - (12*I)*a - 12*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^
2) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(7 - (10*I)*a - 18*a^2
+ 2*(I + 6*a)*b*x))/(24*b^4) + ((3*I + 12*a - (12*I)*a^2 - 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
```

```
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{4b^2} + \frac{\int \frac{x\sqrt{1+ia+ibx}\left(-2\left(1+a^2\right)-(i+6a)bx\right)}{\sqrt{1-ia-ibx}}\,dx}{4b^2}$$

$$= \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}\left(7-10ia-18a^2+2(i\right.}{24b^4}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3}}{4b^2}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3}}{4b^2}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3}}{4b^2}$$

$$= -\frac{\left(3-12ia-12a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3}}{4b^2}$$

Mathematica [A] time = 0.31, size = 176, normalized size = 0.88

$$\frac{\sqrt{b}\sqrt{a^2+2abx+b^2x^2+1}\left(-6ia^3+a^2(44+6ibx)+a\left(-6ib^2x^2-20bx+39i\right)+6ib^3x^3+8b^2x^2-9ibx-16\right)}{24b^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[b]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-16 - (6*I)*a^3 - (9*I)*b*x + 8
*b^2*x^2 + (6*I)*b^3*x^3 + a^2*(44 + (6*I)*b*x) + a*(39*I - 20*b*x - (6*I)*
b^2*x^2)) - 6*(-1)^(1/4)*(-3*I - 12*a + (12*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*Ar
cSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(24*b^(
9/2))
```

fricas [A] time = 0.63, size = 137, normalized size = 0.68

$$\frac{-45i\,a^4+224\,a^3+192i\,a^2+\left(192\,a^3+288i\,a^2-288\,a-72i\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(48i\,b^3x^3\right.}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/192*(-45*I*a^4 + 224*a^3 + 192*I*a^2 + (192*a^3 + 288*I*a^2 - 288*a - 72*
I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (48*I*b^3*x^3 - 16*(
3*I*a - 4)*b^2*x^2 - 48*I*a^3 + (48*I*a^2 - 160*a - 72*I)*b*x + 352*a^2 + 3
12*I*a - 128)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

giac [A] time = 0.87, size = 163, normalized size = 0.81

$$\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{3\,ix}{b}-\frac{3\,ab^5i-4\,b^5}{b^7}\right)x+\frac{6\,a^2b^4i-20\,ab^4-9\,b^4i}{b^7}\right)x-\frac{6\,a^3b^3i-44\,a^2b^3-39\,ab^3i+16\,b^3}{b^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt((b*x + a)^2 + 1)*((2*(3*i*x/b - (3*a*b^5*i - 4*b^5)/b^7)*x + (6*a
^2*b^4*i - 20*a*b^4 - 9*b^4*i)/b^7)*x - (6*a^3*b^3*i - 44*a^2*b^3 - 39*a*b^
3*i + 16*b^3)/b^7) + 1/8*(8*a^3 + 12*a^2*i - 12*a - 3*i)*log(-a*b - (x*abs(
b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

maple [B] time = 0.19, size = 465, normalized size = 2.31

$$\frac{3i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^3\sqrt{b^2}}-\frac{3ix\sqrt{b^2x^2+2abx+a^2+1}}{8b^3}+\frac{13ia\sqrt{b^2x^2+2abx+a^2+1}}{8b^4}-\frac{ia^3\sqrt{b^2x^2+2\ldots}}{4b\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x)
```

```
[Out] 3/8*I/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)-3/8*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/8*I/b^4*a*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)-1/4*I/b^4*a^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/4*I/b*x^3*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)-1/4*I/b^2*a*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/4
*I/b^3*a^2*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/3*x^2/b^2*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)-5/6*a/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+11/6*a^2/b^4*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a^3/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-2/3/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.35, size = 529, normalized size = 2.63

$$\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x^3}{4\,b}-\frac{7i\sqrt{b^2x^2+2\,abx+a^2+1}\,ax^2}{12\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,(-i\,a-1)x^2}{3\,b^2}+\frac{35i\,a^4\,\mathrm{arsinh}\left(\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b - 7/12*I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a*x^2/b^2 - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*
x^2/b^2 + 35/8*I*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2))/b^4 - 5/2*a^3*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2))/b^4 + 35/24*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 -
5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x/b^3 - 15/4*I*(a^2 + 1)*
a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 3/2*(
a^2 + 1)*a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2))/b^4 - 35/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 + 5/2*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^2*(I*a + 1)/b^4 - 1/8*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(3*I*a^2 + 3*I)*x/b^3 + 3/8*I*(a^2 + 1)^2*arcsinh(2*(b^2*x + a*b)/
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 55/24*I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(a^2 + 1)*a/b^4 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*
(I*a + 1)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}{\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{ix^3}{\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{ax^3}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{bx^4}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**3,x)
```

```
[Out] I*(Integral(-I*x**3/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(a*x
**3/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(b*x**4/sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1), x))
```

3.164 $\int e^{i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=171

$$-\frac{\left(-2ia^2+2a+i\right)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^3}-\frac{\left(-2a^2-2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}-\frac{(4a+i)\sqrt{-ia-ibx+1}\,(ia+i}{6b^3}$$

[Out] -1/2*(1-2*I*a-2*a^2)*arcsinh(b*x+a)/b^3-1/6*(I+4*a)*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^3+1/3*x*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2-1/2*(I+2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 90, 80, 50, 53, 619, 215}

$$-\frac{\left(-2ia^2+2a+i\right)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^3}-\frac{\left(-2a^2-2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{-ia-ibx+1}\,(ia+ibx+1}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[E^(I*ArcTan[a + b*x])*x^2,x]

[Out] -((I + 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3) - ((I + 4*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(6*b^3) + (x*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(3*b^2) - ((1 - (2*I)*a - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x^2\,dx=\int\frac{x^2\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$=\frac{x\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{3b^2}+\frac{\int\frac{\sqrt{1+ia+ibx}\left(-1-a^2-(i+4a)bx\right)}{\sqrt{1-ia-ibx}}\,dx}{3b^2}$$

$$=-\frac{(i+4a)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3}+\frac{x\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{3b^2}-\frac{(1-2ia-2}{}$$

$$=-\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3}-\frac{(i+4a)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3}+$$

$$=-\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3}-\frac{(i+4a)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3}+$$

$$=-\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3}-\frac{(i+4a)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3}+$$

$$=-\frac{\left(i+2a-2ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3}-\frac{(i+4a)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3}+$$

Mathematica [A] time = 0.14, size = 135, normalized size = 0.79

$$\frac{\sqrt[4]{-1}\left(2a^2+2ia-1\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{7/2}}+\frac{\sqrt{a^2+2abx+b^2x^2+1}\,\left(2ia^2+a(-9-2ibx)+2ib^2x^2\right.}{6b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4*I + (2*I)*a^2 + 3*b*x + (2*I)*b^2*x^
2 + a*(-9 - (2*I)*b*x)))/(6*b^3) + ((-1)^(1/4)*(-1 + (2*I)*a + 2*a^2)*Sqrt[
(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]
])/b^(7/2)
```

fricas [A] time = 0.48, size = 106, normalized size = 0.62

$$\frac{7ia^3-21a^2-12\left(2a^2+2ia-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2abx+a^2+1}\right)+\sqrt{b^2x^2+2abx+a^2+1}\,\left(8ib^2x^2\right.}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(7*I*a^3 - 21*a^2 - 12*(2*a^2 + 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(8*I*b^2*x^2 - 4
*(2*I*a - 3)*b*x + 8*I*a^2 - 36*a - 16*I) - 9*I*a)/b^3
```

giac [A] time = 0.20, size = 117, normalized size = 0.68

$$\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^3i-3\,b^3}{b^5}\right)x+\frac{2\,a^2b^2i-9\,ab^2-4\,b^2i}{b^5}\right)-\frac{\left(2\,a^2+2\,ai-1\right)\log\left(-ab-\left(x|b|-\sqrt{(bx+a}\right.\right.}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^3*i - 3*b^3)/b^5)*x + (2*a^2*b
^2*i - 9*a*b^2 - 4*b^2*i)/b^5) - 1/2*(2*a^2 + 2*a*i - 1)*log(-a*b - (x*abs(
b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

maple [B] time = 0.18, size = 302, normalized size = 1.77

$$\frac{ix^2\sqrt{b^2x^2+2abx+a^2+1}}{3b}-\frac{iax\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}+\frac{ia^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^3}+\frac{ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2ab\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x)
```

```
[Out] 1/3*I/b*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/b^2*a*x*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+1/3*I/b^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/b^2*a*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-2/3*I/b^3*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 0.35, size = 351, normalized size = 2.05

$$\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x^2}{3\,b}-\frac{5i\,a^3\,\mathrm{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3}+\frac{3\,a^2(i\,a+1)\,\mathrm{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3}-\frac{5i\,\sqrt{b^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/b - 5/2*I*a^3*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3/2*a^2*(I*a + 1)*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 5/6*I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*a*x/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*
a - 1)*x/b^2 + 3/2*I*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2))/b^3 - 1/2*(a^2 + 1)*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*a^2/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)/b^3 - 1/3*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*a^2 + 2*I)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}{\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{ix^2}{\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{ax^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{bx^3}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**2,x)
```

```
[Out] I*(Integral(-I*x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(a*x
**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(b*x**3/sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1), x))
```

3.165 $\int e^{i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2b^2}+\frac{(1-2ia)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^2}-\frac{(2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

```
[Out] -1/2*(I+2*a)*arcsinh(b*x+a)/b^2+1/2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)
/b^2+1/2*(1-2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 80, 50, 53, 619, 215}

$$\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2b^2}+\frac{(1-2ia)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^2}-\frac{(2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + (Sqrt
[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((I + 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$
$$= \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{}{\sqrt{1-ia-}}}{2b}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{}{\sqrt{(1-ia)}}}{2}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\,\mathrm{Subst}\Big(\int}{}$$
$$= \frac{(1-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\sinh^{-1}(a}{2b^2}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.98

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(-ia+ibx+2)}{2b^2} + \frac{(-1)^{3/4}(2a+i)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}\,b^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2 - I*a + I*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + ((-1)^(3/4)
*(I + 2*a)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)
*b]])/(Sqrt[(-I)*b]*b^(3/2))
```

fricas [A] time = 0.43, size = 77, normalized size = 0.70

$$\frac{-3i\,a^2 + (8\,a+4i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}\,(4i\,bx-4i\,a+8)+4\,a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*a^2 + (8*a + 4*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(4*I*b*x - 4*I*a + 8) + 4*a)/b^2
```

giac [A] time = 0.19, size = 76, normalized size = 0.69

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}-\frac{abi-2\,b}{b^3}\right) + \frac{(2\,a+i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b*i - 2*b)/b^3) + 1/2*(2*a + i)*log(-
a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

maple [A] time = 0.17, size = 171, normalized size = 1.55

$$\frac{ix\sqrt{b^2x^2+2abx+a^2+1}}{2b}-\frac{ia\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}+\frac{\sqrt{b^2x^2+2abx+a^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x)
```

```
[Out] 1/2*I/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/b^2*a*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)-1/2*I/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)+1/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a/b*ln((b^2*x+a*b)/(b^2)^(1/2
)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 0.33, size = 209, normalized size = 1.90

$$\frac{3i\,a^2\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^2}-\frac{a(i\,a+1)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b^2}+\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x}{2\,b}-\frac{\left(i\,a^2+i\right)\operatorname{ars}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 3/2*I*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 -
 a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
2 + 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - 1/2*(I*a^2 + I)*arcsinh(2
*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - 3/2*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a + 1)/b
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}{\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{ix}{\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{ax}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{bx^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x,x)
```

```
[Out] I*(Integral(-I*x/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(a*x/sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(b*x**2/sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1), x))
```

3.166 $\int e^{i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh^{-1}(a+bx)}{b}+\frac{i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}$$

```
[Out] arcsinh(b*x+a)/b+I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5093, 50, 53, 619, 215}

$$\frac{\sinh^{-1}(a+bx)}{b}+\frac{i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{i\tan^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx\\ &= \frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\int \frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\\ &= \frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx\\ &= \frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}\\ &= \frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\frac{\sinh^{-1}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.54

$$\frac{\sinh^{-1}(a+bx)+i\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (I*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

fricas [A] time = 0.44, size = 60, normalized size = 1.15

$$\frac{i\,a+2i\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(I*a + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [A] time = 0.48, size = 51, normalized size = 0.98

$$\frac{\sqrt{(bx+a)^2+1}\,i}{b}-\frac{\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt((b*x + a)^2 + 1)*i/b - log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*a
bs(b))/abs(b)
```

maple [A] time = 0.11, size = 69, normalized size = 1.33

$$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 0.33, size = 62, normalized size = 1.19

$$\frac{\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b}+\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b
```

mupad [B] time = 1.09, size = 97, normalized size = 1.87

$$\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\;1\mathrm{i}}{b}+\frac{\operatorname{asinh}\left(a+b\,x\right)}{b}+\frac{a\operatorname{asinh}\left(a+b\,x\right)\;1\mathrm{i}}{b}-\frac{a\,b^2\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right.}{\left(b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)*1i)/b + asinh(a + b*x)/b + (a*asinh(a
+ b*x)*1i)/b - (a*b^2*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*
x)/(b^2)^(1/2))*1i)/(b^2)^(3/2)
```

sympy [A] time = 3.41, size = 36, normalized size = 0.69

$$\begin{cases} \frac{i\sqrt{\left(a+bx\right)^2+1}+\operatorname{asinh}\left(a+bx\right)}{b} & \text{for}\ b\neq 0 \\ \frac{x\left(ia+1\right)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise(((I*sqrt((a + b*x)**2 + 1) + asinh(a + b*x))/b, Ne(b, 0)), (x*(I*
a + 1)/sqrt(a**2 + 1), True))
```

3.167 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=89

$$i\sinh^{-1}(a+bx)-\frac{2\sqrt{-a+i}\,\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}$$

```
[Out] I*arcsinh(b*x+a)-2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I
*a-I*b*x)^(1/2))*(I-a)^(1/2)/(I+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 105, 53, 619, 215, 93, 208}

$$i\sinh^{-1}(a+bx)-\frac{2\sqrt{-a+i}\,\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] I*ArcSinh[a + b*x] - (2*Sqrt[I - a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b
*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I + a]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1+ia+ibx}}{x\sqrt{1-ia-ibx}}\,dx\\
&= -\left((-1-ia)\int \frac{1}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\right)+(ib)\int \frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\\
&= (2(1+ia))\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)+(ib)\int \frac{}{\sqrt{(1-ia)(1+i}}\\
&= -\frac{2\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}+\frac{i\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b}\\
&= i\sinh^{-1}(a+bx)-\frac{2\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 142, normalized size = 1.60

$$\frac{2(-1)^{3/4}\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{b}}-\frac{2\sqrt{-1-ia}\,\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
b*x)])/Sqrt[(-I)*b]])/Sqrt[b] - (2*Sqrt[-1 - I*a]*ArcTanh[(Sqrt[-1 - I*a]*S
qrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 +
 I*a]
```

fricas [B] time = 0.47, size = 155, normalized size = 1.74

$$\frac{1}{2}\sqrt{-\frac{4\,a-4i}{a+i}}\,\log\left(-bx+\frac{1}{2}\,(i\,a-1)\sqrt{-\frac{4\,a-4i}{a+i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\frac{1}{2}\sqrt{-\frac{4\,a-4i}{a+i}}\,\log\left(-bx+\frac{1}{2}\,(-i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-(4*a - 4*I)/(a + I))*log(-b*x + 1/2*(I*a - 1)*sqrt(-(4*a - 4*I)/(
a + I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 1/2*sqrt(-(4*a - 4*I)/(a + I
))*log(-b*x + 1/2*(-I*a + 1)*sqrt(-(4*a - 4*I)/(a + I)) + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)) - I*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

giac [A] time = 3.92, size = 112, normalized size = 1.26

$$-\frac{bi\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}+\frac{(ai+1)\log\left(\left|\frac{-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}}{-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}}\right|\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -b*i*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) + (a*i +
1)*log(abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*
x*abs(b) + 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/sqrt(a^2 + 1)
```

maple [B] time = 0.16, size = 157, normalized size = 1.76

$$\frac{ib\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{i\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\ \sqrt{b^2x^2+2abx+a^2+1}}{x}\right)a}{\sqrt{a^2+1}}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\ \sqrt{b^2x^2+2a}}{x}\right.}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I
/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)*a-1/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x)
```

maxima [B] time = 0.33, size = 233, normalized size = 2.62

$$-\frac{i\,a\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}\right)}{\sqrt{a^2+1}}-\frac{\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\ldots\right.}{\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -I*a*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + I*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

mupad [B] time = 1.15, size = 118, normalized size = 1.33

$$\mathrm{asinh}\,(a+b\,x)\ 1\mathrm{i}-\frac{\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\ \sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}-\frac{a\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\ \sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)\,1\mathrm{i}}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)/(x*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] asinh(a + b*x)*1i - log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2
 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2) - (a*log(a*b + (a^2 + 1)/x + ((a^
2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{b}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int \left(-\frac{i}{x\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int \frac{a}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] I*(Integral(b/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(-I/(x*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a/(x*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)), x))
```

3.168 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=130

$$\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\,(a+i)^{3/2}}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

```
[Out] 2*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/
2))/(I+a)^(3/2)/(I-a)^(1/2)-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)
/x
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5095, 94, 93, 208}

$$\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\,(a+i)^{3/2}}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x)) + ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/(Sqrt[I - a]*(I + a)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{i+a}$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i+a}$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1-ia)x} + \frac{2ib\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}\,(i+a)^{3/2}}$$

Mathematica [A] time = 0.09, size = 120, normalized size = 0.92

$$-i\left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax+ix} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}\,(-1+ia)^{3/2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-I)*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*x + a*x) + (2*b*ArcTanh[(Sqrt[-1
 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/
(Sqrt[-1 - I*a]*(-1 + I*a)^(3/2)))
```

fricas [B] time = 0.50, size = 227, normalized size = 1.75

$$-\frac{2\,(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+\left(a^3+i\,a^2+a+i\right)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}}{b}\right) - 2\,(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log}{(2\,a+2i)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b + (a^3 + I*a^2 + a + I)*sqrt(b^2/(a^4 + 2*I*a^3
+ 2*I*a - 1)))/b) - 2*(a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-
(b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 + I*a^2 + a + I)*sqrt(b
^2/(a^4 + 2*I*a^3 + 2*I*a - 1)))/b) + 2*I*b*x + 2*I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/((2*a + 2*I)*x)
```

giac [A] time = 0.23, size = 145, normalized size = 1.12

$$\frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}\,(a+i)} - \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(ai-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*a
bs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a + i))
 - 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(a*i - 1))
```

maple [B] time = 0.17, size = 236, normalized size = 1.82

$$-\frac{ib\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}-\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,a}{\left(a^2+1\right)x}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{ia^2b\ln\left(\frac{2a^2+2+2a}{\ }\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -I*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)-I/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/(a^2+1)/x*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)+I*a^2*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+a*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a
*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

maxima [B] time = 0.32, size = 239, normalized size = 1.84

$$\frac{a(i\,a+1)b\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{\left(a^2+1\right)^{\frac{3}{2}}}-\frac{i\,b\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] a*(I*a + 1)*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - I*b*arcsinh(2*a*b*x/(sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)/((a^2 + 1)*x)
```

mupad [B] time = 1.68, size = 218, normalized size = 1.68

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)}-\frac{b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)1\mathrm{i}}{\sqrt{a^2+1}}+\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)/(x^2*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] (a^2*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x +
1)^(1/2)))*1i)/(a^2 + 1)^(3/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^
2 + 1)) - (b*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*
b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2) - (a*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(
1/2)*1i)/(x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2
 + b^2*x^2 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{a}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{b}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] I*(Integral(-I/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x))
```

3.169 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}+\frac{(1+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{5/2}}-\frac{(1+2ia)b\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2(-a+i)(a+i)^2x}$$

```
[Out] (1+2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*
b*x)^(1/2))/(I-a)^(3/2)/(I+a)^(5/2)-1/2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(
1/2)/(a^2+1)/x^2-1/2*(1+2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I
-a)/(I+a)^2/x
```

Rubi [A] time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}+\frac{(1+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{5/2}}-\frac{(1+2ia)b\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2(-a+i)(a+i)^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -((1 + (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)*(
I + a)^2*x) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*(1 + a^2)*
x^2) + ((1 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt
[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(3/2)*(I + a)^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1+ia+ibx}}{x^3\sqrt{1-ia-ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} + \frac{((i-2a)b)\int \frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx}{2\left(1+a^2\right)}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} - \frac{\left((i-2a)b^2\right)\int -}{2(i+}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} - \frac{\left((i-2a)b^2\right)\text{Sub}}{}$$

$$= -\frac{(i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x} - \frac{\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2} + \frac{(1+2ia)b^2\tanh}{(i-a)^3}$$

Mathematica [A] time = 0.14, size = 154, normalized size = 0.77

$$\frac{\frac{2(2a-i)b^2\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}\,\sqrt{-1+ia}} - \frac{i\left(a^2-abx+2ibx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{x^2}}{2(a-i)(a+i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (((-I)*(1 + a^2 + (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2
 + (2*(-I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqr
t[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2*(-
I + a)*(I + a)^2)
```

fricas [B] time = 0.45, size = 455, normalized size = 2.26

$$(i\,a+2)b^2x^2 + \sqrt{\frac{\left(4\,a^2-4i\,a-1\right)b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-1}}\left(a^3+i\,a^2+a+i\right)x^2\log\left(-\frac{(2\,a-i)b^3x-\sqrt{b^2x^2+2\,abx+a^2+1}\,(2\,a-i)b^2+\left(a^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((I*a + 2)*b^2*x^2 + sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 +
6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*
```

```
a - I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 + (a^5 + I*a
^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 +
 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) - sqrt((
4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 +
 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*a - I)*b^3*x - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 - (a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a +
I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3
- 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*((I*a + 2)*b*x - I*a^2 - I))/((2*a^3 + 2*I*a^2 + 2*a + 2*I)*x^2)
```

giac [**B**] time = 0.22, size = 475, normalized size = 2.36

$$-\frac{\left(2\,ab^2-b^2i\right)\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\left(a^3+a^2i+a+i\right)\sqrt{a^2+1}}+\frac{4\left(x|b|-\sqrt{(bx+a)^2+1}\right)a^4b^2i+2\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2a^3bi|}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^2 - b^2*i)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqr
t(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/((
a^3 + a^2*i + a + i)*sqrt(a^2 + 1)) + (4*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^4*b^2*i + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*i*abs(b) + 2*a^5*
b*i*abs(b) - 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 + 2*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a^3*b^2 + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*b^2*i
+ 5*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*b^2*i - 2*(x*abs(b) - sqrt((b*x
+ a)^2 + 1))^2*a^2*b*abs(b) + 2*a^4*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)
^2 + 1))^2*a*b*i*abs(b) + 4*a^3*b*i*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2
 + 1))*a*b^2 + (x*abs(b) - sqrt((b*x + a)^2 + 1))*b^2*i - 2*(x*abs(b) - sqr
t((b*x + a)^2 + 1))^2*b*abs(b) + 4*a^2*b*abs(b) + 2*a*b*i*abs(b) + 2*b*abs(
b))/((a^3 + a^2*i + a + i)*((x*abs(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)
^2)
```

maple [**B**] time = 0.18, size = 405, normalized size = 2.01

$$-\frac{ib\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{3ib^2a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{2\left(a^2+1\right)^{\frac{3}{2}}}-\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,a}{2\left(a^2+1\right)x^2}-\frac{\sqrt{b^2x^2+2}}{2\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -I*b/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I*b^2*a/(a^2+1)^(3/2)*ln((
2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*I/(a^
2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+3/2*I*a^2*b/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*a*b/(
a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*I*a^3*b^2/(a^2+1)^(5/2)*ln((2*
a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3/2*a^2*b^2
/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)+1/2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2))/x)
```

maxima [B] time = 0.34, size = 424, normalized size = 2.11

$$-\frac{3\,a^2(i\,a+1)b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{2\left(a^2+1\right)^{\frac{5}{2}}}+\frac{i\,ab^2\operatorname{arsinh}\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*(I*a + 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + I*a*b^2*arcsinh(2*a*b*x
/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2
 + 1)^(3/2) - 1/2*(-I*a - 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + 3/2*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*b/((a^2 + 1)^2*x) - I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*b/((a^2 + 1)*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
I*a + 1)/((a^2 + 1)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{x^3\,\sqrt{{(a+b\,x)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{a}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{b}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] I*(Integral(-I/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**2*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.170 $\int \frac{e^{i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\left(2a-i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{7/2}}+\frac{\left(-2a^2+9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\left(a^2+1\right)^2x}-\frac{(-2a+3i)b\sqrt{-}}{6(1}$$

```
[Out] (2*a-I*(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/
(1-I*a-I*b*x)^(1/2))/(I-a)^(5/2)/(I+a)^(7/2)-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1-I*a)/x^3-1/6*(3*I-2*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)
^(1/2)/(1-I*a)/(a^2+1)/x^2+1/6*(4+9*I*a-2*a^2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I
*a+I*b*x)^(1/2)/(1-I*a)/(a^2+1)^2/x
```

Rubi [A] time = 0.18, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5095, 99, 151, 12, 93, 208}

$$\frac{\left(-2a^2+9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\left(a^2+1\right)^2x}+\frac{\left(2a-i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{7/2}}-\frac{(-2a+3i)b\sqrt{-}}{6(1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 - I*a)*x^3) - ((3*I -
2*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(1 - I*a)*(1 + a^2)*
x^2) + ((4 + (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 - I*a)*(1 + a^2)^2*x) + ((2*a - I*(1 - 2*a^2))*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(5/2)*(I + a)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1+ia+ibx}}{x^4\sqrt{1-ia-ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} + \frac{\int \frac{(3i-2a)b-2b^2x}{x^3\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{3(1-ia)}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} - \frac{\int \frac{\left(4+9ia-2a^2\right)b^2+(3 \cdots}{x^2\sqrt{1-ia-ibx}\,\sqrt{1 \cdots}}}{6(1-ia)\left(1 \cdots\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right) \cdots}{6(1 \cdots}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right) \cdots}{6(1 \cdots}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right) \cdots}{6(1 \cdots}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} + \frac{\left(4+9ia-2a^2\right) \cdots}{6(1 \cdots}$$

Mathematica [A] time = 0.29, size = 247, normalized size = 0.87

$$\frac{(1+4ia)bx(a+bx-i)\sqrt{a^2+2abx+b^2x^2+1} + 2(1-ia)(a-i)(a+bx-i)\sqrt{a^2+2abx+b^2x^2+1} + \frac{3\left(2a^2-2ia- \cdots\right.}{\cdots}}{6\left(a^2+1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] (2*(1 - I*a)*(-I + a)*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1
 + (4*I)*a)*b*x*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3*(-1 -
```

```
(2*I)*a + 2*a^2)*b^2*x^2*(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]*Sqrt[1 + a^2 + 2*a
*b*x + b^2*x^2] - (2*I)*b*x*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)
])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]]))/(Sqrt[-1 - I*a]*(-1 + I*a)^(3/
2)))/(6*(1 + a^2)^2*x^3)
```

fricas [B] time = 0.57, size = 700, normalized size = 2.47

$$\left(-2i\,a^2-9\,a+4i\right)b^3x^3-\sqrt{\frac{\left(4\,a^4-8i\,a^3-8\,a^2+4i\,a+1\right)b^6}{a^{12}+2i\,a^{11}+4\,a^{10}+10i\,a^9+5\,a^8+20i\,a^7+20i\,a^5-5\,a^4+10i\,a^3-4\,a^2+2i\,a-1}}\left(3\,a^5+3i\,a^4+6\,a^3+6i\,a^2+3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((-2*I*a^2 - 9*a + 4*I)*b^3*x^3 - sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1
)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 -
5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(3*a^5 + 3*I*a^4 + 6*a^3 + 6*I*a^2 +
 3*a + 3*I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 + (a^7 + I*a^6 + 3*a^5 + 3*I*a^4 + 3*a^3 +
 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*
I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3
 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + sqrt((4*a^4 - 8*I*a^3
- 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*
I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(3*a^5 + 3*I*a^4
+ 6*a^3 + 6*I*a^2 + 3*a + 3*I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 - (a^7 + I*a^6 + 3*a^5
+ 3*I*a^4 + 3*a^3 + 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a
+ 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5
 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + ((-
2*I*a^2 - 9*a + 4*I)*b^2*x^2 - 2*I*a^4 + (2*I*a^3 + 3*a^2 + 2*I*a + 3)*b*x
- 4*I*a^2 - 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 + 6*I*a^4 + 12*
a^3 + 12*I*a^2 + 6*a + 6*I)*x^3)
```

giac [B] time = 0.29, size = 900, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^3 - 2*a*b^3*i - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 +
 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^
2 + 1)))/((a^5 + a^4*i + 2*a^3 + 2*a^2*i + a + i)*sqrt(a^2 + 1)) + 1/3*(8*(
x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3*i + 24*(x*abs(b) - sqrt((b*x +
a)^2 + 1))*a^7*b^3*i + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^6*b^2*i*ab
s(b) + 8*a^8*b^2*i*abs(b) + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a^2*b^3
- 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 + 18*(x*abs(b) - sqrt((b*
x + a)^2 + 1))*a^6*b^3 - 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a*b^3*i + 3
2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3*i + 54*(x*abs(b) - sqrt((b*x
 + a)^2 + 1))*a^5*b^3*i - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^5*b^2*a
bs(b) + 12*a^7*b^2*abs(b) + 60*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^4*b^2
*i*abs(b) + 20*a^6*b^2*i*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*b^
3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 + 39*(x*abs(b) - sqrt((
b*x + a)^2 + 1))*a^4*b^3 + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^3*i
+ 36*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^3*b^3*i - 24*(x*abs(b) - sqrt((b*
x + a)^2 + 1))^2*a^3*b^2*abs(b) + 36*a^5*b^2*abs(b) + 48*(x*abs(b) - sqrt((
b*x + a)^2 + 1))^2*a^2*b^2*i*abs(b) + 12*a^4*b^2*i*abs(b) + 24*(x*abs(b) -
sqrt((b*x + a)^2 + 1))*a^2*b^3 + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b^3
```

```
*i - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b^2*abs(b) + 36*a^3*b^2*abs(
b) + 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b^2*i*abs(b) - 4*a^2*b^2*i*abs
(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*b^3 + 12*a*b^2*abs(b) - 4*b^2*i*
abs(b))/((a^5 + a^4*i + 2*a^3 + 2*a^2*i + a + i)*((x*abs(b) - sqrt((b*x + a
)^2 + 1))^2 - a^2 - 1)^3)
```

maple [B] time = 0.18, size = 611, normalized size = 2.16

$$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,a}{3\left(a^2+1\right)x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{3\left(a^2+1\right)x^3}+\frac{5ia^2b\sqrt{b^2x^2+2abx+a^2+1}}{6\left(a^2+1\right)^2x^2}+\frac{5ab\sqrt{b^2x^2+2abx+a^2+}}{6\left(a^2+1\right)^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/3*I/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/3/(a^2+1)/x^3*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+5/6*I*a^2*b/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)+5/6*a*b/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/2*I*a^4*b^3/(a^2+1
)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
x)-5/2*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I*b^3/(a^2+1)^
(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
+5/2*a^3*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)-5/2*I*a^3*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-3/2*a*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)+13/6*I*b^2/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
a+2/3*b^2/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*a^2*b^3/(a^2+1)^(5/
2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/
2*I*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.35, size = 644, normalized size = 2.28

$$\frac{5\,a^3(i\,a+1)b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{2\left(a^2+1\right)^{\frac{7}{2}}}-\frac{3i\,a^2b^3\,\mathrm{arsinh}\left(\frac{}{\sqrt{-4\,a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/2*a^3*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*a
bs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/2*I*a^2*b^3*arcsinh(2*
a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))
/(a^2 + 1)^(5/2) - 3/2*a*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*I*b
^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)))/(a^2 + 1)^(3/2) - 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*(
I*a + 1)*b^2/((a^2 + 1)^3*x) + 3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^
2/((a^2 + 1)^2*x) - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*b^2/((
a^2 + 1)^2*x) + 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*b/((a^2 +
 1)^2*x^2) - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b/((a^2 + 1)*x^2) - 1/
3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a + 1)/((a^2 + 1)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{x^4\,\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\right)dx+\int\frac{a}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\,dx+\int\frac{b}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] I*(Integral(-I/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.171 $\int e^{2i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=92

$$\frac{2i(a+i)^4\log(a+bx+i)}{b^5}-\frac{2(1-ia)^3x}{b^4}+\frac{i(a+i)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[Out] -2*(1-I*a)^3*x/b^4+I*(I+a)^2*x^2/b^3+2/3*(1-I*a)*x^3/b^2+1/2*I*x^4/b-1/5*x^
5+2*I*(I+a)^4*ln(I+a+b*x)/b^5
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1-ia)x^3}{3b^2}+\frac{i(a+i)^2x^2}{b^3}-\frac{2(1-ia)^3x}{b^4}+\frac{2i(a+i)^4\log(a+bx+i)}{b^5}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{2i\tan^{-1}(a+bx)}x^4\,dx &= \int \frac{x^4(1+ia+ibx)}{1-ia-ibx}\,dx\\
&= \int \left(\frac{2(-1+ia)^3}{b^4}+\frac{2i(i+a)^2x}{b^3}+\frac{2(1-ia)x^2}{b^2}+\frac{2ix^3}{b}-x^4+\frac{2i(i+a)^4}{b^4(i+a+bx)}\right)dx\\
&= -\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 1.00

$$\frac{2i(a+i)^4\log(a+bx+i)}{b^5}-\frac{2(1-ia)^3x}{b^4}+\frac{i(a+i)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

fricas [A] time = 0.48, size = 105, normalized size = 1.14

$$-\frac{6\,b^5x^5-15i\,b^4x^4+20\,(i\,a-1)b^3x^3-\left(30i\,a^2-60\,a-30i\right)b^2x^2-\left(-60i\,a^3+180\,a^2+180i\,a-60\right)bx-\left(60i\,a^4-\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 - 15*I*b^4*x^4 + 20*(I*a - 1)*b^3*x^3 - (30*I*a^2 - 60*a -
 30*I)*b^2*x^2 - (-60*I*a^3 + 180*a^2 + 180*I*a - 60)*b*x - (60*I*a^4 - 240
*a^3 - 360*I*a^2 + 240*a + 60*I)*log((b*x + a + I)/b))/b^5
```

giac [A] time = 0.12, size = 130, normalized size = 1.41

$$\frac{2\left(a^4i-4\,a^3-6\,a^2i+4\,a+i\right)\log\left(bx+a+i\right)}{b^5}-\frac{6\,b^5x^5-15\,b^4ix^4+20\,ab^3ix^3-30\,a^2b^2ix^2+60\,a^3bix-20\,b^3x^3+}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="giac")
```

```
[Out] 2*(a^4*i - 4*a^3 - 6*a^2*i + 4*a + i)*log(b*x + a + i)/b^5 - 1/30*(6*b^5*x^
5 - 15*b^4*i*x^4 + 20*a*b^3*i*x^3 - 30*a^2*b^2*i*x^2 + 60*a^3*b*i*x - 20*b^
3*x^3 + 60*a*b^2*x^2 + 30*b^2*i*x^2 - 180*a^2*b*x - 180*a*b*i*x + 60*b*x)/b
^5
```

maple [B] time = 0.04, size = 347, normalized size = 3.77

$$-\frac{x^5}{5}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b^5}-\frac{2ix^3a}{3b^2}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^4}{b^5}+\frac{ix^4}{2b}+\frac{2x^3}{3b^2}-\frac{ix^2}{b^3}-\frac{2a\,x^2}{b^3}-\frac{6i\ln\left(b^2x^2+2ab\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x)
```

```
[Out] -1/5*x^5+I/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)-2/3*I/b^2*x^3*a+I/b^5*ln(b^2*x^2+2
*a*b*x+a^2+1)*a^4+1/2*I*x^4/b+2/3/b^2*x^3-I/b^3*x^2-2/b^3*a*x^2-6*I/b^5*ln(
b^2*x^2+2*a*b*x+a^2+1)*a^2+6/b^4*x*a^2-2/b^4*x-8*I/b^5*arctan(1/2*(2*b^2*x+
2*a*b)/b)*a+8*I/b^5*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3-4/b^5*ln(b^2*x^2+2*a*
b*x+a^2+1)*a^3-2*I/b^4*a^3*x+4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a+I/b^3*x^2*a^
2+2/b^5*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^4+6*I/b^4*a*x-12/b^5*arctan(1/2*(2*
b^2*x+2*a*b)/b)*a^2+2/b^5*arctan(1/2*(2*b^2*x+2*a*b)/b)
```

maxima [B] time = 0.45, size = 150, normalized size = 1.63

$$-\frac{6\,b^4x^5-15i\,b^3x^4+20\,(i\,a-1)b^2x^3-\left(30i\,a^2-60\,a-30i\right)bx^2-\left(-60i\,a^3+180\,a^2+180i\,a-60\right)x}{30\,b^4}+\frac{\left(2\,a^4+8i\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 - 15*I*b^3*x^4 + 20*(I*a - 1)*b^2*x^3 - (30*I*a^2 - 60*a -
 30*I)*b*x^2 - (-60*I*a^3 + 180*a^2 + 180*I*a - 60)*x)/b^4 + (2*a^4 + 8*I*a
^3 - 12*a^2 - 8*I*a + 2)*arctan((b^2*x + a*b)/b)/b^5 + (I*a^4 - 4*a^3 - 6*I
*a^2 + 4*a + I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

mupad [B] time = 0.58, size = 201, normalized size = 2.18

$$\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}+\frac{\left(2\,a^4-12\,a^2+2\right)\,1\mathrm{i}}{b^5}\right)-x^4\,\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{4\,b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5}+\frac{x^2\,(-1+a\,1\mathrm{i})^2\,\left(\cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] log(x + (a + 1i)/b)*((8*a - 8*a^3)/b^5 + ((2*a^4 - 12*a^2 + 2)*1i)/b^5) - x
^4*(((a*1i - 1)*1i)/(4*b) - ((a*1i + 1)*1i)/(4*b)) - x^5/5 + (x^2*(a*1i - 1
)^2*(((a*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b))/(2*b^2) - (x^3*(a*1i - 1)*(((a
*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(3*b) + (x*(a*1i - 1)^3*(((a*1i - 1
)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/b^3
```

sympy [A] time = 0.50, size = 114, normalized size = 1.24

$$-\frac{x^5}{5}-x^3\left(\frac{2ia}{3b^2}-\frac{2}{3b^2}\right)-x^2\left(-\frac{ia^2}{b^3}+\frac{2a}{b^3}+\frac{i}{b^3}\right)-x\left(\frac{2ia^3}{b^4}-\frac{6a^2}{b^4}-\frac{6ia}{b^4}+\frac{2}{b^4}\right)+\frac{ix^4}{2b}+\frac{2i\,(a+i)^4\log\left(ia+ibx-1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**4,x)
```

```
[Out] -x**5/5 - x**3*(2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(-I*a**2/b**3 + 2*a/b**
3 + I/b**3) - x*(2*I*a**3/b**4 - 6*a**2/b**4 - 6*I*a/b**4 + 2/b**4) + I*x**
4/(2*b) + 2*I*(a + I)**4*log(I*a + I*b*x - 1)/b**5
```

3.172 $\int e^{2i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=72

$$-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2i(a+i)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[Out] 2*I*(I+a)^2*x/b^3+(1-I*a)*x^2/b^2+2/3*I*x^3/b-1/4*x^4-2*(1-I*a)^3*ln(I+a+b*
x)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{(1-ia)x^2}{b^2}+\frac{2i(a+i)^2x}{b^3}-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x^3\,dx&=\int\frac{x^3(1+ia+ibx)}{1-ia-ibx}\,dx\\&=\int\left(\frac{2i(i+a)^2}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{2ix^2}{b}-x^3+\frac{2(-1+ia)^3}{b^3(i+a+bx)}\right)dx\\&=\frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 1.00

$$-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2i(a+i)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

fricas [A] time = 0.47, size = 77, normalized size = 1.07

$$-\frac{3\,b^4x^4 - 8i\,b^3x^3 + 12\,(i\,a - 1)b^2x^2 - \left(24i\,a^2 - 48\,a - 24i\right)bx - \left(-24i\,a^3 + 72\,a^2 + 72i\,a - 24\right)\log\left(\frac{bx+a+i}{b}\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 - 8*I*b^3*x^3 + 12*(I*a - 1)*b^2*x^2 - (24*I*a^2 - 48*a -
24*I)*b*x - (-24*I*a^3 + 72*a^2 + 72*I*a - 24)*log((b*x + a + I)/b))/b^4
```

giac [A] time = 0.13, size = 88, normalized size = 1.22

$$-\frac{2\left(a^3i - 3\,a^2 - 3\,ai + 1\right)\log\left(bx + a + i\right)}{b^4} - \frac{3\,b^4x^4 - 8\,b^3ix^3 + 12\,ab^2ix^2 - 24\,a^2bix - 12\,b^2x^2 + 48\,abx + 24\,bix}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] -2*(a^3*i - 3*a^2 - 3*a*i + 1)*log(b*x + a + i)/b^4 - 1/12*(3*b^4*x^4 - 8*b
^3*i*x^3 + 12*a*b^2*i*x^2 - 24*a^2*b*i*x - 12*b^2*x^2 + 48*a*b*x + 24*b*i*x
)/b^4
```

maple [B] time = 0.04, size = 255, normalized size = 3.54

$$-\frac{x^4}{4} + \frac{2ix^3}{3b} - \frac{ix^2a}{b^2} + \frac{2ia^2x}{b^3} + \frac{x^2}{b^2} - \frac{2ix}{b^3} - \frac{4ax}{b^3} - \frac{i\ln\left(b^2x^2 + 2abx + a^2 + 1\right)a^3}{b^4} + \frac{3i\ln\left(b^2x^2 + 2abx + a^2 + 1\right)a}{b^4} + \frac{3\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x)
```

```
[Out] -1/4*x^4+2/3*I*x^3/b-I/b^2*x^2*a+2*I/b^3*a^2*x+1/b^2*x^2-2*I/b^3*x-4*a*x/b^
3-I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+3*I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+3
/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-1/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)-6*I/b^4*
arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3+2
*I/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)+6/b^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a
```

maxima [B] time = 0.43, size = 118, normalized size = 1.64

$$-\frac{3\,b^3x^4 - 8i\,b^2x^3 + 12\,(i\,a - 1)bx^2 - \left(24i\,a^2 - 48\,a - 24i\right)x}{12\,b^3} - \frac{\left(2\,a^3 + 6i\,a^2 - 6\,a - 2i\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{\left(-i\,a^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 - 8*I*b^2*x^3 + 12*(I*a - 1)*b*x^2 - (24*I*a^2 - 48*a - 24
*I)*x)/b^3 - (2*a^3 + 6*I*a^2 - 6*a - 2*I)*arctan((b^2*x + a*b)/b)/b^4 + (-
I*a^3 + 3*a^2 + 3*I*a - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

mupad [B] time = 0.53, size = 153, normalized size = 2.12

$$-x^3\left(\frac{(-1 + a\,1\mathrm{i})\,1\mathrm{i}}{3\,b} - \frac{(1 + a\,1\mathrm{i})\,1\mathrm{i}}{3\,b}\right) - \frac{x^4}{4} + \ln\left(x + \frac{a + 1\mathrm{i}}{b}\right)\left(\frac{6\,a^2 - 2}{b^4} + \frac{\left(6\,a - 2\,a^3\right)1\mathrm{i}}{b^4}\right) - \frac{x^2\,(-1 + a\,1\mathrm{i})\left(\frac{(-1+a\,1\mathrm{i})}{b}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] log(x + (a + 1i)/b)*(((6*a - 2*a^3)*1i)/b^4 + (6*a^2 - 2)/b^4) - x^4/4 - x^
3*(((a*1i - 1)*1i)/(3*b) - ((a*1i + 1)*1i)/(3*b)) - (x^2*(a*1i - 1)*(((a*1i
 - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(2*b) + (x*(a*1i - 1)^2*(((a*1i - 1)*1
i)/b - ((a*1i + 1)*1i)/b))/b^2
```

sympy [A] time = 0.38, size = 78, normalized size = 1.08

$$-\frac{x^4}{4} - x^2\left(\frac{ia}{b^2} - \frac{1}{b^2}\right) - x\left(-\frac{2ia^2}{b^3} + \frac{4a}{b^3} + \frac{2i}{b^3}\right) + \frac{2ix^3}{3b} - \frac{2i\,(a+i)^3\log\left(ia + ibx - 1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**3,x)
```

```
[Out] -x**4/4 - x**2*(I*a/b**2 - 1/b**2) - x*(-2*I*a**2/b**3 + 4*a/b**3 + 2*I/b**
3) + 2*I*x**3/(3*b) - 2*I*(a + I)**3*log(I*a + I*b*x - 1)/b**4
```

3.173 $\int e^{2i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=54

$$\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[Out] 2*(1-I*a)*x/b^2+I*x^2/b-1/3*x^3+2*I*(I+a)^2*ln(I+a+b*x)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2(1-ia)x}{b^2}+\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x^2\,dx&=\int\frac{x^2(1+ia+ibx)}{1-ia-ibx}\,dx\\&=\int\left(-\frac{2i(i+a)}{b^2}+\frac{2ix}{b}-x^2+\frac{2i(i+a)^2}{b^2(i+a+bx)}\right)dx\\&=\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.00

$$\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

fricas [A] time = 0.41, size = 53, normalized size = 0.98

$$-\frac{b^3x^3 - 3i\,b^2x^2 + 6\,(i\,a-1)bx - \left(6i\,a^2 - 12\,a - 6i\right)\log\left(\frac{bx+a+i}{b}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*I*b^2*x^2 + 6*(I*a - 1)*b*x - (6*I*a^2 - 12*a - 6*I)*log(
(b*x + a + I)/b))/b^3
```

giac [A] time = 0.14, size = 57, normalized size = 1.06

$$\frac{2\left(a^2i - 2\,a - i\right)\log\left(bx + a + i\right)}{b^3} - \frac{b^3x^3 - 3\,b^2ix^2 + 6\,abix - 6\,bx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*i - 2*a - i)*log(b*x + a + i)/b^3 - 1/3*(b^3*x^3 - 3*b^2*i*x^2 + 6*a
*b*i*x - 6*b*x)/b^3
```

maple [B] time = 0.04, size = 176, normalized size = 3.26

$$-\frac{x^3}{3}+\frac{ix^2}{b}-\frac{2iax}{b^2}+\frac{2x}{b^2}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^3}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b^3}-\frac{2\ln\left(b^2x^2+2abx+a^2+1\right)a}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x)
```

```
[Out] -1/3*x^3+I*x^2/b-2*I/b^2*a*x+2*x/b^2+I/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-I/
b^3*ln(b^2*x^2+2*a*b*x+a^2+1)-2/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a+4*I/b^3*arc
tan(1/2*(2*b^2*x+2*a*b)/b)*a+2/b^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2/b^3*
arctan(1/2*(2*b^2*x+2*a*b)/b)
```

maxima [B] time = 0.42, size = 87, normalized size = 1.61

$$-\frac{b^2x^3 - 3i\,bx^2 + 6\,(i\,a-1)x}{3\,b^2}+\frac{2\left(a^2+2i\,a-1\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}+\frac{\left(i\,a^2-2\,a-i\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 - 3*I*b*x^2 + 6*(I*a - 1)*x)/b^2 + 2*(a^2 + 2*I*a - 1)*arctan
((b^2*x + a*b)/b)/b^3 + (I*a^2 - 2*a - I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/
b^3
```

mupad [B] time = 0.51, size = 107, normalized size = 1.98

$$-\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3}-\frac{\left(2\,a^2-2\right)\,1\mathrm{i}}{b^3}\right)-x^2\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{2\,b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{2\,b}\right)-\frac{x^3}{3}-\frac{x\,(-1+a\,1\mathrm{i})\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})}{b}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] - log(x + (a + 1i)/b)*((4*a)/b^3 - ((2*a^2 - 2)*1i)/b^3) - x^2*(((a*1i - 1)
*1i)/(2*b) - ((a*1i + 1)*1i)/(2*b)) - x^3/3 - (x*(a*1i - 1)*(((a*1i - 1)*1i
)/b - ((a*1i + 1)*1i)/b)*1i)/b
```

sympy [A] time = 0.31, size = 49, normalized size = 0.91

$$-\frac{x^3}{3} - x\left(\frac{2ia}{b^2} - \frac{2}{b^2}\right) + \frac{ix^2}{b} + \frac{2i\,(a+i)^2\log\left(ia+ibx-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**2,x)
```

```
[Out] -x**3/3 - x*(2*I*a/b**2 - 2/b**2) + I*x**2/b + 2*I*(a + I)**2*log(I*a + I*b
*x - 1)/b**3
```

3.174 $\int e^{2i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=37

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

```
[Out] 2*I*x/b-1/2*x^2+2*(1-I*a)*ln(I+a+b*x)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 77}

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}x\,dx &= \int \frac{x(1+ia+ibx)}{1-ia-ibx}\,dx\\ &= \int \left(\frac{2i}{b}-x+\frac{2(1-ia)}{b(i+a+bx)}\right)dx\\ &= \frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

fricas [A] time = 0.50, size = 35, normalized size = 0.95

$$-\frac{b^2x^2 - 4i\,bx + 4\,(i\,a - 1)\log\left(\frac{bx+a+i}{b}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 - 4*I*b*x + 4*(I*a - 1)*log((b*x + a + I)/b))/b^2
```

giac [A] time = 0.13, size = 36, normalized size = 0.97

$$-\frac{2\,(ai-1)\log\left(bx+a+i\right)}{b^2} - \frac{b^2x^2 - 4\,bix}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="giac")
[Out] -2*(a*i - 1)*log(b*x + a + i)/b^2 - 1/2*(b^2*x^2 - 4*b*i*x)/b^2
```

maple [B] time = 0.04, size = 107, normalized size = 2.89

$$-\frac{x^2}{2}+\frac{2ix}{b}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a}{b^2}+\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{b^2}-\frac{2i\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}-\frac{2\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x)
[Out] -1/2*x^2+2*I*x/b-I/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a+1/b^2*ln(b^2*x^2+2*a*b*x
+a^2+1)-2*I/b^2*arctan(1/2*(2*b^2*x+2*a*b)/b)-2/b^2*arctan(1/2*(2*b^2*x+2*a
*b)/b)*a
```

maxima [B] time = 0.44, size = 66, normalized size = 1.78

$$-\frac{bx^2 - 4i\,x}{2\,b} - \frac{(2\,a+2i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2} + \frac{(-i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="maxima")
[Out] -1/2*(b*x^2 - 4*I*x)/b - (2*a + 2*I)*arctan((b^2*x + a*b)/b)/b^2 + (-I*a +
1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

mupad [B] time = 0.13, size = 60, normalized size = 1.62

$$-\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(-\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right)-x\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
[Out] - log(x + (a + 1i)/b)*((a*2i)/b^2 - 2/b^2) - x*(((a*1i - 1)*1i)/b - ((a*1i
+ 1)*1i)/b) - x^2/2
```

sympy [A] time = 0.22, size = 32, normalized size = 0.86

$$-\frac{x^2}{2} + \frac{2ix}{b} - \frac{2i\,(a+i)\log\left(ia+ibx-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x,x)
```

```
[Out] -x**2/2 + 2*I*x/b - 2*I*(a + I)*log(I*a + I*b*x - 1)/b**2
```

3.175 $\int e^{2i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\log(a+bx+i)}{b}$$

```
[Out] -x+2*I*ln(I+a+b*x)/b
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5093, 43}

$$-x+\frac{2i\log(a+bx+i)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + ((2*I)*Log[I + a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2i\tan^{-1}(a+bx)}\,dx&=\int\frac{1+ia+ibx}{1-ia-ibx}\,dx\\&=\int\left(-1+\frac{2i}{i+a+bx}\right)dx\\&=-x+\frac{2i\log(i+a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.60

$$\frac{i\log\left((a+bx)^2+1\right)}{b}+\frac{2\tan^{-1}(a+bx)}{b}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + (2*ArcTan[a + b*x])/b + (I*Log[1 + (a + b*x)^2])/b
```

fricas [A] time = 0.58, size = 22, normalized size = 1.10

$$-\frac{bx-2i\log\left(\frac{bx+a+i}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -(b*x - 2*I*log((b*x + a + I)/b))/b
```

giac [A] time = 0.12, size = 17, normalized size = 0.85

$$-x + \frac{2\,i\log\left(bx + a + i\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -x + 2*i*log(b*x + a + i)/b
```

maple [B] time = 0.04, size = 51, normalized size = 2.55

$$-x + \frac{i\ln\left(b^2x^2 + 2abx + a^2 + 1\right)}{b} + \frac{2\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2),x)
```

```
[Out] -x+I/b*ln(b^2*x^2+2*a*b*x+a^2+1)+2/b*arctan(1/2*(2*b^2*x+2*a*b)/b)
```

maxima [B] time = 0.42, size = 46, normalized size = 2.30

$$-x + \frac{2\,\arctan\left(\frac{b^2x+ab}{b}\right)}{b} + \frac{i\,\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*arctan((b^2*x + a*b)/b)/b + I*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

mupad [B] time = 0.46, size = 21, normalized size = 1.05

$$-x + \frac{\ln\left(x + \frac{a+1\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^2/((a + b*x)^2 + 1),x)
```

```
[Out] (log(x + (a + 1i)/b)*2i)/b - x
```

sympy [A] time = 0.16, size = 17, normalized size = 0.85

$$-x + \frac{2i\log\left(ia + ibx - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2),x)
```

```
[Out] -x + 2*I*log(I*a + I*b*x - 1)/b
```

3.176 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=38

$$\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

```
[Out] (I-a)*ln(x)/(I+a)-2*ln(I+a+b*x)/(1-I*a)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 72}

$$\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] ((I - a)*Log[x])/(I + a) - (2*Log[I + a + b*x])/(1 - I*a)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{1+ia+ibx}{x(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x}-\frac{2ib}{(i+a)(i+a+bx)}\right)dx\\ &= \frac{(i-a)\log(x)}{i+a}-\frac{2\log(i+a+bx)}{1-ia}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.82

$$-\frac{2i\log(a+bx+i)+(a-i)\log(x)}{a+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] -(((-I + a)*Log[x] + (2*I)*Log[I + a + b*x])/(I + a))
```

fricas [A] time = 0.63, size = 27, normalized size = 0.71

$$-\frac{(a-i)\log(x)+2i\log\left(\frac{bx+a+i}{b}\right)}{a+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a - I)*log(x) + 2*I*log((b*x + a + I)/b))/(a + I)
```

giac [A] time = 0.12, size = 36, normalized size = 0.95

$$-\frac{2\,bi\log\left(bx+a+i\right)}{ab+bi}-\frac{(a-i)\log\left(|x|\right)}{a+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] -2*b*i*log(b*x + a + i)/(a*b + b*i) - (a - i)*log(abs(x))/(a + i)
```

maple [B] time = 0.05, size = 149, normalized size = 3.92

$$\frac{2i\ln(x)a}{a^2+1}-\frac{\ln(x)a^2}{a^2+1}+\frac{\ln(x)}{a^2+1}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a}{a^2+1}-\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{a^2+1}+\frac{2i\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{a^2+1}-\frac{2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x)
```

```
[Out] 2*I/(a^2+1)*ln(x)*a-1/(a^2+1)*ln(x)*a^2+1/(a^2+1)*ln(x)-I/(a^2+1)*ln(b^2*x^
2+2*a*b*x+a^2+1)*a-1/(a^2+1)*ln(b^2*x^2+2*a*b*x+a^2+1)+2*I/(a^2+1)*arctan(1
/2*(2*b^2*x+2*a*b)/b)-2/(a^2+1)*arctan(1/2*(2*b^2*x+2*a*b)/b)*a
```

maxima [B] time = 0.41, size = 80, normalized size = 2.11

$$-\frac{(2\,a-2i)\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}-\frac{(i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{\left(a^2-2i\,a-1\right)\log(x)}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(2*a - 2*I)*arctan((b^2*x + a*b)/b)/(a^2 + 1) - (I*a + 1)*log(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(a^2 + 1) - (a^2 - 2*I*a - 1)*log(x)/(a^2 + 1)
```

mupad [B] time = 0.70, size = 32, normalized size = 0.84

$$\ln(x)\left(-1+\frac{2\mathrm{i}}{a+1\mathrm{i}}\right)-\frac{\ln\left(a+b\,x+1\mathrm{i}\right)\,2\mathrm{i}}{a+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^2/(x*((a + b*x)^2 + 1)),x)
```

```
[Out] log(x)*(2i/(a + 1i) - 1) - (log(a + b*x + 1i)*2i)/(a + 1i)
```

sympy [B] time = 0.80, size = 107, normalized size = 2.82

$$-\frac{(a-i)\log\left(\frac{ia^2(a-i)}{a+i}-ia^2-\frac{2a(a-i)}{a+i}+x\left(-iab-3b\right)-\frac{i(a-i)}{a+i}-i\right)}{a+i}-\frac{2i\log\left(-ia^2-\frac{2a^2}{a+i}-\frac{4ia}{a+i}+x\left(-iab-3b\right)-i+\frac{2}{a+i}\right)}{a+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x,x)
```

```
[Out] -(a - I)*log(I*a**2*(a - I)/(a + I) - I*a**2 - 2*a*(a - I)/(a + I) + x*(-I*
a*b - 3*b) - I*(a - I)/(a + I) - I)/(a + I) - 2*I*log(-I*a**2 - 2*a**2/(a +
 I) - 4*I*a/(a + I) + x*(-I*a*b - 3*b) - I + 2/(a + I))/(a + I)
```

3.177 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=55

$$-\frac{2ib\log(x)}{(a+i)^2}+\frac{2ib\log(a+bx+i)}{(a+i)^2}-\frac{-a+i}{(a+i)x}$$

```
[Out] (-I+a)/(I+a)/x-2*I*b*ln(x)/(I+a)^2+2*I*b*ln(I+a+b*x)/(I+a)^2
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2ib\log(x)}{(a+i)^2}+\frac{2ib\log(a+bx+i)}{(a+i)^2}-\frac{-a+i}{(a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -((I - a)/((I + a)*x)) - ((2*I)*b*Log[x])/(I + a)^2 + ((2*I)*b*Log[I + a +
b*x])/(I + a)^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{1+ia+ibx}{x^2(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x^2}-\frac{2ib}{(i+a)^2x}+\frac{2ib^2}{(i+a)^2(i+a+bx)}\right)dx\\ &= -\frac{i-a}{(i+a)x}-\frac{2ib\log(x)}{(i+a)^2}+\frac{2ib\log(i+a+bx)}{(i+a)^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.71

$$\frac{a^2+2ibx\log(a+bx+i)-2ibx\log(x)+1}{(a+i)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (1 + a^2 - (2*I)*b*x*Log[x] + (2*I)*b*x*Log[I + a + b*x])/((I + a)^2*x)
```

fricas [A] time = 0.50, size = 40, normalized size = 0.73

$$\frac{-2i\,bx\log(x) + 2i\,bx\log\left(\frac{bx+a+i}{b}\right) + a^2 + 1}{\left(a^2 + 2i\,a - 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (-2*I*b*x*log(x) + 2*I*b*x*log((b*x + a + I)/b) + a^2 + 1)/((a^2 + 2*I*a -
1)*x)
```

giac [A] time = 0.12, size = 68, normalized size = 1.24

$$-\frac{2\,b^2\log\left(bx+a+i\right)}{a^2bi - 2\,ab - bi} + \frac{2\,b\log\left(|x|\right)}{a^2i - 2\,a - i} - \frac{\left(a^2i + i\right)i}{(a+i)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(b*x + a + i)/(a^2*b*i - 2*a*b - b*i) + 2*b*log(abs(x))/(a^2*i -
2*a - i) - (a^2*i + i)*i/((a + i)^2*x)
```

maple [B] time = 0.05, size = 260, normalized size = 4.73

$$-\frac{2ia}{\left(a^2+1\right)x} + \frac{a^2}{\left(a^2+1\right)x} - \frac{1}{\left(a^2+1\right)x} - \frac{2ib\ln(x)a^2}{\left(a^2+1\right)^2} + \frac{2ib\ln(x)}{\left(a^2+1\right)^2} - \frac{4b\ln(x)a}{\left(a^2+1\right)^2} + \frac{ib\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{\left(a^2+1\right)^2} - \frac{ib\ln\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x)
```

```
[Out] -2*I/(a^2+1)/x*a+1/(a^2+1)/x*a^2-1/(a^2+1)/x-2*I*b/(a^2+1)^2*ln(x)*a^2+2*I*
b/(a^2+1)^2*ln(x)-4*b/(a^2+1)^2*ln(x)*a+I*b/(a^2+1)^2*ln(b^2*x^2+2*a*b*x+a^
2+1)*a^2-I*b/(a^2+1)^2*ln(b^2*x^2+2*a*b*x+a^2+1)+2*b/(a^2+1)^2*ln(b^2*x^2+2
*a*b*x+a^2+1)*a-4*I*b/(a^2+1)^2*arctan(1/2*(2*b^2*x+2*a*b)/b)*a+2*b/(a^2+1)
^2*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2-2*b/(a^2+1)^2*arctan(1/2*(2*b^2*x+2*a*
b)/b)
```

maxima [B] time = 0.42, size = 125, normalized size = 2.27

$$\frac{2\left(a^2 - 2i\,a - 1\right)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4 + 2\,a^2 + 1} + \frac{\left(i\,a^2 + 2\,a - i\right)b\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{a^4 + 2\,a^2 + 1} + \frac{\left(-2i\,a^2 - 4\,a + 2i\right)b\log(x)}{a^4 + 2\,a^2 + 1} + \frac{a^2 -}{\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 - 2*I*a - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + (I*a^2 +
2*a - I)*b*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) + (-2*I*a^2 -
 4*a + 2*I)*b*log(x)/(a^4 + 2*a^2 + 1) + (a^2 - 2*I*a - 1)/((a^2 + 1)*x)
```

mupad [B] time = 0.64, size = 98, normalized size = 1.78

$$\frac{a-\mathrm{i}}{x\,\left(a+1\mathrm{i}\right)} + \frac{b\,\mathrm{atanh}\left(\frac{a^2+a\,2\mathrm{i}-1}{{\left(a+1\mathrm{i}\right)}^2} - \frac{x\,\left(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2\right)}{{\left(a+1\mathrm{i}\right)}^2\,\left(-b\,a^3+1\mathrm{i}\,b\,a^2-b\,a+b\,1\mathrm{i}\right)}\right)\,4\mathrm{i}}{{\left(a+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^2/(x^2*((a + b*x)^2 + 1)),x)
```

```
[Out] (a - 1i)/(x*(a + 1i)) + (b*atanh((a*2i + a^2 - 1)/(a + 1i)^2 - (x*(2*b^2 +
4*a^2*b^2 + 2*a^4*b^2))/((a + 1i)^2*(b*1i - a*b + a^2*b*1i - a^3*b)))*4i)/(
a + 1i)^2
```

sympy [B] time = 0.61, size = 158, normalized size = 2.87

$$-\frac{2ib\log\left(-\frac{2a^3b}{(a+i)^2}-\frac{6ia^2b}{(a+i)^2}+2ab+\frac{6ab}{(a+i)^2}+4b^2x+2ib+\frac{2ib}{(a+i)^2}\right)}{(a+i)^2}+\frac{2ib\log\left(\frac{2a^3b}{(a+i)^2}+\frac{6ia^2b}{(a+i)^2}+2ab-\frac{6ab}{(a+i)^2}+4b^2x+2\right.}{(a+i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**2,x)
```

```
[Out] -2*I*b*log(-2*a**3*b/(a + I)**2 - 6*I*a**2*b/(a + I)**2 + 2*a*b + 6*a*b/(a
+ I)**2 + 4*b**2*x + 2*I*b + 2*I*b/(a + I)**2)/(a + I)**2 + 2*I*b*log(2*a**
3*b/(a + I)**2 + 6*I*a**2*b/(a + I)**2 + 2*a*b - 6*a*b/(a + I)**2 + 4*b**2*
x + 2*I*b - 2*I*b/(a + I)**2)/(a + I)**2 - (a - I)/(x*(-a - I))
```

3.178 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=76

$$-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(a+bx+i)}{(1-ia)^3}+\frac{2ib}{(a+i)^2x}-\frac{-a+i}{2(a+i)x^2}$$

```
[Out] 1/2*(-I+a)/(I+a)/x^2+2*I*b/(I+a)^2/x-2*b^2*ln(x)/(1-I*a)^3+2*b^2*ln(I+a+b*x
)/(1-I*a)^3
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(a+bx+i)}{(1-ia)^3}+\frac{2ib}{(a+i)^2x}-\frac{-a+i}{2(a+i)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -(I - a)/(2*(I + a)*x^2) + ((2*I)*b)/((I + a)^2*x) - (2*b^2*Log[x])/(1 - I*
a)^3 + (2*b^2*Log[I + a + b*x])/(1 - I*a)^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1+ia+ibx}{x^3(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x^3}-\frac{2ib}{(i+a)^2x^2}+\frac{2ib^2}{(i+a)^3x}-\frac{2ib^3}{(i+a)^3(i+a+bx)}\right)dx\\ &= -\frac{i-a}{2(i+a)x^2}+\frac{2ib}{(i+a)^2x}-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(i+a+bx)}{(1-ia)^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.83

$$\frac{(a+i)\left(a^2+4ibx+1\right)-4ib^2x^2\log(a+bx+i)+4ib^2x^2\log(x)}{2(a+i)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((I + a)*(1 + a^2 + (4*I)*b*x) + (4*I)*b^2*x^2*Log[x] - (4*I)*b^2*x^2*Log[I
 + a + b*x])/(2*(I + a)^3*x^2)
```

fricas [A] time = 0.43, size = 70, normalized size = 0.92

$$\frac{4i\,b^2x^2\log(x) - 4i\,b^2x^2\log\left(\frac{bx+a+i}{b}\right) + a^3 - 4\,(-i\,a+1)bx + i\,a^2 + a + i}{\left(2\,a^3 + 6i\,a^2 - 6\,a - 2i\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] (4*I*b^2*x^2*log(x) - 4*I*b^2*x^2*log((b*x + a + I)/b) + a^3 - 4*(-I*a + 1)
*b*x + I*a^2 + a + I)/((2*a^3 + 6*I*a^2 - 6*a - 2*I)*x^2)
```

giac [A] time = 0.12, size = 98, normalized size = 1.29

$$\frac{2\,b^3\log\left(bx + a + i\right)}{a^3bi - 3\,a^2b - 3\,abi + b} - \frac{2\,b^2\log\left(|x|\right)}{a^3i - 3\,a^2 - 3\,ai + 1} + \frac{a^3i - a^2 + ai - 4\,(ab + bi)x - 1}{2\,(a+i)^3\,ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(b*x + a + i)/(a^3*b*i - 3*a^2*b - 3*a*b*i + b) - 2*b^2*log(abs(x)
)/(a^3*i - 3*a^2 - 3*a*i + 1) + 1/2*(a^3*i - a^2 + a*i - 4*(a*b + b*i)*x -
1)/((a + i)^3*i*x^2)
```

maple [B] time = 0.05, size = 406, normalized size = 5.34

$$\frac{6ib^2\arctan\left(\frac{2b^2x+2ab}{2b}\right)a^2}{\left(a^2+1\right)^3} + \frac{a^2}{2\left(a^2+1\right)x^2} - \frac{1}{2\left(a^2+1\right)x^2} - \frac{ib^2\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{\left(a^2+1\right)^3} + \frac{2ib\,a^2}{\left(a^2+1\right)^2x} + \frac{4ba}{\left(a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x)
```

```
[Out] 6*I*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2+1/2/(a^2+1)/x^2*a^2-1/2
/(a^2+1)/x^2-I*b^2/(a^2+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+2*I*b/(a^2+1)^2/
x*a^2+4*b/(a^2+1)^2/x*a+2*I*b^2/(a^2+1)^3*ln(x)*a^3-2*I*b/(a^2+1)^2/x+6*b^2
/(a^2+1)^3*ln(x)*a^2-2*b^2/(a^2+1)^3*ln(x)+3*I*b^2/(a^2+1)^3*ln(b^2*x^2+2*a
*b*x+a^2+1)*a-I/(a^2+1)/x^2*a-3*b^2/(a^2+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2
+b^2/(a^2+1)^3*ln(b^2*x^2+2*a*b*x+a^2+1)-2*I*b^2/(a^2+1)^3*arctan(1/2*(2*b^
2*x+2*a*b)/b)-2*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3-6*I*b^2/(a^
2+1)^3*ln(x)*a+6*b^2/(a^2+1)^3*arctan(1/2*(2*b^2*x+2*a*b)/b)*a
```

maxima [B] time = 0.43, size = 188, normalized size = 2.47

$$-\frac{\left(2\,a^3 - 6i\,a^2 - 6\,a + 2i\right)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6 + 3\,a^4 + 3\,a^2 + 1} + \frac{\left(-2i\,a^3 - 6\,a^2 + 6i\,a + 2\right)b^2\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\left(a^6 + 3\,a^4 + 3\,a^2 + 1\right)} + \frac{\left(2i\,a^3 +\right.}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -(2*a^3 - 6*I*a^2 - 6*a + 2*I)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3
*a^2 + 1) + 1/2*(-2*I*a^3 - 6*a^2 + 6*I*a + 2)*b^2*log(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1) + (2*I*a^3 + 6*a^2 - 6*I*a - 2)*b^2*log(
x)/(a^6 + 3*a^4 + 3*a^2 + 1) + 1/2*(a^4 - 2*I*a^3 + (4*I*a^2 + 8*a - 4*I)*b
*x - 2*I*a - 1)/((a^4 + 2*a^2 + 1)*x^2)
```

mupad [B] time = 0.69, size = 154, normalized size = 2.03

$$\frac{\frac{a-\mathrm{i}}{2\,(a+1\mathrm{i})}+\frac{b\,x\,2\mathrm{i}}{(a+1\mathrm{i})^2}}{x^2}+\frac{b^2\,\mathrm{atanh}\left(\frac{-a^3-a^2\,3\mathrm{i}+3\,a+1\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{x\,\left(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2\right)}{(a+1\mathrm{i})^3\,\left(-b\,a^6+2\mathrm{i}\,b\,a^5-b\,a^4+4\mathrm{i}\,b\,a^3+b\,a^2+2\mathrm{i}\,b\,a+b\right)}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^2/(x^3*((a + b*x)^2 + 1)),x)
```

```
[Out] ((a - 1i)/(2*(a + 1i)) + (b*x*2i)/(a + 1i)^2)/x^2 + (b^2*atanh((3*a - a^2*3
i - a^3 + 1i)/(a + 1i)^3 + (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2 +
 2*a^8*b^2))/((a + 1i)^3*(b + a*b*2i + a^2*b + a^3*b*4i - a^4*b + a^5*b*2i
- a^6*b)))*4i)/(a + 1i)^3
```

sympy [B] time = 0.88, size = 226, normalized size = 2.97

$$\frac{2ib^2\log\left(-\frac{2a^4b^2}{(a+i)^3}-\frac{8ia^3b^2}{(a+i)^3}+\frac{12a^2b^2}{(a+i)^3}+2ab^2+\frac{8iab^2}{(a+i)^3}+4b^3x+2ib^2-\frac{2b^2}{(a+i)^3}\right)}{(a+i)^3}-\frac{2ib^2\log\left(\frac{2a^4b^2}{(a+i)^3}+\frac{8ia^3b^2}{(a+i)^3}-\frac{12a^2b^2}{(a+i)^3}+2ab^2-\right.}{(a+i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**3,x)
```

```
[Out] 2*I*b**2*log(-2*a**4*b**2/(a + I)**3 - 8*I*a**3*b**2/(a + I)**3 + 12*a**2*b
**2/(a + I)**3 + 2*a*b**2 + 8*I*a*b**2/(a + I)**3 + 4*b**3*x + 2*I*b**2 - 2
*b**2/(a + I)**3)/(a + I)**3 - 2*I*b**2*log(2*a**4*b**2/(a + I)**3 + 8*I*a*
*3*b**2/(a + I)**3 - 12*a**2*b**2/(a + I)**3 + 2*a*b**2 - 8*I*a*b**2/(a + I
)**3 + 4*b**3*x + 2*I*b**2 + 2*b**2/(a + I)**3)/(a + I)**3 - (a**2 + 4*I*b*
x + 1)/(x**2*(-2*a**2 - 4*I*a + 2))
```

3.179 $\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4}+\frac{2b^2}{(1-ia)^3x}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

```
[Out] 1/3*(-I+a)/(I+a)/x^3+I*b/(I+a)^2/x^2+2*b^2/(1-I*a)^3/x-2*I*b^3*ln(x)/(I+a)^
4+2*I*b^3*ln(I+a+b*x)/(I+a)^4
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -(I - a)/(3*(I + a)*x^3) + (I*b)/((I + a)^2*x^2) + (2*b^2)/((1 - I*a)^3*x)
- ((2*I)*b^3*Log[x])/(I + a)^4 + ((2*I)*b^3*Log[I + a + b*x])/(I + a)^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1+ia+ibx}{x^4(1-ia-ibx)}\,dx\\
&= \int \left(\frac{i-a}{(i+a)x^4}-\frac{2ib}{(i+a)^2x^3}+\frac{2ib^2}{(i+a)^3x^2}-\frac{2ib^3}{(i+a)^4x}+\frac{2ib^4}{(i+a)^4(i+a+bx)}\right)dx\\
&= -\frac{i-a}{3(i+a)x^3}+\frac{ib}{(i+a)^2x^2}+\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(i+a)^4}+\frac{2ib^3\log(i+a+bx)}{(i+a)^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.95

$$\frac{(a+i)\left(a^3+ia^2+3iabx+a-6ib^2x^2-3bx+i\right)+6ib^3x^3\log(a+bx+i)-6ib^3x^3\log(x)}{3(a+i)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((I + a)*(I + a + I*a^2 + a^3 - 3*b*x + (3*I)*a*b*x - (6*I)*b^2*x^2) - (6*I
)*b^3*x^3*Log[x] + (6*I)*b^3*x^3*Log[I + a + b*x])/(3*(I + a)^4*x^3)
```

fricas [A] time = 0.51, size = 94, normalized size = 1.01

$$\frac{-6i\,b^3x^3\log(x)+6i\,b^3x^3\log\left(\frac{bx+a+i}{b}\right)-6\,(i\,a-1)b^2x^2+a^4+2i\,a^3+\left(3i\,a^2-6\,a-3i\right)bx+2i\,a-1}{\left(3\,a^4+12i\,a^3-18\,a^2-12i\,a+3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] (-6*I*b^3*x^3*log(x) + 6*I*b^3*x^3*log((b*x + a + I)/b) - 6*(I*a - 1)*b^2*x
^2 + a^4 + 2*I*a^3 + (3*I*a^2 - 6*a - 3*I)*b*x + 2*I*a - 1)/((3*a^4 + 12*I*
a^3 - 18*a^2 - 12*I*a + 3)*x^3)
```

giac [A] time = 0.12, size = 136, normalized size = 1.46

$$-\frac{2\,b^4\log\left(bx+a+i\right)}{a^4bi-4\,a^3b-6\,a^2bi+4\,ab+bi}+\frac{2\,b^3\log\left(|x|\right)}{a^4i-4\,a^3-6\,a^2i+4\,a+i}+\frac{a^4i-2\,a^3+6\left(ab^2+b^2i\right)x^2-3\left(a^2b+2\,abi-b\right)x}{3\,(a+i)^4ix^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(b*x + a + i)/(a^4*b*i - 4*a^3*b - 6*a^2*b*i + 4*a*b + b*i) + 2*b
^3*log(abs(x))/(a^4*i - 4*a^3 - 6*a^2*i + 4*a + i) + 1/3*(a^4*i - 2*a^3 + 6
*(a*b^2 + b^2*i)*x^2 - 3*(a^2*b + 2*a*b*i - b)*x - 2*a - i)/((a + i)^4*i*x^
3)
```

maple [B] time = 0.06, size = 560, normalized size = 6.02

$$\frac{2b^3\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{\left(a^2+1\right)^4}+\frac{a^2}{3\left(a^2+1\right)x^3}+\frac{2b^2}{\left(a^2+1\right)^3x}-\frac{8b^3\ln(x)a^3}{\left(a^2+1\right)^4}+\frac{2ba}{\left(a^2+1\right)^2x^2}-\frac{6b^2a^2}{\left(a^2+1\right)^3x}-\frac{4b^3\ln\left(b^2x^2+2abx+\right.}{\left(a^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x)
```

```
[Out] 2*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)/b)+1/3/(a^2+1)/x^3*a^2+2*b^2/(a^
2+1)^3/x-2*I*b^3/(a^2+1)^4*ln(x)-8*b^3/(a^2+1)^4*ln(x)*a^3-2/3*I/(a^2+1)/x^
3*a+2*b/(a^2+1)^2/x^2*a-I*b/(a^2+1)^2/x^2-6*b^2/(a^2+1)^3/x*a^2-4*b^3/(a^2+
1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+2*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)
/b)*a^4-12*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^2+4*b^3/(a^2+1)^4*
ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+I*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)+8*b
^3/(a^2+1)^4*ln(x)*a-1/3/(a^2+1)/x^3+8*I*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+
2*a*b)/b)*a+I*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^4+6*I*b^2/(a^2+1)^3
/x*a-2*I*b^3/(a^2+1)^4*ln(x)*a^4+12*I*b^3/(a^2+1)^4*ln(x)*a^2+I*b/(a^2+1)^2
/x^2*a^2-2*I*b^2/(a^2+1)^3/x*a^3-6*I*b^3/(a^2+1)^4*ln(b^2*x^2+2*a*b*x+a^2+1
)*a^2-8*I*b^3/(a^2+1)^4*arctan(1/2*(2*b^2*x+2*a*b)/b)*a^3
```

maxima [B] time = 0.44, size = 263, normalized size = 2.83

$$\frac{\left(2\,a^4-8i\,a^3-12\,a^2+8i\,a+2\right)b^3\arctan\left(\frac{b^2x+ab}{b}\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1}+\frac{\left(i\,a^4+4\,a^3-6i\,a^2-4\,a+i\right)b^3\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] (2*a^4 - 8*I*a^3 - 12*a^2 + 8*I*a + 2)*b^3*arctan((b^2*x + a*b)/b)/(a^8 + 4
*a^6 + 6*a^4 + 4*a^2 + 1) + (I*a^4 + 4*a^3 - 6*I*a^2 - 4*a + I)*b^3*log(b^2
*x^2 + 2*a*b*x + a^2 + 1)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) + (-2*I*a^4 - 8
*a^3 + 12*I*a^2 + 8*a - 2*I)*b^3*log(x)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) +
 1/3*(a^6 - 2*I*a^5 - (6*I*a^3 + 18*a^2 - 18*I*a - 6)*b^2*x^2 + a^4 - 4*I*a
^3 - (-3*I*a^4 - 6*a^3 - 6*a + 3*I)*b*x - a^2 - 2*I*a - 1)/((a^6 + 3*a^4 +
3*a^2 + 1)*x^3)
```

mupad [B] time = 0.72, size = 199, normalized size = 2.14

$$\frac{\frac{a-\mathrm{i}}{3\,(a+1\mathrm{i})}-\frac{b^2\,x^2\,2\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{b\,x\,1\mathrm{i}}{(a+1\mathrm{i})^2}}{x^3}+\frac{b^3\,\mathrm{atanh}\left(\frac{a^4+a^3\,4\mathrm{i}-6\,a^2-a\,4\mathrm{i}+1}{(a+1\mathrm{i})^4}-\frac{x\left(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2\right)}{(a+1\mathrm{i})^4\left(-b\,a^9+3\mathrm{i}\,b\,a^8+8\mathrm{i}\,b\,a^6+6\,b\,a^5+6\mathrm{i}\,b\,a^4+8\,b\,a^3+3\,b\,a-b\,1\mathrm{i}\right)}\right)4\mathrm{i}}{(a+1\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^2/(x^4*((a + b*x)^2 + 1)),x)
```

```
[Out] ((a - 1i)/(3*(a + 1i)) - (b^2*x^2*2i)/(a + 1i)^3 + (b*x*1i)/(a + 1i)^2)/x^3
 + (b^3*atanh((a^3*4i - 6*a^2 - a*4i + a^4 + 1)/(a + 1i)^4 - (x*(2*b^2 + 12
*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*a^12*b^2)
)/((a + 1i)^4*(3*a*b - b*1i + 8*a^3*b + a^4*b*6i + 6*a^5*b + a^6*b*8i + a^8
*b*3i - a^9*b)))*4i)/(a + 1i)^4
```

sympy [B] time = 1.15, size = 286, normalized size = 3.08

$$-\frac{2ib^3\log\left(-\frac{2a^5b^3}{(a+i)^4}-\frac{10ia^4b^3}{(a+i)^4}+\frac{20a^3b^3}{(a+i)^4}+\frac{20ia^2b^3}{(a+i)^4}+2ab^3-\frac{10ab^3}{(a+i)^4}+4b^4x+2ib^3-\frac{2ib^3}{(a+i)^4}\right)}{(a+i)^4}+\frac{2ib^3\log\left(\frac{2a^5b^3}{(a+i)^4}+\frac{10ia^4b^3}{(a+i)^4}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**4,x)
```

```
[Out] -2*I*b**3*log(-2*a**5*b**3/(a + I)**4 - 10*I*a**4*b**3/(a + I)**4 + 20*a**3
*b**3/(a + I)**4 + 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 - 10*a*b**3/(a + I)
**4 + 4*b**4*x + 2*I*b**3 - 2*I*b**3/(a + I)**4)/(a + I)**4 + 2*I*b**3*log(
2*a**5*b**3/(a + I)**4 + 10*I*a**4*b**3/(a + I)**4 - 20*a**3*b**3/(a + I)**
4 - 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 + 10*a*b**3/(a + I)**4 + 4*b**4*x
+ 2*I*b**3 + 2*I*b**3/(a + I)**4)/(a + I)**4 - (-I*a**3 + a**2 - I*a - 6*b*
*2*x**2 + x*(3*a*b + 3*I*b) + 1)/(x**3*(3*I*a**3 - 9*a**2 - 9*I*a + 3))
```

3.180 $\int e^{3i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=324

$$-\frac{i\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(112ia^3+2\left(-52ia^2+118a+61i\right)bx-422a^2-458ia+163\right)}{40b^5}-\frac{3\left(8ia^4-48a^3-\right.}{}$$

```
[Out] -3/8*(19-68*I*a-88*a^2+48*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5-2*I*x^4*(1+I*a+I*
b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)+3/20*(17*I+16*a)*x^2*(1+I*a+I*b*x)^(3/2)*(
1-I*a-I*b*x)^(1/2)/b^3-11/5*x^3*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2
-1/40*I*(1+I*a+I*b*x)^(3/2)*(163-458*I*a-422*a^2+112*I*a^3+2*(61*I+118*a-52
*I*a^2)*b*x)*(1-I*a-I*b*x)^(1/2)/b^5-3/8*(19*I+68*a-88*I*a^2-48*a^3+8*I*a^4
)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] time = 0.27, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(2\left(-52ia^2+118a+61i\right)bx+112ia^3-422a^2-458ia+163\right)}{40b^5}-\frac{3\left(8ia^4-48a^3-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-3*(19*I + 68*a - (88*I)*a^2 - 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*S
qrt[1 + I*a + I*b*x])/(8*b^5) - ((2*I)*x^4*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt
[1 - I*a - I*b*x]) + (3*(17*I + 16*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
I*b*x)^(3/2))/(20*b^3) - (11*x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3
/2))/(5*b^2) - ((I/40)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(163 -
 (458*I)*a - 422*a^2 + (112*I)*a^3 + 2*(61*I + 118*a - (52*I)*a^2)*b*x))/b^
5 - (3*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b
^5)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{(2i)\int \frac{x^3\sqrt{1+ia+ibx}\left(4(1+ia)+\frac{11ibx}{2}\right)}{\sqrt{1-ia-ibx}}dx}{b}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{11x^3\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{5b^2} + \frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(-\frac{33}{2}(i-a)(1-i}{\sqrt{1-ia-ib}}}{5b^3}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+16a)x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1-ia-ibx}}{5b}$$

$$= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{3(17i+16a)x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1-ia-ibx}}{5b}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}$$

$$= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{8b^5} - \frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}$$

