Computer algebra independent integration tests

3-Logarithms/3.5-Logarithm-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [314]. This is test number [64].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (314)	% 0.00 (0)
Mathematica	% 100.00 (314)	% 0.00 (0)
Maple	% 74.52 (234)	% 25.48 (80)
Maxima	% 69.75 (219)	% 30.25 (95)
Fricas	% 88.85 (279)	% 11.15 (35)
Sympy	% 40.45 (127)	% 59.55 (187)
Giac	% 60.83 (191)	% 39.17 (123)
Mupad	% 58.28 (183)	% 41.72 (131)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

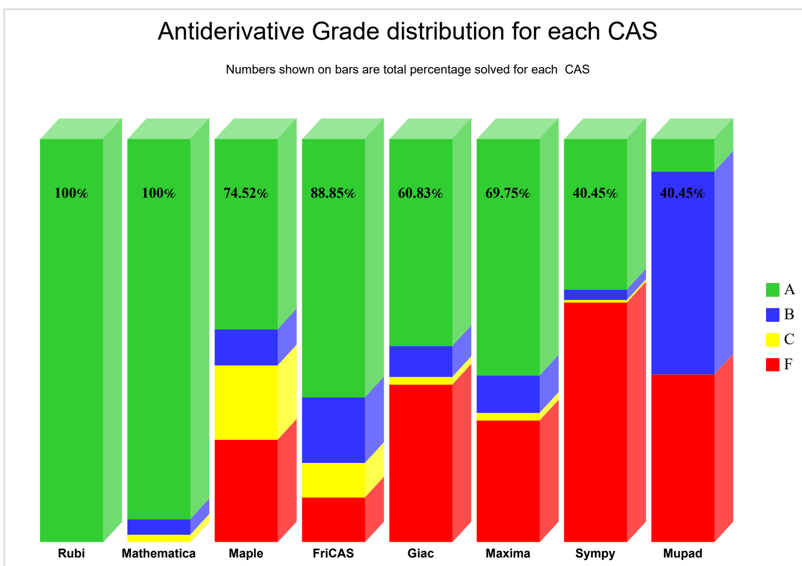
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

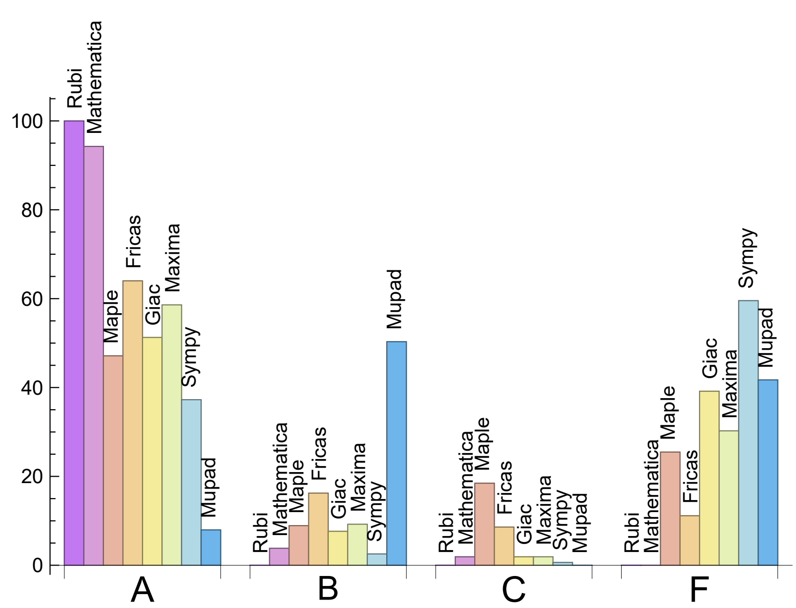
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.27	3.82	1.91	0.00
Maple	47.13	8.92	18.47	25.48
Maxima	58.60	9.24	1.91	30.25
Fricas	64.01	16.24	8.60	11.15
Sympy	37.26	2.55	0.64	59.55
Giac	51.27	7.64	1.91	39.17
Mupad	7.96	50.32	0.00	41.72

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	80	83.75 %	5.00 %	11.25 %
Maxima	95	55.79 %	0.00 %	44.21 %
Fricas	35	80.00 %	0.00 %	20.00 %
Sympy	187	67.91 %	28.34 %	3.74 %
Giac	123	95.12 %	1.63 %	3.25 %
Mupad	131	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