Mathematica [**A**] time = 0.48, size = 249, normalized size = 0.77

$$\frac{3(-1)^{3/4}\left(8a^4+48ia^3-88a^2-68ia+19\right)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}\,b^{9/2}} - \sqrt{ia+ibx+1}\left(8a^5+418ia^4+14ia^3(8bx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] -1/40*(Sqrt[1 + I*a + I*b*x]*(448*I + (418*I)*a^4 + 8*a^5 + 163*b*x + (61*I
)*b^2*x^2 - 34*b^3*x^3 - (22*I)*b^4*x^4 + 8*b^5*x^5 + (14*I)*a^3*(121*I + 8
*b*x) - I*a^2*(2599 - (422*I)*b*x + 52*b^2*x^2) + a*(1763 - (458*I)*b*x + 1
18*b^2*x^2 + (32*I)*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(3/
4)*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[
b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

fricas [**A**] time = 0.50, size = 263, normalized size = 0.81

$$\frac{-62i\,a^6+2687\,a^5+11575i\,a^4-20350\,a^3+\left(-62i\,a^5+2625\,a^4+8950i\,a^3-11400\,a^2-6340i\,a+1280\right)bx-177}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] (-62*I*a^6 + 2687*a^5 + 11575*I*a^4 - 20350*a^3 + (-62*I*a^5 + 2625*a^4 + 8
950*I*a^3 - 11400*a^2 - 6340*I*a + 1280)*b*x - 17740*I*a^2 + (960*a^5 + 672
0*I*a^4 - 16320*a^3 + (960*a^4 + 5760*I*a^3 - 10560*a^2 - 8160*I*a + 2280)*
b*x - 18720*I*a^2 + 10440*a + 2280*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)) + (-64*I*b^5*x^5 - 176*b^4*x^4 + (256*a + 272*I)*b^3*x^3 - 64*
I*a^5 - 8*(52*a^2 + 118*I*a - 61)*b^2*x^2 + 3344*a^4 + 13552*I*a^3 + (896*a
^3 + 3376*I*a^2 - 3664*a - 1304*I)*b*x - 20792*a^2 - 14104*I*a + 3584)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1) + 7620*a + 1280*I)/(320*b^6*x + (320*a + 320*
I)*b^5)
```

giac [A] time = 0.21, size = 345, normalized size = 1.06

$$-\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(\left(\frac{4\,ix}{b}-\frac{4\,ab^{17}i-15\,b^{17}}{b^{19}}\right)x+\frac{4\,a^2b^{16}i-35\,ab^{16}-32\,b^{16}i}{b^{19}}\right)x-\frac{8\,a^3b^{15}i-130\,a^2b^{15}-2}{b^{19}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] -1/40*sqrt((b*x + a)^2 + 1)*((2*((4*i*x/b - (4*a*b^17*i - 15*b^17)/b^19)*x
+ (4*a^2*b^16*i - 35*a*b^16 - 32*b^16*i)/b^19)*x - (8*a^3*b^15*i - 130*a^2*
b^15 - 252*a*b^15*i + 125*b^15)/b^19)*x + (8*a^4*b^14*i - 250*a^3*b^14 - 80
4*a^2*b^14*i + 835*a*b^14 + 288*b^14*i)/b^19) + 1/8*(8*a^4 + 48*a^3*i - 88*
a^2 - 68*a*i + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b +
 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt(
(b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b
) + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqr
t((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

maple [B] time = 0.20, size = 933, normalized size = 2.88

$$\frac{33a^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b^4\sqrt{b^2}}+\frac{89ia^2x^2}{10b^3\sqrt{b^2x^2+2abx+a^2+1}}+\frac{501ia^3x}{10b^4\sqrt{b^2x^2+2abx+a^2+1}}+\frac{}{5b\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x)
```

```
[Out] 33*a^2/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)+89/10*I/b^3*a^2*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+501/10*I/b^4*a^3*x/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-29/10*I/b^2*a*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-367/10*I/b^4*a*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-28/5*I/b^3*x^2/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-1/2*x^3/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2+3/2*a^3/b
^3*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*a^4/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(
b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/4/b*a*x^4/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)-3/4*x^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-103/8*a/b^3*x^2/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)-1/5*I*b*x^6/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-527/8*a^2/b^4*x
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+53/4*a^4/b^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x
+319/10*I/b^5*a^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^5*a^6/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+209/10*I/b^5*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I*a*x^5
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+7/5*I/b*x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-56
/5*I/b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+57/8/b^4*x/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)-57/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^
2)^(1/2)-181/8*a^3/b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+41/4*a^5/b^5/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)-263/8*a/b^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+19/8*x^3/b^2
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/5*I/b^4*a^5*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-18*I/b^4*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)+51/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)
```

maxima [**B**] time = 0.41, size = 3117, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/5*I*b*x^6/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 11/20*I*a*x^5/sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 693/4*I*a^7*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b^2) - 33/20*I*a^2*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b) + 2415/8*I*(a^2 + 1)*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^2) + 231/40*I*a^3*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^2) - 1/5*(-2*I*a^2 - 2*I)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
) + 1/4*(-3*I*a*b^2 - 3*b^2)*x^5/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) -
231/8*I*(a^2 + 1)*a^6/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*b^3) - 315/4*(-3*I*a*b^2 - 3*b^2)*a^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 35*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^
5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 291
9/20*I*(a^2 + 1)^2*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*b^2) - 15*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*x/((a^2*b^2 - (a^2 +
1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 231/8*I*a^4*x^2/(sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b^3) - 111/40*I*(a^2 + 1)*a*x^3/(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b^2) - 3/4*(-3*I*a*b^2 - 3*b^2)*a*x^4/(sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*b^3) + 1/3*(-3*I*a^2*b - 6*a*b + 3*I*b)*x^4/(sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*b^2) + 189/5*I*(a^2 + 1)^2*a^4/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 945/8*(-3*I*a*b^2 - 3*b^2)*(a^2
+ 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4
) - 265/6*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a^3*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 653/40*I*(a^2 + 1)^3*a*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 31/2*(-I*
a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 819/40*I*(a^2 + 1)*a^2*x^2/(sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^3) + 21/8*(-3*I*a*b^2 - 3*b^2)*a^2*x^3/(sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b^4) - 7/6*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*x^3/(sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 1/2*(I*a^3 + 3*a^2 - 3*I*a - 1)*x^3/(sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 693/8*I*a^5*arcsinh(2*(b^2*x + a*b)/s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 105/8*(-3*I*a*b^2 - 3*b^2)*(a^2 +
1)*a^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) +
35/6*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a^4/((a^2*b^2 - (a^2 + 1)*b^2)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 397/40*I*(a^2 + 1)^3*a^2/((a^2*b^2
 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 5/2*(-I*a^3 - 3*
a^2 + 3*I*a + 1)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b^3) - 169/4*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2*a^2*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 61/6*(-3*I*a^
2*b - 6*a*b + 3*I*b)*(a^2 + 1)^2*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^3) - 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)^2*
x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 105/8
*(-3*I*a*b^2 - 3*b^2)*a^3*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) + 35/
6*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^4) - 8/5*I*(a^2 + 1)^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 5/2*(
-I*a^3 - 3*a^2 + 3*I*a + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) -
 5/8*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^4) - 315/4*I*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a
^2 + 1)*b^2))/b^5 + 14*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2*a^3/((a^2*b^2 - (a^
2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 231/4*I*(a^2 + 1)*a^4/
(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 29/6*(-3*I*a^2*b - 6*a*b + 3*I*b)
*(a^2 + 1)^2*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*b^4) + 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)^2*a/((a^2*b^2 - (a^2 +
 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 15/8*(-3*I*a*b^2 - 3*b^2)
```

```
*(a^2 + 1)^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*b^4) + 49/8*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*b^5) - 4/3*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*x^2/(sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b^4) + 315/8*(-3*I*a*b^2 - 3*b^2)*a^4*arcsinh(2*(b^
2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^7 - 35/2*(-3*I*a^2*b - 6*a
*b + 3*I*b)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
/b^6 + 105/8*I*(a^2 + 1)^2*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a
^2 + 1)*b^2))/b^5 + 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 15/8*(-3*I*a*b^2 - 3*b^2)*
(a^2 + 1)^3*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^5) + 819/20*I*(a^2 + 1)^2*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 1
05/4*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2))/b^7 + 15/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*
a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 3/2*(-I
*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2
 + 4*(a^2 + 1)*b^2))/b^5 - 105/4*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a^3/(sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b^7) + 35/3*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 +
 1)*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^6) - 16/5*I*(a^2 + 1)^3/(sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 5*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1
)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) + 15/8*(-3*I*a*b^2 - 3*b^2)*(a^
2 + 1)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^7 +
49/4*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^7) - 8/3*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)^2/(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*b^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx+\dots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**4,x)
```

```
[Out] -I*(Integral(I*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**4/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**4/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)), x) + Integral(-3*b*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**7/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**4/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
```

```
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**6/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**6/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.181 $\int e^{3i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=249

$$-\frac{i\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)}{8b^4}+\frac{3\left(8ia^3-36a^2-44ia+17\right)\sqrt{-ia-}}{8b^4}$$

```
[Out] -3/8*(17*I+44*a-36*I*a^2-8*a^3)*arcsinh(b*x+a)/b^4-2*I*x^3*(1+I*a+I*b*x)^(3
/2)/b/(1-I*a-I*b*x)^(1/2)-9/4*x^2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b
^2-1/8*I*(1+I*a+I*b*x)^(3/2)*(29*I+54*a-22*I*a^2-2*(11-10*I*a)*b*x)*(1-I*a-
I*b*x)^(1/2)/b^4+3/8*(17-44*I*a-36*a^2+8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+
I*b*x)^(1/2)/b^4
```

Rubi [A] time = 0.24, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)}{8b^4}+\frac{3\left(8ia^3-36a^2-44ia+17\right)\sqrt{-ia-}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (3*(17 - (44*I)*a - 36*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^4) - ((2*I)*x^3*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I
*b*x]) - (9*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^2) - ((
I/8)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(29*I + 54*a - (22*I)*a^
2 - 2*(11 - (10*I)*a)*b*x))/b^4 - (3*(17*I + 44*a - (36*I)*a^2 - 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(3(1+ia)+\frac{9ibx}{2}\right)}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{4b^2} + \frac{i\int \frac{x\sqrt{1+ia+ibx}\left(-9i\left(1+a^2\right)b+\frac{3}{2}(\cdots\right.}{\sqrt{1-ia-ibx}}}{2b^3}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}\,(1+ia+ib\cdots}{\cdots}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{\cdots}}{\cdots}$$

Mathematica [**A**] time = 0.28, size = 201, normalized size = 0.81

$$\frac{3\sqrt[4]{-1}\left(8a^3+36ia^2-44a-17i\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4b^{9/2}} + \frac{\sqrt{ia+ibx+1}\left(2a^4+78ia^3+a^2(-233+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(80 + (78*I)*a^3 + 2*a^4 - (29*I)*b*x + 11*b^2*x^2 +
 (6*I)*b^3*x^3 - 2*b^4*x^4 + a^2*(-233 + (22*I)*b*x) - I*a*(237 - (54*I)*b*
x + 10*b^2*x^2)))/(8*b^4*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(-17*I -
 44*a + (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

fricas [**A**] time = 0.51, size = 217, normalized size = 0.87

$$\frac{15i\,a^5 - 495\,a^4 - 1664i\,a^3 + \left(15i\,a^4 - 480\,a^3 - 1184i\,a^2 + 968\,a + 256i\right)bx + 2152\,a^2 - \left(192\,a^4 + 1056i\,a^3 + (19\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] (15*I*a^5 - 495*a^4 - 1664*I*a^3 + (15*I*a^4 - 480*a^3 - 1184*I*a^2 + 968*a
 + 256*I)*b*x + 2152*a^2 - (192*a^4 + 1056*I*a^3 + (192*a^3 + 864*I*a^2 - 1
056*a - 408*I)*b*x - 1920*a^2 - 1464*I*a + 408)*log(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + (-16*I*b^4*x^4 - 48*b^3*x^3 + (80*a + 88*I)*b^2*x^
```

```
2 + 16*I*a^4 - 624*a^3 - 8*(22*a^2 + 54*I*a - 29)*b*x - 1864*I*a^2 + 1896*a
 + 640*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1224*I*a - 256)/(64*b^5*x + (
64*a + 64*I)*b^4)
```

giac [A] time = 0.19, size = 293, normalized size = 1.18

$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{ix}{b}-\frac{ab^{11}i-4\,b^{11}}{b^{13}}\right)x+\frac{2\,a^2b^{10}i-20\,ab^{10}-19\,b^{10}i}{b^{13}}\right)x-\frac{2\,a^3b^9i-44\,a^2b^9-93\,ab^9i+48\,b^9}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*(i*x/b - (a*b^11*i - 4*b^11)/b^13)*x + (2*a^
2*b^10*i - 20*a*b^10 - 19*b^10*i)/b^13)*x - (2*a^3*b^9*i - 44*a^2*b^9 - 93*
a*b^9*i + 48*b^9)/b^13) - 1/8*(8*a^3 + 36*a^2*i - 44*a - 17*i)*log(3*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2
 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*
(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a
)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^
3*abs(b))
```

maple [B] time = 0.19, size = 711, normalized size = 2.86

$$-\frac{3x^2a^2}{2b^2\sqrt{b^2x^2+2abx+a^2+1}}+\frac{a\,x^3}{2b\sqrt{b^2x^2+2abx+a^2+1}}+\frac{3a^3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b^3\sqrt{b^2}}-\frac{33a\ln\left(\frac{b^2x+}{\sqrt{b^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x)
```

```
[Out] -3/2*x^2/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2+1/2/b*a*x^3/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+3*a^3/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/(b^2)^(1/2)-33/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)-265/8*I/b^3*a^2*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-53
/8*I/b^2*a*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-51/8*I/b^3*ln((b^2*x+a*b)/(b^2
)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-155/8*I/b^4*a^3/(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+17/8*I/b*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+10/b^4/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+27/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/4*I*b*x^5/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+1/4*I/b^3*a^4*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/4*I*a*x^4/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+1/4*I/b^4*a^5/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+51/8*I/b^3*x/(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)-157/8*I/b^4*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5*
x^2/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+53/
2*a/b^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-25/2*a^3/b^3/(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*x+1/2*a^2/b^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-19/2*a^4/b^4/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.37, size = 2321, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*b*x^5/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 315/4*I*a^6*x/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/4*I*a*x^4/sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) - 945/8*I*(a^2 + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^
```

```
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 21/8*I*a^2*x^3/(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b) + 105/8*I*(a^2 + 1)*a^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 35*(-3*I*a*b^2 - 3*b^2)*a^5*x/((a^2*b^
2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 15*(-3*I*a^2*b
- 6*a*b + 3*I*b)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*b^2) + 169/4*I*(a^2 + 1)^2*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*x/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 105/8*I*a^3*x^
2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 1/8*(-5*I*a^2 - 5*I)*x^3/(sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 1/3*(-3*I*a*b^2 - 3*b^2)*x^4/(sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^2) - 14*I*(a^2 + 1)^2*a^3/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 265/6*(-3*I*a*b^2 - 3*b^2)*(a^2
 + 1)*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^
3) + 31/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 15/8*I*(a^2 + 1)^3*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 5*(-I*a^3 - 3*a
^2 + 3*I*a + 1)*(a^2 + 1)*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*b) - 49/8*I*(a^2 + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*b^2) - 7/6*(-3*I*a*b^2 - 3*b^2)*a*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*b^3) + 1/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*b^2) - 315/8*I*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2))/b^4 + 35/6*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a^4/((a^2*b^2 - (a^2 +
1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 5/2*(-3*I*a^2*b - 6*a*b +
3*I*b)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^3) + 15/8*I*(a^2 + 1)^3*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^2) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a^2/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 61/6*(-3*I*
a*b^2 - 3*b^2)*(a^2 + 1)^2*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b^3) - 3/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)^2*x/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 35/6*(-3*I*
a*b^2 - 3*b^2)*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 5/2*(-3*I*
a^2*b - 6*a*b + 3*I*b)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + (-I*
a^3 - 3*a^2 + 3*I*a + 1)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 105/
4*I*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
))/b^4 - 29/6*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2*a^2/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 105/4*I*(a^2 + 1)*a^3/(sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 3/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)
^2*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 4/
3*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4
) - 35/2*(-3*I*a*b^2 - 3*b^2)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))/b^6 + 15/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*arcsinh(2*(b
^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 15/8*I*(a^2 + 1)^2*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 - 3*(-I*a^3 -
 3*a^2 + 3*I*a + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
*b^2))/b^4 - 49/4*I*(a^2 + 1)^2*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) +
 15/2*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))/b^6 - 3/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35/3*(-3*I*
a*b^2 - 3*b^2)*(a^2 + 1)*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^6) - 5*(-
3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^5) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^4) - 8/3*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2/(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*b^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**3,x)
```

```
[Out] -I*(Integral(I*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**3/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**3/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)), x) + Integral(-3*b*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**6/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**3/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**5/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.182 $\int e^{3i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=227

$$\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{6b^3}+\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^3}+\frac{(-6a$$

```
[Out] 1/2*(11-18*I*a-6*a^2)*arcsinh(b*x+a)/b^3-I*(I+a)^2*(1+I*a+I*b*x)^(5/2)/b^3/
(1-I*a-I*b*x)^(1/2)+1/6*(11*I+18*a-6*I*a^2)*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*
x)^(1/2)/b^3+1/3*I*(1+I*a+I*b*x)^(5/2)*(1-I*a-I*b*x)^(1/2)/b^3+1/2*(11*I+18
*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 89, 80, 50, 53, 619, 215}

$$\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{6b^3}+\frac{\left(-6ia^2+18a+11i\right)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^3}+\frac{(-6a$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*
b^3) + ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(
3/2))/(6*b^3) - (I*(I + a)^2*(1 + I*a + I*b*x)^(5/2))/(b^3*Sqrt[1 - I*a - I
*b*x]) + ((I/3)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(5/2))/b^3 + ((11 -
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} - \frac{i\int \frac{(1+ia+ibx)^{3/2}\left((3-2ia)(i+a)b-b^2x\right)}{\sqrt{1-ia-ibx}}\,dx}{b^3}$$

$$= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}\,(1+ia+ibx)^{5/2}}{3b^3} + \frac{\left(11-18ia-6a^2\right)\int \frac{(1+ia+ibx}{\sqrt{1-ia-i}}}{3b^2}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-i}}{}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,(1+ia+}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,(1+ia+}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,(1+ia+}{6b^3}$$

$$= \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(11i+18a-6ia^2\right)\sqrt{1-ia-ibx}\,(1+ia+}{6b^3}$$

Mathematica [A] time = 0.24, size = 160, normalized size = 0.70

$$\frac{(-1)^{3/4}\left(6a^2+18ia-11\right)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}\,b^{5/2}} + \frac{\sqrt{ia+ibx+1}\,\left(-2a^3-53ia^2+a(103-16ibx)-2b^3x^3+7\right.}{6b^3\sqrt{-i(a+bx+i)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(52*I - (53*I)*a^2 - 2*a^3 + 19*b*x + (7*I)*b^2*x^2
- 2*b^3*x^3 + a*(103 - (16*I)*b*x)))/(6*b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-
1)^(3/4)*(-11 + (18*I)*a + 6*a^2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I
 + a + b*x)])/Sqrt[(-I)*b]])/(Sqrt[(-I)*b]*b^(5/2))
```

fricas [A] time = 0.46, size = 174, normalized size = 0.77

$$-7i\,a^4+166\,a^3+\left(-7i\,a^3+159\,a^2+249i\,a-96\right)bx+408i\,a^2+\left(72\,a^3+12\left(6\,a^2+18i\,a-11\right)bx+288i\,a^2-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] (-7*I*a^4 + 166*a^3 + (-7*I*a^3 + 159*a^2 + 249*I*a - 96)*b*x + 408*I*a^2 +
 (72*a^3 + 12*(6*a^2 + 18*I*a - 11)*b*x + 288*I*a^2 - 348*a - 132*I)*log(-b
*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-8*I*b^3*x^3 - 28*b^2*x^2 -
8*I*a^3 + (64*a + 76*I)*b*x + 212*a^2 + 412*I*a - 208)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) - 345*a - 96*I)/(24*b^4*x + (24*a + 24*I)*b^3)
```

giac [A] time = 0.19, size = 249, normalized size = 1.10

$$-\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^6i-9\,b^6}{b^8}\right)x+\frac{2\,a^2b^5i-27\,ab^5-28\,b^5i}{b^8}\right)+\frac{\left(6\,a^2+18\,ai-11\right)\log\left|3\left(x|b|-\sqrt{(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^6*i - 9*b^6)/b^8)*x + (2*a^2*
b^5*i - 27*a*b^5 - 28*b^5*i)/b^8) + 1/6*(6*a^2 + 18*a*i - 11)*log(3*(x*abs(
b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2
+ 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(
x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a)
^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2
*abs(b))
```

maple [B] time = 0.18, size = 519, normalized size = 2.29

$$-\frac{ib\,x^4}{3\sqrt{b^2x^2+2abx+a^2+1}}-\frac{ia\,x^3}{3\sqrt{b^2x^2+2abx+a^2+1}}+\frac{26i}{3b^3\sqrt{b^2x^2+2abx+a^2+1}}+\frac{3a\,x^2}{2b\sqrt{b^2x^2+2abx+a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x)
```

```
[Out] -1/3*I*b*x^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I*a*x^3/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+26/3*I/b^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2/b*a*x^2/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)-3/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/(b^2)^(1/2)*a^2-1/3*I/b^2*a^3*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/3*I
/b*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/b^3*a^4/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+11/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-11/2*x/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+53/3*I/b^2*a*x/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)-3/2*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9*I/b^2*a*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+25/3*I/b^3*
a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+17/2*a/b^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
17/2*a^3/b^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+23/2*a^2/b^2/(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)*x
```

maxima [B] time = 0.36, size = 1626, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -35*I*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 1/3*I*b*x^4/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 265/6*I*(a^2 + 1)*a^3*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 7/6*I*a*x^3/s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 35/6*I*(a^2 + 1)*a^4/((a^2*b^2 - (a^2 +
1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 61/6*I*(a^2 + 1)^2*a*x/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(-I*a^3 - 3*a^
2 + 3*I*a + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) - 15*(-3*I*a*b^2 - 3*b^2)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 6*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^3*x/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 35/6*I*a^2*x^2
/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 29/6*I*(a^2 + 1)^2*a^2/((a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + (-I*a^3 - 3*a^2 + 3*
I*a + 1)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) + 31/2*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 5*(-3*I*a^2*b - 6*a*b + 3*I*b)
*(a^2 + 1)*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*b) - 1/3*(-4*I*a^2 - 4*I)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 1/2*
(-3*I*a*b^2 - 3*b^2)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 35/2*I*a
^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 5/2*(-
3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*b^3) + (-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a^2/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - (-I*a^3 - 3*a
^2 + 3*I*a + 1)*(a^2 + 1)*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*b) - 3/2*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)^2*x/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 5/2*(-3*I*a*b^2 - 3*b^2)
*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + (-3*I*a^2*b - 6*a*b + 3*I*
b)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 15/2*I*(a^2 + 1)*a*arcsinh
(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3/2*(-3*I*a*b^2
- 3*b^2)*(a^2 + 1)^2*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*b^3) - 35/3*I*(a^2 + 1)*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3
) + 15/2*(-3*I*a*b^2 - 3*b^2)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))/b^5 - 3*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + (-I*a^3 - 3*a^2 + 3*I*a +
1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 8/3*I*
(a^2 + 1)^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 3/2*(-3*I*a*b^2 - 3*b
^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b
^5 - 5*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^5) + 2*(-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**2,x)
```

```
[Out] -I*(Integral(I*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**2/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**2/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)), x) + Integral(-3*b*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**4/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.183 $\int e^{3i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=163

$$-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}-\frac{(3-2ia)\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2b^2}-\frac{3(3-2ia)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^2}+\frac{3(2}{}$$

```
[Out] 3/2*(3*I+2*a)*arcsinh(b*x+a)/b^2-(1-I*a)*(1+I*a+I*b*x)^(5/2)/b^2/(1-I*a-I*b
*x)^(1/2)-1/2*(3-2*I*a)*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2-3/2*(3-
2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 78, 50, 53, 619, 215}

$$-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}-\frac{(3-2ia)\sqrt{-ia-ibx+1}\,(ia+ibx+1)^{3/2}}{2b^2}-\frac{3(3-2ia)\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{2b^2}+\frac{3(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (-3*(3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((
3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((1 -
 I*a)*(1 + I*a + I*b*x)^(5/2))/(b^2*Sqrt[1 - I*a - I*b*x]) + (3*(3*I + 2*a)
*ArcSinh[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3i+2a)\int \frac{(1+ia+ibx)^{3/2}}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{(3-2ia)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\int \frac{\sqrt{1+}}{\sqrt{1-}}}{2b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-}{b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-}{b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-}{b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-}{b}$$

Mathematica [A] time = 0.18, size = 132, normalized size = 0.81

$$\frac{\sqrt{ia+ibx+1}\,\left(a^2+15ia-b^2x^2+5ibx-14\right)}{2b^2\sqrt{-i(a+bx+i)}} + \frac{3\sqrt[4]{-1}\,(2a+3i)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-14 + (15*I)*a + a^2 + (5*I)*b*x - b^2*x^2))/(2*b^2
*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh
[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

fricas [A] time = 0.51, size = 137, normalized size = 0.84

$$\frac{3i\,a^3 + \left(3i\,a^2 - 44\,a - 32i\right)bx - 47\,a^2 - \left((24\,a+36i)bx + 24\,a^2 + 60i\,a - 36\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a}\right.}{8\,b^3x + (8\,a+8i)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] (3*I*a^3 + (3*I*a^2 - 44*a - 32*I)*b*x - 47*a^2 - ((24*a + 36*I)*b*x + 24*a
^2 + 60*I*a - 36)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(-4*I*b^2*x^2 + 4*I*a^2 - 20*b*x - 60*a - 56*I
) - 76*I*a + 32)/(8*b^3*x + (8*a + 8*I)*b^2)
```

giac [A] time = 0.19, size = 214, normalized size = 1.31

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}-\frac{ab^2i-6b^2}{b^4}\right)-\frac{(2a+3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+2\left(x|b|-\sqrt{(bx+a)^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b^2*i - 6*b^2)/b^4) - 1/2*(2*a + 3*i
)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + 2*(x*abs(b) - sq
rt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))
^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 4*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1
))*abs(b))/(b*abs(b))
```

maple [B] time = 0.18, size = 358, normalized size = 2.20

$$-\frac{10ax}{b\sqrt{b^2x^2+2abx+a^2+1}}+\frac{9i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}+\frac{ia^3}{2b^2\sqrt{b^2x^2+2abx+a^2+1}}-\frac{ib\,x^3}{2\sqrt{b^2x^2+2abx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x)
```

```
[Out] -10*a/b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+9/2*I/b*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/b^2*a^3/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)-1/2*I*b*x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7/b^2/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+3*a/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/
2))/(b^2)^(1/2)+1/2*I/b*a^2*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*x^2/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)-1/2*I*a*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I/b^2*a/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9/2*I/b*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7*a^2
/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.37, size = 1116, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 15*I*a^4*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
- 31/2*I*(a^2 + 1)*a^2*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)) - 1/2*I*b*x^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 5/2*I*(a^2
+ 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*
(-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)) + 6*(-3*I*a*b^2 - 3*b^2)*a^3*x/((a^2*b^2 - (a^2 + 1)*
b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/2*I*(a^2 + 1)^2*b*x/((a^2*b^2
 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*
I*a + 1)*a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + 5/2*I*a*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3/2*I*(a^2 + 1)^2*a/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^
2 + 3*I*a + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
```

```
+ 1)) + (-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 5*(-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*a*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 15/2*I*a^
2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 + (-3*I*a
*b^2 - 3*b^2)*(a^2 + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*b^2) - (-3*I*a^2*b - 6*a*b + 3*I*b)*(a^2 + 1)*a/((a^2*b^2 - (
a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + (-3*I*a*b^2 - 3*b^2)*x
^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 1/2*(-3*I*a^2 - 3*I)*arcsinh(2
*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 + 5*I*(a^2 + 1)*a/(s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 3*(-3*I*a*b^2 - 3*b^2)*a*arcsinh(2*
(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + (-3*I*a^2*b - 6*a*b
 + 3*I*b)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 +
 (I*a^3 + 3*a^2 - 3*I*a - 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 2*(-
3*I*a*b^2 - 3*b^2)*(a^2 + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{ix}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x,x)
```

```
[Out] -I*(Integral(I*x/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x/(a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)), x) + Integral(a**3*x/(a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
) + Integral(-3*b*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**
4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x/(a**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(-3*I*b**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
+ Integral(3*a*b**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b
*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2
*b*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x**2/(a**2*sqr
```

```
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)), x))
```

3.184 $\int e^{3i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=94

$$-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}-\frac{3i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

```
[Out] -3*arcsinh(b*x+a)/b-2*I*(1+I*a+I*b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)-3*I*(1-I*
a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5093, 47, 50, 53, 619, 215}

$$-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}-\frac{3i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-3*I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - ((2*I)*(1 + I*a +
I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) - (3*ArcSinh[a + b*x])/b
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{3i\tan^{-1}(a+bx)}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$
$$= -\frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$
$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx$$
$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}$$
$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right.}{2b^2}$$
$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\sinh^{-1}(a+bx)}{b}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.48

$$-\frac{3\sinh^{-1}(a+bx)}{b} + \frac{\sqrt{(a+bx)^2+1}\left(\frac{4}{a+bx+i}-i\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x]),x]
[Out] (Sqrt[1 + (a + b*x)^2]*(-I + 4/(I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

fricas [A] time = 0.52, size = 102, normalized size = 1.09

$$\frac{(-i\,a+8)bx - i\,a^2 + (6\,bx+6\,a+6i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}\,(-2i\,bx -}{2\,b^2x+(2\,a+2i)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
[Out] ((-I*a + 8)*b*x - I*a^2 + (6*b*x + 6*a + 6*I)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-2*I*b*x - 2*I*a
+ 10) + 9*a + 8*I)/(2*b^2*x + (2*a + 2*I)*b)
```

giac [B] time = 0.21, size = 183, normalized size = 1.95

$$-\frac{\sqrt{(bx+a)^2+1}\,i}{b} + \frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2 ab + a^3b + 2\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2 bi + 2\,a^2bi + \left(x|b|-\sqrt{(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt((b*x + a)^2 + 1)*i/b + log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b
 + a^3*b + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*i + 2*a^2*b*i + (x*abs(
b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^2*abs(b) + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*ab
s(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b)
```

maple [B] time = 0.11, size = 362, normalized size = 3.85

$$-\frac{5i}{b\sqrt{b^2x^2+2abx+a^2+1}}+\frac{3a}{b\sqrt{b^2x^2+2abx+a^2+1}}+\frac{3a^2x}{\sqrt{b^2x^2+2abx+a^2+1}}+\frac{3a^3}{b\sqrt{b^2x^2+2abx+a^2+1}}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -5*I/b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/b*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*
a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/b*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-4*I/
b*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*a^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I
/b*a^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-I*b*x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-
5*I*a*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*(1+I*a)^3*(2*b^2*x+2*a*b)/(4*b^2*(a
^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*x/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)
```

maxima [B] time = 0.34, size = 740, normalized size = 7.87

$$-\frac{6i\,a^3b^2x}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}+\frac{5i\left(a^2+1\right)ab^2x}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}-\frac{i\,(a}{\left(a^2b^2-\left(a^2+1\right)b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -6*I*a^3*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + 5*I*(a^2 + 1)*a*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)) - I*(a^2 + 1)*a^2*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) - 2*(-3*I*a*b^2 - 3*b^2)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-3*I*a^2*b - 6*a*b + 3*I*b)*a*b*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 1/4*(-4*I*a
^3 - 12*a^2 + 12*I*a + 4)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)) - I*b*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + (-3*I*a^2*
b - 6*a*b + 3*I*b)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)) - (-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-3*I*a*b^2 - 3*b^2)*(a^2 + 1)*x/((a^2*b^
2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*I*a*arcsinh(2*(b^
2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b - (-3*I*a*b^2 - 3*b^2)*(a^
2 + 1)*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) +
(-2*I*a^2 - 2*I)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + (-3*I*a*b^2 - 3*b^
2)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - (-3*I*
a^2*b - 6*a*b + 3*I*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(a**2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)), x) + Integral(a**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + In
tegral(-3*I*a**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(-3*I*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
3*a*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2*b*x/(a**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x/(a**2*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)), x))
```

3.185 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-i\sinh^{-1}(a+bx)-\frac{2(-a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}$$

```
[Out] -I*arcsinh(b*x+a)-2*(I-a)^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-
a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I+a)^(3/2)+4*(1+I*a+I*b*x)^(1/2)/(1-I*a)/(1-
I*a-I*b*x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5095, 98, 157, 53, 619, 215, 93, 208}

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-i\sinh^{-1}(a+bx)-\frac{2(-a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (4*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - I*ArcSinh[a +
 b*x] - (2*(I - a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I + a)^(3/2)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1+ia+ibx)^{3/2}}{x(1-ia-ibx)^{3/2}}\,dx\\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2\int \frac{\frac{1}{2}i(i-a)^2b-\frac{1}{2}(1-ia)b^2x}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)b}\\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{(i-a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{1-ia} - (ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ib}\ldots}\\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{\left(2(i-a)^2\right)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1-ia} - (ib)\int \frac{\ldots}{\sqrt{(1-\ldots}}\\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2(i-a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}} - \frac{i\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+\ldots\right.}{2b}\\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - i\sinh^{-1}(a+bx) - \frac{2(i-a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 196, normalized size = 1.46

$$\frac{2\left(\frac{\sqrt[4]{-1}(a+i)(-ib)^{3/2}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{3/2}} + \frac{2i\sqrt{ia+ibx+1}}{\sqrt{-i(a+bx+i)}} + \frac{\sqrt{-1-ia}\,(a-i)\tanh^{-1}\left(\frac{\sqrt{-1-ia}\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}}\right)}{a+i}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(((2*I)*Sqrt[1 + I*a + I*b*x])/Sqrt[(-I)*(I + a + b*x)] + ((-1)^(1/4)*(I
 + a)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])
/Sqrt[(-I)*b]])/b^(3/2) + (Sqrt[-1 - I*a]*(-I + a)*ArcTanh[(Sqrt[-1 - I*a]*
Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1
+ I*a]))/(I + a)
```

fricas [B] time = 0.51, size = 381, normalized size = 2.84

$$-\frac{\left((a+i)bx+a^2+2i\,a-1\right)\sqrt{-\frac{4\,a^3-12i\,a^2-12\,a+4i}{a^3+3i\,a^2-3\,a-i}}\,\log\left(-\frac{(2\,a-2i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}\,(2\,a-2i)-\left(i\,a^2-2\,a-i\right)\sqrt{-\frac{4\,a^3-12i\,a^2-12\,a+4i}{a^3+3i\,a^2-3\,a-i}}}{2\,a-2i}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^
3 + 3*I*a^2 - 3*a - I))*log(-((2*a - 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(2*a - 2*I) - (I*a^2 - 2*a - I)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I
)/(a^3 + 3*I*a^2 - 3*a - I)))/(2*a - 2*I)) - ((a + I)*b*x + a^2 + 2*I*a - 1
)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^3 + 3*I*a^2 - 3*a - I))*log(-((2
*a - 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - 2*I) - (-I*a^2 + 2
*a + I)*sqrt(-(4*a^3 - 12*I*a^2 - 12*a + 4*I)/(a^3 + 3*I*a^2 - 3*a - I)))/(
2*a - 2*I)) + 8*b*x + (2*(-I*a + 1)*b*x - 2*I*a^2 + 4*a + 2*I)*log(-b*x - a
 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 8*a + 8*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1) + 8*I)/((2*a + 2*I)*b*x + 2*a^2 + 4*I*a - 2)
```

giac [B] time = 0.36, size = 259, normalized size = 1.93

$$-\frac{\left(a^2 i+2\,a-i\right)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}\,(a+i)}-\frac{b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab-a^3b-2\left(x|b|-\sqrt{(b\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -(a^2*i + 2*a - i)*log(abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a
^2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqr
t(a^2 + 1)*(a + i)) - 1/3*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b
 - a^3*b - 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*i - 2*a^2*b*i - (x*abs(
b) - sqrt((b*x + a)^2 + 1))^3*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^2*abs(b) - 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) + a*b + (x*ab
s(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(i*abs(b))
```

maple [B] time = 0.17, size = 818, normalized size = 6.10

$$\frac{1}{\left(a^2+1\right)\sqrt{b^2x^2+2abx+a^2+1}}+\frac{ia^5}{\left(a^2+1\right)\sqrt{b^2x^2+2abx+a^2+1}}+\frac{ia^4bx}{\left(a^2+1\right)\sqrt{b^2x^2+2abx+a^2+1}}+\frac{\cdots}{\left(4b^2\left(a^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] 1/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*a^5/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)+I*a^4*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+6*I*(1+I*a)^2*b*(2
*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I*a
```

```
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*b*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-4*I/(a
^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-1/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x
+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3*a^2/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^3*b/(a^2+1)/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*x-a*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3*I*a^2*b/(a^2
+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^3+2*I*a^3/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)-I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)+I*b*x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I/(a^2+1)^(3/2)*ln((2*a^
2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a+3*I/(a^2+1)
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+2*I*b*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-
4*a^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a^4/(a^2+1)/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)+3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))/x)*a^2
```

maxima [B] time = 0.34, size = 738, normalized size = 5.51

$$\frac{2i\,a^2b^3x}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}-\frac{\left(4i\,a^2+4i\right)b^3x}{4\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}+\frac{\left(-i\,a^3-3\right.}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*I*a^2*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
 - 1/4*(4*I*a^2 + 4*I)*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + I*(a^2 + 1)*a*b^2/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^
2 + 3*I*a + 1)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(a^2 + 1)) + (-3*I*a*b^2 - 3*b^2)*a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (-3*I*a^2*b - 6*a*b + 3*I*b)*b^2*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-3*I*a*b^2 -
 3*b^2)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 (-3*I*a^2*b - 6*a*b + 3*I*b)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) - (-I*a^3 - 3*a^2 + 3*I*a + 1)*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3
/2) + (-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2
+ 1)) - (-3*I*a*b^2 - 3*b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - I*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+2abx-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] -I*(Integral(I/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(a**
2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b
**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)),
x) + Integral(-3*b*x/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**
3*x**3/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a
**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a*b**2*x**2/(a**2*x*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2*b*x/(a**2*x*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**
2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)), x) + Integral(-6*I*a*b*x/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
))
```

3.186 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=176

$$-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}-\frac{6ib\sqrt{ia+ibx+1}}{(a+i)^2\sqrt{-ia-ibx+1}}+\frac{6i\sqrt{-a+i}\,b\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(a+i)^{5/2}}$$

```
[Out] 6*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/
2))*(I-a)^(1/2)/(I+a)^(5/2)-(1+I*a+I*b*x)^(3/2)/(1-I*a)/x/(1-I*a-I*b*x)^(1/
2)-6*I*b*(1+I*a+I*b*x)^(1/2)/(I+a)^2/(1-I*a-I*b*x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5095, 94, 93, 208}

$$-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}-\frac{6ib\sqrt{ia+ibx+1}}{(a+i)^2\sqrt{-ia-ibx+1}}+\frac{6i\sqrt{-a+i}\,b\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(a+i)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((-6*I)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*Sqrt[1 - I*a - I*b*x]) - (1 + I
*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x]) + ((6*I)*Sqrt[I - a]*
b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I
*b*x])])/(I + a)^(5/2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3b)\int \frac{\sqrt{1+ia+ibx}}{x(1-ia-ibx)^{3/2}}\,dx}{i+a}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3(i-a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)^2}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(6(i-a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx,\right.}{(i+a)^2}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} + \frac{6i\sqrt{i-a}\,b\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{5/2}}$$

Mathematica [A] time = 0.17, size = 145, normalized size = 0.82

$$\frac{\frac{\sqrt{ia+ibx+1}\left(a^2+abx-5ibx+1\right)}{x\sqrt{-i(a+bx+i)}}+\frac{6i\sqrt{-1-ia}\,b\tanh^{-1}\left(\frac{\sqrt{-1-ia}\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}}}{(a+i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x + a*b*x))/(x*Sqrt[(-I)*(I + a
+ b*x)]) + ((6*I)*Sqrt[-1 - I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a
 + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 + I*a])/(I + a)^
2
```

fricas [B] time = 0.47, size = 404, normalized size = 2.30

$$-\frac{2\,(-i\,a-5)b^2x^2-\left(2i\,a^2+8\,a+10i\right)bx-\left(\left(a^2+2i\,a-1\right)bx^2+\left(a^3+3i\,a^2-3\,a-i\right)x\right)\sqrt{\frac{(36\,a-36i)b^2}{a^5+5i\,a^4-10\,a^3-10i\,a^2+5\,a+}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(-I*a - 5)*b^2*x^2 - (2*I*a^2 + 8*a + 10*I)*b*x - ((a^2 + 2*I*a - 1)*b*
x^2 + (a^3 + 3*I*a^2 - 3*a - I)*x)*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 -
10*a^3 - 10*I*a^2 + 5*a + I))*log(-1/6*(6*b^2*x + (a^3 + 3*I*a^2 - 3*a - I)
*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - 6*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + ((a^2 + 2*I*a - 1)*b*x^2 + (a^3 +
 3*I*a^2 - 3*a - I)*x)*sqrt((36*a - 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*
I*a^2 + 5*a + I))*log(-1/6*(6*b^2*x - (a^3 + 3*I*a^2 - 3*a - I)*sqrt((36*a
- 36*I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - 6*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*(-I*a -
5)*b*x - 2*I*a^2 - 2*I))/(2*(a^2 + 2*I*a - 1)*b*x^2 + (2*a^3 + 6*I*a^2 - 6*
a - 2*I)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.17, size = 1358, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 9*I*a^2*b/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)+I/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-9*a^4*b^2/
(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3*a^2*b^2/(a^2+1)^2/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)*x+3*I*a^2*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^2*a/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
*x+12*a^2*b^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-12*I*b/(a^2+1)/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*a^2-3*I/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+1
2*I*a^4*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9*I*a^2*b/(a^2+1)^2/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)-3*I*a^6*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-
3*I*a^4*b/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)+6*a*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+5*I*b*a^4/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)-3*I*a^5*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+9*I*a^3*b^2/(a
^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+5*I*a^3*b^2/(a^2+1)/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*x-9*I*a*b^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-6*I*a*b^
2*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1
/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-
3*a*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+12*a^3*b/(a^2+1)^2/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)+3*I*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I*b/(a^2+
1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/x)-8*b/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+12*b*a^3/(a^2+1)/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)+I*b*a^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-6*b^2*(2*b^2*x+2*
a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*a*b/(a^2+1)^
(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
-2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^2+3/(a^2+1)/x/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)*a^2-9*a^5*b/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9*a^3*b/(
a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)
```

maxima [B] time = 0.35, size = 993, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -I*a*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - I*a^2*b^3/((a^2*b^2 - (a^2 + 1)*b
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3
*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
^2) + (-3*I*a^2*b - 6*a*b + 3*I*b)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^
4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1))
 + (-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3
/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) -
```

```
(-3*I*a*b^2 - 3*b^2)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)) - (-3*I*a*b^2 - 3*b^2)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)) + 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b*arcsinh(2*
a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))
/(a^2 + 1)^(5/2) + I*b/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3*(-I*a^3 - 3*a^
2 + 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - (-3*I*
a^2*b - 6*a*b + 3*I*b)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + (-3*I*a^2*b - 6*a*b + 3
*I*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - (-I*a^3 - 3*a^2 + 3*I
*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i})^3}{x^2\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.187 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=264

$$-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}+\frac{3(-2a+3i)b^2\sqrt{ia+ibx+1}}{(1+ia)(a+i)^3\sqrt{-ia-ibx+1}}+\frac{3(3+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\,(a+i)^{7/2}}+\frac{(-2a}{2(1+ia)}$$

```
[Out] 3*(3+2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-
I*b*x)^(1/2))/(I+a)^(7/2)/(I-a)^(1/2)+1/2*(3*I-2*a)*b*(1+I*a+I*b*x)^(3/2)/(
1+I*a)/(I+a)^2/x/(1-I*a-I*b*x)^(1/2)-1/2*(1+I*a+I*b*x)^(5/2)/(a^2+1)/x^2/(1
-I*a-I*b*x)^(1/2)+3*(3*I-2*a)*b^2*(1+I*a+I*b*x)^(1/2)/(1+I*a)/(I+a)^3/(1-I*
a-I*b*x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}+\frac{3(-2a+3i)b^2\sqrt{ia+ibx+1}}{(1+ia)(a+i)^3\sqrt{-ia-ibx+1}}+\frac{3(3+2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\,(a+i)^{7/2}}+\frac{(-2a}{2(1+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (3*(3*I - 2*a)*b^2*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*(I + a)^3*Sqrt[1 - I*a
 - I*b*x]) + ((3*I - 2*a)*b*(1 + I*a + I*b*x)^(3/2))/(2*(1 + I*a)*(I + a)^2
*x*Sqrt[1 - I*a - I*b*x]) - (1 + I*a + I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[1
 - I*a - I*b*x]) + (3*(3 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
 I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]*(I + a)^(7/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{x^3(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} + \frac{((3i-2a)b)\int \frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx}{2\left(1+a^2\right)}$$

$$= -\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1-ia-ibx}} - \frac{\left(3(3i-2a)b^2\right)\int \frac{\sqrt{1+i}}{x(1-ia}}{2(i+a)\left(1+a^2\right.}$$

$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/}}{2\left(1+a^2\right)x^2\sqrt{1-ia}}$$

$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/}}{2\left(1+a^2\right)x^2\sqrt{1-ia}}$$

$$= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}} - \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\left(1+a^2\right)x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/}}{2\left(1+a^2\right)x^2\sqrt{1-ia}}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.73

$$\frac{\frac{\sqrt{ia+ibx+1}\left(a^3+ia^2-ab^2x^2+5iabx+a+14ib^2x^2-5bx+i\right)}{x^2\sqrt{-i(a+bx+i)}} - \frac{6i\sqrt{-1-ia}\,(2a-3i)b^2\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1+ia}\,(a-i)}}{2(a+i)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(I + a + I*a^2 + a^3 - 5*b*x + (5*I)*a*b*x + (14*I)
*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[(-I)*(I + a + b*x)]) - ((6*I)*Sqrt[-1 - I*
a]*(-3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt
[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 + I*a]*(-I + a)))/(2*(I + a)^3
)
```

fricas [B] time = 0.56, size = 580, normalized size = 2.20

$$(-i\,a-14)b^3x^3 + \left(-i\,a^2-13\,a-14i\right)b^2x^2 - 3\left(\left(a^3+3i\,a^2-3\,a-i\right)bx^3 + \left(a^4+4i\,a^3-6\,a^2-4i\,a+1\right)x^2\right)\sqrt{\frac{}{a^8+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((-I*a - 14)*b^3*x^3 + (-I*a^2 - 13*a - 14*I)*b^2*x^2 - 3*((a^3 + 3*I*a^2 -
 3*a - I)*b*x^3 + (a^4 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12
*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*
a - 1))*log(-((6*a - 9*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a -
9*I)*b^2 + 3*(a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I
*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a
- 1)))/((6*a - 9*I)*b^2)) + 3*((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (a^4 + 4*I
*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7
 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))*log(-((6*a - 9*I)*b^
3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a - 9*I)*b^2 - 3*(a^5 + 3*I*a^4
- 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 -
 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1)))/((6*a - 9*I)*b^2)) +
((-I*a - 14)*b^2*x^2 + I*a^3 - (5*a + 5*I)*b*x - a^2 + I*a - 1)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((2*a^3 + 6*I*a^2 - 6*a - 2*I)*b*x^3 + (2*a^4 + 8*I
*a^3 - 12*a^2 - 8*I*a + 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 1955, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] -75/2*a^4*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-15/2*a^2*b^2/(a^2+1)^
(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
+29*a^2*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15*b^2*a^2/(a^2+1)/(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)-45/2*a^2*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*
(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2/(a^2+1)/x^2/(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)*a^2+45/2*a^6*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
+45/2*a^4*b^2/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x)+15/2*a^2*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-30*a^4*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*I*a^2*b/(a^2+1)^2/
x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9*I*b^3*a^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*x-31/2*I*a^4*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+57/2*I*a^
2*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-5/2*I*a^4*b/(a^2+1)^2/x/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*I*a^6*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*x-45/2*I*a^4*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3*I*b/(a^
2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2-2*I*b^3*(2*b^2*x+2*a*b)/(4*b^2*(a^
2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-30*I*a^5*b^2/(a^2+1)^3/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-27/2*I*a*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-31/2*I*a^5*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+27/2*I*a*b^2/(a^2+
1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/x)-45/2*I*a^3*b^2/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/x)+1/2*I/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
*a^3-3/2*I/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+9*I*b^2*a^3/(a^2+1)/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-21/2*I*a^3*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*
b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+39*I*a^3*b^2/(a^2+1)^
2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+45/2*a^5*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*x-15/2*a^3*b/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+13/2*a*b^3
```

```
/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+5/2*a*b/(a^2+1)^2/x/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)-15/2*a^3*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-6*
I/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b^3-3*I*b/(a^2+1)/x/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+45/2*I*a^3*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/
2*I*a^7*b^2/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*I*a^5*b^2/(a^2+1)^
(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
+6*b/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-75/2*a^3*b^3/(a^2+1)^2/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*x+15*b^3*a/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x
+3*I*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/x)*a-9*I*b^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+3*b^2/(
a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)+3/2*b^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/x)-3/2*b^2/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*
b^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)
```

maxima [B] time = 0.37, size = 1541, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 15/2*(-I*a^3 - 3*a^2 + 3*I*a +
 1)*a^4*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
^2 + 1)^3) - 3*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)
*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*b^5*x/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a^2 +
3*I*a + 1)*a*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*(a^2 + 1)^2) - 3*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^3*b^3/((a^2*b^2 - (a^2
 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*a*b^4/((a^2*b
^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a
^2 + 3*I*a + 1)*a^2*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + 1)^2) + (-3*I*a*b^2 - 3*b^2)*a*b^3*x/((a^2*b^2 - (a^2 + 1)
*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-3*I*a^2*b - 6*a*b
+ 3*I*b)*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(a^2 + 1)) + (-3*I*a*b^2 - 3*b^2)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*
b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1))
 - 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*a
bs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) + 1
5/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(a^2 + 1)^3) + 3*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*b*arcsinh(2*a*b*x/(sqrt(-4
*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/
2) + 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(
x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - 3*(-
3*I*a^2*b - 6*a*b + 3*I*b)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
^2) - 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*(a^2 + 1)^2) - (-3*I*a*b^2 - 3*b^2)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + (-3*I*a
*b^2 - 3*b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 5/2*(-I*a^3 -
 3*a^2 + 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x) -
 (-3*I*a^2*b - 6*a*b + 3*I*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*
x) - 1/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
^2 + 1)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x^3\,\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2+2ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] -I*(Integral(I/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**3*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**3*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.188 $\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=338

$$\frac{\left(-2a^2+51ia+52\right)b^3\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^4\sqrt{-ia-ibx+1}}-\frac{\left(-6ia^2-18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{9/2}}+\frac{(19+16ia)b^2\sqrt{ia+}}{6(-a+i)(a+i)^3x\sqrt{-i}}$$

```
[Out] -(11*I-18*a-6*I*a^2)*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2
)/(1-I*a-I*b*x)^(1/2))/(I-a)^(3/2)/(I+a)^(9/2)+1/6*(52+51*I*a-2*a^2)*b^3*(1
+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^4/(1-I*a-I*b*x)^(1/2)-1/3*(I-a)*(1+I*a+I*b*x)
^(1/2)/(I+a)/x^3/(1-I*a-I*b*x)^(1/2)+7/6*I*b*(1+I*a+I*b*x)^(1/2)/(I+a)^2/x^
2/(1-I*a-I*b*x)^(1/2)+1/6*(19+16*I*a)*b^2*(1+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^3
/x/(1-I*a-I*b*x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 98, 151, 152, 12, 93, 208}

$$\frac{\left(-2a^2+51ia+52\right)b^3\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^4\sqrt{-ia-ibx+1}}-\frac{\left(-6ia^2-18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{9/2}}+\frac{(19+16ia)b^2\sqrt{ia+}}{6(-a+i)(a+i)^3x\sqrt{-i}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((52 + (51*I)*a - 2*a^2)*b^3*Sqrt[1 + I*a + I*b*x])/(6*(I - a)*(I + a)^4*Sq
rt[1 - I*a - I*b*x]) - ((I - a)*Sqrt[1 + I*a + I*b*x])/(3*(I + a)*x^3*Sqrt[
1 - I*a - I*b*x]) + (((7*I)/6)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*x^2*Sqrt
[1 - I*a - I*b*x]) + ((19 + (16*I)*a)*b^2*Sqrt[1 + I*a + I*b*x])/(6*(I - a)
*(I + a)^3*x*Sqrt[1 - I*a - I*b*x]) - ((11*I - 18*a - (6*I)*a^2)*b^3*ArcTan
h[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])
/((I - a)^(3/2)*(I + a)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1+ia+ibx)^{3/2}}{x^4(1-ia-ibx)^{3/2}}\,dx$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} - \frac{\int \frac{-7(i-a)b+6b^2x}{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}\,dx}{3(1-ia)}$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{\int \frac{-\left(19+35ia-16a^2\right)b^2-14(i-a)b^3x}{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}\,dx}{6(1-ia)\left(1+a^2\right)}$$
$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} -$$
$$= \frac{\left(52+51ia-2a^2\right)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}}$$
$$= \frac{\left(52+51ia-2a^2\right)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}}$$
$$= \frac{\left(52+51ia-2a^2\right)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}}$$
$$= \frac{\left(52+51ia-2a^2\right)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}}$$

Mathematica [A] time = 0.43, size = 282, normalized size = 0.83

$$-\frac{-i\left(6a^2-18ia-11\right)b^2x^2\left(i\sqrt{-1+ia}\sqrt{ia+ibx+1}\left(a^2+abx-5ibx+1\right)-6\sqrt{-1-ia}\,bx\sqrt{-i(a+bx+i)}\,\tanh\right.}{6(-1+ia)^{5/2}\left(a^2+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -1/6*(2*(-1 + I*a)^(3/2)*(1 + I*a)*(I + a)^2*(1 + I*a + I*b*x)^(5/2) + (3*I
 - 4*a)*(-1 + I*a)^(5/2)*b*x*(1 + I*a + I*b*x)^(5/2) - I*(-11 - (18*I)*a +
6*a^2)*b^2*x^2*(I*Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x
 + a*b*x) - 6*Sqrt[-1 - I*a]*b*x*Sqrt[(-I)*(I + a + b*x)]*ArcTanh[(Sqrt[-1
- I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]))/
((-1 + I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[(-I)*(I + a + b*x)])
```

fricas [B] time = 0.55, size = 853, normalized size = 2.52

$$\frac{\left(2i\,a^2+51\,a-52i\right)b^4x^4+\left(2i\,a^3+49\,a^2-i\,a+52\right)b^3x^3+\sqrt{\frac{\left(36\,a^4-216i\,a^3-456\,a^2+396i\,a+121\right)b^6}{a^{12}+6i\,a^{11}-12\,a^{10}-2i\,a^9-27\,a^8-36i\,a^7-36i\,a^5+27\,a^4-2i\,a^3+12\,a^2\cdots}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((2*I*a^2 + 51*a - 52*I)*b^4*x^4 + (2*I*a^3 + 49*a^2 - I*a + 52)*b^3*x^3 +
sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 -
```

```
12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^
2 + 6*I*a - 1))*((3*a^5 + 9*I*a^4 - 6*a^3 + 6*I*a^2 - 9*a - 3*I)*b*x^4 + (3
*a^6 + 12*I*a^5 - 15*a^4 - 15*a^2 - 12*I*a + 3)*x^3)*log(-((6*a^2 - 18*I*a
- 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 - 18*I*a - 11)*b^3 +
 (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I*a^2 - 3*a - I)*sqrt((36*a^4 - 2
16*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^
9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1)))
/((6*a^2 - 18*I*a - 11)*b^3)) - sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*
a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*
I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1))*((3*a^5 + 9*I*a^4 - 6*a^3 +
 6*I*a^2 - 9*a - 3*I)*b*x^4 + (3*a^6 + 12*I*a^5 - 15*a^4 - 15*a^2 - 12*I*a
+ 3)*x^3)*log(-((6*a^2 - 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*(6*a^2 - 18*I*a - 11)*b^3 - (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I
*a^2 - 3*a - I)*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^
12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 -
 2*I*a^3 + 12*a^2 + 6*I*a - 1)))/((6*a^2 - 18*I*a - 11)*b^3)) + ((2*I*a^2 +
 51*a - 52*I)*b^3*x^3 + 2*I*a^5 + (16*a^2 - 3*I*a + 19)*b^2*x^2 - 2*a^4 + 4
*I*a^3 - (7*a^3 + 7*I*a^2 + 7*a + 7*I)*b*x - 4*a^2 + 2*I*a - 2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((6*a^5 + 18*I*a^4 - 12*a^3 + 12*I*a^2 - 18*a - 6*I
)*b*x^4 + (6*a^6 + 24*I*a^5 - 30*a^4 - 30*a^2 - 24*I*a + 6)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 2624, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] 35/2*a^3*b^3/(a^2+1)^(9/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)-15/2*a*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+135/2*a^3*b^3/(a^2+1)^(7/2)*ln((2
*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-18*a*b^3/(
a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)+7*I*b^3*a^2/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+45*I*a^2*b^3/(a^2
+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-45*I*a^2*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+
2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-187/6*I*b^3/(a^2+
1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^4+41*I*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)*a^2+30*I*a^4*b^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+125/3*I*a^6*b^3/(a^2+1)^3/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)-110*I*a^4*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-2
7/2*I*b^3*a^2/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x)+1/3*I/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-
I/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+70*I*a^6*b^3/(a^2+1)^4/(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-105/2*I*a^4*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)-35/2*I*a^8*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*b^2/(a^2+1)/x
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+6*b^4/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x+6*b^3/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-35/2*a^3*b^3/(a^2+1)^4/(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)+70*a^5*b^3/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-260/3*a^3*b^3/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+15/2*a*b^3/(a^2+1)^
3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(
```

```
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I*b^3/(a^2+1)/(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)-9/2*I*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I*b^3
/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)+1/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2-105/2*a^7*b^3/(a^
2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-105/2*a^5*b^3/(a^2+1)^(9/2)*ln((2*a^2+
2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+205/2*a^5*b^3/(
a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-56*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)*a^3+4/3*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+8/3*b^4/(a
^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+62/3*b^3/(a^2+1)^2/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)*a-35/2*I*a^6*b^3/(a^2+1)^(9/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+105/2*I*a^4*b^3/(a^2+1)^(9/2)*ln((2
*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+7/6*a*b/(a
^2+1)^2/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35/6*a^2*b^2/(a^2+1)^3/x/(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+35/2*a^4*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x-3/2*I*b/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*b/(a^2+1)/x^2/(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)*a-19*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a
^2-56*b^4/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2-7/2*a^3*b/(a^2+1)^2
/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+35/2*a^4*b^2/(a^2+1)^3/x/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)-105/2*a^6*b^4/(a^2+1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+205/
2*a^4*b^4/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-115/6*a^2*b^4/(a^2+1)^3
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/3/(a^2+1)/x^3/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+3*I*b^2/(a^2+1)/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+35/6*I*a^5*b^2/(a^2+1
)^3/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-7/6*I*a^4*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+7/2*I*a^2*b/(a^2+1)^2/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35
/2*I*a^3*b^2/(a^2+1)^3/x/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-35/2*I*a^7*b^4/(a^2+
1)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+105/2*I*a^5*b^4/(a^2+1)^4/(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*x+3/2*I*b/(a^2+1)/x^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2+7
*I*b^4*a/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+55/2*I*b^4/(a^2+1)^2/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*x*a+23/2*I*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a-187/6*I*b^4/(a^2+1)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^3+125/3*I*
a^5*b^4/(a^2+1)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-80*I*a^3*b^4/(a^2+1)^3/(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-53/6*I*b^2/(a^2+1)^2/x/(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*a^3
```

maxima [B] time = 0.38, size = 2327, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -35/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^4) - 35/2*(-I*a^3 - 3*a^2 + 3*I*a
+ 1)*a^5*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
a^2 + 1)^4) + 15/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^3*b^5*x/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - I*a*b^6*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 115/6*(
-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 15/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^
4*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1
)^3) - I*a^2*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*(a^2 + 1)) + 115/6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5/((a^2*b^2 - (a^2
 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - 3*(-3*I*a*b^2 -
 3*b^2)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*(a^2 + 1)^2) - 13/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*b^5*x/((a^2*b^2 - (a
^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 8/3*(-I*a^3 -
 3*a^2 + 3*I*a + 1)*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(a^2 + 1)^2) - 3*(-3*I*a*b^2 - 3*b^2)*a^3*b^3/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 13/2*(-3*I*a^2*b
 - 6*a*b + 3*I*b)*a^2*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
```

```
 + a^2 + 1)*(a^2 + 1)^2) - 8/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^5/((a^2*b^2
 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + 2*(-3*I*
a*b^2 - 3*b^2)*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(a^2 + 1)) + 35/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^3*arcsinh(2*a*b
*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a
^2 + 1)^(9/2) + 2*(-3*I*a*b^2 - 3*b^2)*a*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 35/2*(-I*a^3 - 3*a^2 + 3*I*a +
1)*a^3*b^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^4) - 15/2*(-3*I*a^2
*b - 6*a*b + 3*I*b)*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) + I*b^3*arcsinh(2*a*
b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(
a^2 + 1)^(3/2) - 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3*arcsinh(2*a*b*x/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 +
1)^(7/2) + 15/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a^2*b^2/(sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*(a^2 + 1)^3) - I*b^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
 1)) + 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(a^2 + 1)^3) + 3*(-3*I*a*b^2 - 3*b^2)*a*b*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2)
 + 3/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - 3*(-3*
I*a*b^2 - 3*b^2)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 3/2*
(-3*I*a^2*b - 6*a*b + 3*I*b)*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
1)^2) - 35/6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^2/(sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + 1)^3*x) + 5/2*(-3*I*a^2*b - 6*a*b + 3*I*b)*a*b/(sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x) + 4/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b
^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x) + 7/6*(-I*a^3 - 3*a^2
+ 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x^2) - (-3*
I*a*b^2 - 3*b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x) - 1/2*(-3*
I*a^2*b - 6*a*b + 3*I*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x^2)
- 1/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2
+ 1)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i})^3}{x^4\left((a + b\,x)^2 + 1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^2x^4\sqrt{a^2 + 2abx + b^2x^2 + 1} + 2abx^5\sqrt{a^2 + 2abx + b^2x^2 + 1} + b^2x^6\sqrt{a^2 + 2abx + b^2x^2 + 1} + x^4\sqrt{a^2 + 2ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] -I*(Integral(I/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**
```

```
2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**4*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**4*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

3.189 $\int e^{-i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=276

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13\right)bx-86ia^2-114a+19i\right)}{120b^5}-\frac{\left(8ia^4+16a^3-24ia^2-\right.}{}$$

```
[Out] 1/8*(3+12*I*a-24*a^2-16*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5+1/20*(I-8*a)*x^2*(1
-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/5*x^3*(1-I*a-I*b*x)^(3/2)*(1+I*
a+I*b*x)^(1/2)/b^2-1/120*(1-I*a-I*b*x)^(3/2)*(19*I-114*a-86*I*a^2+96*a^3+2*
(13+14*I*a-36*a^2)*b*x)*(1+I*a+I*b*x)^(1/2)/b^5-1/8*(3*I-12*a-24*I*a^2+16*a
^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] time = 0.22, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5095, 100, 153, 147, 50, 53, 619, 215}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(2\left(-36a^2+14ia+13\right)bx+96a^3-86ia^2-114a+19i\right)}{120b^5}-\frac{\left(8ia^4+16a^3-24ia^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -((3*I - 12*a - (24*I)*a^2 + 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt
[1 + I*a + I*b*x])/(8*b^5) + ((I - 8*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1
+ I*a + I*b*x])/(20*b^3) + (x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(19*I - 114*a
- (86*I)*a^2 + 96*a^3 + 2*(13 + (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 +
(12*I)*a - 24*a^2 - (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x^2\sqrt{1-ia-ibx}\left(-3\left(1+a^2\right)+(i-8a)bx\right)}{\sqrt{1+ia+ibx}}dx}{5b^2}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}}{}}{}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx}{}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/}}{20b^3}$$

$$= -\frac{\left(3i-12a-24ia^2+16a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/}}{20b^3}$$

Mathematica [A] time = 0.72, size = 248, normalized size = 0.90

$$\frac{\sqrt[4]{-1}\left(-8ia^4-16a^3+24ia^2+12a-3i\right)\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}\,b^{9/2}} + \frac{i\sqrt{ia+ibx+1}\left(24ia^5+226a^4+2a^3(72bx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/120)*Sqrt[1 + I*a + I*b*x]*(-64 + 226*a^4 + (24*I)*a^5 + (109*I)*b*x +
77*b^2*x^2 - (62*I)*b^3*x^3 - 54*b^4*x^4 + (24*I)*b^5*x^5 + 2*a^3*(-41*I +
72*b*x) + a^2*(57 - (346*I)*b*x - 84*b^2*x^2) + a*(-211*I - 346*b*x + (154*
I)*b^2*x^2 + 64*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(1/4)*(-3
*I + 12*a + (24*I)*a^2 - 16*a^3 - (8*I)*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

fricas [A] time = 0.47, size = 175, normalized size = 0.63

$$\frac{-186i\,a^5-1345\,a^4+1730i\,a^3+1320\,a^2-\left(960\,a^4-1920i\,a^3-2880\,a^2+1440i\,a+360\right)\log\left(-bx-a+\sqrt{b^2x^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/960*(-186*I*a^5 - 1345*a^4 + 1730*I*a^3 + 1320*a^2 - (960*a^4 - 1920*I*a^
3 - 2880*a^2 + 1440*I*a + 360)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (-192*I*b^4*x^4 - 48*(-4*I*a - 5)*b^3*x^3 + (-192*I*a^2 - 560*a + 2
56*I)*b^2*x^2 - 192*I*a^4 - 2000*a^3 + (192*I*a^3 + 1040*a^2 - 928*I*a - 36
0)*b*x + 2656*I*a^2 + 2200*a - 512*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3
00*I*a)/b^5
```

giac [A] time = 0.16, size = 214, normalized size = 0.78

$$-\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\left(\frac{4\,ix}{b}-\frac{4\,ab^7i+5\,b^7}{b^9}\right)x+\frac{12\,a^2b^6i+35\,ab^6-16\,b^6i}{b^9}\right)x-\frac{24\,a^3b^5i+130\,a^2b^5-116\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*sqrt((b*x + a)^2 + 1)*((2*(3*(4*i*x/b - (4*a*b^7*i + 5*b^7)/b^9)*x +
 (12*a^2*b^6*i + 35*a*b^6 - 16*b^6*i)/b^9)*x - (24*a^3*b^5*i + 130*a^2*b^5
- 116*a*b^5*i - 45*b^5)/b^9)*x + (24*a^4*b^4*i + 250*a^3*b^4 - 332*a^2*b^4*
i - 275*a*b^4 + 64*b^4*i)/b^9) - 1/8*(8*a^4 - 16*a^3*i - 24*a^2 + 12*a*i +
3)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

maple [B] time = 0.22, size = 1208, normalized size = 4.38

$$\frac{3a^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b^4\sqrt{b^2}}+\frac{3a^2x\sqrt{b^2x^2+2abx+a^2+1}}{b^4}-\frac{i\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}\,a^4}{b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 3*a^2/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)+3*a^2/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I/b^4*a^3*x*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)-5/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)+2*I/b^4*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)-4*I/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+
((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^3+4*I/b^4*ln((I*
b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
/(b^2)^(1/2)*a+3/5*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-5/2*I/b^5*a^2*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+2*I/b^5*a^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-6/5*I
/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/5*I/b^3*x^2*(b^2*x^2+2*a*b*x+a^2+1
)^(3/2)+1/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^4-6/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(
1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^2-I/b^5*((x
-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^4+6*I/b^5*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2)*a^2-5/2*I/b^4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-4/b^5
*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^3+4/b^5*((x-(I-a)/b)^2*b^2+2
*I*b*(x-(I-a)/b))^(1/2)*a+1/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I
-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-I/b^5*((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(1/2)+7/15*I/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/4/b^4*
x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-13/12/b^5*a*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+3
/b^5*a^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)-5/8/b^5*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-5/8/b^4*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 0.47, size = 456, normalized size = 1.65

$$\frac{2i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^3x}{b^4}-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x^2}{5\,b^3}+\frac{a^4\operatorname{arsinh}(bx+a)}{b^5}+\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 - 1/5*I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)*x^2/b^3 + a^4*arcsinh(b*x + a)/b^5 + I*sqrt(b^2*x^2 + 2*a*
```

```
b*x + a^2 + 1)*a^4/b^5 + 3/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4
+ 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^4 - 2*I*a^3*arcsinh(b*x + a)/
b^5 - 6/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^3/b^5 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 -
5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^4 - 3*a^2*arcsinh(b*x + a)/b^
5 - 13/12*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 + 7/2*I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*a^2/b^5 - 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 +
3/2*I*a*arcsinh(b*x + a)/b^5 + 7/15*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b
^5 + 27/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 3/8*arcsinh(b*x + a)/b^
5 - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{{\left(a+b\,x\right)}^2+1}}{1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
[Out] int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{x^{4}\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}{a+bx-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
[Out] -I*Integral(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

3.190 $\int e^{-i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=201

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)}{24b^4}-\frac{\left(-8ia^3-12a^2+12ia+3\right)\sqrt{-ia-ibx+}}{8b^4}$$

```
[Out] -1/8*(3*I-12*a-12*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4+1/4*x^2*(1-I*a-I*b*x)^(3/
2)*(1+I*a+I*b*x)^(1/2)/b^2-1/24*(1-I*a-I*b*x)^(3/2)*(7+10*I*a-18*a^2-2*(I-6
*a)*b*x)*(1+I*a+I*b*x)^(1/2)/b^4-1/8*(3+12*I*a-12*a^2-8*I*a^3)*(1-I*a-I*b*x
)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
```

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 100, 147, 50, 53, 619, 215}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)}{24b^4}-\frac{\left(-8ia^3-12a^2+12ia+3\right)\sqrt{-ia-ibx+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -((3 + (12*I)*a - 12*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a +
I*b*x])/(8*b^4) + (x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^
2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(7 + (10*I)*a - 18*a^2
- 2*(I - 6*a)*b*x))/(24*b^4) - ((3*I - 12*a - (12*I)*a^2 + 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
```

```
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}\left(-2\left(1+a^2\right)+(i-6a)bx\right)}{\sqrt{1+ia+ibx}}\,dx}{4b^2}$$

$$= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}\left(7+10ia-18a^2-2(i-\right.}{24b^4}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}$$

$$= -\frac{\left(3+12ia-12a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}$$

Mathematica [A] time = 0.52, size = 202, normalized size = 1.00

$$\frac{(-1)^{3/4}\left(8ia^3+12a^2-12ia-3\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4b^{9/2}} + \frac{\sqrt{ia+ibx+1}\left(6a^4-38ia^3+5a^2(1-6ibx)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-16 - (38*I)*a^3 + 6*a^4 + (25*I)*b*x + 17*b^2*x^2
- (14*I)*b^3*x^3 - 6*b^4*x^4 + 5*a^2*(1 - (6*I)*b*x) + I*a*(-23 + (50*I)*b*
x + 18*b^2*x^2)))/(24*b^4*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(3/4)*(-3 - (12
*I)*a + 12*a^2 + (8*I)*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

fricas [A] time = 0.52, size = 137, normalized size = 0.68

$$\frac{45i\,a^4 + 224\,a^3 - 192i\,a^2 + \left(192\,a^3 - 288i\,a^2 - 288\,a + 72i\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \left(-48i\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(45*I*a^4 + 224*a^3 - 192*I*a^2 + (192*a^3 - 288*I*a^2 - 288*a + 72*I
)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-48*I*b^3*x^3 - 16*(
-3*I*a - 4)*b^2*x^2 + 48*I*a^3 + (-48*I*a^2 - 160*a + 72*I)*b*x + 352*a^2 -
 312*I*a - 128)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

giac [A] time = 0.15, size = 163, normalized size = 0.81

$$-\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\left(\frac{3\,ix}{b} - \frac{3\,ab^5i + 4\,b^5}{b^7}\right)x + \frac{6\,a^2b^4i + 20\,ab^4 - 9\,b^4i}{b^7}\right)x - \frac{6\,a^3b^3i + 44\,a^2b^3 - 39\,ab^3i - 16}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt((b*x + a)^2 + 1)*((2*(3*i*x/b - (3*a*b^5*i + 4*b^5)/b^7)*x + (6*
a^2*b^4*i + 20*a*b^4 - 9*b^4*i)/b^7)*x - (6*a^3*b^3*i + 44*a^2*b^3 - 39*a*b
^3*i - 16*b^3)/b^7) + 1/8*(8*a^3 - 12*a^2*i - 12*a + 3*i)*log(-a*b - (x*abs
(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

maple [B] time = 0.19, size = 894, normalized size = 4.45

$$\frac{i\sqrt{\left(x-\frac{i-a}{b}\right)^2 b^2 + 2ib\left(x-\frac{i-a}{b}\right)}\,a^3}{b^4} + \frac{5ia\sqrt{b^2x^2+2abx+a^2+1}}{8b^4} - \frac{3ia^3\sqrt{b^2x^2+2abx+a^2+1}}{2b^4} - \frac{3ax\sqrt{b^2x^2+2a}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] I/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^3+5/8*I/b^4*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)*a-3/2*I/b^4*a^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*
x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x
^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3*I/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I
-a)/b))^(1/2)*a+5/8*I/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)-1/4*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/3/b^4*(b^2
*x^2+2*a*b*x+a^2+1)^(3/2)-3/2*I/b^3*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2
+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/
b))^(1/2)+3/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^2+3*I/b^3*ln(
(I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/
2))/(b^2)^(1/2)*a^2-1/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^3+3/b^3*ln((I*b+(x-(I-a)/b)*
b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a
-I/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-
a)/b))^(1/2))/(b^2)^(1/2)-3/2*I/b^3*a^2*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/8
```

```
*I/b^3*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/4*I/b^4*a*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)-3/2*a^2/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.44, size = 308, normalized size = 1.53

$$-\frac{3i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2x}{2\,b^3}-\frac{a^3\,\mathrm{arsinh}\,(bx+a)}{b^4}-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^3}{2\,b^4}-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x}{4\,b^3}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 - a^3*arcsinh(b*x + a)/b
^4 - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 - 1/4*I*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3
 + 3/2*I*a^2*arcsinh(b*x + a)/b^4 + 3/4*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)*a/b^4 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 5/8*I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*x/b^3 + 3/2*a*arcsinh(b*x + a)/b^4 + 1/3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)/b^4 - 19/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/
b^4 - 3/8*I*arcsinh(b*x + a)/b^4 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

```
[Out] int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{x^3\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

3.191 $\int e^{-i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=171

$$\frac{\left(-2ia^2-2a+i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}-\frac{\left(-2a^2+2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}+\frac{(-4a+i)\sqrt{ia+ibx+1}\,(-ia}{6b^3}$$

```
[Out] -1/2*(1+2*I*a-2*a^2)*arcsinh(b*x+a)/b^3+1/6*(I-4*a)*(1-I*a-I*b*x)^(3/2)*(1+
I*a+I*b*x)^(1/2)/b^3+1/3*x*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2+1/2*
(I-2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 90, 80, 50, 53, 619, 215}

$$\frac{\left(-2ia^2-2a+i\right)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}-\frac{\left(-2a^2+2ia+1\right)\sinh^{-1}(a+bx)}{2b^3}+\frac{x\sqrt{ia+ibx+1}\,(-ia-ibx+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I - 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3)
 + ((I - 4*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(6*b^3) + (x*(
1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(3*b^2) - ((1 + (2*I)*a - 2*a
^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} + \frac{\int \frac{\sqrt{1-ia-ibx}\left(-1-a^2+(i-4a)bx\right)}{\sqrt{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{\left(1+2ia-2a^2\right)}{2b}$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} +$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} +$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} +$$

$$= -\frac{\left(2a-i\left(1-2a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} +$$

Mathematica [A] time = 0.30, size = 162, normalized size = 0.95

$$\frac{\sqrt[4]{-1}\left(2a^2-2ia-1\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{7/2}} + \frac{i\sqrt{ia+ibx+1}\left(2ia^3+7a^2+a(8bx+5i)+2ib^3x^3-5b^2\right.}{6b^3\sqrt{-i(a+bx+i)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/6)*Sqrt[1 + I*a + I*b*x]*(4 + 7*a^2 + (2*I)*a^3 - (7*I)*b*x - 5*b^2*x^2
 + (2*I)*b^3*x^3 + a*(5*I + 8*b*x)))/(b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-1)
^(1/4)*(-1 - (2*I)*a + 2*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqr
t[(-I)*(I + a + b*x)]])/Sqrt[(-I)*b]])/b^(7/2)
```

fricas [A] time = 0.45, size = 106, normalized size = 0.62

$$\frac{-7i\,a^3-21\,a^2-12\left(2\,a^2-2i\,a-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\sqrt{b^2x^2+2\,abx+a^2+1}\left(-8i\,b^2x^2-\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(-7*I*a^3 - 21*a^2 - 12*(2*a^2 - 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2 -
 4*(-2*I*a - 3)*b*x - 8*I*a^2 - 36*a + 16*I) + 9*I*a)/b^3
```

giac [A] time = 0.14, size = 117, normalized size = 0.68

$$-\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b}-\frac{2\,ab^3i+3\,b^3}{b^5}\right)x+\frac{2\,a^2b^2i+9\,ab^2-4\,b^2i}{b^5}\right)-\frac{\left(2\,a^2-2\,ai-1\right)\log\left(-ab-\left(x|b|-\sqrt{(b\right.\right.}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^3*i + 3*b^3)/b^5)*x + (2*a^2*
b^2*i + 9*a*b^2 - 4*b^2*i)/b^5) - 1/2*(2*a^2 - 2*a*i - 1)*log(-a*b - (x*abs
(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

maple [B] time = 0.18, size = 605, normalized size = 3.54

$$-\frac{i\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^3}+\frac{iax\sqrt{b^2x^2+2abx+a^2+1}}{b^2}+\frac{ia^2\sqrt{b^2x^2+2abx+a^2+1}}{b^3}+\frac{ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/3*I/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+I/b^2*a*x*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+I/b^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/b^2*a*ln((b^2*x+a*b)/(b^2)^
(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*x/b^2*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+1/2/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+1/2/b^2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/b^3*((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^2+I/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)
/b))^(1/2)-2/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+1/b^2*ln((I*
b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
/(b^2)^(1/2)*a^2-1/b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*
b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-2*I/b^2*ln((I*b+(x-(I-a)/b)*b^2)/
(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a
```

maxima [A] time = 0.44, size = 161, normalized size = 0.94

$$\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,ax}{b^2}+\frac{a^2\operatorname{arsinh}(bx+a)}{b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,x}{2\,b^2}-\frac{i\,a\operatorname{arsinh}(bx+a)}{b^3}-\frac{i\left(b^2x^2+2\,ab\right.}{3\,l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^2 + a^2*arcsinh(b*x + a)/b^3 + 1/
2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - I*a*arcsinh(b*x + a)/b^3 - 1/3*
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a/b^3 - 1/2*arcsinh(b*x + a)/b^3 + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
[Out] int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
[Out] -I*Integral(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

3.192 $\int e^{-i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{2b^2}+\frac{(1+2ia)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^2}+\frac{(-2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

```
[Out] 1/2*(I-2*a)*arcsinh(b*x+a)/b^2+1/2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/
b^2+1/2*(1+2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 80, 50, 53, 619, 215}

$$\frac{\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{2b^2}+\frac{(1+2ia)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^2}+\frac{(-2a+i)\sinh^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((1 + (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((1 -
 I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((I - 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b}$$

$$= \frac{(1+2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\int \frac{\cdots}{\sqrt{1-ia-ib\cdots}}}{2b}$$

$$= \frac{(1+2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\int \frac{\cdots}{\sqrt{(1-ia)(1\cdots}}}{2b\cdots}$$

$$= \frac{(1+2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(1+2ia)\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\sinh^{-1}(a\cdots}{2b^2}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.19

$$\frac{\sqrt{ia+ibx+1}\left(a^2-ia-b^2x^2-3ibx+2\right)}{2b^2\sqrt{-i(a+bx+i)}} + \frac{(-1)^{3/4}(1+2ia)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(2 - I*a + a^2 - (3*I)*b*x - b^2*x^2))/(2*b^2*Sqrt[(
-I)*(I + a + b*x)]) + ((-1)^(3/4)*(1 + (2*I)*a)*Sqrt[(-I)*b]*ArcSinh[((1/2
+ I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

fricas [A] time = 0.47, size = 77, normalized size = 0.70

$$\frac{3i\,a^2 + (8\,a-4i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}\,(-4i\,bx+4i\,a+8)+4\,a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*a^2 + (8*a - 4*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-4*I*b*x + 4*I*a + 8) + 4*a)/b^2
```

giac [A] time = 0.16, size = 78, normalized size = 0.71

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}-\frac{abi+2\,b}{b^3}\right) + \frac{(2\,a-i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(i*x/b - (a*b*i + 2*b)/b^3) + 1/2*(2*a - i)*log(
-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

maple [B] time = 0.18, size = 350, normalized size = 3.18

$$-\frac{ix\sqrt{b^2x^2+2abx+a^2+1}}{2b}-\frac{ia\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}+\frac{i\sqrt{\left(x-\frac{i-a}{b}\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*I/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/b^2*a*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)-1/2*I/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(
b^2)^(1/2)+I/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+1/b^2*((x-(I
-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)-1/b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1
/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a+I/b*ln((I*b+
(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(
b^2)^(1/2)
```

maxima [A] time = 0.43, size = 97, normalized size = 0.88

$$-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x}{2\,b}-\frac{a\,\mathrm{arsinh}\left(bx+a\right)}{b^2}+\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,a}{2\,b^2}+\frac{i\,\mathrm{arsinh}\left(bx+a\right)}{2\,b^2}+\frac{\sqrt{b^2x^2+2\,abx}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - a*arcsinh(b*x + a)/b^2 + 1/2
*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^2 + 1/2*I*arcsinh(b*x + a)/b^2 + s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{{\left(a+b\,x\right)}^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

```
[Out] int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

3.193 $\int e^{-i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh^{-1}(a+bx)}{b}-\frac{i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}$$

```
[Out] arcsinh(b*x+a)/b-I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5093, 50, 53, 619, 215}

$$\frac{\sinh^{-1}(a+bx)}{b}-\frac{i\sqrt{-ia-ibx+1}\,\sqrt{ia+ibx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{-i\tan^{-1}(a+bx)}\,dx=\int\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\int\frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\int\frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx$$
$$=-\frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{2b^2}$$
$$=-\frac{i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b}+\frac{\sinh^{-1}(a+bx)}{b}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.54

$$\frac{\sinh^{-1}(a+bx)-i\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-I)*ArcTan[a + b*x]),x]
[Out] ((-I)*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

fricas [A] time = 0.55, size = 60, normalized size = 1.15

$$\frac{-i\,a-2i\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(-I*a - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [A] time = 0.13, size = 52, normalized size = 1.00

$$-\frac{\sqrt{(bx+a)^2+1}\,i}{b}-\frac{\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -sqrt((b*x + a)^2 + 1)*i/b - log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*
abs(b))/abs(b)
```

maple [B] time = 0.11, size = 122, normalized size = 2.35

$$-\frac{i\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{b}+\frac{\ln\left(\frac{ib+\left(x-\frac{i-a}{b}\right)b^2}{\sqrt{b^2}}+\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] -I/b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+ln((I*b+(x-(I-a)/b)*b^2)/(
b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 0.43, size = 35, normalized size = 0.67

$$\frac{\operatorname{arsinh}(bx+a)}{b}-\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x + a)/b - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

3.194 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=89

$$-i\sinh^{-1}(a+bx)-\frac{2\sqrt{a+i}\,\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}$$

```
[Out] -I*arcsinh(b*x+a)-2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-
I*a-I*b*x)^(1/2))*(I+a)^(1/2)/(I-a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5095, 105, 53, 619, 215, 93, 208}

$$-i\sinh^{-1}(a+bx)-\frac{2\sqrt{a+i}\,\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a + b*x] - (2*Sqrt[I + a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I - a]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{\sqrt{1-ia-ibx}}{x\sqrt{1+ia+ibx}}\,dx\\
&= -\left((-1+ia)\int \frac{1}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\right)-(ib)\int \frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\\
&= (2(1-ia))\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)-(ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)}\cdots}\\
&= -\frac{2\sqrt{i+a}\,\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}-\frac{i\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b}\\
&= -i\sinh^{-1}(a+bx)-\frac{2\sqrt{i+a}\,\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 142, normalized size = 1.60

$$\frac{2\sqrt[4]{-1}\,(-ib)^{3/2}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{3/2}}-\frac{2\sqrt{-1+ia}\,\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*(-1)^(1/4)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a
+ b*x)])/Sqrt[(-I)*b]])/b^(3/2) - (2*Sqrt[-1 + I*a]*ArcTanh[(Sqrt[-1 - I*a]
*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1
 - I*a]
```

fricas [B] time = 0.47, size = 155, normalized size = 1.74

$$-\frac{1}{2}\sqrt{-\frac{4\,a+4i}{a-i}}\,\log\left(-bx+\frac{1}{2}\,(i\,a+1)\sqrt{-\frac{4\,a+4i}{a-i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\frac{1}{2}\sqrt{-\frac{4\,a+4i}{a-i}}\,\log\left(-bx+\frac{1}{2}\,(-i\,a\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(4*a + 4*I)/(a - I))*log(-b*x + 1/2*(I*a + 1)*sqrt(-(4*a + 4*I)/
(a - I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 1/2*sqrt(-(4*a + 4*I)/(a -
I))*log(-b*x + 1/2*(-I*a - 1)*sqrt(-(4*a + 4*I)/(a - I)) + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)) + I*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

giac [A] time = 0.15, size = 77, normalized size = 0.87

$$\frac{bi\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}+\frac{2\,(a+i)\arctan\left(-\frac{\left(x|b|-\sqrt{(bx+a)^2+1}\right)i}{\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] b*i*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) + 2*(a + i
)*arctan(-(x*abs(b) - sqrt((b*x + a)^2 + 1))*i/sqrt(a^2 + 1))/sqrt(a^2 + 1)
```

maple [B] time = 0.18, size = 283, normalized size = 3.18

$$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{i-a}+\frac{iab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{(i-a)\sqrt{b^2}}-\frac{i\sqrt{a^2+1}\ \ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\ \sqrt{b^2x^2+2abx}}{x}\right.}{i-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] I/(I-a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I/(I-a)*a*b*ln((b^2*x+a*b)/(b^2)^(1/2
)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)*(a^2+1)^(1/2)*ln((2*a^
2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I/(I-a)*((x-(
I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/(I-a)*b*ln((I*b+(x-(I-a)/b)*b^2)/(
b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2+1}}{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax+bx^2-ix}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x + b*x**2 - I*x), x)
```

3.195 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}-\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}$$

```
[Out] -2*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1
/2))/(I-a)^(3/2)/(I+a)^(1/2)-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1+I*a
)/x
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5095, 94, 93, 208}

$$-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}-\frac{2ib\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x)) - ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/((I - a)^(3/2)*Sqrt[I + a])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1+ia)x} + \frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{i-a}$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1+ia)x} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i-a}$$
$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{(1+ia)x} - \frac{2ib\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}\sqrt{i+a}}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.92

$$i\left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{(a-i)x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{(-1-ia)^{3/2}\sqrt{-1+ia}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] I*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/((-I + a)*x) + (2*b*ArcTanh[(Sqrt[-1 -
 I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/((
-1 - I*a)^(3/2)*Sqrt[-1 + I*a]))
```

fricas [B] time = 0.49, size = 227, normalized size = 1.75

$$-\frac{2\,(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+\left(a^3-i\,a^2+a-i\right)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}}{b}\right)-2\,(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\Bigg(}{(2\,a-2i)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b + (a^3 - I*a^2 + a - I)*sqrt(b^2/(a^4 - 2*I*a^3
- 2*I*a - 1)))/b) - 2*(a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-
(b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 - I*a^2 + a - I)*sqrt(b
^2/(a^4 - 2*I*a^3 - 2*I*a - 1)))/b) - 2*I*b*x - 2*I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/((2*a - 2*I)*x)
```

giac [A] time = 0.23, size = 147, normalized size = 1.13

$$\frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}\,(a-i)} + \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(ai+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*a
bs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a - i))
 + 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(a*i + 1))
```

maple [B] time = 0.18, size = 602, normalized size = 4.63

$$\frac{ib\sqrt{b^2x^2+2abx+a^2+1}}{(i-a)^2}+\frac{ib^2a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{(i-a)^2\sqrt{b^2}}-\frac{ib\sqrt{a^2+1}\ \ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\ \sqrt{b^2x^2+2abx}}{x}\right.}{(i-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] I*b/(I-a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+I*b^2/(I-a)^2*a*ln((b^2*x+a*b)/(b
^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*b/(I-a)^2*(a^2+1)^(1
/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I
/(I-a)/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+2*I/(I-a)*a*b/(a^2+1)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)+I/(I-a)*a^2*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)*a*b/(a^2+1)^(1/2)*ln((2*
a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I/(I-a)*b^2
/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/(I-a)*b^2/(a^2+1)*ln((b^2*x+a*b)
/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*b/(I-a)^2*((x-(I-
a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+b^2/(I-a)^2*ln((I*b+(x-(I-a)/b)*b^2)/(
b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2+1}}{x^2\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^2+bx^3-ix^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**2 + b*x**3 - I*x**2)
, x)
```

3.196 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2\left(a^2+1\right)x^2}+\frac{(1-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{3/2}}+\frac{(1-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+}}{2(-a+i)^2(a+i)x}$$

```
[Out] (1-2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*
b*x)^(1/2))/(I-a)^(5/2)/(I+a)^(3/2)-1/2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(
1/2)/(a^2+1)/x^2+1/2*(1-2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I
-a)^2/(I+a)/x
```

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2\left(a^2+1\right)x^2}+\frac{(1-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{3/2}}+\frac{(1-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+}}{2(-a+i)^2(a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((1 - (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)^2*
(I + a)*x) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*(1 + a^2)*x
^2) + ((1 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(5/2)*(I + a)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^3\sqrt{1+ia+ibx}}\,dx$$

$$= -\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} - \frac{((i+2a)b)\int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx}{2\left(1+a^2\right)}$$

$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{\left((i+2a)b^2\right)\int \frac{}{x\sqrt{1\ldots}}}{2(i-a)\ldots}$$

$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{\left((i+2a)b^2\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{(1-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{(1-2ia)b^2\tanh^{-1}\ldots}{(i-a)^{5/2}(\ldots}$$

Mathematica [A] time = 0.15, size = 154, normalized size = 0.77

$$\frac{\frac{i\left(a^2-abx-2ibx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{x^2} + \frac{2(2a+i)b^2\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}\,\sqrt{-1+ia}}}{2(a-i)^2(a+i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((I*(1 + a^2 - (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 +
(2*(I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1
 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2*(-I +
a)^2*(I + a))
```

fricas [B] time = 0.52, size = 455, normalized size = 2.26

$$(-i\,a+2)b^2x^2 + \sqrt{\frac{\left(4\,a^2+4i\,a-1\right)b^4}{a^8-2i\,a^7+2\,a^6-6i\,a^5-6i\,a^3-2\,a^2-2i\,a-1}}\left(a^3-i\,a^2+a-i\right)x^2\log\left(-\frac{(2\,a+i)b^3x-\sqrt{b^2x^2+2\,abx+a^2+1}\,(2\,a+i)b^2+\left(a^5-i\,\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((-I*a + 2)*b^2*x^2 + sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 -
 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2
```

```
*a + I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 + (a^5 - I*
a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7
+ 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) - sqrt(
(4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2
- 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2*a + I)*b^3*x - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 - (a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a -
 I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3
 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*((-I*a + 2)*b*x + I*a^2 + I))/((2*a^3 - 2*I*a^2 + 2*a - 2*I)*x^2)
```

giac [B] time = 0.26, size = 481, normalized size = 2.39

$$-\frac{\left(2\,ab^2+b^2i\right)\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\left(a^3-a^2i+a-i\right)\sqrt{a^2+1}}-\frac{4\left(x|b|-\sqrt{(bx+a)^2+1}\right)a^4b^2i+2\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2a^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^2 + b^2*i)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqr
t(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/((
a^3 - a^2*i + a - i)*sqrt(a^2 + 1)) - (4*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^4*b^2*i + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*i*abs(b) + 2*a^5*
b*i*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 - 2*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a^3*b^2 + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*b^2*i
+ 5*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*b^2*i + 2*(x*abs(b) - sqrt((b*x
+ a)^2 + 1))^2*a^2*b*abs(b) - 2*a^4*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)
^2 + 1))^2*a*b*i*abs(b) + 4*a^3*b*i*abs(b) - 2*(x*abs(b) - sqrt((b*x + a)^2
 + 1))*a*b^2 + (x*abs(b) - sqrt((b*x + a)^2 + 1))*b^2*i + 2*(x*abs(b) - sqr
t((b*x + a)^2 + 1))^2*b*abs(b) - 4*a^2*b*abs(b) + 2*a*b*i*abs(b) - 2*b*abs(
b))/((a^3 - a^2*i + a - i)*((x*abs(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)
^2)
```

maple [B] time = 0.18, size = 1146, normalized size = 5.70

$$-\frac{ib^2a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(i-a)^2\sqrt{a^2+1}}+\frac{ib^3a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2(i-a)\left(a^2+1\right)\sqrt{b^2}}-\frac{ia^2b^2\sqrt{b^2x^2+2abx+\cdots}}{(i-a)\left(a^2+1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -I*b^2/(I-a)^2*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2))/x)+1/2*I/(I-a)*b^3/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)*a^2*b^2/(a^2+1)^2*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I/(I-a)*a^2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2
*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^3/(I-a)^3*a*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*b/(I-
a)^2/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-I*b^2/(I-a)^3*((x-(I-a)/b)^2*b
^2+2*I*b*(x-(I-a)/b))^(1/2)+I*b^3/(I-a)^2*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^
(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+2*I*b^2/(I-a)^2*a/(a^2+1)*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/(I-a)*b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*a
*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b^3/(I-a)^3*ln((I*b+
(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(
b^2)^(1/2)-I*b^2/(I-a)^3*(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*
```

```
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I*b^3/(I-a)^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)*x-1/2*I/(I-a)/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I/(I
-a)*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/2*I/(I-a)*a*b^3/(a^2+
1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+
1/2*I/(I-a)*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I/(I-a)*a*b/(a^2+
1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+I*b^2/(I-a)^3*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+I*b^3/(I-a)^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-a)*a^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a+b\,x)^2+1}}{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^3+bx^4-ix^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**3 + b*x**4 - I*x**3)
, x)
```

3.197 $\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\left(2a+i\left(1-2a^2\right)\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}(a+i)^{5/2}}+\frac{\left(-2a^2-9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1+ia)\left(a^2+1\right)^2x}-\frac{\sqrt{-ia-ibx}}{3(}$$

```
[Out] (2*a+I*(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/
(1-I*a-I*b*x)^(1/2))/(I-a)^(7/2)/(I+a)^(5/2)-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1+I*a)/x^3+1/6*(3-2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)
^(1/2)/(I-a)^2/(I+a)/x^2+1/6*(4-9*I*a-2*a^2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1+I*a)/(a^2+1)^2/x
```

Rubi [A] time = 0.18, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5095, 99, 151, 12, 93, 208}

$$\frac{\left(-2a^2-9ia+4\right)b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1+ia)\left(a^2+1\right)^2x}+\frac{\left(-2ia^2+2a+i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}(a+i)^{5/2}}+\frac{(3-2ia)b\sqrt{}}{6(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(3*(1 + I*a)*x^3) + ((3 - (2
*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(I - a)^2*(I + a)*
x^2) + ((4 - (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*
x])/(6*(1 + I*a)*(1 + a^2)^2*x) + ((I + 2*a - (2*I)*a^2)*b^3*ArcTanh[(Sqrt[
I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a
)^(7/2)*(I + a)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1-ia-ibx}}{x^4\sqrt{1+ia+ibx}}\,dx$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{\int \frac{-(3i+2a)b-2b^2x}{x^3\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{3(1+ia)}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} - \frac{\int \frac{\left(4-9ia-2a^2\right)b^2-(3i+}{x^2\sqrt{1-ia-ibx}\,\sqrt{1+i}}}{6(1+ia)\left(1+\right.}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2}{6(1+}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2}{6(1+}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2}{6(1+}$$

$$= -\frac{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{\left(4-9ia-2a^2\right)b^2}{6(1+}$$

Mathematica [A] time = 0.30, size = 247, normalized size = 0.87

$$\frac{(1-4ia)bx(a+bx+i)\sqrt{a^2+2abx+b^2x^2+1} + 2(1+ia)(a+i)(a+bx+i)\sqrt{a^2+2abx+b^2x^2+1} + \frac{3\left(2a^2+2ia-1\right)b}{}}{6\left(a^2+1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] (2*(1 + I*a)*(I + a)*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 -
 (4*I)*a)*b*x*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3*(-1 + (2
```

```
*I)*a + 2*a^2)*b^2*x^2*(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]*Sqrt[1 + a^2 + 2*a*b*
x + b^2*x^2] + (2*I)*b*x*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/
(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]))/((-1 - I*a)^(3/2)*Sqrt[-1 + I*a])
)/(6*(1 + a^2)^2*x^3)
```

fricas [B] time = 0.52, size = 700, normalized size = 2.47

$$\left(2i\,a^{2}-9\,a-4i\right)b^{3}x^{3}-\sqrt{\frac{\left(4\,a^{4}+8i\,a^{3}-8\,a^{2}-4i\,a+1\right)b^{6}}{a^{12}-2i\,a^{11}+4\,a^{10}-10i\,a^{9}+5\,a^{8}-20i\,a^{7}-20i\,a^{5}-5\,a^{4}-10i\,a^{3}-4\,a^{2}-2i\,a-1}}\left(3\,a^{5}-3i\,a^{4}+6\,a^{3}-6i\,a^{2}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((2*I*a^2 - 9*a - 4*I)*b^3*x^3 - sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)
*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5
*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(3*a^5 - 3*I*a^4 + 6*a^3 - 6*I*a^2 +
3*a - 3*I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 + (a^7 - I*a^6 + 3*a^5 - 3*I*a^4 + 3*a^3 -
3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I
*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3
- 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + sqrt((4*a^4 + 8*I*a^3 -
 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I
*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(3*a^5 - 3*I*a^4 +
 6*a^3 - 6*I*a^2 + 3*a - 3*I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 - (a^7 - I*a^6 + 3*a^5 -
 3*I*a^4 + 3*a^3 - 3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a +
 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5
- 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + ((2*
I*a^2 - 9*a - 4*I)*b^2*x^2 + 2*I*a^4 + (-2*I*a^3 + 3*a^2 - 2*I*a + 3)*b*x +
 4*I*a^2 + 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 - 6*I*a^4 + 12*a
^3 - 12*I*a^2 + 6*a - 6*I)*x^3)
```

giac [B] time = 0.23, size = 906, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^3 + 2*a*b^3*i - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 +
 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^
2 + 1)))/((a^5 - a^4*i + 2*a^3 - 2*a^2*i + a - i)*sqrt(a^2 + 1)) - 1/3*(8*(
x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3*i + 24*(x*abs(b) - sqrt((b*x +
a)^2 + 1))*a^7*b^3*i + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^6*b^2*i*ab
s(b) + 8*a^8*b^2*i*abs(b) - 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a^2*b^3
+ 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 - 18*(x*abs(b) - sqrt((b*
x + a)^2 + 1))*a^6*b^3 - 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a*b^3*i + 3
2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3*i + 54*(x*abs(b) - sqrt((b*x
 + a)^2 + 1))*a^5*b^3*i + 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^5*b^2*a
bs(b) - 12*a^7*b^2*abs(b) + 60*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^4*b^2
*i*abs(b) + 20*a^6*b^2*i*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*b^
3 + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 - 39*(x*abs(b) - sqrt((
b*x + a)^2 + 1))*a^4*b^3 + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^3*i
+ 36*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^3*b^3*i + 24*(x*abs(b) - sqrt((b*
x + a)^2 + 1))^2*a^3*b^2*abs(b) - 36*a^5*b^2*abs(b) + 48*(x*abs(b) - sqrt((
b*x + a)^2 + 1))^2*a^2*b^2*i*abs(b) + 12*a^4*b^2*i*abs(b) - 24*(x*abs(b) -
sqrt((b*x + a)^2 + 1))*a^2*b^3 + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b^3
```

```
*i + 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b^2*abs(b) - 36*a^3*b^2*abs(
b) + 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b^2*i*abs(b) - 4*a^2*b^2*i*abs
(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*b^3 - 12*a*b^2*abs(b) - 4*b^2*i*
abs(b))/((a^5 - a^4*i + 2*a^3 - 2*a^2*i + a - i)*((x*abs(b) - sqrt((b*x + a
)^2 + 1))^2 - a^2 - 1)^3)
```

maple [B] time = 0.20, size = 1738, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*I*b/(I-a)^2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+I*b^4/(I-a)^3/(a
^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I*b^4/(I-a)^3/(a^2+1)*ln((b^2*x+a*b)/
(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*b^2/(I-a)^3/(a^2+1
)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+2*I*b^3/(I-a)^3*a/(a^2+1)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)-I*b^3/(I-a)^3*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I*b^3/(I-a)^2*a^2/(a^2+1)^2*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/2*I*b^3/(I-a)^2*a^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*
b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I/(I-a)*a^3*b^3/(a^2+
1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*I/(I-a)*a^3*b^3/(a^2+1)^(5/2)*ln((2*
a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*I/(I-a)
*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*I/(I-a)*a*b^3/(a^2+1)^(3
/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+I
*b^4/(I-a)^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-1/2*I*b^3/(I-a)^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-I*b^3/(I-a)^4*(a^2+1)^(1/2)*ln((2*a^2+
2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/3*I/(I-a)/(a^
2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+I*b^3/(I-a)^4*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+b^4/(I-a)^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-I*b^3/(I-a)^4*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2)+1/2*I*b^3/(I-a)^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)+1/2*I*b^4/(I-a)^2/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+
a^2+1)^(1/2))/(b^2)^(1/2)+I*b^4/(I-a)^3*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1
/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)*a*b/(a^2+1)^2/x^
2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I/(I-a)*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*
a*b*x+a^2+1)^(3/2)+1/2*I/(I-a)*a^4*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)
+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)*a^2*b^4/(a^2+1)^3*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*I/(I-a)*a^2*b^4/(a^2+1)^3*ln((b^2*x+a*b)
/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-a)*a^2*b^4
/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)+1/2*I*b^2/(I-a)^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*I*b
^4/(I-a)^2*a^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)-1/2*I*b^4/(I-a)^2*a/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)*x-1/2*I*b^4/(I-a)^2*a/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a+b\,x)^2+1}}{x^4\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)),x)
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^4+bx^5-ix^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**4,x)
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**4 + b*x**5 - I*x**4)
, x)
```

3.198 $\int e^{-2i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=99

$$-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{2(1+ia)^3x}{b^4}-\frac{i(-a+i)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[Out] -2*(1+I*a)^3*x/b^4-I*(I-a)^2*x^2/b^3+2/3*(1+I*a)*x^3/b^2-1/2*I*x^4/b-1/5*x^
5-2*I*(I-a)^4*ln(I-a-b*x)/b^5
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5095, 77}

$$\frac{2(1+ia)x^3}{3b^2}-\frac{i(-a+i)^2x^2}{b^3}-\frac{2(1+ia)^3x}{b^4}-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(I - a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
- ((I/2)*x^4)/b - x^5/5 - ((2*I)*(I - a)^4*Log[I - a - b*x])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x^4\,dx &= \int \frac{x^4(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(\frac{2(-1-ia)^3}{b^4}-\frac{2i(-i+a)^2x}{b^3}+\frac{2(1+ia)x^2}{b^2}-\frac{2ix^3}{b}-x^4-\frac{2i(-i+a)^4}{b^4(-i+a+bx)}\right)dx\\ &= -\frac{2(1+ia)^3x}{b^4}-\frac{i(i-a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(i-a)^4\log(i-a-bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.96

$$-\frac{2i(a-i)^4\log(-a-bx+i)}{b^5}-\frac{2(1+ia)^3x}{b^4}-\frac{i(a-i)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(-I + a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
 - ((I/2)*x^4)/b - x^5/5 - ((2*I)*(-I + a)^4*Log[I - a - b*x])/b^5
```

fricas [A] time = 0.46, size = 105, normalized size = 1.06

$$-\frac{6\,b^5x^5+15i\,b^4x^4+20\,(-i\,a-1)b^3x^3-\left(-30i\,a^2-60\,a+30i\right)b^2x^2-\left(60i\,a^3+180\,a^2-180i\,a-60\right)bx-\left(-60\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 15*I*b^4*x^4 + 20*(-I*a - 1)*b^3*x^3 - (-30*I*a^2 - 60*a
 + 30*I)*b^2*x^2 - (60*I*a^3 + 180*a^2 - 180*I*a - 60)*b*x - (-60*I*a^4 - 2
40*a^3 + 360*I*a^2 + 240*a - 60*I)*log((b*x + a - I)/b))/b^5
```

giac [B] time = 0.12, size = 235, normalized size = 2.37

$$\frac{2\left(a^4i+4\,a^3-6\,a^2i-4\,a+i\right)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b^5}+\frac{(bix+ai+1)^5\left(\frac{15\,(2\,ab-3\,bi)i}{(bix+ai+1)b}-\frac{20\left(3\,a^2b^2-10\,ab^2i-7\,b^2\right)i^2}{(bix+ai+1)^2b^2}+\frac{60\left(a^3b^3-\right.}{(}\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 2*(a^4*i + 4*a^3 - 6*a^2*i - 4*a + i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))
/b^5 + 1/30*(b*i*x + a*i + 1)^5*(15*(2*a*b - 3*b*i)*i/((b*i*x + a*i + 1)*b)
 - 20*(3*a^2*b^2 - 10*a*b^2*i - 7*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) + 60*(
a^3*b^3 - 6*a^2*b^3*i - 9*a*b^3 + 4*b^3*i)*i^3/((b*i*x + a*i + 1)^3*b^3) -
30*(a^4*b^4 - 12*a^3*b^4*i - 30*a^2*b^4 + 28*a*b^4*i + 9*b^4)*i^4/((b*i*x +
 a*i + 1)^4*b^4) - 6)/(b^5*i^5)
```

maple [B] time = 0.06, size = 292, normalized size = 2.95

$$-\frac{x^5}{5}+\frac{8i\arctan(bx+a)\,a}{b^5}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^4}{b^5}-\frac{ix^2a^2}{b^3}-\frac{6iax}{b^4}+\frac{2x^3}{3b^2}+\frac{2ix^3a}{3b^2}-\frac{2a\,x^2}{b^3}+\frac{6i\ln\left(b^2x^2+2ab\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/5*x^5+8*I/b^5*arctan(b*x+a)*a-I/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a^4-I/b^3*
x^2*a^2-6*I/b^4*a*x+2/3/b^2*x^3+2/3*I/b^2*x^3*a-2/b^3*a*x^2+6*I/b^5*ln(b^2*
x^2+2*a*b*x+a^2+1)*a^2+6/b^4*x*a^2-2/b^4*x+2/b^5*arctan(b*x+a)*a^4-I/b^5*ln
(b^2*x^2+2*a*b*x+a^2+1)-12/b^5*arctan(b*x+a)*a^2+I/b^3*x^2+2*I/b^4*a^3*x-4/
b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+2/b^5*arctan(b*x+a)-1/2*I*x^4/b-8*I/b^5*a
rctan(b*x+a)*a^3+4/b^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a
```

maxima [A] time = 0.32, size = 102, normalized size = 1.03

$$-\frac{6\,b^4x^5+15i\,b^3x^4-20\,(i\,a+1)b^2x^3+\left(30i\,a^2+60\,a-30i\right)bx^2+\left(-60i\,a^3-180\,a^2+180i\,a+60\right)x}{30\,b^4}+\frac{\left(-2i\,a^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 + 15*I*b^3*x^4 - 20*(I*a + 1)*b^2*x^3 + (30*I*a^2 + 60*a -
 30*I)*b*x^2 + (-60*I*a^3 - 180*a^2 + 180*I*a + 60)*x)/b^4 + (-2*I*a^4 - 8*
a^3 + 12*I*a^2 + 8*a - 2*I)*log(I*b*x + I*a + 1)/b^5
```

mupad [B] time = 0.17, size = 165, normalized size = 1.67

$$\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}-\frac{\left(2\,a^4-12\,a^2+2\right)\,1\mathrm{i}}{b^5}\right)+x^4\,\left(\frac{a-\mathrm{i}}{4\,b}-\frac{a+1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5}+\frac{x^2\,\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,{\left(a-\mathrm{i}\right)}^2}{2\,b^2}-\frac{x^3\,\left(\frac{a-\mathrm{i}}{b}-\frac{a}{}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] log(x + (a - 1i)/b)*((8*a - 8*a^3)/b^5 - ((2*a^4 - 12*a^2 + 2)*1i)/b^5) + x
^4*((a - 1i)/(4*b) - (a + 1i)/(4*b)) - x^5/5 + (x^2*((a - 1i)/b - (a + 1i)/
b)*(a - 1i)^2)/(2*b^2) - (x^3*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/(3*b) - (
x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^3)/b^3
```

sympy [A] time = 0.52, size = 117, normalized size = 1.18

$$-\frac{x^5}{5}-x^3\left(-\frac{2ia}{3b^2}-\frac{2}{3b^2}\right)-x^2\left(\frac{ia^2}{b^3}+\frac{2a}{b^3}-\frac{i}{b^3}\right)-x\left(-\frac{2ia^3}{b^4}-\frac{6a^2}{b^4}+\frac{6ia}{b^4}+\frac{2}{b^4}\right)-\frac{ix^4}{2b}-\frac{2i\left(a-i\right)^4\log\left(ia+ibx+1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**5/5 - x**3*(-2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(I*a**2/b**3 + 2*a/b**
3 - I/b**3) - x*(-2*I*a**3/b**4 - 6*a**2/b**4 + 6*I*a/b**4 + 2/b**4) - I*x*
*4/(2*b) - 2*I*(a - I)**4*log(I*a + I*b*x + 1)/b**5
```

3.199 $\int e^{-2i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=77

$$-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2i(-a+i)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[Out] -2*I*(I-a)^2*x/b^3+(1+I*a)*x^2/b^2-2/3*I*x^3/b-1/4*x^4-2*(1+I*a)^3*ln(I-a-b
*x)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{(1+ia)x^2}{b^2}-\frac{2i(-a+i)^2x}{b^3}-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-2i\tan^{-1}(a+bx)}x^3\,dx &= \int \frac{x^3(1-ia-ibx)}{1+ia+ibx}\,dx\\
&= \int \left(-\frac{2i(-i+a)^2}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{2ix^2}{b}-x^3+\frac{2(-1-ia)^3}{b^3(-i+a+bx)}\right)dx\\
&= -\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.00

$$-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2i(-a+i)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

fricas [A] time = 0.44, size = 77, normalized size = 1.00

$$-\frac{3\,b^4x^4+8i\,b^3x^3+12\,(-i\,a-1)b^2x^2-\left(-24i\,a^2-48\,a+24i\right)bx-\left(24i\,a^3+72\,a^2-72i\,a-24\right)\log\left(\frac{bx+a-i}{b}\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 + 8*I*b^3*x^3 + 12*(-I*a - 1)*b^2*x^2 - (-24*I*a^2 - 48*a
+ 24*I)*b*x - (24*I*a^3 + 72*a^2 - 72*I*a - 24)*log((b*x + a - I)/b))/b^4
```

giac [B] time = 0.13, size = 173, normalized size = 2.25

$$-\frac{2\left(a^3i+3\,a^2-3\,ai-1\right)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b^4}+\frac{(bix+ai+1)^4\left(\frac{4\,(3\,ab-5\,bi)i}{(bix+ai+1)b}-\frac{18\left(a^2b^2-4\,ab^2i-3\,b^2\right)i^2}{(bix+ai+1)^2b^2}+\frac{12\left(a^3b^3-9\,a^2b^3i-15\,ab^3\right.}{(bix+ai+1)^3b^3}\right.}{12\,b^4i^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -2*(a^3*i + 3*a^2 - 3*a*i - 1)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^4 +
1/12*(b*i*x + a*i + 1)^4*(4*(3*a*b - 5*b*i)*i/((b*i*x + a*i + 1)*b) - 18*(a
^2*b^2 - 4*a*b^2*i - 3*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) + 12*(a^3*b^3 - 9
*a^2*b^3*i - 15*a*b^3 + 7*b^3*i)*i^3/((b*i*x + a*i + 1)^3*b^3) - 3)/(b^4*i^
4)
```

maple [B] time = 0.06, size = 211, normalized size = 2.74

$$-\frac{x^4}{4}-\frac{2ix^3}{3b}+\frac{ix^2a}{b^2}-\frac{2ia^2x}{b^3}+\frac{x^2}{b^2}+\frac{2ix}{b^3}-\frac{4ax}{b^3}-\frac{2\arctan(bx+a)\,a^3}{b^4}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{b^4}+\frac{6\arctan(bx+a)\,a}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/4*x^4-2/3*I*x^3/b+I/b^2*x^2*a-2*I/b^3*a^2*x+1/b^2*x^2+2*I/b^3*x-4*a*x/b^
3-2/b^4*arctan(b*x+a)*a^3+I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^3+6/b^4*arctan(
b*x+a)*a+6*I/b^4*arctan(b*x+a)*a^2-3*I/b^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+3/b^
4*ln(b^2*x^2+2*a*b*x+a^2+1)*a^2-2*I/b^4*arctan(b*x+a)-1/b^4*ln(b^2*x^2+2*a*
b*x+a^2+1)
```

maxima [A] time = 0.32, size = 74, normalized size = 0.96

$$-\frac{i\left(-3i\,b^3x^4+8\,b^2x^3-(12\,a-12i)bx^2+24\left(a^2-2i\,a-1\right)x\right)}{12\,b^3}+\frac{\left(2i\,a^3+6\,a^2-6i\,a-2\right)\log\left(i\,bx+i\,a+1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/12*I*(-3*I*b^3*x^4 + 8*b^2*x^3 - (12*a - 12*I)*b*x^2 + 24*(a^2 - 2*I*a -
 1)*x)/b^3 + (2*I*a^3 + 6*a^2 - 6*I*a - 2)*log(I*b*x + I*a + 1)/b^4
```

mupad [B] time = 0.54, size = 129, normalized size = 1.68

$$x^3\left(\frac{a-\mathrm{i}}{3\,b}-\frac{a+1\mathrm{i}}{3\,b}\right)-\frac{x^4}{4}-\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(-\frac{6\,a^2-2}{b^4}+\frac{\left(6\,a-2\,a^3\right)\,1\mathrm{i}}{b^4}\right)-\frac{x^2\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})}{2\,b}+\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] x^3*((a - 1i)/(3*b) - (a + 1i)/(3*b)) - x^4/4 - log(x + (a - 1i)/b)*(((6*a
- 2*a^3)*1i)/b^4 - (6*a^2 - 2)/b^4) - (x^2*((a - 1i)/b - (a + 1i)/b)*(a - 1
i))/(2*b) + (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^2)/b^2
```

sympy [A] time = 0.49, size = 80, normalized size = 1.04

$$-\frac{x^4}{4} - x^2\left(-\frac{ia}{b^2} - \frac{1}{b^2}\right) - x\left(\frac{2ia^2}{b^3} + \frac{4a}{b^3} - \frac{2i}{b^3}\right) - \frac{2ix^3}{3b} + \frac{2i(a-i)^3\log(ia + ibx + 1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**4/4 - x**2*(-I*a/b**2 - 1/b**2) - x*(2*I*a**2/b**3 + 4*a/b**3 - 2*I/b**
3) - 2*I*x**3/(3*b) + 2*I*(a - I)**3*log(I*a + I*b*x + 1)/b**4
```

3.200 $\int e^{-2i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=59

$$-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[Out] 2*(1+I*a)*x/b^2-I*x^2/b-1/3*x^3-2*I*(I-a)^2*ln(I-a-b*x)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5095, 77}

$$\frac{2(1+ia)x}{b^2}-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}-\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (2*(1 + I*a)*x)/b^2 - (I*x^2)/b - x^3/3 - ((2*I)*(I - a)^2*Log[I - a - b*x]
)/b^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x^2\,dx &= \int \frac{x^2(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(\frac{2i(-i+a)}{b^2}-\frac{2ix}{b}-x^2-\frac{2i(-i+a)^2}{b^2(-i+a+bx)}\right)dx\\ &= \frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{2i(i-a)^2\log(i-a-bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.93

$$\frac{bx\left(6ia-b^2x^2-3ibx+6\right)-6i(a-i)^2\log(-a-bx+i)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (b*x*(6 + (6*I)*a - (3*I)*b*x - b^2*x^2) - (6*I)*(-I + a)^2*Log[I - a - b*x
])/(3*b^3)
```

fricas [A] time = 0.53, size = 53, normalized size = 0.90

$$-\frac{b^3x^3 + 3i\,b^2x^2 + 6\,(-i\,a - 1)bx - \left(-6i\,a^2 - 12\,a + 6i\right)\log\left(\frac{bx+a-i}{b}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + 3*I*b^2*x^2 + 6*(-I*a - 1)*b*x - (-6*I*a^2 - 12*a + 6*I)*lo
g((b*x + a - I)/b))/b^3
```

giac [B] time = 0.14, size = 120, normalized size = 2.03

$$\frac{2\left(a^2i + 2\,a - i\right)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b^3} + \frac{(bix + ai + 1)^3\left(\frac{3\,(ab-2\,bi)i}{(bix+ai+1)b} - \frac{3\left(a^2b^2-6\,ab^2i-5\,b^2\right)i^2}{(bix+ai+1)^2b^2} - 1\right)}{3\,b^3i^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 2*(a^2*i + 2*a - i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^3 + 1/3*(b*i*x
+ a*i + 1)^3*(3*(a*b - 2*b*i)*i/((b*i*x + a*i + 1)*b) - 3*(a^2*b^2 - 6*a*b^
2*i - 5*b^2)*i^2/((b*i*x + a*i + 1)^2*b^2) - 1)/(b^3*i^3)
```

maple [B] time = 0.06, size = 143, normalized size = 2.42

$$-\frac{x^3}{3}-\frac{ix^2}{b}+\frac{2iax}{b^2}+\frac{2x}{b^2}+\frac{2\arctan(bx+a)\,a^2}{b^3}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^3}-\frac{2\arctan(bx+a)}{b^3}-\frac{4i\arctan(bx+}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/3*x^3-I*x^2/b+2*I/b^2*a*x+2*x/b^2+2/b^3*arctan(b*x+a)*a^2-I/b^3*ln(b^2*x
^2+2*a*b*x+a^2+1)*a^2-2/b^3*arctan(b*x+a)-4*I/b^3*arctan(b*x+a)*a+I/b^3*ln(
b^2*x^2+2*a*b*x+a^2+1)-2/b^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a
```

maxima [A] time = 0.32, size = 52, normalized size = 0.88

$$-\frac{b^2x^3 + 3i\,bx^2 + 6\,(-i\,a - 1)x}{3\,b^2} + \frac{\left(-2i\,a^2 - 4\,a + 2i\right)\log\left(i\,bx + i\,a + 1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 + 3*I*b*x^2 + 6*(-I*a - 1)*x)/b^2 + (-2*I*a^2 - 4*a + 2*I)*lo
g(I*b*x + I*a + 1)/b^3
```

mupad [B] time = 0.54, size = 90, normalized size = 1.53

$$-\ln\left(x + \frac{a - \mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3} + \frac{\left(2\,a^2 - 2\right)\,1\mathrm{i}}{b^3}\right) + x^2\left(\frac{a - \mathrm{i}}{2\,b} - \frac{a + 1\mathrm{i}}{2\,b}\right) - \frac{x^3}{3} - \frac{x\left(\frac{a-\mathrm{i}}{b} - \frac{a+1\mathrm{i}}{b}\right)(a - \mathrm{i})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] x^2*((a - 1i)/(2*b) - (a + 1i)/(2*b)) - log(x + (a - 1i)/b)*((4*a)/b^3 + ((
2*a^2 - 2)*1i)/b^3) - x^3/3 - (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/b
```

sympy [A] time = 0.36, size = 53, normalized size = 0.90

$$-\frac{x^3}{3} - x\left(-\frac{2ia}{b^2} - \frac{2}{b^2}\right) - \frac{ix^2}{b} - \frac{2i\,(a-i)^2\log\left(ia + ibx + 1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**3/3 - x*(-2*I*a/b**2 - 2/b**2) - I*x**2/b - 2*I*(a - I)**2*log(I*a + I*
b*x + 1)/b**3
```

3.201 $\int e^{-2i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=40

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

```
[Out] -2*I*x/b-1/2*x^2+2*(1+I*a)*ln(I-a-b*x)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 77}

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}x\,dx &= \int \frac{x(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(-\frac{2i}{b}-x+\frac{2(1+ia)}{b(-i+a+bx)}\right)dx\\ &= -\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

fricas [A] time = 0.44, size = 35, normalized size = 0.88

$$-\frac{b^2x^2 + 4i\,bx + 4\,(-i\,a - 1)\log\left(\frac{bx+a-i}{b}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 4*I*b*x + 4*(-I*a - 1)*log((b*x + a - I)/b))/b^2
```

giac [B] time = 0.14, size = 80, normalized size = 2.00

$$-\frac{i\left(\frac{4\,(a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b} - \frac{(bix+ai+1)^2\left(i-\frac{2\,(abi+3\,b)i}{(bix+ai+1)b}\right)}{bi^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*i*(4*(a - i)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b - (b*i*x + a*i +
1)^2*(i - 2*(a*b*i + 3*b)*i/((b*i*x + a*i + 1)*b))/(b*i^2))/b
```

maple [B] time = 0.06, size = 85, normalized size = 2.12

$$-\frac{x^2}{2}-\frac{2ix}{b}-\frac{2\arctan\left(bx+a\right)a}{b^2}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a}{b^2}+\frac{2i\arctan\left(bx+a\right)}{b^2}+\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

```
[Out] -1/2*x^2-2*I*x/b-2/b^2*arctan(b*x+a)*a+I/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)*a+2*
I/b^2*arctan(b*x+a)+1/b^2*ln(b^2*x^2+2*a*b*x+a^2+1)
```

maxima [A] time = 0.32, size = 36, normalized size = 0.90

$$\frac{i\left(i\,bx^2 - 4\,x\right)}{2\,b} - \frac{2\,(-i\,a - 1)\log\left(i\,bx + i\,a + 1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*I*(I*b*x^2 - 4*x)/b - 2*(-I*a - 1)*log(I*b*x + I*a + 1)/b^2
```

mupad [B] time = 0.50, size = 51, normalized size = 1.28

$$\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right)-\frac{x^2}{2}+x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] log(x + (a - 1i)/b)*((a*2i)/b^2 + 2/b^2) - x^2/2 + x*((a - 1i)/b - (a + 1i)
/b)
```

sympy [A] time = 0.25, size = 32, normalized size = 0.80

$$-\frac{x^2}{2} - \frac{2ix}{b} + \frac{2i(a-i)\log(ia+ibx+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**2/2 - 2*I*x/b + 2*I*(a - I)*log(I*a + I*b*x + 1)/b**2
```

3.202 $\int e^{-2i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=23

$$-x-\frac{2i\log(-a-bx+i)}{b}$$

```
[Out] -x-2*I*ln(I-a-b*x)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5093, 43}

$$-x-\frac{2i\log(-a-bx+i)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a - b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2i\tan^{-1}(a+bx)}\,dx&=\int\frac{1-ia-ibx}{1+ia+ibx}\,dx\\&=\int\left(-1-\frac{2i}{-i+a+bx}\right)dx\\&=-x-\frac{2i\log(i-a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.39

$$-\frac{i\log\left((a+bx)^2+1\right)}{b}+\frac{2\tan^{-1}(a+bx)}{b}-x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + (2*ArcTan[a + b*x])/b - (I*Log[1 + (a + b*x)^2])/b
```

fricas [A] time = 0.51, size = 22, normalized size = 0.96

$$-\frac{bx+2i\log\left(\frac{bx+a-i}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -(b*x + 2*I*log((b*x + a - I)/b))/b
```

giac [A] time = 0.13, size = 38, normalized size = 1.65

$$\frac{(bix+ai+1)i}{b}+\frac{2\,i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
[Out] (b*i*x + a*i + 1)*i/b + 2*i*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b
```

maple [A] time = 0.05, size = 40, normalized size = 1.74

$$-x-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{2\arctan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
[Out] -x-I/b*ln(b^2*x^2+2*a*b*x+a^2+1)+2/b*arctan(b*x+a)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.83

$$-x-\frac{2i\,\log\left(i\,bx+i\,a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
[Out] -x - 2*I*log(I*b*x + I*a + 1)/b
```

mupad [B] time = 0.06, size = 21, normalized size = 0.91

$$-x-\frac{\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)/(a*1i + b*x*1i + 1)^2,x)
[Out] - x - (log(x + (a - 1i)/b)*2i)/b
```

sympy [A] time = 0.22, size = 19, normalized size = 0.83

$$-x-\frac{2i\log\left(ia+ibx+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
[Out] -x - 2*I*log(I*a + I*b*x + 1)/b
```

3.203 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=41

$$\frac{(a+i)\log(x)}{-a+i}-\frac{2\log(-a-bx+i)}{1+ia}$$

```
[Out] (I+a)*ln(x)/(I-a)-2*ln(I-a-b*x)/(1+I*a)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 72}

$$\frac{(a+i)\log(x)}{-a+i}-\frac{2\log(-a-bx+i)}{1+ia}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
[Out] ((I + a)*Log[x])/(I - a) - (2*Log[I - a - b*x])/(1 + I*a)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{1-ia-ibx}{x(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x}+\frac{2ib}{(-i+a)(-i+a+bx)}\right)dx\\ &= \frac{(i+a)\log(x)}{i-a}-\frac{2\log(i-a-bx)}{1+ia}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.83

$$\frac{2i\log(-a-bx+i)-(a+i)\log(x)}{a-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
[Out] (-((I + a)*Log[x]) + (2*I)*Log[I - a - b*x])/(-I + a)
```

fricas [A] time = 0.44, size = 27, normalized size = 0.66

$$-\frac{(a+i)\log(x)-2i\log\left(\frac{bx+a-i}{b}\right)}{a-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="fricas")
[Out] -((a + I)*log(x) - 2*I*log((b*x + a - I)/b))/(a - I)
```

giac [B] time = 0.14, size = 77, normalized size = 1.88

$$bi\left(\frac{(ai-1)\log\left(-\frac{ai^2}{bix+ai+1}+i-\frac{i}{bix+ai+1}\right)}{ab-bi}-\frac{i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}\,|b|}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="giac")
[Out] b*i*((a*i - 1)*log(-a*i^2/(b*i*x + a*i + 1) + i - i/(b*i*x + a*i + 1))/(a*b
 - b*i) - i*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b)
```

maple [A] time = 0.06, size = 74, normalized size = 1.80

$$-\frac{\ln(x)a^2}{(i-a)^2}-\frac{\ln(x)}{(i-a)^2}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{i-a}+\frac{2\arctan(bx+a)}{i-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x)
[Out] -1/(I-a)^2*ln(x)*a^2-1/(I-a)^2*ln(x)-I/(I-a)*ln(b^2*x^2+2*a*b*x+a^2+1)+2/(I
-a)*arctan(b*x+a)
```

maxima [A] time = 0.33, size = 47, normalized size = 1.15

$$-\frac{2(-ia-1)\log(ibx+ia+1)}{a^2-2ia-1}-\frac{\left(a^2+1\right)\log(x)}{a^2-2ia-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="maxima")
[Out] -2*(-I*a - 1)*log(I*b*x + I*a + 1)/(a^2 - 2*I*a - 1) - (a^2 + 1)*log(x)/(a^
2 - 2*I*a - 1)
```

mupad [B] time = 0.72, size = 34, normalized size = 0.83

$$-\frac{2\ln(a+bx-\mathrm{i})}{1+a\,1\mathrm{i}}+\ln(x)\left(\frac{2}{1+a\,1\mathrm{i}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)/(x*(a*1i + b*x*1i + 1)^2),x)
[Out] log(x)*(2/(a*1i + 1) - 1) - (2*log(a + b*x - 1i))/(a*1i + 1)
```

sympy [B] time = 0.97, size = 102, normalized size = 2.49

$$-\frac{(a+i)\log\left(ia^2-\frac{ia^2(a+i)}{a-i}-\frac{2a(a+i)}{a-i}+x(iab-3b)+i+\frac{i(a+i)}{a-i}\right)}{a-i}+\frac{2i\log\left(ia^2-\frac{2a^2}{a-i}+\frac{4ia}{a-i}+x(iab-3b)+i+\frac{2}{a-i}\right)}{a-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x,x)
```

```
[Out] -(a + I)*log(I*a**2 - I*a**2*(a + I)/(a - I) - 2*a*(a + I)/(a - I) + x*(I*a
*b - 3*b) + I + I*(a + I)/(a - I))/(a - I) + 2*I*log(I*a**2 - 2*a**2/(a - I
) + 4*I*a/(a - I) + x*(I*a*b - 3*b) + I + 2/(a - I))/(a - I)
```

3.204 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=62

$$\frac{2ib\log(x)}{(-a+i)^2} - \frac{2ib\log(-a-bx+i)}{(-a+i)^2} - \frac{a+i}{(-a+i)x}$$

```
[Out] (-I-a)/(I-a)/x+2*I*b*ln(x)/(I-a)^2-2*I*b*ln(I-a-b*x)/(I-a)^2
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2ib\log(x)}{(-a+i)^2} - \frac{2ib\log(-a-bx+i)}{(-a+i)^2} - \frac{a+i}{(-a+i)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((I + a)/((I - a)*x)) + ((2*I)*b*Log[x])/(I - a)^2 - ((2*I)*b*Log[I - a -
b*x])/(I - a)^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^2} dx &= \int \frac{1-ia-ibx}{x^2(1+ia+ibx)} dx \\
&= \int \left(\frac{-i-a}{(-i+a)x^2} + \frac{2ib}{(-i+a)^2x} - \frac{2ib^2}{(-i+a)^2(-i+a+bx)}\right) dx \\
&= -\frac{i+a}{(i-a)x} + \frac{2ib\log(x)}{(i-a)^2} - \frac{2ib\log(i-a-bx)}{(i-a)^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.68

$$\frac{a^2 - 2ibx\log(-a-bx+i) + 2ibx\log(x) + 1}{(a-i)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] (1 + a^2 + (2*I)*b*x*Log[x] - (2*I)*b*x*Log[I - a - b*x])/((-I + a)^2*x)
```

fricas [A] time = 0.45, size = 40, normalized size = 0.65

$$\frac{2i\,bx\log(x) - 2i\,bx\log\left(\frac{bx+a-i}{b}\right) + a^2 + 1}{\left(a^2 - 2i\,a - 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*I*b*x*log(x) - 2*I*b*x*log((b*x + a - I)/b) + a^2 + 1)/((a^2 - 2*I*a - 1
)*x)
```

giac [B] time = 0.12, size = 106, normalized size = 1.71

$$-\frac{2\,b^2\log\left(-\frac{ai}{bix+ai+1} + \frac{i^2}{bix+ai+1} + 1\right)}{a^2bi + 2\,ab - bi} - \frac{ab + bi}{(a-i)^2\left(\frac{ai}{bix+ai+1} - \frac{i^2}{bix+ai+1} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^2*b*i + 2
*a*b - b*i) - (a*b + b*i)/((a - i)^2*(a*i/(b*i*x + a*i + 1) - i^2/(b*i*x +
a*i + 1) - 1))
```

maple [B] time = 0.06, size = 152, normalized size = 2.45

$$\frac{a^2}{x\,(i-a)^2} + \frac{1}{x\,(i-a)^2} - \frac{2ib\ln(x)a}{(i-a)^3} - \frac{2b\ln(x)}{(i-a)^3} + \frac{ib\ln\left(b^2x^2 + 2abx + a^2 + 1\right)a}{(i-a)^3} + \frac{b\ln\left(b^2x^2 + 2abx + a^2 + 1\right)}{(i-a)^3} - \frac{2b\,\mathrm{arcta}}{(i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x)
```

```
[Out] 1/x/(I-a)^2*a^2+1/x/(I-a)^2-2*I*b/(I-a)^3*ln(x)*a-2*b/(I-a)^3*ln(x)+I*b/(I-
a)^3*ln(b^2*x^2+2*a*b*x+a^2+1)*a+b/(I-a)^3*ln(b^2*x^2+2*a*b*x+a^2+1)-2*b/(I
-a)^3*arctan(b*x+a)*a+2*I*b/(I-a)^3*arctan(b*x+a)
```

maxima [B] time = 0.32, size = 113, normalized size = 1.82

$$-\frac{(2\,a - 2i)b\log(i\,bx + i\,a + 1)}{-i\,a^3 - 3\,a^2 + 3i\,a + 1} + \frac{(2\,a - 2i)b\log(x)}{-i\,a^3 - 3\,a^2 + 3i\,a + 1} + \frac{a^3 + \left(a^2 + 1\right)bx - i\,a^2 + a - i}{\left(a^2 - 2i\,a - 1\right)bx^2 + \left(a^3 - 3i\,a^2 - 3\,a + i\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*a - 2*I)*b*log(I*b*x + I*a + 1)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (2*a - 2
*I)*b*log(x)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (a^3 + (a^2 + 1)*b*x - I*a^2 +
a - I)/((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

mupad [B] time = 0.65, size = 100, normalized size = 1.61

$$\frac{-1 + a\,1\mathrm{i}}{x\,(1 + a\,1\mathrm{i})} - \frac{4\,b\,\mathrm{atan}\left(\frac{a^2\,1\mathrm{i}+2\,a-\mathrm{i}}{(a-\mathrm{i})^2} + \frac{x\left(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2\right)}{(a-\mathrm{i})^2\left(-1\mathrm{i}\,b\,a^3+b\,a^2-1\mathrm{i}\,b\,a+b\right)}\right)}{(a-\mathrm{i})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)/(x^2*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] (a*1i - 1)/(x*(a*1i + 1)) - (4*b*atan((2*a + a^2*1i - 1i)/(a - 1i)^2 + (x*(
2*b^2 + 4*a^2*b^2 + 2*a^4*b^2))/((a - 1i)^2*(b - a*b*1i + a^2*b - a^3*b*1i)
)))/(a - 1i)^2
```

sympy [B] time = 0.74, size = 156, normalized size = 2.52

$$\frac{2ib\log\left(-\frac{2a^3b}{(a-i)^2}+\frac{6ia^2b}{(a-i)^2}+2ab+\frac{6ab}{(a-i)^2}+4b^2x-2ib-\frac{2ib}{(a-i)^2}\right)}{(a-i)^2}-\frac{2ib\log\left(\frac{2a^3b}{(a-i)^2}-\frac{6ia^2b}{(a-i)^2}+2ab-\frac{6ab}{(a-i)^2}+4b^2x-2ib\right.}{(a-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**2,x)
```

```
[Out] 2*I*b*log(-2*a**3*b/(a - I)**2 + 6*I*a**2*b/(a - I)**2 + 2*a*b + 6*a*b/(a -
 I)**2 + 4*b**2*x - 2*I*b - 2*I*b/(a - I)**2)/(a - I)**2 - 2*I*b*log(2*a**3
*b/(a - I)**2 - 6*I*a**2*b/(a - I)**2 + 2*a*b - 6*a*b/(a - I)**2 + 4*b**2*x
 - 2*I*b + 2*I*b/(a - I)**2)/(a - I)**2 - (a + I)/(x*(-a + I))
```

3.205 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=83

$$-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(-a-bx+i)}{(1+ia)^3}-\frac{2ib}{(-a+i)^2x}+\frac{-a-i}{2(-a+i)x^2}$$

```
[Out] 1/2*(-I-a)/(I-a)/x^2-2*I*b/(I-a)^2/x-2*b^2*ln(x)/(1+I*a)^3+2*b^2*ln(I-a-b*x
)/(1+I*a)^3
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(-a-bx+i)}{(1+ia)^3}-\frac{2ib}{(-a+i)^2x}-\frac{a+i}{2(-a+i)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] -(I + a)/(2*(I - a)*x^2) - ((2*I)*b)/((I - a)^2*x) - (2*b^2*Log[x])/(1 + I*
a)^3 + (2*b^2*Log[I - a - b*x])/(1 + I*a)^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{1-ia-ibx}{x^3(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^3}+\frac{2ib}{(-i+a)^2x^2}-\frac{2ib^2}{(-i+a)^3x}+\frac{2ib^3}{(-i+a)^3(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{2(i-a)x^2}-\frac{2ib}{(i-a)^2x}-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(i-a-bx)}{(1+ia)^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.80

$$\frac{(a-i)\left(a^2-4ibx+1\right)+4ib^2x^2\log(-a-bx+i)-4ib^2x^2\log(x)}{2(a-i)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((-I + a)*(1 + a^2 - (4*I)*b*x) - (4*I)*b^2*x^2*Log[x] + (4*I)*b^2*x^2*Log[
I - a - b*x])/(2*(-I + a)^3*x^2)
```

fricas [A] time = 0.46, size = 70, normalized size = 0.84

$$\frac{-4i\,b^2x^2\log(x)+4i\,b^2x^2\log\left(\frac{bx+a-i}{b}\right)+a^3-4\,(i\,a+1)bx-i\,a^2+a-i}{\left(2\,a^3-6i\,a^2-6\,a+2i\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] (-4*I*b^2*x^2*log(x) + 4*I*b^2*x^2*log((b*x + a - I)/b) + a^3 - 4*(I*a + 1)
*b*x - I*a^2 + a - I)/((2*a^3 - 6*I*a^2 - 6*a + 2*I)*x^2)
```

giac [B] time = 0.14, size = 157, normalized size = 1.89

$$\frac{2\,b^3\log\left(-\frac{ai}{bix+ai+1}+\frac{i^2}{bix+ai+1}+1\right)}{a^3bi+3\,a^2b-3\,abi-b}-\frac{\frac{2\left(ab^3i-3\,b^3\right)i^2}{(bix+ai+1)b}+\frac{ab^2i-5\,b^2}{ai+1}}{2\,(a-i)^2\left(\frac{ai}{bix+ai+1}-\frac{i^2}{bix+ai+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^3*b*i + 3*
a^2*b - 3*a*b*i - b) - 1/2*(2*(a*b^3*i - 3*b^3)*i^2/((b*i*x + a*i + 1)*b) +
 (a*b^2*i - 5*b^2)/(a*i + 1))/((a - i)^2*(a*i/(b*i*x + a*i + 1) - i^2/(b*i*
x + a*i + 1) - 1)^2)
```

maple [B] time = 0.06, size = 246, normalized size = 2.96

$$-\frac{2ib^2\ln(x)a}{(i-a)^4}-\frac{2b^2\ln(x)}{(i-a)^4}-\frac{2ib\,a^2}{(i-a)^4\,x}+\frac{2ib}{(i-a)^4\,x}-\frac{4ba}{(i-a)^4\,x}-\frac{ia^3}{(i-a)^4\,x^2}+\frac{a^4}{2\,(i-a)^4\,x^2}-\frac{ia}{(i-a)^4\,x^2}-\frac{1}{2\,(i-a)^4\,x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x)
```

```
[Out] -2*I*b^2/(I-a)^4*ln(x)*a-2*b^2/(I-a)^4*ln(x)-2*I*b/(I-a)^4/x*a^2+2*I*b/(I-a
)^4/x-4*b/(I-a)^4/x*a-I/(I-a)^4/x^2*a^3+1/2/(I-a)^4/x^2*a^4-I/(I-a)^4/x^2*a
-1/2/(I-a)^4/x^2+I*b^2/(I-a)^4*ln(b^2*x^2+2*a*b*x+a^2+1)*a+b^2/(I-a)^4*ln(b
^2*x^2+2*a*b*x+a^2+1)-2*b^2/(I-a)^4*arctan(b*x+a)*a+2*I*b^2/(I-a)^4*arctan(
b*x+a)
```

maxima [B] time = 0.34, size = 163, normalized size = 1.96

$$-\frac{2\,(-i\,a-1)b^2\log\left(i\,bx+i\,a+1\right)}{a^4-4i\,a^3-6\,a^2+4i\,a+1}-\frac{2\,(i\,a+1)b^2\log(x)}{a^4-4i\,a^3-6\,a^2+4i\,a+1}+\frac{4\,(-i\,a-1)b^2x^2+a^4-2i\,a^3+\left(a^3-5i\,a^2-7\,a\right.}{\left(2\,a^3-6i\,a^2-6\,a+2i\right)bx^3+\left(2\,a^4-8i\,a^3-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*(-I*a - 1)*b^2*log(I*b*x + I*a + 1)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)
- 2*(I*a + 1)*b^2*log(x)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1) + (4*(-I*a - 1
)*b^2*x^2 + a^4 - 2*I*a^3 + (a^3 - 5*I*a^2 - 7*a + 3*I)*b*x - 2*I*a - 1)/((
2*a^3 - 6*I*a^2 - 6*a + 2*I)*b*x^3 + (2*a^4 - 8*I*a^3 - 12*a^2 + 8*I*a + 2)
*x^2)
```

mupad [B] time = 0.70, size = 156, normalized size = 1.88

$$\frac{\frac{a+1\mathrm{i}}{2\,(a-\mathrm{i})}-\frac{b\,x\,2\mathrm{i}}{(a-\mathrm{i})^2}}{x^2}-\frac{b^2\,\mathrm{atanh}\left(\frac{-a^3+a^2\,3\mathrm{i}+3\,a-\mathrm{i}}{(a-\mathrm{i})^3}-\frac{x\,\left(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2\right)}{(a-\mathrm{i})^3\,\left(b\,a^6+2\mathrm{i}\,b\,a^5+b\,a^4+4\mathrm{i}\,b\,a^3-b\,a^2+2\mathrm{i}\,b\,a-b\right)}\right)\,4\mathrm{i}}{(a-\mathrm{i})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)/(x^3*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] ((a + 1i)/(2*(a - 1i)) - (b*x*2i)/(a - 1i)^2)/x^2 - (b^2*atanh((3*a + a^2*3
i - a^3 - 1i)/(a - 1i)^3 - (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2 +
 2*a^8*b^2))/((a - 1i)^3*(a*b*2i - b - a^2*b + a^3*b*4i + a^4*b + a^5*b*2i
+ a^6*b)))*4i)/(a - 1i)^3
```

sympy [B] time = 0.96, size = 226, normalized size = 2.72

$$-\frac{2ib^2\log\left(-\frac{2a^4b^2}{(a-i)^3}+\frac{8ia^3b^2}{(a-i)^3}+\frac{12a^2b^2}{(a-i)^3}+2ab^2-\frac{8iab^2}{(a-i)^3}+4b^3x-2ib^2-\frac{2b^2}{(a-i)^3}\right)}{(a-i)^3}+\frac{2ib^2\log\left(\frac{2a^4b^2}{(a-i)^3}-\frac{8ia^3b^2}{(a-i)^3}-\frac{12a^2b^2}{(a-i)^3}+2ab^2\right.}{(a-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**3,x)
```

```
[Out] -2*I*b**2*log(-2*a**4*b**2/(a - I)**3 + 8*I*a**3*b**2/(a - I)**3 + 12*a**2*
b**2/(a - I)**3 + 2*a*b**2 - 8*I*a*b**2/(a - I)**3 + 4*b**3*x - 2*I*b**2 -
2*b**2/(a - I)**3)/(a - I)**3 + 2*I*b**2*log(2*a**4*b**2/(a - I)**3 - 8*I*a
**3*b**2/(a - I)**3 - 12*a**2*b**2/(a - I)**3 + 2*a*b**2 + 8*I*a*b**2/(a -
I)**3 + 4*b**3*x - 2*I*b**2 + 2*b**2/(a - I)**3)/(a - I)**3 - (a**2 - 4*I*b
*x + 1)/(x**2*(-2*a**2 + 4*I*a + 2))
```

3.206 $\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=104

$$\frac{2ib^3\log(x)}{(-a+i)^4}-\frac{2ib^3\log(-a-bx+i)}{(-a+i)^4}+\frac{2b^2}{(1+ia)^3x}-\frac{ib}{(-a+i)^2x^2}+\frac{-a-i}{3(-a+i)x^3}$$

```
[Out] 1/3*(-I-a)/(I-a)/x^3-I*b/(I-a)^2/x^2+2*b^2/(1+I*a)^3/x+2*I*b^3*ln(x)/(I-a)^
4-2*I*b^3*ln(I-a-b*x)/(I-a)^4
```

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5095, 77}

$$\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(-a+i)^4}-\frac{2ib^3\log(-a-bx+i)}{(-a+i)^4}-\frac{ib}{(-a+i)^2x^2}-\frac{a+i}{3(-a+i)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -(I + a)/(3*(I - a)*x^3) - (I*b)/((I - a)^2*x^2) + (2*b^2)/((1 + I*a)^3*x)
+ ((2*I)*b^3*Log[x])/(I - a)^4 - ((2*I)*b^3*Log[I - a - b*x])/(I - a)^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{1-ia-ibx}{x^4(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^4}+\frac{2ib}{(-i+a)^2x^3}-\frac{2ib^2}{(-i+a)^3x^2}+\frac{2ib^3}{(-i+a)^4x}-\frac{2ib^4}{(-i+a)^4(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{3(i-a)x^3}-\frac{ib}{(i-a)^2x^2}+\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(i-a)^4}-\frac{2ib^3\log(i-a-bx)}{(i-a)^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.88

$$\frac{(a-i)\left(a^3-ia^2-3iabx+a+6ib^2x^2-3bx-i\right)-6ib^3x^3\log(-a-bx+i)+6ib^3x^3\log(x)}{3(a-i)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

[Out] ((-I + a)*(-I + a - I*a^2 + a^3 - 3*b*x - (3*I)*a*b*x + (6*I)*b^2*x^2) + (6*I)*b^3*x^3*Log[x] - (6*I)*b^3*x^3*Log[I - a - b*x])/(3*(-I + a)^4*x^3)

fricas [A] time = 0.43, size = 94, normalized size = 0.90

$$\frac{6i\,b^3x^3\log(x) - 6i\,b^3x^3\log\left(\frac{bx+a-i}{b}\right) - 6\,(-i\,a-1)b^2x^2 + a^4 - 2i\,a^3 + \left(-3i\,a^2 - 6\,a + 3i\right)bx - 2i\,a - 1}{\left(3\,a^4 - 12i\,a^3 - 18\,a^2 + 12i\,a + 3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="fricas")

[Out] (6*I*b^3*x^3*log(x) - 6*I*b^3*x^3*log((b*x + a - I)/b) - 6*(-I*a - 1)*b^2*x^2 + a^4 - 2*I*a^3 + (-3*I*a^2 - 6*a + 3*I)*b*x - 2*I*a - 1)/((3*a^4 - 12*I*a^3 - 18*a^2 + 12*I*a + 3)*x^3)

giac [B] time = 0.12, size = 202, normalized size = 1.94

$$-\frac{2\,b^4\log\left(-\frac{ai}{bix+ai+1} + \frac{i^2}{bix+ai+1} + 1\right)}{a^4bi + 4\,a^3b - 6\,a^2bi - 4\,ab + bi} - \frac{\frac{3\left(ab^4i-8\,b^4\right)i^2}{(bix+ai+1)b} + \frac{ab^3i-10\,b^3}{ai+1} + \frac{3\left(a^2b^5+4\,ab^5i+5\,b^5\right)i^2}{(bix+ai+1)^2b^2}}{3\,(a-i)^3\left(\frac{ai}{bix+ai+1} - \frac{i^2}{bix+ai+1} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="giac")

[Out] -2*b^4*log(-a*i/(b*i*x + a*i + 1) + i^2/(b*i*x + a*i + 1) + 1)/(a^4*b*i + 4*a^3*b - 6*a^2*b*i - 4*a*b + b*i) - 1/3*(3*(a*b^4*i - 8*b^4)*i^2/((b*i*x + a*i + 1)*b) + (a*b^3*i - 10*b^3)/(a*i + 1) + 3*(a^2*b^5 + 4*a*b^5*i + 5*b^5)*i^2/((b*i*x + a*i + 1)^2*b^2))/((a - i)^3*(a*i/(b*i*x + a*i + 1) - i^2/(b*i*x + a*i + 1) - 1)^3)

maple [B] time = 0.06, size = 349, normalized size = 3.36

$$\frac{ia^4}{(i-a)^5\,x^3} - \frac{a^5}{3\,(i-a)^5\,x^3} + \frac{2ib^3\arctan(bx+a)}{(i-a)^5} + \frac{2a^3}{3\,(i-a)^5\,x^3} + \frac{ib^3\ln\left(b^2x^2+2abx+a^2+1\right)a}{(i-a)^5} + \frac{a}{(i-a)^5\,x^3} - \frac{i}{3\,(i-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x)

[Out] I/(I-a)^5/x^3*a^4-1/3/(I-a)^5/x^3*a^5+2*I*b^3/(I-a)^5*arctan(b*x+a)+2/3/(I-a)^5/x^3*a^3+I*b^3/(I-a)^5*ln(b^2*x^2+2*a*b*x+a^2+1)*a+1/(I-a)^5/x^3*a-1/3*I/(I-a)^5/x^3-2*b^3/(I-a)^5*ln(x)+I*b/(I-a)^5/x^2*a^3+2*I*b^2/(I-a)^5/x-4*b^2/(I-a)^5/x*a-2*I*b^2/(I-a)^5/x*a^2-3*I*b/(I-a)^5/x^2*a+3*b/(I-a)^5/x^2*a^2-b/(I-a)^5/x^2+2/3*I/(I-a)^5/x^3*a^2+b^3/(I-a)^5*ln(b^2*x^2+2*a*b*x+a^2+1)-2*b^3/(I-a)^5*arctan(b*x+a)*a-2*I*b^3/(I-a)^5*ln(x)*a

maxima [B] time = 0.34, size = 222, normalized size = 2.13

$$\frac{(2\,a-2i)b^3\log(i\,bx+i\,a+1)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1} - \frac{(2\,a-2i)b^3\log(x)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1} - \frac{(6\,a-6i)b^3x^3 - i\,a^5 + 3\left(a^2-2i\,a-1\right.}{\left(3i\,a^4+12\,a^3-18i\,a^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="maxima")

[Out] (2*a - 2*I)*b^3*log(I*b*x + I*a + 1)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I*a + 1) - (2*a - 2*I)*b^3*log(x)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I*a + 1) - ((6*a - 6*I)*b^3*x^3 - I*a^5 + 3*(a^2 - 2*I*a - 1)*b^2*x^2 - 3*a^

```
4 + 2*I*a^3 - (I*a^4 + 5*a^3 - 9*I*a^2 - 7*a + 2*I)*b*x - 2*a^2 + 3*I*a + 1
)/((3*I*a^4 + 12*a^3 - 18*I*a^2 - 12*a + 3*I)*b*x^4 + (3*I*a^5 + 15*a^4 - 3
0*I*a^3 - 30*a^2 + 15*I*a + 3)*x^3)
```

mupad [B] time = 0.77, size = 199, normalized size = 1.91

$$\frac{\frac{a+1\mathrm{i}}{3\,(a-\mathrm{i})}+\frac{b^2\,x^2\,2\mathrm{i}}{(a-\mathrm{i})^3}-\frac{b\,x\,1\mathrm{i}}{(a-\mathrm{i})^2}}{x^3}-\frac{4\,b^3\,\mathrm{atan}\left(\frac{\left(a^4-a^3\,4\mathrm{i}-6\,a^2+a\,4\mathrm{i}+1\right)\,1\mathrm{i}}{(a-\mathrm{i})^4}+\frac{x\,\left(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2\right)}{(a-\mathrm{i})^4\,\left(-1\mathrm{i}\,b\,a^9+3\,b\,a^8+8\,b\,a^6+6\mathrm{i}\,b\,a^5+6\,b\,a^4+8\mathrm{i}\,b\,a^3+3\mathrm{i}\,b\,a-b\right)}\right)}{(a-\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)/(x^4*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] ((a + 1i)/(3*(a - 1i)) + (b^2*x^2*2i)/(a - 1i)^3 - (b*x*1i)/(a - 1i)^2)/x^3
 - (4*b^3*atan(((a*4i - 6*a^2 - a^3*4i + a^4 + 1)*1i)/(a - 1i)^4 + (x*(2*b^
2 + 12*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*a^1
2*b^2))/((a - 1i)^4*(a*b*3i - b + a^3*b*8i + 6*a^4*b + a^5*b*6i + 8*a^6*b +
 3*a^8*b - a^9*b*1i))))/(a - 1i)^4
```

sympy [B] time = 1.40, size = 286, normalized size = 2.75

$$\frac{2ib^3\log\left(-\frac{2a^5b^3}{(a-i)^4}+\frac{10ia^4b^3}{(a-i)^4}+\frac{20a^3b^3}{(a-i)^4}-\frac{20ia^2b^3}{(a-i)^4}+2ab^3-\frac{10ab^3}{(a-i)^4}+4b^4x-2ib^3+\frac{2ib^3}{(a-i)^4}\right)}{(a-i)^4}-\frac{2ib^3\log\left(\frac{2a^5b^3}{(a-i)^4}-\frac{10ia^4b^3}{(a-i)^4}-\frac{2\ldots}{(\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**4,x)
```

```
[Out] 2*I*b**3*log(-2*a**5*b**3/(a - I)**4 + 10*I*a**4*b**3/(a - I)**4 + 20*a**3*
b**3/(a - I)**4 - 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 - 10*a*b**3/(a - I)*
*4 + 4*b**4*x - 2*I*b**3 + 2*I*b**3/(a - I)**4)/(a - I)**4 - 2*I*b**3*log(2
*a**5*b**3/(a - I)**4 - 10*I*a**4*b**3/(a - I)**4 - 20*a**3*b**3/(a - I)**4
 + 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 + 10*a*b**3/(a - I)**4 + 4*b**4*x -
 2*I*b**3 - 2*I*b**3/(a - I)**4)/(a - I)**4 - (-I*a**3 - a**2 - I*a + 6*b**
2*x**2 + x*(-3*a*b + 3*I*b) - 1)/(x**3*(3*I*a**3 + 9*a**2 - 9*I*a - 3))
```

3.207 $\int e^{-3i\tan^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=324

$$\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)bx-422a^2+458ia+163\right)}{40b^5}+\frac{3\left(8ia^4+48a^3-\right.}{}$$

```
[Out] -3/8*(19+68*I*a-88*a^2-48*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5+2*I*x^4*(1-I*a-I*
b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)-3/20*(17*I-16*a)*x^2*(1-I*a-I*b*x)^(3/2)*(
1+I*a+I*b*x)^(1/2)/b^3-11/5*x^3*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2
+1/40*I*(1-I*a-I*b*x)^(3/2)*(163+458*I*a-422*a^2-112*I*a^3-2*(61*I-118*a-52
*I*a^2)*b*x)*(1+I*a+I*b*x)^(1/2)/b^5+3/8*(19*I-68*a-88*I*a^2+48*a^3+8*I*a^4
)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] time = 0.28, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-2\left(-52ia^2-118a+61i\right)bx-112ia^3-422a^2+458ia+163\right)}{40b^5}+\frac{3\left(8ia^4+48a^3-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*x^4*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(19*I -
68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^5) - (3*(17*I - 16*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I
*a + I*b*x])/(20*b^3) - (11*x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) + ((I/40)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(163 +
(458*I)*a - 422*a^2 - (112*I)*a^3 - 2*(61*I - 118*a - (52*I)*a^2)*b*x))/b^5
 - (3*(19 + (68*I)*a - 88*a^2 - (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^
5)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^3\sqrt{1-ia-ibx}\left(4(1-ia)-\frac{11ibx}{2}\right)}{\sqrt{1+ia+ibx}}\,dx}{b}$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(\frac{33}{2}(1+ia)(i+a)b\right.}{\sqrt{1+ia+ibx}}}{5b^3}$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/}}{5b^2}$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/}}{5b^2}$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} -$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} -$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} -$$
$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} -$$

Mathematica [A] time = 0.52, size = 299, normalized size = 0.92

$$\frac{\frac{30\sqrt[4]{-1}\left(8ia^4+48a^3-88ia^2-68a+19i\right)\sqrt{b}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}} + \frac{8ia^6+a^5(410+8ibx)+2a^4(265bx-638i)+a^3\left(60b^2x^2-2004ibx-905\right)-a^2\left(20b^3\right.}{}}{40b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((448*I + (8*I)*a^6 + 285*b*x + (224*I)*b^2*x^2 + 95*b^3*x^3 - (56*I)*b^4*x
^4 - 30*b^5*x^5 + (8*I)*b^6*x^6 + a^5*(410 + (8*I)*b*x) + 2*a^4*(-638*I + 2
65*b*x) + a^3*(-905 - (2004*I)*b*x + 60*b^2*x^2) - a^2*(836*I + 2635*b*x +
(356*I)*b^2*x^2 + 20*b^3*x^3) + a*(-1315 + (1468*I)*b*x - 515*b^2*x^2 + (11
6*I)*b^3*x^3 + 10*b^4*x^4 + (8*I)*b^5*x^5))/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^
2] + (30*(-1)^(1/4)*(19*I - 68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[b]
*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/Sqrt
[(-I)*b])/(40*b^5)
```

fricas [A] time = 0.50, size = 263, normalized size = 0.81

$$\frac{62i\,a^6 + 2687\,a^5 - 11575i\,a^4 - 20350\,a^3 + \left(62i\,a^5 + 2625\,a^4 - 8950i\,a^3 - 11400\,a^2 + 6340i\,a + 1280\right)bx + 17740i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (62*I*a^6 + 2687*a^5 - 11575*I*a^4 - 20350*a^3 + (62*I*a^5 + 2625*a^4 - 895
0*I*a^3 - 11400*a^2 + 6340*I*a + 1280)*b*x + 17740*I*a^2 + (960*a^5 - 6720*
I*a^4 - 16320*a^3 + (960*a^4 - 5760*I*a^3 - 10560*a^2 + 8160*I*a + 2280)*b*
x + 18720*I*a^2 + 10440*a - 2280*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)) + (64*I*b^5*x^5 - 176*b^4*x^4 + (256*a - 272*I)*b^3*x^3 + 64*I*a
^5 - 8*(52*a^2 - 118*I*a - 61)*b^2*x^2 + 3344*a^4 - 13552*I*a^3 + (896*a^3
- 3376*I*a^2 - 3664*a + 1304*I)*b*x - 20792*a^2 + 14104*I*a + 3584)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1) + 7620*a - 1280*I)/(320*b^6*x + (320*a - 320*I)*
b^5)
```

giac [A] time = 0.20, size = 353, normalized size = 1.09

$$\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(\left(\frac{4\,ix}{b}-\frac{4\,ab^{17}i+15\,b^{17}}{b^{19}}\right)x+\frac{4\,a^2b^{16}i+35\,ab^{16}-32\,b^{16}i}{b^{19}}\right)x-\frac{8\,a^3b^{15}i+130\,a^2b^{15}-25}{b^{19}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*sqrt((b*x + a)^2 + 1)*((2*((4*i*x/b - (4*a*b^17*i + 15*b^17)/b^19)*x +
 (4*a^2*b^16*i + 35*a*b^16 - 32*b^16*i)/b^19)*x - (8*a^3*b^15*i + 130*a^2*b
^15 - 252*a*b^15*i - 125*b^15)/b^19)*x + (8*a^4*b^14*i + 250*a^3*b^14 - 804
*a^2*b^14*i - 835*a*b^14 + 288*b^14*i)/b^19) + 1/8*(8*a^4*i + 48*a^3 - 88*a
^2*i - 68*a + 19*i)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b*i + a^3*
b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*i*abs(b) + 3*(x*abs(b) - sqrt((b
*x + a)^2 + 1))*a^2*i*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2
*a^2*b - a*b*i + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*abs(b) - (x*abs(b)
- sqrt((b*x + a)^2 + 1))*i*abs(b))/(b^4*i*abs(b))
```

maple [B] time = 0.20, size = 2058, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 2*I/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^4+18*I/b^5*((x-(I-a)/
b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^4-24*I/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-
(I-a)/b))^(1/2)*a^2-6*I/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)
/b))^(5/2)-22*I/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^2-I/b^5*a
^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/2*I/b^5*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-3/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I
-a)/b))^(1/2))/(b^2)^(1/2)*a^4+33/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+
((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^2-1/b^8/(x-I/b+a
/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a^4+6/b^8/(x-I/b+a/b)^3*(
(x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a^2-3/b^4*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2)*x*a^4+33/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1
/2)*x*a^2-12/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*
a^3+20/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a-I/b^
4*a*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/2*I/b^4*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*x-3/2*I/b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)+18*I/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^
2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^3+18*I/b^4*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2)*x*a^3+24*I/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(5/2)*a^2-24*I/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2
)*x*a-24*I/b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*
b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a-2*I/b^7/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^
```

```
2+2*I*b*(x-(I-a)/b))^(5/2)*a^4+4*I/b^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(5/2)*a^3-4*I/b^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x
-(I-a)/b))^(5/2)*a-9/8/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9/8/b^5*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)*a-9/8/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(b^2)^(1/2)+4*I/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3
/2)-16/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a+12/b^5*((x-(I-a)/b
)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^3-1/b^8/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(5/2)-6/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)
*x-3/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^5+1/5*I/b^5*(b^2*x^2
+2*a*b*x+a^2+1)^(5/2)+33/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^
3-6/b^5*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-6/b^4*ln((I*b+(x-(I-a
)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1
/2)-3/4/b^4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3/4/b^5*a*(b^2*x^2+2*a*b*x+a^2+
1)^(3/2)
```

maxima [B] time = 0.48, size = 1368, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^4/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 -
2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b
^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 +
2*a*b*x + a^2 + 1)^(3/2)*a^3/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) + 6*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^4/(I*b^6*x + I*a*b^5 + b^5) - 6*I*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a^2/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I
*a*b^5 - b^5) - 12*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^6*x + 2
*I*a*b^5 + 2*b^5) + 24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^6*x + I*a
*b^5 + b^5) - 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^7*x^2 + 2*a*b^6*x
+ a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) - 12*(b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2)*a/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 36*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a^2/(I*b^6*x + I*a*b^5 + b^5) + I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*I*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 24*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^6*x + I*a*b^5 + b^5) - 3*a^4*arcsinh(b*x
+ a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^6*x + I*a*b^5 + b^5)
- I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4 - 3*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 4*I*b*x + 4*I*a + 3)*a^2*x/b^4 + 18*I*a^3*arcsinh(b*x + a)/b^5 + I
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 + 6*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*a^3/b^5 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^
3/b^5 - 3/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 - 3/2*I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a*x/b^4 + 6*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
+ 4*I*a + 3)*a*x/b^4 + 3*a^2*arcsin(I*b*x + I*a + 2)/b^5 + 36*a^2*arcsinh(b
*x + a)/b^5 + 1/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2)/b^5 + 13/4*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 - 39/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*a^2/b^5 + 12*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b
^5 - 9/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 3*sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 4*I*b*x + 4*I*a + 3)*x/b^4 - 6*I*a*arcsin(I*b*x + I*a + 2)/b^5 -
63/2*I*a*arcsinh(b*x + a)/b^5 - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^5
 - 153/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 15*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^5 - 3*arcsin(I*b*x + I*a + 2)/b^5 - 81/
8*arcsinh(b*x + a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5 - 6*I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
[Out] int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
[Out] Timed out
```

3.208 $\int e^{-3i\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=249

$$-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{8b^4}+\frac{3\left(-8ia^3-36a^2+44ia+17\right)\sqrt{-ia-i}}{8b^4}$$

```
[Out] 3/8*(17*I-44*a-36*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4+2*I*x^3*(1-I*a-I*b*x)^(3/
2)/b/(1+I*a+I*b*x)^(1/2)-9/4*x^2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^
2-1/8*I*(1-I*a-I*b*x)^(3/2)*(29*I-54*a-22*I*a^2+2*(11+10*I*a)*b*x)*(1+I*a+I
*b*x)^(1/2)/b^4+3/8*(17+44*I*a-36*a^2-8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I
*b*x)^(1/2)/b^4
```

Rubi [A] time = 0.25, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5095, 97, 153, 147, 50, 53, 619, 215}

$$-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{8b^4}+\frac{3\left(-8ia^3-36a^2+44ia+17\right)\sqrt{-ia-i}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*x^3*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(17 + (4
4*I)*a - 36*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(
8*b^4) - (9*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^2) - ((
I/8)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(29*I - 54*a - (22*I)*a^
2 + 2*(11 + (10*I)*a)*b*x))/b^4 + (3*(17*I - 44*a - (36*I)*a^2 + 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(3(1-ia)-\frac{9ibx}{2}\right)}{\sqrt{1+ia+ibx}}dx}{b}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i\int \frac{x\sqrt{1-ia-ibx}\left(9i(1+a^2)b+\frac{3}{2}(11+10\right.}{\sqrt{1+ia+ibx}}}{2b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-}{}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-}{}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-}{}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(17+44ia-36a^2-8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-}{}$$

Mathematica [A] time = 0.39, size = 244, normalized size = 0.98

$$\frac{3\sqrt[4]{-1}\left(8a^3-36ia^2-44a+17i\right)\sqrt{-ib}\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{4b^{9/2}} + \frac{-2ia^5+a^4(-76-2ibx)-5a^3(20bx-31i)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (80 - (2*I)*a^5 - (51*I)*b*x + 40*b^2*x^2 - (17*I)*b^3*x^3 - 8*b^4*x^4 + (2
*I)*b^5*x^5 + a^4*(-76 - (2*I)*b*x) - 5*a^3*(-31*I + 20*b*x) + a^2*(4 + (26
5*I)*b*x - 12*b^2*x^2) + a*(157*I + 212*b*x + (53*I)*b^2*x^2 + 4*b^3*x^3 +
(2*I)*b^4*x^4))/(8*b^4*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3*(-1)^(1/4)*(
17*I - 44*a - (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]
*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

fricas [A] time = 0.44, size = 217, normalized size = 0.87

$$\frac{-15i\,a^5-495\,a^4+1664i\,a^3+\left(-15i\,a^4-480\,a^3+1184i\,a^2+968\,a-256i\right)bx+2152\,a^2-\left(192\,a^4-1056i\,a^3+(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (-15*I*a^5 - 495*a^4 + 1664*I*a^3 + (-15*I*a^4 - 480*a^3 + 1184*I*a^2 + 968
*a - 256*I)*b*x + 2152*a^2 - (192*a^4 - 1056*I*a^3 + (192*a^3 - 864*I*a^2 -
 1056*a + 408*I)*b*x - 1920*a^2 + 1464*I*a + 408)*log(-b*x - a + sqrt(b^2*x
```

```
^2 + 2*a*b*x + a^2 + 1)) + (16*I*b^4*x^4 - 48*b^3*x^3 + (80*a - 88*I)*b^2*x
^2 - 16*I*a^4 - 624*a^3 - 8*(22*a^2 - 54*I*a - 29)*b*x + 1864*I*a^2 + 1896*
a - 640*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1224*I*a - 256)/(64*b^5*x +
(64*a - 64*I)*b^4)
```

giac [A] time = 0.18, size = 303, normalized size = 1.22

$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2x\left(\frac{x}{bi}-\frac{ab^{11}-4b^{11}i}{b^{13}i}\right)+\frac{2a^2b^{10}-20ab^{10}i-19b^{10}}{b^{13}i}\right)x-\frac{2a^3b^9-44a^2b^9i-93ab^9+48}{b^{13}i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*x*(x/(b*i) - (a*b^11 - 4*b^11*i)/(b^13*i)) +
 (2*a^2*b^10 - 20*a*b^10*i - 19*b^10)/(b^13*i))*x - (2*a^3*b^9 - 44*a^2*b^9
*i - 93*a*b^9 + 48*b^9*i)/(b^13*i)) - 1/8*(8*a^3 - 36*a^2*i - 44*a + 17*i)*
log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b - 2*(x*abs(b) - sqrt
((b*x + a)^2 + 1))^2*b*i - 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3
*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 4*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))
*abs(b))/(b^3*abs(b))
```

maple [B] time = 0.19, size = 1529, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -27/2*I/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(
x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^2-3*I/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(5/2)*a^2+2*I/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*
b*(x-(I-a)/b))^(5/2)*a^3-12*I/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x
-(I-a)/b))^(5/2)*a-27/2*I/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x
*a^2+4/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-2*I/b^4*((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^3+1/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+
2*I*b*(x-(I-a)/b))^(5/2)*a^3-3/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(
x-(I-a)/b))^(5/2)*a+I/b^7/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b
))^(5/2)+9/b^6/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a^
2+3/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x*a^3-33/2/b^3*((x-(I-a
)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x*a+1/4*I/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(
3/2)*x+1/4*I/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*a+3/8*I/b^3*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)*x+3/8*I/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-27/2*I/b^4*((x-(I
-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^3+11*I/b^4*((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(3/2)*a+6*I/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x
+6*I/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+6*I/b^3*ln((I*b+(x-(
I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)
^(1/2)+3/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*
(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^3-33/2/b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)
^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a-5/b^6/(x-
I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-9/b^4*((x-(I-a)/b)^2
*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^2+3/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b
))^(1/2)*a^4-33/2/b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^2+3/8*I
/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 0.47, size = 979, normalized size = 3.93

$$-\frac{i\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}a^3}{b^6x^2+2ab^5x+a^2b^4-2ib^5x-2iab^4-b^4}-\frac{3\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}a^2}{b^6x^2+2ab^5x+a^2b^4-2ib^5x-2iab^4-b^4}-\frac{3\left(b^2x^2+2abx+a^2\right.}{2ib^5x+2iab^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 -
 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(
b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^5*x + 2*I*a*b^4 + 2*b^4) - 6*I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^5*x + I*a*b^4 + b^4) + 3*I*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^(3/2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*
a*b^4 - b^4) + 6*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^5*x + 2*I*a
*b^4 + 2*b^4) - 18*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/(I*b^5*x + I*a*b^4
 + b^4) + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^6*x^2 + 2*a*b^5*x + a^2*b^
4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) + 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2
*I*b^5*x + 2*I*a*b^4 + 2*b^4) + 18*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I
*b^5*x + I*a*b^4 + b^4) + 3*a^3*arcsinh(b*x + a)/b^4 + 6*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(I*b^5*x + I*a*b^4 + b^4) + 1/4*I*(b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)
*a*x/b^3 - 27/2*I*a^2*arcsinh(b*x + a)/b^4 - 3/4*I*(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2)*a/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 3/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b^4 + 3/8*I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*x/b^3 - 3/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*
b*x + 4*I*a + 3)*x/b^3 - 3/2*a*arcsin(I*b*x + I*a + 2)/b^4 - 18*a*arcsinh(b
*x + a)/b^4 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^4 + 75/8*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b^4 - 9/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
 + 4*I*a + 3)*a/b^4 + 3/2*I*arcsin(I*b*x + I*a + 2)/b^4 + 63/8*I*arcsinh(b*
x + a)/b^4 + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4 - 3*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