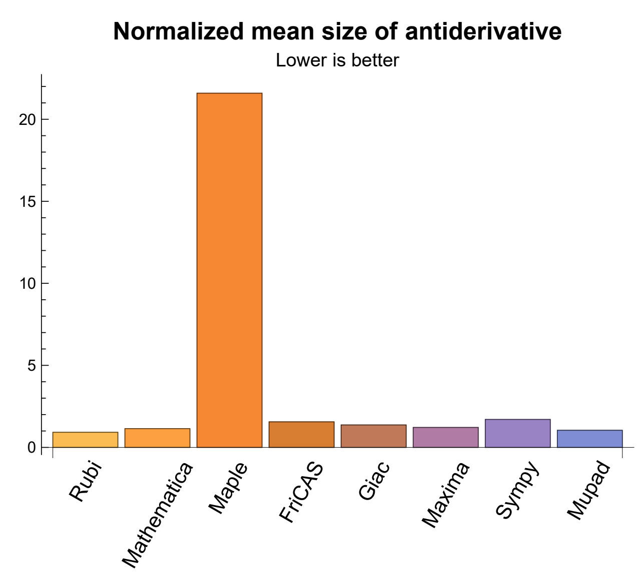
1.3 Performance

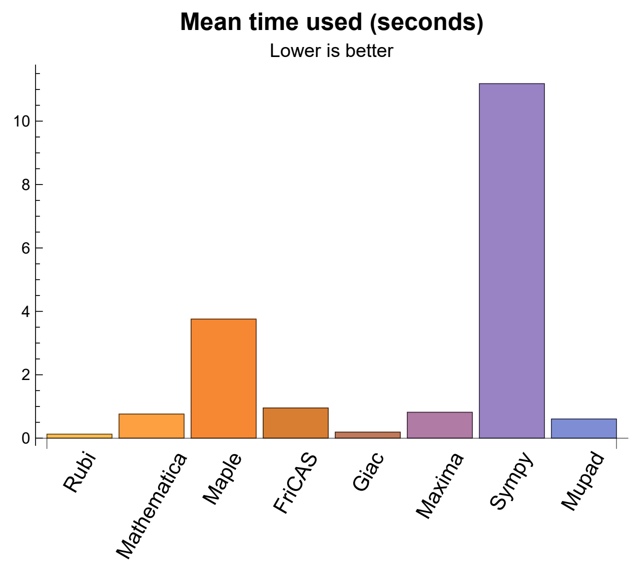
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	69.56	0.92	41.00	1.00
Mathematica	0.76	79.27	1.14	42.00	1.00
Maple	3.76	7027.59	21.59	43.00	1.17
Maxima	0.82	53.19	1.22	36.00	1.00
Fricas	0.95	122.99	1.56	45.00	1.06
Sympy	11.18	68.83	1.70	26.00	1.00
Giac	0.19	105.81	1.37	33.00	1.00
Mupad	0.60	107.11	1.04	22.00	0.87

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {40, 100, 280, 281}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