```
[Out] int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.209 $\int e^{-3i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{6b^3}-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^3}+\frac{(-6}{}$$

```
[Out] 1/2*(11+18*I*a-6*a^2)*arcsinh(b*x+a)/b^3+I*(I-a)^2*(1-I*a-I*b*x)^(5/2)/b^3/
(1+I*a+I*b*x)^(1/2)-1/6*(11*I-18*a-6*I*a^2)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*
x)^(1/2)/b^3-1/3*I*(1-I*a-I*b*x)^(5/2)*(1+I*a+I*b*x)^(1/2)/b^3-1/2*(11*I-18
*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5095, 89, 80, 50, 53, 619, 215}

$$-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{6b^3}-\frac{\left(-6ia^2-18a+11i\right)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^3}+\frac{(-6}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(I - a)^2*(1 - I*a - I*b*x)^(5/2))/(b^3*Sqrt[1 + I*a + I*b*x]) - ((11*I
- 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3) -
((11*I - 18*a - (6*I)*a^2)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(
6*b^3) - ((I/3)*(1 - I*a - I*b*x)^(5/2)*Sqrt[1 + I*a + I*b*x])/b^3 + ((11 +
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{i\int \frac{(1-ia-ibx)^{3/2}\left(-(i-a)(3+2ia)b-b^2x\right)}{\sqrt{1+ia+ibx}}\,dx}{b^3}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{\left(11+18ia-6a^2\right)\int \frac{(1-ia-ibx)^{3}}{\sqrt{1+ia+ib}}}{3b^2}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} - \frac{i(1-ia-}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(1\right.\right.}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(1\right.\right.}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(1\right.\right.}{}$$

$$= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{\left(18a-i\left(11-6a^2\right)\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{\left(18a-i\left(1\right.\right.}{}$$

Mathematica [A] time = 0.32, size = 198, normalized size = 0.86

$$\frac{\sqrt[4]{-1}\left(-6a^2+18ia+11\right)\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{b^{7/2}} + \frac{2ia^4+a^3(51+2ibx)+a^2(69bx-50i)+a\left(2ib^3x^3+\right.}{6b^3\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*a^4 + a^3*(51 + (2*I)*b*x) + a^2*(-50*I + 69*b*x) + a*(51 - (106*I)*
b*x + 9*b^2*x^2 + (2*I)*b^3*x^3) + I*(-52 + (33*I)*b*x - 26*b^2*x^2 + (9*I)
*b^3*x^3 + 2*b^4*x^4))/(6*b^3*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ((-1)^(1
/4)*(11 + (18*I)*a - 6*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

fricas [A] time = 0.44, size = 174, normalized size = 0.76

$$\frac{7i\,a^4 + 166\,a^3 + \left(7i\,a^3 + 159\,a^2 - 249i\,a - 96\right)bx - 408i\,a^2 + \left(72\,a^3 + 12\left(6\,a^2 - 18i\,a - 11\right)bx - 288i\,a^2 - 348\,a \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (7*I*a^4 + 166*a^3 + (7*I*a^3 + 159*a^2 - 249*I*a - 96)*b*x - 408*I*a^2 + (
72*a^3 + 12*(6*a^2 - 18*I*a - 11)*b*x - 288*I*a^2 - 348*a + 132*I)*log(-b*x
 - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (8*I*b^3*x^3 - 28*b^2*x^2 + 8*I
*a^3 + (64*a - 76*I)*b*x + 212*a^2 - 412*I*a - 208)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1) - 345*a + 96*I)/(24*b^4*x + (24*a - 24*I)*b^3)
```

giac [A] time = 0.18, size = 257, normalized size = 1.12

$$\frac{1}{6}\sqrt{(bx+a)^2+1}\left(\left(\frac{2\,ix}{b} - \frac{2\,ab^6i + 9\,b^6}{b^8}\right)x + \frac{2\,a^2b^5i + 27\,ab^5 - 28\,b^5i}{b^8}\right) + \frac{\left(6\,a^2i + 18\,a - 11\,i\right)\log\left(3\left(x|b| - \sqrt{(b \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt((b*x + a)^2 + 1)*((2*i*x/b - (2*a*b^6*i + 9*b^6)/b^8)*x + (2*a^2*b
^5*i + 27*a*b^5 - 28*b^5*i)/b^8) + 1/6*(6*a^2*i + 18*a - 11*i)*log(3*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a*b*i + a^3*b*i + (x*abs(b) - sqrt((b*x + a)
^2 + 1))^3*i*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) + 2
*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*a^2*b - a*b*i + 4*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a*abs(b) - (x*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b
))/(b^2*i*abs(b))
```

maple [B] time = 0.18, size = 1026, normalized size = 4.48

$$\frac{2i\left(\left(x-\frac{i-a}{b}\right)^2 b^2 + 2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}} a^2}{b^3} - \frac{11i\left(\left(x-\frac{i-a}{b}\right)^2 b^2 + 2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3b^3} - \frac{3\ln\left(\frac{ib+\left(x-\frac{i-a}{b}\right)b^2}{\sqrt{b^2}} + \sqrt{\left(x-\frac{i-a}{b}\right)^2 b^2 \cdots}\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 2*I/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)*a^2-11/3*I/b^3*((x-(I-a
)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-3/b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1
/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a^2+9*I/b^3*((
x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^2+2*I/b^6/(x-I/b+a/b)^3*((x-(I-
a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a-2*I/b^5/(x-I/b+a/b)^2*((x-(I-a)/b)^2
*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a^2-6/b^5*a/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+
2*I*b*(x-(I-a)/b))^(5/2)+9*I/b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(
I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a+1/b^6/(x-I/b+a/b)^3*(
```

```
(x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-1/b^6/(x-I/b+a/b)^3*((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)*a^2-3/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)
/b))^(1/2)*x*a^2+9*I/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x*a+4*
I/b^5/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-3/b^3*((x-(
I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a^3+11/2/b^2*((x-(I-a)/b)^2*b^2+2*I*
b*(x-(I-a)/b))^(1/2)*x+11/2/b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)
*a+11/2/b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(
x-(I-a)/b))^(1/2))/(b^2)^(1/2)+6/b^3*a*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b)
)^(3/2)
```

maxima [B] time = 0.44, size = 624, normalized size = 2.72

$$\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3}+\frac{2\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3}+\frac{2\left(b^2x^2+2\,abx+a^2+1\right.}{2i\,b^4x+2i\,ab^3+2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 -
2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^5
*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^4*x + 2*I*a*b^3 + 2*b^3) + 6*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a^2/(I*b^4*x + I*a*b^3 + b^3) - I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b
^3) - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^4*x + 2*I*a*b^3 + 2*b^
3) + 12*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^4*x + I*a*b^3 + b^3) - 3*a
^2*arcsinh(b*x + a)/b^3 - 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^4*x +
I*a*b^3 + b^3) - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*x
/b^2 + 9*I*a*arcsinh(b*x + a)/b^3 + 1/3*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)/b^3 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3 - 1/2*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^3 + 1/2*arcsin(I*b*x + I*a + 2)/b^3
+ 6*arcsinh(b*x + a)/b^3 - 3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3 + I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

```
[Out] int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.210 $\int e^{-3i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=163

$$-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}-\frac{(3+2ia)\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{2b^2}-\frac{3(3+2ia)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^2}$$

```
[Out] -3/2*(3*I-2*a)*arcsinh(b*x+a)/b^2-(1+I*a)*(1-I*a-I*b*x)^(5/2)/b^2/(1+I*a+I*
b*x)^(1/2)-1/2*(3+2*I*a)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2-3/2*(3
+2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5095, 78, 50, 53, 619, 215}

$$-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}-\frac{(3+2ia)\sqrt{ia+ibx+1}\,(-ia-ibx+1)^{3/2}}{2b^2}-\frac{3(3+2ia)\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] -(((1 + I*a)*(1 - I*a - I*b*x)^(5/2))/(b^2*Sqrt[1 + I*a + I*b*x])) - (3*(3
+ (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((3 + (2*
I)*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) - (3*(3*I - 2*
a)*ArcSinh[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{(3i-2a)\int \frac{(1-ia-ibx)^{3/2}}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3(3i-2a))\int \frac{\sqrt{1-ia-}}{\sqrt{1+ia+}}}{2b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx}{2b^2}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx}{2b^2}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx}{2b^2}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx}{2b^2}$$

Mathematica [A] time = 0.33, size = 157, normalized size = 0.96

$$\frac{i\left(-a^3+a^2(-bx+14i)+a\left(b^2x^2+20ibx-1\right)+b^3x^3+6ib^2x^2+9bx+14i\right)}{2b^2\sqrt{a^2+2abx+b^2x^2+1}}+\frac{3\sqrt[4]{-1}\,(2a-3i)\sqrt{-ib}\,\sinh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{}\right.}{b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((I/2)*(14*I - a^3 + 9*b*x + (6*I)*b^2*x^2 + b^3*x^3 + a^2*(14*I - b*x) + a
*(-1 + (20*I)*b*x + b^2*x^2)))/(b^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3
*(-1)^(1/4)*(-3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I
)*(I + a + b*x)]])/Sqrt[(-I)*b]])/b^(5/2)
```

fricas [A] time = 0.48, size = 137, normalized size = 0.84

$$\frac{-3i\,a^3+\left(-3i\,a^2-44\,a+32i\right)bx-47\,a^2-\left((24\,a-36i)bx+24\,a^2-60i\,a-36\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a}\right.}{8\,b^3x+(8\,a-8i)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (-3*I*a^3 + (-3*I*a^2 - 44*a + 32*I)*b*x - 47*a^2 - ((24*a - 36*I)*b*x + 24
*a^2 - 60*I*a - 36)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(4*I*b^2*x^2 - 4*I*a^2 - 20*b*x - 60*a + 56*
I) + 76*I*a + 32)/(8*b^3*x + (8*a - 8*I)*b^2)
```

giac [A] time = 0.15, size = 219, normalized size = 1.34

$$-\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{bi}-\frac{ab^2-6b^2i}{b^4i}\right)-\frac{(2a-3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b-2\left(x|b|-\sqrt{(bx+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(x/(b*i) - (a*b^2 - 6*b^2*i)/(b^4*i)) - 1/2*(2*a
 - 3*i)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b - 2*(x*abs(b
) - sqrt((b*x + a)^2 + 1))^2*b*i - 2*a^2*b*i + (x*abs(b) - sqrt((b*x + a)^2
 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 4*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))*a*i*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)
^2 + 1))*abs(b))/(b*abs(b))
```

maple [B] time = 0.18, size = 676, normalized size = 4.15

$$\frac{2ia\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b^4\left(x-\frac{i}{b}+\frac{a}{b}\right)^2}+\frac{3\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b^4\left(x-\frac{i}{b}+\frac{a}{b}\right)^2}-\frac{3\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{b^2}-\frac{2i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] 2*I/b^4*a/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+3/b^4/(
x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-3/b^2*((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-2*I/b^2*a*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)
/b))^(3/2)-9/2*I/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-9/2*I/b*
ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^
(1/2))/(b^2)^(1/2)+1/b^5*a/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/
b))^(5/2)-I/b^5/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-9
/2*I/b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x+3/b*a*((x-(I-a)/b)^2*b
^2+2*I*b*(x-(I-a)/b))^(1/2)*x+3/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(
1/2)*a^2+3/b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*
(x-(I-a)/b))^(1/2))/(b^2)^(1/2)*a
```

maxima [B] time = 0.44, size = 293, normalized size = 1.80

$$-\frac{i\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}a}{b^4x^2+2ab^3x+a^2b^2-2ib^3x-2iab^2-b^2}-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{b^4x^2+2ab^3x+a^2b^2-2ib^3x-2iab^2-b^2}-\frac{\left(b^2x^2+2abx+a^2+\right.}{2ib^3x+2iab^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 - 2
*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2 - 2*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x +
 a^2 + 1)^(3/2)/(2*I*b^3*x + 2*I*a*b^2 + 2*b^2) - 6*I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/(I*b^3*x + I*a*b^2 + b^2) + 3*a*arcsinh(b*x + a)/b^2 - 6*sqr
```

```
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^3*x + I*a*b^2 + b^2) - 9/2*I*arcsinh(b*
x + a)/b^2 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
[Out] int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
[Out] Timed out
```

3.211 $\int e^{-3i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=94

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}+\frac{3i\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

```
[Out] -3*arcsinh(b*x+a)/b+2*I*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3*I*(1-I*
a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5093, 47, 50, 53, 619, 215}

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}+\frac{3i\sqrt{ia+ibx+1}\,\sqrt{-ia-ibx+1}}{b}-\frac{3\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + ((3*I)*Sqrt[1 -
 I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - (3*ArcSinh[a + b*x])/b
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-3i\tan^{-1}(a+bx)}\,dx &= \int \frac{(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - 3\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}\\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}{b} - \frac{3\sinh^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.48

$$-\frac{3\sinh^{-1}(a+bx)}{b} + \frac{\sqrt{(a+bx)^2+1}\left(\frac{4}{a+bx-i}+i\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*I)*ArcTan[a + b*x]),x]
[Out] (Sqrt[1 + (a + b*x)^2]*(I + 4/(-I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

fricas [A] time = 0.47, size = 102, normalized size = 1.09

$$\frac{(i\,a+8)bx + i\,a^2 + (6\,bx + 6\,a - 6i)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,(2i\,bx + 2i}{2\,b^2x + (2\,a - 2i)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
[Out] ((I*a + 8)*b*x + I*a^2 + (6*b*x + 6*a - 6*I)*log(-b*x - a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b*x + 2*I*a +
10) + 9*a - 8*I)/(2*b^2*x + (2*a - 2*I)*b)
```

giac [B] time = 0.17, size = 185, normalized size = 1.97

$$\frac{\sqrt{(bx+a)^2+1}\,i}{b} + \frac{\log\left(3\left(x|b| - \sqrt{(bx+a)^2+1}\right)^2 abi + a^3bi + \left(x|b| - \sqrt{(bx+a)^2+1}\right)^3 i|b| + 3\left(x|b| - \sqrt{(bx+a)^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt((b*x + a)^2 + 1)*i/b + log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b*
i + a^3*b*i + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*i*abs(b) + 3*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))
^2*b + 2*a^2*b - a*b*i + 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*abs(b) - (x
*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b))/abs(b)
```

maple [B] time = 0.12, size = 329, normalized size = 3.50

$$-\frac{\left(\left(x-\frac{i-a}{b}\right)^2 b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b^4\left(x-\frac{i}{b}+\frac{a}{b}\right)^3}-\frac{2i\left(\left(x-\frac{i-a}{b}\right)^2 b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b^3\left(x-\frac{i}{b}+\frac{a}{b}\right)^2}+\frac{2i\left(\left(x-\frac{i-a}{b}\right)^2 b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{b}-3\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

```
[Out] -1/b^4/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-2*I/b^3/(x
-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+2*I/b*((x-(I-a)/b)^
2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2
)*x-3/b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-3*ln((I*b+(x-(I-a)/b)
*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 0.44, size = 103, normalized size = 1.10

$$\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^3x^2+2\,ab^2x+a^2b-2i\,b^2x-2i\,ab-b}-\frac{3\,\operatorname{arsinh}\left(bx+a\right)}{b}+\frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^2x+i\,ab+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b - 2*I*b^
2*x - 2*I*a*b - b) - 3*arcsinh(b*x + a)/b + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(I*b^2*x + I*a*b + b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3,x)
```

```
[Out] int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx+\int\frac{a^2\sqrt{a^2+}}{a^3+3a^2bx-3ia^2+3ab^2x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a
**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x +
 I), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2
```

```
*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x*
*2 - 3*b*x + I), x) + Integral(b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x
**3 - 3*I*b**2*x**2 - 3*b*x + I), x) + Integral(2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x
 - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x))
```

3.212 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}+i\sinh^{-1}(a+bx)-\frac{2(a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}$$

```
[Out] I*arcsinh(b*x+a)-2*(I+a)^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a
)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(3/2)+4*(1-I*a-I*b*x)^(1/2)/(1+I*a)/(1+I
*a+I*b*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5095, 98, 157, 53, 619, 215, 93, 208}

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}+i\sinh^{-1}(a+bx)-\frac{2(a+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+i}\,\sqrt{ia+ibx+1}}{\sqrt{-a+i}\,\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (4*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) + I*ArcSinh[a +
 b*x] - (2*(I + a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I - a)^(3/2)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x}\,dx &= \int \frac{(1-ia-ibx)^{3/2}}{x(1+ia+ibx)^{3/2}}\,dx \\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + \frac{2\int \frac{-\frac{1}{2}i(i+a)^2b-\frac{1}{2}(1+ia)b^2x}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{(i-a)b} \\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{(i+a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\,\sqrt{1+ia+ibx}}\,dx}{1+ia} + (ib)\int \frac{1}{\sqrt{1-ia-ibx}\,\sqrt{1+ia+\ldots}} \\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{\left(2(i+a)^2\right)\operatorname{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1+ia} + (ib)\int \frac{\ldots}{\sqrt{(1\ldots}} \\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{2(i+a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}} + \frac{i\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab\ldots\right)}{2b} \\
&= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + i\sinh^{-1}(a+bx) - \frac{2(i+a)^{3/2}\tanh^{-1}\left(\frac{\sqrt{i+a}\,\sqrt{1+ia+ibx}}{\sqrt{i-a}\,\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.66, size = 189, normalized size = 1.41

$$
\frac{2\left(\frac{\sqrt{-1+ia}\,(a+i)\tanh^{-1}\left(\frac{\sqrt{-1-ia}\,\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\,\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}} - \frac{2\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\right)}{a-i} + \frac{2(-1)^{3/4}\sqrt{-ib}\,\sinh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\,\sqrt{-i(a+bx+i)}}{\sqrt{-ib}}\right)}{\sqrt{b}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
b*x)])/Sqrt[(-I)*b]])/Sqrt[b] + (2*((-2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/
(-I + a + b*x) + (Sqrt[-1 + I*a]*(I + a)*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*
(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a]))/(
-I + a)
```

fricas [B] time = 0.49, size = 381, normalized size = 2.84

$$\frac{\left((a-i)bx+a^2-2i\,a-1\right)\sqrt{-\frac{4\,a^3+12i\,a^2-12\,a-4i}{a^3-3i\,a^2-3\,a+i}}\,\log\left(-\frac{(2\,a+2i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}\,(2\,a+2i)-\left(i\,a^2+2\,a-i\right)\sqrt{-\frac{4\,a^3+12i\,a^2-12\,a-4i}{a^3-3i\,a^2-3\,a+i}}}{2\,a+2i}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3
 - 3*I*a^2 - 3*a + I))*log(-((2*a + 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*(2*a + 2*I) - (I*a^2 + 2*a - I)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)
/(a^3 - 3*I*a^2 - 3*a + I)))/(2*a + 2*I)) - ((a - I)*b*x + a^2 - 2*I*a - 1)
*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3 - 3*I*a^2 - 3*a + I))*log(-((2*
a + 2*I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + 2*I) - (-I*a^2 - 2*
a + I)*sqrt(-(4*a^3 + 12*I*a^2 - 12*a - 4*I)/(a^3 - 3*I*a^2 - 3*a + I)))/(2
*a + 2*I)) - 8*b*x - (2*(I*a + 1)*b*x + 2*I*a^2 + 4*a - 2*I)*log(-b*x - a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*a - 8*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) + 8*I)/((2*a - 2*I)*b*x + 2*a^2 - 4*I*a - 2)
```

giac [B] time = 0.33, size = 263, normalized size = 1.96

$$\frac{\left(a^2i-2\,a-i\right)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}\,(a-i)}+\frac{b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2abi-a^3bi-\left(x|b|-\sqrt{(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] (a^2*i - 2*a - i)*log(abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^
2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt
(a^2 + 1)*(a - i)) + 1/3*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b*
i - a^3*b*i - (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*i*abs(b) - 3*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^2*i*abs(b) - 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))
^2*b - 2*a^2*b + a*b*i - 4*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a*abs(b) + (x
*abs(b) - sqrt((b*x + a)^2 + 1))*i*abs(b))/(i*abs(b))
```

maple [B] time = 0.14, size = 1278, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] -1/(I-a)^2/b^2/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+1/
(I-a)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-3/2*I/(I-a)^3*a*b*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I/(I-a)^3*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/3*I/(I-a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1
/2*I/(I-a)^3*a*b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+I/(I-a)^3*(a^2+1)^(1/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*I/(
```

```
I-a)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-I/(I-a)^3*a^3*b*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+2*I/(I-a)*(
(x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)+1/3*I/(I-a)^3*((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(3/2)-3/2*I/(I-a)^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3
/2*I/(I-a)^2*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-1/(I-a)/b^3/(x
-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-2*I/(I-a)/b^2/(x-I/
b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+I/(I-a)^3*(a^2+1)^(1/2
)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2
-3/(I-a)*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-3/(I-a)*((x-(I-a)/
b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-3/(I-a)*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2
)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)+3/2*I/(I-a
)^2*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a
)/b))^(1/2))/(b^2)^(1/2)-1/2/(I-a)^3*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b)
)^(1/2)*x-1/2/(I-a)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-1/2/(I-
a)^3*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-
a)/b))^(1/2))/(b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx+\int \frac{}{a^3x+3a^2bx^2-3ia^2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 -
3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*x*
*3 - 3*b*x**2 + I*x), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*
a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x) + Integral(b**2*x**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x +
 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**
2 + I*x), x) + Integral(2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*
x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**
3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x))
```

3.213 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=178

$$-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}+\frac{6ib\sqrt{-ia-ibx+1}}{(-a+i)^2\sqrt{ia+ibx+1}}-\frac{6i\sqrt{a+i}\,b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}}$$

```
[Out] -6*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1
/2))*(I+a)^(1/2)/(I-a)^(5/2)-(1-I*a-I*b*x)^(3/2)/(1+I*a)/x/(1+I*a+I*b*x)^(1
/2)+6*I*b*(1-I*a-I*b*x)^(1/2)/(I-a)^2/(1+I*a+I*b*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5095, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}+\frac{6ib\sqrt{-ia-ibx+1}}{(-a+i)^2\sqrt{ia+ibx+1}}-\frac{6i\sqrt{a+i}\,b\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((6*I)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*Sqrt[1 + I*a + I*b*x]) - (1 - I*
a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x]) - ((6*I)*Sqrt[I + a]*b
*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/(I - a)^(5/2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}\,dx$$
$$= -\frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3b)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/2}}\,dx}{i-a}$$
$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3(i+a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i-a)^2}$$
$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(6(i+a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \right.}{(i-a)^2}$$
$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{i+a}\,b\tanh^{-1}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}}$$

Mathematica [A] time = 0.17, size = 145, normalized size = 0.81

$$\frac{\frac{\sqrt{-i(a+bx+i)}\left(a^2+abx+5ibx+1\right)}{x\sqrt{ia+ibx+1}} - \frac{6i\sqrt{-1+ia}\,b\tanh^{-1}\left(\frac{\sqrt{-1-ia}\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}}}{(a-i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*b*x + a*b*x))/(x*Sqrt[1 + I*a +
 I*b*x]) - ((6*I)*Sqrt[-1 + I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a
 + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a])/(-I + a)
^2
```

fricas [B] time = 0.46, size = 404, normalized size = 2.27

$$-\frac{2\,(i\,a-5)b^2x^2 - \left(-2i\,a^2+8\,a-10i\right)bx - \left(\left(a^2-2i\,a-1\right)bx^2+\left(a^3-3i\,a^2-3\,a+i\right)x\right)\sqrt{\frac{(36\,a+36i)b^2}{a^5-5i\,a^4-10\,a^3+10i\,a^2+5\,a-i}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(I*a - 5)*b^2*x^2 - (-2*I*a^2 + 8*a - 10*I)*b*x - ((a^2 - 2*I*a - 1)*b*
x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 -
10*a^3 + 10*I*a^2 + 5*a - I))*log(-1/6*(6*b^2*x + (a^3 - 3*I*a^2 - 3*a + I)
*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - 6*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + ((a^2 - 2*I*a - 1)*b*x^2 + (a^3 -
 3*I*a^2 - 3*a + I)*x)*sqrt((36*a + 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*
I*a^2 + 5*a - I))*log(-1/6*(6*b^2*x - (a^3 - 3*I*a^2 - 3*a + I)*sqrt((36*a
+ 36*I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - 6*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*(I*a - 5
)*b*x + 2*I*a^2 + 2*I))/(2*(a^2 - 2*I*a - 1)*b*x^2 + (2*a^3 - 6*I*a^2 - 6*a
 + 2*I)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.19, size = 1917, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 3*I/(I-a)^3*a*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/x)-I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2
)*x-3/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3/2*I/(I-a)^3
*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2
)^(1/2)-9/2*I/(I-a)^3*a^3*b/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*I/(I-a)
^4*b^2*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)+3*I/(I-a)^4*b*(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2-2*I/(I-a)^3*a*b/(a^2+1)*(b^2*x^2+2*a*b*x+a
^2+1)^(3/2)-3/2*I/(I-a)^4*b^2*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/2*I/(I-a)
^4*b^2*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1
/2)-9/2*I/(I-a)^4*b*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*I/(I-a)^4*b*(a^2+1)
^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x
)-9/2*I/(I-a)^3*a*b/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3*I/(I-a)^3*a^3*b
/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)+I/(I-a)^4*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-3/2/(I-a)
^4*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-3/2/(I-a)^4*b*((x-(I-a
)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-3/2/(I-a)^4*b^2*ln((I*b+(x-(I-a)/b)*b
^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-1/
(I-a)^2/b^2/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-3/(I-
a)^2*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-3/(I-a)^2*b*((x-(I-a
)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+2*I/(I-a)^2*b*((x-(I-a)/b)^2*b^2+2*I*
b*(x-(I-a)/b))^(3/2)-I/(I-a)^4*b*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3*I/(I-a)^4*
b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-2/(I-a)^3/b/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^
2+2*I*b*(x-(I-a)/b))^(5/2)+3*I/(I-a)^3*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/
b))^(1/2)*a+3*I/(I-a)^3*b^2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/
b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-2*I/(I-a)^2/b/(x-I/b+a/b)^2*
((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+I/(I-a)^3/(a^2+1)/x*(b^2*x^2+2*
a*b*x+a^2+1)^(5/2)+3*I/(I-a)^3*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1
/2)*x-3/2*I/(I-a)^3*a^2*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/2*I/(
I-a)^3*a^2*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)-3*I/(I-a)^3*a^4*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3*b^2/(I-a)^2*ln((I*b+(x-(I-a)/b)*
b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)+2
/(I-a)^3*b*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**2,x)
[Out] Timed out
```

3.214 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=264

$$-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}-\frac{3(2a+3i)b^2\sqrt{-ia-ibx+1}}{(1+ia)^3(a+i)\sqrt{ia+ibx+1}}+\frac{3(3-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}\sqrt{a+i}}+\frac{(3-2i\ldots}{2(-a+i)\ldots}$$

```
[Out] 3*(3-2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-
I*b*x)^(1/2))/(I-a)^(7/2)/(I+a)^(1/2)+1/2*(3-2*I*a)*b*(1-I*a-I*b*x)^(3/2)/(
I-a)^2/(I+a)/x/(1+I*a+I*b*x)^(1/2)-1/2*(1-I*a-I*b*x)^(5/2)/(a^2+1)/x^2/(1+I
*a+I*b*x)^(1/2)-3*(3*I+2*a)*b^2*(1-I*a-I*b*x)^(1/2)/(1+I*a)^3/(I+a)/(1+I*a+
I*b*x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5095, 96, 94, 93, 208}

$$-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}-\frac{3(2a+3i)b^2\sqrt{-ia-ibx+1}}{(1+ia)^3(a+i)\sqrt{ia+ibx+1}}+\frac{3(3-2ia)b^2\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}\sqrt{a+i}}+\frac{(3-2i\ldots}{2(-a+i)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] (-3*(3*I + 2*a)*b^2*Sqrt[1 - I*a - I*b*x])/((1 + I*a)^3*(I + a)*Sqrt[1 + I*
a + I*b*x]) + ((3 - (2*I)*a)*b*(1 - I*a - I*b*x)^(3/2))/(2*(I - a)^2*(I + a
)*x*Sqrt[1 + I*a + I*b*x]) - (1 - I*a - I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[
1 + I*a + I*b*x]) + (3*(3 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a
+ I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(7/2)*Sqrt[I + a])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-ia-ibx)^{3/2}}{x^3(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} - \frac{((3i+2a)b)\int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}\,dx}{2\left(1+a^2\right)}$$

$$= \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}} + \frac{\left(3(3i+2a)b^2\right)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/}}}{2(i-a)^2(i+a)}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ib}}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ib}}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ib}}$$

Mathematica [A] time = 0.27, size = 194, normalized size = 0.73

$$\frac{\frac{\sqrt{-i(a+bx+i)}\left(a^3-ia^2-ab^2x^2-5iabx+a-14ib^2x^2-5bx-i\right)}{x^2\sqrt{ia+ibx+1}}+\frac{6i\sqrt{-1+ia}\,(2a+3i)b^2\tanh^{-1}\left(\frac{\sqrt{-1-ia}\sqrt{-i(a+bx+i)}}{\sqrt{-1+ia}\sqrt{ia+ibx+1}}\right)}{\sqrt{-1-ia}\,(a+i)}}{2(a-i)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(-I + a - I*a^2 + a^3 - 5*b*x - (5*I)*a*b*x - (1
4*I)*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[1 + I*a + I*b*x]) + ((6*I)*Sqrt[-1 + I
*a]*(3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt
[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*(I + a)))/(2*(-I + a)^3
)
```

fricas [B] time = 0.49, size = 580, normalized size = 2.20

$$(i\,a-14)b^3x^3 + \left(i\,a^2-13\,a+14i\right)b^2x^2 - 3\left(\left(a^3-3i\,a^2-3\,a+i\right)bx^3 + \left(a^4-4i\,a^3-6\,a^2+4i\,a+1\right)x^2\right)\sqrt{\frac{}{a^8-6i\,a^7-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] ((I*a - 14)*b^3*x^3 + (I*a^2 - 13*a + 14*I)*b^2*x^2 - 3*((a^3 - 3*I*a^2 - 3
*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I
*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a
- 1))*log(-((6*a + 9*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a + 9*
I)*b^2 + 3*(a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a
 - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a -
1)))/((6*a + 9*I)*b^2)) + 3*((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a
^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 -
 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))*log(-((6*a + 9*I)*b^3*
x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a + 9*I)*b^2 - 3*(a^5 - 3*I*a^4 -
2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 1
4*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1)))/((6*a + 9*I)*b^2)) + ((
I*a - 14)*b^2*x^2 - I*a^3 - (5*a - 5*I)*b*x - a^2 - I*a - 1)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))/((2*a^3 - 6*I*a^2 - 6*a + 2*I)*b*x^3 + (2*a^4 - 8*I*a^
3 - 12*a^2 + 8*I*a + 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.20, size = 3042, normalized size = 11.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] -9/4*I/(I-a)^3*b^2/(a^2+1)*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I/(I-a)^3*
b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2))/x)*a^2-9/2*I/(I-a)^4*b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-
9/2*I/(I-a)^4*b^3/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)+3*I/(I-a)^4*b/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-
6*I/(I-a)^4*b^2*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-27/2*I/(I-a)^4*b^2*
a^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/(I-a)^3/b/(x-I/b+a/b)^3*((x-(I-
a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-3/(I-a)^3*b^3*((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(1/2)*x-3/(I-a)^3*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(
1/2)*a+9/2*I/(I-a)^4*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+9/2*
I/(I-a)^4*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-9/2*I/(I-a)^4*b
^3*a^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-27/2*I/(I-a)^4*b^3*a^2/(a^2+
1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9*
I/(I-a)^4*b^3*a^4/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)-3/4*I/(I-a)^3*b^3/(a^2+1)*a*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*x-9/4*I/(I-a)^3*b^3/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^3*b^3/(a^2+1)*a^3*ln((b^2*x+a*b)/(
b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*I/(I-a)^3*a*b/(a^
2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-3/4*I/(I-a)^3*a^3*b^3/(a^2+1)^2*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)*x-9/4*I/(I-a)^3*a^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/
(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^3*a^5*b^
3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)
^(1/2)-1/2*I/(I-a)^3*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x-3/4*I/
(I-a)^3*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-3/4*I/(I-a)^3*a*b^3
/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)-27/2*I/(I-a)^4*b^2*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9*I/(I-a)^
```

```
4*b^2*a^3/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x)+9*I/(I-a)^4*b^2*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a
^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-3*I/(I-a)^4*b^3/(a^2+1)*(b^2*
x^2+2*a*b*x+a^2+1)^(3/2)*x-3*I/(I-a)^5*b^3*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
x-9*I/(I-a)^5*b^3*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2
))/(b^2)^(1/2)-6*I/(I-a)^5*b^3*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*I/(I-a)^5*b^2*(a^2+1)^(1/2)*ln((2*a^2+2+2*a
*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2-I/(I-a)^3*a^2*b^
2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-9/4*I/(I-a)^3*a^4*b^2/(a^2+1)^2*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2)-9/4*I/(I-a)^3*a^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)+3/2*I/(I-a)^3*a^4*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+3/2*I/(I-a)^3*a^2*b^2/(a^2+1
)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
x)+9/2*I/(I-a)^4*b^3*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^
2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)-9*I/(I-a)^5*b^2*a^2*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+6*I/(I-a)^5*b^2*(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+1/2*I/(I-a)^3/(a^2+1)/x^2*(b^2*x^2+
2*a*b*x+a^2+1)^(5/2)-1/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2
)-3/2*I/(I-a)^3*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/2*I/(I-a)^3*b^2
/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/x)-3/(I-a)^4/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/
2)+3/(I-a)^4*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-3/(I-a)^3*b^3*
ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^
(1/2))/(b^2)^(1/2)-2*I/(I-a)^3/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I
-a)/b))^(5/2)+2*I/(I-a)^3*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-2
*I/(I-a)^5*b^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-6*I/(I-a)^5*b^2*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)-3/(I-a)^5*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x
-3/(I-a)^5*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-3/(I-a)^5*b^3*
ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^
(1/2))/(b^2)^(1/2)+2*I/(I-a)^5*b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3
/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^3), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**3,x)
[Out] Timed out
```

3.215 $\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-2a^2-51ia+52\right)b^3\sqrt{-ia-ibx+1}}{6(-a+i)^4(a+i)\sqrt{ia+ibx+1}}+\frac{\left(-6ia^2+18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{9/2}(a+i)^{3/2}}+\frac{(19-16ia)b^2\sqrt{-i}}{6(-a+i)^3(a+i)x\sqrt{}}$$

```
[Out] (11*I+18*a-6*I*a^2)*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)
/(1-I*a-I*b*x)^(1/2))/(I-a)^(9/2)/(I+a)^(3/2)-1/6*(52-51*I*a-2*a^2)*b^3*(1-
I*a-I*b*x)^(1/2)/(I-a)^4/(I+a)/(1+I*a+I*b*x)^(1/2)-1/3*(I+a)*(1-I*a-I*b*x)^
(1/2)/(I-a)/x^3/(1+I*a+I*b*x)^(1/2)-7/6*I*b*(1-I*a-I*b*x)^(1/2)/(I-a)^2/x^2
/(1+I*a+I*b*x)^(1/2)+1/6*(19-16*I*a)*b^2*(1-I*a-I*b*x)^(1/2)/(I-a)^3/(I+a)/
x/(1+I*a+I*b*x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5095, 98, 151, 152, 12, 93, 208}

$$-\frac{\left(-2a^2-51ia+52\right)b^3\sqrt{-ia-ibx+1}}{6(-a+i)^4(a+i)\sqrt{ia+ibx+1}}+\frac{\left(-6ia^2+18a+11i\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{9/2}(a+i)^{3/2}}+\frac{(19-16ia)b^2\sqrt{-i}}{6(-a+i)^3(a+i)x\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -((52 - (51*I)*a - 2*a^2)*b^3*Sqrt[1 - I*a - I*b*x])/(6*(I - a)^4*(I + a)*S
qrt[1 + I*a + I*b*x]) - ((I + a)*Sqrt[1 - I*a - I*b*x])/(3*(I - a)*x^3*Sqrt
[1 + I*a + I*b*x]) - (((7*I)/6)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*x^2*Sqr
t[1 + I*a + I*b*x]) + ((19 - (16*I)*a)*b^2*Sqrt[1 - I*a - I*b*x])/(6*(I - a
)^3*(I + a)*x*Sqrt[1 + I*a + I*b*x]) + ((11*I + 18*a - (6*I)*a^2)*b^3*ArcTa
nh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])]
)/((I - a)^(9/2)*(I + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1-ia-ibx)^{3/2}}{x^4(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{\int \frac{7(i+a)b+6b^2x}{x^3\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}\,dx}{3(1+ia)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{\int \frac{-\left(19-35ia-16a^2\right)b^2+14(i+a)b^3x}{x^2\sqrt{1-ia-ibx}\,(1+ia+ibx)^{3/2}}\,dx}{6(1+ia)\left(1+a^2\right)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+i}}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+i}}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+i}}$$

$$= -\frac{\left(52-51ia-2a^2\right)b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+i}}$$

Mathematica [A] time = 0.42, size = 275, normalized size = 0.81

$$-\frac{i\left(6a^2+18ia-11\right)b^2x^2\left(-i\sqrt{-1-ia}\sqrt{-i(a+bx+i)}\left(a^2+abx+5ibx+1\right)-6\sqrt{-1+ia}\,bx\sqrt{ia+ibx+1}\ \tan}{6(-1-ia)^{5/2}\left(a^2+1\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -1/6*(-2*(-1 - I*a)^(7/2)*(1 - I*a)*((-I)*(I + a + b*x))^(5/2) - (-1 - I*a)
^(5/2)*(3*I + 4*a)*b*x*((-I)*(I + a + b*x))^(5/2) + I*(-11 + (18*I)*a + 6*a
^2)*b^2*x^2*((-I)*Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*
b*x + a*b*x) - 6*Sqrt[-1 + I*a]*b*x*Sqrt[1 + I*a + I*b*x]*ArcTanh[(Sqrt[-1
- I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]))/
((-1 - I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[1 + I*a + I*b*x])
```

fricas [B] time = 0.53, size = 853, normalized size = 2.52

$$\left(-2i\,a^2+51\,a+52i\right)b^4x^4+\left(-2i\,a^3+49\,a^2+i\,a+52\right)b^3x^3+\sqrt{\frac{\left(36\,a^4+216i\,a^3-456\,a^2-396i\,a+121\right)b^6}{a^{12}-6i\,a^{11}-12\,a^{10}+2i\,a^9-27\,a^8+36i\,a^7+36i\,a^5+27\,a^4+2i\,a^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] ((-2*I*a^2 + 51*a + 52*I)*b^4*x^4 + (-2*I*a^3 + 49*a^2 + I*a + 52)*b^3*x^3
+ sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11
```

```
- 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*
a^2 - 6*I*a - 1))*((3*a^5 - 9*I*a^4 - 6*a^3 - 6*I*a^2 - 9*a + 3*I)*b*x^4 +
(3*a^6 - 12*I*a^5 - 15*a^4 - 15*a^2 + 12*I*a + 3)*x^3)*log(-((6*a^2 + 18*I*
a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 + 18*I*a - 11)*b^3
 + (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 - I*a^2 - 3*a + I)*sqrt((36*a^4 +
 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*
a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)
))/((6*a^2 + 18*I*a - 11)*b^3)) - sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*
I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 3
6*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1))*((3*a^5 - 9*I*a^4 - 6*a^3
 - 6*I*a^2 - 9*a + 3*I)*b*x^4 + (3*a^6 - 12*I*a^5 - 15*a^4 - 15*a^2 + 12*I*
a + 3)*x^3)*log(-((6*a^2 + 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(6*a^2 + 18*I*a - 11)*b^3 - (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 -
 I*a^2 - 3*a + I)*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(
a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4
 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)))/((6*a^2 + 18*I*a - 11)*b^3)) + ((-2*I*a^
2 + 51*a + 52*I)*b^3*x^3 - 2*I*a^5 + (16*a^2 + 3*I*a + 19)*b^2*x^2 - 2*a^4
- 4*I*a^3 - (7*a^3 - 7*I*a^2 + 7*a - 7*I)*b*x - 4*a^2 - 2*I*a - 2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((6*a^5 - 18*I*a^4 - 12*a^3 - 12*I*a^2 - 18*a +
6*I)*b*x^4 + (6*a^6 - 24*I*a^5 - 30*a^4 - 30*a^2 + 24*I*a + 6)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.20, size = 4390, normalized size = 12.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] -3/2*I/(I-a)^3*a^4*b^4/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)-9/2*I/(I-a)^4*b^4*a^5/(a^2+1)^2*ln((b^2*x+a*b)/
(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^4*b^4*a/
(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x-9/2*I/(I-a)^4*b^4/(a^2+1)*a^3*ln(
(b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3*b^4/(I
-a)^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a
)/b))^(1/2))/(b^2)^(1/2)-3/2*I/(I-a)^4*b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)-9*I/(I-a)^5*b^4*a^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-27*I/(I-a
)^5*b^4*a^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2
))/(b^2)^(1/2)-9/4*I/(I-a)^3*a^2*b^4/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(
b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9*I/(I-a)^5*b^4/(a^2+1)*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9*I/(I-a)^5*b^
4/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+18*I/(I-a)^5*b^3*a^3/(a^2+1)^(1/2
)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+18*
I/(I-a)^5*b^3*a/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/x)+2/3*I/(I-a)^3*b^2/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2
+1)^(5/2)+6*I/(I-a)^5*b^2/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-12*I/(I-a
)^5*b^3*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-6*I/(I-a)^5*b^4/(a^2+1)*(b^
2*x^2+2*a*b*x+a^2+1)^(3/2)*x+3/4*I/(I-a)^3*a^5*b^3/(a^2+1)^3*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)+3/4*I/(I-a)^3*a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-7/6*I/(I-a)^3*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-9/4*I/(I-a)^
3*a^3*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/2*I/(I-a)^3*a*b^3/(a^2+
```

```
1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-2/3*I/(I-a)^3*b^4/(a^2+1)^2*(b^2*x^2+2*a
*b*x+a^2+1)^(3/2)*x-I/(I-a)^3*b^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x
-27*I/(I-a)^5*b^3*a^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-27*I/(I-a)^5*b^
3*a/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+10*I/(I-a)^6*b^3*(a^2+1)^(1/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)*a^2-I/(
I-a)^3*b^4/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/
2))/(b^2)^(1/2)+3/2*I/(I-a)^3*a*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^
2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-5*I/(I-a)^6*b^4*a*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*x-15*I/(I-a)^6*b^4*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-10*I/(I-a)^6*b^4*a^3*ln((b^2*x+a*b)/(b^2)
^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*I/(I-a)^3*a^3*b^3/(a^
2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
))/x)+3/2*I/(I-a)^3*a^3*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*I/(I-a)^3*a^5*b^3/(a^2+1)^(5/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+1/3*I/(
I-a)^3*a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-3*I/(I-a)^4*b^3*a^2/
(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-27/4*I/(I-a)^4*b^3*a^4/(a^2+1)^2*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)-27/4*I/(I-a)^4*b^3*a^2/(a^2+1)^2*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)+9/2*I/(I-a)^4*b^3*a^4/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+9/2*I/(I-a)^4*b^3*a^2/(a^2+1
)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
x)-27/4*I/(I-a)^4*b^3/(a^2+1)*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I/(I-a)
^4*b^3/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/x)*a^2+3/2*I/(I-a)^4*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(5/
2)-10/3*I/(I-a)^6*b^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-10*I/(I-a)^6*b^3*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)-5/(I-a)^6*b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))
^(1/2)*x-5/(I-a)^6*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a-5/(I-a
)^6*b^4*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I
-a)/b))^(1/2))/(b^2)^(1/2)+10/3*I/(I-a)^6*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(
I-a)/b))^(3/2)-3/(I-a)^4*b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x-
3/(I-a)^4*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+2*I/(I-a)^4*b^3
*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)-4/(I-a)^5*b/(x-I/b+a/b)^2*((x-
(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-9/4*I/(I-a)^4*b^4*a^3/(a^2+1)^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/6*I/(I-a)^3*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a
*b*x+a^2+1)^(5/2)+3/2*I/(I-a)^4*b^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(
5/2)-9/4*I/(I-a)^4*b^4*a/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*I/(I
-a)^3*a^6*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/(b^2)^(1/2)-18*I/(I-a)^5*b^4*a^4/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/6*I/(I-a)^3*a^2*b^4/(a^2+1)^3*
(b^2*x^2+2*a*b*x+a^2+1)^(3/2)*x-9/4*I/(I-a)^4*b^4/(a^2+1)*a*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*x-27/4*I/(I-a)^4*b^4/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/6*I/(I-a)^3*a*b/(a^2+1)^2/x^2*(b
^2*x^2+2*a*b*x+a^2+1)^(5/2)-27/4*I/(I-a)^4*b^4*a^3/(a^2+1)^2*ln((b^2*x+a*b)
/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-9/4*I/(I-a)^4*b^4*a
/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)+1/4*I/(I-a)^3*a^2*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/4*I/(I-a)^3*a^2*b^4/(a^2+1)^3*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*x+1/4*I/(I-a)^3*a^4*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*x+3/4*I/(I-a)^3*a^4*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/4*I/(I-a)^3*a^2*b^4/(a^2+1)^2*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*x-1/(I-a)^4/(x-I/b+a/b)^3*((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(5/2)+4/(I-a)^5*b^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/
2)-9/2*I/(I-a)^4*b^3/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+9/2*I/(I-a)^4*b^
3/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/x)+1/3*I/(I-a)^3/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(5/2)-15*I/(I-
a)^6*b^3*a^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+10*I/(I-a)^6*b^3*(a^2+1)^(1/2)*l
n((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+6*I/(I
-a)^5*b^4*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*x+6*I/(I-a)^5*b^3*((x
-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)*a+6*I/(I-a)^5*b^4*ln((I*b+(x-(I-a)
```

```
/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/
2)-2*I/(I-a)^4*b/(x-I/b+a/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**4,x)
[Out] Timed out
```

3.216 $\int e^{\frac{1}{2} i \tan^{-1}(a+bx)} x^2 \, dx$

Optimal. Leaf size=494

$$-\frac{\left(-8ia^2+4a+3i\right)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{8b^3}-\frac{\left(-8ia^2+4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\cdots$$

```
[Out] -1/8*(3*I+4*a-8*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3-1/12*(I+
8*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(5/4)/b^3+1/3*x*(1-I*a-I*b*x)^(3/4)*
(1+I*a+I*b*x)^(5/4)/b^2+1/16*(3*I+4*a-8*I*a^2)*arctan(1-(1-I*a-I*b*x)^(1/4)
*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(3*I+4*a-8*I*a^2)*arctan(1+(
1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/32*(3*I+4*a-8
*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)+1/32*(3*I+4*a-8*I*a^2)*ln(1+(1-I*a-I
*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(
1/2))/b^3*2^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-8ia^2+4a+3i\right)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{8b^3}-\frac{\left(-8ia^2+4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -((3*I + 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/
(8*b^3) - ((I + 8*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(12*b
^3) + (x*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(3*b^2) + ((3*I +
 4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I
*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I + 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*
I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] -
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3
) + ((3*I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqr
t[2]*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx=\int \frac{x^2\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$

$$=\frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2}+\frac{\int \frac{\sqrt[4]{1+ia+ibx}\left(-1-a^2-\frac{1}{2}(i+8a)bx\right)}{\sqrt[4]{1-ia-ibx}}\,dx}{3b^2}$$

$$=-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3}+\frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2}-\frac{(3-4i}{}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

$$=-\frac{\left(3i+4a-8ia^2\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3}-\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx}{12b^3}$$

Mathematica [C] time = 0.09, size = 121, normalized size = 0.24

$$\frac{(-i(a+bx+i))^{3/4}\left(2i\sqrt[4]{2}\left(8a^2+4ia-3\right){}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)-i\sqrt[4]{ia+ibx+1}\left(8a^2+a(4bx-7i)-\right.\right.}{12b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*((-I)*(1 + I*a + I*b*x)^(1/4)*(1 + 8*a^2 + (5*I
)*b*x - 4*b^2*x^2 + a*(-7*I + 4*b*x)) + (2*I)*2^(1/4)*(-3 + (4*I)*a + 8*a^2
)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3)
```

fricas [A] time = 0.51, size = 554, normalized size = 1.12

$$3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}\,\log\left(\frac{i\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}+\left(8\,a^2+4i\,a-3\right)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2+4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24}{b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((I*b^
3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a -
3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I*a
- 3)) - 3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((-I
*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a
 - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I
*a - 3)) + 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)*log
((I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^2 + 4
*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 +
 4*I*a - 3)) - 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)
*log((-I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^
2 + 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*
a^2 + 4*I*a - 3)) + 2*(8*b^3*x^3 - 2*I*b^2*x^2 + 8*a^3 + (8*I*a - 1)*b*x +
34*I*a^2 - 37*a - 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.03Done
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{\frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2 + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x**2,x)
```

```
[Out] Timed out
```

3.217 $\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}+\frac{(1-4ia)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{4b^2}+\frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}\right.}{8\sqrt{2}b^2}$$

```
[Out] 1/4*(1-4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x)
^(3/4)*(1+I*a+I*b*x)^(5/4)/b^2-1/8*(1-4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/8*(1-4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/16*(1-4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)-1/16*(1-4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}+\frac{(1-4ia)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{4b^2}+\frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b^2) - ((1 - (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) + ((1 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx=\int \frac{x\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}dx$$
$$=\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}-\frac{(i+4a)\int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}dx}{4b}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}-\frac{(i+4a)\int \frac{}{\sqrt[4]{1-i}}}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}+\frac{(1-4ia)\,\mathrm{Subs}}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}+\frac{(1-4ia)\,\mathrm{Subs}}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}-\frac{(1-4ia)\,\mathrm{Subst}}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}+\frac{(1-4ia)\,\mathrm{Subs}}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}+\frac{(1-4ia)\log\Big(}{}$$
$$=\frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2}-\frac{(1-4ia)\tan^{-1}}{4}$$

Mathematica [C] time = 0.05, size = 81, normalized size = 0.20

$$\frac{(-i(a+bx+i))^{3/4}\left(2\sqrt[4]{2}(1-4ia)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)+3(ia+ibx+1)^{5/4}\right)}{6b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(1 + I*a + I*b*x)^(5/4) + 2*2^(1/4)*(1 - (4*
I)*a)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(6*b^2)
```

fricas [A] time = 0.50, size = 415, normalized size = 1.01

$$-\frac{b^2\sqrt{\frac{16ia^2-8a-i}{b^4}}\log\left(\frac{ib^2\sqrt{\frac{16ia^2-8a-i}{b^4}}+(4a+i)\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}}{4a+i}\right)-b^2\sqrt{\frac{16ia^2-8a-i}{b^4}}\log\left(\frac{-ib^2\sqrt{\frac{16ia^2-8a-i}{b^4}}+(4a+i)\sqrt{\frac{i\sqrt{b^2x^2+2abx}}{bx+a+i}}}{4a+i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((I*b^2*sqrt((16*I*a^2 - 8*a -
I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/(4*a + I)) - b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((-I*b^2*sqrt((16*I*a^
```

```
2 - 8*a - I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
 + a + I)))/(4*a + I)) + b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log((I*b^2*sqr
t((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)))/(4*a + I)) - b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log
((-I*b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + I)) - 2*(2*b^2*x^2 - 2*a^2 - I*b*
x - 5*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.21Done
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1+i\left(bx+a\right)}{\sqrt{1+\left(bx+a\right)^{2}}}}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{\left(bx+a\right)^{2}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{\left(a+b\,x\right)^{2}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x,x)
```

```
[Out] Timed out
```

3.218 $\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[Out] I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-1/2*I*arctan(1-(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/2*I*arctan(1+(1-I*a-I*b*x)^(1/
4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2^
(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2
)-1/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - (I*ArcTan[1 - (Sqrt
[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*Arc
Tan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) + ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) - ((I/2)*L
og[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*
b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{\frac{1}{2}i\tan^{-1}(a+bx)}dx = \int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}dx$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}dx$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}dx, x\right.}{b}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{i\,\text{Subst}\left(\int \frac{}{1+\sqrt{2}}\right.}{}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-i}}{\sqrt{1+ia+i}}\right.}{2\sqrt{}}$$
$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} -$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{5}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{5b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[5/4, 2, 9/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

fricas [A] time = 0.45, size = 257, normalized size = 0.76

$$\frac{b\sqrt{\frac{i}{b^2}}\log\left(ib\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right) - b\sqrt{\frac{i}{b^2}}\log\left(-ib\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right) + b\sqrt{-\frac{i}{b^2}}\log\left(ib\sqrt{-\frac{}{b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*sqrt(I/b^2)*log(I*b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-I*b*sqrt(I/b^2) + sqrt(I*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2)*log(I*b*sqrt(-
I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(
-I/b^2)*log(-I*b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I))) + (2*b*x + 2*a + 2*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I)))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueEvaluation time: 0.54Done
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{i\,(a+bx)+1}{\sqrt{(a+bx)^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)
```

3.219 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{-a+i}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right.}{\sqrt[4]{a+i}}$$

```
[Out] -2*(I-a)^(1/4)*arctan((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x
+a))^(1/4))/(I+a)^(1/4)-2*(I-a)^(1/4)*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/
4)/(I-a)^(1/4)/(1-I*(b*x+a))^(1/4))/(I+a)^(1/4)-1/2*ln(1-(1+I*(b*x+a))^(1/4
)*2^(1/2)/(1-I*(b*x+a))^(1/4)+(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*2^(1
/2)+1/2*ln(1+(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-I*(b*x+a))^(1/4)+(1+I*(b*x+a))^
(1/2)/(1-I*(b*x+a))^(1/2))*2^(1/2)-arctan(1-(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-
I*(b*x+a))^(1/4))*2^(1/2)+arctan(1+(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-I*(b*x+a)
)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5094, 445, 481, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{-a+i}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right.}{\sqrt[4]{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (-2*(I - a)^(1/4)*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(
1/4)*(1 - I*(a + b*x))^(1/4))])/(I + a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] - (2*(I - a)^(1/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a +
b*x))^(1/4))])/(I + a)^(1/4) - Log[1 - (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1
 - I*(a + b*x))^(1/4) + Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2
] + Log[1 + (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4) + Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 445

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
```

```
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx = 8\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)\left(1-ia-\frac{1+ia}{x^4}\right)x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= 8\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)\left(-1-ia+(1-ia)x^4\right)}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + (4(1+ia))\,\text{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4}\,dx\right.$$

$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) -$$

$$= -\frac{2\sqrt[4]{i-a}\,\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\,\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+}{-1-\sqrt{2}}\right.}{}$$

$$= -\frac{2\sqrt[4]{i-a}\,\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\,\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\log\left(1-\frac{\sqrt{2}\,\sqrt[4]{1+i}}{\sqrt[4]{1-i(a}}\right.}{}$$

$$= -\frac{2\sqrt[4]{i-a}\,\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \sqrt{2}\,\tan^{-1}\left(1+\right.$$

Mathematica [C] time = 0.10, size = 124, normalized size = 0.31

$$\frac{2}{3}(-i(a+bx+i))^{3/4}\left(\frac{2(a-i)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)(ia+ibx+1)^{3/4}} - \sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(-(2^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (
-1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[3/4, 1, 7/4, (1 + a
^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^
(3/4))))/3
```

fricas [A] time = 0.49, size = 414, normalized size = 1.05

$$\frac{1}{2}\sqrt{4i}\,\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-\frac{1}{2}\sqrt{4i}\,\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
```

```
2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log
(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))
 - (-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)) + (-(a - I)/(a + I))^(1/4)) - I*(-(a - I)/(a + I))^(1/4)*log(s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + I*(-(a - I)/(a + I
))^(1/4)) + I*(-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(b*x + a + I)) - I*(-(a - I)/(a + I))^(1/4)) + (-(a - I)/(a + I))^
(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-(a -
I)/(a + I))^(1/4))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.06Done
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

3.220 $\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt[4]{1+i(a+bx)}\,(a+bx+i)}{(a+i)x\sqrt[4]{1-i(a+bx)}}+\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}+\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}$$

```
[Out] -(I+a+b*x)*(1+I*(b*x+a))^(1/4)/(I+a)/x/(1-I*(b*x+a))^(1/4)+I*b*arctan((I+a)
^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x+a))^(1/4))/(I-a)^(3/4)/(I+
a)^(5/4)+I*b*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x+
a))^(1/4))/(I-a)^(3/4)/(I+a)^(5/4)
```

Rubi [A] time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5094, 263, 288, 212, 208, 205}

$$-\frac{\sqrt[4]{1+i(a+bx)}\,(a+bx+i)}{(a+i)x\sqrt[4]{1-i(a+bx)}}+\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}+\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((I + a + b*x)*(1 + I*(a + b*x))^(1/4))/((I + a)*x*(1 - I*(a + b*x))^(1/4
))) + (I*b*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1
 - I*(a + b*x))^(1/4))])/((I - a)^(3/4)*(I + a)^(5/4)) + (I*b*ArcTanh[((I +
 a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))]
)/((I - a)^(3/4)*(I + a)^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx = (8ib)\,\text{Subst}\left(\int \frac{1}{\left(1-ia-\frac{1+ia}{x^4}\right)^2 x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= (8ib)\,\text{Subst}\left(\int \frac{x^4}{\left(-1-ia+(1-ia)x^4\right)^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{i+a}$$

$$= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}\,x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{i-a}\,(1-ia)} + \frac{b\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} + \frac{ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}} + \frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}}$$

Mathematica [C] time = 0.03, size = 110, normalized size = 0.54

$$\frac{(-i(a+bx+i))^{3/4}\left(2ibx\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+3(a+i)(a+bx-i)\right)}{3(a+i)^2x(ia+ibx+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(I + a)*(-I + a + b*x) + (2*I)*b*x*Hypergeom
etric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])
)/(3*(I + a)^2*x*(1 + I*a + I*b*x)^(3/4))
```

fricas [B] time = 0.61, size = 612, normalized size = 2.99

$$\left(-\frac{b^4}{16\,a^8+32i\,a^7+32\,a^6+96i\,a^5+96i\,a^3-32\,a^2+32i\,a-16}\right)^{\frac{1}{4}}(-i\,a+1)x\log\left(\frac{b\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+2\left(-\frac{b^4}{16\,a^8+32i\,a^7+32\,a^6+96i\,a^5+96i\,a^3-32\,a^2+3}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] ((-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a
- 16))^(1/4)*(-I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)) + 2*(-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 -
 32*a^2 + 32*I*a - 16))^(1/4)*(a^2 + 1))/b) + (-b^4/(16*a^8 + 32*I*a^7 + 32
*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(I*a - 1)*x*log((
b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - 2*(-b^4/(16*a^8
 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(
a^2 + 1))/b) + (-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32
*a^2 + 32*I*a - 16))^(1/4)*(a + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)) + (-b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 9
6*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(2*I*a^2 + 2*I))/b) - (-b^4/(16*a^8
+ 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 16))^(1/4)*(a
 + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-
b^4/(16*a^8 + 32*I*a^7 + 32*a^6 + 96*I*a^5 + 96*I*a^3 - 32*a^2 + 32*I*a - 1
6))^(1/4)*(-2*I*a^2 - 2*I))/b) - (b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.19Done
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2,x)
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
[Out] Timed out
```

3.221 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$-\frac{\left(-24ia^2+36a+17i\right)\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{24b^3}+\frac{\left(-24ia^2+36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right.}{16\sqrt{2}\,b^3}$$

```
[Out] -1/24*(17*I+36*a-24*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3-1/12
*(3*I+8*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(7/4)/b^3+1/3*x*(1-I*a-I*b*x)^
(1/4)*(1+I*a+I*b*x)^(7/4)/b^2+1/16*(17*I+36*a-24*I*a^2)*arctan(1-(1-I*a-I*b
*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(17*I+36*a-24*I*a^2
)*arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)+1/3
2*(17*I+36*a-24*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)
+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)-1/32*(17*I+36*a-24*I*
a^2)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/
2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(-24ia^2+36a+17i\right)\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{24b^3}+\frac{\left(-24ia^2+36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right.}{16\sqrt{2}\,b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -((17*I + 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4
))/(24*b^3) - ((3*I + 8*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))
/(12*b^3) + (x*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(3*b^2) + (
(17*I + 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
+ I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*ArcTan
[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]
*b^3) + ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 +
I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/
(16*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x
]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*
x)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{x\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{3b^2} + \frac{\int \frac{(1+ia+ibx)^{3/4}\left(-1-a^2-\frac{1}{2}(3i+8a)bx\right)}{(1-ia-ibx)^{3/4}}\,dx}{3b^2}$$

$$= -\frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17-36ia-}{}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(17i+36a-24ia^2\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx}{12b^3}$$

Mathematica [C] time = 0.09, size = 121, normalized size = 0.24

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(2i2^{3/4}\left(24a^2+36ia-17\right){}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)-i(ia+ibx+1)^{3/4}\left(8a^2+a(4bx-5i)-\right.}{12b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*((-I)*(1 + I*a + I*b*x)^(3/4)*(3 + 8*a^2 + (7*I
)*b*x - 4*b^2*x^2 + a*(-5*I + 4*b*x)) + (2*I)*2^(3/4)*(-17 + (36*I)*a + 24*
a^2)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3)
```

fricas [A] time = 0.48, size = 561, normalized size = 1.14

$$\frac{3\,b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}\,\log\left(\frac{b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}+\left(24\,a^2+36i\,a-17\right)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2+36i\,a-17}\right)-3\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6)*
log((b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6) + (
24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I
)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2
 + 1224*a + 289*I)/b^6)*log(-(b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 +
 1224*a + 289*I)/b^6) - (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((-576*I*a
^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)*log((b^3*sqrt((-576*I*a^4
 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6) + (24*a^2 + 36*I*a - 17)*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 1
7)) + 3*b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
*log(-(b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
- (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/(24*a^2 + 36*I*a - 17)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(8*I*b
^2*x^2 - 2*(4*I*a - 7)*b*x + 8*I*a^2 - 46*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/b^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.42Done
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="maxim
a")
[Out] integrate(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{\sqrt{(a + b\,x)^2 + 1}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x**2,x)
[Out] Timed out
```

3.222 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{7/4}}{2b^2}+\frac{(3-4ia)\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{4b^2}-\frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-i\ldots}}{\sqrt[4]{ia+\ldots}}\right.}{8\sqrt{2}b^2}$$

```
[Out] 1/4*(3-4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x)
^(1/4)*(1+I*a+I*b*x)^(7/4)/b^2-3/8*(3-4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+3/8*(3-4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/16*(3-4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)+3/16*(3-4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{7/4}}{2b^2}+\frac{(3-4ia)\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{4b^2}-\frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-i\ldots}}{\sqrt[4]{ia+\ldots}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((3 - (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b^2) - (3*(3 - (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 - (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} - \frac{(3i+4a)\int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx}{4b}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} - \frac{(3(3i+4a))\int}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{S}}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{S}}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{S}}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\,\mathrm{S}}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\log}{}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\tan}{}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.19

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(2\,2^{3/4}(3-4ia)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)+(ia+ibx+1)^{7/4}\right)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*((1 + I*a + I*b*x)^(7/4) + 2*2^(3/4)*(3 - (4*I)
*a)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(2*b^2)
```

fricas [A] time = 0.46, size = 421, normalized size = 1.03

$$-\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}\,\log\left(\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}+(12\,a+9i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{12\,a+9i}\right)-b^2\sqrt{\frac{144i\,a^2-216\,a-81i}{b^4}}\,\log\left(-\frac{b^2\sqrt{\frac{144i\,a^2-216\,a-}{b^4}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4)*log((b^2*sqrt((144*I*a^2 - 2
16*a - 81*I)/b^4) + (12*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/(12*a + 9*I)) - b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4)*log
(-(b^2*sqrt((144*I*a^2 - 216*a - 81*I)/b^4) - (12*a + 9*I)*sqrt(I*sqrt(b^2*
```

```
x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a + 9*I)) - b^2*sqrt((-144*I*a
^2 + 216*a + 81*I)/b^4)*log((b^2*sqrt((-144*I*a^2 + 216*a + 81*I)/b^4) + (1
2*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a +
 9*I)) + b^2*sqrt((-144*I*a^2 + 216*a + 81*I)/b^4)*log(-(b^2*sqrt((-144*I*a
^2 + 216*a + 81*I)/b^4) - (12*a + 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I)))/(12*a + 9*I)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2
*I*b*x - 2*I*a + 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/b^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.42Done
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\frac{1+i\left(bx+a\right)}{\sqrt{1+\left(bx+a\right)^{2}}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x,x)
```

```
[Out] Timed out
```

3.223 $\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$\frac{i\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}\,b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}\,b}$$

```
[Out] I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-3/2*I*arctan(1-(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/2*I*arctan(1+(1-I*a-I*b*x)^(1/
4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)-3/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2^
(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2
)+3/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{i\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}\,b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - ((3*I)*ArcTan[1 - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((
3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])
/(Sqrt[2]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*
x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b
) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[
2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{\frac{3}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{3}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.}{b}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}\,b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ib}}{\sqrt{1+ia+ib}}\right.}{2\sqrt{}}$$
$$= \frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}\,b} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}\,b} -$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{7}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{7b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[7/4, 2, 11/4, -
E^((2*I)*ArcTan[a + b*x])])/b
```

fricas [A] time = 0.46, size = 268, normalized size = 0.79

$$b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{-\frac{9i}{b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*b*sqrt(9*I/b^2) + s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-9*I/b^2)*
log(1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I))) + b*sqrt(-9*I/b^2)*log(-1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.24Done
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^{2}}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^{2}+1}}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

3.224 $\int \frac{e^{\frac{3}{2} i \tan^{-1}(a+bx)}}{x} \, dx$

Optimal. Leaf size=427

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}+\frac{2(-a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right.}{(a+i)^{3/4}}$$

```
[Out] 2*(I-a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b
*x)^(1/4))/(I+a)^(3/4)-2*(I-a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4
)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I+a)^(3/4)+1/2*ln(1-(1-I*a-I*b*x)^(1/4)
*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/
2)-1/2*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(
1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I
*a+I*b*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)
^(1/4))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5095, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 205, 208}

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}+\frac{2(-a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+}}\right.}{(a+i)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(I - a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1
/4)*(1 - I*a - I*b*x)^(1/4))])/(I + a)^(3/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I - a)^(3/4)*A
rcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*
b*x)^(1/4))])/(I + a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
 - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
- I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2} i \tan^{-1}(a+bx)}}{x} dx = \int \frac{(1+ia+ibx)^{3/4}}{x(1-ia-ibx)^{3/4}} dx$$

$$= -\left((-1-ia)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}} dx\right) + (ib)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ib\ldots}}$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}} dx, x, \sqrt[4]{1-ia-ibx}\right)\right) + (4(1+ia))\,\text{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia\ldots}\right.$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\right) - \frac{(2(i-a))\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}\,x^2} dx, x, \frac{\sqrt[4]{1\ldots}}{\sqrt[4]{1\ldots}}\right.}{\sqrt{i+a}}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} - \frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} - 2\,\text{Subst}\left(\int \frac{1-\ldots}{1+\ldots}\right.$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} - \frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\ldots}{-1-\sqrt{2}\ldots}\right.}{\ldots}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} - \frac{2(i-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-\ldots}}{\sqrt{1+ia+\ldots}}\right.}{\ldots}$$

$$= \frac{2(i-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \sqrt{2}\tan^{-1}\left(1+\ldots\right.$$

Mathematica [C] time = 0.10, size = 122, normalized size = 0.29

$$2\sqrt[4]{-i(a+bx+i)}\left(\frac{2(a-i)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)\sqrt[4]{ia+ibx+1}} - 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] 2*((-I)*(I + a + b*x))^(1/4)*(-(2^(3/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (-
1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[1/4, 1, 5/4, (1 + a^
2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^(
1/4)))
```

fricas [B] time = 0.51, size = 690, normalized size = 1.62

$$\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-\frac{1}{2}\sqrt{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I
) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*
I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I))) - (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log
(((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^
(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(a^2 - 2*I*a - 1)) + (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3
*a - I))^(1/4)*log(-((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3
*I*a^2 - 3*a - I))^(3/4) - (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) + I*(-(a^3 - 3*I*a^2 - 3*a +
I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((I*a^2 - 2*a - I)*(-(a^3 - 3*I*a^2
 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) - I*(-(a^
3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((-I*a^2 + 2*a
+ I)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 -
2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 -
2*I*a - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
```

```
c index_m operator + Error: Bad Argument ValueWarning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Arg
ument ValueWarning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming 0=[0,0,0]in
dex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 2.32Don
e
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^{2}}}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^{2}+1}}\right)^{3/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.225 $\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=211

$$-\frac{\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{(1-ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\,\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}\,(a+i)^{7/4}}+\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\,\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}\,(a+i)^{7/4}}$$

```
[Out] -(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/(1-I*a)/x-3*I*b*arctan((I+a)^(1/4)
*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(1/4)/(I+a)^(7/
4)+3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^
(1/4))/(I-a)^(1/4)/(I+a)^(7/4)
```

Rubi [A] time = 0.11, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5095, 94, 93, 298, 205, 208}

$$-\frac{\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{(1-ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\,\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}\,(a+i)^{7/4}}+\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\,\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\,\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}\,(a+i)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/((1 - I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(7/4)) + ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(1/4)*(I + a)^(7/4))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1+ia+ibx)^{3/4}}{x^2(1-ia-ibx)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(3b)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{2(i+a)}\\
&= -\frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(6b)\,\text{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i+a}\\
&= -\frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{(1-ia)x} + \frac{(3ib)\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}\,x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/2}} - \frac{(3ib)\,\text{S}\cdots}{\cdots}\\
&= -\frac{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}\,(i+a)^{7/4}} + \frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ib\cdots}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ib\cdots}}\right)}{\sqrt[4]{i-a}\,(i+a)^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 106, normalized size = 0.50

$$\frac{\sqrt[4]{-i(a+bx+i)}\left(6ibx\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{(a+i)^2x\sqrt[4]{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x + (6*I)*b*x*Hypergeome
tric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))
/((I + a)^2*x*(1 + I*a + I*b*x)^(1/4))
```

fricas [B] time = 0.48, size = 711, normalized size = 3.37

$$\frac{3\left(-\frac{b^4}{16a^8+96ia^7-224a^6-224ia^5-224ia^3+224a^2+96ia-16}\right)^{\frac{1}{4}}(-ia+1)x\log\left(\frac{b^3\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}+\left(8a^6+32ia^5-40a^4-40a^2-32ia+\cdots\right.}{b^3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="frica
s")
```

```
[Out] (3*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 9
6*I*a - 16))^(1/4)*(-I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)) + (8*a^6 + 32*I*a^5 - 40*a^4 - 40*a^2 - 32*I*a + 8)*(
-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*
a - 16))^(3/4))/b^3) + 3*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 2
24*I*a^3 + 224*a^2 + 96*I*a - 16))^(1/4)*(I*a - 1)*x*log((b^3*sqrt(I*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (8*a^6 + 32*I*a^5 - 40*a^4 - 4
0*a^2 - 32*I*a + 8)*(-b^4/(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*
a^3 + 224*a^2 + 96*I*a - 16))^(3/4))/b^3) - 3*(-b^4/(16*a^8 + 96*I*a^7 - 22
4*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16))^(1/4)*(a + I)*x*log
((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (8*I*a^6 -
32*a^5 - 40*I*a^4 - 40*I*a^2 + 32*a + 8*I)*(-b^4/(16*a^8 + 96*I*a^7 - 224*a
^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16))^(3/4))/b^3) + 3*(-b^4/
(16*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 1
6))^(1/4)*(a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)) + (-8*I*a^6 + 32*a^5 + 40*I*a^4 + 40*I*a^2 - 32*a - 8*I)*(-b^4/(1
6*a^8 + 96*I*a^7 - 224*a^6 - 224*I*a^5 - 224*I*a^3 + 224*a^2 + 96*I*a - 16)
)^(3/4))/b^3) - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.26Done
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{{\left(a+b\,x\right)}^2+1}}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.226 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$\frac{\left(-8ia^2-4a+3i\right)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{8b^3}+\frac{\left(-8ia^2-4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}\,b^3}-\frac{(-8ia}{}$$

```
[Out] 1/8*(3*I-4*a-8*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/12*(I-8
*a)*(1-I*a-I*b*x)^(5/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/3*x*(1-I*a-I*b*x)^(5/4)*(
1+I*a+I*b*x)^(3/4)/b^2+1/16*(3*I-4*a-8*I*a^2)*arctan(1-(1-I*a-I*b*x)^(1/4)*
2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(3*I-4*a-8*I*a^2)*arctan(1+(1
-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)+1/32*(3*I-4*a-8*
I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(
1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)-1/32*(3*I-4*a-8*I*a^2)*ln(1+(1-I*a-I*
b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1
/2))/b^3*2^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\left(-8ia^2-4a+3i\right)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{8b^3}+\frac{\left(-8ia^2-4a+3i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}\,b^3}-\frac{(-8ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((3*I - 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(
8*b^3) + ((I - 8*a)*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(12*b^
3) + (x*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(3*b^2) + ((3*I -
4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*
b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I - 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) + ((3*I
 - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3)
 - ((3*I - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*
b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt
[2]*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} + \frac{\int \frac{\sqrt[4]{1-ia-ibx}\left(-1-a^2+\frac{1}{2}(i-8a)bx\right)}{\sqrt[4]{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{\left(3+4ia-\right.}{}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

$$= -\frac{\left(4a-i\left(3-8a^2\right)\right)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx}{12b^3}$$

Mathematica [C] time = 0.10, size = 99, normalized size = 0.20

$$\frac{(-i(a+bx+i))^{5/4}\left(3\,2^{3/4}\left(8ia^2+4a-3i\right)\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{1}{2}i(a+bx+i)\right)+5(ia+ibx+1)^{3/4}(-8a+4bx+i)\right)}{60b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(5/4)*(5*(1 + I*a + I*b*x)^(3/4)*(I - 8*a + 4*b*x) +
3*2^(3/4)*(-3*I + 4*a + (8*I)*a^2)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2*I
)*(I + a + b*x)]))/(60*b^3)
```

fricas [A] time = 0.48, size = 561, normalized size = 1.14

$$3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}\,\log\left(\frac{b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}+\left(8\,a^2-4i\,a-3\right)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2-4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2\cdots}{b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log((b^3*
sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) + (8*a^2 - 4*I*a - 3)
*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
3)) - 3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log(-(b^3
*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) - (8*a^2 - 4*I*a - 3
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
 3)) - 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log((b^
3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) + (8*a^2 - 4*I*a -
 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a
 - 3)) + 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log(-
(b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) - (8*a^2 - 4*I*
a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*
I*a - 3)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2 - 2*(-4*I*a -
 5)*b*x - 8*I*a^2 - 26*a + 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/b^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.67Done
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxim
a")
[Out] integrate(x^2/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
[Out] Integral(x**2/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
)
```

3.227 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}+\frac{(1+4ia)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{4b^2}+\frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-\ldots}}{\sqrt[4]{ia-\ldots}}\right.}{8\sqrt{2}b^2}$$

```
[Out] 1/4*(1+4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x)
^(5/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/8*(1+4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-1/8*(1+4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/16*(1+4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)-1/16*(1+4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}+\frac{(1+4ia)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{4b^2}+\frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-\ldots}}{\sqrt[4]{ia-\ldots}}\right.}{8\sqrt{2}b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((1 + (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(2*b^2) + ((1 + (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) - ((1 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$
$$= \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx}{4b}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{S}}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{S}}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{S}}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\,\mathrm{S}}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(1+4ia)\,\mathrm{l}}{}$$
$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(1+4ia)\,\mathrm{t}}{}$$

Mathematica [C] time = 0.06, size = 84, normalized size = 0.20

$$-\frac{i(-i(a+bx+i))^{5/4}\left(2^{3/4}(4a-i)\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{1}{2}i(a+bx+i)\right)+5i(ia+ibx+1)^{3/4}\right)}{10b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-1/10*I)*((-I)*(I + a + b*x))^(5/4)*((5*I)*(1 + I*a + I*b*x)^(3/4) + 2^(3
/4)*(-I + 4*a)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2*I)*(I + a + b*x)]))/b
^2
```

fricas [A] time = 0.45, size = 421, normalized size = 1.03

$$-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}+(4\,a-i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-i}\right)-b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}-(4\,a-i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,a}}{bx+a}}}{4\,a-i}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log((b^2*sqrt((16*I*a^2 + 8*a - I)
/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/
```

```
(4*a - I)) - b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log(-(b^2*sqrt((16*I*a^2 +
8*a - I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(4*a - I)) - b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log((b^2*sqrt((-16
*I*a^2 - 8*a + I)/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)))/(4*a - I)) + b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log(-(b^2
*sqrt((-16*I*a^2 - 8*a + I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(4*a - I)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(-2*I*b*x + 2*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.96Done
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
[Out] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
[Out] Integral(x/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

3.228 $\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=338

$$-\frac{i\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[Out] -I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-1/2*I*arctan(1-(1-I*a-I*b*x)^(
1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/2*I*arctan(1+(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)-1/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/
2)+1/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)
^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5093, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{-ia-ibx+1}\,(ia+ibx+1)^{3/4}}{b}-\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}+\frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - (I*ArcTan[1 - (S
qrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqr
t[2]*b) - ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((I/2
)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a -
 I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{-\frac{1}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{1}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.}{b}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\right.}{}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right.}{2\sqrt{}}$$
$$= -\frac{i\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}{b} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} -$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.13

$$-\frac{8ie^{\frac{3}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{3b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-1/2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[3/4, 2, 7/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

fricas [A] time = 0.48, size = 266, normalized size = 0.79

$$\frac{b\sqrt{\frac{i}{b^2}}\log\left(b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{i}{b^2}}\log\left(b\sqrt{-\frac{i}{b^2}}+\sqrt{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(b*sqrt(I/b^2)*log(b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-I/b^2)*log(b*sqrt(-I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2
)*log(-b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I))) - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(b*x + a + I)))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueEvaluation time: 1.31Done
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{i(a+bx)+1}{\sqrt{(a+bx)^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)
```

3.229 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{a+i}\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}}$$

```
[Out] -2*(I+a)^(1/4)*arctan((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x
+a))^(1/4))/(I-a)^(1/4)-2*(I+a)^(1/4)*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/
4)/(I+a)^(1/4)/(1+I*(b*x+a))^(1/4))/(I-a)^(1/4)-1/2*ln(1-(1-I*(b*x+a))^(1/4
)*2^(1/2)/(1+I*(b*x+a))^(1/4)+1/(1+I*(b*x+a))^(1/2)*(1-I*(b*x+a))^(1/2))*2^
(1/2)+1/2*ln(1+(1-I*(b*x+a))^(1/4)*2^(1/2)/(1+I*(b*x+a))^(1/4)+1/(1+I*(b*x+
a))^(1/2)*(1-I*(b*x+a))^(1/2))*2^(1/2)-arctan(1-(1-I*(b*x+a))^(1/4)*2^(1/2)
/(1+I*(b*x+a))^(1/4))*2^(1/2)+arctan(1+(1-I*(b*x+a))^(1/4)*2^(1/2)/(1+I*(b*
x+a))^(1/4))*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5094, 481, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}-\frac{2\sqrt[4]{a+i}\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-2*(I + a)^(1/4)*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(
1/4)*(1 + I*(a + b*x))^(1/4))])/(I - a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] - (2*(I + a)^(1/4)*
ArcTanh[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a +
b*x))^(1/4))])/(I - a)^(1/4) - Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a
+ b*x)] - (Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2
] + Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a + b*x)] + (Sqrt[2]*(1 - I*(
a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x}\,dx = -\left(8\,\text{Subst}\left(\int \frac{x^4}{(1+x^4)\left(1-ia-(1+ia)x^4\right)}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - (4(1-ia))\,\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,d$$
$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)$$
$$= -\frac{2\sqrt[4]{i+a}\,\tan^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\,\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{}}\right.$$
$$= -\frac{2\sqrt[4]{i+a}\,\tan^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\,\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \log\left(1+\frac{\sqrt{1-i(}}{\sqrt{1+i(}}\right.$$
$$= -\frac{2\sqrt[4]{i+a}\,\tan^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + \sqrt{2}\,\tan^{-1}\left(1\right.$$

Mathematica [C] time = 0.04, size = 126, normalized size = 0.32

$$\frac{2\sqrt[4]{-i(a+bx+i)}\left(2^{3/4}\sqrt[4]{ia+ibx+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{1}{2}i(a+bx+i)\right)-2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)\right)}{\sqrt[4]{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(1/4)*(2^(3/4)*(1 + I*a + I*b*x)^(1/4)*Hypergeometr
ic2F1[1/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[1/4, 1,
5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(1 + I*a + I*b*
x)^(1/4)
```

fricas [A] time = 0.54, size = 470, normalized size = 1.19

$$-\frac{1}{2}\sqrt{4i}\,\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{4i}\,\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*
I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4
*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
 a + I))) + (-(a + I)/(a - I))^(1/4)*log(((a - I)*(-(a + I)/(a - I))^(3/4)
+ (a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
 - (-(a + I)/(a - I))^(1/4)*log(-((a - I)*(-(a + I)/(a - I))^(3/4) - (a + I
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) - I*(-(
a + I)/(a - I))^(1/4)*log(((I*a + 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqr
```

```
t(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) + I*(-(a + I
)/(a - I))^(1/4)*log(((-I*a - 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.67Done
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^{2}}}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^{2}+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(1/(x*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))),
x)
```

3.230 $\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=210

$$-\frac{\sqrt[4]{1-i(a+bx)}\,(-a-bx+i)}{(-a+i)x\sqrt[4]{1+i(a+bx)}}-\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}-\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}$$

```
[Out] -(I-a-b*x)*(1-I*(b*x+a))^(1/4)/(I-a)/x/(1+I*(b*x+a))^(1/4)-I*b*arctan((I-a)
^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x+a))^(1/4))/(I-a)^(5/4)/(I+
a)^(3/4)-I*b*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x+
a))^(1/4))/(I-a)^(5/4)/(I+a)^(3/4)
```

Rubi [A] time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5094, 288, 212, 208, 205}

$$-\frac{\sqrt[4]{1-i(a+bx)}\,(-a-bx+i)}{(-a+i)x\sqrt[4]{1+i(a+bx)}}-\frac{ib\tan^{-1}\left(\frac{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}-\frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{-a+i}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\,\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((I - a - b*x)*(1 - I*(a + b*x))^(1/4))/((I - a)*x*(1 + I*(a + b*x))^(1/4
))) - (I*b*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1
 + I*(a + b*x))^(1/4))])/((I - a)^(5/4)*(I + a)^(3/4)) - (I*b*ArcTanh[((I -
 a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))]
)/((I - a)^(5/4)*(I + a)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5094

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[(x^(2/(I*n))*(1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m)/(1 + x^(2/(I*n)))^(m + 2), x], x, (1 - I*c*(a + b*x))^
((I*n)/2)/(1 + I*c*(a + b*x))^((I*n)/2)], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx = -\left((8ib)\,\text{Subst}\left(\int \frac{x^4}{\left(1-ia-(1+ia)x^4\right)^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{i-a}$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{i+a}-\sqrt{i-a}\,x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{(1+ia)\sqrt{i+a}} - \frac{b\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{ib\tan^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}} - \frac{ib\tanh^{-1}\left(\frac{\sqrt[4]{i-a}\,\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\,\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}}$$

Mathematica [C] time = 0.03, size = 107, normalized size = 0.51

$$-\frac{\sqrt[4]{-i(a+bx+i)}\left(-2ibx\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{\left(a^2+1\right)x\sqrt[4]{ia+ibx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(1/4)))
```

fricas [B] time = 0.47, size = 728, normalized size = 3.47

$$\left(-\frac{b^4}{16\,a^8-32i\,a^7+32\,a^6-96i\,a^5-96i\,a^3-32\,a^2-32i\,a-16}\right)^{\frac{1}{4}}(-i\,a-1)x\log\left(\frac{b^3\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(8\,a^6-16i\,a^5+8\,a^4-32i\,a^3-8\,a^2-16i\,a-8\right)\left(\cdots\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] ((-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a
- 16))^(1/4)*(-I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)) + (8*a^6 - 16*I*a^5 + 8*a^4 - 32*I*a^3 - 8*a^2 - 16*I*a - 8
)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a
 - 16))^(3/4))/b^3) + (-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a
^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (8*a^6 - 16*I*a^5 + 8*a^4 - 32*I*a^3
- 8*a^2 - 16*I*a - 8)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a
```

```
^3 - 32*a^2 - 32*I*a - 16))^(3/4))/b^3) - (-b^4/(16*a^8 - 32*I*a^7 + 32*a^6
 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(a - I)*x*log((b^3*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (8*I*a^6 + 16*a^5 +
 8*I*a^4 + 32*a^3 - 8*I*a^2 + 16*a - 8*I)*(-b^4/(16*a^8 - 32*I*a^7 + 32*a^6
 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(3/4))/b^3) + (-b^4/(16*a^8
 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(1/4)*(
a - I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) +
 (-8*I*a^6 - 16*a^5 - 8*I*a^4 - 32*a^3 + 8*I*a^2 - 16*a + 8*I)*(-b^4/(16*a^
8 - 32*I*a^7 + 32*a^6 - 96*I*a^5 - 96*I*a^3 - 32*a^2 - 32*I*a - 16))^(3/4))
/b^3) + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)))/((a - I)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.74Done
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{{\left(a+b\,x\right)}^2+1}}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
[Out] Timed out
```

3.231 $\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=494

$$\frac{\left(-24ia^2-36a+17i\right)\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{3/4}}{24b^3}-\frac{\left(-24ia^2-36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}\,b^3}$$

```
[Out] 1/24*(17*I-36*a-24*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/12*
(3*I-8*a)*(1-I*a-I*b*x)^(7/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/3*x*(1-I*a-I*b*x)^(
7/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/16*(17*I-36*a-24*I*a^2)*arctan(1-(1-I*a-I*b*
x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(17*I-36*a-24*I*a^2)
*arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/32
*(17*I-36*a-24*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+
(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)+1/32*(17*I-36*a-24*I*a
^2)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2
)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5095, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(-24ia^2-36a+17i\right)\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{3/4}}{24b^3}-\frac{\left(-24ia^2-36a+17i\right)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}\,b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((17*I - 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4)
)/(24*b^3) + ((3*I - 8*a)*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/
(12*b^3) + (x*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(3*b^2) + ((
17*I - 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
 I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I - 36*a - (24*I)*a^2)*ArcTan[
1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*
b^3) - ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I
*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(
16*Sqrt[2]*b^3) + ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]
/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x
)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$

$$= \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} + \frac{\int \frac{(1-ia-ibx)^{3/4}\left(-1-a^2+\frac{1}{2}(3i-8a)bx\right)}{(1+ia+ibx)^{3/4}}\,dx}{3b^2}$$

$$= \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} + \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} - \frac{(17+36ia-}{}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

$$= -\frac{\left(36a-i\left(17-24a^2\right)\right)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia}}{12b^3}$$

Mathematica [C] time = 0.09, size = 98, normalized size = 0.20

$$\frac{(-i(a+bx+i))^{7/4}\left(\sqrt[4]{2}\left(24ia^2+36a-17i\right){}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{1}{2}i(a+bx+i)\right)+7\sqrt[4]{ia+ibx+1}\,(-8a+4bx+3i)\right)}{84b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(7/4)*(7*(1 + I*a + I*b*x)^(1/4)*(3*I - 8*a + 4*b*x)
+ 2^(1/4)*(-17*I + 36*a + (24*I)*a^2)*Hypergeometric2F1[3/4, 7/4, 11/4, (-1
/2*I)*(I + a + b*x)]))/(84*b^3)
```

fricas [**A**] time = 0.47, size = 555, normalized size = 1.12

$$\frac{3\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}\,\log\left(\frac{i\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}+\left(24\,a^2-36i\,a-17\right)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2-36i\,a-17}\right)-3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6)*
log((I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6) +
 (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/(24*a^2 - 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6)*log((-I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I*a - 17)) + 3*b^3*sqrt((-576
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6)*log((I*b^3*sqrt((-576
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6) + (24*a^2 - 36*I*a -
17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I
*a - 17)) - 3*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I
)/b^6)*log((-I*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*
I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/(24*a^2 - 36*I*a - 17)) - 2*(8*b^3*x^3 + 22*I*b^2*x^2 + 8*a^
3 - (40*I*a + 37)*b*x - 38*I*a^2 + 23*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)))/b^3
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueWarning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Arg
```

```
ument ValueWarning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming 0=[0,0,0]in
dex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 3.06Don
e
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x^2/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

3.232 $\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{7/4}}{2b^2}+\frac{(3+4ia)\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{3/4}}{4b^2}-\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-i}}{\sqrt[4]{ia+}}\right.}{8\sqrt{2}\,b^2}$$

```
[Out] 1/4*(3+4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x)
^(7/4)*(1+I*a+I*b*x)^(1/4)/b^2+3/8*(3+4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/8*(3+4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/16*(3+4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)+3/16*(3+4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5095, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{7/4}}{2b^2}+\frac{(3+4ia)\sqrt[4]{ia+ibx+1}\,(-ia-ibx+1)^{3/4}}{4b^2}-\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-i}}{\sqrt[4]{ia+}}\right.}{8\sqrt{2}\,b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((3 + (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(2*b^2) + (3*(3 + (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 + (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}x\,dx = \int \frac{x(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$
$$= \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3i-4a)\int \frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx}{4b}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3(3i-4a))}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3(3+4ia))}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{3(3+4ia)\,\mathrm{lo}}{}$$
$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{3(3+4ia)\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.04, size = 84, normalized size = 0.20

$$-\frac{i(-i(a+bx+i))^{7/4}\left(\sqrt[4]{2}\,(4a-3i)\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{1}{2}i(a+bx+i)\right)+7i\sqrt[4]{ia+ibx+1}\right)}{14b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-1/14*I)*((-I)*(I + a + b*x))^(7/4)*((7*I)*(1 + I*a + I*b*x)^(1/4) + 2^(1
/4)*(-3*I + 4*a)*Hypergeometric2F1[3/4, 7/4, 11/4, (-1/2*I)*(I + a + b*x)])
)/b^2
```

fricas [A] time = 0.63, size = 415, normalized size = 1.01

$$-\frac{b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}\,\log\left(\frac{i\,b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}+(12\,a-9i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{12\,a-9i}\right)-b^2\sqrt{\frac{144i\,a^2+216\,a-81i}{b^4}}\,\log\left(\frac{-i\,b^2\sqrt{\frac{144i\,a^2+216}{b^4}}}{}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4)*log((I*b^2*sqrt((144*I*a^2 +
 216*a - 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

```
/(b*x + a + I)))/(12*a - 9*I)) - b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4)*l
og((-I*b^2*sqrt((144*I*a^2 + 216*a - 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(12*a - 9*I)) + b^2*sqrt((-144
*I*a^2 - 216*a + 81*I)/b^4)*log((I*b^2*sqrt((-144*I*a^2 - 216*a + 81*I)/b^4
) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(
12*a - 9*I)) - b^2*sqrt((-144*I*a^2 - 216*a + 81*I)/b^4)*log((-I*b^2*sqrt((
-144*I*a^2 - 216*a + 81*I)/b^4) + (12*a - 9*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(12*a - 9*I)) + 2*(2*b^2*x^2 - 2*a^2 + 7*I*b*x
 + 3*I*a - 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueWarning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Arg
ument ValueWarning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming 0=[0,0,0]in
dex.cc index_m operator + Error: Bad Argument ValueWarning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming 0=[0,0,0]index.cc index_m operator + Error: Ba
d Argument ValueWarning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming 0=[0,0
,0]index.cc index_m operator + Error: Bad Argument ValueEvaluation time: 3.
08Done
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{{\left(a+b\,x\right)}^2+1}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

3.233 $\int e^{-\frac{3}{2} i \tan^{-1}(a+bx)} \, dx$

Optimal. Leaf size=338

$$-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[Out] -I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-3/2*I*arctan(1-(1-I*a-I*b*x)^(
1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/2*I*arctan(1+(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/
2)-3/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)
^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5093, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}+\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}-\frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - ((3*I)*ArcTan[1
- (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) +
 ((3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4
)])/(Sqrt[2]*b) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\int e^{-\frac{3}{2}i\tan^{-1}(a+bx)}\,dx = \int \frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}\,dx$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(6i)\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(6i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{(3i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{(3i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.}{b}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{(3i)\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}\,b} - \frac{3i\log\left(1+\frac{\sqrt{1-i}}{\sqrt{1+i}}\right.}{2}$$
$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}\,b} + \frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}\,b}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.13

$$-\frac{8ie^{\frac{1}{2}i\tan^{-1}(a+bx)}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};-e^{2i\tan^{-1}(a+bx)}\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-8*I)*E^((I/2)*ArcTan[a + b*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*
ArcTan[a + b*x])])/b
```

fricas [A] time = 0.48, size = 258, normalized size = 0.76

$$\frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}ib\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}ib\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{9i}{b^2}}\log\left(\frac{1}{3}ib\sqrt{-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*I*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*I*b*sqrt(9*I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-9*I/b
^2)*log(1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I))) - b*sqrt(-9*I/b^2)*log(-1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - (2*b*x + 2*a + 2*I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.87Done
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
[Out] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
[Out] Timed out
```

3.234 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{2(a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right.}{(-a+i)^{3/4}}$$

```
[Out] -2*(I+a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*
b*x)^(1/4))/(I-a)^(3/4)-2*(I+a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/
4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(3/4)+1/2*ln(1-(1-I*a-I*b*x)^(1/4
)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1
/2)-1/2*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+
I*a+I*b*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x
)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5095, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 208, 205}

$$\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}-\frac{2(a+i)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right.}{(-a+i)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-2*(I + a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(
1/4)*(1 - I*a - I*b*x)^(1/4))])/(I - a)^(3/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I + a)^(3/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I
*b*x)^(1/4))])/(I - a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
 I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2
] - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
 - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-ia-ibx)^{3/4}}{x(1+ia+ibx)^{3/4}}\,dx$$

$$= -\left((-1+ia)\int \frac{1}{x\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}\,dx\right) - (ib)\int \frac{1}{\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)$$

$$= 4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right) + (4(1-ia))\,\mathrm{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)$$

$$= 4\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \frac{(2(i+a))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}\,x^2}\,dx, x, \frac{\sqrt[4]{1+ia-}}{\sqrt[4]{1-ia-}}\right.}{\sqrt{i-a}}$$

$$= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - 2\,\mathrm{Subst}\left(\int$$

$$= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} + \frac{\mathrm{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \frac{2(i+a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1-}}\right.}{}$$

$$= -\frac{2(i+a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) + \sqrt{2}\tan^{-1}\left(1-\right.$$

Mathematica [C] time = 0.04, size = 128, normalized size = 0.30

$$\frac{2(-i(a+bx+i))^{3/4}\left(\sqrt[4]{2}\,(ia+ibx+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{1}{2}i(a+bx+i)\right)-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)\right)}{3(ia+ibx+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(2^(1/4)*(1 + I*a + I*b*x)^(3/4)*Hypergeometr
ic2F1[3/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[3/4, 1,
7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(3*(1 + I*a + I
*b*x)^(3/4))
```

fricas [B] time = 0.54, size = 629, normalized size = 1.47

$$-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-\frac{1}{2}\sqrt{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*lo
g(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
) + (-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a +
I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (a - I)*(-(a^3
 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - (-(a^3 +
 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (a - I)*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) + I*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (I*a + 1)*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - I*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)) + (-I*a - 1)*(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3
- 3*I*a^2 - 3*a + I))^(1/4))/(a + I))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.81Done
```

maple **[F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^{2}}}\right)^{\frac{3}{2}} x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^{2}+1}}\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxim
a")
[Out] integrate(1/(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^{2}+1}}\right)^{3/2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
[Out] Timed out
```

3.235 $\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=211

$$-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}-\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}$$

```
[Out] -(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/(1+I*a)/x-3*I*b*arctan((I+a)^(1/4)
*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(7/4)/(I+a)^(1/
4)-3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^
(1/4))/(I-a)^(7/4)/(I+a)^(1/4)
```

Rubi [A] time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5095, 94, 93, 212, 208, 205}

$$-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}-\frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}-\frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/((1 + I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(7/4)*(I + a)^(1/4)) - ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(7/4)*(I + a)^(1/4))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}i\tan^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1-ia-ibx)^{3/4}}{x^2(1+ia+ibx)^{3/4}}\,dx$$
$$= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(3b)\int \frac{1}{x\sqrt[4]{1-ia-ibx}\,(1+ia+ibx)^{3/4}}\,dx}{2(i-a)}$$
$$= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(6b)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i-a}$$
$$= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{(3ib)\,\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}\,x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/2}} - \frac{(3ib)\,\text{S}}{}$$
$$= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{3ib\tan^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}} - \frac{3ib\tanh^{-1}\left(\frac{\sqrt[4]{i+a}\,\sqrt[4]{1+ia+i}}{\sqrt[4]{i-a}\,\sqrt[4]{1-ia-i}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}}$$

Mathematica [C] time = 0.03, size = 107, normalized size = 0.51

$$-\frac{(-i(a+bx+i))^{3/4}\left(-2ibx\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+a^2+abx+ibx+1\right)}{\left(a^2+1\right)x(ia+ibx+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((((-I)*(I + a + b*x))^(3/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(3/4)))
```

fricas [B] time = 0.50, size = 626, normalized size = 2.97

$$3\left(-\frac{b^4}{16a^8-96ia^7-224a^6+224ia^5+224ia^3+224a^2-96ia-16}\right)^{\frac{1}{4}}(-ia-1)x\log\left(\frac{b\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}+2\left(-\frac{b^4}{16a^8-96ia^7-224a^6+224ia^5+224i\cdots}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] (3*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 9
6*I*a - 16))^(1/4)*(-I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I)) + 2*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 22
4*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(16
*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))
^(1/4)*(I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)) - 2*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224
*a^2 - 96*I*a - 16))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(16*a^8 - 96*I*a
^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a - I
)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-b^4/
(16*a^8 - 96*I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 1
6))^(1/4)*(2*I*a^2 + 4*a - 2*I))/b) - 3*(-b^4/(16*a^8 - 96*I*a^7 - 224*a^6
+ 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(a - I)*x*log((b*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-b^4/(16*a^8 - 96*
I*a^7 - 224*a^6 + 224*I*a^5 + 224*I*a^3 + 224*a^2 - 96*I*a - 16))^(1/4)*(-2
*I*a^2 - 4*a + 2*I))/b) + (b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(b*x + a + I)))/((a - I)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument ValueWarn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming 0=[0,0,0]index.cc index_m
operator + Error: Bad Argument ValueWarning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument Valu
eWarning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming 0=[0,0,0]index.cc ind
ex_m operator + Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming 0=[0,0,0]index.cc index_m operator + Error: Bad Argument
 ValueWarning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming 0=[0,0,0]index.c
c index_m operator + Error: Bad Argument ValueEvaluation time: 1.81Done
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.236 $\int e^{n\tan^{-1}(a+bx)}x^m\,dx$

Optimal. Leaf size=140

$$\frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}F_1\left(m+1;-\frac{in}{2},\frac{in}{2};m+2;-\frac{bx}{a+i},\frac{bx}{i-a}\right)}{m+1}$$

```
[Out] x^(1+m)*(1-I*a-I*b*x)^(1/2*I*n)*(1-b*x/(I-a))^(1/2*I*n)*AppellF1(1+m,1/2*I*
n,-1/2*I*n,2+m,b*x/(I-a),-b*x/(I+a))/(1+m)/((1+I*a+I*b*x)^(1/2*I*n))/((1+b*
x/(I+a))^(1/2*I*n))
```

Rubi [A] time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5095, 135, 133}

$$\frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}F_1\left(m+1;-\frac{in}{2},\frac{in}{2};m+2;-\frac{bx}{a+i},\frac{bx}{i-a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - I*a - I*b*x)^((I/2)*n)*(1 - (b*x)/(I - a))^((I/2)*n)*Appell
F1[1 + m, (-I/2)*n, (I/2)*n, 2 + m, -((b*x)/(I + a)), (b*x)/(I - a)])/((1 +
 m)*(1 + I*a + I*b*x)^((I/2)*n)*(1 + (b*x)/(I + a))^((I/2)*n))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^m\,dx = \int x^m(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \left((1-ia-ibx)^{\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}}\right)\int x^m(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{\frac{in}{2}}\,dx$$
$$= \left((1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}}\left(1+\frac{ibx}{1+ia}\right)^{\frac{in}{2}}\right)\int x^m\left(1-\frac{ibx}{1-ia}\right)^{\frac{in}{2}}\left(1\right.$$
$$= \frac{x^{1+m}(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{bx}{i-a}\right)^{\frac{in}{2}}\left(1+\frac{bx}{i+a}\right)^{-\frac{in}{2}}F_1\left(1+m;-\frac{in}{2},\frac{in}{2};2+m;-\right.}{1+m}$$

Mathematica [F] time = 0.94, size = 0, normalized size = 0.00

$$\int e^{n\tan^{-1}(a+bx)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^m,x]
[Out] Integrate[E^(n*ArcTan[a + b*x])*x^m, x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m e^{(n\arctan(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int e^{n\arctan(bx+a)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^m,x)
[Out] int(exp(n*arctan(b*x+a))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{(n\arctan(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m e^{n \operatorname{atan}(a+b x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atan(a + b*x)),x)
```

```
[Out] int(x^m*exp(n*atan(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**m,x)
```

```
[Out] Timed out
```

3.237 $\int e^{n\tan^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=260

$$-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}\left(-18a^2+2bx(6a+n)-10an-n^2+6\right)(ia+ibx+1)^{1-\frac{in}{2}}}{24b^4}+\frac{2^{-2-\frac{in}{2}}\left(24a^3+36a^2n-12a\left(2\right.\right.}{}$$

```
[Out] 1/4*x^2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2-1/24*(1-I*a
-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)*(6-18*a^2-10*a*n-n^2+2*b*(6*a
+n)*x)/b^4+1/3*2^(-2-1/2*I*n)*(24*a^3+36*a^2*n-12*a*(-n^2+2)-n*(-n^2+8))*(1
-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*
I*a-1/2*I*b*x)/b^4/(2*I-n)
```

Rubi [A] time = 0.19, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 100, 147, 69}

$$\frac{2^{-2-\frac{in}{2}}\left(36a^2n+24a^3-12a\left(2-n^2\right)-n\left(8-n^2\right)\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^4(-n+2i)}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (x^2*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(4*b^
2) - ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)*(6 -
18*a^2 - 10*a*n - n^2 + 2*b*(6*a + n)*x))/(24*b^4) + (2^(-2 - (I/2)*n)*(24*
a^3 + 36*a^2*n - 12*a*(2 - n^2) - n*(8 - n^2))*(1 - I*a - I*b*x)^(1 + (I/2)
*n)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/
2])/(3*b^4*(2*I - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^3\,dx = \int x^3(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} + \frac{\int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-2\left(1+a^2\right)-\right.}{4b^2}$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10a\right.}{24b^4}$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10a\right.}{24b^4}$$

Mathematica [A] time = 0.38, size = 272, normalized size = 1.05

$$\frac{(-i(a+bx+i))^{1+\frac{in}{2}}\left(b^2(-n+2i)x^2(ia+ibx+1)^{1-\frac{in}{2}}-2^{3-\frac{in}{2}}(6a+n)\,{}_2F_1\left(\frac{in}{2}-2,\frac{in}{2}+1;\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(b^2*(2*I - n)*x^2*(1 + I*a + I*b*x)^(1
 - (I/2)*n) - 2^(3 - (I/2)*n)*(6*a + n)*Hypergeometric2F1[-2 + (I/2)*n, 1 +
 (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)] + 2^(3 - (I/2)*n)*(1 + I*a)*
(-I + 5*a + n)*Hypergeometric2F1[-1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (-
1/2*I)*(I + a + b*x)] + 2^(1 - (I/2)*n)*(-I + a)^2*(-2*I + 4*a + n)*Hyperge
ometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)]))/(4*
b^4*(2*I - n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3e^{(n\arctan(bx+a))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \arctan(bx+a)} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^3,x)
[Out] int(exp(n*arctan(b*x+a))*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{(n \arctan(bx+a))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="maxima")
[Out] integrate(x^3*e^(n*arctan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \, \mathrm{e}^{n \, \mathrm{atan}(a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(n*atan(a + b*x)),x)
[Out] int(x^3*exp(n*atan(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**3,x)
[Out] Timed out
```

3.238 $\int e^{n\tan^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=220

$$\frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^3(-n+2i)}-\frac{(4a+n)(-ia-ibx+1)^{1+}}{6b^3}$$

```
[Out] -1/6*(4*a+n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^3+1/3*x*
(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+1/3*(-6*a^2-6*a*n-n
^2+2)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],
1/2-1/2*I*a-1/2*I*b*x)/(2^(1/2*I*n))/b^3/(2*I-n)
```

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 90, 80, 69}

$$\frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{3b^3(-n+2i)}-\frac{(4a+n)(-ia-ibx+1)^{1+}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -((4*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)
)/(6*b^3) + (x*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)
*n))/(3*b^2) + ((2 - 6*a^2 - 6*a*n - n^2)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*H
ypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(
3*2^((I/2)*n)*b^3*(2*I - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
```

```
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tan^{-1}(a+bx)}x^2\,dx = \int x^2(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-1-a^2-b(4a\right.}{3b^2}$$
$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} - \left(\right.$$
$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{2}{}$$

Mathematica [A] time = 0.16, size = 160, normalized size = 0.73

$$\frac{(-i(a+bx+i))^{1+\frac{in}{2}}\left(\frac{2^{1-\frac{in}{2}}(6a^2+6an+n^2-2)\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)}{n-2i}-\left((4a+n)(ia+ibx+1)^{1-\frac{in}{2}}\right)+2bx(ia+ibx\right.}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(-((4*a + n)*(1 + I*a + I*b*x)^(1 - (I/
2)*n)) + 2*b*x*(1 + I*a + I*b*x)^(1 - (I/2)*n) + (2^(1 - (I/2)*n)*(-2 + 6*a
^2 + 6*a*n + n^2)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-1/
2*I)*(I + a + b*x)])/(-2*I + n)))/(6*b^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2e^{(n\arctan(bx+a))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\arctan(bx+a)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x^2,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(n*arctan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \, \mathrm{e}^{n \operatorname{atan}(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atan(a + b*x)),x)
```

```
[Out] int(x^2*exp(n*atan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{n \operatorname{atan}{(a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*atan(a + b*x)), x)
```

3.239 $\int e^{n\tan^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=147

$$\frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

```
[Out] 1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+(2*a+n)*(1-I*a-
I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*I*a-1
/2*I*b*x)/(2^(1/2*I*n))/b^2/(2*I-n)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5095, 80, 69}

$$\frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*b^2) +
 ((2*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
(I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(2^((I/2)*n)*b^2*(2*I - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(a+bx)}x\,dx &= \int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\\ &= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2}-\frac{(2a+n)\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx}{2b}\\ &= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2}+\frac{2^{-\frac{in}{2}}(2a+n)(1-ia-ibx)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2}\right.}{b^2(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 128, normalized size = 0.87

$$\frac{i(-i(a+bx+i))^{1+\frac{in}{2}}\left(\frac{2^{1-\frac{in}{2}}(2a+n)\,_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;-\frac{1}{2}i(a+bx+i)\right)}{-2-in}+(a+bx-i)(ia+ibx+1)^{-\frac{in}{2}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x,x]
```

```
[Out] ((I/2)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((-I + a + b*x)/(1 + I*a + I*b*x)
^((I/2)*n) + (2^(1 - (I/2)*n)*(2*a + n)*Hypergeometric2F1[1 + (I/2)*n, (I/2
)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)])/(-2 - I*n)))/b^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(xe^{\left(n\arctan(bx+a)\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(n*arctan(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\arctan(bx+a)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))*x,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\left(n\arctan(bx+a)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(n*arctan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atan(a + b*x)),x)
```

```
[Out] int(x*exp(n*atan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))*x,x)
[Out] Integral(x*exp(n*atan(a + b*x)), x)
```

3.240 $\int e^{n\tan^{-1}(a+bx)}\,dx$

Optimal. Leaf size=91

$$-\frac{2^{1-\frac{in}{2}}(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b(-n+2i)}$$

```
[Out] -2^(1-1/2*I*n)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+
1/2*I*n],1/2-1/2*I*a-1/2*I*b*x)/b/(2*I-n)
```

Rubi [A] time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5093, 69}

$$-\frac{2^{1-\frac{in}{2}}(-ia-ibx+1)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(-ia-ibx+1)\right)}{b(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I
/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(b*(2*I - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5093

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^((I*n)/2)/(1 + I*a*c + I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(a+bx)}\,dx &= \int (1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\\ &= -\frac{2^{1-\frac{in}{2}}(1-ia-ibx)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2};\frac{1}{2}(1-ia-ibx)\right)}{b(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.66

$$\frac{4e^{(n+2i)\tan^{-1}(a+bx)}\,{}_2F_1\left(2,1-\frac{in}{2};2-\frac{in}{2};-e^{2i\tan^{-1}(a+bx)}\right)}{b(n+2i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a + b*x]),x]
```

```
[Out] (4*E^((2*I + n)*ArcTan[a + b*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2
)*n, -E^((2*I)*ArcTan[a + b*x])])/(b*(2*I + n))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(n \arctan(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="fricas")
[Out] integral(e^(n*arctan(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n \arctan(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a)),x)
[Out] int(exp(n*arctan(b*x+a)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n \arctan(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\, \mathrm{atan}(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a + b*x)),x)
[Out] int(exp(n*atan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{atan}{(a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a)),x)
[Out] Integral(exp(n*atan(a + b*x)), x)
```

3.241 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=191

$$\frac{2i(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{n}-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(-$$

```
[Out] 2*I*(1-I*a-I*b*x)^(1/2*I*n)*hypergeom([1, 1/2*I*n],[1+1/2*I*n],(I-a)*(1-I*a
-I*b*x)/(I+a)/(1+I*a+I*b*x))/n/((1+I*a+I*b*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*(1
-I*a-I*b*x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a-
1/2*I*b*x)/n
```

Rubi [A] time = 0.07, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5095, 105, 69, 131}

$$\frac{2i(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{n}-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(-$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeometric2F1[1, (I/2)*n, 1 + (I/2)*
n, ((I - a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/(n*(1 + I*a +
I*b*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeo
metric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a - I*b*x)/2])/n
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx$$
$$= -\left((-1+ia)\int \frac{(1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx\right) - (ib)\int (1-ia-ibx)^{-1+\frac{in}{2}}(1+ia$$
$$= \frac{2i(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2};1+\frac{in}{2};\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}}{}$$

Mathematica [A] time = 0.05, size = 170, normalized size = 0.89

$$\frac{2i(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{\frac{in}{2}}\left({}_2F_1\left(1,\frac{in}{2};\frac{in}{2}+1;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)-2^{-\frac{in}{2}}(ia+ibx+1)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;-\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*((-I)*(I + a + b*x))^((I/2)*n)*(Hypergeometric2F1[1, (I/2)*n, 1 + (I
/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)] - ((1 + I*a + I
*b*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (-1/2*I)*(
I + a + b*x)])/2^((I/2)*n)))/(n*(1 + I*a + I*b*x)^((I/2)*n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(bx+a))}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atan}(a+b x)}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a + b*x))/x,x)
```

```
[Out] int(exp(n*atan(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atan}(a+bx)}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x,x)
```

```
[Out] Integral(exp(n*atan(a + b*x))/x, x)
```

3.242 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=128

$$-\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

```
[Out] -4*b*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2, 1+1
/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I+a)^2/(2*I-n
)
```

Rubi [A] time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5095, 131}

$$-\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-4*b*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(-1 - (I/2)*n)*Hype
rgeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((I
+ a)*(1 + I*a + I*b*x))])/((I + a)^2*(2*I - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2}\,dx\\ &= -\frac{4b(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,1+\frac{in}{2};2+\frac{in}{2};\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i+a)^2(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 125, normalized size = 0.98

$$-\frac{4ib(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{1+\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)}{(a+i)^2(n-2i)(a+bx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((-4*I)*b*((-I)*(I + a + b*x))^(1 + (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)
*n, 2 + (I/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I
+ a)^2*(-2*I + n)*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(bx+a))}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(bx+a)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x^2,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(bx+a))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atan}(a+b\,x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a + b*x))/x^2,x)
```

```
[Out] int(exp(n*atan(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x**2,x)
```

```
[Out] Integral(exp(n*atan(a + b*x))/x**2, x)
```

3.243 $\int \frac{e^{n\tan^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=207

$$-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}-\frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\right.}{(-a+i)(a+i)^3(-n+2i)}$$

```
[Out] -1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/(a^2+1)/x^2-2*b^2*
(2*a-n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2,
1+1/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I-a)/(I+a)
^3/(2*I-n)
```

Rubi [A] time = 0.11, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5095, 96, 131}

$$-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}-\frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\right.}{(-a+i)(a+i)^3(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*(1 +
a^2)*x^2) - (2*b^2*(2*a - n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b
*x)^(-1 - (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*
(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I - a)*(I + a)^3*(2*I -
n))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5095

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[((d + e*x)^m*(1 - I*a*c - I*b*c*x)^((I*n)/2))/(1 + I*a*c
+ I*b*c*x)^((I*n)/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^3}\,dx$$
$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2\left(1+a^2\right)x^2} - \frac{(b(2a-n))\int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2}\,dx}{2\left(1+a^2\right)}$$
$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2\left(1+a^2\right)x^2} + \frac{2b^2(2a-n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,{}_2F_1\left(\right.}{(i+a)^2\left(1+a^2\right)(2i-n)}$$

Mathematica [A] time = 0.08, size = 173, normalized size = 0.84

$$-\frac{i(ia+ibx+1)^{-\frac{in}{2}}(-i(a+bx+i))^{1+\frac{in}{2}}\left(4b^2x^2(n-2a)\,{}_2F_1\left(2,\frac{in}{2}+1;\frac{in}{2}+2;\frac{a^2+bxa-ibx+1}{a^2+bxa+ibx+1}\right)+(a+i)^2(n-2i)(a+bx\right.}{2(a-i)(a+i)^3(n-2i)x^2(a+bx-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((-1/2*I)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((I + a)^2*(-2*I + n)*(-I + a
+ b*x)^2 + 4*b^2*(-2*a + n)*x^2*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)
*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/((-I + a)*(I + a
)^3*(-2*I + n)*x^2*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(bx+a))}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(b*x + a))/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(bx+a)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(b*x+a))/x^3,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(bx+a))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(b*x + a))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atan}(a+b\,x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a + b*x))/x^3,x)
```

```
[Out] int(exp(n*atan(a + b*x))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(b*x+a))/x**3,x)
```

```
[Out] Timed out
```

3.244 $\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=102

$$\frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(2p+(2+i))}$$

```
[Out] I*2^(1-1/2*I+p)*(1-I*a*x)^(1+1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I-p, 1
+1/2*I+p],[2+1/2*I+p],1/2-1/2*I*a*x)/a/(2+I+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5076, 5073, 69}

$$\frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(2p+(2+i))}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^((1 - I/2) + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeo
metric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((2 + I
) + 2*p)*(1 + a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\
&= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{\frac{i}{2}+p}(1+iax)^{-\frac{i}{2}+p}\,dx\\
&= \frac{i2^{\left(1-\frac{i}{2}\right)+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p;\left(2+\frac{i}{2}\right)\right.}{a((2+i)+2p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 102, normalized size = 1.00

$$\frac{i2^{p-\frac{i}{2}}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right);p+\left(2+\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a\left(p+\left(1+\frac{i}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-1/2*I + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeomet
ric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((1 + I/2)
 + p)*(1 + a^2*x^2)^p)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^p e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int e^{\arctan(ax)}\left(a^2c\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^p e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^p,x)
```

```
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(atan(a*x)), x)
```

3.245 $\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/37+6/37*I)*2^(3-1/2*I)*c^2*(1-I*a*x)^(3+1/2*I)*hypergeom([3+1/2*I, -2+1/
2*I],[4+1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx&=c^2\int(1-iax)^{2+\frac{i}{2}}(1+iax)^{2-\frac{i}{2}}\,dx\\&=\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.00

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,{}_2F_1\left(-2+\frac{i}{2},3+\frac{i}{2};4+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{\arctan(ax)}\left(a^2c\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^2 e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2e^{\operatorname{atan}(ax)}\,dx + \int a^4x^4e^{\operatorname{atan}(ax)}\,dx + \int e^{\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*exp(atan(a*x)), x) + Integral(a**4*x**4*exp(atan
(a*x)), x) + Integral(exp(atan(a*x)), x))
```

3.246 $\int e^{\tan^{-1}(ax)} \left(c + a^2cx^2\right) dx$

Optimal. Leaf size=61

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/17+4/17*I)*2^(2-1/2*I)*c*(1-I*a*x)^(2+1/2*I)*hypergeom([2+1/2*I, -1+1/2*
I],[3+1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx &= c\int (1-iax)^{1+\frac{i}{2}}(1+iax)^{1-\frac{i}{2}}\,dx\\ &= \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.00

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,{}_2F_1\left(-1+\frac{i}{2},2+\frac{i}{2};3+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{\arctan(ax)}\left(a^2c\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2x^2e^{\mathrm{atan}\,(ax)}\,dx+\int e^{\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(atan(a*x)), x) + Integral(exp(atan(a*x)), x))
```

3.247 $\int e^{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+2/5*I)*2^(1-1/2*I)*(1-I*a*x)^(1+1/2*I)*hypergeom([1/2*I, 1+1/2*I],[2+1
/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5061, 69}

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x],x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I/2)*(1 - I*a*x)^(1 + I/2)*Hypergeometric2F1[I/2, 1
 + I/2, 2 + I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\,dx &= \int (1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,{}_2F_1\left(\frac{i}{2},1+\frac{i}{2};2+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.75

$$\frac{\left(\frac{4}{5}-\frac{8i}{5}\right)e^{(1+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x],x]
```

```
[Out] ((4/5 - (8*I)/5)*E^((1 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
 - I/2, -E^((2*I)*ArcTan[a*x])])/a
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(e^{(\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x)),x)
[Out] int(exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\mathrm{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x)),x)
[Out] int(exp(atan(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{atan}{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x)),x)
[Out] Integral(exp(atan(a*x)), x)
```

3.248 $\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{e^{\tan^{-1}(ax)}}{ac}$$

```
[Out] exp(arctan(a*x))/a/c
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5071}

$$\frac{e^{\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] E^ArcTan[a*x]/(a*c)
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{e^{\tan^{-1}(ax)}}{ac}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 2.69

$$\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] (1 - I*a*x)^(I/2)/(a*c*(1 + I*a*x)^(I/2))
```

fricas [A] time = 0.43, size = 12, normalized size = 0.92

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

giac [A] time = 0.13, size = 12, normalized size = 0.92

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

maple [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\mathrm{e}^{\arctan(ax)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] exp(arctan(a*x))/a/c
```

maxima [A] time = 0.44, size = 12, normalized size = 0.92

$$\frac{e^{(\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(arctan(a*x))/(a*c)
```

mupad [B] time = 0.53, size = 12, normalized size = 0.92

$$\frac{\mathrm{e}^{\mathrm{atan}(a\,x)}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] exp(atan(a*x))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{e^{\operatorname{atan}{(ax)}}}{ac} & \text{for}\: c \neq 0 \\ \tilde{\infty} \int e^{\operatorname{atan}{(ax)}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((exp(atan(a*x))/(a*c), Ne(c, 0)), (zoo*Integral(exp(atan(a*x)), x
), True))
```

3.249 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} \, dx$

Optimal. Leaf size=50

$$\frac{(2ax+1)e^{\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}+\frac{2e^{\tan^{-1}(ax)}}{5ac^2}$$

```
[Out] 2/5*exp(arctan(a*x))/a/c^2+1/5*exp(arctan(a*x))*(2*a*x+1)/a/c^2/(a^2*x^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{(2ax+1)e^{\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}+\frac{2e^{\tan^{-1}(ax)}}{5ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcTan[a*x])/(5*a*c^2) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*c^2*(1 + a^2
*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}+\frac{2\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{5c}\\ &= \frac{2e^{\tan^{-1}(ax)}}{5ac^2}+\frac{e^{\tan^{-1}(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 1.20

$$\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\left(2a^2x^2+2ax+3\right)}{5c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1 - I*a*x)^(I/2)*(3 + 2*a*x + 2*a^2*x^2))/(5*c^2*(1 + I*a*x)^(I/2)*(a + a
^3*x^2))
```

fricas [A] time = 0.53, size = 39, normalized size = 0.78

$$\frac{\left(2\,a^{2}x^{2}+2\,ax+3\right)e^{\left(\arctan\left(ax\right)\right)}}{5\,\left(a^{3}c^{2}x^{2}+ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*(2*a^2*x^2 + 2*a*x + 3)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 39, normalized size = 0.78

$$\frac{e^{\arctan\left(ax\right)}\left(2a^{2}x^{2}+2ax+3\right)}{5\left(a^{2}x^{2}+1\right)a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x)
[Out] 1/5*exp(arctan(a*x))*(2*a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\arctan\left(ax\right)\right)}}{\left(a^{2}cx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

mupad [B] time = 0.56, size = 44, normalized size = 0.88

$$\frac{e^{\mathrm{atan}\left(a\,x\right)}\left(\frac{3}{5\,a^{3}\,c^{2}}+\frac{2\,x}{5\,a^{2}\,c^{2}}+\frac{2\,x^{2}}{5\,a\,c^{2}}\right)}{\frac{1}{a^{2}}+x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] (exp(atan(a*x))*(3/(5*a^3*c^2) + (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))/(1
/a^2 + x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2a^{2}x^{2}e^{\operatorname{atan}{\left(ax\right)}}}{5a^{3}c^{2}x^{2}+5ac^{2}}+\frac{2axe^{\operatorname{atan}{\left(ax\right)}}}{5a^{3}c^{2}x^{2}+5ac^{2}}+\frac{3e^{\operatorname{atan}{\left(ax\right)}}}{5a^{3}c^{2}x^{2}+5ac^{2}} & \text{for}\: c \neq 0 \\ \tilde{\infty}\int e^{\operatorname{atan}{\left(ax\right)}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((2*a**2*x**2*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 2*a*x
*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 3*exp(atan(a*x))/(5*a**3*c*
*2*x**2 + 5*a*c**2), Ne(c, 0)), (zoo*Integral(exp(atan(a*x)), x), True))
```

3.250 $\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal. Leaf size=83

$$\frac{12(2ax+1)e^{\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}+\frac{(4ax+1)e^{\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}+\frac{24e^{\tan^{-1}(ax)}}{85ac^3}$$

```
[Out] 24/85*exp(arctan(a*x))/a/c^3+1/17*exp(arctan(a*x))*(4*a*x+1)/a/c^3/(a^2*x^2
+1)^2+12/85*exp(arctan(a*x))*(2*a*x+1)/a/c^3/(a^2*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{12(2ax+1)e^{\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}+\frac{(4ax+1)e^{\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}+\frac{24e^{\tan^{-1}(ax)}}{85ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (24*E^ArcTan[a*x])/(85*a*c^3) + (E^ArcTan[a*x]*(1 + 4*a*x))/(17*a*c^3*(1 +
a^2*x^2)^2) + (12*E^ArcTan[a*x]*(1 + 2*a*x))/(85*a*c^3*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{17c}\\ &= \frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\tan^{-1}(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{85c^2}\\ &= \frac{24e^{\tan^{-1}(ax)}}{85ac^3}+\frac{e^{\tan^{-1}(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\tan^{-1}(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.29, size = 114, normalized size = 1.37

$$\frac{\frac{5(4ax+1)e^{\tan^{-1}(ax)}}{\left(a^2x^2+1\right)^2}+\frac{24(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax+(1-i))}{ax-i}+(12-24i)(1-iax)^{-1+\frac{i}{2}}(1+iax)^{-1-\frac{i}{2}}}{85ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((12 - 24*I)/((1 - I*a*x)^(1 - I/2)*(1 + I*a*x)^(1 + I/2)) + (24*(1 - I*a*x
)^(I/2)*((1 - I) + a*x))/((1 + I*a*x)^(I/2)*(-I + a*x)) + (5*E^ArcTan[a*x]*
(1 + 4*a*x))/(1 + a^2*x^2)^2)/(85*a*c^3)
```

fricas [A] time = 0.54, size = 66, normalized size = 0.80

$$\frac{\left(24\,a^4x^4+24\,a^3x^3+60\,a^2x^2+44\,ax+41\right)e^{(\arctan(ax))}}{85\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/85*(24*a^4*x^4 + 24*a^3*x^3 + 60*a^2*x^2 + 44*a*x + 41)*e^(arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 55, normalized size = 0.66

$$\frac{\mathrm{e}^{\arctan(ax)}\left(24a^4x^4+24a^3x^3+60a^2x^2+44ax+41\right)}{85\left(a^2x^2+1\right)^2a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/85*exp(arctan(a*x))*(24*a^4*x^4+24*a^3*x^3+60*a^2*x^2+44*a*x+41)/(a^2*x^2
+1)^2/a/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

mupad [B] time = 0.62, size = 74, normalized size = 0.89

$$\frac{24\,\mathrm{e}^{\mathrm{atan}(a\,x)}}{85\,a\,c^3}+\frac{12\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(2\,a\,x+1\right)}{85\,a\,c^3\,\left(a^2\,x^2+1\right)}+\frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(4\,a\,x+1\right)}{17\,a\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (24*exp(atan(a*x)))/(85*a*c^3) + (12*exp(atan(a*x))*(2*a*x + 1))/(85*a*c^3*
(a^2*x^2 + 1)) + (exp(atan(a*x))*(4*a*x + 1))/(17*a*c^3*(a^2*x^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{24a^4x^4e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{24a^3x^3e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{60a^2x^2e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{44axe^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{41e^{\operatorname{atan}(a}}{85a^5c^3x^4+170a^3c} \\ \tilde{\infty} \int e^{\operatorname{atan}(ax)}\, dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((24*a**4*x**4*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a**3*c**3*x
**2 + 85*a*c**3) + 24*a**3*x**3*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a**
3*c**3*x**2 + 85*a*c**3) + 60*a**2*x**2*exp(atan(a*x))/(85*a**5*c**3*x**4 +
 170*a**3*c**3*x**2 + 85*a*c**3) + 44*a*x*exp(atan(a*x))/(85*a**5*c**3*x**4
 + 170*a**3*c**3*x**2 + 85*a*c**3) + 41*exp(atan(a*x))/(85*a**5*c**3*x**4 +
 170*a**3*c**3*x**2 + 85*a*c**3), Ne(c, 0)), (zoo*Integral(exp(atan(a*x)),
x), True))
```

3.251 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=116

$$\frac{72(2ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}+\frac{30(4ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}+\frac{(6ax+1)e^{\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}+\frac{144e^{\tan^{-1}(ax)}}{629ac^4}$$

```
[Out] 144/629*exp(arctan(a*x))/a/c^4+1/37*exp(arctan(a*x))*(6*a*x+1)/a/c^4/(a^2*x
^2+1)^3+30/629*exp(arctan(a*x))*(4*a*x+1)/a/c^4/(a^2*x^2+1)^2+72/629*exp(ar
ctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{72(2ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}+\frac{30(4ax+1)e^{\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}+\frac{(6ax+1)e^{\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}+\frac{144e^{\tan^{-1}(ax)}}{629ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (144*E^ArcTan[a*x])/(629*a*c^4) + (E^ArcTan[a*x]*(1 + 6*a*x))/(37*a*c^4*(1
+ a^2*x^2)^3) + (30*E^ArcTan[a*x]*(1 + 4*a*x))/(629*a*c^4*(1 + a^2*x^2)^2)
+ (72*E^ArcTan[a*x]*(1 + 2*a*x))/(629*a*c^4*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^3}\,dx}{37c}$$
$$= \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2} + \frac{360\int \frac{e^{\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx}{629c^2}$$
$$= \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2} + \frac{72e^{\tan^{-1}(ax)}(1+2ax)}{629ac^4(1+a^2x^2)} + \frac{144\int \frac{e^{\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{629c^3}$$
$$= \frac{144e^{\tan^{-1}(ax)}}{629ac^4} + \frac{e^{\tan^{-1}(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30e^{\tan^{-1}(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2} + \frac{72e^{\tan^{-1}(ax)}(1+2ax)}{629ac^4(1+a^2x^2)}$$

Mathematica [C] time = 0.30, size = 123, normalized size = 1.06

$$\frac{17c(6ax+1)e^{\tan^{-1}(ax)} + 6(a^2cx^2+c)\left(5(4ax+1)e^{\tan^{-1}(ax)} + 12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax-i)(ax+i)(2a^2x^2+2ax\cdot\right.}{629ac^2(a^2cx^2+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (17*c*E^ArcTan[a*x]*(1 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4
*a*x) + (12*(1 - I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))
/(1 + I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

fricas [A] time = 0.47, size = 93, normalized size = 0.80

$$\frac{(144\,a^6x^6 + 144\,a^5x^5 + 504\,a^4x^4 + 408\,a^3x^3 + 606\,a^2x^2 + 366\,ax + 263)e^{(\arctan(ax))}}{629\,(a^7c^4x^6 + 3\,a^5c^4x^4 + 3\,a^3c^4x^2 + ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/629*(144*a^6*x^6 + 144*a^5*x^5 + 504*a^4*x^4 + 408*a^3*x^3 + 606*a^2*x^2
+ 366*a*x + 263)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x
^2 + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 71, normalized size = 0.61

$$\frac{e^{\arctan(ax)}(144a^6x^6 + 144a^5x^5 + 504a^4x^4 + 408a^3x^3 + 606a^2x^2 + 366ax + 263)}{629(a^2x^2+1)^3\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] 1/629*exp(arctan(a*x))*(144*a^6*x^6+144*a^5*x^5+504*a^4*x^4+408*a^3*x^3+606
*a^2*x^2+366*a*x+263)/(a^2*x^2+1)^3/a/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2 + c\right)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

mupad [B] time = 0.69, size = 104, normalized size = 0.90

$$\frac{144\,\mathrm{e}^{\mathrm{atan}(a\,x)}}{629\,a\,c^4}+\frac{72\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(2\,a\,x+1\right)}{629\,a\,c^4\,\left(a^2\,x^2+1\right)}+\frac{30\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(4\,a\,x+1\right)}{629\,a\,c^4\,{\left(a^2\,x^2+1\right)}^2}+\frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(6\,a\,x+1\right)}{37\,a\,c^4\,{\left(a^2\,x^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (144*exp(atan(a*x)))/(629*a*c^4) + (72*exp(atan(a*x))*(2*a*x + 1))/(629*a*c
^4*(a^2*x^2 + 1)) + (30*exp(atan(a*x))*(4*a*x + 1))/(629*a*c^4*(a^2*x^2 + 1
)^2) + (exp(atan(a*x))*(6*a*x + 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{144a^6x^6e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{144a^5x^5e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{504a^4x^4e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} \\ \tilde{\infty} \int e^{\operatorname{atan}(ax)}\, dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise((144*a**6*x**6*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**
4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 144*a**5*x**5*exp(atan(a*x))/(
629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4
) + 504*a**4*x**4*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4
+ 1887*a**3*c**4*x**2 + 629*a*c**4) + 408*a**3*x**3*exp(atan(a*x))/(629*a**
7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 606
*a**2*x**2*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*
a**3*c**4*x**2 + 629*a*c**4) + 366*a*x*exp(atan(a*x))/(629*a**7*c**4*x**6 +
 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 263*exp(atan(a*x
))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*
c**4), Ne(c, 0)), (zoo*Integral(exp(atan(a*x)), x), True))
```

3.252 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^5}\,dx$

Optimal. Leaf size=149

$$\frac{4032(2ax+1)e^{\tan^{-1}(ax)}}{40885ac^5\left(a^2x^2+1\right)}+\frac{336(4ax+1)e^{\tan^{-1}(ax)}}{8177ac^5\left(a^2x^2+1\right)^2}+\frac{56(6ax+1)e^{\tan^{-1}(ax)}}{2405ac^5\left(a^2x^2+1\right)^3}+\frac{(8ax+1)e^{\tan^{-1}(ax)}}{65ac^5\left(a^2x^2+1\right)^4}+\frac{8064e^{\tan^{-1}(ax)}}{40885ac^5}$$

```
[Out] 8064/40885*exp(arctan(a*x))/a/c^5+1/65*exp(arctan(a*x))*(8*a*x+1)/a/c^5/(a^
2*x^2+1)^4+56/2405*exp(arctan(a*x))*(6*a*x+1)/a/c^5/(a^2*x^2+1)^3+336/8177*
exp(arctan(a*x))*(4*a*x+1)/a/c^5/(a^2*x^2+1)^2+4032/40885*exp(arctan(a*x))*
(2*a*x+1)/a/c^5/(a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5070, 5071}

$$\frac{4032(2ax+1)e^{\tan^{-1}(ax)}}{40885ac^5\left(a^2x^2+1\right)}+\frac{336(4ax+1)e^{\tan^{-1}(ax)}}{8177ac^5\left(a^2x^2+1\right)^2}+\frac{56(6ax+1)e^{\tan^{-1}(ax)}}{2405ac^5\left(a^2x^2+1\right)^3}+\frac{(8ax+1)e^{\tan^{-1}(ax)}}{65ac^5\left(a^2x^2+1\right)^4}+\frac{8064e^{\tan^{-1}(ax)}}{40885ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

```
[Out] (8064*E^ArcTan[a*x])/(40885*a*c^5) + (E^ArcTan[a*x]*(1 + 8*a*x))/(65*a*c^5*
(1 + a^2*x^2)^4) + (56*E^ArcTan[a*x]*(1 + 6*a*x))/(2405*a*c^5*(1 + a^2*x^2)
^3) + (336*E^ArcTan[a*x]*(1 + 4*a*x))/(8177*a*c^5*(1 + a^2*x^2)^2) + (4032*
E^ArcTan[a*x]*(1 + 2*a*x))/(40885*a*c^5*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^5}\,dx = \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4} + \frac{56\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx}{65c}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3} + \frac{336\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{481c^2}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2} + \frac{4032\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{8177c^3}$$

$$= \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2} + \frac{4032e^{\tan^{-1}(ax)}(1+2}{40885ac^5\left(1+a^2x\right.}$$

$$= \frac{8064e^{\tan^{-1}(ax)}}{40885ac^5} + \frac{e^{\tan^{-1}(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4} + \frac{56e^{\tan^{-1}(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3} + \frac{336e^{\tan^{-1}(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2} + \frac{40}{4}$$

Mathematica [C] time = 0.32, size = 153, normalized size = 1.03

$$\frac{629(8ax+1)e^{\tan^{-1}(ax)} + \frac{56\left(a^2cx^2+c\right)\left(17c(6ax+1)e^{\tan^{-1}(ax)}+6\left(a^2cx^2+c\right)\left(5(4ax+1)e^{\tan^{-1}(ax)}+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(ax-i)(ax+i)\left(2a^2x^2+2a\right.\right.\right.}{c^2}}{40885ac\left(a^2cx^2+c\right)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

```
[Out] (629*E^ArcTan[a*x]*(1 + 8*a*x) + (56*(c + a^2*c*x^2)*(17*c*E^ArcTan[a*x]*(1
 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a*x
)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))))/
c^2)/(40885*a*c*(c + a^2*c*x^2)^4)
```

fricas [A] time = 0.52, size = 120, normalized size = 0.81

$$\frac{\left(8064\,a^8x^8 + 8064\,a^7x^7 + 36288\,a^6x^6 + 30912\,a^5x^5 + 62160\,a^4x^4 + 43344\,a^3x^3 + 48664\,a^2x^2 + 25528\,ax + 15\right.}{40885\left(a^9c^5x^8 + 4\,a^7c^5x^6 + 6\,a^5c^5x^4 + 4\,a^3c^5x^2 + ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] 1/40885*(8064*a^8*x^8 + 8064*a^7*x^7 + 36288*a^6*x^6 + 30912*a^5*x^5 + 6216
0*a^4*x^4 + 43344*a^3*x^3 + 48664*a^2*x^2 + 25528*a*x + 15357)*e^(arctan(a*
x))/(a^9*c^5*x^8 + 4*a^7*c^5*x^6 + 6*a^5*c^5*x^4 + 4*a^3*c^5*x^2 + a*c^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 87, normalized size = 0.58

$$\frac{e^{\arctan(ax)}\left(8064a^8x^8+8064a^7x^7+36288a^6x^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+25528ax+\right.}{40885\left(a^2x^2+1\right)^4c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x)
```

```
[Out] 1/40885*exp(arctan(a*x))*(8064*a^8*x^8+8064*a^7*x^7+36288*a^6*x^6+30912*a^5
*x^5+62160*a^4*x^4+43344*a^3*x^3+48664*a^2*x^2+25528*a*x+15357)/(a^2*x^2+1)
^4/c^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^5, x)
```

mupad [B] time = 0.74, size = 134, normalized size = 0.90

$$\frac{e^{\operatorname{atan}(a\,x)}\left(\frac{15357}{40885\,a^9\,c^5}+\frac{25528\,x}{40885\,a^8\,c^5}+\frac{8064\,x^8}{40885\,a\,c^5}+\frac{8064\,x^7}{40885\,a^2\,c^5}+\frac{36288\,x^6}{40885\,a^3\,c^5}+\frac{30912\,x^5}{40885\,a^4\,c^5}+\frac{336\,x^4}{221\,a^5\,c^5}+\frac{43344\,x^3}{40885\,a^6\,c^5}+\frac{48664\,x^2}{40885\,a^7\,c^5}\right)}{\frac{1}{a^8}+x^8+\frac{4\,x^6}{a^2}+\frac{6\,x^4}{a^4}+\frac{4\,x^2}{a^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^5,x)
```

```
[Out] (exp(atan(a*x))*(15357/(40885*a^9*c^5) + (25528*x)/(40885*a^8*c^5) + (8064*
x^8)/(40885*a*c^5) + (8064*x^7)/(40885*a^2*c^5) + (36288*x^6)/(40885*a^3*c^
5) + (30912*x^5)/(40885*a^4*c^5) + (336*x^4)/(221*a^5*c^5) + (43344*x^3)/(4
0885*a^6*c^5) + (48664*x^2)/(40885*a^7*c^5)))/(1/a^8 + x^8 + (4*x^6)/a^2 +
(6*x^4)/a^4 + (4*x^2)/a^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{8064a^8x^8e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+40885ac^5} + \frac{8064a^7x^7e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+40885ac^5} + \\ \tilde{\infty}\int e^{\operatorname{atan}(ax)}\,dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**5,x)
```

```
[Out] Piecewise((8064*a**8*x**8*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**
7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5)
 + 8064*a**7*x**7*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x
**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 36288
*a**6*x**6*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 2
45310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 30912*a**5*x
**5*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a
**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 62160*a**4*x**4*exp
(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**
```

```
5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 43344*a**3*x**3*exp(atan(a
*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4
+ 163540*a**3*c**5*x**2 + 40885*a*c**5) + 48664*a**2*x**2*exp(atan(a*x))/(4
0885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 16354
0*a**3*c**5*x**2 + 40885*a*c**5) + 25528*a*x*exp(atan(a*x))/(40885*a**9*c**
5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x
**2 + 40885*a*c**5) + 15357*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a
**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**
5), Ne(c, 0)), (zoo*Integral(exp(atan(a*x)), x), True))
```

3.253 $\int e^{\tan^{-1}(ax)} \left(c + a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=98

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (1/13+5/13*I)*2^(3/2-1/2*I)*c*(1-I*a*x)^(5/2+1/2*I)*hypergeom([5/2+1/2*I, -
3/2+1/2*I],[7/2+1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{i}{2}}(1+iax)^{\frac{3}{2}-\frac{i}{2}}dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.02, size = 98, normalized size = 1.00

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2};\frac{7}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int e^{\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}e^{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(atan(a*x)), x)
```

3.254 $\int e^{\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=97

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (1/5+3/5*I)*2^(1/2-1/2*I)*(1-I*a*x)^(3/2+1/2*I)*hypergeom([3/2+1/2*I, -1/2+
1/2*I],[5/2+1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx &= \frac{\sqrt{c+a^2cx^2}\int e^{\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+\frac{i}{2}}(1+iax)^{\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\;{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 97, normalized size = 1.00

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2};\frac{5}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{a^2cx^2+c}\,e^{(\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{\arctan(ax)}\sqrt{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2cx^2+c}\,e^{(\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}\, e^{\operatorname{atan}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(atan(a*x)), x)
```

3.255 $\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (1+I)*2^(-1/2-1/2*I)*(1-I*a*x)^(1/2+1/2*I)*hypergeom([1/2+1/2*I, 1/2+1/2*I]
,[3/2+1/2*I],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{i}{2}}(1+iax)^{-\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\ {}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 93, normalized size = 1.00

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\ {}_2F_1\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2};\frac{3}{2}+\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 + 
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.256 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} dx$

Optimal. Leaf size=35

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] 1/2*exp(arctan(a*x))*(a*x+1)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5069}

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} dx = \frac{e^{\tan^{-1}(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{(ax+1)e^{\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.51, size = 42, normalized size = 1.20

$$\frac{\sqrt{a^2cx^2+c}\,(ax+1)e^{(\arctan(ax))}}{2\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x + 1)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 37, normalized size = 1.06

$$\frac{\left(a^2x^2+1\right)(ax+1)\,\mathrm{e}^{\arctan(ax)}}{2a\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
[Out] 1/2*(a^2*x^2+1)*(a*x+1)*exp(arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 0.60, size = 33, normalized size = 0.94

$$\frac{\mathrm{e}^{\operatorname{atan}(a\,x)}\left(\frac{x}{2\,c}+\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(atan(a*x))*(x/(2*c) + 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.257 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 1/10*exp(arctan(a*x))*(3*a*x+1)/a/c/(a^2*c*x^2+c)^(3/2)+3/10*exp(arctan(a*x
))*(a*x+1)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5069}

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*
x]*(1 + a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= \frac{e^{\tan^{-1}(ax)}(1+3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3e^{\tan^{-1}(ax)}(1+ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.83

$$\frac{\left(3a^3x^3+3a^2x^2+6ax+4\right)e^{\tan^{-1}(ax)}}{10c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^ArcTan[a*x]*(4 + 6*a*x + 3*a^2*x^2 + 3*a^3*x^3))/(10*c^2*(a + a^3*x^2)*S
qrt[c + a^2*c*x^2])
```

fricas [A] time = 0.48, size = 70, normalized size = 0.97

$$\frac{\left(3\,a^{3}x^{3}+3\,a^{2}x^{2}+6\,ax+4\right)\sqrt{a^{2}cx^{2}+c}\,e^{\left(\arctan\left(ax\right)\right)}}{10\,\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 4)*sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 54, normalized size = 0.75

$$\frac{\left(a^{2}x^{2}+1\right)\left(3a^{3}x^{3}+3a^{2}x^{2}+6ax+4\right)\mathrm{e}^{\arctan\left(ax\right)}}{10a\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3+3*a^2*x^2+6*a*x+4)*exp(arctan(a*x))/a/(a^2*c*x^
2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{\left(\arctan\left(ax\right)\right)}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 0.65, size = 78, normalized size = 1.08

$$\frac{\mathrm{e}^{\mathrm{atan}\left(a\,x\right)}\,\left(\frac{2}{5\,a^3\,c^2}+\frac{3\,x^3}{10\,c^2}+\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] (exp(atan(a*x))*(2/(5*a^3*c^2) + (3*x^3)/(10*c^2) + (3*x)/(5*a^2*c^2) + (3*
x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.258 $\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=108

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+1)e^{\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] 1/26*exp(arctan(a*x))*(5*a*x+1)/a/c/(a^2*c*x^2+c)^(5/2)+1/13*exp(arctan(a*x
))*(3*a*x+1)/a/c^2/(a^2*c*x^2+c)^(3/2)+3/13*exp(arctan(a*x))*(a*x+1)/a/c^3/
(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5069}

$$\frac{3(ax+1)e^{\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+1)e^{\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 5*a*x))/(26*a*c*(c + a^2*c*x^2)^(5/2)) + (E^ArcTan[a*x]
*(1 + 3*a*x))/(13*a*c^2*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*x]*(1 + a*x)
)/(13*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx &= \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{10\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{13c}\\
&= \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{e^{\tan^{-1}(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c^2}\\
&= \frac{e^{\tan^{-1}(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{e^{\tan^{-1}(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{3e^{\tan^{-1}(ax)}(1+ax)}{13ac^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.73

$$\frac{\left(6a^5x^5+6a^4x^4+18a^3x^3+14a^2x^2+17ax+9\right)e^{\tan^{-1}(ax)}}{26ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^ArcTan[a*x]*(9 + 17*a*x + 14*a^2*x^2 + 18*a^3*x^3 + 6*a^4*x^4 + 6*a^5*x^
5))/(26*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.66, size = 97, normalized size = 0.90

$$\frac{\left(6\,a^5x^5+6\,a^4x^4+18\,a^3x^3+14\,a^2x^2+17\,ax+9\right)\sqrt{a^2cx^2+c}\,e^{(\arctan(ax))}}{26\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/26*(6*a^5*x^5 + 6*a^4*x^4 + 18*a^3*x^3 + 14*a^2*x^2 + 17*a*x + 9)*sqrt(a^
2*c*x^2 + c)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 70, normalized size = 0.65

$$\frac{\left(a^2x^2+1\right)\left(6a^5x^5+6a^4x^4+18a^3x^3+14a^2x^2+17ax+9\right)\mathrm{e}^{\arctan(ax)}}{26a\left(a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5+6*a^4*x^4+18*a^3*x^3+14*a^2*x^2+17*a*x+9)*exp(a
rctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 0.67, size = 120, normalized size = 1.11

$$\frac{e^{\operatorname{atan}(a\,x)}\,\left(\frac{9}{26\,a^5\,c^3}+\frac{3\,x^5}{13\,c^3}+\frac{17\,x}{26\,a^4\,c^3}+\frac{3\,x^4}{13\,a\,c^3}+\frac{9\,x^3}{13\,a^2\,c^3}+\frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] (exp(atan(a*x))*(9/(26*a^5*c^3) + (3*x^5)/(13*c^3) + (17*x)/(26*a^4*c^3) +
(3*x^4)/(13*a*c^3) + (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c + a^
2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2)^(1/
2))/a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.259 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p{}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

```
[Out] I*2^(-I+p)*(1-I*a*x)^(1+I+p)*(a^2*c*x^2+c)^p*hypergeom([I-p, 1+I+p],[2+I+p]
,1/2-1/2*I*a*x)/a/(1+I+p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p{}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{i+p}(1+iax)^{-i+p}\,dx\\ &= \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p{}_2F_1\left(i-p,(1+i)+p;(2+i)+p\right.}{a((1+i)+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 1.00

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p {}_2F_1\left(i-p,p+(1+i);p+(2+i);\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{2\,\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

```
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{2\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(2*atan(a*x)), x)
```

3.260 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+3/5*I)*2^(1-I)*c^2*(1-I*a*x)^(3+I)*hypergeom([3+I, -2+I],[4+I],1/2-1/2
*I*a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx&=c^2\int(1-iax)^{2+i}(1+iax)^{2-i}\,dx\\&=\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,{}_2F_1\left(-2+i,3+i;4+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)e^{(2\,\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^2e^{(2\,\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{2\,\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2e^{2\,\mathrm{atan}\,(ax)}\,dx+\int a^4x^4e^{2\,\mathrm{atan}\,(ax)}\,dx+\int e^{2\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(2*a**2*x**2*exp(2*atan(a*x)), x) + Integral(a**4*x**4*exp(2*
atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

3.261 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=51

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+2/5*I)*2^(1-I)*c*(1-I*a*x)^(2+I)*hypergeom([2+I, -1+I],[3+I],1/2-1/2*I
*a*x)/a
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx &= c\int(1-iax)^{1+i}(1+iax)^{1-i}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,{}_2F_1\left(-1+i,2+i;3+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2 + c\right)e^{(2\ \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2 + c\right)e^{(2\ \arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2x^2e^{2\,\mathrm{atan}\,(ax)}\,dx + \int e^{2\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(2*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

3.262 $\int e^{2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=46

$$\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1+I)*2^(-1-I)*(1-I*a*x)^(1+I)*hypergeom([I, 1+I],[2+I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5061, 69}

$$\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x]),x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1 + I)*Hypergeometric2F1[I, 1 + I, 2 + I, (1 - I*a*x)
/2])/(2^(1 + I)*a)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\,dx &= \int (1-iax)^i(1+iax)^{-i}\,dx\\ &= \frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,{}_2F_1\left(i,1+i;2+i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.80

$$\frac{(1-i)e^{(2+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1-i,2;2-i;-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x]),x]
```

```
[Out] ((1 - I)*E^((2 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I, 2, 2 - I, -E^((
2*I)*ArcTan[a*x])])/a
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(2\ \arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(e^(2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x)),x)
```

```
[Out] int(exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{2\,\mathrm{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x)),x)
```

```
[Out] int(exp(2*atan(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{2\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x)),x)
```

```
[Out] Integral(exp(2*atan(a*x)), x)
```

3.263 $\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{2\tan^{-1}(ax)}}{2ac}$$

```
[Out] 1/2*exp(2*arctan(a*x))/a/c
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$\frac{e^{2\tan^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] E^(2*ArcTan[a*x])/(2*a*c)
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx=\frac{e^{2\tan^{-1}(ax)}}{2ac}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 1.89

$$\frac{(1-iax)^i(1+iax)^{-i}}{2ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] (1 - I*a*x)^I/(2*a*c*(1 + I*a*x)^I)
```

fricas [A] time = 0.56, size = 15, normalized size = 0.83

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

giac [A] time = 0.12, size = 15, normalized size = 0.83

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

maple [A] time = 0.04, size = 16, normalized size = 0.89

$$\frac{e^{2\arctan(ax)}}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] 1/2*exp(2*arctan(a*x))/a/c
```

maxima [A] time = 0.43, size = 15, normalized size = 0.83

$$\frac{e^{(2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

mupad [B] time = 0.53, size = 15, normalized size = 0.83

$$\frac{e^{2\,\mathrm{atan}(a\,x)}}{2\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] exp(2*atan(a*x))/(2*a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{e^{2\,\mathrm{atan}\,(ax)}}{2ac} & \text{for } c \neq 0 \\ \tilde{\infty} \int e^{2\,\mathrm{atan}\,(ax)}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((exp(2*atan(a*x))/(2*a*c), Ne(c, 0)), (zoo*Integral(exp(2*atan(a*
x)), x), True))
```

3.264 $\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal. Leaf size=53

$$\frac{(ax+1)e^{2\tan^{-1}(ax)}}{4ac^2(a^2x^2+1)}+\frac{e^{2\tan^{-1}(ax)}}{8ac^2}$$

```
[Out] 1/8*exp(2*arctan(a*x))/a/c^2+1/4*exp(2*arctan(a*x))*(a*x+1)/a/c^2/(a^2*x^2+
1)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{(ax+1)e^{2\tan^{-1}(ax)}}{4ac^2(a^2x^2+1)}+\frac{e^{2\tan^{-1}(ax)}}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] E^(2*ArcTan[a*x])/(8*a*c^2) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*c^2*(1 + a
^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^2}\,dx &= \frac{e^{2\tan^{-1}(ax)}(1+ax)}{4ac^2(1+a^2x^2)}+\frac{\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{4c}\\ &= \frac{e^{2\tan^{-1}(ax)}}{8ac^2}+\frac{e^{2\tan^{-1}(ax)}(1+ax)}{4ac^2(1+a^2x^2)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 1.04

$$\frac{(1-iax)^i(1+iax)^{-i}(a^2x^2+2ax+3)}{8c^2(a^3x^2+a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

[Out] ((1 - I*a*x)^I*(3 + 2*a*x + a^2*x^2))/(8*c^2*(1 + I*a*x)^I*(a + a^3*x^2))

fricas [A] time = 0.48, size = 40, normalized size = 0.75

$$\frac{\left(a^2x^2 + 2\,ax + 3\right)e^{\left(2\,\arctan(ax)\right)}}{8\left(a^3c^2x^2 + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] 1/8*(a^2*x^2 + 2*a*x + 3)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.04, size = 40, normalized size = 0.75

$$\frac{e^{2\arctan(ax)}\left(a^2x^2 + 2ax + 3\right)}{8\left(a^2x^2 + 1\right)a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x)

[Out] 1/8*exp(2*arctan(a*x))*(a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(2\,\arctan(ax)\right)}}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)

mupad [B] time = 0.58, size = 46, normalized size = 0.87

$$\frac{e^{2\,\mathrm{atan}(a\,x)}\left(\frac{3}{8\,a^3\,c^2} + \frac{x}{4\,a^2\,c^2} + \frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2} + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^2,x)

[Out] (exp(2*atan(a*x))*(3/(8*a^3*c^2) + x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^2 + x^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^2x^2e^{2\,\mathrm{atan}\,(ax)}}{8a^3c^2x^2+8ac^2} + \frac{2axe^{2\,\mathrm{atan}\,(ax)}}{8a^3c^2x^2+8ac^2} + \frac{3e^{2\,\mathrm{atan}\,(ax)}}{8a^3c^2x^2+8ac^2} & \text{for } c \neq 0 \\ \tilde{\infty} \int e^{2\,\mathrm{atan}\,(ax)}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise(((a**2*x**2*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 2*a*x
*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 3*exp(2*atan(a*x))/(8*a**
3*c**2*x**2 + 8*a*c**2), Ne(c, 0)), (zoo*Integral(exp(2*atan(a*x)), x), Tru
e))
```

3.265 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} \, dx$

Optimal. Leaf size=88

$$\frac{3(ax+1)e^{2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}+\frac{(2ax+1)e^{2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}+\frac{3e^{2\tan^{-1}(ax)}}{40ac^3}$$

```
[Out] 3/40*exp(2*arctan(a*x))/a/c^3+1/10*exp(2*arctan(a*x))*(2*a*x+1)/a/c^3/(a^2*
x^2+1)^2+3/20*exp(2*arctan(a*x))*(a*x+1)/a/c^3/(a^2*x^2+1)
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5070, 5071}

$$\frac{3(ax+1)e^{2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}+\frac{(2ax+1)e^{2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}+\frac{3e^{2\tan^{-1}(ax)}}{40ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*E^(2*ArcTan[a*x]))/(40*a*c^3) + (E^(2*ArcTan[a*x])*(1 + 2*a*x))/(10*a*c^
3*(1 + a^2*x^2)^2) + (3*E^(2*ArcTan[a*x])*(1 + a*x))/(20*a*c^3*(1 + a^2*x^2
))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3} \, dx &= \frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2} \, dx}{5c}\\
&= \frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\tan^{-1}(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}+\frac{3\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2} \, dx}{20c^2}\\
&= \frac{3e^{2\tan^{-1}(ax)}}{40ac^3}+\frac{e^{2\tan^{-1}(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\tan^{-1}(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [**C**] time = 0.14, size = 86, normalized size = 0.98

$$\frac{4(2ax+1)e^{2\tan^{-1}(ax)}+3(1-iax)^{i}(1+iax)^{-i}\left(a^{2}x^{2}+1\right)\left(a^{2}x^{2}+2ax+3\right)}{40ac^{3}\left(a^{2}x^{2}+1\right)^{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4*E^(2*ArcTan[a*x])*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(1 + a^2*x^2)*(3 + 2*a*
x + a^2*x^2))/(1 + I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

fricas [**A**] time = 0.56, size = 68, normalized size = 0.77

$$\frac{\left(3\,a^{4}x^{4}+6\,a^{3}x^{3}+12\,a^{2}x^{2}+14\,ax+13\right)e^{(2\,\arctan(ax))}}{40\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^4*x^4 + 6*a^3*x^3 + 12*a^2*x^2 + 14*a*x + 13)*e^(2*arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.04, size = 57, normalized size = 0.65

$$\frac{\mathrm{e}^{2\arctan(ax)}\left(3a^{4}x^{4}+6a^{3}x^{3}+12a^{2}x^{2}+14ax+13\right)}{40\left(a^{2}x^{2}+1\right)^{2}a\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/40*exp(2*arctan(a*x))*(3*a^4*x^4+6*a^3*x^3+12*a^2*x^2+14*a*x+13)/(a^2*x^2
+1)^2/a/c^3
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(2\,\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

mupad [**B**] time = 0.62, size = 79, normalized size = 0.90

$$\frac{3\,\mathrm{e}^{2\,\mathrm{atan}(a\,x)}}{40\,a\,c^{3}}+\frac{3\,\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(a\,x+1\right)}{20\,a\,c^{3}\,\left(a^{2}\,x^{2}+1\right)}+\frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(2\,a\,x+1\right)}{10\,a\,c^{3}\,\left(a^{2}\,x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (3*exp(2*atan(a*x)))/(40*a*c^3) + (3*exp(2*atan(a*x))*(a*x + 1))/(20*a*c^3*
(a^2*x^2 + 1)) + (exp(2*atan(a*x))*(2*a*x + 1))/(10*a*c^3*(a^2*x^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{3a^4x^4e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{6a^3x^3e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{12a^2x^2e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{14axe^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{13e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} \\ \tilde{\infty}\int e^{2\operatorname{atan}(ax)}\,dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((3*a**4*x**4*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a**3*c**3*x
**2 + 40*a*c**3) + 6*a**3*x**3*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a**
3*c**3*x**2 + 40*a*c**3) + 12*a**2*x**2*exp(2*atan(a*x))/(40*a**5*c**3*x**4
 + 80*a**3*c**3*x**2 + 40*a*c**3) + 14*a*x*exp(2*atan(a*x))/(40*a**5*c**3*x
**4 + 80*a**3*c**3*x**2 + 40*a*c**3) + 13*exp(2*atan(a*x))/(40*a**5*c**3*x*
*4 + 80*a**3*c**3*x**2 + 40*a*c**3), Ne(c, 0)), (zoo*Integral(exp(2*atan(a*
x)), x), True))
```

3.266 $\int \frac{e^{2\tan^{-1}(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal. Leaf size=123

$$\frac{9(ax+1)e^{2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}+\frac{3(2ax+1)e^{2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}+\frac{(3ax+1)e^{2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}+\frac{9e^{2\tan^{-1}(ax)}}{160ac^4}$$

```
[Out] 9/160*exp(2*arctan(a*x))/a/c^4+1/20*exp(2*arctan(a*x))*(3*a*x+1)/a/c^4/(a^2
*x^2+1)^3+3/40*exp(2*arctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)^2+9/80*exp(2*
arctan(a*x))*(a*x+1)/a/c^4/(a^2*x^2+1)
```

Rubi [A] time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{9(ax+1)e^{2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}+\frac{3(2ax+1)e^{2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}+\frac{(3ax+1)e^{2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}+\frac{9e^{2\tan^{-1}(ax)}}{160ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (9*E^(2*ArcTan[a*x]))/(160*a*c^4) + (E^(2*ArcTan[a*x])*(1 + 3*a*x))/(20*a*c
^4*(1 + a^2*x^2)^3) + (3*E^(2*ArcTan[a*x])*(1 + 2*a*x))/(40*a*c^4*(1 + a^2*
x^2)^2) + (9*E^(2*ArcTan[a*x])*(1 + a*x))/(80*a*c^4*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}$$
$$= \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}$$
$$= \frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9e^{2\tan^{-1}(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)}+\frac{9\int \frac{e^{2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{80c^3}$$
$$= \frac{9e^{2\tan^{-1}(ax)}}{160ac^4}+\frac{e^{2\tan^{-1}(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\tan^{-1}(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9e^{2\tan^{-1}(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.27, size = 122, normalized size = 0.99

$$\frac{8c(3ax+1)e^{2\tan^{-1}(ax)}+3\left(a^2cx^2+c\right)\left(4(2ax+1)e^{2\tan^{-1}(ax)}+3(1-iax)^i(1+iax)^{-i}(ax-i)(ax+i)\left(a^2x^2+2ax\right.\right.}{160ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (8*c*E^(2*ArcTan[a*x])*(1 + 3*a*x) + 3*(c + a^2*c*x^2)*(4*E^(2*ArcTan[a*x])
*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(-I + a*x)*(I + a*x)*(3 + 2*a*x + a^2*x^2))
/(1 + I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

fricas [A] time = 0.48, size = 95, normalized size = 0.77

$$\frac{\left(9\,a^6x^6+18\,a^5x^5+45\,a^4x^4+60\,a^3x^3+75\,a^2x^2+66\,ax+47\right)e^{(2\arctan(ax))}}{160\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/160*(9*a^6*x^6 + 18*a^5*x^5 + 45*a^4*x^4 + 60*a^3*x^3 + 75*a^2*x^2 + 66*a
*x + 47)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 + a
*c^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 73, normalized size = 0.59

$$\frac{e^{2\arctan(ax)}\left(9a^6x^6+18a^5x^5+45a^4x^4+60a^3x^3+75a^2x^2+66ax+47\right)}{160\left(a^2x^2+1\right)^3 a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] 1/160*exp(2*arctan(a*x))*(9*a^6*x^6+18*a^5*x^5+45*a^4*x^4+60*a^3*x^3+75*a^2
*x^2+66*a*x+47)/(a^2*x^2+1)^3/a/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(2\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

mupad [B] time = 0.67, size = 111, normalized size = 0.90

$$\frac{9\,e^{2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4}+\frac{9\,e^{2\,\mathrm{atan}(a\,x)}\,(a\,x+1)}{80\,a\,c^4\,\left(a^2\,x^2+1\right)}+\frac{3\,e^{2\,\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{40\,a\,c^4\,\left(a^2\,x^2+1\right)^2}+\frac{e^{2\,\mathrm{atan}(a\,x)}\,(3\,a\,x+1)}{20\,a\,c^4\,\left(a^2\,x^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (9*exp(2*atan(a*x)))/(160*a*c^4) + (9*exp(2*atan(a*x))*(a*x + 1))/(80*a*c^4
*(a^2*x^2 + 1)) + (3*exp(2*atan(a*x))*(2*a*x + 1))/(40*a*c^4*(a^2*x^2 + 1)^
2) + (exp(2*atan(a*x))*(3*a*x + 1))/(20*a*c^4*(a^2*x^2 + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{9a^6x^6e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{18a^5x^5e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{45a^4x^4e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{}{160a^7c^4} \\ \tilde{\infty}\int e^{2\,\mathrm{atan}\,(ax)}\,dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise((9*a**6*x**6*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4
*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 18*a**5*x**5*exp(2*atan(a*x))/(1
60*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) +
 45*a**4*x**4*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 4
80*a**3*c**4*x**2 + 160*a*c**4) + 60*a**3*x**3*exp(2*atan(a*x))/(160*a**7*c
**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 75*a**2*
x**2*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c
**4*x**2 + 160*a*c**4) + 66*a*x*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*
a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 47*exp(2*atan(a*x))/(16
0*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4), N
e(c, 0)), (zoo*Integral(exp(2*atan(a*x)), x), True))
```

3.267 $\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (2/29+5/29*I)*2^(5/2-I)*c*(1-I*a*x)^(5/2+I)*hypergeom([5/2+I, -3/2+I],[7/2+
I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{2\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}+i}(1+iax)^{\frac{3}{2}-i}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\;{}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.00

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}+i,\frac{5}{2}+i;\frac{7}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\ \arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\ \arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} e^{2 \operatorname{atan}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(2*atan(a*x)), x)
```

3.268 $\int e^{2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=87

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (2/13+3/13*I)*2^(3/2-I)*(1-I*a*x)^(3/2+I)*hypergeom([3/2+I, -1/2+I],[5/2+I]
,1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx &= \frac{\sqrt{c+a^2cx^2}\int e^{2\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+i}(1+iax)^{\frac{1}{2}-i}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\;{}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{1}{2}+i,\frac{3}{2}+i;\frac{5}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{2\arctan(ax)}\sqrt{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}\,e^{2\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(2*atan(a*x)), x)
```

3.269 $\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (2/5+1/5*I)*2^(1/2-I)*(1-I*a*x)^(1/2+I)*hypergeom([1/2+I, 1/2+I],[3/2+I],1/
2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{2\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+i}(1+iax)^{-\frac{1}{2}-i}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\;{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}+i,\frac{1}{2}+i;\frac{3}{2}+i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.270 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

[Out] 1/5*exp(2*arctan(a*x))*(a*x+2)/a/c/(a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5069}

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]

[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{e^{2\tan^{-1}(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{(ax+2)e^{2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]

[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])

fricas [A] time = 0.47, size = 44, normalized size = 1.19

$$\frac{\sqrt{a^2cx^2+c}\,(ax+2)e^{(2\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x + 2)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 39, normalized size = 1.05

$$\frac{\left(a^{2} x^{2}+1\right) \left(a x +2\right) {\mathrm e}^{2 \arctan \left(a x \right)}}{5 a \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
[Out] 1/5*(a^2*x^2+1)*(a*x+2)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(2 \arctan\left(ax\right)\right)}}{\left(a^{2} c x^{2} + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 0.61, size = 35, normalized size = 0.95

$$\frac{{\mathrm{e}}^{2\,\mathrm{atan}\left(a\,x\right)}\,\left(\frac{x}{5\,c}+\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(2*atan(a*x))*(x/(5*c) + 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2 \operatorname{atan}{\left(ax \right)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.271 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{6(ax+2)e^{2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 1/13*exp(2*arctan(a*x))*(3*a*x+2)/a/c/(a^2*c*x^2+c)^(3/2)+6/65*exp(2*arctan
(a*x))*(a*x+2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$\frac{6(ax+2)e^{2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*Ar
cTan[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{e^{2\tan^{-1}(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c}\\ &= \frac{e^{2\tan^{-1}(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6e^{2\tan^{-1}(ax)}(2+ax)}{65ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.82

$$\frac{\left(6a^3x^3+12a^2x^2+21ax+22\right)e^{2\tan^{-1}(ax)}}{65c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(22 + 21*a*x + 12*a^2*x^2 + 6*a^3*x^3))/(65*c^2*(a + a^3
*x^2)*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.43, size = 72, normalized size = 0.95

$$\frac{\left(6\,a^{3}x^{3}+12\,a^{2}x^{2}+21\,ax+22\right)\sqrt{a^{2}cx^{2}+c}\,e^{(2\,\arctan(ax))}}{65\,\left(a^{5}c^{3}x^{4}+2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/65*(6*a^3*x^3 + 12*a^2*x^2 + 21*a*x + 22)*sqrt(a^2*c*x^2 + c)*e^(2*arctan
(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 56, normalized size = 0.74

$$\frac{\left(a^{2}x^{2}+1\right)\left(6a^{3}x^{3}+12a^{2}x^{2}+21ax+22\right)\mathrm{e}^{2\arctan(ax)}}{65a\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3+12*a^2*x^2+21*a*x+22)*exp(2*arctan(a*x))/a/(a^2
*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(2\,\arctan(ax))}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 0.22, size = 80, normalized size = 1.05

$$\frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(\frac{22}{65\,a^3\,c^2}+\frac{6\,x^3}{65\,c^2}+\frac{21\,x}{65\,a^2\,c^2}+\frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] (exp(2*atan(a*x))*(22/(65*a^3*c^2) + (6*x^3)/(65*c^2) + (21*x)/(65*a^2*c^2)
 + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(
1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.272 $\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{24(ax+2)e^{2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}+\frac{20(3ax+2)e^{2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+2)e^{2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] 1/29*exp(2*arctan(a*x))*(5*a*x+2)/a/c/(a^2*c*x^2+c)^(5/2)+20/377*exp(2*arct
an(a*x))*(3*a*x+2)/a/c^2/(a^2*c*x^2+c)^(3/2)+24/377*exp(2*arctan(a*x))*(a*x
+2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$\frac{24(ax+2)e^{2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}+\frac{20(3ax+2)e^{2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+2)e^{2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 5*a*x))/(29*a*c*(c + a^2*c*x^2)^(5/2)) + (20*E^(2*A
rcTan[a*x])*(2 + 3*a*x))/(377*a*c^2*(c + a^2*c*x^2)^(3/2)) + (24*E^(2*ArcTa
n[a*x])*(2 + a*x))/(377*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx &= \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}+\frac{20\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{29c}\\ &= \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}+\frac{20e^{2\tan^{-1}(ax)}(2+3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{120\int \frac{e^{2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{377c^2}\\ &= \frac{e^{2\tan^{-1}(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}+\frac{20e^{2\tan^{-1}(ax)}(2+3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{24e^{2\tan^{-1}(ax)}(2+ax)}{377ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.71

$$\frac{\left(24a^5x^5+48a^4x^4+108a^3x^3+136a^2x^2+149ax+114\right)e^{2\tan^{-1}(ax)}}{377ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(114 + 149*a*x + 136*a^2*x^2 + 108*a^3*x^3 + 48*a^4*x^4
+ 24*a^5*x^5))/(377*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.42, size = 99, normalized size = 0.87

$$\frac{\left(24\,a^5x^5+48\,a^4x^4+108\,a^3x^3+136\,a^2x^2+149\,ax+114\right)\sqrt{a^2cx^2+c}\,e^{(2\,\arctan(ax))}}{377\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/377*(24*a^5*x^5 + 48*a^4*x^4 + 108*a^3*x^3 + 136*a^2*x^2 + 149*a*x + 114)
*sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3
*c^4*x^2 + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 72, normalized size = 0.63

$$\frac{\left(a^2x^2+1\right)\left(24a^5x^5+48a^4x^4+108a^3x^3+136a^2x^2+149ax+114\right)\mathrm{e}^{2\arctan(ax)}}{377a\left(a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5+48*a^4*x^4+108*a^3*x^3+136*a^2*x^2+149*a*x+11
4)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(2\,\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 0.69, size = 122, normalized size = 1.07

$$\frac{e^{2\,\mathrm{atan}(a\,x)}\,\left(\frac{114}{377\,a^5\,c^3}+\frac{24\,x^5}{377\,c^3}+\frac{149\,x}{377\,a^4\,c^3}+\frac{48\,x^4}{377\,a\,c^3}+\frac{108\,x^3}{377\,a^2\,c^3}+\frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] (exp(2*atan(a*x))*(114/(377*a^5*c^3) + (24*x^5)/(377*c^3) + (149*x)/(377*a^
4*c^3) + (48*x^4)/(377*a*c^3) + (108*x^3)/(377*a^2*c^3) + (136*x^2)/(377*a^
3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c
 + a^2*c*x^2)^(1/2))/a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.273 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{2^{p+\left(1+\frac{i}{2}\right)}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(-2ip-(1+2i))}$$

```
[Out] 2^(1+1/2*I+p)*(1-I*a*x)^(1-1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([-1/2*I-p, 1-
1/2*I+p],[2-1/2*I+p],1/2-1/2*I*a*x)/a/(-1-2*I-2*I*p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{2^{p+\left(1+\frac{i}{2}\right)}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a(-2ip-(1+2i))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (2^((1 + I/2) + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeome
tric2F1[-I/2 - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((-1 - 2
*I) - (2*I)*p)*(1 + a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\
&= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{-\frac{i}{2}+p}(1+iax)^{\frac{i}{2}+p}\,dx\\
&= \frac{2^{\left(1+\frac{i}{2}\right)+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p;\left(2-\right.\right.}{a((-1-2i)-2ip)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 102, normalized size = 1.01

$$\frac{i2^{p+\frac{i}{2}}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right);p+\left(2-\frac{i}{2}\right);\frac{1}{2}(1-iax)\right)}{a\left(p+\left(1-\frac{i}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (I*2^(I/2 + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeometric
2F1[-1/2*I - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((1 - I/2)
 + p)*(1 + a^2*x^2)^p)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^pe^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\left(a^2c\,x^2+c\right)^p\mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^pe^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\mathrm{e}^{-\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^p,x)
```

```
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^p, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{-\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(atan(a*x)),x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(-atan(a*x)), x)
```

3.274 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=63

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/37+6/37*I)*2^(3+1/2*I)*c^2*(1-I*a*x)^(3-1/2*I)*hypergeom([3-1/2*I, -2-1
/2*I],[4-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int(1-iax)^{2-\frac{i}{2}}(1+iax)^{2+\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.00

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,{}_2F_1\left(-2-\frac{i}{2},3-\frac{i}{2};4-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)e^{(-\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2 + c\right)^2 \mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2 + c\right)^2 e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^2,x)
```

```
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2e^{-\operatorname{atan}(ax)}\,dx + \int a^4x^4e^{-\operatorname{atan}(ax)}\,dx + \int e^{-\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/exp(atan(a*x)),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2*exp(-atan(a*x)), x) + Integral(a**4*x**4*exp(-at
an(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

3.275 $\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=61

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/17+4/17*I)*2^(2+1/2*I)*c*(1-I*a*x)^(2-1/2*I)*hypergeom([2-1/2*I, -1-1/2
*I],[3-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)dx &= c\int(1-iax)^{1-\frac{i}{2}}(1+iax)^{1+\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.00

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,{}_2F_1\left(-1-\frac{i}{2},2-\frac{i}{2};3-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)e^{\left(-\arctan(ax)\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)\mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)e^{\left(-\arctan(ax)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2x^2e^{-\operatorname{atan}(ax)}\,dx+\int e^{-\operatorname{atan}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/exp(atan(a*x)),x)
[Out] c*(Integral(a**2*x**2*exp(-atan(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

3.276 $\int e^{-\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+2/5*I)*2^(1+1/2*I)*(1-I*a*x)^(1-1/2*I)*hypergeom([-1/2*I, 1-1/2*I],[2
-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5061, 69}

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I/2)*(1 - I*a*x)^(1 - I/2)*Hypergeometric2F1[-I/2,
 1 - I/2, 2 - I/2, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\,dx\\ &= -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,{}_2F_1\left(-\frac{i}{2},1-\frac{i}{2};2-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.75

$$-\frac{\left(\frac{4}{5}+\frac{8i}{5}\right)e^{(-1+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1+\frac{i}{2},2;2+\frac{i}{2};-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcTan[a*x]),x]
```

```
[Out] ((-4/5 - (8*I)/5)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -E^((2*I)*ArcTan[a
*x])])/(a*E^((1 - 2*I)*ArcTan[a*x]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{\left(-\arctan\left(ax\right)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(-arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-arctan(a*x)),x)
[Out] int(exp(-arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(-\arctan\left(ax\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{-\mathrm{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x)),x)
[Out] int(exp(-atan(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-\operatorname{atan}{\left(ax \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-atan(a*x)),x)
[Out] Integral(exp(-atan(a*x)), x)
```

3.277 $\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\tan^{-1}(ax)}}{ac}$$

```
[Out] -1/a/c/exp(arctan(a*x))
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5071}

$$-\frac{e^{-\tan^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
```

```
[Out] -(1/(a*c*E^ArcTan[a*x]))
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-\tan^{-1}(ax)}}{ac}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 2.25

$$-\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
```

```
[Out] -((1 + I*a*x)^(I/2)/(a*c*(1 - I*a*x)^(I/2)))
```

fricas [A] time = 0.47, size = 15, normalized size = 0.94

$$-\frac{e^{(-\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -e^(-arctan(a*x))/(a*c)
```

giac [A] time = 0.11, size = 15, normalized size = 0.94

$$-\frac{e^{(-\arctan(ax))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -e^(-arctan(a*x))/(a*c)
```

maple [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{e^{-\arctan(ax)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] -1/a/c/exp(arctan(a*x))
```

maxima [A] time = 0.35, size = 23, normalized size = 1.44

$$-\frac{2\,e^{(-\arctan(ax))}}{a^3cx^2 + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -2*e^(-arctan(a*x))/(a^3*c*x^2 + a*c)
```

mupad [B] time = 0.53, size = 15, normalized size = 0.94

$$-\frac{e^{-\mathrm{atan}(a\,x)}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] -exp(-atan(a*x))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{e^{-\operatorname{atan}(ax)}}{ac} & \text{for } c \neq 0 \\ \tilde{\infty} \int e^{-\operatorname{atan}(ax)}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((-exp(-atan(a*x))/(a*c), Ne(c, 0)), (zoo*Integral(exp(-atan(a*x))
, x), True))
```

3.278 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{(1-2ax)e^{-\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}-\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}$$

```
[Out] -2/5/a/c^2/exp(arctan(a*x))+1/5*(2*a*x-1)/a/c^2/exp(arctan(a*x))/(a^2*x^2+1
)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-2ax)e^{-\tan^{-1}(ax)}}{5ac^2\left(a^2x^2+1\right)}-\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

```
[Out] -2/(5*a*c^2*E^ArcTan[a*x]) - (1 - 2*a*x)/(5*a*c^2*E^ArcTan[a*x]*(1 + a^2*x^
2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{e^{-\tan^{-1}(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)}+\frac{2\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{5c}\\ &= -\frac{2e^{-\tan^{-1}(ax)}}{5ac^2}-\frac{e^{-\tan^{-1}(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] time = 0.03, size = 60, normalized size = 1.11

$$-\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(2a^2x^2-2ax+3\right)}{5c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/5*((1 + I*a*x)^(I/2)*(3 - 2*a*x + 2*a^2*x^2))/(c^2*(1 - I*a*x)^(I/2)*(a
+ a^3*x^2))
```

fricas [A] time = 0.57, size = 41, normalized size = 0.76

$$-\frac{\left(2\,a^2x^2 - 2\,ax + 3\right)e^{(-\arctan(ax))}}{5\,\left(a^3c^2x^2 + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/5*(2*a^2*x^2 - 2*a*x + 3)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 41, normalized size = 0.76

$$-\frac{\left(2a^2x^2 - 2ax + 3\right)\mathrm{e}^{-\arctan(ax)}}{5\left(a^2x^2+1\right)c^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x)
[Out] -1/5*(2*a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(arctan(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

mupad [B] time = 0.56, size = 47, normalized size = 0.87

$$-\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(\frac{3}{5\,a^3\,c^2}-\frac{2\,x}{5\,a^2\,c^2}+\frac{2\,x^2}{5\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] -(exp(-atan(a*x))*(3/(5*a^3*c^2) - (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))/
(1/a^2 + x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2a^2x^2}{5a^3c^2x^2e^{\operatorname{atan}{(ax)}}+5ac^2e^{\operatorname{atan}{(ax)}}} + \frac{2ax}{5a^3c^2x^2e^{\operatorname{atan}{(ax)}}+5ac^2e^{\operatorname{atan}{(ax)}}} - \frac{3}{5a^3c^2x^2e^{\operatorname{atan}{(ax)}}+5ac^2e^{\operatorname{atan}{(ax)}}} & \text{for}\: c \neq 0 \\ \tilde{\infty} \int e^{-\operatorname{atan}{(ax)}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-2*a**2*x**2/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(ata
n(a*x))) + 2*a*x/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x))
) - 3/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x))), Ne(c, 0)
), (zoo*Integral(exp(-atan(a*x)), x), True))
```

3.279 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{(1-4ax)e^{-\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}-\frac{12(1-2ax)e^{-\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}-\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}$$

```
[Out] -24/85/a/c^3/exp(arctan(a*x))+1/17*(4*a*x-1)/a/c^3/exp(arctan(a*x))/(a^2*x^
2+1)^2-12/85*(-2*a*x+1)/a/c^3/exp(arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-4ax)e^{-\tan^{-1}(ax)}}{17ac^3\left(a^2x^2+1\right)^2}-\frac{12(1-2ax)e^{-\tan^{-1}(ax)}}{85ac^3\left(a^2x^2+1\right)}-\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

```
[Out] -24/(85*a*c^3*E^ArcTan[a*x]) - (1 - 4*a*x)/(17*a*c^3*E^ArcTan[a*x]*(1 + a^2
*x^2)^2) - (12*(1 - 2*a*x))/(85*a*c^3*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= -\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{17c}\\
&= -\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\tan^{-1}(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{85c^2}\\
&= -\frac{24e^{-\tan^{-1}(ax)}}{85ac^3}-\frac{e^{-\tan^{-1}(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\tan^{-1}(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 91, normalized size = 1.02

$$\frac{5(4ax-1)e^{-\tan^{-1}(ax)}-12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(a^2x^2+1\right)\left(2a^2x^2-2ax+3\right)}{85ac^3\left(a^2x^2+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((5*(-1 + 4*a*x))/E^ArcTan[a*x] - (12*(1 + I*a*x)^(I/2)*(1 + a^2*x^2)*(3 -
2*a*x + 2*a^2*x^2))/(1 - I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

fricas [A] time = 0.46, size = 68, normalized size = 0.76

$$-\frac{\left(24\,a^4x^4 - 24\,a^3x^3 + 60\,a^2x^2 - 44\,ax + 41\right)e^{\left(-\arctan\left(ax\right)\right)}}{85\,\left(a^5c^3x^4 + 2\,a^3c^3x^2 + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/85*(24*a^4*x^4 - 24*a^3*x^3 + 60*a^2*x^2 - 44*a*x + 41)*e^(-arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 57, normalized size = 0.64

$$-\frac{\left(24a^4x^4 - 24a^3x^3 + 60a^2x^2 - 44ax + 41\right)\mathrm{e}^{-\arctan\left(ax\right)}}{85\left(a^2x^2+1\right)^2c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/85*(24*a^4*x^4-24*a^3*x^3+60*a^2*x^2-44*a*x+41)/(a^2*x^2+1)^2/c^3/exp(ar
ctan(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(-\arctan\left(ax\right)\right)}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

mupad [B] time = 0.62, size = 80, normalized size = 0.90

$$\frac{12\,\mathrm{e}^{-\mathrm{atan}\left(a\,x\right)}\,\left(2\,a\,x-1\right)}{85\,a\,c^3\,\left(a^2\,x^2+1\right)}-\frac{24\,\mathrm{e}^{-\mathrm{atan}\left(a\,x\right)}}{85\,a\,c^3}+\frac{\mathrm{e}^{-\mathrm{atan}\left(a\,x\right)}\,\left(4\,a\,x-1\right)}{17\,a\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (12*exp(-atan(a*x))*(2*a*x - 1))/(85*a*c^3*(a^2*x^2 + 1)) - (24*exp(-atan(a
*x)))/(85*a*c^3) + (exp(-atan(a*x))*(4*a*x - 1))/(17*a*c^3*(a^2*x^2 + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.280 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4} dx$

Optimal. Leaf size=124

$$-\frac{(1-6ax)e^{-\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}-\frac{72(1-2ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}-\frac{30(1-4ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}-\frac{144e^{-\tan^{-1}(ax)}}{629ac^4}$$

```
[Out] -144/629/a/c^4/exp(arctan(a*x))+1/37*(6*a*x-1)/a/c^4/exp(arctan(a*x))/(a^2*
x^2+1)^3-30/629*(-4*a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)^2-72/629*(-2*
a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-6ax)e^{-\tan^{-1}(ax)}}{37ac^4\left(a^2x^2+1\right)^3}-\frac{72(1-2ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)}-\frac{30(1-4ax)e^{-\tan^{-1}(ax)}}{629ac^4\left(a^2x^2+1\right)^2}-\frac{144e^{-\tan^{-1}(ax)}}{629ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] -144/(629*a*c^4*E^ArcTan[a*x]) - (1 - 6*a*x)/(37*a*c^4*E^ArcTan[a*x]*(1 + a
^2*x^2)^3) - (30*(1 - 4*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2)^2) - (
72*(1 - 2*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} + \frac{30\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{37c}$$
$$= -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} + \frac{360\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{629c^2}$$
$$= -\frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} - \frac{72e^{-\tan^{-1}(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)} + \frac{144\int \frac{e^{-\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{629c^3}$$
$$= -\frac{144e^{-\tan^{-1}(ax)}}{629ac^4} - \frac{e^{-\tan^{-1}(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3} - \frac{30e^{-\tan^{-1}(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2} - \frac{72e^{-\tan^{-1}(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.39, size = 127, normalized size = 1.02

$$\frac{17c(6ax-1)e^{-\tan^{-1}(ax)} - 6\left(a^2cx^2+c\right)\left(5(1-4ax)e^{-\tan^{-1}(ax)} + 12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(ax-i)(ax+i)\left(2a^2x^2-2a\right.\right.}{629ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((17*c*(-1 + 6*a*x))/E^ArcTan[a*x] - 6*(c + a^2*c*x^2)*((5*(1 - 4*a*x))/E^A
rcTan[a*x] + (12*(1 + I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 - 2*a*x + 2*a^2*
x^2))/(1 - I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

fricas [A] time = 0.46, size = 95, normalized size = 0.77

$$-\frac{\left(144\,a^6x^6 - 144\,a^5x^5 + 504\,a^4x^4 - 408\,a^3x^3 + 606\,a^2x^2 - 366\,ax + 263\right)e^{(-\arctan(ax))}}{629\left(a^7c^4x^6 + 3\,a^5c^4x^4 + 3\,a^3c^4x^2 + ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/629*(144*a^6*x^6 - 144*a^5*x^5 + 504*a^4*x^4 - 408*a^3*x^3 + 606*a^2*x^2
 - 366*a*x + 263)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 73, normalized size = 0.59

$$-\frac{\left(144a^6x^6 - 144a^5x^5 + 504a^4x^4 - 408a^3x^3 + 606a^2x^2 - 366ax + 263\right)\mathrm{e}^{-\arctan(ax)}}{629\left(a^2x^2+1\right)^3c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] -1/629*(144*a^6*x^6-144*a^5*x^5+504*a^4*x^4-408*a^3*x^3+606*a^2*x^2-366*a*x
+263)/(a^2*x^2+1)^3/c^4/exp(arctan(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

mupad [B] time = 0.68, size = 112, normalized size = 0.90

$$\frac{72\,e^{-\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{629\,a\,c^4\,\left(a^2\,x^2+1\right)}-\frac{144\,e^{-\mathrm{atan}(a\,x)}}{629\,a\,c^4}+\frac{30\,e^{-\mathrm{atan}(a\,x)}\,(4\,a\,x-1)}{629\,a\,c^4\,\left(a^2\,x^2+1\right)^2}+\frac{e^{-\mathrm{atan}(a\,x)}\,(6\,a\,x-1)}{37\,a\,c^4\,\left(a^2\,x^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (72*exp(-atan(a*x))*(2*a*x - 1))/(629*a*c^4*(a^2*x^2 + 1)) - (144*exp(-atan
(a*x)))/(629*a*c^4) + (30*exp(-atan(a*x))*(4*a*x - 1))/(629*a*c^4*(a^2*x^2
+ 1)^2) + (exp(-atan(a*x))*(6*a*x - 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.281 $\int e^{-\tan^{-1}(ax)} \left(c + a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=98

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (-1/13+5/13*I)*2^(3/2+1/2*I)*c*(1-I*a*x)^(5/2-1/2*I)*hypergeom([5/2-1/2*I,
-3/2-1/2*I],[7/2-1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}-\frac{i}{2}}(1+iax)^{\frac{3}{2}+\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$=-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\ {}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax\right.}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.00

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2};\frac{7}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(atan(a*x)),x)
[Out] Timed out
```

3.282 $\int e^{-\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (-1/5+3/5*I)*2^(1/2+1/2*I)*(1-I*a*x)^(3/2-1/2*I)*hypergeom([3/2-1/2*I, -1/2
-1/2*I],[5/2-1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx &= \frac{\sqrt{c+a^2cx^2}\int e^{-\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}-\frac{i}{2}}(1+iax)^{\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}\\
&= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\ {}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 97, normalized size = 1.00

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,{}_2F_1\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2};\frac{5}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{a^2c\,x^2+c}\;\mathrm{e}^{-\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}\,e^{-\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(atan(a*x)),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(-atan(a*x)), x)
```

3.283 $\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (-1+I)*2^(-1/2+1/2*I)*(1-I*a*x)^(1/2-1/2*I)*hypergeom([1/2-1/2*I, 1/2-1/2*I
],[3/2-1/2*I],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-\frac{i}{2}}(1+iax)^{-\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\;{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.00

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2};\frac{3}{2}-\frac{i}{2};\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(-atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.284 $\int \frac{e^{-\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(1-ax)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[Out] 1/2*(a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5069}

$$-\frac{(1-ax)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
[Out] -(1 - a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{-\tan^{-1}(ax)}(1-ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{(ax-1)e^{-\tan^{-1}(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
[Out] (-1 + a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.43, size = 44, normalized size = 1.16

$$\frac{\sqrt{a^2cx^2+c}\,(ax-1)e^{(-\arctan(ax))}}{2\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x - 1)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 39, normalized size = 1.03

$$\frac{\left(a^{2} x^{2}+1\right) \left(a x -1\right) {\mathrm e}^{-\arctan \left(a x \right)}}{2 a \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
[Out] 1/2*(a^2*x^2+1)*(a*x-1)/a/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(-\arctan\left(ax\right)\right)}}{\left(a^{2} c x^{2} + c\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 0.63, size = 35, normalized size = 0.92

$$\frac{{\mathrm{e}}^{-\mathrm{atan}\left(a\,x\right)}\,\left(\frac{x}{2\,c}-\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(-atan(a*x))*(x/(2*c) - 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{- \operatorname{atan}{\left(a x \right)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(-atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.285 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 1/10*(3*a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/10*(-a*x+1)/a/c^2
/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{10ac^2\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{10ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(1 - 3*a*x)/(10*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(
10*a*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= -\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\tan^{-1}(ax)}(1-ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.81

$$\frac{\left(3a^3x^3-3a^2x^2+6ax-4\right)e^{-\tan^{-1}(ax)}}{10c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-4 + 6*a*x - 3*a^2*x^2 + 3*a^3*x^3)/(10*c^2*E^ArcTan[a*x]*(a + a^3*x^2)*Sq
rt[c + a^2*c*x^2])
```

fricas [A] time = 0.42, size = 72, normalized size = 0.94

$$\frac{\left(3\,a^3x^3-3\,a^2x^2+6\,ax-4\right)\sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))}}{10\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 - 3*a^2*x^2 + 6*a*x - 4)*sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x
))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 56, normalized size = 0.73

$$\frac{\left(a^2x^2+1\right)\left(3a^3x^3-3a^2x^2+6ax-4\right)\mathrm{e}^{-\arctan(ax)}}{10a\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3-3*a^2*x^2+6*a*x-4)/a/exp(arctan(a*x))/(a^2*c*x^
2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 0.65, size = 81, normalized size = 1.05

$$-\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\left(\frac{2}{5\,a^3\,c^2}-\frac{3\,x^3}{10\,c^2}-\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp(-atan(a*x))*(2/(5*a^3*c^2) - (3*x^3)/(10*c^2) - (3*x)/(5*a^2*c^2) + (
3*x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.286 $\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(1-5ax)e^{-\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] 1/26*(5*a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2)+1/13*(3*a*x-1)/a/c^
2/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/13*(-a*x+1)/a/c^3/exp(arctan(a*x))
/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5070, 5069}

$$-\frac{3(1-ax)e^{-\tan^{-1}(ax)}}{13ac^3\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\tan^{-1}(ax)}}{13ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(1-5ax)e^{-\tan^{-1}(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(1 - 5*a*x)/(26*a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)) - (1 - 3*a*x)/(13
*a*c^2*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(13*a*c^3*E^Arc
Tan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx &= -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{10\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{13c}\\ &= -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}-\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{-\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c^2}\\ &= -\frac{e^{-\tan^{-1}(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}-\frac{e^{-\tan^{-1}(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\tan^{-1}(ax)}(1-ax)}{13ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.70

$$\frac{\left(6a^5x^5-6a^4x^4+18a^3x^3-14a^2x^2+17ax-9\right)e^{-\tan^{-1}(ax)}}{26ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-9 + 17*a*x - 14*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 + 6*a^5*x^5)/(26*a*c^3*E
^ArcTan[a*x]*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.42, size = 99, normalized size = 0.86

$$\frac{\left(6\,a^5x^5-6\,a^4x^4+18\,a^3x^3-14\,a^2x^2+17\,ax-9\right)\sqrt{a^2cx^2+c}\,e^{(-\arctan(ax))}}{26\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/26*(6*a^5*x^5 - 6*a^4*x^4 + 18*a^3*x^3 - 14*a^2*x^2 + 17*a*x - 9)*sqrt(a^
2*c*x^2 + c)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2
+ a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 72, normalized size = 0.63

$$\frac{\left(a^2x^2+1\right)\left(6a^5x^5-6a^4x^4+18a^3x^3-14a^2x^2+17ax-9\right)\mathrm{e}^{-\arctan(ax)}}{26a\left(a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5-6*a^4*x^4+18*a^3*x^3-14*a^2*x^2+17*a*x-9)/a/exp
(arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(-\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 0.67, size = 123, normalized size = 1.07

$$-\frac{e^{-\mathrm{atan}(a\,x)}\,\left(\frac{9}{26\,a^5\,c^3}-\frac{3\,x^5}{13\,c^3}-\frac{17\,x}{26\,a^4\,c^3}+\frac{3\,x^4}{13\,a\,c^3}-\frac{9\,x^3}{13\,a^2\,c^3}+\frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] -(exp(-atan(a*x))*(9/(26*a^5*c^3) - (3*x^5)/(13*c^3) - (17*x)/(26*a^4*c^3)
+ (3*x^4)/(13*a*c^3) - (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c +
a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2)^(
1/2))/a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.287 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

```
[Out] I*2^(I+p)*(1-I*a*x)^(1-I+p)*(a^2*c*x^2+c)^p*hypergeom([-I-p, 1-I+p],[2-I+p]
,1/2-1/2*I*a*x)/a/(1-I+p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{-i+p}(1+iax)^{i+p}\,dx\\ &= \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(-i-p,(1-i)+p;(2-i)+\right.}{a((1-i)+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 1.00

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,{}_2F_1\left(-p-i,p+(1-i);p+(2-i);\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^p\mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,{\left(c\,a^2\,x^2+c\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

```
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{-2\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*atan(a*x)),x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(-2*atan(a*x)), x)
```

3.288 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+3/5*I)*2^(1+I)*c^2*(1-I*a*x)^(3-I)*hypergeom([3-I, -2-I],[4-I],1/2-1/
2*I*a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^2\,dx &= c^2\int(1-iax)^{2-i}(1+iax)^{2+i}\,dx\\ &= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,{}_2F_1\left(-2-i,3-i;4-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)e^{(-2\,\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)^2 \mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^2 e^{(-2\,\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2e^{-2\,\mathrm{atan}\,(ax)}\,dx + \int a^4x^4e^{-2\,\mathrm{atan}\,(ax)}\,dx + \int e^{-2\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/exp(2*atan(a*x)),x)
[Out] c**2*(Integral(2*a**2*x**2*exp(-2*atan(a*x)), x) + Integral(a**4*x**4*exp(-
2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)), x))
```

3.289 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal. Leaf size=51

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+2/5*I)*2^(1+I)*c*(1-I*a*x)^(2-I)*hypergeom([-1-I, 2-I],[3-I],1/2-1/2*
I*a*x)/a
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx&=c\int(1-iax)^{1-i}(1+iax)^{1+i}\,dx\\&=-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,{}_2F_1\left(-1-i,2-i;3-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a^2c\,x^2+c\right)\mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2x^2e^{-2\,\mathrm{atan}\,(ax)}\,dx+\int e^{-2\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/exp(2*atan(a*x)),x)
[Out] c*(Integral(a**2*x**2*exp(-2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)), x
))
```

3.290 $\int e^{-2\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=46

$$-\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1+I)*2^(-1+I)*(1-I*a*x)^(1-I)*hypergeom([-I, 1-I],[2-I],1/2-1/2*I*a*x)/a
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5061, 69}

$$-\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcTan[a*x]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1 - I)*Hypergeometric2F1[-I, 1 - I, 2 - I, (1 - I*a*
x)/2])/(2^(1 - I)*a)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tan^{-1}(ax)}\,dx &= \int (1-iax)^{-i}(1+iax)^{i}\,dx\\ &= -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,{}_2F_1\left(-i,1-i;2-i;\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.80

$$-\frac{(1+i)e^{(-2+2i)\tan^{-1}(ax)}\,{}_2F_1\left(1+i,2;2+i;-e^{2i\tan^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-2*ArcTan[a*x]),x]
```

```
[Out] ((-1 - I)*Hypergeometric2F1[1 + I, 2, 2 + I, -E^((2*I)*ArcTan[a*x])])/(a*E^
((2 - 2*I)*ArcTan[a*x]))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(-2\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(-2*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{-2 \arctan(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*arctan(a*x)),x)
[Out] int(exp(-2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(-2 \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x)),x)
[Out] int(exp(-2*atan(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-2 \operatorname{atan}{(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*atan(a*x)),x)
[Out] Integral(exp(-2*atan(a*x)), x)
```

3.291 $\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

```
[Out] -1/2/a/c/exp(2*arctan(a*x))
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5071}

$$-\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

```
[Out] -1/(2*a*c*E^(2*ArcTan[a*x]))
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\tan^{-1}(ax)}}{2ac}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 1.89

$$-\frac{(1-iax)^{-i}(1+iax)^i}{2ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

```
[Out] -1/2*(1 + I*a*x)^I/(a*c*(1 - I*a*x)^I)
```

fricas [A] time = 0.44, size = 15, normalized size = 0.83

$$-\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

giac [A] time = 0.13, size = 15, normalized size = 0.83

$$-\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

maple [A] time = 0.04, size = 18, normalized size = 1.00

$$-\frac{e^{-2\arctan(ax)}}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] -1/2/a/c/exp(2*arctan(a*x))
```

maxima [A] time = 0.34, size = 23, normalized size = 1.28

$$-\frac{e^{(-2\arctan(ax))}}{a^3cx^2 + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -e^(-2*arctan(a*x))/(a^3*c*x^2 + a*c)
```

mupad [B] time = 0.53, size = 15, normalized size = 0.83

$$-\frac{e^{-2\,\mathrm{atan}(a\,x)}}{2\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] -exp(-2*atan(a*x))/(2*a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{e^{-2\,\mathrm{atan}\,(ax)}}{2ac} & \text{for}\ c \neq 0 \\ \tilde{\infty} \int e^{-2\,\mathrm{atan}\,(ax)}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((-exp(-2*atan(a*x))/(2*a*c), Ne(c, 0)), (zoo*Integral(exp(-2*atan
(a*x)), x), True))
```

3.292 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=54

$$-\frac{(1-ax)e^{-2\tan^{-1}(ax)}}{4ac^2\left(a^2x^2+1\right)}-\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}$$

```
[Out] -1/8/a/c^2/exp(2*arctan(a*x))+1/4*(a*x-1)/a/c^2/exp(2*arctan(a*x))/(a^2*x^2
+1)
```

<u>**Rubi**</u> **[A]** time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5070, 5071}

$$-\frac{(1-ax)e^{-2\tan^{-1}(ax)}}{4ac^2\left(a^2x^2+1\right)}-\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/(8*a*c^2*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*c^2*E^(2*ArcTan[a*x])*(1 +
a^2*x^2))
```

<u>Rule 5070</u>

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

<u>Rule 5071</u>

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

<u>Rubi steps</u>

$$\begin{aligned}\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}+\frac{\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{4c}\\ &= -\frac{e^{-2\tan^{-1}(ax)}}{8ac^2}-\frac{e^{-2\tan^{-1}(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}\end{aligned}$$

<u>**Mathematica**</u> **[C]** time = 0.02, size = 55, normalized size = 1.02

$$-\frac{(1-iax)^{-i}(1+iax)^{i}\left(a^2x^2-2ax+3\right)}{8c^2\left(a^3x^2+a\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

[Out] -1/8*((1 + I*a*x)^I*(3 - 2*a*x + a^2*x^2))/(c^2*(1 - I*a*x)^I*(a + a^3*x^2)
)

fricas [A] time = 0.47, size = 40, normalized size = 0.74

$$-\frac{\left(a^{2} x^{2}-2\, a x+3\right) e^{\left(-2\, \arctan\left(a x\right)\right)}}{8\, \left(a^{3} c^{2} x^{2}+a c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] -1/8*(a^2*x^2 - 2*a*x + 3)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.04, size = 42, normalized size = 0.78

$$-\frac{\left(a^{2} x^{2}-2 a x+3\right) \mathrm{e}^{-2 \arctan\left(a x\right)}}{8 \left(a^{2} x^{2}+1\right) c^{2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x)

[Out] -1/8*(a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(2*arctan(a*x))/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(-2\, \arctan\left(a x\right)\right)}}{\left(a^{2} c x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)

mupad [B] time = 0.57, size = 47, normalized size = 0.87

$$-\frac{\mathrm{e}^{-2\,\mathrm{atan}\left(a\,x\right)}\,\left(\frac{3}{8\,a^3\,c^2}-\frac{x}{4\,a^2\,c^2}+\frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^2,x)

[Out] -(exp(-2*atan(a*x))*(3/(8*a^3*c^2) - x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^2
 + x^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} - \frac{a^{2} x^{2}}{8 a^{3} c^{2} x^{2} e^{2 \operatorname{atan}{\left(a x \right)}} + 8 a c^{2} e^{2 \operatorname{atan}{\left(a x \right)}}} + \frac{2 a x}{8 a^{3} c^{2} x^{2} e^{2 \operatorname{atan}{\left(a x \right)}} + 8 a c^{2} e^{2 \operatorname{atan}{\left(a x \right)}}} - \frac{3}{8 a^{3} c^{2} x^{2} e^{2 \operatorname{atan}{\left(a x \right)}} + 8 a c^{2} e^{2 \operatorname{atan}{\left(a x \right)}}} & \text{for}\: c \neq 0 \\\tilde{\infty} \int e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**2*x**2/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*a
tan(a*x))) + 2*a*x/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*atan
(a*x))) - 3/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*atan(a*x)))
, Ne(c, 0)), (zoo*Integral(exp(-2*atan(a*x)), x), True))
```

3.293 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{(1-2ax)e^{-2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}-\frac{3(1-ax)e^{-2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}-\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}$$

```
[Out] -3/40/a/c^3/exp(2*arctan(a*x))+1/10*(2*a*x-1)/a/c^3/exp(2*arctan(a*x))/(a^2
*x^2+1)^2-3/20*(-a*x+1)/a/c^3/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-2ax)e^{-2\tan^{-1}(ax)}}{10ac^3\left(a^2x^2+1\right)^2}-\frac{3(1-ax)e^{-2\tan^{-1}(ax)}}{20ac^3\left(a^2x^2+1\right)}-\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] -3/(40*a*c^3*E^(2*ArcTan[a*x])) - (1 - 2*a*x)/(10*a*c^3*E^(2*ArcTan[a*x])*(
1 + a^2*x^2)^2) - (3*(1 - a*x))/(20*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{5c}\\
&= -\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}-\frac{3e^{-2\tan^{-1}(ax)}(1-ax)}{20ac^3\left(1+a^2x^2\right)}+\frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{20c^2}\\
&= -\frac{3e^{-2\tan^{-1}(ax)}}{40ac^3}-\frac{e^{-2\tan^{-1}(ax)}(1-2ax)}{10ac^3\left(1+a^2x^2\right)^2}-\frac{3e^{-2\tan^{-1}(ax)}(1-ax)}{20ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 85, normalized size = 0.96

$$\frac{(8ax-4)e^{-2\tan^{-1}(ax)}-3(1-iax)^{-i}(1+iax)^{i}\left(a^2x^2+1\right)\left(a^2x^2-2ax+3\right)}{40ac^3\left(a^2x^2+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] ((-4 + 8*a*x)/E^(2*ArcTan[a*x]) - (3*(1 + I*a*x)^I*(1 + a^2*x^2)*(3 - 2*a*x
 + a^2*x^2))/(1 - I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

fricas [A] time = 0.44, size = 68, normalized size = 0.76

$$-\frac{\left(3 a^4 x^4 - 6 a^3 x^3 + 12 a^2 x^2 - 14 a x + 13\right) e^{(-2 \arctan(ax))}}{40 \left(a^5 c^3 x^4 + 2 a^3 c^3 x^2 + a c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^4*x^4 - 6*a^3*x^3 + 12*a^2*x^2 - 14*a*x + 13)*e^(-2*arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 59, normalized size = 0.66

$$-\frac{\left(3a^4x^4 - 6a^3x^3 + 12a^2x^2 - 14ax + 13\right) e^{-2\arctan(ax)}}{40 \left(a^2x^2 + 1\right)^2 c^3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x)
```

```
[Out] -1/40*(3*a^4*x^4-6*a^3*x^3+12*a^2*x^2-14*a*x+13)/(a^2*x^2+1)^2/c^3/exp(2*ar
ctan(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-2 \arctan(ax))}}{\left(a^2 c x^2 + c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

mupad [B] time = 0.62, size = 79, normalized size = 0.89

$$\frac{3\,e^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{20\,a\,c^3\,\left(a^2\,x^2+1\right)} - \frac{3\,e^{-2\,\mathrm{atan}(a\,x)}}{40\,a\,c^3} + \frac{e^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{10\,a\,c^3\,\left(a^2\,x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

```
[Out] (3*exp(-2*atan(a*x))*(a*x - 1))/(20*a*c^3*(a^2*x^2 + 1)) - (3*exp(-2*atan(a
*x)))/(40*a*c^3) + (exp(-2*atan(a*x))*(2*a*x - 1))/(10*a*c^3*(a^2*x^2 + 1)^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.294 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=124

$$-\frac{(1-3ax)e^{-2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}-\frac{9(1-ax)e^{-2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}-\frac{3(1-2ax)e^{-2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4}$$

```
[Out] -9/160/a/c^4/exp(2*arctan(a*x))+1/20*(3*a*x-1)/a/c^4/exp(2*arctan(a*x))/(a^
2*x^2+1)^3-3/40*(-2*a*x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)^2-9/80*(-a*
x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$-\frac{(1-3ax)e^{-2\tan^{-1}(ax)}}{20ac^4\left(a^2x^2+1\right)^3}-\frac{9(1-ax)e^{-2\tan^{-1}(ax)}}{80ac^4\left(a^2x^2+1\right)}-\frac{3(1-2ax)e^{-2\tan^{-1}(ax)}}{40ac^4\left(a^2x^2+1\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] -9/(160*a*c^4*E^(2*ArcTan[a*x])) - (1 - 3*a*x)/(20*a*c^4*E^(2*ArcTan[a*x])*
(1 + a^2*x^2)^3) - (3*(1 - 2*a*x))/(40*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2
)^2) - (9*(1 - a*x))/(80*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}$$

$$= -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}$$

$$= -\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}+\frac{9\int \frac{e^{-2\tan^{-1}(ax)}}{c+a^2cx^2}\,dx}{80c^3}$$

$$= -\frac{9e^{-2\tan^{-1}(ax)}}{160ac^4}-\frac{e^{-2\tan^{-1}(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\tan^{-1}(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}-\frac{9e^{-2\tan^{-1}(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}$$

Mathematica [C] time = 0.37, size = 121, normalized size = 0.98

$$\frac{8c(3ax-1)e^{-2\tan^{-1}(ax)}-3\left(a^2cx^2+c\right)\left((4-8ax)e^{-2\tan^{-1}(ax)}+3(1-iax)^{-i}(1+iax)^{i}(ax-i)(ax+i)\left(a^2x^2-2ax\right.\right.}{160ac^2\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((8*c*(-1 + 3*a*x))/E^(2*ArcTan[a*x]) - 3*(c + a^2*c*x^2)*((4 - 8*a*x)/E^(2
*ArcTan[a*x]) + (3*(1 + I*a*x)^I*(-I + a*x)*(I + a*x)*(3 - 2*a*x + a^2*x^2)
)/(1 - I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

fricas [A] time = 0.49, size = 95, normalized size = 0.77

$$-\frac{\left(9\,a^{6}x^{6}-18\,a^{5}x^{5}+45\,a^{4}x^{4}-60\,a^{3}x^{3}+75\,a^{2}x^{2}-66\,ax+47\right)e^{(-2\,\arctan(ax))}}{160\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/160*(9*a^6*x^6 - 18*a^5*x^5 + 45*a^4*x^4 - 60*a^3*x^3 + 75*a^2*x^2 - 66*
a*x + 47)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 75, normalized size = 0.60

$$-\frac{\left(9a^{6}x^{6}-18a^{5}x^{5}+45a^{4}x^{4}-60a^{3}x^{3}+75a^{2}x^{2}-66ax+47\right)\mathrm{e}^{-2\arctan(ax)}}{160\left(a^{2}x^{2}+1\right)^{3}c^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x)

[Out] -1/160*(9*a^6*x^6-18*a^5*x^5+45*a^4*x^4-60*a^3*x^3+75*a^2*x^2-66*a*x+47)/(a^2*x^2+1)^3/c^4/exp(2*arctan(a*x))/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-2\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")

[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)

mupad [B] time = 0.67, size = 111, normalized size = 0.90

$$\frac{9\,e^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{80\,a\,c^4\,\left(a^2\,x^2+1\right)}-\frac{9\,e^{-2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4}+\frac{3\,e^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{40\,a\,c^4\,\left(a^2\,x^2+1\right)^2}+\frac{e^{-2\,\mathrm{atan}(a\,x)}\,(3\,a\,x-1)}{20\,a\,c^4\,\left(a^2\,x^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^4,x)

[Out] (9*exp(-2*atan(a*x))*(a*x - 1))/(80*a*c^4*(a^2*x^2 + 1)) - (9*exp(-2*atan(a*x)))/(160*a*c^4) + (3*exp(-2*atan(a*x))*(2*a*x - 1))/(40*a*c^4*(a^2*x^2 + 1)^2) + (exp(-2*atan(a*x))*(3*a*x - 1))/(20*a*c^4*(a^2*x^2 + 1)^3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)

[Out] Timed out

3.295 $\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (-2/29+5/29*I)*2^(5/2+I)*c*(1-I*a*x)^(5/2-I)*hypergeom([5/2-I, -3/2-I],[7/2
-I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-2\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}-i}(1+iax)^{\frac{3}{2}+i}\,dx}{\sqrt{1+a^2x^2}}\\
&= -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\;{}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 1.00

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{3}{2}-i,\frac{5}{2}-i;\frac{7}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(2*atan(a*x)),x)
[Out] Timed out
```

3.296 $\int e^{-2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[Out] (-2/13+3/13*I)*2^(3/2+I)*(1-I*a*x)^(3/2-I)*hypergeom([3/2-I, -1/2-I],[5/2-I
],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\;{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx &= \frac{\sqrt{c+a^2cx^2}\int e^{-2\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}-i}(1+iax)^{\frac{1}{2}+i}\,dx}{\sqrt{1+a^2x^2}}\\
&= -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\;{}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\ {}_2F_1\left(-\frac{1}{2}-i,\frac{3}{2}-i;\frac{5}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{a^2cx^2+c}\,e^{(-2\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{a^2c\,x^2+c}\,\mathrm{e}^{-2\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2cx^2+c}\,e^{(-2\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}e^{-2\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(2*atan(a*x)),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(-2*atan(a*x)), x)
```

3.297 $\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (-2/5+1/5*I)*2^(1/2+I)*(1-I*a*x)^(1/2-I)*hypergeom([1/2-I, 1/2-I],[3/2-I],1
/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-2\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-i}(1+iax)^{-\frac{1}{2}+i}\,dx}{\sqrt{c+a^2cx^2}}\\
&= -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\;{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,{}_2F_1\left(\frac{1}{2}-i,\frac{1}{2}-i;\frac{3}{2}-i;\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{e}^{-2\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-2\,\mathrm{atan}\,(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(-2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.298 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(2-ax)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

```
[Out] 1/5*(a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5069}

$$-\frac{(2-ax)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] -(2 - a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{-2\tan^{-1}(ax)}(2-ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{(ax-2)e^{-2\tan^{-1}(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] (-2 + a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.43, size = 44, normalized size = 1.16

$$\frac{\sqrt{a^2cx^2+c}\,(ax-2)e^{(-2\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x - 2)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.04, size = 41, normalized size = 1.08

$$\frac{(a^2x^2+1)(ax-2)\,\mathrm{e}^{-2\arctan(ax)}}{5a\,(a^2c\,x^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x)
[Out] 1/5*(a^2*x^2+1)*(a*x-2)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 0.18, size = 35, normalized size = 0.92

$$\frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\left(\frac{x}{5\,c}-\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(-2*atan(a*x))*(x/(5*c) - 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

3.299 $\int \frac{e^{-2\tan^{-1}(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{6(2-ax)e^{-2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 1/13*(3*a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-6/65*(-a*x+2)/a/c
^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{6(2-ax)e^{-2\tan^{-1}(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\tan^{-1}(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(2 - 3*a*x)/(13*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 - a*x
))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(2-3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c}\\ &= -\frac{e^{-2\tan^{-1}(ax)}(2-3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}-\frac{6e^{-2\tan^{-1}(ax)}(2-ax)}{65ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.81

$$\frac{\left(6a^3x^3-12a^2x^2+21ax-22\right)e^{-2\tan^{-1}(ax)}}{65c^2\left(a^3x^2+a\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-22 + 21*a*x - 12*a^2*x^2 + 6*a^3*x^3)/(65*c^2*E^(2*ArcTan[a*x])*(a + a^3*
x^2)*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.51, size = 72, normalized size = 0.94

$$\frac{\left(6\,a^3x^3-12\,a^2x^2+21\,ax-22\right)\sqrt{a^2cx^2+c}\,e^{(-2\,\arctan(ax))}}{65\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/65*(6*a^3*x^3 - 12*a^2*x^2 + 21*a*x - 22)*sqrt(a^2*c*x^2 + c)*e^(-2*arcta
n(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 58, normalized size = 0.75

$$\frac{\left(a^2x^2+1\right)\left(6a^3x^3-12a^2x^2+21ax-22\right)\mathrm{e}^{-2\arctan(ax)}}{65a\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3-12*a^2*x^2+21*a*x-22)/a/exp(2*arctan(a*x))/(a^2
*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(-2\,\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 0.63, size = 81, normalized size = 1.05

$$-\frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{22}{65\,a^3\,c^2}-\frac{6\,x^3}{65\,c^2}-\frac{21\,x}{65\,a^2\,c^2}+\frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp(-2*atan(a*x))*(22/(65*a^3*c^2) - (6*x^3)/(65*c^2) - (21*x)/(65*a^2*c^
2) + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.300 $\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{24(2-ax)e^{-2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}-\frac{20(2-3ax)e^{-2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(2-5ax)e^{-2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] 1/29*(5*a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2)-20/377*(-3*a*x+2)
/a/c^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-24/377*(-a*x+2)/a/c^3/exp(2*a
rctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5070, 5069}

$$-\frac{24(2-ax)e^{-2\tan^{-1}(ax)}}{377ac^3\sqrt{a^2cx^2+c}}-\frac{20(2-3ax)e^{-2\tan^{-1}(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}-\frac{(2-5ax)e^{-2\tan^{-1}(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(2 - 5*a*x)/(29*a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)) - (20*(2 - 3*
a*x))/(377*a*c^2*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (24*(2 - a*x))/
(377*a*c^3*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx &= -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}+\frac{20\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{29c}\\
&= -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}-\frac{20e^{-2\tan^{-1}(ax)}(2-3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{120\int \frac{e^{-2\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{377c^2}\\
&= -\frac{e^{-2\tan^{-1}(ax)}(2-5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}-\frac{20e^{-2\tan^{-1}(ax)}(2-3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}}-\frac{24e^{-2\tan^{-1}(ax)}(2-ax)}{377ac^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 81, normalized size = 0.70

$$\frac{\left(24a^5x^5-48a^4x^4+108a^3x^3-136a^2x^2+149ax-114\right)e^{-2\tan^{-1}(ax)}}{377ac^3\left(a^2x^2+1\right)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-114 + 149*a*x - 136*a^2*x^2 + 108*a^3*x^3 - 48*a^4*x^4 + 24*a^5*x^5)/(377
*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.50, size = 99, normalized size = 0.86

$$\frac{\left(24\,a^5x^5-48\,a^4x^4+108\,a^3x^3-136\,a^2x^2+149\,ax-114\right)\sqrt{a^2cx^2+c}\,e^{(-2\,\arctan(ax))}}{377\left(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/377*(24*a^5*x^5 - 48*a^4*x^4 + 108*a^3*x^3 - 136*a^2*x^2 + 149*a*x - 114)
*sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^
3*c^4*x^2 + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.04, size = 74, normalized size = 0.64

$$\frac{\left(a^2x^2+1\right)\left(24a^5x^5-48a^4x^4+108a^3x^3-136a^2x^2+149ax-114\right)\mathrm{e}^{-2\arctan(ax)}}{377a\left(a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5-48*a^4*x^4+108*a^3*x^3-136*a^2*x^2+149*a*x-11
4)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{(-2\,\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 0.66, size = 123, normalized size = 1.07

$$-\frac{e^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{114}{377\,a^5\,c^3}-\frac{24\,x^5}{377\,c^3}-\frac{149\,x}{377\,a^4\,c^3}+\frac{48\,x^4}{377\,a\,c^3}-\frac{108\,x^3}{377\,a^2\,c^3}+\frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] -(exp(-2*atan(a*x))*(114/(377*a^5*c^3) - (24*x^5)/(377*c^3) - (149*x)/(377*
a^4*c^3) + (48*x^4)/(377*a*c^3) - (108*x^3)/(377*a^2*c^3) + (136*x^2)/(377*
a^3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*
(c + a^2*c*x^2)^(1/2))/a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.301 $\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

```
[Out] -2*I/a/(1-I*a*x)^2+4*I/a/(1-I*a*x)+I*ln(I+a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*Log[I + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{(1+iax)^2}{(1-iax)^3}\,dx\\ &= \int \left(\frac{4}{(1-iax)^3}-\frac{4}{(1-iax)^2}+\frac{1}{1-iax}\right)dx\\ &= -\frac{2i}{a(1-iax)^2}+\frac{4i}{a(1-iax)}+\frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.84

$$\frac{i\left(4iax+(ax+i)^2\log(ax+i)-2\right)}{a(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a*x)^2)
```

fricas [A] time = 0.51, size = 53, normalized size = 1.06

$$-\frac{4\,ax-\left(i\,a^2x^2-2\,ax-i\right)\log\left(\frac{ax+i}{a}\right)+2i}{a^3x^2+2i\,a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -(4*a*x - (I*a^2*x^2 - 2*a*x - I)*log((a*x + I)/a) + 2*I)/(a^3*x^2 + 2*I*a^
2*x - a)
```

giac [A] time = 0.15, size = 30, normalized size = 0.60

$$\frac{i\log\left(ax+i\right)}{a}-\frac{2\left(2\,ax+i\right)}{\left(ax+i\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] i*log(a*x + i)/a - 2*(2*a*x + i)/((a*x + i)^2*a)
```

maple [A] time = 0.06, size = 45, normalized size = 0.90

$$\frac{-4x-\frac{2i}{a}}{\left(ax+i\right)^2}+\frac{i\ln\left(a^2x^2+1\right)}{2a}+\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^3,x)
```

```
[Out] (-4*x-2*I/a)/(I+a*x)^2+1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

maxima [A] time = 0.43, size = 63, normalized size = 1.26

$$-\frac{32\,a^3x^3-48i\,a^2x^2-16i}{8\left(a^5x^4+2\,a^3x^2+a\right)}+\frac{\arctan\left(ax\right)}{a}+\frac{i\,\log\left(a^2x^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(32*a^3*x^3 - 48*I*a^2*x^2 - 16*I)/(a^5*x^4 + 2*a^3*x^2 + a) + arctan(
a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

mupad [B] time = 0.13, size = 49, normalized size = 0.98

$$\frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a}-\frac{\frac{4\,x}{a^2}+\frac{2\mathrm{i}}{a^3}}{x^2-\frac{1}{a^2}+\frac{x\,2\mathrm{i}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^5/(a^2*x^2 + 1)^3,x)
```

```
[Out] (log(x + 1i/a)*1i)/a - ((4*x)/a^2 + 2i/a^3)/((x*2i)/a - 1/a^2 + x^2)
```

sympy [A] time = 0.31, size = 37, normalized size = 0.74

$$-\frac{-4ax-2i}{-a^3x^2-2ia^2x+a}+\frac{i\log\left(iax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**3,x)
```

```
[Out] -(-4*a*x - 2*I)/(-a**3*x**2 - 2*I*a**2*x + a) + I*log(I*a*x - 1)/a
```

3.302 $\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}+\frac{\sinh^{-1}(ax)}{a}$$

```
[Out] -2/3*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)^(3/2)+arcsinh(a*x)/a+2*I*(1+I*a*x)^(1/2)
/a/(1-I*a*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}+\frac{\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - (((2*I)/3)*(1 + I*a*x)^(3/2))
/(a*(1 - I*a*x)^(3/2)) + ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{(1+iax)^{3/2}}{(1-iax)^{5/2}}\,dx\\ &= -\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} - \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1-iax}\,\sqrt{1+iax}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \frac{\sinh^{-1}(ax)}{a}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.66

$$-\frac{4i\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (((-4*I)/3)*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*a*x)/2])/(a*
(1 - I*a*x)^(3/2))
```

fricas [A] time = 0.43, size = 86, normalized size = 1.18

$$-\frac{8\,a^2x^2+16i\,ax+\left(3\,a^2x^2+6i\,ax-3\right)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}\,(8\,ax+4i)-8}{3\,a^3x^2+6i\,a^2x-3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
[Out] -(8*a^2*x^2 + 16*I*a*x + (3*a^2*x^2 + 6*I*a*x - 3)*log(-a*x + sqrt(a^2*x^2
+ 1)) + sqrt(a^2*x^2 + 1)*(8*a*x + 4*I) - 8)/(3*a^3*x^2 + 6*I*a^2*x - 3*a)
```

giac [A] time = 0.15, size = 24, normalized size = 0.33

$$-\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="giac")
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

maple [A] time = 0.18, size = 113, normalized size = 1.55

$$\frac{7x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}}-\frac{7x}{3\sqrt{a^2x^2+1}}-\frac{a^2x^3}{3\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{4ia\,x^2}{\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{4i}{3a\left(a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x)
```

```
[Out] 7/3*x/(a^2*x^2+1)^(3/2)-7/3*x/(a^2*x^2+1)^(1/2)-1/3*a^2*x^3/(a^2*x^2+1)^(3/
2)+ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)+4*I*a*x^2/(a^2*x^2+1
)^(3/2)+4/3*I/a/(a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.32, size = 112, normalized size = 1.53

$$-\frac{1}{3}a^4x\left(\frac{3x^2}{(a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{2}{(a^2x^2+1)^{\frac{3}{2}}a^4}\right)+\frac{4i\,ax^2}{(a^2x^2+1)^{\frac{3}{2}}}-\frac{5x}{3\sqrt{a^2x^2+1}}+\frac{\operatorname{arsinh}(ax)}{a}+\frac{7x}{3(a^2x^2+1)^{\frac{3}{2}}}+\frac{4i}{3(a^2x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^4*x*(3*x^2/((a^2*x^2 + 1)^(3/2)*a^2) + 2/((a^2*x^2 + 1)^(3/2)*a^4))
+ 4*I*a*x^2/(a^2*x^2 + 1)^(3/2) - 5/3*x/sqrt(a^2*x^2 + 1) + arcsinh(a*x)/a
+ 7/3*x/(a^2*x^2 + 1)^(3/2) + 4/3*I/((a^2*x^2 + 1)^(3/2)*a)
```

mupad [B] time = 0.53, size = 92, normalized size = 1.26

$$\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}}-\frac{8\sqrt{a^2x^2+1}}{3\left(x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}+\frac{a\sqrt{a^2x^2+1}\,4\mathrm{i}}{3\left(a^4x^2+a^3x\,2\mathrm{i}-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(5/2),x)
```

```
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2)
*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3*x
*2i - a^2 + a^4*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-i)^4}{(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral((a*x - I)**4/(a**2*x**2 + 1)**(5/2), x)
```

3.303 $\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

```
[Out] 2/a/(I+a*x)-I*ln(I+a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1+iax}{(1-iax)^2}\,dx\\ &= \int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx\\ &= \frac{2}{a(i+ax)}-\frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

fricas [A] time = 0.41, size = 31, normalized size = 1.03

$$\frac{(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)+2}{a^2x+i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="fricas")
[Out] ((-I*a*x + 1)*log((a*x + I)/a) + 2)/(a^2*x + I*a)
```

giac [A] time = 0.12, size = 25, normalized size = 0.83

$$-\frac{i\log\left(ax+i\right)}{a}+\frac{2}{(ax+i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="giac")
[Out] -i*log(a*x + i)/a + 2/((a*x + i)*a)
```

maple [A] time = 0.06, size = 40, normalized size = 1.33

$$\frac{2}{a\left(ax+i\right)}-\frac{i\ln\left(a^{2}x^{2}+1\right)}{2a}-\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^2,x)
[Out] 2/a/(I+a*x)-1/2*I/a*ln(a^2*x^2+1)-arctan(a*x)/a
```

maxima [A] time = 0.43, size = 44, normalized size = 1.47

$$\frac{4\,ax-4i}{2\left(a^{3}x^{2}+a\right)}-\frac{\arctan\left(ax\right)}{a}-\frac{i\,\log\left(a^{2}x^{2}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*(4*a*x - 4*I)/(a^3*x^2 + a) - arctan(a*x)/a - 1/2*I*log(a^2*x^2 + 1)/a
```

mupad [B] time = 0.49, size = 28, normalized size = 0.93

$$\frac{2}{x\,a^{2}+a\,1\mathrm{i}}-\frac{\ln\left(a\,x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^2,x)
[Out] 2/(a*1i + a^2*x) - (log(a*x + 1i)*1i)/a
```

sympy [A] time = 0.16, size = 19, normalized size = 0.63

$$\frac{2}{a^{2}x+ia}-\frac{i\log\left(ax+i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**2,x)
[Out] 2/(a**2*x + I*a) - I*log(a*x + I)/a
```

3.304 $\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sinh^{-1}(ax)}{a}-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

```
[Out] -arcsinh(a*x)/a-2*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{5073, 47, 41, 215\}$

$$-\frac{\sinh^{-1}(ax)}{a}-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 1.27

$$-\frac{2i\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}+\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((-2*I)*(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x] + ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]])
)/a
```

fricas [A] time = 0.43, size = 54, normalized size = 1.32

$$\frac{2\,ax + (ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right) + 2\,\sqrt{a^2x^2+1} + 2i}{a^2x + i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) + 2*
I)/(a^2*x + I*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.17, size = 63, normalized size = 1.54

$$\frac{2x}{\sqrt{a^2x^2+1}} - \frac{\ln\left(\frac{x\,a^2}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} - \frac{2i}{a\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x)
```

```
[Out] 2*x/(a^2*x^2+1)^(1/2)-ln(x*a^2/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-2
*I/a/(a^2*x^2+1)^(1/2)
```

maxima [A]   time = 0.33, size = 40, normalized size = 0.98

$$\frac{2\,x}{\sqrt{a^2x^2+1}} - \frac{\operatorname{arsinh}\left(ax\right)}{a} - \frac{2i}{\sqrt{a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x/sqrt(a^2*x^2 + 1) - arcsinh(a*x)/a - 2*I/(sqrt(a^2*x^2 + 1)*a)
```

mupad [B]   time = 0.49, size = 55, normalized size = 1.34

$$-\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{2\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
- asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2x^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx - \int\left(-\frac{2iax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx - \int\left(-\frac{1}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(3/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)
), x) - Integral(-1/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x)
```

3.305 $\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{i\log(ax+i)}{a}$$

```
[Out] I*ln(I+a*x)/a
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5073, 31}

$$\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*Log[I + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1}{1-iax}\,dx\\ &= \frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{i\log(ax+i)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*Log[I + a*x])/a
```

fricas [A] time = 0.43, size = 15, normalized size = 1.00

$$\frac{i\log\left(\frac{ax+i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
[Out] I*log((a*x + I)/a)/a
```

giac [A] time = 0.11, size = 13, normalized size = 0.87

$$\frac{i\log\left(-aix+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
[Out] i*log(-a*i*x + 1)/a
```

maple [A] time = 0.04, size = 26, normalized size = 1.73

$$\frac{i\ln\left(a^2x^2+1\right)}{2a}+\frac{\arctan\left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1),x)
[Out] 1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

maxima [B] time = 0.43, size = 24, normalized size = 1.60

$$\frac{\arctan\left(ax\right)}{a}+\frac{i\log\left(a^2x^2+1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
[Out] arctan(a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

mupad [B] time = 0.47, size = 15, normalized size = 1.00

$$\frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1),x)
[Out] (log(x + 1i/a)*1i)/a
```

sympy [A] time = 0.06, size = 10, normalized size = 0.67

$$\frac{i\log\left(iax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1),x)
[Out] I*log(I*a*x - 1)/a
```

3.306 $\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{i\log(-ax+i)}{a}$$

[Out] -I*ln(I-a*x)/a

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 31}

$$-\frac{i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]

[Out] ((-I)*Log[I - a*x])/a

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1}{1+iax}\,dx\\ &= -\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{i\log(-ax+i)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]

[Out] ((-I)*Log[I - a*x])/a

fricas [A] time = 0.39, size = 15, normalized size = 0.94

$$-\frac{i\log\left(\frac{ax-i}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="fricas")
```

```
[Out] -I*log((a*x - I)/a)/a
```

giac [A] time = 0.11, size = 13, normalized size = 0.81

$$-\frac{i \log \left(aix + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="giac")
```

```
[Out] -i*log(a*i*x + 1)/a
```

maple [A] time = 0.03, size = 26, normalized size = 1.62

$$-\frac{i \ln \left(a^{2} x^{2}+1\right)}{2 a}+\frac{\arctan \left(a x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x),x)
```

```
[Out] -1/2*I/a*ln(a^2*x^2+1)+arctan(a*x)/a
```

maxima [A] time = 0.32, size = 12, normalized size = 0.75

$$-\frac{i \log \left(i\, a x + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*x + 1)/a
```

mupad [B] time = 0.48, size = 15, normalized size = 0.94

$$-\frac{\ln \left(x-\frac{1\mathrm{i}}{a}\right)\, 1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x*1i + 1),x)
```

```
[Out] -(log(x - 1i/a)*1i)/a
```

sympy [A] time = 0.05, size = 14, normalized size = 0.88

$$-\frac{i \log \left(- i a x - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x),x)
```

```
[Out] -I*log(-I*a*x - 1)/a
```

3.307 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sinh^{-1}(ax)}{a}+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

[Out] -arcsinh(a*x)/a+2*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$-\frac{\sinh^{-1}(ax)}{a}+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]

[Out] ((2*I)*Sqrt[1 - I*a*x])/(a*Sqrt[1 + I*a*x]) - ArcSinh[a*x]/a

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1-iax}\,\sqrt{1+iax}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \frac{\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 1.37

$$\frac{2\left(\sqrt{a^2x^2+1}+(-1-iax)\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (2*(Sqrt[1 + a^2*x^2] + (-1 - I*a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*(
-I + a*x))
```

fricas [A] time = 0.48, size = 54, normalized size = 1.32

$$\frac{2\,ax+(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+2\sqrt{a^2x^2+1}-2i}{a^2x-i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) - 2*
I)/(a^2*x - I*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.18, size = 143, normalized size = 3.49

$$-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{a^3\left(x-\frac{i}{a}\right)^2}+\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a}-\frac{\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x)
```

```
[Out] -I/a^3/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+I/a*((x-I/a)^2*a^2+2*I
*a*(x-I/a))^(1/2)-ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-
I/a))^(1/2))/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 33, normalized size = 0.80

$$-\frac{\operatorname{arsinh}(ax)}{a} + \frac{2i\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a*x)/a + 2*I*sqrt(a^2*x^2 + 1)/(I*a^2*x + a)
```

mupad [B] time = 0.49, size = 56, normalized size = 1.37

$$-\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{2\sqrt{a^2x^2+1}}{\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^2,x)
```

```
[Out] - asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)
*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{a^2x^2+1}}{a^2x^2-2iax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(a**2*x**2 + 1)/(a**2*x**2 - 2*I*a*x - 1), x)
```

3.308 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

```
[Out] -2/a/(I-a*x)+I*ln(I-a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx &= \int \frac{1-iax}{(1+iax)^2}\,dx\\ &= \int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)dx\\ &= -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

fricas [A] time = 0.51, size = 31, normalized size = 0.97

$$\frac{(i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{a^2x-i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] ((I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2*x - I*a)
```

giac [A] time = 0.14, size = 28, normalized size = 0.88

$$\frac{i \log \left(ax - i\right)}{a} + \frac{2}{(ax - i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] i*log(a*x - i)/a + 2/((a*x - i)*a)
```

maple [A] time = 0.05, size = 41, normalized size = 1.28

$$-\frac{2}{a\left(-ax + i\right)} + \frac{i \ln \left(a^{2} x^{2} + 1\right)}{2a} - \frac{\arctan \left(ax\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1),x)
```

```
[Out] -2/a/(-a*x+I)+1/2*I/a*ln(a^2*x^2+1)-arctan(a*x)/a
```

maxima [A] time = 0.32, size = 41, normalized size = 1.28

$$-\frac{4 \left(-i\, ax - 1\right)}{2i\, a^{3} x^{2} + 4\, a^{2} x - 2i\, a} + \frac{i\, \log \left(i\, ax + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(2*I*a^3*x^2 + 4*a^2*x - 2*I*a) + I*log(I*a*x + 1)/a
```

mupad [B] time = 0.49, size = 29, normalized size = 0.91

$$-\frac{2}{-a^{2}\, x + a\, 1\mathrm{i}} + \frac{\ln \left(a\, x - \mathrm{i}\right)\, 1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/(a*x*1i + 1)^3,x)
```

```
[Out] (log(a*x - 1i)*1i)/a - 2/(a*1i - a^2*x)
```

sympy [A] time = 0.16, size = 19, normalized size = 0.59

$$\frac{2}{a^{2} x - ia} + \frac{i \log \left(ax - i\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1),x)
```

```
[Out] 2/(a**2*x - I*a) + I*log(a*x - I)/a
```

3.309 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=73

$$\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}+\frac{\sinh^{-1}(ax)}{a}$$

```
[Out] 2/3*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^(3/2)+arcsinh(a*x)/a-2*I*(1-I*a*x)^(1/2)/
a/(1+I*a*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5073, 47, 41, 215}

$$\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}+\frac{\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(3/2)) - ((2*I)*Sqrt[1 - I*a*x
])/(a*Sqrt[1 + I*a*x]) + ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(1-iax)^{3/2}}{(1+iax)^{5/2}}\,dx$$
$$= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx$$
$$= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx$$
$$= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \frac{\sinh^{-1}(ax)}{a}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 1.12

$$\frac{2i\left(\frac{2\sqrt{1+iax}\left(2a^2x^2+iax+1\right)}{\sqrt{1-iax}\,(ax-i)^2}+3\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (((2*I)/3)*((2*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2))/(Sqrt[1 - I*a*x]*(-
I + a*x)^2) + 3*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.48, size = 86, normalized size = 1.18

$$-\frac{8\,a^2x^2 - 16i\,ax + \left(3\,a^2x^2 - 6i\,ax - 3\right)\log\left(-ax + \sqrt{a^2x^2+1}\right) + \sqrt{a^2x^2+1}\,(8\,ax - 4i) - 8}{3\,a^3x^2 - 6i\,a^2x - 3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(8*a^2*x^2 - 16*I*a*x + (3*a^2*x^2 - 6*I*a*x - 3)*log(-a*x + sqrt(a^2*x^2
+ 1)) + sqrt(a^2*x^2 + 1)*(8*a*x - 4*I) - 8)/(3*a^3*x^2 - 6*I*a^2*x - 3*a)
```

giac [A] time = 0.15, size = 24, normalized size = 0.33

$$-\frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

maple [B] time = 0.19, size = 262, normalized size = 3.59

$$\frac{i\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{3a^5\left(x-\frac{i}{a}\right)^4} + \frac{\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{3a^4\left(x-\frac{i}{a}\right)^3} + \frac{2i\left(\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{3a^3\left(x-\frac{i}{a}\right)^2} - \frac{2i\left(\left(x-\frac{i}{a}\right)^2 a^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/3*I/a^5/(x-I/a)^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+1/3/a^4/(x-I/a)^3*(
(x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+2/3*I/a^3/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a
*(x-I/a))^(5/2)-2/3*I/a*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+((x-I/a)^2*a^2+
2*I*a*(x-I/a))^(1/2)*x+ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*
a*(x-I/a))^(1/2))/(a^2)^(1/2)
```

maxima [B] time = 0.42, size = 108, normalized size = 1.48

$$\frac{i\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{-3i\,a^{4}x^{3}-9\,a^{3}x^{2}+9i\,a^{2}x+3\,a}+\frac{\operatorname{arsinh}\left(ax\right)}{a}-\frac{2i\,\sqrt{a^{2}x^{2}+1}}{3\,a^{3}x^{2}-6i\,a^{2}x-3\,a}-\frac{7i\,\sqrt{a^{2}x^{2}+1}}{3i\,a^{2}x+3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(a^2*x^2 + 1)^(3/2)/(-3*I*a^4*x^3 - 9*a^3*x^2 + 9*I*a^2*x + 3*a) + arcsin
h(a*x)/a - 2*I*sqrt(a^2*x^2 + 1)/(3*a^3*x^2 - 6*I*a^2*x - 3*a) - 7*I*sqrt(a
^2*x^2 + 1)/(3*I*a^2*x + 3*a)
```

mupad [B] time = 0.10, size = 93, normalized size = 1.27

$$\frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}}+\frac{8\,\sqrt{a^2\,x^2+1}}{3\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}+\frac{a\,\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{3\,\left(-a^4\,x^2+a^3\,x\,2{}\mathrm{i}+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^4,x)
```

```
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) + (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2)
*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3*x
*2i + a^2 - a^4*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(ax-i\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**(3/2)/(a*x - I)**4, x)
```

3.310 $\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}}{a(1-iax)^2\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] -2*I*(a^2*x^2+1)^(1/2)/a/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)+4*I*(a^2*x^2+1)^(1
/2)/a/(1-I*a*x)/(a^2*c*x^2+c)^(1/2)+I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 43}

$$\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)\sqrt{a^2cx^2+c}}-\frac{2i\sqrt{a^2x^2+1}}{a(1-iax)^2\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2])/(a*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2]) + ((4*I)*S
qrt[1 + a^2*x^2])/(a*(1 - I*a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2
]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1+iax)^2}{(1-iax)^3}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \left(\frac{4}{(1-iax)^3}-\frac{4}{(1-iax)^2}+\frac{1}{1-iax}\right)dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2i\sqrt{1+a^2x^2}}{a(1-iax)^2\sqrt{c+a^2cx^2}}+\frac{4i\sqrt{1+a^2x^2}}{a(1-iax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.53

$$\frac{i\sqrt{a^2x^2+1}\left(4iax+(ax+i)^2\log(ax+i)-2\right)}{a(ax+i)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a
*x)^2*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.60, size = 366, normalized size = 2.79

$$\frac{-4i\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\,ax^2+\left(i\,a^4cx^4-2\,a^3cx^3-2\,acx-i\,c\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(i\,a^6x^2-2\,a^5x-2i\,a^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(i\,a^9cx^4-\cdots\right.}{8\,a^3x^3+8i\,a^2x^2+8\,ax+\cdots}\right.}{2\,a^4c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (-4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a*x^2 + (I*a^4*c*x^4 - 2*a^3*c*
x^3 - 2*a*c*x - I*c)*sqrt(1/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c
*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I))
 + (-I*a^4*c*x^4 + 2*a^3*c*x^3 + 2*a*c*x + I*c)*sqrt(1/(a^2*c))*log(((I*a^6
*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (-I*a^9*c
*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 +
 8*I*a^2*x^2 + 8*a*x + 8*I)))/(2*a^4*c*x^4 + 4*I*a^3*c*x^3 + 4*I*a*c*x - 2*
c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

maple [A] time = 0.20, size = 84, normalized size = 0.64

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(i\ln\left(ax+i\right)x^2a^2-2\ln\left(ax+i\right)xa-i\ln\left(ax+i\right)-4ax-2i\right)}{\sqrt{a^2x^2+1}\,ca\left(ax+i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I+a*x)*x^2*a^2-2*ln(I+a*x)*
x*a-I*ln(I+a*x)-4*a*x-2*I)/c/a/(I+a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(i\,ax+1\right)^5}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(1+a\,x\,1{}\mathrm{i}\right)}^5}{\sqrt{c\,a^2\,x^2+c}\,{\left(a^2\,x^2+1\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{a^4x^4\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}\right)dx+\int\frac{}{a^4x^4\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(-I/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**
2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt
(a**2*c*x**2 + c)), x) + Integral(5*a*x/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt
(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) +
 sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x**3/(a
**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
), x) + Integral(a**5*x**5/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x*
*2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(10*I*a**2*x**2/(a**4*x**4*sqr
t(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Inte
gral(-5*I*a**4*x**4/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) +
2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)
*sqrt(a**2*c*x**2 + c)), x))
```

3.311 $\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2ic(1+iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

```
[Out] -2/3*I*c*(1+I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)+arctanh(a*x*c^(1/2)/(a^2*c*x^2+c
)^(1/2))/a/c^(1/2)+2*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5075, 669, 653, 217, 206}

$$-\frac{2ic(1+iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (((-2*I)/3)*c*(1 + I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) + ((2*I)*(1 + I*a*x)
)/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*S
qrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
```

```
& ILtQ[(I*n)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c^2\int \frac{(1+iax)^4}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx\\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)\\
&= -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.74

$$
-\frac{4i\sqrt{2a^2x^2+2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}\sqrt{a^2cx^2+c}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (((-4*I)/3)*Sqrt[2 + 2*a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*
a*x)/2])/(a*(1 - I*a*x)^(3/2)*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.61, size = 187, normalized size = 1.95

$$
\frac{\left(3\,a^3cx^2+6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(3\,a^3cx^2+6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx-\sqrt{a^2c\right.}}{\right.}\right.}{6\,a^3cx^2+12i\,a^2cx-6\,ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] ((3*a^3*c*x^2 + 6*I*a^2*c*x - 3*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(
a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - (3*a^3*c*x^2 + 6*I*a^2*c*x - 3*a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - sqrt(a^2*c*x^2 + c)*(16*a*x + 8*I))/(6*a^3*c*x^2 + 12*I*a^2*c*x -
 6*a*c)
```

giac [A] time = 0.28, size = 139, normalized size = 1.45

$$
-\frac{\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}}-\frac{8\left(3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^2 i-2\,ci-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)\sqrt{c}\right)}{3\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)i-\sqrt{c}\right)^3 ai}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 8/3*(3*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*i - 2*c*i - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 + c))*sqrt(c))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*i - sqrt(c))^3*
a*i)
```

maple [B] time = 0.24, size = 800, normalized size = 8.33

$$\frac{\ln\left(\frac{x a^2 c}{\sqrt{a^2 c}} + \sqrt{a^2 c x^2 + c}\right)}{\sqrt{a^2 c}} - \frac{2\left(i\sqrt{-a^2}+a\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2 a^2 c + 2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}{a^3 c\left(x-\frac{\sqrt{-a^2}}{a^2}\right)} + \frac{2i\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2 a^2 c - 2c\sqrt{-\cdots}}}{3a^3 c\left(x+\frac{\sqrt{-a^2}}{a^2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-2/a^3*(I*(-a^2)
^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(
1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)+2/3*I/a^3/c/(x+(-a^2)^(1/2)/a^2)^2*((x+(-a
^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)-2/3/a^2
/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)^2*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-
a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)-2/3*I/a^3/c/(x+(-a^2)^(1/2)/a^2)*((x
+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)*(-a
^2)^(1/2)+2/3/a^2/c/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*
(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+2/3*I/a^3/c/(x-(-a^2)^(1/2)/a^2)^2
*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)
+2/3/a^2/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^2)^2*((x-(-a^2)^(1/2)/a^2)^2*a^2*
c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)+2/3*I/a^3/c/(x-(-a^2)^(1/2)/
a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(
1/2)*(-a^2)^(1/2)+2/3/a^2/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^
2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)+2/a^3*(I*(-a^2)^(1/2)-a)/c
/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a
^2)^(1/2)/a^2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^4}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1+a\,x\,1\mathrm{i})^4}{\sqrt{c\,a^2\,x^2+c}\,\left(a^2\,x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2),x)
```

```
[Out] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-i)^4}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral((a*x - I)**4/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**2), x)
```

3.312 $\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{a^2x^2+1}}{a(ax+i)\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] 2*(a^2*x^2+1)^(1/2)/a/(I+a*x)/(a^2*c*x^2+c)^(1/2)-I*ln(I+a*x)*(a^2*x^2+1)^(
1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 43}

$$\frac{2\sqrt{a^2x^2+1}}{a(ax+i)\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/(a*(I + a*x)*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1+iax}{(1-iax)^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{2\sqrt{1+a^2x^2}}{a(i+ax)\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.65

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2}{ax+i}-i\log(ax+i)\right)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] (Sqrt[1 + a^2*x^2]*(2/(I + a*x) - I*Log[I + a*x]))/(a*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.57, size = 360, normalized size = 4.29

$$\frac{\left(-i\,a^3cx^3+a^2cx^2-i\,acx+c\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{\left(i\,a^6x^2-2\,a^5x-2i\,a^4\right)\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}+\left(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\,a^3x^3+8i\,a^2x^2+8\,ax+8i}\right)+\left(i\,a^3\right.}{2\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
[Out] ((-I*a^3*c*x^3 + a^2*c*x^2 - I*a*c*x + c)*sqrt(1/(a^2*c))*log(((I*a^6*x^2 -
 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 -
2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^
2*x^2 + 8*a*x + 8*I)) + (I*a^3*c*x^3 - a^2*c*x^2 + I*a*c*x - c)*sqrt(1/(a^2
*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2
+ 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c
)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt
(a^2*x^2 + 1)*x)/(2*a^3*c*x^3 + 2*I*a^2*c*x^2 + 2*a*c*x + 2*I*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.16, size = 61, normalized size = 0.73

$$\frac{\left(-i\ln\left(ax+i\right)xa+\ln\left(ax+i\right)+2\right)\sqrt{c\left(a^{2}x^{2}+1\right)}}{\sqrt{a^{2}x^{2}+1}\;ca\left(ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-I*ln(I+a*x)*x*a+ln(I+a*x)+2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/a/
(I+a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(i\,ax+1\right)^{3}}{\sqrt{a^{2}cx^{2}+c}\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1+a\,x\,1\mathrm{i}\right)^{3}}{\sqrt{c\,a^{2}\,x^{2}+c}\,\left(a^{2}\,x^{2}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}+\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}}\,dx+\int\left(-\frac{3ax}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}+\sqrt{a^{2}x^{2}+1}\,\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(
a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**2*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**
2 + c)), x) + Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c
*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*I
*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2
*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

3.313 $\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}-\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*I*(1+I*a*x)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5075, 653, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}-\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*(1 + I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
 + a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rubi steps

$$\int \frac{e^{2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx$$
$$= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)$$
$$= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

Mathematica [**A**] time = 0.04, size = 91, normalized size = 1.44

$$-\frac{2i\sqrt{a^2x^2+1}\left(\sqrt{1+iax}+\sqrt{1-iax}\,\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}\,\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*(Sqrt[1 + I*a*x] + Sqrt[1 - I*a*x]*ArcSin[Sqrt[1
- I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*Sqrt[c + a^2*c*x^2])
```

fricas [**B**] time = 0.43, size = 152, normalized size = 2.41

$$-\frac{\left(a^2cx+i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(a^2cx+i\,ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-4\sqrt{a^2cx^2+}}{2\left(a^2cx+i\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/2*((a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c
)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*
c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c)
)/(a^2*c*x + I*a*c)
```

giac [**A**] time = 0.17, size = 72, normalized size = 1.14

$$\frac{\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{4}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)i-\sqrt{c}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 + c))*i - sqrt(c))*a)
```

maple [**B**] time = 0.18, size = 204, normalized size = 3.24

$$-\frac{\ln\left(\frac{x\,a^2c}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+c}\right)}{\sqrt{a^2c}}+\frac{\left(i\sqrt{-a^2}+a\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}{a^3c\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}-\frac{\left(i\sqrt{-a^2}-a\right)\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2}}{a^3c\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+1/a^3*(I*(-a^2
)^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^
(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-1/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1/2
)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^2}{\sqrt{a^2cx^2+c}\,\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*x + 1)^2/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(1+a\,x\,1{}\mathrm{i}\right)}^2}{\sqrt{c\,a^2\,x^2+c}\,\left(a^2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)),x)
```

```
[Out] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\,dx-\int \left(-\frac{2iax}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\right)dx-\int \left(-\frac{1}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*x**2 + c
)), x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*x
**2 + c)), x) - Integral(-1/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*
x**2 + c)), x)
```

3.314 $\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 31}

$$\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1-iax}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{i\sqrt{1+a^2x^2}\,\log(i+ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{i\sqrt{a^2x^2+1}\,\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.48, size = 255, normalized size = 6.07

$$\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(ia^6x^2-2a^5x-2ia^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(ia^9cx^4-2a^8cx^3+ia^7cx^2-2a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8a^3x^3+8ia^2x^2+8ax+8i}\right)-\frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 +
 c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*
x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I)) - 1/2*I*sqrt(1
/(a^2*c))*log(((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(
a^2*c)))/(8*a^3*x^3 + 8*I*a^2*x^2 + 8*a*x + 8*I))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{iax+1}{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((I*a*x + 1)/(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)), x)
```

maple [A] time = 0.10, size = 53, normalized size = 1.26

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(i\ln\left(a^2x^2+1\right)+2\arctan\left(ax\right)\right)}{2\sqrt{a^2x^2+1}\,ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(a^2*x^2+1)+2*arctan(a*x))
/c/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1+a\,x\,1\mathrm{i}}{\sqrt{c\,a^2\,x^2+c}\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}\right)dx+\int\frac{ax}{\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(-I/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a
*x/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

3.315 $\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=43

$$-\frac{i\sqrt{a^2x^2+1}\,\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] -I*ln(I-a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 31}

$$-\frac{i\sqrt{a^2x^2+1}\,\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+iax}\,dx}{\sqrt{c+a^2cx^2}}\\
&= -\frac{i\sqrt{1+a^2x^2}\,\log(i-ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$-\frac{i\sqrt{a^2x^2+1}\,\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.54, size = 255, normalized size = 5.93

$$\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(-ia^6x^2-2a^5x+2ia^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(ia^9cx^4+2a^8cx^3+ia^7cx^2+2a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8a^3x^3-8ia^2x^2+8ax-8i}\right)-\frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2
+ c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c
*x)*sqrt(1/(a^2*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I)) - 1/2*I*sqrt(
1/(a^2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a
^2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1
/(a^2*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 42, normalized size = 0.98

$$-\frac{i\sqrt{c\left(a^2x^2+1\right)}\,\ln\left(iax+1\right)}{\sqrt{a^2x^2+1}\,ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -I/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c*ln(1+I*a*x)/a
```

maxima [A] time = 0.34, size = 15, normalized size = 0.35

$$-\frac{i\log\left(iax+1\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] -I*log(I*a*x + 1)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a^2\,x^2+1}}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)),x)
```

```
[Out] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^2x^2+1}}{ax\sqrt{a^2cx^2+c}-i\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x*sqrt(a**2*c*x**2 + c) - I*sqrt(a**2*c*
x**2 + c)), x)
```

3.316 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}+\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*I*(1-I*a*x)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5074, 653, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}+\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((2*I)*(1 - I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
+ a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rubi steps

$$\int \frac{e^{-2i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)$$
$$= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.06, size = 117, normalized size = 1.86

$$\frac{2\sqrt{a^2x^2+1}\left((1-iax)\sqrt{1+iax}-i\sqrt{1-iax}\,(ax-i)\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}\,(ax-i)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*((1 - I*a*x)*Sqrt[1 + I*a*x] - I*Sqrt[1 - I*a*x]*(-I +
 a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*(-I + a*x)*Sqrt[
c + a^2*c*x^2])
```

fricas [B] time = 0.55, size = 152, normalized size = 2.41

$$-\frac{\left(a^2cx-i\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(a^2cx-i\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-4\sqrt{a^2cx}}{2\left(a^2cx-i\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*((a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c
)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*
c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c)
)/(a^2*c*x - I*a*c)
```

giac [A] time = 0.20, size = 70, normalized size = 1.11

$$\frac{\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}}+\frac{4}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)i+\sqrt{c}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) + 4/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 + c))*i + sqrt(c))*a)
```

maple [A] time = 0.22, size = 87, normalized size = 1.38

$$-\frac{\ln\left(\frac{x a^2 c}{\sqrt{a^2 c}}+\sqrt{a^2 c\, x^2+c}\right)}{\sqrt{a^2 c}}+\frac{2\sqrt{\left(x-\frac{i}{a}\right)^2 a^2 c+2 i a c\left(x-\frac{i}{a}\right)}}{a^2 c\left(x-\frac{i}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+2/a^2/c/(x-I/a
)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)
```

maxima [A] time = 0.42, size = 40, normalized size = 0.63

$$\frac{2 i \sqrt{a^2 c x^2+c}}{i a^2 c x+a c}-\frac{\operatorname{arsinh}(a x)}{a \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*I*sqrt(a^2*c*x^2 + c)/(I*a^2*c*x + a*c) - arcsinh(a*x)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a^2\,x^2+1}{\sqrt{c\,a^2\,x^2+c}\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2),x)
```

```
[Out] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 x^2}{a^2 x^2 \sqrt{a^2 c x^2+c}-2 i a x \sqrt{a^2 c x^2+c}-\sqrt{a^2 c x^2+c}}\, dx-\int \frac{1}{a^2 x^2 \sqrt{a^2 c x^2+c}-2 i a x \sqrt{a^2 c x^2+c}-\sqrt{a^2 c x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) - 2*I*a*x*sqrt(a**2*c*
x**2 + c) - sqrt(a**2*c*x**2 + c)), x) - Integral(1/(a**2*x**2*sqrt(a**2*c*
x**2 + c) - 2*I*a*x*sqrt(a**2*c*x**2 + c) - sqrt(a**2*c*x**2 + c)), x)
```

3.317 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(I-a*x)/(a^2*c*x^2+c)^(1/2)+I*ln(I-a*x)*(a^2*x^2+1)^
(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5076, 5073, 43}

$$\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a(-ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*
x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1-iax}{(1+iax)^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{a(i-ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.70

$$\frac{\sqrt{a^2x^2+1}\left(\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}\right)}{\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/(a*(I - a*x)) + (I*Log[I - a*x])/a))/Sqrt[c + a^2*c*
x^2]
```

fricas [B] time = 0.62, size = 360, normalized size = 4.19

$$\frac{\left(-i\,a^3cx^3-a^2cx^2-i\,acx-c\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{\left(-i\,a^6x^2-2\,a^5x+2i\,a^4\right)\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}+\left(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\,a^3x^3-8i\,a^2x^2+8\,ax-8i}\right)+\left(i\,a^3c\right.}{2\,a^3cx^3-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] ((-I*a^3*c*x^3 - a^2*c*x^2 - I*a*c*x - c)*sqrt(1/(a^2*c))*log(((-I*a^6*x^2
- 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 +
 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c))))/(8*a^3*x^3 - 8*I*a
^2*x^2 + 8*a*x - 8*I)) + (I*a^3*c*x^3 + a^2*c*x^2 + I*a*c*x + c)*sqrt(1/(a^
2*c))*log(((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2
*c)))/(8*a^3*x^3 - 8*I*a^2*x^2 + 8*a*x - 8*I)) - 4*I*sqrt(a^2*c*x^2 + c)*sq
rt(a^2*x^2 + 1)*x)/(2*a^3*c*x^3 - 2*I*a^2*c*x^2 + 2*a*c*x - 2*I*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}\,(i\,ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/(sqrt(a^2*c*x^2 + c)*(I*a*x + 1)^3), x)
```

maple [A] time = 0.16, size = 66, normalized size = 0.77

$$\frac{\left(-i\ln\left(-ax+i\right)xa-\ln\left(-ax+i\right)-2\right)\sqrt{c\left(a^2x^2+1\right)}}{\sqrt{a^2x^2+1}\;ca\left(-ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-I*ln(-a*x+I)*x*a-ln(-a*x+I)-2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/
a/(-a*x+I)
```

maxima [A] time = 0.34, size = 35, normalized size = 0.41

$$\frac{i\log\left(iax+1\right)}{a\sqrt{c}}+\frac{2}{a^2\sqrt{c}\,x-ia\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] I*log(I*a*x + 1)/(a*sqrt(c)) + 2/(a^2*sqrt(c)*x - I*a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2\,x^2+1\right)^{3/2}}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3),x)
```

```
[Out] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^3\sqrt{a^2cx^2+c}-3ia^2x^2\sqrt{a^2cx^2+c}-3ax\sqrt{a^2cx^2+c}+i\sqrt{a^2cx^2+c}}\,dx+\int \frac{}{a^3x^3\sqrt{a^2cx^2+c}-3ia^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a**2*c*x**2 + c) - 3*I*a**2
*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*x**2 + c) + I*sqrt(a**2*c*x
**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a**2
*c*x**2 + c) - 3*I*a**2*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*x**2
 + c) + I*sqrt(a**2*c*x**2 + c)), x))
```

3.318 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=96

$$\frac{2ic(1-iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

```
[Out] 2/3*I*c*(1-I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)+arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)
^(1/2))/a/c^(1/2)-2*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5074, 669, 653, 217, 206}

$$\frac{2ic(1-iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}+\frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (((2*I)/3)*c*(1 - I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) - ((2*I)*(1 - I*a*x))
/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*Sq
rt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
```

```
IGtQ[(I*n)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-4i\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= c^2\int \frac{(1-iax)^4}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\ &= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\ &= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx\\ &= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right)\\ &= \frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \frac{\tanh^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 1.38

$$\frac{2\sqrt{a^2x^2+1}\left(2i\sqrt{1+iax}\left(2a^2x^2+iax+1\right)+3i\sqrt{1-iax}\,(ax-i)^2\sin^{-1}\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-iax}\,(ax-i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*((2*I)*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2) + (3*I)
*Sqrt[1 - I*a*x]*(-I + a*x)^2*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(3*a*Sqrt[1
 - I*a*x]*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.51, size = 187, normalized size = 1.95

$$\frac{\left(3\,a^3cx^2-6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}\,a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-\left(3\,a^3cx^2-6i\,a^2cx-3\,ac\right)\sqrt{\frac{1}{a^2c}}\,\log\left(\frac{2\left(a^2cx-\sqrt{a^2c\cdots}\right.}{\cdots}\right.\cdots}{6\,a^3cx^2-12i\,a^2cx-6\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((3*a^3*c*x^2 - 6*I*a^2*c*x - 3*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(
a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - (3*a^3*c*x^2 - 6*I*a^2*c*x - 3*a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - sqrt(a^2*c*x^2 + c)*(16*a*x - 8*I))/(6*a^3*c*x^2 - 12*I*a^2*c*x -
 6*a*c)
```

giac [A] time = 0.25, size = 137, normalized size = 1.43

$$-\frac{\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}}+\frac{8\left(3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^2i-2\,ci+3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)\sqrt{c}\right)}{3\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)i+\sqrt{c}\right)^3 ai}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) + 8/3*(3*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*i - 2*c*i + 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 + c))*sqrt(c))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*i + sqrt(c))^3*
a*i)
```

maple [A] time = 0.19, size = 136, normalized size = 1.42

$$\frac{\ln\left(\frac{x\,a^2c}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+c}\right)}{\sqrt{a^2c}}-\frac{8\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3a^2c\left(x-\frac{i}{a}\right)}-\frac{4i\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3a^3c\left(x-\frac{i}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] ln(x*a^2*c/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-8/3/a^2/c/(x-I/
a)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-4/3*I/a^3/c/(x-I/a)^2*((x-I/a)^2
*a^2*c+2*I*a*c*(x-I/a))^(1/2)
```

maxima [A] time = 0.42, size = 76, normalized size = 0.79

$$-\frac{4i\sqrt{a^2cx^2+c}}{3\left(a^3cx^2-2i\,a^2cx-ac\right)}-\frac{8i\sqrt{a^2cx^2+c}}{3i\,a^2cx+3\,ac}+\frac{\operatorname{arsinh}(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -4/3*I*sqrt(a^2*c*x^2 + c)/(a^3*c*x^2 - 2*I*a^2*c*x - a*c) - 8*I*sqrt(a^2*c
*x^2 + c)/(3*I*a^2*c*x + 3*a*c) + arcsinh(a*x)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2\,x^2+1\right)^2}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4),x)
```

```
[Out] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{c\left(a^2x^2+1\right)}\,(ax-i)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**2/(sqrt(c*(a**2*x**2 + 1))*(a*x - I)**4), x)
```

3.319 $\int \frac{e^{5i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}$$

```
[Out] -2/3/a/(I+a*x)^3-1/2*I/a/(I+a*x)^2
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 43}

$$-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] -2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{5i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= \int \frac{1+iax}{(1-iax)^4}\,dx\\ &= \int \left(\frac{2}{(i+ax)^4}+\frac{i}{(i+ax)^3}\right)dx\\ &= -\frac{2}{3a(i+ax)^3}-\frac{i}{2a(i+ax)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.69

$$-\frac{1+3iax}{6a(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] -1/6*(1 + (3*I)*a*x)/(a*(I + a*x)^3)
```

fricas [A] time = 0.42, size = 35, normalized size = 1.00

$$\frac{-3i\,ax-1}{6\,a^4x^3+18i\,a^3x^2-18\,a^2x-6i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="fricas")
[Out] (-3*I*a*x - 1)/(6*a^4*x^3 + 18*I*a^3*x^2 - 18*a^2*x - 6*I*a)
```

giac [A] time = 0.12, size = 19, normalized size = 0.54

$$-\frac{3\,aix+1}{6\,(ax+i)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="giac")
[Out] -1/6*(3*a*i*x + 1)/((a*x + i)^3*a)
```

maple [A] time = 0.06, size = 20, normalized size = 0.57

$$\frac{-\frac{ix}{2}-\frac{1}{6a}}{(ax+i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^4,x)
[Out] (-1/2*I*x-1/6/a)/(I+a*x)^3
```

maxima [B] time = 0.43, size = 59, normalized size = 1.69

$$\frac{-24i\,a^4x^4-80\,a^3x^3+96i\,a^2x^2+48\,ax-8i}{48\left(a^7x^6+3\,a^5x^4+3\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="maxima")
[Out] 1/48*(-24*I*a^4*x^4 - 80*a^3*x^3 + 96*I*a^2*x^2 + 48*a*x - 8*I)/(a^7*x^6 +
3*a^5*x^4 + 3*a^3*x^2 + a)
```

mupad [B] time = 0.10, size = 21, normalized size = 0.60

$$-\frac{3\,a\,x-\mathrm{i}}{6\,a\,(-1+a\,x\,1\mathrm{i})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^5/(a^2*x^2 + 1)^4,x)
[Out] -(3*a*x - 1i)/(6*a*(a*x*1i - 1)^3)
```

sympy [A] time = 0.31, size = 37, normalized size = 1.06

$$\frac{3iax+1}{-6a^4x^3-18ia^3x^2+18a^2x+6ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**4,x)
[Out] (3*I*a*x + 1)/(-6*a**4*x**3 - 18*I*a**3*x**2 + 18*a**2*x + 6*I*a)
```

3.320 $\int \frac{e^{4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

```
[Out] -1/5*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)^(5/2)-1/15*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)
^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-I/5)*(1 + I*a*x)^(3/2))/(a*(1 - I*a*x)^(5/2)) - ((I/15)*(1 + I*a*x)^(3/2
))/(a*(1 - I*a*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{\sqrt{1+iax}}{(1-iax)^{7/2}}\,dx\\
&= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+iax}}{(1-iax)^{5/2}}\,dx\\
&= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.70

$$\frac{(1+iax)^{3/2}(ax+4i)}{15a\sqrt{1-iax}\,(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x))/(15*a*Sqrt[1 - I*a*x]*(I + a*x)^2)
```

fricas [A] time = 0.44, size = 76, normalized size = 1.13

$$-\frac{a^3x^3+3i\,a^2x^2-3\,ax+\left(a^2x^2+3i\,ax+4\right)\sqrt{a^2x^2+1}-i}{15\,a^4x^3+45i\,a^3x^2-45\,a^2x-15i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 + 3*I*a*x + 4)*sqrt(a^2*x^2 + 1)
 - I)/(15*a^4*x^3 + 45*I*a^3*x^2 - 45*a^2*x - 15*I*a)
```

giac [B] time = 0.17, size = 111, normalized size = 1.66

$$-\frac{2\left(5\,a^3i\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-15\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3-4\,a^4+25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2-15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4\right)}{15\left(ai+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^3*i*(sqrt(a^2 + 1/x^2) - 1/x) - 15*a*i*(sqrt(a^2 + 1/x^2) - 1/x)
^3 - 4*a^4 + 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 - 15*(sqrt(a^2 + 1/x^2) - 1
/x)^4)/(a*i + sqrt(a^2 + 1/x^2) - 1/x)^5
```

maple [B] time = 0.18, size = 269, normalized size = 4.01

$$\frac{x}{5\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{a^2x^2+1}}+a^4\left(-\frac{x^3}{2a^2\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{-\frac{3x}{8a^2\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{x}{5\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{\cdots}}\right)}{8a^2}}{a^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x)
```

```
[Out] 1/5*x/(a^2*x^2+1)^(5/2)+4/15*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^(1/2)+a
^4*(-1/2*x^3/a^2/(a^2*x^2+1)^(5/2)+3/2/a^2*(-1/4*x/a^2/(a^2*x^2+1)^(5/2)+1/
4/a^2*(1/5*x/(a^2*x^2+1)^(5/2)+4/15*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^
(1/2))))-4*I*a^3*(-1/3*x^2/a^2/(a^2*x^2+1)^(5/2)-2/15/a^4/(a^2*x^2+1)^(5/2)
)-6*a^2*(-1/4*x/a^2/(a^2*x^2+1)^(5/2)+1/4/a^2*(1/5*x/(a^2*x^2+1)^(5/2)+4/15
*x/(a^2*x^2+1)^(3/2)+8/15*x/(a^2*x^2+1)^(1/2)))-4/5*I/a/(a^2*x^2+1)^(5/2)
```

maxima [B] time = 0.33, size = 95, normalized size = 1.42

$$-\frac{a^2x^3}{2\left(a^2x^2+1\right)^{\frac{5}{2}}}-\frac{x}{15\sqrt{a^2x^2+1}}+\frac{4i\,ax^2}{3\left(a^2x^2+1\right)^{\frac{5}{2}}}-\frac{x}{30\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{11\,x}{10\left(a^2x^2+1\right)^{\frac{5}{2}}}-\frac{4i}{15\left(a^2x^2+1\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x^3/(a^2*x^2 + 1)^(5/2) - 1/15*x/sqrt(a^2*x^2 + 1) + 4/3*I*a*x^2/(
a^2*x^2 + 1)^(5/2) - 1/30*x/(a^2*x^2 + 1)^(3/2) + 11/10*x/(a^2*x^2 + 1)^(5/
2) - 4/15*I/((a^2*x^2 + 1)^(5/2)*a)
```

mupad [B] time = 0.52, size = 41, normalized size = 0.61

$$\frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2\,1\mathrm{i}-3\,a\,x+4\mathrm{i}\right)}{15\,a\,{\left(-1+a\,x\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(7/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*(a*x*1i - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-i\right)^4}{\left(a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral((a*x - I)**4/(a**2*x**2 + 1)**(7/2), x)
```

3.321 $\int \frac{e^{3i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{i}{2a(1-iax)^2}$$

```
[Out] -1/2*I/a/(1-I*a*x)^2
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 32}

$$-\frac{i}{2a(1-iax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] (-I/2)/(a*(1 - I*a*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= \int \frac{1}{(1-iax)^3}\,dx\\ &= -\frac{i}{2a(1-iax)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{i}{2a(ax+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] (I/2)/(a*(I + a*x)^2)
```

fricas [A] time = 0.38, size = 22, normalized size = 1.16

$$\frac{i}{2\,a^3x^2+4i\,a^2x-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] I/(2*a^3*x^2 + 4*I*a^2*x - 2*a)
```

giac [A] time = 0.14, size = 13, normalized size = 0.68

$$\frac{i}{2\,(ax+i)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/2*i/((a*x + i)^2*a)
```

maple [A] time = 0.06, size = 15, normalized size = 0.79

$$\frac{i}{2a\,(ax+i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^3,x)
```

```
[Out] 1/2*I/a/(I+a*x)^2
```

maxima [B] time = 0.43, size = 35, normalized size = 1.84

$$\frac{4i\,a^2x^2+8\,ax-4i}{8\left(a^5x^4+2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*I*a^2*x^2 + 8*a*x - 4*I)/(a^5*x^4 + 2*a^3*x^2 + a)
```

mupad [B] time = 0.50, size = 24, normalized size = 1.26

$$\frac{1\mathrm{i}}{2\,\left(a^3\,x^2+a^2\,x\,2\mathrm{i}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^3,x)
```

```
[Out] 1i/(2*(a^2*x*2i - a + a^3*x^2))
```

sympy [A] time = 0.19, size = 20, normalized size = 1.05

$$\frac{i}{2a^3x^2+4ia^2x-2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**3,x)
```

```
[Out] I/(2*a**3*x**2 + 4*I*a**2*x - 2*a)
```

3.322 $\int \frac{e^{2i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

```
[Out] -1/3*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^(3/2)-1/3*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-I/3)*Sqrt[1 + I*a*x])/(a*(1 - I*a*x)^(3/2)) - ((I/3)*Sqrt[1 + I*a*x])/(a
*Sqrt[1 - I*a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= \int \frac{1}{(1-iax)^{5/2}\sqrt{1+iax}}\,dx\\
&= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-iax)^{3/2}\sqrt{1+iax}}\,dx\\
&= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.72

$$\frac{(2-iax)\sqrt{1+iax}}{3a\sqrt{1-iax}\,(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x])/(3*a*Sqrt[1 - I*a*x]*(I + a*x))
```

fricas [A] time = 0.47, size = 51, normalized size = 0.76

$$\frac{a^2x^2+2i\,ax+\sqrt{a^2x^2+1}\,(ax+2i)-1}{3\,a^3x^2+6i\,a^2x-3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 + 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x + 2*I) - 1)/(3*a^3*x^2 + 6*I*a^
2*x - 3*a)
```

giac [A] time = 0.14, size = 66, normalized size = 0.99

$$\frac{2\left(3\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-2\,a^2+3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2\right)}{3\left(ai+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*i*(sqrt(a^2 + 1/x^2) - 1/x) - 2*a^2 + 3*(sqrt(a^2 + 1/x^2) - 1/x)^
2)/(a*i + sqrt(a^2 + 1/x^2) - 1/x)^3
```

maple [B] time = 0.18, size = 104, normalized size = 1.55

$$-a^2\left(-\frac{x}{2a^2\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{\frac{x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{a^2x^2+1}}}{2a^2}\right)-\frac{2i}{3a\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x)
```

```
[Out] -a^2*(-1/2*x/a^2/(a^2*x^2+1)^(3/2)+1/2/a^2*(1/3*x/(a^2*x^2+1)^(3/2)+2/3*x/(
a^2*x^2+1)^(1/2)))-2/3*I/a/(a^2*x^2+1)^(3/2)+1/3*x/(a^2*x^2+1)^(3/2)+2/3*x/
(a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.32, size = 45, normalized size = 0.67

$$\frac{x}{3\sqrt{a^2x^2+1}}+\frac{2\,x}{3\left(a^2x^2+1\right)^{\frac{3}{2}}}-\frac{2i}{3\left(a^2x^2+1\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/sqrt(a^2*x^2 + 1) + 2/3*x/(a^2*x^2 + 1)^(3/2) - 2/3*I/((a^2*x^2 + 1)^
(3/2)*a)
```

mupad [B] time = 0.50, size = 33, normalized size = 0.49

$$\frac{\sqrt{a^2\,x^2+1}\,\,\left(-2+a\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3\,a\,{\left(-1+a\,x\,1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(5/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(a*x*1i - 2)*1i)/(3*a*(a*x*1i - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2x^2}{a^4x^4\sqrt{a^2x^2+1}+2a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx-\int\left(-\frac{2iax}{a^4x^4\sqrt{a^2x^2+1}+2a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(5/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*
x**2 + 1) + sqrt(a**2*x**2 + 1)), x) - Integral(-2*I*a*x/(a**4*x**4*sqrt(a*
*2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) -
 Integral(-1/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**2 +
1) + sqrt(a**2*x**2 + 1)), x)
```

3.323 $\int \frac{e^{i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}(ax)}{2a}+\frac{1}{2a(ax+i)}$$

```
[Out] 1/2/a/(I+a*x)+1/2*arctan(a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 44, 203}

$$\frac{\tan^{-1}(ax)}{2a}+\frac{1}{2a(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1-iax)^2(1+iax)}\,dx\\ &= \int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx\\ &= \frac{1}{2a(i+ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx\\ &= \frac{1}{2a(i+ax)}+\frac{\tan^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.75

$$\frac{\tan^{-1}(ax)+\frac{1}{ax+i}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

fricas [B] time = 0.45, size = 49, normalized size = 1.75

$$\frac{(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{4\,a^2x+4i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((I*a*x - 1)*log((a*x + I)/a) + (-I*a*x + 1)*log((a*x - I)/a) + 2)/(4*a^2*x
 + 4*I*a)
```

giac [A] time = 0.14, size = 41, normalized size = 1.46

$$\frac{i\log(ax+i)}{4\,a}+\frac{\log(-aix-1)}{4\,ai}+\frac{1}{2\,(ax+i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*i*log(a*x + i)/a + 1/4*log(-a*i*x - 1)/(a*i) + 1/2/((a*x + i)*a)
```

maple [A] time = 0.04, size = 38, normalized size = 1.36

$$\frac{2a^2x-2ia}{4a^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^2,x)
```

```
[Out] 1/4*(2*a^2*x-2*I*a)/a^2/(a^2*x^2+1)+1/2*arctan(a*x)/a
```

maxima [A] time = 0.43, size = 28, normalized size = 1.00

$$\frac{ax-i}{2\left(a^3x^2+a\right)}+\frac{\arctan(ax)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x - I)/(a^3*x^2 + a) + 1/2*arctan(a*x)/a
```

mupad [B] time = 0.48, size = 23, normalized size = 0.82

$$\frac{1}{2\,\left(x\,a^2+a\,1\mathrm{i}\right)}+\frac{\mathrm{atan}(a\,x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1)^2,x)
```

```
[Out] 1/(2*(a*1i + a^2*x)) + atan(a*x)/(2*a)
```

sympy [A] time = 0.21, size = 34, normalized size = 1.21

$$\frac{1}{2a^2x+2ia}-\frac{\frac{i\log\left(x-\frac{i}{a}\right)}{4}-\frac{i\log\left(x+\frac{i}{a}\right)}{4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**2,x)
```

```
[Out] 1/(2*a**2*x + 2*I*a) - (I*log(x - I/a)/4 - I*log(x + I/a)/4)/a
```

3.324 $\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

```
[Out] -1/2/a/(I-a*x)+1/2*arctan(a*x)/a
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 44, 203}

$$\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(2*a*(I - a*x)) + ArcTan[a*x]/(2*a)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1-iax)(1+iax)^2}\,dx\\
&= \int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx\\
&= -\frac{1}{2a(i-ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx\\
&= -\frac{1}{2a(i-ax)}+\frac{\tan^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.72

$$\frac{\tan^{-1}(ax)+\frac{1}{ax-i}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] ((-I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

fricas [B] time = 0.56, size = 49, normalized size = 1.69

$$\frac{(i\,ax+1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax-1)\log\left(\frac{ax-i}{a}\right)+2}{4\,a^2x-4i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
[Out] ((I*a*x + 1)*log((a*x + I)/a) + (-I*a*x - 1)*log((a*x - I)/a) + 2)/(4*a^2*x
 - 4*I*a)
```

giac [A] time = 0.13, size = 44, normalized size = 1.52

$$-\frac{i\log(ax-i)}{4\,a}-\frac{\log(aix-1)}{4\,ai}+\frac{1}{2\,(ax-i)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
[Out] -1/4*i*log(a*x - i)/a - 1/4*log(a*i*x - 1)/(a*i) + 1/2/((a*x - i)*a)
```

maple [A] time = 0.10, size = 25, normalized size = 0.86

$$-\frac{1}{2a\,(-ax+i)}+\frac{\arctan(ax)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)/(a^2*x^2+1),x)
[Out] -1/2/a/(-a*x+I)+1/2*arctan(a*x)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.48, size = 25, normalized size = 0.86

$$\frac{\operatorname{atan}(a\,x)}{2\,a}-\frac{1}{2\,\left(-a^2\,x+a\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2*x^2 + 1)*(a*x*1i + 1)),x)
[Out] atan(a*x)/(2*a) - 1/(2*(a*1i - a^2*x))
```

sympy [**A**] time = 0.20, size = 34, normalized size = 1.17

$$\frac{1}{2a^2x - 2ia} + \frac{-\frac{i\log\left(x-\frac{i}{a}\right)}{4} + \frac{i\log\left(x+\frac{i}{a}\right)}{4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)/(a**2*x**2+1),x)
```

```
[Out] 1/(2*a**2*x - 2*I*a) + (-I*log(x - I/a)/4 + I*log(x + I/a)/4)/a
```

3.325 $\int \frac{e^{-2i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

```
[Out] 1/3*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(3/2)+1/3*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*Sqrt[1 - I*a*x])/(a*(1 + I*a*x)^(3/2)) + ((I/3)*Sqrt[1 - I*a*x])/(a*
Sqrt[1 + I*a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= \int \frac{1}{\sqrt{1-iax}\,(1+iax)^{5/2}}\,dx\\ &= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-iax}\,(1+iax)^{3/2}}\,dx\\ &= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}+\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.72

$$\frac{\sqrt{1-iax}\,(2+iax)}{3a\sqrt{1+iax}\,(ax-i)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]

[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x))/(3*a*Sqrt[1 + I*a*x]*(-I + a*x))

fricas [A] time = 0.50, size = 51, normalized size = 0.76

$$\frac{a^2x^2-2i\,ax+\sqrt{a^2x^2+1}\,(ax-2i)-1}{3\,a^3x^2-6i\,a^2x-3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] (a^2*x^2 - 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x - 2*I) - 1)/(3*a^3*x^2 - 6*I*a^2*x - 3*a)

giac [A] time = 0.16, size = 66, normalized size = 0.99

$$\frac{2\left(3\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)+2\,a^2-3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2\right)}{3\left(ai-\sqrt{a^2+\frac{1}{x^2}}+\frac{1}{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] 2/3*(3*a*i*(sqrt(a^2 + 1/x^2) - 1/x) + 2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)^2)/(a*i - sqrt(a^2 + 1/x^2) + 1/x)^3

maple [A] time = 0.18, size = 93, normalized size = 1.39

$$-\frac{\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3a\left(x-\frac{i}{a}\right)^2}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3\left(x-\frac{i}{a}\right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x)

[Out] -1/a^2*(1/3*I/a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)-1/3/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))

maxima [A] time = 0.41, size = 59, normalized size = 0.88

$$-\frac{i\sqrt{a^2x^2+1}}{3\,a^3x^2-6i\,a^2x-3\,a}+\frac{i\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -I*sqrt(a^2*x^2 + 1)/(3*a^3*x^2 - 6*I*a^2*x - 3*a) + I*sqrt(a^2*x^2 + 1)/(3*I*a^2*x + 3*a)

mupad [B] time = 0.06, size = 31, normalized size = 0.46

$$-\frac{\sqrt{a^2\,x^2+1}\,\left(a\,x-2{}\mathrm{i}\right)}{3\,a\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2*x^2 + 1)^(1/2)*(a*x*1i + 1)^2),x)
```

```
[Out] -((a^2*x^2 + 1)^(1/2)*(a*x - 2i))/(3*a*(a*x*1i + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}x^{2}\sqrt{a^{2}x^{2}+1}-2iax\sqrt{a^{2}x^{2}+1}-\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a**2*x**2 + 1) - 2*I*a*x*sqrt(a**2*x**2 + 1) -
sqrt(a**2*x**2 + 1)), x)
```

3.326 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{i}{2a(1+iax)^2}$$

```
[Out] 1/2*I/a/(1+I*a*x)^2
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5073, 32}

$$\frac{i}{2a(1+iax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (I/2)/(a*(1 + I*a*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{1}{(1+iax)^3}\,dx\\ &= \frac{i}{2a(1+iax)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$-\frac{i}{2a(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (-1/2*I)/(a*(-I + a*x)^2)
```

fricas [A] time = 0.37, size = 22, normalized size = 1.16

$$-\frac{i}{2\,a^3x^2-4i\,a^2x-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="fricas")
[Out] -I/(2*a^3*x^2 - 4*I*a^2*x - 2*a)
```

giac [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{i}{2\,(aix+1)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="giac")
[Out] 1/2*i/((a*i*x + 1)^2*a)
```

maple [A] time = 0.03, size = 16, normalized size = 0.84

$$\frac{i}{2a\,(iax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3,x)
[Out] 1/2*I/a/(1+I*a*x)^2
```

maxima [A] time = 0.32, size = 13, normalized size = 0.68

$$\frac{i}{2\,(i\,ax+1)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="maxima")
[Out] 1/2*I/((I*a*x + 1)^2*a)
```

mupad [B] time = 0.05, size = 24, normalized size = 1.26

$$\frac{1\mathrm{i}}{2\,\left(-a^3\,x^2+a^2\,x\,2\mathrm{i}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x*1i + 1)^3,x)
[Out] 1i/(2*(a + a^2*x*2i - a^3*x^2))
```

sympy [A] time = 0.17, size = 22, normalized size = 1.16

$$-\frac{i}{2a^3x^2-4ia^2x-2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3,x)
[Out] -I/(2*a**3*x**2 - 4*I*a**2*x - 2*a)
```

3.327 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

```
[Out] 1/5*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^(5/2)+1/15*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^
(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5073, 45, 37}

$$\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/5)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(5/2)) + ((I/15)*(1 - I*a*x)^(3/2)
)/(a*(1 + I*a*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx &= \int \frac{\sqrt{1-iax}}{(1+iax)^{7/2}}\,dx\\
&= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1-iax}}{(1+iax)^{5/2}}\,dx\\
&= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}+\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.70

$$\frac{(1-iax)^{3/2}(ax-4i)}{15a\sqrt{1+iax}\,(ax-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x))/(15*a*Sqrt[1 + I*a*x]*(-I + a*x)^2)
```

fricas [A] time = 0.46, size = 76, normalized size = 1.13

$$-\frac{a^3x^3-3i\,a^2x^2-3\,ax+\left(a^2x^2-3i\,ax+4\right)\sqrt{a^2x^2+1}\,+i}{15\,a^4x^3-45i\,a^3x^2-45\,a^2x+15i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 - 3*I*a*x + 4)*sqrt(a^2*x^2 + 1)
 + I)/(15*a^4*x^3 - 45*I*a^3*x^2 - 45*a^2*x + 15*I*a)
```

giac [B] time = 0.20, size = 111, normalized size = 1.66

$$-\frac{2\left(5\,a^3i\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)-15\,ai\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3+4\,a^4-25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4\right)}{15\left(ai-\sqrt{a^2+\frac{1}{x^2}}+\frac{1}{x}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^3*i*(sqrt(a^2 + 1/x^2) - 1/x) - 15*a*i*(sqrt(a^2 + 1/x^2) - 1/x)
^3 + 4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 15*(sqrt(a^2 + 1/x^2) - 1
/x)^4)/(a*i - sqrt(a^2 + 1/x^2) + 1/x)^5
```

maple [A] time = 0.18, size = 92, normalized size = 1.37

$$\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{i}{a}\right)^4}-\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{i}{a}\right)^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^4*(1/5*I/a/(x-I/a)^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)-1/15/(x-I/a)^3
*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2))
```

maxima [B] time = 0.32, size = 100, normalized size = 1.49

$$\frac{2i\,\sqrt{a^2x^2+1}}{-5i\,a^4x^3-15\,a^3x^2+15i\,a^2x+5\,a}+\frac{i\,\sqrt{a^2x^2+1}}{15\,a^3x^2-30i\,a^2x-15\,a}-\frac{i\,\sqrt{a^2x^2+1}}{15i\,a^2x+15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

[Out] 2*I*sqrt(a^2*x^2 + 1)/(-5*I*a^4*x^3 - 15*a^3*x^2 + 15*I*a^2*x + 5*a) + I*sqrt(a^2*x^2 + 1)/(15*a^3*x^2 - 30*I*a^2*x - 15*a) - I*sqrt(a^2*x^2 + 1)/(15*I*a^2*x + 15*a)

mupad [B] time = 0.52, size = 40, normalized size = 0.60

$$\frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2-a\,x\,3\mathrm{i}+4\right)\,1\mathrm{i}}{15\,a\,{\left(1+a\,x\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^4,x)

[Out] ((a^2*x^2 + 1)^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*(a*x*1i + 1)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}}{(ax-i)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(1/2),x)

[Out] Integral(sqrt(a**2*x**2 + 1)/(a*x - I)**4, x)

3.328 $\int \frac{e^{5i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(ax+i)^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3ac(ax+i)^3\sqrt{a^2cx^2+c}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)^3/(a^2*c*x^2+c)^(1/2)-1/2*I*(a^2*x^2+1)^
(1/2)/a/c/(I+a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 43}

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(ax+i)^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3ac(ax+i)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*c*(I + a*x)^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqr
t[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{5i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\tan^{-1}(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1+iax}{(1-iax)^4}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \left(\frac{2}{(i+ax)^4}+\frac{i}{(i+ax)^3}\right)dx}{c\sqrt{c+a^2cx^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}}{3ac(i+ax)^3\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.59

$$
-\frac{i(3ax-i)\sqrt{a^2x^2+1}}{6ac(ax+i)^3\sqrt{a^2cx^2+c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-1/6*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^3*Sqrt[c + a^2*c*x
^2])
```

fricas [A] time = 0.49, size = 101, normalized size = 1.06

$$
\frac{\sqrt{a^2cx^2+c}\left(i\,a^2x^3-3\,ax^2-6i\,x\right)\sqrt{a^2x^2+1}}{6\,a^5c^2x^5+18i\,a^4c^2x^4-12\,a^3c^2x^3+12i\,a^2c^2x^2-18\,ac^2x-6i\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(I*a^2*x^3 - 3*a*x^2 - 6*I*x)*sqrt(a^2*x^2 + 1)/(6*a^5*
c^2*x^5 + 18*I*a^4*c^2*x^4 - 12*a^3*c^2*x^3 + 12*I*a^2*c^2*x^2 - 18*a*c^2*x
 - 6*I*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(i\,ax+1)^5}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

maple [A] time = 0.16, size = 48, normalized size = 0.51

$$
-\frac{\sqrt{c\left(a^2x^2+1\right)}\,(3iax+1)}{6\sqrt{a^2x^2+1}\,c^2a\,(ax+i)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/6/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*a*x+1)/c^2/a/(I+a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^5}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

mupad [B] time = 1.62, size = 48, normalized size = 0.51

$$-\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\left(3\,a\,x-\mathrm{i}\right)}{6\,a\,c^2\,\sqrt{a^2\,x^2+1}\,{\left(-1+a\,x\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(5/2)),x)
```

```
[Out] -((c*(a^2*x^2 + 1))^(1/2)*(3*a*x - 1i))/(6*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x*1
i - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{a^6cx^6\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(-I/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a
**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a
**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**
2 + c)), x) + Integral(5*a*x/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x
**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2
*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*s
qrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x**3/(a**6*c*x**6*sqrt(a**2*x
**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**
2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c
*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a**5*x**5/(a**6*
c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a*
*2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integra
l(10*I*a**2*x**2/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3
*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x
**2 + c)), x) + Integral(-5*I*a**4*x**4/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 +
c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

3.329 $\int \frac{e^{4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{ic(1+iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1+iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] -1/3*I*c*(1+I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)+1/15*I*c*(1+I*a*x)^5/a/(a^2*c*x^
2+c)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5075, 659, 651}

$$\frac{ic(1+iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1+iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-I/3)*c*(1 + I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) + ((I/15)*c*(1 + I*a*x)^
5)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c^2\int \frac{(1+iax)^4}{\left(c+a^2cx^2\right)^{7/2}}\,dx\\
&= -\frac{ic(1+iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}}-\frac{1}{3}c^2\int \frac{(1+iax)^5}{\left(c+a^2cx^2\right)^{7/2}}\,dx\\
&= -\frac{ic(1+iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}}+\frac{ic(1+iax)^5}{15a\left(c+a^2cx^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.12

$$\frac{(1+iax)^{3/2}(ax+4i)\sqrt{a^2x^2+1}}{15ac\sqrt{1-iax}\,(ax+i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 - I*a*x]*(
I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.52, size = 67, normalized size = 0.97

$$-\frac{\sqrt{a^2cx^2+c}\left(a^2x^2+3i\,ax+4\right)}{15\,a^4c^2x^3+45i\,a^3c^2x^2-45\,a^2c^2x-15i\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 3*I*a*x + 4)/(15*a^4*c^2*x^3 + 45*I*a^3*c^2
*x^2 - 45*a^2*c^2*x - 15*I*a*c^2)
```

giac [B] time = 0.16, size = 135, normalized size = 1.96

$$-\frac{2\left(5\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^2ci-15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}+c^2i+5\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)c^{\frac{3}{2}}\right)}{15\left(\sqrt{c}\,i+\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -2/15*(5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*c*i - 15*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 + c))^3*sqrt(c) + c^2*i + 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
+ c))*c^(3/2))/((sqrt(c)*i + sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^5*a*c)
```

maple [B] time = 0.19, size = 940, normalized size = 13.62

$$\frac{x}{c\sqrt{a^2cx^2+c}}-\frac{2\left(i\sqrt{-a^2}+a\right)\left(-\frac{1}{5c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}-\frac{3a^2\left(-\frac{1}{3c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2\cdots}}\cdots\right)}{\cdots}\cdots\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x/c/(a^2*c*x^2+c)^(1/2)-2/a^3*(I*(-a^2)^(1/2)+a)*(-1/5/c/(-a^2)^(1/2)/(x-(-
a^2)^(1/2)/a^2)^2/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^
(1/2)/a^2))^(1/2)-3/5*a^2/(-a^2)^(1/2)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)
/a^2)/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^
(1/2)-1/3/c^2/(-a^2)^(1/2)*(2*(x-(-a^2)^(1/2)/a^2)*a^2*c+2*c*(-a^2)^(1/2))/
```

```
((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2))
)-2/a*(I*(-a^2)^(1/2)+a)/(-a^2)^(1/2)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/
a^2)/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(
1/2)-1/3/c^2/(-a^2)^(1/2)*(2*(x-(-a^2)^(1/2)/a^2)*a^2*c+2*c*(-a^2)^(1/2))/(
(x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2))+
2/a^3*(I*(-a^2)^(1/2)-a)*(1/5/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)^2/((x+(-a
^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+3/5*a^2
/(-a^2)^(1/2)*(1/3/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)/((x+(-a^2)^(1/2)/a^2
)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+1/3/c^2/(-a^2)^(1/2)
*(2*(x+(-a^2)^(1/2)/a^2)*a^2*c-2*c*(-a^2)^(1/2))/((x+(-a^2)^(1/2)/a^2)^2*a^
2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)))-2/a*(I*(-a^2)^(1/2)-a)/(
-a^2)^(1/2)*(1/3/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)/((x+(-a^2)^(1/2)/a^2)^
2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+1/3/c^2/(-a^2)^(1/2)*(
2*(x+(-a^2)^(1/2)/a^2)*a^2*c-2*c*(-a^2)^(1/2))/((x+(-a^2)^(1/2)/a^2)^2*a^2*
c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^4}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^2), x)
```

mupad [B] time = 0.98, size = 46, normalized size = 0.67

$$\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\left(a^2\,x^2\,1\mathrm{i}-3\,a\,x+4\mathrm{i}\right)}{15\,a\,c^2\,{\left(-1+a\,x\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^2),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*c^2*(a*x*1i - 1)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-i)^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x - I)**4/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)**2), x)
```

3.330 $\int \frac{e^{3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

```
[Out] -1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 32}

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= -\frac{i\sqrt{1+a^2x^2}}{2ac(1-iax)^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.16

$$\frac{i\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{2ac^2(ax-i)(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((I/2)*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*c^2*(-I + a*x)*(I + a*x)^3
)
```

fricas [A] time = 0.48, size = 71, normalized size = 1.45

$$\frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\left(i\,ax^2-2\,x\right)}{2\,a^4c^2x^4+4i\,a^3c^2x^3+4i\,ac^2x-2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(I*a*x^2 - 2*x)/(2*a^4*c^2*x^4 + 4*I*
a^3*c^2*x^3 + 4*I*a*c^2*x - 2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^3/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.10, size = 44, normalized size = 0.90

$$-\frac{(ax+i)\,(iax+1)^3}{2a\left(a^2x^2+1\right)^{\frac{3}{2}}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(I+a*x)/a*(1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.25, size = 41, normalized size = 0.84

$$\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,1\mathrm{i}}{2\,a\,c^2\,\sqrt{a^2\,x^2+1}\,\left(a\,x+1\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*1i)/(2*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{a^4cx^4\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}\,+2a^2cx^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}\,+c\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}\,dx+\int\left(-\frac{}{a^4cx^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a
**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1
)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**4*c*x**4*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a**
3*x**3/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x*
*2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a
**2*c*x**2 + c)), x) + Integral(-3*I*a**2*x**2/(a**4*c*x**4*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

3.331 $\int \frac{e^{2i\tan^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x}{3c\sqrt{a^2cx^2+c}}-\frac{2i(1+iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

[Out] -2/3*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5075, 653, 191}

$$\frac{x}{3c\sqrt{a^2cx^2+c}}-\frac{2i(1+iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-2*I)/3)*(1 + I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*
x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5075

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^((I*n)/2), Int[(c + d*x^2)^(p + (I*n)/2)/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[(I*n)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1+iax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\
&= -\frac{2i(1+iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\
&= -\frac{2i(1+iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.44

$$\frac{(2-iax)\sqrt{1+iax}\sqrt{a^2x^2+1}}{3ac\sqrt{1-iax}\,(ax+i)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x]*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 - I*a*x]*(I +
 a*x)*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.55, size = 47, normalized size = 0.87

$$\frac{\sqrt{a^2cx^2+c}\,(ax+2i)}{3\,a^3c^2x^2+6i\,a^2c^2x-3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x + 2*I)/(3*a^3*c^2*x^2 + 6*I*a^2*c^2*x - 3*a*c^2)
```

giac [A] time = 0.18, size = 76, normalized size = 1.41

$$-\frac{2\,\sqrt{a^2c}\left(\sqrt{c}\,i+3\,\sqrt{a^2c}\,x-3\,\sqrt{a^2cx^2+c}\right)}{3\left(\sqrt{c}\,i+\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^3a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a^2*c)*(sqrt(c)*i + 3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c))/((sq
rt(c)*i + sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*a^2*c)
```

maple [B] time = 0.18, size = 398, normalized size = 7.37

$$-\frac{x}{c\sqrt{a^2c\,x^2+c}}+\frac{\left(i\sqrt{-a^2}+a\right)\left(-\frac{1}{3c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}-\frac{2\left(x-\frac{\sqrt{-a^2}}{a^2}\right)a^2c+2c\sqrt{-a^2}}{3c^2\sqrt{-a^2}\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(\right.}}\right.}{a\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -x/c/(a^2*c*x^2+c)^(1/2)+1/a*(I*(-a^2)^(1/2)+a)/(-a^2)^(1/2)*(-1/3/c/(-a^2)
^(1/2)/(x-(-a^2)^(1/2)/a^2)/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*
(x-(-a^2)^(1/2)/a^2))^(1/2)-1/3/c^2/(-a^2)^(1/2)*(2*(x-(-a^2)^(1/2)/a^2)*a^
2*c+2*c*(-a^2)^(1/2))/((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a
^2)^(1/2)/a^2))^(1/2))+1/a*(I*(-a^2)^(1/2)-a)/(-a^2)^(1/2)*(1/3/c/(-a^2)^(1
/2)/(x+(-a^2)^(1/2)/a^2)/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+
(-a^2)^(1/2)/a^2))^(1/2)+1/3/c^2/(-a^2)^(1/2)*(2*(x+(-a^2)^(1/2)/a^2)*a^2*c
-2*c*(-a^2)^(1/2))/((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)
^(1/2)/a^2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*x + 1)^2/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)), x)
```

mupad [B] time = 0.65, size = 32, normalized size = 0.59

$$\frac{a^3\,x^3+3\,a\,x-2\mathrm{i}}{3\,a\,{\left(c\,\left(a^2\,x^2+1\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)),x)
```

```
[Out] (3*a*x + a^3*x^3 - 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\,dx-\int\left(-\frac{2iax}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt
(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x) - Integral(-2*I*a*x/(a**4*
c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt
(a**2*c*x**2 + c)), x) - Integral(-1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2
*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x)
```

3.332 $\int \frac{e^{i \tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}}{2ac(ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] 1/2*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/2*arctan(a*x)*(a^2*
x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5076, 5073, 44, 203}

$$\frac{\sqrt{a^2x^2+1}\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}}{2ac(ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2
]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\tan^{-1}(ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [**A**] time = 0.04, size = 51, normalized size = 0.58

$$\frac{\sqrt{a^2x^2+1}\left(\tan^{-1}(ax)+\frac{1}{ax+i}\right)}{2ac\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*((I + a*x)^(-1) + ArcTan[a*x]))/(2*a*c*Sqrt[c + a^2*c*x^
2])
```

fricas [**B**] time = 0.48, size = 317, normalized size = 3.60

$$\frac{\left(-i\,a^3c^2x^3+a^2c^2x^2-i\,ac^2x+c^2\right)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{8\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\,a^6x+\left(4i\,a^{10}c^2x^4-4i\,a^6c^2\right)\sqrt{\frac{1}{a^2c^3}}}{2\left(a^4x^4+2\,a^2x^2+1\right)}\right)+\left(i\,a^3c^2x^3-a^2c^2x^2+i\,a\right.}{2\left(4\,a^3c^2x^3+4i\,a^2c^2x^2+4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*((-I*a^3*c^2*x^3 + a^2*c^2*x^2 - I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log
(1/2*(8*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (4*I*a^10*c^2*x^4 - 4
*I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^3*c^2*x^3
- a^2*c^2*x^2 + I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log(1/2*(8*sqrt(a^2*c*x^
2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (-4*I*a^10*c^2*x^4 + 4*I*a^6*c^2)*sqrt(1/(
a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1)*x)/(4*a^3*c^2*x^3 + 4*I*a^2*c^2*x^2 + 4*a*c^2*x + 4*I*c^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.18, size = 54, normalized size = 0.61

$$\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(\arctan\left(ax\right)x^{2}a^{2}+ax+\arctan\left(ax\right)-i\right)}{2\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2/(a^2*x^2+1)^(3/2)*(c*(a^2*x^2+1))^(1/2)/a*(arctan(a*x)*x^2*a^2+a*x+arct
an(a*x)-I)/c^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1+a\,x\,1\mathrm{i}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{a^{2}cx^{2}\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}+c\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}}\right)dx+\int\frac{ax}{a^{2}cx^{2}\sqrt{a^{2}x^{2}+1}\,\sqrt{a^{2}cx^{2}+c}+c\sqrt{a^{2}x^{2}

3.333 $\int \frac{e^{-i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a^2x^2+1}\,\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}}{2ac(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -1/2*(a^2*x^2+1)^(1/2)/a/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)+1/2*arctan(a*x)*(a^2
*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5076, 5073, 44, 203}

$$\frac{\sqrt{a^2x^2+1}\,\tan^{-1}(ax)}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}}{2ac(-ax+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^
2]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx}{c\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

$$= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\tan^{-1}(ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.67

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\tan^{-1}(ax)}{2a}-\frac{1}{2a(-ax+i)}\right)}{c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c +
a^2*c*x^2])
```

fricas [B] time = 0.48, size = 317, normalized size = 3.56

$$\frac{\left(-i\,a^3c^2x^3-a^2c^2x^2-i\,ac^2x-c^2\right)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{8\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}\,a^6x+\left(4i\,a^{10}c^2x^4-4i\,a^6c^2\right)\sqrt{\frac{1}{a^2c^3}}}{2\left(a^4x^4+2\,a^2x^2+1\right)}\right)+\left(i\,a^3c^2x^3+a^2c^2x^2+\cdots\right.}{2\left(4\,a^3c^2x^3-4i\,a^2c^2x^2+4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*((-I*a^3*c^2*x^3 - a^2*c^2*x^2 - I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log
(1/2*(8*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (4*I*a^10*c^2*x^4 - 4
*I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^3*c^2*x^3
+ a^2*c^2*x^2 + I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log(1/2*(8*sqrt(a^2*c*x^
2 + c)*sqrt(a^2*x^2 + 1)*a^6*x + (-4*I*a^10*c^2*x^4 + 4*I*a^6*c^2)*sqrt(1/(
a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^
2 + 1)*x)/(4*a^3*c^2*x^3 - 4*I*a^2*c^2*x^2 + 4*a*c^2*x - 4*I*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.20, size = 86, normalized size = 0.97

$$-\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\;\left(i\ln\left(ax+i\right)xa-i\ln\left(-ax+i\right)xa+\ln\left(ax+i\right)-\ln\left(-ax+i\right)+2\right)}{4\sqrt{a^{2}x^{2}+1}\;c^{2}a\left(-ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I+a*x)*x*a-I*ln(-a*x+I)*
x*a+ln(I+a*x)-ln(-a*x+I)+2)/c^2/a/(-a*x+I)
```

maxima [A] time = 0.33, size = 52, normalized size = 0.58

$$\frac{\sqrt{c}}{2\,a^{2}c^{2}x-2i\,ac^{2}}-\frac{i\,\log\left(ax-i\right)}{4\,ac^{\frac{3}{2}}}+\frac{i\,\log\left(i\,ax-1\right)}{4\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] sqrt(c)/(2*a^2*c^2*x - 2*I*a*c^2) - 1/4*I*log(a*x - I)/(a*c^(3/2)) + 1/4*I*
log(I*a*x - 1)/(a*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2\,x^2+1}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}\,\left(1+a\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
```

```
[Out] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int\frac{\sqrt{a^{2}x^{2}+1}}{a^{3}cx^{3}\sqrt{a^{2}cx^{2}+c}-ia^{2}cx^{2}\sqrt{a^{2}cx^{2}+c}+acx\sqrt{a^{2}cx^{2}+c}-ic\sqrt{a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) - I*a**2
*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sqrt(a**2
*c*x**2 + c)), x)
```

3.334 $\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x}{3c\sqrt{a^2cx^2+c}}+\frac{2i(1-iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

```
[Out] 2/3*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5074, 653, 191}

$$\frac{x}{3c\sqrt{a^2cx^2+c}}+\frac{2i(1-iax)}{3a\left(a^2cx^2+c\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*x
^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{(1-iax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{2i(1-iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2i(1-iax)}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.44

$$\frac{\sqrt{1-iax}\,(2+iax)\sqrt{a^2x^2+1}}{3ac\sqrt{1+iax}\,(ax-i)\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x)*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 + I*a*x]*(-I
+ a*x)*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.46, size = 47, normalized size = 0.87

$$\frac{\sqrt{a^2cx^2+c}\,(ax-2i)}{3\,a^3c^2x^2-6i\,a^2c^2x-3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c*x^2 + c)*(a*x - 2*I)/(3*a^3*c^2*x^2 - 6*I*a^2*c^2*x - 3*a*c^2)
```

giac [A] time = 0.25, size = 75, normalized size = 1.39

$$-\frac{2\sqrt{a^2c}\left(\sqrt{c}\,i-3\sqrt{a^2c}\,x+3\sqrt{a^2cx^2+c}\right)}{3\left(\sqrt{c}\,i-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right)^3a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -2/3*sqrt(a^2*c)*(sqrt(c)*i - 3*sqrt(a^2*c)*x + 3*sqrt(a^2*c*x^2 + c))/((sq
rt(c)*i - sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c))^3*a^2*c)
```

maple [B] time = 0.18, size = 137, normalized size = 2.54

$$-\frac{x}{c\sqrt{a^2c\,x^2+c}}-\frac{2i\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^2c+2iac\right)}{3a\,c^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -x/c/(a^2*c*x^2+c)^(1/2)-2*I/a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*
(x-I/a))^(1/2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a
*c*(x-I/a))^(1/2))
```

maxima [A] time = 0.33, size = 59, normalized size = 1.09

$$\frac{x}{3\sqrt{a^2cx^2+c}\,c}+\frac{2i}{3i\sqrt{a^2cx^2+c}\,a^2cx+3\sqrt{a^2cx^2+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*x/(sqrt(a^2*c*x^2 + c)*c) + 2*I/(3*I*sqrt(a^2*c*x^2 + c)*a^2*c*x + 3*sq
rt(a^2*c*x^2 + c)*a*c)
```

mupad [B] time = 0.65, size = 32, normalized size = 0.59

$$\frac{a^3\,x^3+3\,a\,x+2\mathrm{i}}{3\,a\,\left(c\,\left(a^2\,x^2+1\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^2),x)
```

```
[Out] (3*a*x + a^3*x^3 + 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3cx^3\sqrt{a^2cx^2+c}-2iacx\sqrt{a^2cx^2+c}-c\sqrt{a^2cx^2+c}}\,dx-\int\frac{}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x**3*sq
rt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*c*x**2
+ c)), x) - Integral(1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x**3
*sqrt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*c*x*
*2 + c)), x)
```

3.335 $\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

[Out] 1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1+I*a*x)^2/(a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5076, 5073, 32}

$$\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]

[Out] ((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 + I*a*x)^2*Sqrt[c + a^2*c*x^2])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{i\sqrt{1+a^2x^2}}{2ac(1+iax)^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.16

$$-\frac{i\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{2ac^2(ax-i)^3(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*c^2*(-I + a*x)^3*(I + a
*x))
```

fricas [**A**] time = 0.54, size = 71, normalized size = 1.45

$$\frac{\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}\left(-i\,ax^2-2\,x\right)}{2\,a^4c^2x^4-4i\,a^3c^2x^3-4i\,ac^2x-2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-I*a*x^2 - 2*x)/(2*a^4*c^2*x^4 - 4*I
*a^3*c^2*x^3 - 4*I*a*c^2*x - 2*c^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.04, size = 45, normalized size = 0.92

$$\frac{(-ax+i)\left(a^2x^2+1\right)^{\frac{3}{2}}}{2a\,(iax+1)^3\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/2*(-a*x+I)/a/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2)
```

maxima [**A**] time = 0.34, size = 29, normalized size = 0.59

$$\frac{1}{2i\,a^3c^{\frac{3}{2}}x^2+4\,a^2c^{\frac{3}{2}}x-2i\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] 1/(2*I*a^3*c^(3/2)*x^2 + 4*a^2*c^(3/2)*x - 2*I*a*c^(3/2))
```

mupad [**B**] time = 1.07, size = 49, normalized size = 1.00

$$-\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\sqrt{a^2\,x^2+1}}{2\,a\,c^2\,\left(a\,x+1{}\mathrm{i}\right)\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^3),x)
```

```
[Out] -((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2))/(2*a*c^2*(a*x + 1i)*(a*x*1i
+ 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int \frac{\sqrt{a^2x^2+1}}{a^5cx^5\sqrt{a^2cx^2+c} - 3ia^4cx^4\sqrt{a^2cx^2+c} - 2a^3cx^3\sqrt{a^2cx^2+c} - 2ia^2cx^2\sqrt{a^2cx^2+c} - 3acx\sqrt{a^2cx^2+c} + ic\sqrt{a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**5*c*x**5*sqrt(a**2*c*x**2 + c) - 3*I*a*
*4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*a**3*c*x**3*sqrt(a**2*c*x**2 + c) - 2*I
*a**2*c*x**2*sqrt(a**2*c*x**2 + c) - 3*a*c*x*sqrt(a**2*c*x**2 + c) + I*c*sq
rt(a**2*c*x**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**5*c*x
**5*sqrt(a**2*c*x**2 + c) - 3*I*a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*a**3*
c*x**3*sqrt(a**2*c*x**2 + c) - 2*I*a**2*c*x**2*sqrt(a**2*c*x**2 + c) - 3*a*
c*x*sqrt(a**2*c*x**2 + c) + I*c*sqrt(a**2*c*x**2 + c)), x))
```

3.336 $\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{ic(1-iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1-iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}$$

```
[Out] 1/3*I*c*(1-I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)-1/15*I*c*(1-I*a*x)^5/a/(a^2*c*x^2
+c)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5074, 659, 651}

$$\frac{ic(1-iax)^4}{3a\left(a^2cx^2+c\right)^{5/2}}-\frac{ic(1-iax)^5}{15a\left(a^2cx^2+c\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*c*(1 - I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) - ((I/15)*c*(1 - I*a*x)^5
)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 5074

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^((I*n)/2), Int[(c + d*x^2)^(p - (I*n)/2)*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[(I*n)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-4i\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx &= c^2\int \frac{(1-iax)^4}{\left(c+a^2cx^2\right)^{7/2}}\,dx\\
&= \frac{ic(1-iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}}-\frac{1}{3}c^2\int \frac{(1-iax)^5}{\left(c+a^2cx^2\right)^{7/2}}\,dx\\
&= \frac{ic(1-iax)^4}{3a\left(c+a^2cx^2\right)^{5/2}}-\frac{ic(1-iax)^5}{15a\left(c+a^2cx^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.12

$$\frac{(1-iax)^{3/2}(ax-4i)\sqrt{a^2x^2+1}}{15ac\sqrt{1+iax}\,(ax-i)^2\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 + I*a*x]*
(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

fricas [A] time = 0.72, size = 67, normalized size = 0.97

$$-\frac{\sqrt{a^2cx^2+c}\left(a^2x^2-3i\,ax+4\right)}{15\,a^4c^2x^3-45i\,a^3c^2x^2-45\,a^2c^2x+15i\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -sqrt(a^2*c*x^2 + c)*(a^2*x^2 - 3*I*a*x + 4)/(15*a^4*c^2*x^3 - 45*I*a^3*c^2
*x^2 - 45*a^2*c^2*x + 15*I*a*c^2)
```

giac [B] time = 0.17, size = 134, normalized size = 1.94

$$-\frac{2\left(5\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^2ci+15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}+c^2i-5\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\right)c^{\frac{3}{2}}\right)}{15\left(\sqrt{c}\,i-\sqrt{a^2c}\,x+\sqrt{a^2cx^2+c}\right)^5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] -2/15*(5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2*c*i + 15*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 + c))^3*sqrt(c) + c^2*i - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
+ c))*c^(3/2))/((sqrt(c)*i - sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c))^5*a*c)
```

maple [B] time = 0.18, size = 307, normalized size = 4.45

$$\frac{x}{c\sqrt{a^2c\,x^2+c}}-\frac{4\left(\frac{i}{5ac\left(x-\frac{i}{a}\right)^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{3ia\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^2c+2iac\right)}{3a\,c^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}\right)}{5}\right)}{a^2}+\ \cdots 4i\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x/c/(a^2*c*x^2+c)^(1/2)-4/a^2*(1/5*I/a/c/(x-I/a)^2/((x-I/a)^2*a^2*c+2*I*a*c
*(x-I/a))^(1/2)+3/5*I*a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a)
)^(1/2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I
/a))^(1/2)))+4*I/a*(1/3*I/a/c/(x-I/a)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/
2)+1/3*I/a/c^2*(2*(x-I/a)*a^2*c+2*I*a*c)/((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^
(1/2))
```

maxima [B] time = 0.33, size = 120, normalized size = 1.74

$$-\frac{x}{15\sqrt{a^2cx^2+c}\,c}-\frac{4i}{5\sqrt{a^2cx^2+c}\,a^3cx^2-10i\sqrt{a^2cx^2+c}\,a^2cx-5\sqrt{a^2cx^2+c}\,ac}-\frac{8i}{15i\sqrt{a^2cx^2+c}\,a^2cx+15\sqrt{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/15*x/(sqrt(a^2*c*x^2 + c)*c) - 4*I/(5*sqrt(a^2*c*x^2 + c)*a^3*c*x^2 - 10
*I*sqrt(a^2*c*x^2 + c)*a^2*c*x - 5*sqrt(a^2*c*x^2 + c)*a*c) - 8*I/(15*I*sqr
t(a^2*c*x^2 + c)*a^2*c*x + 15*sqrt(a^2*c*x^2 + c)*a*c)
```

mupad [B] time = 1.00, size = 45, normalized size = 0.65

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2-a\,x\,3\mathrm{i}+4\right)1\mathrm{i}}{15\,a\,c^2\,(1+a\,x\,1\mathrm{i})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^4),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*c^2*(a*x*1i + 1)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2x^2+1\right)^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}(ax-i)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**2/((c*(a**2*x**2 + 1))**(3/2)*(a*x - I)**4), x)
```

3.337 $\int e^{n \tan^{-1}(ax)} \left(c + a^2cx^2\right)^2 \, dx$

Optimal. Leaf size=86

$$-\frac{c^2 2^{3-\frac{in}{2}} (1 - iax)^{3+\frac{in}{2}} \, _2F_1\left(\frac{in}{2} - 2, \frac{in}{2} + 3; \frac{in}{2} + 4; \frac{1}{2}(1 - iax)\right)}{a(-n + 6i)}$$

```
[Out] -2^(3-1/2*I*n)*c^2*(1-I*a*x)^(3+1/2*I*n)*hypergeom([-2+1/2*I*n, 3+1/2*I*n],
[4+1/2*I*n],1/2-1/2*I*a*x)/a/(6*I-n)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5073, 69}

$$-\frac{c^2 2^{3-\frac{in}{2}} (1 - iax)^{3+\frac{in}{2}} \, _2F_1\left(\frac{in}{2} - 2, \frac{in}{2} + 3; \frac{in}{2} + 4; \frac{1}{2}(1 - iax)\right)}{a(-n + 6i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] -((2^(3 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(6*I - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{n \tan^{-1}(ax)} \left(c + a^2cx^2\right)^2 \, dx &= c^2 \int (1 - iax)^{2+\frac{in}{2}} (1 + iax)^{2-\frac{in}{2}} \, dx \\
&= -\frac{2^{3-\frac{in}{2}} c^2 (1 - iax)^{3+\frac{in}{2}} \, _2F_1\left(-2 + \frac{in}{2}, 3 + \frac{in}{2}; 4 + \frac{in}{2}; \frac{1}{2}(1 - iax)\right)}{a(6i - n)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 1.05

$$\frac{ic^2 2^{2-\frac{in}{2}} (1 - iax)^{3+\frac{in}{2}} \, _2F_1\left(\frac{in}{2} - 2, \frac{in}{2} + 3; \frac{in}{2} + 4; \frac{1}{2}(1 - iax)\right)}{a\left(3 + \frac{in}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] (I*2^(2 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(3 + (I/2)*n))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)e^{(n\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^2 e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int 2a^2x^2e^{n\,\mathrm{atan}\,(ax)}\,dx + \int a^4x^4e^{n\,\mathrm{atan}\,(ax)}\,dx + \int e^{n\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(2*a**2*x**2*exp(n*atan(a*x)), x) + Integral(a**4*x**4*exp(n*
atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

3.338 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal. Leaf size=84

$$-\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

```
[Out] -2^(2-1/2*I*n)*c*(1-I*a*x)^(2+1/2*I*n)*hypergeom([2+1/2*I*n, -1+1/2*I*n],[3
+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-n)
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5073, 69}

$$-\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] -((2^(2 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(4*I - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)\,dx &= c\int (1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx\\ &= -\frac{2^{2-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(-1+\frac{in}{2},2+\frac{in}{2};3+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a(4i-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.05

$$\frac{ic2^{1-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-1,\frac{in}{2}+2;\frac{in}{2}+3;\frac{1}{2}(1-iax)\right)}{a\left(2+\frac{in}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] (I*2^(1 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(2 + (I/2)*n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\arctan(ax)}\left(a^2c\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int a^2x^2e^{n\,\mathrm{atan}\,(ax)}\,dx+\int e^{n\,\mathrm{atan}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(n*atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

3.339 $\int e^{n\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

```
[Out] -2^(1-1/2*I*n)*(1-I*a*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*
I*n],1/2-1/2*I*a*x)/a/(2*I-n)
```

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5061, 69}

$$-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2};\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
 (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a*(2*I - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5061

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^((I*n)/2)/(1
 + I*a*x)^((I*n)/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\,dx &= \int (1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx\\ &= -\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(1+\frac{in}{2},\frac{in}{2};2+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a(2i-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.69

$$\frac{4e^{(n+2i)\tan^{-1}(ax)}\,{}_2F_1\left(2,1-\frac{in}{2};2-\frac{in}{2};-e^{2i\tan^{-1}(ax)}\right)}{a(n+2i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x]),x]
```

```
[Out] (4*E^((2*I + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2)*n,
 -E^((2*I)*ArcTan[a*x])])/(a*(2*I + n))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(n\arctan(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x)),x)
[Out] int(exp(n*arctan(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x)),x)
[Out] int(exp(n*atan(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x)),x)
[Out] Integral(exp(n*atan(a*x)), x)
```

3.340 $\int \frac{e^{n\tan^{-1}(ax)}x^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=131

$$\frac{i\left(n^2-2\right)e^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};-e^{2i\tan^{-1}(ax)}\right)}{a^4cn}+\frac{\left(-in^2+n+2i\right)e^{n\tan^{-1}(ax)}}{2a^4cn}-\frac{nxe^{n\tan^{-1}(ax)}}{2a^3c}+\frac{x^2e^{n\tan^{-1}(ax)}}{2a^2c}$$

```
[Out] 1/2*exp(n*arctan(a*x))*(2*I+n-I*n^2)/a^4/c/n-1/2*exp(n*arctan(a*x))*n*x/a^3
/c+1/2*exp(n*arctan(a*x))*x^2/a^2/c+I*exp(n*arctan(a*x))*(n^2-2)*hypergeom(
[1, -1/2*I*n],[1-1/2*I*n],-(1+I*a*x)^2/(a^2*x^2+1))/a^4/c/n
```

Rubi [A] time = 0.23, antiderivative size = 206, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5082, 100, 143, 69}

$$\frac{2^{-1-\frac{in}{2}}\left(2-n^2\right)(1-iax)^{1+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{a^4c(2+in)}+\frac{i(1+iax)^{-\frac{in}{2}}\left(ian^2x-n^2-in+2\right)(1-iax)^{\frac{in}{2}}}{2a^4cn}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] (x^2*(1 - I*a*x)^((I/2)*n))/(2*a^2*c*(1 + I*a*x)^((I/2)*n)) + ((I/2)*(1 - I
*a*x)^((I/2)*n)*(2 - I*n - n^2 + I*a*n^2*x))/(a^4*c*n*(1 + I*a*x)^((I/2)*n)
) + (2^(-1 - (I/2)*n)*(2 - n^2)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1
[1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a^4*c*(2 + I*n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
```

```
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^3}{c+a^2cx^2}\,dx = \frac{\int x^3(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$
$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-2-anx)\,dx}{2a^2c}$$
$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2-in-n^2+ian^2x\right)}{2a^4cn} - \frac{\left(i\left(2-n^2\right)\right)\int$$
$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2-in-n^2+ian^2x\right)}{2a^4cn} + \frac{2^{-1-\frac{in}{2}}\left(2-n\right.}{}$$

Mathematica [A] time = 0.13, size = 141, normalized size = 1.08

$$\frac{(1-iax)^{\frac{in}{2}}\left(\frac{\left(a^2nx^2-\left(n^2(ax+i)\right)+n+2i\right)(1+iax)^{-\frac{in}{2}}}{n}+\frac{2^{-\frac{in}{2}}\left(n^2-2\right)(ax+i)\,{}_2F_1\left(\frac{in}{2}+1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1}{2}(1-iax)\right)}{n-2i}\right)}{2a^4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2*I + n + a^2*n*x^2 - n^2*(I + a*x))/(n*(1 + I*a*x
)^((I/2)*n)) + ((-2 + n^2)*(I + a*x)*Hypergeometric2F1[1 + (I/2)*n, 1 + (I/
2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(2^((I/2)*n)*(-2*I + n))))/(2*a^4*c)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^3e^{(n\arctan(ax))}}{a^2cx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x^3}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{(n \arctan(ax))}}{a^2cx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\, \mathrm{e}^{n\, \mathrm{atan}(a\, x)}}{c\, a^2\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
```

```
[Out] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3 e^{n \operatorname{atan}{(ax)}}}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(x**3*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.341 $\int \frac{e^{n\tan^{-1}(ax)}x^2}{c+a^2cx^2}\,dx$

Optimal. Leaf size=164

$$\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^3c}-\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}$$

```
[Out] -(1+I*n)*(1-I*a*x)^(1/2*I*n)/a^3/c/n/((1+I*a*x)^(1/2*I*n))+x*(1-I*a*x)^(1/2
*I*n)/a^2/c/((1+I*a*x)^(1/2*I*n))+I*2^(1-1/2*I*n)*(1-I*a*x)^(1/2*I*n)*hyper
geom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^3/c
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5082, 90, 79, 69}

$$\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^3c}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}-\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

```
[Out] -(((1 + I*n)*(1 - I*a*x)^((I/2)*n))/(a^3*c*n*(1 + I*a*x)^((I/2)*n))) + (x*(
1 - I*a*x)^((I/2)*n))/(a^2*c*(1 + I*a*x)^((I/2)*n)) + (I*2^(1 - (I/2)*n)*(1
 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I
*a*x)/2])/(a^3*c)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
```

```
[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\tan^{-1}(ax)}x^2}{c+a^2cx^2}\,dx &= \frac{\int x^2(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}\\
&= \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}+\frac{\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-1-anx)\,dx}{a^2c}\\
&= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}+\frac{(in)\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{i}{2}}}{a^2c}\\
&= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}+\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};1+\right.}{a^3c}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 121, normalized size = 0.74

$$
\frac{(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2in(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)+2^{\frac{in}{2}}(-1+n(ax-i))\right)}{a^3cn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*(2^((I/2)*n)*(-1 + n*(-I + a*x)) + (2*I)*n*(1 + I*a*
x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2
]))/(a^3*c*n*(2 + (2*I)*a*x)^((I/2)*n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
sage_0x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$
\int \frac{e^{n\arctan(ax)}x^2}{a^2c\,x^2+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{(n\arctan(ax))}}{a^2cx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2 e^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c),x)
[Out] Integral(x**2*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.342 $\int \frac{e^{n\tan^{-1}(ax)}x}{c+a^2cx^2}\,dx$

Optimal. Leaf size=122

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^2cn}$$

```
[Out] I*(1-I*a*x)^(1/2*I*n)/a^2/c/n/((1+I*a*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*(1-I*a*
x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^2/c/
n
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5082, 79, 69}

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(a^2*c*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)
*n)*(1 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n,
(1 - I*a*x)/2])/(a^2*c*n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x}{c+a^2cx^2}\,dx = \frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i\int(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx}{ac}$$
$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};1+\frac{in}{2};\frac{1}{2}(1-iax)\right)}{a^2cn}$$

Mathematica [A] time = 0.06, size = 109, normalized size = 0.89

$$\frac{i(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}-2(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2},\frac{in}{2};\frac{in}{2}+1;\frac{1}{2}(1-iax)\right)\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
[Out] (I*(1 - I*a*x)^((I/2)*n)*(2^((I/2)*n) - 2*(1 + I*a*x)^((I/2)*n)*Hypergeomet
ric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2]))/(a^2*c*n*(2 + (2*I)*
a*x)^((I/2)*n))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{xe^{(n\arctan(ax))}}{a^2cx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{xe^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c),x)
[Out] Integral(x*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.343 $\int \frac{e^{n\tan^{-1}(ax)}}{c+a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\tan^{-1}(ax)}}{acn}$$

```
[Out] exp(n*arctan(a*x))/a/c/n
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5071}

$$\frac{e^{n\tan^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] E^(n*ArcTan[a*x])/(a*c*n)
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{c+a^2cx^2}\,dx = \frac{e^{n\tan^{-1}(ax)}}{acn}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 2.33

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] (1 - I*a*x)^((I/2)*n)/(a*c*n*(1 + I*a*x)^((I/2)*n))
```

fricas [A] time = 0.42, size = 17, normalized size = 0.94

$$\frac{e^{(n\arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] e^(n*arctan(a*x))/(a*c*n)
```

giac [A] time = 0.12, size = 17, normalized size = 0.94

$$\frac{e^{(n\arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] e^(n*arctan(a*x))/(a*c*n)
```

maple [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{e^{n \arctan(ax)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c),x)
```

```
[Out] exp(n*arctan(a*x))/a/c/n
```

maxima [A] time = 0.43, size = 17, normalized size = 0.94

$$\frac{e^{(n \arctan(ax))}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] e^(n*arctan(a*x))/(a*c*n)
```

mupad [B] time = 0.63, size = 17, normalized size = 0.94

$$\frac{e^{n \operatorname{atan}(a\,x)}}{a\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2),x)
```

```
[Out] exp(n*atan(a*x))/(a*c*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty}x & \text{for } c = 0 \wedge n = 0 \\ \tilde{\infty} \int e^{n \operatorname{atan}(ax)}\, dx & \text{for } c = 0 \\ \frac{\operatorname{atan}(ax)}{ac} & \text{for } n = 0 \\ \frac{e^{n \operatorname{atan}(ax)}}{acn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c),x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(n, 0)), (zoo*Integral(exp(n*atan(a*x)), x),
 Eq(c, 0)), (atan(a*x)/(a*c), Eq(n, 0)), (exp(n*atan(a*x))/(a*c*n), True))
```

3.344 $\int \frac{e^{n\tan^{-1}(ax)}}{x(c+a^2cx^2)}\,dx$

Optimal. Leaf size=65

$$\frac{ie^{n\tan^{-1}(ax)}}{cn}-\frac{2ie^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};e^{2i\tan^{-1}(ax)}\right)}{cn}$$

```
[Out] I*exp(n*arctan(a*x))/c/n-2*I*exp(n*arctan(a*x))*hypergeom([1, -1/2*I*n],[1-
1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Rubi [B] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 2.03, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5082, 96, 131}

$$\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{2(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n)) - (2*(1 - I*a*x)^(1 +
 (I/2)*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 +
(I/2)*n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}$$

$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} + \frac{\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}$$

$$= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{2(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,1+\frac{in}{2};2+\frac{in}{2};\frac{1-iax}{1+iax}\right)}{c(2+in)}$$

Mathematica [A] time = 0.04, size = 120, normalized size = 1.85

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2(n-ianx)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+(2+in)(ax-i)\right)}{cn(n-2i)(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2 + I*n)*(-I + a*x) + 2*(n - I*a*n*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(c*n*(-2*I + n)*(
1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^3+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{x\left(a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)),x)
```

```
[Out] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^{n \operatorname{atan}{(ax)}}}{a^{2}x^{3}+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**3 + x), x)/c
```

3.345 $\int \frac{e^{n\tan^{-1}(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal. Leaf size=90

$$-\frac{2ia\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};\frac{2i}{ax+i}-1\right)e^{n\tan^{-1}(ax)}}{c}+\frac{ia(n+i)e^{n\tan^{-1}(ax)}}{cn}-\frac{e^{n\tan^{-1}(ax)}}{cx}$$

```
[Out] I*a*exp(n*arctan(a*x))*(I+n)/c/n-exp(n*arctan(a*x))/c/x-2*I*a*exp(n*arctan(
a*x))*hypergeom([1, -1/2*I*n],[1-1/2*I*n],-1+2*I/(I+a*x))/c
```

Rubi [A] time = 0.14, antiderivative size = 180, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5082, 129, 155, 12, 131}

$$-\frac{2an(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}-\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] -((a*(1 - I*n)*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n))) - (1 - I
*a*x)^((I/2)*n)/(c*x*(1 + I*a*x)^((I/2)*n)) - (2*a*n*(1 - I*a*x)^(1 + (I/2)
*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*
n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^2}\,dx}{c}$$

$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} - \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-an+a^2x\right)}{x}\,dx}{c}$$

$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} + \frac{\int \frac{a^2n^2(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{acn}$$

$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} + \frac{(an)\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,d}{c}$$

$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} - \frac{2an(1-iax)^{1+\frac{in}{2}}(1+iax)^{-}}{c(}$$

Mathematica [A] time = 0.06, size = 142, normalized size = 1.58

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2an^2x(1-iax)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+(n-2i)(1+iax)(n(ax+i)+iax)\right)}{cn(n-2i)x(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((-2*I + n)*(1 + I*a*x)*(I*a*x + n*(I + a*x)) + 2*a*
n^2*x*(1 - I*a*x)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/
(I - a*x)]))/(c*n*(-2*I + n)*x*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{e^{n \arctan(ax)}}{x^2 \left(a^2 c\, x^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n \arctan(ax))}}{\left(a^2 c x^2 + c\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atan}(a\, x)}}{x^2 \left(c\, a^2\, x^2 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^{n \operatorname{atan}{(ax)}}}{a^2 x^4 + x^2}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**4 + x**2), x)/c
```

3.346 $\int \frac{e^{n\tan^{-1}(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal. Leaf size=126

$$-\frac{ia^2(n^2-2)e^{n\tan^{-1}(ax)}\,{}_2F_1\left(1,-\frac{in}{2};1-\frac{in}{2};e^{2i\tan^{-1}(ax)}\right)}{cn}+\frac{ia^2(n^2+in-2)e^{n\tan^{-1}(ax)}}{2cn}-\frac{e^{n\tan^{-1}(ax)}}{2cx^2}-\frac{ane^{n\tan^{-1}(ax)}}{2cx}$$

```
[Out] 1/2*I*a^2*exp(n*arctan(a*x))*(-2+I*n+n^2)/c/n-1/2*exp(n*arctan(a*x))/c/x^2-
1/2*a*exp(n*arctan(a*x))*n/c/x-I*a^2*exp(n*arctan(a*x))*(n^2-2)*hypergeom([
1, -1/2*I*n],[1-1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.92, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5082, 129, 151, 155, 12, 131}

$$\frac{a^2(2-n^2)(1-iax)^{1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{1-iax}{iax+1}\right)}{c(2+in)}-\frac{a^2(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -(a^2*(2*I + n - I*n^2)*(1 - I*a*x)^((I/2)*n))/(2*c*n*(1 + I*a*x)^((I/2)*n)
) - (1 - I*a*x)^((I/2)*n)/(2*c*x^2*(1 + I*a*x)^((I/2)*n)) - (a*n*(1 - I*a*x
)^((I/2)*n))/(2*c*x*(1 + I*a*x)^((I/2)*n)) + (a^2*(2 - n^2)*(1 - I*a*x)^(1
+ (I/2)*n)*(1 + I*a*x)^(-1 - (I/2)*n)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 +
 (I/2)*n, (1 - I*a*x)/(1 + I*a*x)])/(c*(2 + I*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^3}\,dx}{c}$$

$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-an+2a^2x\right)}{x^2}\,dx}{2c}$$

$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} + \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\left(-a^2\left(2-n^2\right)-a^3n}{x}}{2c}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax}{2cx}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax}{2cx}$$

$$= -\frac{a^2\left(2i+n-in^2\right)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax}{2cx}$$

Mathematica [A] time = 0.08, size = 174, normalized size = 1.38

$$\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2a^2n\left(n^2-2\right)x^2(1-iax)\,{}_2F_1\left(1,\frac{in}{2}+1;\frac{in}{2}+2;\frac{ax+i}{i-ax}\right)+i(n-2i)(ax-i)\left(in\left(a^2x^2+1\right)-2a\right.}{2cn(n-2i)x^2(ax-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*(I*(-2*I + n)*(-I + a*x)*(-2*a^2*x^2 + a*n^2*x*(I +
a*x) + I*n*(1 + a^2*x^2)) + 2*a^2*n*(-2 + n^2)*x^2*(1 - I*a*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(2*c*n*(-2*I + n)
*x^2*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{x^3\left(a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)),x)
[Out] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{a^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c),x)
[Out] Integral(exp(n*atan(a*x))/(a**2*x**5 + x**3), x)/c
```

3.347 $\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=181

$$\frac{360(2ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+4\right)\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)}+\frac{30(4ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)^2}+\frac{(6ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+36\right)\left(a^2x^2+1\right)^3}+$$

```
[Out] 720*exp(n*arctan(a*x))/a/c^4/n/(n^2+36)/(n^4+20*n^2+64)+exp(n*arctan(a*x))*
(6*a*x+n)/a/c^4/(n^2+36)/(a^2*x^2+1)^3+30*exp(n*arctan(a*x))*(4*a*x+n)/a/c^
4/(n^2+16)/(n^2+36)/(a^2*x^2+1)^2+360*exp(n*arctan(a*x))*(2*a*x+n)/a/c^4/(n
^2+36)/(n^4+20*n^2+64)/(a^2*x^2+1)
```

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5070, 5071}

$$\frac{360(2ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+4\right)\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)}+\frac{30(4ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+16\right)\left(n^2+36\right)\left(a^2x^2+1\right)^2}+\frac{(6ax+n)e^{n\tan^{-1}(ax)}}{ac^4\left(n^2+36\right)\left(a^2x^2+1\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcTan[a*x]))/(a*c^4*n*(4 + n^2)*(16 + n^2)*(36 + n^2)) + (E^(n*A
rcTan[a*x])*(n + 6*a*x))/(a*c^4*(36 + n^2)*(1 + a^2*x^2)^3) + (30*E^(n*ArcT
an[a*x])*(n + 4*a*x))/(a*c^4*(16 + n^2)*(36 + n^2)*(1 + a^2*x^2)^2) + (360*
E^(n*ArcTan[a*x])*(n + 2*a*x))/(a*c^4*(4 + n^2)*(16 + n^2)*(36 + n^2)*(1 +
a^2*x^2))
```

Rule 5070

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*c*(n^2 +
 4*(p + 1)^2)), x] + Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 + 4*(p + 1)^2)), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3}+\frac{30\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{c\left(36+n^2\right)}$$

$$= \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3}+\frac{30e^{n\tan^{-1}(ax)}(n+4ax)}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1+a^2x^2\right)^2}+\frac{360\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{c^2\left(16+n^2\right)\left(36+n^2\right)}$$

$$= \frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3}+\frac{30e^{n\tan^{-1}(ax)}(n+4ax)}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1+a^2x^2\right)^2}+\frac{360e^{n\tan^{-1}(ax)}(}{ac^4\left(4+n^2\right)\left(16+n^2\right)\left(3\right.}$$

$$= \frac{720e^{n\tan^{-1}(ax)}}{ac^4n\left(4+n^2\right)\left(16+n^2\right)\left(36+n^2\right)}+\frac{e^{n\tan^{-1}(ax)}(n+6ax)}{ac^4\left(36+n^2\right)\left(1+a^2x^2\right)^3}+\frac{30e^{n\tan^{-1}(ax)}(n+4a}{ac^4\left(16+n^2\right)\left(36+n^2\right)\left(1\right.}$$

Mathematica [C] time = 0.52, size = 165, normalized size = 0.91

$$\frac{(6ax+n)e^{n\tan^{-1}(ax)}+\frac{30\left(a^2cx^2+c\right)\left(12(ax-i)(ax+i)(1-iax)^{\frac{in}{2}}\left(2a^2x^2+2anx+n^2+2\right)(1+iax)^{-\frac{in}{2}}+n(n-2i)(n+2i)(4ax+n)e^{n\tan^{-1}(ax)}\right)}{cn\left(n^4+20n^2+64\right)}}{ac\left(n^2+36\right)\left(a^2cx^2+c\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (E^(n*ArcTan[a*x])*(n + 6*a*x) + (30*(c + a^2*c*x^2)*(E^(n*ArcTan[a*x])*n*(
-2*I + n)*(2*I + n)*(n + 4*a*x) + (12*(1 - I*a*x)^((I/2)*n)*(-I + a*x)*(I +
 a*x)*(2 + n^2 + 2*a*n*x + 2*a^2*x^2))/(1 + I*a*x)^((I/2)*n)))/(c*n*(64 + 2
0*n^2 + n^4)))/(a*c*(36 + n^2)*(c + a^2*c*x^2)^3)
```

fricas [A] time = 0.45, size = 298, normalized size = 1.65

$$\frac{\left(720\,a^6x^6+720\,a^5nx^5+n^6+360\left(a^4n^2+6\,a^4\right)x^4+50\,n^4+120\left(a^3n^3+16\,a^3n\right)x^3+30\left(a^2n^4+\right.\right.}{ac^4n^7+56\,ac^4n^5+784\,ac^4n^3+\left(a^7c^4n^7+56\,a^7c^4n^5+784\,a^7c^4n^3+2304\,a^7c^4n\right)x^6+2304\,ac^4n+3\left(a^5c^4n^7+56\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 + 6*a^4)*x^4 + 50*n^4 + 1
20*(a^3*n^3 + 16*a^3*n)*x^3 + 30*(a^2*n^4 + 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 + 6*(a*n^5 + 40*a*n^3 + 264*a*n)*x + 720)*e^(n*arctan(a*x))/(a*c^4*n^7
+ 56*a*c^4*n^5 + 784*a*c^4*n^3 + (a^7*c^4*n^7 + 56*a^7*c^4*n^5 + 784*a^7*c^
4*n^3 + 2304*a^7*c^4*n)*x^6 + 2304*a*c^4*n + 3*(a^5*c^4*n^7 + 56*a^5*c^4*n^
5 + 784*a^5*c^4*n^3 + 2304*a^5*c^4*n)*x^4 + 3*(a^3*c^4*n^7 + 56*a^3*c^4*n^5
 + 784*a^3*c^4*n^3 + 2304*a^3*c^4*n)*x^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 166, normalized size = 0.92

$$\frac{\left(720a^6x^6+720a^5x^5n+360a^4n^2x^4+120a^3n^3x^3+2160a^4x^4+30a^2n^4x^2+1920x^3a^3n+6a\,n^5x+840a^2n^2x^2\right.}{\left(a^2x^2+1\right)^3c^4an\left(n^6+56n^4+784n^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x)
```

```
[Out] (720*a^6*x^6+720*a^5*n*x^5+360*a^4*n^2*x^4+120*a^3*n^3*x^3+2160*a^4*x^4+30*
a^2*n^4*x^2+1920*a^3*n*x^3+6*a*n^5*x+840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*a
^2*x^2+50*n^4+1584*a*n*x+544*n^2+720)*exp(n*arctan(a*x))/(a^2*x^2+1)^3/c^4/
a/n/(n^6+56*n^4+784*n^2+2304)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

mupad [B] time = 0.87, size = 281, normalized size = 1.55

$$\frac{e^{n\,\mathrm{atan}(a\,x)}\left(\frac{720\,x^5}{a^2\,c^4\,\left(n^6+56\,n^4+784\,n^2+2304\right)}+\frac{n^6+50\,n^4+544\,n^2+720}{a^7\,c^4\,n\,\left(n^6+56\,n^4+784\,n^2+2304\right)}+\frac{720\,x^6}{a\,c^4\,n\,\left(n^6+56\,n^4+784\,n^2+2304\right)}+\frac{6\,x\,\left(n^4+40\,n^2+2\right.}{a^6\,c^4\,\left(n^6+56\,n^4+784\,n\right.}\right.}{\frac{1}{a^6}+x^6+\frac{3\,x^4}{a^2}+\frac{3\,x^2}{a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (exp(n*atan(a*x))*((720*x^5)/(a^2*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (5
44*n^2 + 50*n^4 + n^6 + 720)/(a^7*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304)) +
(720*x^6)/(a*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304)) + (6*x*(40*n^2 + n^4 +
264))/(a^6*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (120*x^3*(n^2 + 16))/(a^4
*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (360*x^4*(n^2 + 6))/(a^3*c^4*n*(784
*n^2 + 56*n^4 + n^6 + 2304)) + (30*x^2*(28*n^2 + n^4 + 72))/(a^5*c^4*n*(784
*n^2 + 56*n^4 + n^6 + 2304))))/(1/a^6 + x^6 + (3*x^4)/a^2 + (3*x^2)/a^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.348 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

```
[Out] -2^(5/2-1/2*I*n)*c*(1-I*a*x)^(5/2+1/2*I*n)*hypergeom([5/2+1/2*I*n, -3/2+1/2
*I*n],[7/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(5*I-n)/(a^2*x^2+1
)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5076, 5073, 69}

$$-\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(a*(5*I
 - n)*Sqrt[1 + a^2*x^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\tan^{-1}(ax)}\left(1+a^2x^2\right)^{3/2}dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}dx}{\sqrt{1+a^2x^2}}$$
$$=-\frac{2^{\frac{5}{2}-\frac{in}{2}}c(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\ {}_2F_1\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in);\frac{1}{2}(7+in);\frac{1}{2}(1-\right.}{a(5i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.12, size = 118, normalized size = 0.98

$$\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{5}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+5),\frac{1}{2}i(n+3i);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{a(n-5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^(5/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(5 + I*n)/2, (I/2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2])/(a*(
-5*I + n)*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2 + c\right)^{\frac{3}{2}} e^{(n \arctan(ax))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atan}(a x)} \left(c a^2 x^2 + c\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(a^2x^2 + 1\right)\right)^{\frac{3}{2}} e^{n \operatorname{atan}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(n*atan(a*x)), x)
```

3.349 $\int e^{n\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

```
[Out] -2^(3/2-1/2*I*n)*(1-I*a*x)^(3/2+1/2*I*n)*hypergeom([3/2+1/2*I*n, -1/2+1/2*I
*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(3*I-n)/(a^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(3/2 - (I/2)*n)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(a*(3*I -
 n)*Sqrt[1 + a^2*x^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{n\tan^{-1}(ax)}\sqrt{c+a^2cx^2}\,dx &= \frac{\sqrt{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}\\
&= -\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in);\frac{1}{2}(5+in);\frac{1}{2}(1-iax\right.}{a(3i-n)\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 117, normalized size = 0.98

$$\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{3}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+3),\frac{1}{2}i(n+i);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{a(n-3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(3/2 - (I/2)*n)*(1 - I*a*x)^(3/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(3 + I*n)/2, (I/2)*(I + n), (5 + I*n)/2, (1 - I*a*x)/2])/(a*(-3*I
 + n)*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\sqrt{a^2c\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

3.350 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([1/2+1/2*I*n, 1/2+1/2*I*
n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(I-n)/(a^2*c*x^2+c)^(1/
2)
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 0.98

$$\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{2},\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{1}{2}-\frac{iax}{2}\right)}{a(n-i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.351 $\int e^{n\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=283

$$\frac{cx\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2\sqrt{a^2x^2+1}}+\frac{c2^{\frac{3}{2}-\frac{in}{2}}\left(5-n^2\right)\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+\right.}{15a^3(-n+5i)\sqrt{a^2x^2+1}}$$

```
[Out] -1/30*c*n*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1/2*I*n)*(a^2*c*x^2+c)^(1/
2)/a^3/(a^2*x^2+1)^(1/2)+1/6*c*x*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1/2
*I*n)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/15*2^(3/2-1/2*I*n)*c*(-n^
2+5)*(1-I*a*x)^(5/2+1/2*I*n)*hypergeom([5/2+1/2*I*n, -3/2+1/2*I*n],[7/2+1/2
*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a^3/(5*I-n)/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 80, 69}

$$\frac{c2^{\frac{3}{2}-\frac{in}{2}}\left(5-n^2\right)\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(5+in)}\,{}_2F_1\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5);\frac{1}{2}(in+7);\frac{1}{2}(1-iax)\right)}{15a^3(-n+5i)\sqrt{a^2x^2+1}}-\frac{cn\sqrt{a^2cx^2+c}\,(1-\ldots}{30a^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -(c*n*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n)/2)*Sqrt[c + a^2*c*x^
2])/(30*a^3*Sqrt[1 + a^2*x^2]) + (c*x*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)
^((5 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(6*a^2*Sqrt[1 + a^2*x^2]) + (2^(3/2 - (
I/2)*n)*c*(5 - n^2)*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeom
etric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(15*a^3*(5
*I - n)*Sqrt[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\tan^{-1}(ax)}x^2\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\left(c\sqrt{c+a^2cx^2}\right)\int x^2(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}}+\frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(}{6a^2\sqrt{1+}}$$
$$=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-}}{6a^2\sqrt{1+a^2x^2}}$$
$$=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-}}{6a^2\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.27, size = 217, normalized size = 0.77

$$\frac{c2^{-1-\frac{in}{2}}(ax+i)^2\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(n-5i)\sqrt{1+iax}\,(ax-i)^2(5ax-n)-4\sqrt{2}\left(n^2-5\right)(1+}{15a^3(n-5i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(-1 - (I/2)*n)*c*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)^2*Sqrt[c + a^2*c*
x^2]*(2^((I/2)*n)*(-5*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)^2*(-n + 5*a*x) - 4*
Sqrt[2]*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(5 + I*n)/2, (I/
2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2]))/(15*a^3*(-5*I + n)*(1 + I*a*x)^
((I/2)*n)*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^4+cx^2\right)\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{n \arctan(ax)} x^2 \left(a^2 c\, x^2 + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^2 e^{(n \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, e^{n \operatorname{atan}(a\, x)} \left(c\, a^2\, x^2 + c\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

3.352 $\int e^{n\tan^{-1}(ax)}x^2\sqrt{c+a^2cx^2}\,dx$

Optimal. Leaf size=280

$$\frac{x\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2\sqrt{a^2x^2+1}}+\frac{2^{-\frac{1}{2}-\frac{in}{2}}\left(3-n^2\right)\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}\right.}{3a^3(-n+3i)\sqrt{a^2x^2+1}}$$

```
[Out] -1/12*n*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I*n)*(a^2*c*x^2+c)^(1/2)
/a^3/(a^2*x^2+1)^(1/2)+1/4*x*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I*n
)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/3*2^(-1/2-1/2*I*n)*(-n^2+3)*(
1-I*a*x)^(3/2+1/2*I*n)*hypergeom([3/2+1/2*I*n, -1/2+1/2*I*n],[5/2+1/2*I*n],
1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a^3/(3*I-n)/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 80, 69}

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}\left(3-n^2\right)\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^3(-n+3i)\sqrt{a^2x^2+1}}-\frac{n\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}}}{12a^3\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -(n*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/2)*Sqrt[c + a^2*c*x^2]
)/(12*a^3*Sqrt[1 + a^2*x^2]) + (x*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3
 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(4*a^2*Sqrt[1 + a^2*x^2]) + (2^(-1/2 - (I/2
)*n)*(3 - n^2)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeometric
2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(3*a^3*(3*I - n
)*Sqrt[1 + a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tan^{-1}(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\frac{\sqrt{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}x^2\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{\sqrt{c+a^2cx^2}\int x^2(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$=\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}}+\frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+ i}{4a^2\sqrt{1+a^2x}}$$
$$=-\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}}+\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in}}{4a^2\sqrt{1+a^2x^2}}$$
$$=-\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}}+\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in}}{4a^2\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.22, size = 214, normalized size = 0.76

$$\frac{2^{-2-\frac{in}{2}}(ax+i)\sqrt{a^2cx^2+c}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(n-3i)\sqrt{1+iax}\,(ax-i)(3ax-n)-2i\sqrt{2}\left(n^2-3\right)(1+}{3a^3(n-3i)\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)*Sqrt[c + a^2*c*x^2]
*(2^((I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)*(-n + 3*a*x) - (2*I)*Sq
rt[2]*(-3 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(3 + I*n)/2, (I/2)
*(I + n), (5 + I*n)/2, (1 - I*a*x)/2]))/(3*a^3*(-3*I + n)*(1 + I*a*x)^((I/2
)*n)*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,x^2e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int e^{n \arctan(ax)} x^2 \sqrt{a^2 c\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 c x^2 + c}\, x^2 e^{(n \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, e^{n \operatorname{atan}(a\, x)} \sqrt{c\, a^2\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c \left(a^2 x^2 + 1\right)}\, e^{n \operatorname{atan}{(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

3.353 $\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=322

$$\frac{x^2\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2\sqrt{a^2cx^2+c}}+\frac{2^{-\frac{1}{2}-\frac{in}{2}}n\left(5-n^2\right)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+\right.}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{a^2cx^2+c}}$$

```
[Out] 1/3*x^2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a
^2/(a^2*c*x^2+c)^(1/2)-1/6*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*
(4-I*n-n^2+a*(1+I*n)*n*x)*(a^2*x^2+1)^(1/2)/a^4/(1+I*n)/(a^2*c*x^2+c)^(1/2)
+1/3*2^(-1/2-1/2*I*n)*n*(-n^2+5)*(1-I*a*x)^(3/2+1/2*I*n)*hypergeom([3/2+1/2
*I*n, 1/2+1/2*I*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a^4/(4*n-
I*(-n^2+3))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 100, 146, 69}

$$\frac{2^{-\frac{1}{2}-\frac{in}{2}}n\left(5-n^2\right)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(3+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+3);\frac{1}{2}(in+5);\frac{1}{2}(1-iax)\right)}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,(1-iax}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])
/(3*a^2*Sqrt[c + a^2*c*x^2]) - ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 -
 I*n)/2)*(4 - I*n - n^2 + a*(1 + I*n)*n*x)*Sqrt[1 + a^2*x^2])/(6*a^4*(1 + I
*n)*Sqrt[c + a^2*c*x^2]) + (2^(-1/2 - (I/2)*n)*n*(5 - n^2)*(1 - I*a*x)^((3
+ I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(1 + I*n)/2, (3 + I*n)/2, (5
+ I*n)/2, (1 - I*a*x)/2])/(3*a^4*(4*n - I*(3 - n^2))*Sqrt[c + a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
```

```
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x^3(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-2}{3a^2\sqrt{c+a^2cx^2}}$$
$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a\right.}{6a^4(1+in)\sqrt{c+a^2cx^2}}$$
$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a\right.}{6a^4(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.32, size = 248, normalized size = 0.77

$$\frac{2^{-\frac{3}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{1}{2}+\frac{in}{2}}(n-3i)\sqrt{1+iax}\,\left(n\left(2a^2x^2+iax+1\right)-2i\left(a^2x^2-2\right)-\left(n^2(ax+\right.\right.\right.}{3a^4\left(n^2-4in-3\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-3/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2 +
 (I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-(n^2*(I + a*x)) - (2*I)*(-2 + a^2*x^
2) + n*(1 + I*a*x + 2*a^2*x^2)) + 2*n*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*(I +
 a*x)*Hypergeometric2F1[1/2 + (I/2)*n, 3/2 + (I/2)*n, 5/2 + (I/2)*n, 1/2 -
(I/2)*a*x]))/(3*a^4*(-3 - (4*I)*n + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c + a^2
*c*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, e^{n\, \mathrm{atan}(a\, x)}}{\sqrt{c\, a^2\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{atan}{(ax)}}}{\sqrt{c \left(a^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.354 $\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=291

$$\frac{x\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2\sqrt{a^2cx^2+c}}-\frac{i2^{\frac{1}{2}-\frac{in}{2}}\left(1-n^2\right)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(i\right.}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

```
[Out] -1/2*(1+I*n)*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1
/2)/a^3/(I+n)/(a^2*c*x^2+c)^(1/2)+1/2*x*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(
1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*2^(1/2-1/2*I*n)*(-
n^2+1)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([-1/2+1/2*I*n, 1/2+1/2*I*n],[3/2+1
/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a^3/(n^2+1)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5085, 5082, 90, 79, 69}

$$-\frac{i2^{\frac{1}{2}-\frac{in}{2}}\left(1-n^2\right)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}+\frac{x\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}}}{2a^2\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((1 + I*n)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^
2*x^2])/(2*a^3*(I + n)*Sqrt[c + a^2*c*x^2]) + (x*(1 - I*a*x)^((1 + I*n)/2)*
(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2*a^2*Sqrt[c + a^2*c*x^2]) -
(I*2^(1/2 - (I/2)*n)*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(
a^3*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^2(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-1}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+}}{2a^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+}}{2a^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.18, size = 206, normalized size = 0.71

$$\frac{2^{-1-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2i\sqrt{2}\left(n^2-1\right)(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}i(n+i);\frac{1}{2}(in+3);\frac{1}{2}(1-i\right.\right.}{a^3\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-1 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*n
)*(-I + n)*Sqrt[1 + I*a*x]*(-1 + I*a*x + n*(-I + a*x)) + (2*I)*Sqrt[2]*(-1
+ n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 + I*n)/2, (I/2)*(I + n),
(3 + I*n)/2, (1 - I*a*x)/2]))/(a^3*(1 + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c +
 a^2*c*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n \arctan(ax)} x^2}{\sqrt{a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\, e^{n \operatorname{atan}(a\, x)}}{\sqrt{c\, a^2\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n \operatorname{atan}{(ax)}}}{\sqrt{c \left(a^2 x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.355 $\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)\sqrt{a^2cx^2+c}}-\frac{i2^{\frac{3}{2}-\frac{in}{2}}n\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3)\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

```
[Out] (1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(1-I*
n)/(a^2*c*x^2+c)^(1/2)-I*2^(3/2-1/2*I*n)*n*(1-I*a*x)^(1/2+1/2*I*n)*hypergeo
m([-1/2+1/2*I*n, 1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2
)/a^2/(n^2+1)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5085, 5082, 79, 69}

$$\frac{\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)\sqrt{a^2cx^2+c}}-\frac{i2^{\frac{3}{2}-\frac{in}{2}}n\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1);\frac{1}{2}(in+3)\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(a^
2*(1 - I*n)*Sqrt[c + a^2*c*x^2]) - (I*2^(3/2 - (I/2)*n)*n*(1 - I*a*x)^((1 +
 I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3
+ I*n)/2, (1 - I*a*x)/2])/(a^2*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
```

```
m, n, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{\left(n\sqrt{1+a^2x^2}\right)\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}i(i+n)}\,dx}{a(1-in)\sqrt{c+a^2cx^2}}$$
$$= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(\frac{1}{2}(-1+in)\right.}{a^2\left(1+n^2\right)\sqrt{c+a^2}}$$

Mathematica [A] time = 0.15, size = 187, normalized size = 0.93

$$\frac{i2^{-\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(2^{\frac{1}{2}+\frac{in}{2}}(n-i)\sqrt{1+iax}-4n(1+iax)^{\frac{in}{2}}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}i(n+i);\frac{1}{2}(in+\right.\right.}{a^2\left(n^2+1\right)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*2^(-1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2
 + (I/2)*n)*(-I + n)*Sqrt[1 + I*a*x] - 4*n*(1 + I*a*x)^((I/2)*n)*Hypergeome
tric2F1[(1 + I*n)/2, (I/2)*(I + n), (3 + I*n)/2, (1 - I*a*x)/2]))/(a^2*(1 +
 n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,e^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.356 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[Out] -2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([1/2+1/2*I*n, 1/2+1/2*I*
n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(I-n)/(a^2*c*x^2+c)^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}\,{}_2F_1\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\ {}_2F_1\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 117, normalized size = 0.98

$$\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}\ {}_2F_1\left(\frac{in}{2}+\frac{1}{2},\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{1}{2}-\frac{iax}{2}\right)}{a(n-i)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.357 $\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

```
[Out] -2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*hypergeom([1, 1/2+1/2*I
*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(1+I*n)/(a^2*c*x^2
+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 131}

$$-\frac{2\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((
1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,{}_2F_1\left(1,\frac{1}{2}(1+in);\frac{1}{2}(3+in);\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 0.99

$$\frac{2\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)}{(-1-in)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^
2]*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)])
/((-1 - I*n)*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}}{a^2cx^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.358 $\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=196

$$-\frac{2an\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+$$

```
[Out] -(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x/(a^2*c
*x^2+c)^(1/2)-2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*hyperg
eom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(
1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5085, 5082, 96, 131}

$$-\frac{2an\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -(((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(
x*Sqrt[c + a^2*c*x^2])) - (2*a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1
 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2,
 (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
```

```
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x^2\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^2}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} + \frac{\left(an\sqrt{1+a^2x^2}\right)\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,d}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} - \frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a}}{(1+in)\sqrt{c}}$$

Mathematica [A] time = 0.07, size = 142, normalized size = 0.72

$$\frac{\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(2anx\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)-(n-i)(ax-i)\right)}{(-1-in)x\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^2]
*(-((-I + n)*(-I + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2
+ (I/2)*n, (I + a*x)/(I - a*x)]))/((-1 - I*n)*x*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}}{a^2cx^4+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atan}{(ax)}}}{x^2 \sqrt{c \left(a^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.359 $\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=281

$$\frac{a^2(1-n^2)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{an\sqrt{a^2x^2+1}\,(1-iax)}{2x\sqrt{a^2c}}$$

```
[Out] -1/2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x^2/
(a^2*c*x^2+c)^(1/2)-1/2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)
*(a^2*x^2+1)^(1/2)/x/(a^2*c*x^2+c)^(1/2)+a^2*(-n^2+1)*(1-I*a*x)^(1/2+1/2*I*
n)*(1+I*a*x)^(-1/2-1/2*I*n)*hypergeom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a
*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5085, 5082, 129, 151, 12, 131}

$$\frac{a^2(1-n^2)\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,{}_2F_1\left(1,\frac{1}{2}(in+1);\frac{1}{2}(in+3);\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}-\frac{an\sqrt{a^2x^2+1}\,(1-iax)}{2x\sqrt{a^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2
*x^2*Sqrt[c + a^2*c*x^2]) - (a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1
- I*n)/2)*Sqrt[1 + a^2*x^2])/(2*x*Sqrt[c + a^2*c*x^2]) + (a^2*(1 - n^2)*(1
- I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hyperge
ometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n
)*Sqrt[c + a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{x^3\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^3}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(-an+a^2x\right)}{x^2}\,d}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} -$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} -$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} +$$

Mathematica [A] time = 0.08, size = 159, normalized size = 0.57

$$\frac{i\sqrt{a^2x^2+1}\,(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\left(2a^2\left(n^2-1\right)x^2\,{}_2F_1\left(1,\frac{in}{2}+\frac{1}{2};\frac{in}{2}+\frac{3}{2};\frac{ax+i}{i-ax}\right)-(n-i)(ax-i)(anx+1)\right)}{2(n-i)x^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((I/2)*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^
2*x^2]*(-((-I + n)*(-I + a*x)*(1 + a*n*x)) + 2*a^2*(-1 + n^2)*x^2*Hypergeom
etric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)]))/((-I + n)*
x^2*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,e^{(n\arctan(ax))}}{a^2cx^5+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x^3\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.360 $\int e^{n\tan^{-1}(ax)}\sqrt[3]{c+a^2cx^2}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}\,(1-iax)^{\frac{1}{6}(8+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+14);\frac{1}{2}(1-iax)\right)}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

```
[Out] -3*2^(4/3-1/2*I*n)*(1-I*a*x)^(4/3+1/2*I*n)*(a^2*c*x^2+c)^(1/3)*hypergeom([4
/3+1/2*I*n, -1/3+1/2*I*n],[7/3+1/2*I*n],1/2-1/2*I*a*x)/a/(8*I-3*n)/(a^2*x^2
+1)^(1/3)
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}\,(1-iax)^{\frac{1}{6}(8+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+14);\frac{1}{2}(1-iax)\right)}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^((8 + (3*I)*n)/6)*(c + a^2*c*x^2)^(1/3)*H
ypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (14 + (3*I)*n)/6, (1 -
I*a*x)/2])/(a*(8*I - 3*n)*(1 + a^2*x^2)^(1/3))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\sqrt[3]{c+a^2cx^2}\,dx&=\frac{\sqrt[3]{c+a^2cx^2}\int e^{n\tan^{-1}(ax)}\sqrt[3]{1+a^2x^2}\,dx}{\sqrt[3]{1+a^2x^2}}\\&=\frac{\sqrt[3]{c+a^2cx^2}\int(1-iax)^{\frac{1}{3}+\frac{in}{2}}(1+iax)^{\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{1+a^2x^2}}\\&=-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(8+3in)}\sqrt[3]{c+a^2cx^2}\,{}_2F_1\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in);\frac{1}{6}(14+3in);\frac{1}{2}\right.}{a(8i-3n)\sqrt[3]{1+a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 120, normalized size = 1.00

$$\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}\,(1-iax)^{\frac{4}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-\frac{1}{3},\frac{in}{2}+\frac{4}{3};\frac{in}{2}+\frac{7}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-8i)\sqrt[3]{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^(4/3 + (I/2)*n)*(c + a^2*c*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 7/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*(-8*I + 3*n)*(1 + a^2*x^2)^(1/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3),x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c\left(a^2x^2+1\right)}\,e^{n\,\mathrm{atan}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/3),x)
[Out] Integral((c*(a**2*x**2 + 1))**(1/3)*exp(n*atan(a*x)), x)
```

3.361 $\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{\frac{1}{6}(4+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+10);\frac{1}{2}(1-iax)\right)}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

```
[Out] -3*2^(2/3-1/2*I*n)*(1-I*a*x)^(2/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([2/3
+1/2*I*n, 1/3+1/2*I*n],[5/3+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-3*n)/(a^2*c*x^2+
c)^(1/3)
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{\frac{1}{6}(4+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+10);\frac{1}{2}(1-iax)\right)}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^((4 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (10 + (3*I)*n)/6, (1 - I*a
*x)/2])/(a*(4*I - 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\sqrt[3]{1+a^2x^2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{1}{3}+\frac{in}{2}}(1+iax)^{-\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$
$$= -\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(4+3in)}\sqrt[3]{1+a^2x^2}\,{}_2F_1\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in);\frac{1}{6}(10+3in);\frac{1}{2}(1-iax)\right)}{a(4i-3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 1.00

$$\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{\frac{2}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{3},\frac{in}{2}+\frac{2}{3};\frac{in}{2}+\frac{5}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-4i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^(2/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 5/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-4*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{\left(a^2c\,x^2+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n \arctan(ax))}}{\left(a^2cx^2 + c\right)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atan}(a x)}}{\left(c a^2 x^2 + c\right)^{1/3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt[3]{c\left(a^2x^2 + 1\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/3),x)
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(1/3), x)
```

3.362 $\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=120

$$-\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}\left(a^2x^2+1\right)^{2/3}(1-iax)^{\frac{1}{6}(2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+8);\frac{1}{2}(1-iax)\right)}{a(-3n+2i)\left(a^2cx^2+c\right)^{2/3}}$$

```
[Out] -3*2^(1/3-1/2*I*n)*(1-I*a*x)^(1/3+1/2*I*n)*(a^2*x^2+1)^(2/3)*hypergeom([2/3
+1/2*I*n, 1/3+1/2*I*n],[4/3+1/2*I*n],1/2-1/2*I*a*x)/a/(2*I-3*n)/(a^2*c*x^2+
c)^(2/3)
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5076, 5073, 69}

$$-\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}\left(a^2x^2+1\right)^{2/3}(1-iax)^{\frac{1}{6}(2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4);\frac{1}{6}(3in+8);\frac{1}{2}(1-iax)\right)}{a(-3n+2i)\left(a^2cx^2+c\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (-3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^((2 + (3*I)*n)/6)*(1 + a^2*x^2)^(2/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (8 + (3*I)*n)/6, (1 - I*a*
x)/2])/(a*(2*I - 3*n)*(c + a^2*c*x^2)^(2/3))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \frac{(1+a^2x^2)^{2/3}\int \frac{e^{n\tan^{-1}(ax)}}{(1+a^2x^2)^{2/3}}\,dx}{(c+a^2cx^2)^{2/3}}$$

$$= \frac{(1+a^2x^2)^{2/3}\int (1-iax)^{-\frac{2}{3}+\frac{in}{2}}(1+iax)^{-\frac{2}{3}-\frac{in}{2}}\,dx}{(c+a^2cx^2)^{2/3}}$$

$$= -\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(2+3in)}(1+a^2x^2)^{2/3}\,{}_2F_1\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in);\frac{1}{6}(8+3in);\frac{1}{2}(1-iax)\right)}{a(2i-3n)(c+a^2cx^2)^{2/3}}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 1.00

$$\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}(a^2x^2+1)^{2/3}(1-iax)^{\frac{1}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}+\frac{1}{3},\frac{in}{2}+\frac{2}{3};\frac{in}{2}+\frac{4}{3};\frac{1}{2}-\frac{iax}{2}\right)}{a(3n-2i)(a^2cx^2+c)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^(1/3 + (I/2)*n)*(1 + a^2*x^2)^(2/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 4/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-2*I + 3*n)*(c + a^2*c*x^2)^(2/3))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(n \arctan\left(ax\right)\right)}}{\left(a^{2}cx^{2}+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3),x)
```

```
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atan}{\left(ax \right)}}}{\left(c \left(a^{2}x^{2}+1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(2/3),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(2/3), x)
```

3.363 $\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=123

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{\frac{1}{6}(-2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+4);\frac{1}{2}(1-iax)\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

```
[Out] 3*2^(-1/3-1/2*I*n)*(1-I*a*x)^(-1/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([4/
3+1/2*I*n, -1/3+1/2*I*n],[2/3+1/2*I*n],1/2-1/2*I*a*x)/a/c/(2*I+3*n)/(a^2*c*
x^2+c)^(1/3)
```

Rubi [A] time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5076, 5073, 69}

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{\frac{1}{6}(-2+3in)}\,{}_2F_1\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8);\frac{1}{6}(3in+4);\frac{1}{2}(1-iax)\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^((-2 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hy
pergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (4 + (3*I)*n)/6, (1 - I*
a*x)/2])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}}{\left(c+a^2cx^2\right)^{4/3}}\,dx = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}}{\left(1+a^2x^2\right)^{4/3}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{4}{3}+\frac{in}{2}}(1+iax)^{-\frac{4}{3}-\frac{in}{2}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$
$$= \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(-2+3in)}\sqrt[3]{1+a^2x^2}\;{}_2F_1\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in);\frac{1}{6}(4+3in);\frac{1}{2}(1-iax\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 123, normalized size = 1.00

$$\frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}\,(1-iax)^{-\frac{1}{3}+\frac{in}{2}}\,{}_2F_1\left(\frac{in}{2}-\frac{1}{3},\frac{in}{2}+\frac{4}{3};\frac{in}{2}+\frac{2}{3};\frac{1}{2}-\frac{iax}{2}\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^(-1/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 2/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2cx^2+c\right)^{\frac{2}{3}}e^{(n\arctan(ax))}}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(2/3)*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x
^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}}{\left(a^2c\,x^2+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

```
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{\left(c\,a^2\,x^2+c\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3),x)
```

```
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(4/3),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(4/3), x)
```

3.364 $\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx$

Optimal. Leaf size=49

$$\frac{cx^{m+1}F_1\left(m+1;-\frac{in}{2}-1,\frac{in}{2}-1;m+2;iax,-iax\right)}{m+1}$$

```
[Out] c*x^(1+m)*AppellF1(1+m,-1+1/2*I*n,-1-1/2*I*n,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5082, 133}

$$\frac{cx^{m+1}F_1\left(m+1;-\frac{in}{2}-1,\frac{in}{2}-1;m+2;iax,-iax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, -1 - (I/2)*n, -1 + (I/2)*n, 2 + m, I*a*x, (-I)
*a*x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx &= c\int x^m(1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx\\ &= \frac{cx^{1+m}F_1\left(1+m;-1-\frac{in}{2},-1+\frac{in}{2};2+m;iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.52, size = 0, normalized size = 0.00

$$\int e^{n\tan^{-1}(ax)}x^m\left(c+a^2cx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

```
[Out] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2), x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)x^me^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int e^{n \arctan(ax)} x^m \left(a^2 c\, x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2 + c\right)x^m e^{(n \arctan(ax))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\, e^{n \operatorname{atan}(a\,x)} \left(c\,a^2\,x^2 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2),x)
```

```
[Out] int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int x^m e^{n \operatorname{atan}{(ax)}}\, dx + \int a^2x^2x^m e^{n \operatorname{atan}{(ax)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m*(a**2*c*x**2+c),x)
```

```
[Out] c*(Integral(x**m*exp(n*atan(a*x)), x) + Integral(a**2*x**2*x**m*exp(n*atan(
a*x)), x))
```

3.365 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{c+a^2cx^2}\,dx$

Optimal. Leaf size=51

$$\frac{x^{m+1}F_1\left(m+1;1-\frac{in}{2},\frac{in}{2}+1;m+2;iax,-iax\right)}{c(m+1)}$$

```
[Out] x^(1+m)*AppellF1(1+m,1+1/2*I*n,1-1/2*I*n,2+m,-I*a*x,I*a*x)/c/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1}F_1\left(m+1;1-\frac{in}{2},\frac{in}{2}+1;m+2;iax,-iax\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1 - (I/2)*n, 1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c*(1 + m))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(ax)}x^m}{c+a^2cx^2}\,dx &= \frac{\int x^m(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}\\ &= \frac{x^{1+m}F_1\left(1+m;1-\frac{in}{2},1+\frac{in}{2};2+m;iax,-iax\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.17, size = 96, normalized size = 1.88

$$\frac{x^m\left(1-e^{2i\tan^{-1}(ax)}\right)^{-m}\left(1+e^{2i\tan^{-1}(ax)}\right)^m e^{n\tan^{-1}(ax)}F_1\left(-\frac{in}{2};m,-m;1-\frac{in}{2};-e^{2i\tan^{-1}(ax)},e^{2i\tan^{-1}(ax)}\right)}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

```
[Out] (E^(n*ArcTan[a*x])*(1 + E^((2*I)*ArcTan[a*x]))^m*x^m*AppellF1[(-1/2*I)*n, m
, -m, 1 - (I/2)*n, -E^((2*I)*ArcTan[a*x]), E^((2*I)*ArcTan[a*x])])/(a*c*(1
- E^((2*I)*ArcTan[a*x]))^m*n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m e^{(n\arctan(ax))}}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)} x^m}{a^2 c\, x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n\arctan(ax))}}{a^2cx^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\, e^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m e^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c),x)
[Out] Integral(x**m*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.366 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{x^{m+1}F_1\left(m+1;2-\frac{in}{2},\frac{in}{2}+2;m+2;iax,-iax\right)}{c^2(m+1)}$$

```
[Out] x^(1+m)*AppellF1(1+m,2+1/2*I*n,2-1/2*I*n,2+m,-I*a*x,I*a*x)/c^2/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1}F_1\left(m+1;2-\frac{in}{2},\frac{in}{2}+2;m+2;iax,-iax\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 2 - (I/2)*n, 2 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^2*(1 + m))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx &= \frac{\int x^m(1-iax)^{-2+\frac{in}{2}}(1+iax)^{-2-\frac{in}{2}}\,dx}{c^2}\\ &= \frac{x^{1+m}F_1\left(1+m;2-\frac{in}{2},2+\frac{in}{2};2+m;iax,-iax\right)}{c^2(1+m)}\end{aligned}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2, x]

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m e^{(n \arctan(ax))}}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] integral(x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{e^{n \arctan(ax)} x^m}{\left(a^2 c\, x^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)

[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2cx^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\, e^{n\, \mathrm{atan}(a\, x)}}{\left(c\, a^2\, x^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2,x)

[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m e^{n \operatorname{atan}{(ax)}}}{a^4x^4+2a^2x^2+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**2,x)

[Out] Integral(x**m*exp(n*atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2

3.367 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=51

$$\frac{x^{m+1}F_1\left(m+1;3-\frac{in}{2},\frac{in}{2}+3;m+2;iax,-iax\right)}{c^3(m+1)}$$

```
[Out] x^(1+m)*AppellF1(1+m,3+1/2*I*n,3-1/2*I*n,2+m,-I*a*x,I*a*x)/c^3/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5082, 133}

$$\frac{x^{m+1}F_1\left(m+1;3-\frac{in}{2},\frac{in}{2}+3;m+2;iax,-iax\right)}{c^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3 - (I/2)*n, 3 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^3*(1 + m))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\int x^m(1-iax)^{-3+\frac{in}{2}}(1+iax)^{-3-\frac{in}{2}}\,dx}{c^3}\\ &= \frac{x^{1+m}F_1\left(1+m;3-\frac{in}{2},3+\frac{in}{2};2+m;iax,-iax\right)}{c^3(1+m)}\end{aligned}$$

Mathematica [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

```
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m e^{(n\arctan(ax))}}{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(a^2c\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,e^{n\,\mathrm{atan}(a\,x)}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3,x)
```

```
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(1-in),\frac{1}{2}(in+1);m+2;iax,-iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/2+1/2*I*n,1/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(1-in),\frac{1}{2}(in+1);m+2;iax,-iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (1 - I*n)/2, (1 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/((1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1+a^2x^2}\,F_1\left(1+m;\frac{1}{2}(1-in),\frac{1}{2}(1+in);2+m;iax,-iax\right)}{(1+m)\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2], x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\arctan(ax)}x^m}{\sqrt{a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{n \operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**m*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.369 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(3-in),\frac{1}{2}(in+3);m+2;iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] x^(1+m)*AppellF1(1+m,3/2+1/2*I*n,3/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/c/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(3-in),\frac{1}{2}(in+3);m+2;iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (3 - I*n)/2, (3 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{3}{2}+\frac{in}{2}}(1+iax)^{-\frac{3}{2}-\frac{in}{2}}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{x^{1+m}\sqrt{1+a^2x^2}\,F_1\left(1+m;\frac{1}{2}(3-in),\frac{1}{2}(3+in);2+m;iax,-iax\right)}{c(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m e^{(n\arctan(ax))}}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\, \mathrm{e}^{n\, \mathrm{atan}(a\, x)}}{\left(c\, a^2\, x^2 + c\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{n \operatorname{atan}{(ax)}}}{\left(c \left(a^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**m*exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.370 $\int \frac{e^{n\tan^{-1}(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(5-in),\frac{1}{2}(in+5);m+2;iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

```
[Out] x^(1+m)*AppellF1(1+m,5/2+1/2*I*n,5/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/c^2/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5085, 5082, 133}

$$\frac{\sqrt{a^2x^2+1}\,x^{m+1}F_1\left(m+1;\frac{1}{2}(5-in),\frac{1}{2}(in+5);m+2;iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (5 - I*n)/2, (5 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c^2*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(1+a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{5}{2}+\frac{in}{2}}(1+iax)^{-\frac{5}{2}-\frac{in}{2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+a^2x^2}\,F_1\left(1+m;\frac{1}{2}(5-in),\frac{1}{2}(5+in);2+m;iax,-iax\right)}{c^2(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tan^{-1}(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2), x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,x^m e^{(n\arctan(ax))}}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m e^{n \operatorname{atan}(a x)}}{\left(c a^2 x^2 + c\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.371 $\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=115

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\,{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

```
[Out] 2^(1-1/2*I*n+p)*(1-I*a*x)^(1+1/2*I*n+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I*n-
p, 1+1/2*I*n+p],[2+1/2*I*n+p],1/2-1/2*I*a*x)/a/(n-2*I*(1+p))/((a^2*x^2+1)^p
)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5076, 5073, 69}

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\,{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{n\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{\frac{in}{2}+p}(1+iax)^{-\frac{in}{2}+p}\,dx\\ &= \frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,{}_2F_1\left(\frac{in}{2}-p,1+\frac{in}{2}+p;2+\frac{in}{2}+p\right.}{a(n-2i(1+p))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 1.00

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}{}_2F_1\left(\frac{in}{2}-p,\frac{in}{2}+p+1;\frac{in}{2}+p+2;\frac{1}{2}(1-iax)\right)}{a(n-2i(p+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+c\right)^pe^{(n\arctan(ax))},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^pe^{(n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

```
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a^2x^2+1\right)\right)^p e^{n\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(n*atan(a*x)), x)
```

3.372 $\int e^{-2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[Out] I*(1-I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5076, 5073, 32}

$$\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2ip\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{2p}\,dx\\ &= \frac{i(1-iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.74

$$\frac{(ax+i)\left(a^2cx^2+c\right)^p e^{-2ip\tan^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
[Out] ((I + a*x)*(c + a^2*c*x^2)^p)/(E^((2*I)*p*ArcTan[a*x])*(a + 2*a*p))
```

fricas [A] time = 0.63, size = 42, normalized size = 0.79

$$\frac{(ax+i)\left(a^2cx^2+c\right)^p\left(-\frac{ax+i}{ax-i}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="fricas")
[Out] (a*x + I)*(a^2*c*x^2 + c)^p*(-(a*x + I)/(a*x - I))^p/(2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.10, size = 41, normalized size = 0.77

$$\frac{(ax+i)\left(a^2c\,x^2+c\right)^p\,\mathrm{e}^{-2ip\arctan(ax)}}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x)
[Out] (I+a*x)/a/(1+2*p)*(a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x))
```

maxima [A] time = 0.36, size = 76, normalized size = 1.43

$$\frac{(ac^px+i\,c^p)\left(a^2x^2+1\right)^p\cos\left(2\,p\arctan\left(ax\right)\right)-(i\,ac^px-c^p)\left(a^2x^2+1\right)^p\sin\left(2\,p\arctan\left(ax\right)\right)}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="maxima")
[Out] ((a*c^p*x + I*c^p)*(a^2*x^2 + 1)^p*cos(2*p*arctan(a*x)) - (I*a*c^p*x - c^p)
*(a^2*x^2 + 1)^p*sin(2*p*arctan(a*x)))/(2*a*p + a)
```

mupad [B] time = 0.66, size = 54, normalized size = 1.02

$$\left(\frac{x\,\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}+\frac{\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\,{\left(c\,a^2\,x^2+c\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
[Out] ((x*exp(-p*atan(a*x)*2i))/(2*p + 1) + (exp(-p*atan(a*x)*2i)*1i)/(a*(2*p + 1
)))*(c + a^2*c*x^2)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt{c}} & \text{for}\ a = 0 \wedge p = -\frac{1}{2} \\ c^p x & \text{for}\ a = 0 \\ \int \frac{e^{i \operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\, dx & \text{for}\ p = -\frac{1}{2} \\ \frac{iax(a^2cx^2+c)^p}{2iape^{2ip\operatorname{atan}(ax)}+iae^{2ip\operatorname{atan}(ax)}} - \frac{(a^2cx^2+c)^p}{2iape^{2ip\operatorname{atan}(ax)}+iae^{2ip\operatorname{atan}(ax)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*I*p*atan(a*x)),x)
```

```
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (I*a*x*(a**2*
c*x**2 + c)**p/(2*I*a*p*exp(2*I*p*atan(a*x)) + I*a*exp(2*I*p*atan(a*x))) -
(a**2*c*x**2 + c)**p/(2*I*a*p*exp(2*I*p*atan(a*x)) + I*a*exp(2*I*p*atan(a*x
))), True))
```

3.373 $\int e^{2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[Out] -I*(1+I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5076, 5073, 32}

$$-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] ((-I)*(1 + I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5073

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*n)/2), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5076

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPart[p], Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2ip\tan^{-1}(ax)}\left(c+a^2cx^2\right)^p\,dx &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2ip\tan^{-1}(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1+iax)^{2p}\,dx\\ &= -\frac{i(1+iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.74

$$\frac{(ax-i)\left(a^2cx^2+c\right)^p e^{2ip\tan^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
[Out] (E^((2*I)*p*ArcTan[a*x])*(-I + a*x)*(c + a^2*c*x^2)^p)/(a + 2*a*p)
```

fricas [A] time = 0.44, size = 44, normalized size = 0.83

$$\frac{(ax-i)\left(a^2cx^2+c\right)^p}{\left(2\,ap+a\right)\left(-\frac{ax+i}{ax-i}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] (a*x - I)*(a^2*c*x^2 + c)^p/((2*a*p + a)*(-(a*x + I)/(a*x - I))^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.09, size = 41, normalized size = 0.77

$$-\frac{(-ax+i)\,\mathrm{e}^{2ip\arctan(ax)}\left(a^2c\,x^2+c\right)^p}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x)
[Out] -(-a*x+I)/a/(1+2*p)*exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2+c\right)^pe^{\left(2i\,p\arctan(ax)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*I*p*arctan(a*x)), x)
```

mupad [B] time = 0.59, size = 54, normalized size = 1.02

$$\left(\frac{x\,\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}-\frac{\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\left(c\,a^2\,x^2+c\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
[Out] ((x*exp(p*atan(a*x)*2i))/(2*p + 1) - (exp(p*atan(a*x)*2i)*1i)/(a*(2*p + 1))
)*(c + a^2*c*x^2)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt{c}} & \text{for } a = 0 \wedge p = -\frac{1}{2} \\ c^p x & \text{for } a = 0 \\ \int \frac{e^{-i \operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\, dx & \text{for } p = -\frac{1}{2} \\ \frac{ax(a^2cx^2+c)^p e^{2ip \operatorname{atan}(ax)}}{2ap+a} - \frac{i(a^2cx^2+c)^p e^{2ip \operatorname{atan}(ax)}}{2ap+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*I*p*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

```
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(-I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (a*x*(a**2*c
*x**2 + c)**p*exp(2*I*p*atan(a*x))/(2*a*p + a) - I*(a**2*c*x**2 + c)**p*exp
(2*I*p*atan(a*x))/(2*a*p + a), True))
```

3.374 $\int e^{in\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx$

Optimal. Leaf size=60

$$\frac{i(1-ianx)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

```
[Out] I*exp(I*n*arctan(a*x))*(1-I*a*n*x)/a^3/c/n/(-n^2+1)/((a^2*c*x^2+c)^(1/2*n^2
))
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {5079}

$$\frac{i(1-ianx)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] (I*E^(I*n*ArcTan[a*x])*(1 - I*a*n*x))/(a^3*c*n*(1 - n^2)*(c + a^2*c*x^2)^(n
^2/2))
```

Rule 5079

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> -Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]))/(a*d*n*(n^
2 + 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && EqQ[n^2 - 2*(p +
1), 0] && !IntegerQ[I*n]
```

Rubi steps

$$\int e^{in\tan^{-1}(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{ie^{in\tan^{-1}(ax)}(1-ianx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.92

$$-\frac{(anx+i)\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}e^{in\tan^{-1}(ax)}}{a^3cn\left(n^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] -((E^(I*n*ArcTan[a*x])*(I + a*n*x))/(a^3*c*n*(-1 + n^2)*(c + a^2*c*x^2)^(n^
2/2)))
```

fricas [A] time = 0.47, size = 78, normalized size = 1.30

$$-\frac{\left(a^3nx^3+ia^2x^2+anx+i\right)\left(a^2cx^2+c\right)^{-\frac{1}{2}n^2-1}}{\left(a^3n^3-a^3n\right)\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"fricas")
```

```
[Out] -(a^3*n*x^3 + I*a^2*x^2 + a*n*x + I)*(a^2*c*x^2 + c)^(-1/2*n^2 - 1)/((a^3*n
^3 - a^3*n)*(-(a*x + I)/(a*x - I))^(1/2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.14, size = 62, normalized size = 1.03

$$\frac{(-ax+i)\,(ax+i)\,(nax+i)\,\mathrm{e}^{in\arctan(ax)}\left(a^2c\,x^2+c\right)^{-1-\frac{n^2}{2}}}{a^3n\left(n^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x)
```

```
[Out] (-a*x+I)*(I+a*x)*(n*a*x+I)*exp(I*n*arctan(a*x))*(a^2*c*x^2+c)^(-1-1/2*n^2)/
a^3/n/(n^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2+c\right)^{-\frac{1}{2}\,n^2-1}x^2e^{(i\,n\arctan(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*e^(I*n*arctan(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{\left(c\,a^2\,x^2+c\right)^{\frac{n^2}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1),x)
```

```
[Out] int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(I*n*atan(a*x))*x**2*(a**2*c*x**2+c)**(-1-1/2*n**2),x)
```

```
[Out] Timed out
```

3.375 $\int \frac{e^{6 i \tan^{-1}(ax)} x^2}{\left(c+a^2 c x^2\right)^{19}} \, dx$

Optimal. Leaf size=38

$$-\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

```
[Out] 1/210*(-I-6*a*x)/a^3/c^19/(1-I*a*x)^21/(1+I*a*x)^15
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

```
[Out] -(I + 6*a*x)/(210*a^3*c^19*(1 - I*a*x)^21*(1 + I*a*x)^15)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{6i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{19}}\,dx &= \frac{\int \frac{x^2}{(1-iax)^{22}(1+iax)^{16}}\,dx}{c^{19}}\\ &= -\frac{i+6ax}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}} \end{aligned}$$

Mathematica [A] time = 1.23, size = 36, normalized size = 0.95

$$\frac{6ax+i}{210a^3c^{19}(ax-i)^{15}(ax+i)^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

```
[Out] (I + 6*a*x)/(210*a^3*c^19*(-I + a*x)^15*(I + a*x)^21)
```

fricas [B] time = 1.34, size = 379, normalized size = 9.97

$$\overline{210\,a^{39}c^{19}x^{36}+1260i\,a^{38}c^{19}x^{35}+14700i\,a^{36}c^{19}x^{33}-22050\,a^{35}c^{19}x^{32}+70560i\,a^{34}c^{19}x^{31}-188160\,a^{33}c^{19}x^{30}+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="fric
as")
```

```
[Out] (6*a*x + I)/(210*a^39*c^19*x^36 + 1260*I*a^38*c^19*x^35 + 14700*I*a^36*c^19
*x^33 - 22050*a^35*c^19*x^32 + 70560*I*a^34*c^19*x^31 - 188160*a^33*c^19*x^
30 + 151200*I*a^32*c^19*x^29 - 819000*a^31*c^19*x^28 - 58800*I*a^30*c^19*x^
27 - 2257920*a^29*c^19*x^26 - 1375920*I*a^28*c^19*x^25 - 4204200*a^27*c^19*
x^24 - 4586400*I*a^26*c^19*x^23 - 5241600*a^25*c^19*x^22 - 9129120*I*a^24*c
^19*x^21 - 3783780*a^23*c^19*x^20 - 12612600*I*a^22*c^19*x^19 - 12612600*I*
a^20*c^19*x^17 + 3783780*a^19*c^19*x^16 - 9129120*I*a^18*c^19*x^15 + 524160
0*a^17*c^19*x^14 - 4586400*I*a^16*c^19*x^13 + 4204200*a^15*c^19*x^12 - 1375
920*I*a^14*c^19*x^11 + 2257920*a^13*c^19*x^10 - 58800*I*a^12*c^19*x^9 + 819
000*a^11*c^19*x^8 + 151200*I*a^10*c^19*x^7 + 188160*a^9*c^19*x^6 + 70560*I*
a^8*c^19*x^5 + 22050*a^7*c^19*x^4 + 14700*I*a^6*c^19*x^3 + 1260*I*a^4*c^19*
x - 210*a^3*c^19)
```

giac [B] time = 0.14, size = 318, normalized size = 8.37

$$-\frac{358229025\,a^{14}x^{14}-5340869100\,a^{13}ix^{13}-37114698075\,a^{12}x^{12}+159416118225\,a^{11}ix^{11}+473088806190\,a^{10}x^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="giac
")
```

```
[Out] -1/901943132160*(358229025*a^14*x^14 - 5340869100*a^13*i*x^13 - 37114698075
*a^12*x^12 + 159416118225*a^11*i*x^11 + 473088806190*a^10*x^10 - 1026819468
675*a^9*i*x^9 - 1682288472150*a^8*x^8 + 2115551402250*a^7*i*x^7 + 205443504
6125*a^6*x^6 - 1535397250002*a^5*i*x^5 - 870854759775*a^4*x^4 + 36430753320
5*a^3*i*x^3 + 106553746740*a^2*x^2 - 19571887695*a*i*x - 1710785408)/((a*x
- i)^15*a^3*c^19) + 1/901943132160*(358229025*a^20*x^20 + 7555375800*a^19*i
*x^19 - 75901131600*a^18*x^18 - 483051354975*a^17*i*x^17 + 2184946607340*a^
16*x^16 + 7469205450840*a^15*i*x^15 - 20031221295000*a^14*x^14 - 4317700403
7300*a^13*i*x^13 + 76013078916950*a^12*x^12 + 110448380006328*a^11*i*x^11 -
 133277726128008*a^10*x^10 - 133908931763530*a^9*i*x^9 + 111933156213900*a^
8*x^8 + 77492989590120*a^7*i*x^7 - 44041557267624*a^6*x^6 - 20244576347604*
a^5*i*x^5 + 7349182966545*a^4*x^4 + 2026362494800*a^3*i*x^3 - 396520754280*
a^2*x^2 - 48177926223*a*i*x + 2584181888)/((a*x + i)^21*a^3*c^19)
```

maple [A] time = 0.37, size = 34, normalized size = 0.89

$$\frac{\frac{x}{35a^2}+\frac{i}{210a^3}}{c^{19}\left(ax+i\right)^{21}\left(ax-i\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x)
```

```
[Out] 1/c^19*(1/35*x/a^2+1/210*I/a^3)/(I+a*x)^21/(a*x-I)^15
```

maxima [B] time = 0.54, size = 292, normalized size = 7.68

$$\frac{2077582196}{727153768692449280\left(a^{45}c^{19}x^{42}+21\,a^{43}c^{19}x^{40}+210\,a^{41}c^{19}x^{38}+1330\,a^{39}c^{19}x^{36}+5985\,a^{37}c^{19}x^{34}+20349\,a^{35}c^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="maxi
ma")
```

```
[Out] 1/727153768692449280*(20775821962641408*a^7*x^7 - 121192294782074880*I*a^6*
x^6 - 290861507476979712*a^5*x^5 + 363576884346224640*I*a^4*x^4 + 242384589
564149760*a^3*x^3 - 72715376869244928*I*a^2*x^2 - 3462636993773568*I)/(a^45
*c^19*x^42 + 21*a^43*c^19*x^40 + 210*a^41*c^19*x^38 + 1330*a^39*c^19*x^36 +
 5985*a^37*c^19*x^34 + 20349*a^35*c^19*x^32 + 54264*a^33*c^19*x^30 + 116280
*a^31*c^19*x^28 + 203490*a^29*c^19*x^26 + 293930*a^27*c^19*x^24 + 352716*a^
25*c^19*x^22 + 352716*a^23*c^19*x^20 + 293930*a^21*c^19*x^18 + 203490*a^19*
c^19*x^16 + 116280*a^17*c^19*x^14 + 54264*a^15*c^19*x^12 + 20349*a^13*c^19*
x^10 + 5985*a^11*c^19*x^8 + 1330*a^9*c^19*x^6 + 210*a^7*c^19*x^4 + 21*a^5*c
^19*x^2 + a^3*c^19)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^6)/((c + a^2*c*x^2)^19*(a^2*x^2 + 1)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [B] time = 6.50, size = 437, normalized size = 11.50

$$-\frac{}{-210a^{39}c^{19}x^{36}-1260ia^{38}c^{19}x^{35}-14700ia^{36}c^{19}x^{33}+22050a^{35}c^{19}x^{32}-70560ia^{34}c^{19}x^{31}+188160a^{33}c^{19}x^{30}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**2/(a**2*c*x**2+c)**19,x)
```

```
[Out] -(6*a*x + I)/(-210*a**39*c**19*x**36 - 1260*I*a**38*c**19*x**35 - 14700*I*a
**36*c**19*x**33 + 22050*a**35*c**19*x**32 - 70560*I*a**34*c**19*x**31 + 18
8160*a**33*c**19*x**30 - 151200*I*a**32*c**19*x**29 + 819000*a**31*c**19*x*
*28 + 58800*I*a**30*c**19*x**27 + 2257920*a**29*c**19*x**26 + 1375920*I*a**
28*c**19*x**25 + 4204200*a**27*c**19*x**24 + 4586400*I*a**26*c**19*x**23 +
5241600*a**25*c**19*x**22 + 9129120*I*a**24*c**19*x**21 + 3783780*a**23*c**
19*x**20 + 12612600*I*a**22*c**19*x**19 + 12612600*I*a**20*c**19*x**17 - 37
83780*a**19*c**19*x**16 + 9129120*I*a**18*c**19*x**15 - 5241600*a**17*c**19
*x**14 + 4586400*I*a**16*c**19*x**13 - 4204200*a**15*c**19*x**12 + 1375920*
I*a**14*c**19*x**11 - 2257920*a**13*c**19*x**10 + 58800*I*a**12*c**19*x**9
- 819000*a**11*c**19*x**8 - 151200*I*a**10*c**19*x**7 - 188160*a**9*c**19*x
**6 - 70560*I*a**8*c**19*x**5 - 22050*a**7*c**19*x**4 - 14700*I*a**6*c**19*
x**3 - 1260*I*a**4*c**19*x + 210*a**3*c**19)
```

3.376 $\int \frac{e^{4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx$

Optimal. Leaf size=38

$$-\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

[Out] 1/60*(-I-4*a*x)/a^3/c^9/(1-I*a*x)^10/(1+I*a*x)^6

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

Antiderivative was successfully verified.

[In] Int[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]

[Out] -(I + 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^10*(1 + I*a*x)^6)

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx &= \frac{\int \frac{x^2}{(1-iax)^{11}(1+iax)^7}\,dx}{c^9}\\ &= -\frac{i+4ax}{60a^3c^9(1-iax)^{10}(1+iax)^6}\end{aligned}$$

Mathematica [A] time = 0.23, size = 36, normalized size = 0.95

$$-\frac{4ax+i}{60a^3c^9(ax-i)^6(ax+i)^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]

[Out] -1/60*(I + 4*a*x)/(a^3*c^9*(-I + a*x)^6*(I + a*x)^10)

fricas [B] time = 0.41, size = 171, normalized size = 4.50

$$-\frac{4}{60\,a^{19}c^9x^{16}+240i\,a^{18}c^9x^{15}+1200i\,a^{16}c^9x^{13}-1200\,a^{15}c^9x^{12}+2160i\,a^{14}c^9x^{11}-3840\,a^{13}c^9x^{10}+1200i\,a^{12}c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -(4*a*x + I)/(60*a^19*c^9*x^16 + 240*I*a^18*c^9*x^15 + 1200*I*a^16*c^9*x^13
 - 1200*a^15*c^9*x^12 + 2160*I*a^14*c^9*x^11 - 3840*a^13*c^9*x^10 + 1200*I*
a^12*c^9*x^9 - 5400*a^11*c^9*x^8 - 1200*I*a^10*c^9*x^7 - 3840*a^9*c^9*x^6 -
 2160*I*a^8*c^9*x^5 - 1200*a^7*c^9*x^4 - 1200*I*a^6*c^9*x^3 - 240*I*a^4*c^9
*x + 60*a^3*c^9)
```

giac [B] time = 0.15, size = 151, normalized size = 3.97

$$-\frac{2145\,a^5x^5-12540\,a^4ix^4-30030\,a^3x^3+37080\,a^2ix^2+23841\,ax-6476\,i}{983040\,(ax-i)^6a^3c^9}+\frac{2145\,a^9x^9+21780\,a^8ix^8-99660}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 - 12540*a^4*i*x^4 - 30030*a^3*x^3 + 37080*a^2*i*x^2
 + 23841*a*x - 6476*i)/((a*x - i)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 217
80*a^8*i*x^8 - 99660*a^7*x^7 - 270480*a^6*i*x^6 + 481446*a^5*x^5 + 584920*a
^4*i*x^4 - 486220*a^3*x^3 - 265680*a^2*i*x^2 + 84065*a*x + 9908*i)/((a*x +
i)^10*a^3*c^9)
```

maple [A] time = 0.13, size = 35, normalized size = 0.92

$$-\frac{\frac{i}{60a^3}+\frac{x}{15a^2}}{c^9\,(ax+i)^{10}\,(ax-i)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x)
```

```
[Out] -1/c^9*(1/60*I/a^3+1/15*x/a^2)/(I+a*x)^10/(a*x-I)^6
```

maxima [B] time = 0.43, size = 155, normalized size = 4.08

$$-\frac{5505024\,a^5x^5-20643840i\,a^4x^4-27525120\,a^3x^3+13762560i\,a^2x^2+1}{82575360\left(a^{23}c^9x^{20}+10\,a^{21}c^9x^{18}+45\,a^{19}c^9x^{16}+120\,a^{17}c^9x^{14}+210\,a^{15}c^9x^{12}+252\,a^{13}c^9x^{10}+210\,a^{11}c^9x^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] -1/82575360*(5505024*a^5*x^5 - 20643840*I*a^4*x^4 - 27525120*a^3*x^3 + 1376
2560*I*a^2*x^2 + 1376256*I)/(a^23*c^9*x^20 + 10*a^21*c^9*x^18 + 45*a^19*c^9
*x^16 + 120*a^17*c^9*x^14 + 210*a^15*c^9*x^12 + 252*a^13*c^9*x^10 + 210*a^1
1*c^9*x^8 + 120*a^9*c^9*x^6 + 45*a^7*c^9*x^4 + 10*a^5*c^9*x^2 + a^3*c^9)
```

mupad [B] time = 3.58, size = 160, normalized size = 4.21

$$-\frac{4\,a^5\,x^5-a^4\,x^4\,15\mathrm{i}-20\,a^3\,x^3+a^2\,x^2\,10\mathrm{i}+1\mathrm{i}}{60\,a^{23}\,c^9\,x^{20}+600\,a^{21}\,c^9\,x^{18}+2700\,a^{19}\,c^9\,x^{16}+7200\,a^{17}\,c^9\,x^{14}+12600\,a^{15}\,c^9\,x^{12}+15120\,a^{13}\,c^9\,x^{10}+12600}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^4)/((c + a^2*c*x^2)^9*(a^2*x^2 + 1)^2),x)
```

```
[Out] -(a^2*x^2*10i - 20*a^3*x^3 - a^4*x^4*15i + 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 60
0*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8 +
15120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^19*
c^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

sympy [B] time = 1.77, size = 192, normalized size = 5.05

$$\frac{4ax + \cdots}{-60a^{19}c^9x^{16} - 240ia^{18}c^9x^{15} - 1200ia^{16}c^9x^{13} + 1200a^{15}c^9x^{12} - 2160ia^{14}c^9x^{11} + 3840a^{13}c^9x^{10} - 1200ia^{12}c^9x^9 + 5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] (4*a*x + I)/(-60*a**19*c**9*x**16 - 240*I*a**18*c**9*x**15 - 1200*I*a**16*c
**9*x**13 + 1200*a**15*c**9*x**12 - 2160*I*a**14*c**9*x**11 + 3840*a**13*c*
*9*x**10 - 1200*I*a**12*c**9*x**9 + 5400*a**11*c**9*x**8 + 1200*I*a**10*c**
9*x**7 + 3840*a**9*c**9*x**6 + 2160*I*a**8*c**9*x**5 + 1200*a**7*c**9*x**4
+ 1200*I*a**6*c**9*x**3 + 240*I*a**4*c**9*x - 60*a**3*c**9)
```

3.377 $\int \frac{e^{2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=38

$$-\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

```
[Out] 1/6*(-I-2*a*x)/a^3/c^3/(1-I*a*x)^3/(1+I*a*x)
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$-\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] -(I + 2*a*x)/(6*a^3*c^3*(1 - I*a*x)^3*(1 + I*a*x))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^2}{(1-iax)^4(1+iax)^2}\,dx}{c^3}\\ &= -\frac{i+2ax}{6a^3c^3(1-iax)^3(1+iax)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.95

$$\frac{2ax+i}{6a^3c^3(ax-i)(ax+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (I + 2*a*x)/(6*a^3*c^3*(-I + a*x)*(I + a*x)^3)
```

fricas [A] time = 0.54, size = 49, normalized size = 1.29

$$\frac{2\,ax+i}{6\,a^{7}c^{3}x^{4}+12i\,a^{6}c^{3}x^{3}+12i\,a^{4}c^{3}x-6\,a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] (2*a*x + I)/(6*a^7*c^3*x^4 + 12*I*a^6*c^3*x^3 + 12*I*a^4*c^3*x - 6*a^3*c^3)
```

giac [A] time = 0.12, size = 48, normalized size = 1.26

$$-\frac{1}{16\,(ax-i)a^{3}c^{3}}+\frac{3\,a^{2}x^{2}+12\,aix-5}{48\,(ax+i)^{3}a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x - i)*a^3*c^3) + 1/48*(3*a^2*x^2 + 12*a*i*x - 5)/((a*x + i)^3*a^
3*c^3)
```

maple [A] time = 0.06, size = 34, normalized size = 0.89

$$\frac{\frac{x}{3a^{2}}+\frac{i}{6a^{3}}}{c^{3}\,(ax+i)^{3}\,(ax-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(1/3*x/a^2+1/6*I/a^3)/(I+a*x)^3/(a*x-I)
```

maxima [B] time = 0.42, size = 62, normalized size = 1.63

$$\frac{16\,a^{3}x^{3}-24i\,a^{2}x^{2}-8i}{48\,\left(a^{9}c^{3}x^{6}+3\,a^{7}c^{3}x^{4}+3\,a^{5}c^{3}x^{2}+a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/48*(16*a^3*x^3 - 24*I*a^2*x^2 - 8*I)/(a^9*c^3*x^6 + 3*a^7*c^3*x^4 + 3*a^5
*c^3*x^2 + a^3*c^3)
```

mupad [B] time = 0.64, size = 47, normalized size = 1.24

$$\frac{\frac{x}{3\,a^{6}\,c^{3}}+\frac{1\mathrm{i}}{6\,a^{7}\,c^{3}}}{\frac{x\,2\mathrm{i}}{a^{3}}-\frac{1}{a^{4}}+x^{4}+\frac{x^{3}\,2\mathrm{i}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^2)/((c + a^2*c*x^2)^3*(a^2*x^2 + 1)),x)
```

```
[Out] (1i/(6*a^7*c^3) + x/(3*a^6*c^3))/((x*2i)/a^3 - 1/a^4 + x^4 + (x^3*2i)/a)
```

sympy [A] time = 0.44, size = 53, normalized size = 1.39

$$-\frac{2ax+i}{-6a^{7}c^{3}x^{4}-12ia^{6}c^{3}x^{3}-12ia^{4}c^{3}x+6a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2/(a**2*c*x**2+c)**3,x)
```

```
[Out] -(2*a*x + I)/(-6*a**7*c**3*x**4 - 12*I*a**6*c**3*x**3 - 12*I*a**4*c**3*x +
6*a**3*c**3)
```

3.378 $\int \frac{e^{-2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=38

$$\frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

```
[Out] 1/6*(I-2*a*x)/a^3/c^3/(1-I*a*x)/(1+I*a*x)^3
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$\frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] (I - 2*a*x)/(6*a^3*c^3*(1 - I*a*x)*(1 + I*a*x)^3)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^2}{(1-iax)^2(1+iax)^4}\,dx}{c^3}\\ &= \frac{i-2ax}{6a^3c^3(1-iax)(1+iax)^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.95

$$\frac{2ax-i}{6a^3c^3(ax-i)^3(ax+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] (-I + 2*a*x)/(6*a^3*c^3*(-I + a*x)^3*(I + a*x))
```

fricas [A] time = 0.52, size = 49, normalized size = 1.29

$$\frac{2\,ax-i}{6\,a^7c^3x^4-12i\,a^6c^3x^3-12i\,a^4c^3x-6\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] (2*a*x - I)/(6*a^7*c^3*x^4 - 12*I*a^6*c^3*x^3 - 12*I*a^4*c^3*x - 6*a^3*c^3)
```

giac [B] time = 0.13, size = 93, normalized size = 2.45

$$\frac{1}{32\,a^3c^3\left(i-\frac{2\,i}{aix+1}\right)}+\frac{\frac{3\,a^3c^6i^2}{aix+1}+\frac{4\,a^3c^6i^4}{(aix+1)^3}+\frac{6\,a^3c^6i^2}{(aix+1)^2}}{48\,a^6c^9i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32/(a^3*c^3*(i - 2*i/(a*i*x + 1))) + 1/48*(3*a^3*c^6*i^2/(a*i*x + 1) + 4*
a^3*c^6*i^4/(a*i*x + 1)^3 + 6*a^3*c^6*i^2/(a*i*x + 1)^2)/(a^6*c^9*i)
```

maple [A] time = 0.10, size = 62, normalized size = 1.63

$$\frac{-\frac{1}{16a^3(ax+i)}-\frac{i}{8a^3(-ax+i)^2}-\frac{1}{12a^3(-ax+i)^3}-\frac{1}{16a^3(-ax+i)}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/16/a^3/(I+a*x)-1/8*I/a^3/(-a*x+I)^2-1/12/a^3/(-a*x+I)^3-1/16/a^3/
(-a*x+I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.59, size = 65, normalized size = 1.71

$$\frac{2\,a^3\,x^3+a^2\,x^2\,3\mathrm{i}+1\mathrm{i}}{6\,a^9\,c^3\,x^6+18\,a^7\,c^3\,x^4+18\,a^5\,c^3\,x^2+6\,a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1))/((c + a^2*c*x^2)^3*(a*x*1i + 1)^2),x)
```

```
[Out] (a^2*x^2*3i + 2*a^3*x^3 + 1i)/(6*a^3*c^3 + 18*a^5*c^3*x^2 + 18*a^7*c^3*x^4
+ 6*a^9*c^3*x^6)
```

sympy [A] time = 0.40, size = 53, normalized size = 1.39

$$-\frac{2ax-i}{-6a^7c^3x^4+12ia^6c^3x^3+12ia^4c^3x+6a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**3,x)
```

```
[Out] -(2*a*x - I)/(-6*a**7*c**3*x**4 + 12*I*a**6*c**3*x**3 + 12*I*a**4*c**3*x +
6*a**3*c**3)
```

3.379 $\int \frac{e^{-4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx$

Optimal. Leaf size=38

$$\frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

```
[Out] 1/60*(I-4*a*x)/a^3/c^9/(1-I*a*x)^6/(1+I*a*x)^10
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5082, 81}

$$\frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^6*(1 + I*a*x)^10)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-4i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^9}\,dx &= \frac{\int \frac{x^2}{(1-iax)^7(1+iax)^{11}}\,dx}{c^9}\\ &= \frac{i-4ax}{60a^3c^9(1-iax)^6(1+iax)^{10}}\end{aligned}$$

Mathematica [A] time = 0.23, size = 36, normalized size = 0.95

$$\frac{-4ax+i}{60a^3c^9(ax-i)^{10}(ax+i)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(-I + a*x)^10*(I + a*x)^6)
```

fricas [B] time = 0.51, size = 171, normalized size = 4.50

$$-\frac{4\,a\cdots}{60\,a^{19}c^9x^{16}-240i\,a^{18}c^9x^{15}-1200i\,a^{16}c^9x^{13}-1200\,a^{15}c^9x^{12}-2160i\,a^{14}c^9x^{11}-3840\,a^{13}c^9x^{10}-1200i\,a^{12}c^9x^9-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -(4*a*x - I)/(60*a^19*c^9*x^16 - 240*I*a^18*c^9*x^15 - 1200*I*a^16*c^9*x^13
 - 1200*a^15*c^9*x^12 - 2160*I*a^14*c^9*x^11 - 3840*a^13*c^9*x^10 - 1200*I*
a^12*c^9*x^9 - 5400*a^11*c^9*x^8 + 1200*I*a^10*c^9*x^7 - 3840*a^9*c^9*x^6 +
 2160*I*a^8*c^9*x^5 - 1200*a^7*c^9*x^4 + 1200*I*a^6*c^9*x^3 + 240*I*a^4*c^9
*x + 60*a^3*c^9)
```

giac [B] time = 0.14, size = 151, normalized size = 3.97

$$-\frac{2145\,a^5x^5+12540\,a^4ix^4-30030\,a^3x^3-37080\,a^2ix^2+23841\,ax+6476\,i}{983040\,(ax+i)^6a^3c^9}+\frac{2145\,a^9x^9-21780\,a^8ix^8-99660\,a^7\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 + 12540*a^4*i*x^4 - 30030*a^3*x^3 - 37080*a^2*i*x^2
 + 23841*a*x + 6476*i)/((a*x + i)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 217
80*a^8*i*x^8 - 99660*a^7*x^7 + 270480*a^6*i*x^6 + 481446*a^5*x^5 - 584920*a
^4*i*x^4 - 486220*a^3*x^3 + 265680*a^2*i*x^2 + 84065*a*x - 9908*i)/((a*x -
i)^10*a^3*c^9)
```

maple [B] time = 0.11, size = 218, normalized size = 5.74

$$\frac{\frac{13i}{16384a^3(ax+i)^4}-\frac{i}{12288a^3(ax+i)^6}-\frac{121i}{65536a^3(ax+i)^2}-\frac{7}{20480a^3(ax+i)^5}+\frac{11}{8192a^3(ax+i)^3}-\frac{143}{65536a^3(ax+i)}+\frac{21i}{8192a^3(-ax+i)^4}+\frac{i}{1280a^3(-ax\cdots}\cdots}{c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(13/16384*I/a^3/(I+a*x)^4-1/12288*I/a^3/(I+a*x)^6-121/65536*I/a^3/(I+
a*x)^2-7/20480/a^3/(I+a*x)^5+11/8192/a^3/(I+a*x)^3-143/65536/a^3/(I+a*x)+21
/8192*I/a^3/(-a*x+I)^4+1/1280*I/a^3/(-a*x+I)^10-1/1024*I/a^3/(-a*x+I)^8-7/6
144*I/a^3/(-a*x+I)^6-165/65536*I/a^3/(-a*x+I)^2+1/768/a^3/(-a*x+I)^9-21/102
40/a^3/(-a*x+I)^5+11/4096/a^3/(-a*x+I)^3-143/65536/a^3/(-a*x+I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.40, size = 159, normalized size = 4.18

$$\frac{-4\,a^5\,x^5-a^4\,x^4\,15\mathrm{i}+20\,a^3\,x^3+a^2\,x^2\,10\mathrm{i}+1\mathrm{i}}{60\,a^{23}\,c^9\,x^{20}+600\,a^{21}\,c^9\,x^{18}+2700\,a^{19}\,c^9\,x^{16}+7200\,a^{17}\,c^9\,x^{14}+12600\,a^{15}\,c^9\,x^{12}+15120\,a^{13}\,c^9\,x^{10}+12600\,a^{11}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^2)/((c + a^2*c*x^2)^9*(a*x*1i + 1)^4),x)
```

```
[Out] (a^2*x^2*10i + 20*a^3*x^3 - a^4*x^4*15i - 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 600
*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8 + 1
5120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^19*c
^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

sympy [B] time = 1.49, size = 192, normalized size = 5.05

$$\frac{4ax}{-60a^{19}c^9x^{16} + 240ia^{18}c^9x^{15} + 1200ia^{16}c^9x^{13} + 1200a^{15}c^9x^{12} + 2160ia^{14}c^9x^{11} + 3840a^{13}c^9x^{10} + 1200ia^{12}c^9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] (4*a*x - I)/(-60*a**19*c**9*x**16 + 240*I*a**18*c**9*x**15 + 1200*I*a**16*c
**9*x**13 + 1200*a**15*c**9*x**12 + 2160*I*a**14*c**9*x**11 + 3840*a**13*c*
*9*x**10 + 1200*I*a**12*c**9*x**9 + 5400*a**11*c**9*x**8 - 1200*I*a**10*c**
9*x**7 + 3840*a**9*c**9*x**6 - 2160*I*a**8*c**9*x**5 + 1200*a**7*c**9*x**4
- 1200*I*a**6*c**9*x**3 - 240*I*a**4*c**9*x - 60*a**3*c**9)
```

3.380 $\int \frac{e^{5i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

```
[Out] -1/120*(I+5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^15/(1+I*a*x)^10/(a^2*
c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5085, 5082, 81}

$$-\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

```
[Out] -((I + 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^15*(1 + I*a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{5i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{27/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{16}(1+iax)^{11}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= -\frac{(i+5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.57, size = 63, normalized size = 0.97

$$\frac{(1-5iax)\sqrt{a^2x^2+1}}{120a^3c^{13}(ax-i)^{10}(ax+i)^{15}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

```
[Out] ((1 - (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^10*(I + a*x)^1
5*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.61, size = 497, normalized size = 7.65

$$\frac{\left(i\,a^{22}x^{25} - \cdots\right.}{120\,a^{27}c^{14}x^{27} + 600i\,a^{26}c^{14}x^{26} + 120\,a^{25}c^{14}x^{25} + 5400i\,a^{24}c^{14}x^{24} - 6000\,a^{23}c^{14}x^{23} + 19920i\,a^{22}c^{14}x^{22} - 39600\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] (I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 - 50*I*a^18*x^21 - 126*a^17*x^20
- 280*I*a^16*x^19 - 160*a^15*x^18 - 765*I*a^14*x^17 + 105*a^13*x^16 - 1248*
I*a^12*x^15 + 720*a^11*x^14 - 1260*I*a^10*x^13 + 1260*a^9*x^12 - 720*I*a^8*
x^11 + 1248*a^7*x^10 - 105*I*a^6*x^9 + 765*a^5*x^8 + 160*I*a^4*x^7 + 280*a^
3*x^6 + 126*I*a^2*x^5 + 50*a*x^4 + 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x
^2 + 1)/(120*a^27*c^14*x^27 + 600*I*a^26*c^14*x^26 + 120*a^25*c^14*x^25 + 5
400*I*a^24*c^14*x^24 - 6000*a^23*c^14*x^23 + 19920*I*a^22*c^14*x^22 - 39600
*a^21*c^14*x^21 + 34320*I*a^20*c^14*x^20 - 125400*a^19*c^14*x^19 + 6600*I*a
^18*c^14*x^18 - 241560*a^17*c^14*x^17 - 99000*I*a^16*c^14*x^16 - 300960*a^1
5*c^14*x^15 - 237600*I*a^14*c^14*x^14 - 237600*a^13*c^14*x^13 - 300960*I*a^
12*c^14*x^12 - 99000*a^11*c^14*x^11 - 241560*I*a^10*c^14*x^10 + 6600*a^9*c^
14*x^9 - 125400*I*a^8*c^14*x^8 + 34320*a^7*c^14*x^7 - 39600*I*a^6*c^14*x^6
+ 19920*a^5*c^14*x^5 - 6000*I*a^4*c^14*x^4 + 5400*a^3*c^14*x^3 + 120*I*a^2*
c^14*x^2 + 600*a*c^14*x + 120*I*c^14)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^5x^2}{\left(a^2cx^2+c\right)^{\frac{27}{2}}\left(a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*a*x + 1)^5*x^2/((a^2*c*x^2 + c)^(27/2)*(a^2*x^2 + 1)^(5/2)), x
)
```

maple [A] time = 0.18, size = 58, normalized size = 0.89

$$\frac{(-ax+i)(ax+i)(5ax+i)(iax+1)^5}{120a^3\left(a^2x^2+1\right)^{\frac{5}{2}}\left(a^2c\,x^2+c\right)^{\frac{27}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x)
```

```
[Out] 1/120*(-a*x+I)*(I+a*x)*(I+5*a*x)*(1+I*a*x)^5/a^3/(a^2*x^2+1)^(5/2)/(a^2*c*x
^2+c)^(27/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 2.21, size = 46, normalized size = 0.71

$$-\frac{c\,(a\,x-\mathrm{i})^5\,(5\,a\,x+1\mathrm{i})\,1\mathrm{i}}{120\,a^3\left(c\,\left(a^2\,x^2+1\right)\right)^{29/2}\,\sqrt{a^2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^5)/((c + a^2*c*x^2)^(27/2)*(a^2*x^2 + 1)^(5/2)),x)
```

```
[Out] -(c*(a*x - 1i)^5*(5*a*x + 1i)*1i)/(120*a^3*(c*(a^2*x^2 + 1))^(29/2)*(a^2*x^
2 + 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)*x**2/(a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

```
[Out] -1/24*(I+3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^6/(1+I*a*x)^3/(a^2*c*x^
2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$-\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

```
[Out] -((I + 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^6*(1 + I*a*x)^3*Sq
rt[c + a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^7(1+iax)^4}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= -\frac{(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 1.00

$$\frac{i(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(ax-i)^3(ax+i)^6\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

```
[Out] ((I/24)*(I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^3*(I + a*x)^6*Sq
rt[c + a^2*c*x^2])
```

fricas [B] time = 0.49, size = 193, normalized size = 2.97

$$\frac{\left(i\,a^6x^9-3\,a^5x^8-8\,a^3x^6-6i\,a^2x^5-6\,ax^4-8i\,x^3\right)\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}}{24\,a^{11}c^6x^{11}+72i\,a^{10}c^6x^{10}+24\,a^9c^6x^9+264i\,a^8c^6x^8-144\,a^7c^6x^7+336i\,a^6c^6x^6-336\,a^5c^6x^5+144i\,a^4c^6x^4-264}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] (I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 - 6*I*a^2*x^5 - 6*a*x^4 - 8*I*x^3)*sqrt(
a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(24*a^11*c^6*x^11 + 72*I*a^10*c^6*x^10 + 2
4*a^9*c^6*x^9 + 264*I*a^8*c^6*x^8 - 144*a^7*c^6*x^7 + 336*I*a^6*c^6*x^6 - 3
36*a^5*c^6*x^5 + 144*I*a^4*c^6*x^4 - 264*a^3*c^6*x^3 - 24*I*a^2*c^6*x^2 - 7
2*a*c^6*x - 24*I*c^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)^3x^2}{\left(a^2cx^2+c\right)^{\frac{11}{2}}\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*a*x + 1)^3*x^2/((a^2*c*x^2 + c)^(11/2)*(a^2*x^2 + 1)^(3/2)), x
)
```

maple [A] time = 0.15, size = 58, normalized size = 0.89

$$\frac{(-ax+i)\,(ax+i)\,(3ax+i)\,(iax+1)^3}{24a^3\left(a^2x^2+1\right)^{\frac{3}{2}}\left(a^2c\,x^2+c\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x)
```

```
[Out] 1/24*(-a*x+I)*(I+a*x)*(I+3*a*x)*(1+I*a*x)^3/a^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^
2+c)^(11/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.60, size = 48, normalized size = 0.74

$$\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,{\left(a\,x-\mathrm{i}\right)}^3\,\left(3\,a\,x+1\mathrm{i}\right)\,1\mathrm{i}}{24\,a^3\,c^6\,{\left(a^2\,x^2+1\right)}^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1)^3)/((c + a^2*c*x^2)^(11/2)*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a*x - 1i)^3*(3*a*x + 1i)*1i)/(24*a^3*c^6*(a^2*x^2
 + 1)^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2/(a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

3.382 $\int \frac{e^{i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{a^2x^2+1}}{2a^3c(ax+i)\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[Out] -1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/4*I*ln(I-a*x)*(a
^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+3/4*I*ln(I+a*x)*(a^2*x^2+1)^(1/2)
/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 88}

$$-\frac{\sqrt{a^2x^2+1}}{2a^3c(ax+i)\sqrt{a^2cx^2+c}}+\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}+\frac{3i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a^3*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + ((I/4)*Sqrt[1
+ a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*Sqrt[1 +
a^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{\sqrt{1+a^2x^2}\int \frac{e^{i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$=\frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$=\frac{\sqrt{1+a^2x^2}\int \left(\frac{i}{4a^2(-i+ax)}+\frac{1}{2a^2(i+ax)^2}+\frac{3i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$=-\frac{\sqrt{1+a^2x^2}}{2a^3c(i+ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}}+\frac{3i\sqrt{1+a^2x^2}\,\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.52

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2}{ax+i}+i\log(-ax+i)+3i\log(ax+i)\right)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(-2/(I + a*x) + I*Log[I - a*x] + (3*I)*Log[I + a*x]))/(4
*a^3*c*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\left(-3i\,a^5c^2x^3+3\,a^4c^2x^2-3i\,a^3c^2x+3\,a^2c^2\right)\sqrt{\frac{1}{a^6c^3}}\,\log\left(\frac{i\,\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}\,a^3cx\sqrt{\frac{1}{a^6c^3}}+i\,a^2x^3+i\,x}{a^3x^3+i\,a^2x^2+ax+i}\right)+\left(3i\,a^5c^2x^3-3\,a^4c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*((-3*I*a^5*c^2*x^3 + 3*a^4*c^2*x^2 - 3*I*a^3*c^2*x + 3*a^2*c^2)*sqrt(1/
(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6
*c^3)) + I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (3*I*a^5*c^2*x
^3 - 3*a^4*c^2*x^2 + 3*I*a^3*c^2*x - 3*a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3
+ I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*
a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2
+ a*x - I)) + (-I*a^5*c^2*x^3 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1
/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a
^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + (4*I*a^5*c^2
*x^3 - 4*a^4*c^2*x^2 + 4*I*a^3*c^2*x - 4*a^2*c^2)*sqrt(1/(a^6*c^3))*log((sq
rt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x
)/(a^2*x^2 + 1)) + (-4*I*a^5*c^2*x^3 + 4*a^4*c^2*x^2 - 4*I*a^3*c^2*x + 4*a^
2*c^2)*sqrt(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*
x*sqrt(1/(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)*x + 2*(4*a^5*c^2*x^3 + 4*I*a^4*c^2*x^2 + 4*a^3*c^2*x + 4
*I*a^2*c^2)*integral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(2*I*a*x + 1
```

```
)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(4*a^5*c^2*x^3 + 4*I*a^4*c^2
*x^2 + 4*a^3*c^2*x + 4*I*a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,ax+1)x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((I*a*x + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(a^2*x^2 + 1)), x)
```

maple [A] time = 0.17, size = 87, normalized size = 0.61

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(3i\ln\left(ax+i\right)xa+i\ln\left(-ax+i\right)xa-3\ln\left(ax+i\right)-\ln\left(-ax+i\right)-2\right)}{4\sqrt{a^2x^2+1}\,c^2a^3\left(ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*ln(I+a*x)*x*a+I*ln(-a*x+I)
*x*a-3*ln(I+a*x)-ln(-a*x+I)-2)/c^2/a^3/(I+a*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(1+a\,x\,1{}\mathrm{i}\right)}{{\left(c\,a^2\,x^2+c\right)}^{3/2}\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{ix^2}{a^2cx^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}\right)dx+\int\frac{ax^3}{a^2cx^2\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(-I*x**2/(a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
+ c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a*x**3/(a**2*
c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c)), x))
```

3.383 $\int \frac{e^{-i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{a^2x^2+1}}{2a^3c(-ax+i)\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[Out] 1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)-3/4*I*ln(I-a*x)*(a^
2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-1/4*I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/
a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 88}

$$\frac{\sqrt{a^2x^2+1}}{2a^3c(-ax+i)\sqrt{a^2cx^2+c}}-\frac{3i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}}-\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a^3*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*Sqrt
[1 + a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) - ((I/4)*Sqrt[1 + a
^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \left(\frac{1}{2a^2(-i+ax)^2}-\frac{3i}{4a^2(-i+ax)}-\frac{i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}}{2a^3c(i-ax)\sqrt{c+a^2cx^2}}-\frac{3i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.52

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2}{-ax+i}-3i\log(-ax+i)-i\log(ax+i)\right)}{4a^3c\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(2/(I - a*x) - (3*I)*Log[I - a*x] - I*Log[I + a*x]))/(4*
a^3*c*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\frac{\left(i\,a^5c^2x^3+a^4c^2x^2+i\,a^3c^2x+a^2c^2\right)\sqrt{\frac{1}{a^6c^3}}\,\log\left(\frac{i\,\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+1}\,a^3cx\sqrt{\frac{1}{a^6c^3}}+i\,a^2x^3+i\,x}{a^3x^3+i\,a^2x^2+ax+i}\right)+\left(-i\,a^5c^2x^3-a^4c^2x^2-i\,a^3c^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*((I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3)
)*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) +
I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (-I*a^5*c^2*x^3 - a^4*c
^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 +
c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)/(a^3*x^3
+ I*a^2*x^2 + a*x + I)) + (-3*I*a^5*c^2*x^3 - 3*a^4*c^2*x^2 - 3*I*a^3*c^2*x
 - 3*a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1
)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x -
 I)) + (3*I*a^5*c^2*x^3 + 3*a^4*c^2*x^2 + 3*I*a^3*c^2*x + 3*a^2*c^2)*sqrt(1
/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a
^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + (-4*I*a^5*c^
2*x^3 - 4*a^4*c^2*x^2 - 4*I*a^3*c^2*x - 4*a^2*c^2)*sqrt(1/(a^6*c^3))*log((s
qrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 +
x)/(a^2*x^2 + 1)) + (4*I*a^5*c^2*x^3 + 4*a^4*c^2*x^2 + 4*I*a^3*c^2*x + 4*a^
2*c^2)*sqrt(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*
x*sqrt(1/(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)*x + 2*(4*a^5*c^2*x^3 - 4*I*a^4*c^2*x^2 + 4*a^3*c^2*x - 4
*I*a^2*c^2)*integral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-2*I*a*x +
```

```
1)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(4*a^5*c^2*x^3 - 4*I*a^4*c^
2*x^2 + 4*a^3*c^2*x - 4*I*a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+1}\,x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)), x)
```

maple [A] time = 0.18, size = 86, normalized size = 0.60

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(i\ln\left(ax+i\right)xa+3i\ln\left(-ax+i\right)xa+\ln\left(ax+i\right)+3\ln\left(-ax+i\right)+2\right)}{4\sqrt{a^2x^2+1}\,c^2a^3\left(-ax+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I+a*x)*x*a+3*I*ln(-a*x+I)
*x*a+ln(I+a*x)+3*ln(-a*x+I)+2)/c^2/a^3/(-a*x+I)
```

maxima [A] time = 0.34, size = 55, normalized size = 0.38

$$-\frac{\sqrt{c}}{2\,a^4c^2x-2i\,a^3c^2}-\frac{3i\,\log\left(ax-i\right)}{4\,a^3c^{\frac{3}{2}}}-\frac{i\,\log\left(i\,ax-1\right)}{4\,a^3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -sqrt(c)/(2*a^4*c^2*x - 2*I*a^3*c^2) - 3/4*I*log(a*x - I)/(a^3*c^(3/2)) - 1
/4*I*log(I*a*x - 1)/(a^3*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{a^2\,x^2+1}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}\,\left(1+a\,x\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
```

```
[Out] int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-i\int \frac{x^2\sqrt{a^2x^2+1}}{a^3cx^3\sqrt{a^2cx^2+c}-ia^2cx^2\sqrt{a^2cx^2+c}+acx\sqrt{a^2cx^2+c}-ic\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) - I
*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sqrt
(a**2*c*x**2 + c)), x)
```

3.384 $\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

```
[Out] 1/24*(I-3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^3/(1+I*a*x)^6/(a^2*c*x^2
+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$\frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((I - 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^3*(1 + I*a*x)^6*Sqr
t[c + a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{11/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^4(1+iax)^7}\,dx}{c^5\sqrt{c+a^2cx^2}}$$
$$= \frac{(i-3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.12, size = 65, normalized size = 1.00

$$-\frac{i(3ax-i)\sqrt{a^2x^2+1}}{24a^3c^5(ax-i)^6(ax+i)^3\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((-1/24*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^6*(I + a*x)^
3*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.50, size = 193, normalized size = 2.97

$$\frac{\left(-i\,a^6x^9-3\,a^5x^8-8\,a^3x^6+6i\,a^2x^5-6\,ax^4+8i\,x^3\right)\sqrt{a^2cx^2+c}\,\sqrt{a^2x^2+}}{24\,a^{11}c^6x^{11}-72i\,a^{10}c^6x^{10}+24\,a^9c^6x^9-264i\,a^8c^6x^8-144\,a^7c^6x^7-336i\,a^6c^6x^6-336\,a^5c^6x^5-144i\,a^4c^6x^4-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] (-I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 + 6*I*a^2*x^5 - 6*a*x^4 + 8*I*x^3)*sqrt
(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(24*a^11*c^6*x^11 - 72*I*a^10*c^6*x^10 +
24*a^9*c^6*x^9 - 264*I*a^8*c^6*x^8 - 144*a^7*c^6*x^7 - 336*I*a^6*c^6*x^6 -
336*a^5*c^6*x^5 - 144*I*a^4*c^6*x^4 - 264*a^3*c^6*x^3 + 24*I*a^2*c^6*x^2 -
72*a*c^6*x + 24*I*c^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(a^2cx^2+c\right)^{\frac{11}{2}}(i\,ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^2/((a^2*c*x^2 + c)^(11/2)*(I*a*x + 1)^3), x
)
```

maple [A] time = 0.15, size = 58, normalized size = 0.89

$$-\frac{(-ax+i)\,(ax+i)\,(-3ax+i)\left(a^2x^2+1\right)^{\frac{3}{2}}}{24a^3\,(iax+1)^3\left(a^2c\,x^2+c\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(-a*x+I)*(I+a*x)*(I-3*a*x)*(a^2*x^2+1)^(3/2)/a^3/(1+I*a*x)^3/(a^2*c*x
^2+c)^(11/2)
```

maxima [A] time = 0.36, size = 93, normalized size = 1.43

$$\frac{3\,ax-i}{24i\,a^{12}c^{\frac{11}{2}}x^9+72\,a^{11}c^{\frac{11}{2}}x^8+192\,a^9c^{\frac{11}{2}}x^6-144i\,a^8c^{\frac{11}{2}}x^5+144\,a^7c^{\frac{11}{2}}x^4-192i\,a^6c^{\frac{11}{2}}x^3-72i\,a^4c^{\frac{11}{2}}x-24\,a^3c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] (3*a*x - I)/(24*I*a^12*c^(11/2)*x^9 + 72*a^11*c^(11/2)*x^8 + 192*a^9*c^(11/
2)*x^6 - 144*I*a^8*c^(11/2)*x^5 + 144*a^7*c^(11/2)*x^4 - 192*I*a^6*c^(11/2)
*x^3 - 72*I*a^4*c^(11/2)*x - 24*a^3*c^(11/2))
```

mupad [B] time = 1.59, size = 57, normalized size = 0.88

$$\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\sqrt{a^2\,x^2+1}\,\left(1+a\,x\,3\mathrm{i}\right)\,1\mathrm{i}}{24\,a^3\,c^6\,{\left(a\,x+1\mathrm{i}\right)}^4\,{\left(1+a\,x\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^(3/2))/((c + a^2*c*x^2)^(11/2)*(a*x*1i + 1)^3),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2)*(a*x*3i + 1)*1i)/(24*a^3*c^6*(
a*x + 1i)^4*(a*x*1i + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

3.385 $\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

```
[Out] 1/120*(I-5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^10/(1+I*a*x)^15/(a^2*c
*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5085, 5082, 81}

$$\frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((I - 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^10*(1 + I*a*x)^15
*Sqrt[c + a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5082

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + (I*n)/2)*(1 + I*a*x)^(p - (I*
n)/2), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5085

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 + a^2*x^2)^FracPar
t[p], Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{\left(c+a^2cx^2\right)^{27/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-5i\tan^{-1}(ax)}x^2}{\left(1+a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{11}(1+iax)^{16}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{(i-5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.60, size = 63, normalized size = 0.97

$$\frac{(1+5iax)\sqrt{a^2x^2+1}}{120a^3c^{13}(ax-i)^{15}(ax+i)^{10}\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^15*(I + a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

fricas [B] time = 0.66, size = 497, normalized size = 7.65

$$\frac{\left(-i\,a^{22}x^{25} - 5\,a\right.}{120\,a^{27}c^{14}x^{27} - 600i\,a^{26}c^{14}x^{26} + 120\,a^{25}c^{14}x^{25} - 5400i\,a^{24}c^{14}x^{24} - 6000\,a^{23}c^{14}x^{23} - 19920i\,a^{22}c^{14}x^{22} - 39600\,a^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] (-I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 + 50*I*a^18*x^21 - 126*a^17*x^20
 + 280*I*a^16*x^19 - 160*a^15*x^18 + 765*I*a^14*x^17 + 105*a^13*x^16 + 1248
*I*a^12*x^15 + 720*a^11*x^14 + 1260*I*a^10*x^13 + 1260*a^9*x^12 + 720*I*a^8
*x^11 + 1248*a^7*x^10 + 105*I*a^6*x^9 + 765*a^5*x^8 - 160*I*a^4*x^7 + 280*a
^3*x^6 - 126*I*a^2*x^5 + 50*a*x^4 - 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1)/(120*a^27*c^14*x^27 - 600*I*a^26*c^14*x^26 + 120*a^25*c^14*x^25 -
5400*I*a^24*c^14*x^24 - 6000*a^23*c^14*x^23 - 19920*I*a^22*c^14*x^22 - 3960
0*a^21*c^14*x^21 - 34320*I*a^20*c^14*x^20 - 125400*a^19*c^14*x^19 - 6600*I*
a^18*c^14*x^18 - 241560*a^17*c^14*x^17 + 99000*I*a^16*c^14*x^16 - 300960*a^
15*c^14*x^15 + 237600*I*a^14*c^14*x^14 - 237600*a^13*c^14*x^13 + 300960*I*a
^12*c^14*x^12 - 99000*a^11*c^14*x^11 + 241560*I*a^10*c^14*x^10 + 6600*a^9*c
^14*x^9 + 125400*I*a^8*c^14*x^8 + 34320*a^7*c^14*x^7 + 39600*I*a^6*c^14*x^6
 + 19920*a^5*c^14*x^5 + 6000*I*a^4*c^14*x^4 + 5400*a^3*c^14*x^3 - 120*I*a^2
*c^14*x^2 + 600*a*c^14*x - 120*I*c^14)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-2,[2,1,2
```

```
]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2
,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[2,2,2]%%%}+%%%{-4,[2,1,
2]%%%}+%%%{2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]
%%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%%{-
2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%
%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[2,2,2]%%%}+%%%{
-4,[2,1,2]%%%}+%%%{2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1
,[0,0,0]%%%}] at parameters values [7,-27,26]ext_reduce Error: Bad Argument
 Typeext_reduce Error: Bad Argument Typeext_reduce Error: Bad Argument Type
ext_reduce Error: Bad Argument TypeEvaluation time: 4.04Done
```

maple [A] time = 0.16, size = 58, normalized size = 0.89

$$-\frac{(-ax+i)\,(ax+i)\,(-5ax+i)\left(a^2x^2+1\right)^{\frac{5}{2}}}{120a^3\,(iax+1)^5\left(a^2c\,x^2+c\right)^{\frac{27}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(-a*x+I)*(I+a*x)*(I-5*a*x)*(a^2*x^2+1)^(5/2)/a^3/(1+I*a*x)^5/(a^2*c*
x^2+c)^(27/2)
```

maxima [B] time = 0.45, size = 274, normalized size = 4.22

$$\overline{120\,a^{28}c^{14}x^{25} - 600i\,a^{27}c^{14}x^{24} - 4800i\,a^{25}c^{14}x^{22} - 6000\,a^{24}c^{14}x^{21} - 15120i\,a^{23}c^{14}x^{20} - 33600\,a^{22}c^{14}x^{19} - 1920}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] (5*I*a*sqrt(c)*x + sqrt(c))/(120*a^28*c^14*x^25 - 600*I*a^27*c^14*x^24 - 48
00*I*a^25*c^14*x^22 - 6000*a^24*c^14*x^21 - 15120*I*a^23*c^14*x^20 - 33600*
a^22*c^14*x^19 - 19200*I*a^21*c^14*x^18 - 91800*a^20*c^14*x^17 + 12600*I*a^
19*c^14*x^16 - 149760*a^18*c^14*x^15 + 86400*I*a^17*c^14*x^14 - 151200*a^16
*c^14*x^13 + 151200*I*a^15*c^14*x^12 - 86400*a^14*c^14*x^11 + 149760*I*a^13
*c^14*x^10 - 12600*a^12*c^14*x^9 + 91800*I*a^11*c^14*x^8 + 19200*a^10*c^14*
x^7 + 33600*I*a^9*c^14*x^6 + 15120*a^8*c^14*x^5 + 6000*I*a^7*c^14*x^4 + 480
0*a^6*c^14*x^3 + 600*a^4*c^14*x - 120*I*a^3*c^14)
```

mupad [B] time = 3.09, size = 47, normalized size = 0.72

$$\frac{c^2\,\sqrt{a^2\,x^2+1}\,\left(a\,x+1\mathrm{i}\right)^5\,\left(1+a\,x\,5\mathrm{i}\right)}{120\,a^3\,{\left(c\,\left(a^2\,x^2+1\right)\right)}^{31/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 + 1)^(5/2))/((c + a^2*c*x^2)^(27/2)*(a*x*1i + 1)^5),x)
```

```
[Out] (c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^5*(a*x*5i + 1))/(120*a^3*(c*(a^2*x^2 +
1))^(31/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+I*a*x)**5*(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```