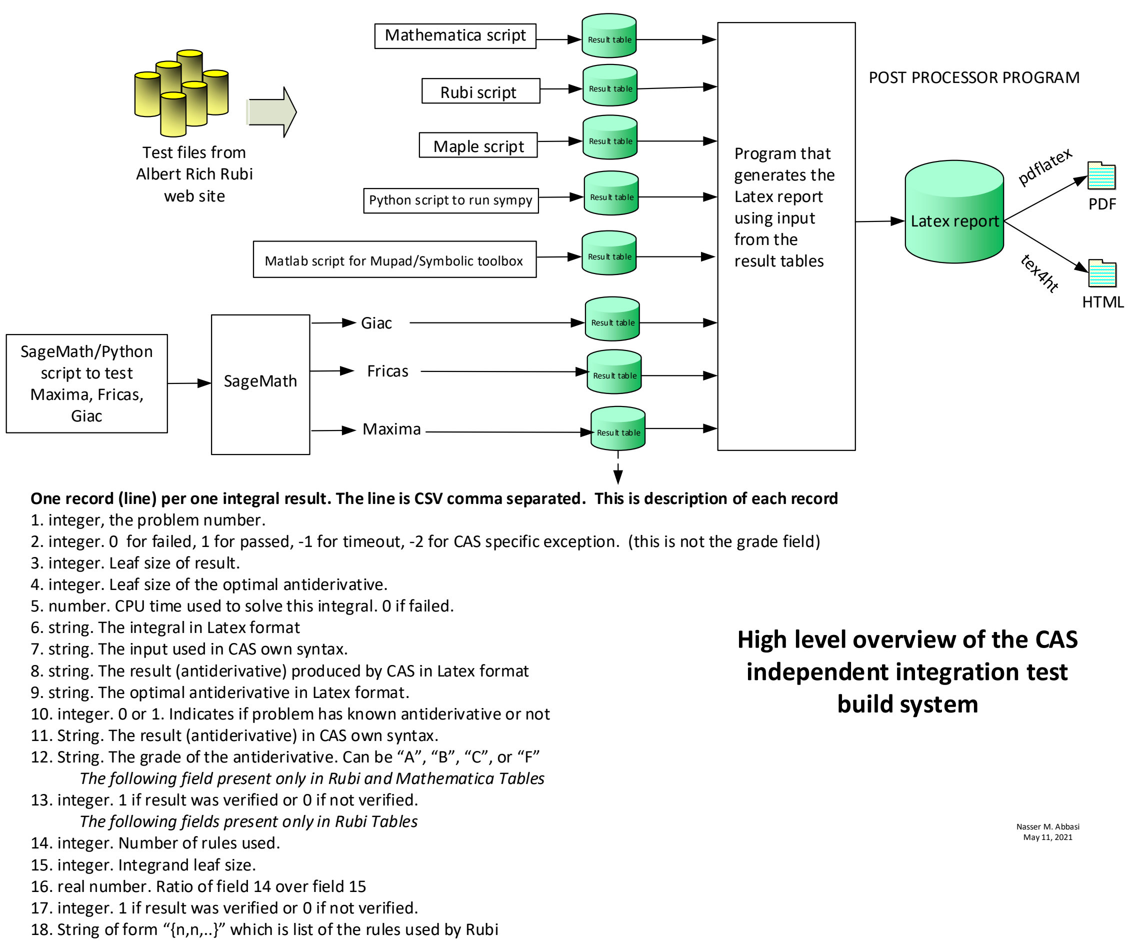
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { 40, 41, 42, 43, 44, 45, 134, 189, 278, 279, 280, 281 }

C grade: { 108, 109, 110, 111, 112, 276 }

F grade: { }

2.1.3 Maple

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 19, 25, 30, 34, 35, 36, 37, 39, 51, 55, 56, 58, 64, 75, 81, 86, 98, 104, 105, 113, 114, 115, 116, 117, 121, 122, 126, 127, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 168, 171, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 207, 209, 210, 211, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 298, 299, 308, 309, 313, 314 }

B grade: { 24, 91, 118, 119, 120, 123, 124, 125, 128, 131, 160, 161, 162, 164, 165, 167, 170, 173, 174, 176, 177, 191, 208, 212, 246, 249, 278, 294 }

C grade: { 9, 10, 11, 17, 18, 20, 21, 22, 23, 26, 27, 28, 29, 38, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 87, 88, 89, 90, 94, 95, 150, 151, 154, 155, 156, 157, 158, 159, 179, 182, 189, 192, 194, 201, 202, 204, 205, 215, 216, 218, 219, 276, 277, 307, 312 }

F grade: { 2, 3, 4, 16, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 92, 93, 96, 97, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 149, 152, 163, 166, 169, 172, 175, 178, 193, 203, 206, 217, 220, 263, 264, 265, 279, 280, 281, 293, 295, 297, 300, 301, 302, 303, 304, 305, 306, 310, 311 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 25, 27, 29, 34, 35, 36, 37, 38, 39, 51, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 96, 105, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 254, 259, 262, 267, 268, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 299, 302, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { 9, 10, 11, 22, 23, 24, 26, 28, 58, 121, 161, 162, 163, 176, 177, 178, 182, 189, 191, 192, 193, 194, 221, 237, 246, 269, 301, 303, 304 }

C grade: { 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 16, 17, 18, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 128, 149, 150, 152, 231, 232, 233, 234, 251, 252, 253, 255, 256, 257, 258, 260, 261, 263, 264, 265, 266, 277, 278, 279, 280, 281, 293, 300 }

2.1.5 FriCAS

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 16, 19, 20, 21, 25, 26, 27, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 121, 122, 126, 127, 129, 130, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 201, 203, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 308, 309, 313, 314 }

B grade: { 9, 10, 11, 17, 18, 22, 23, 24, 28, 89, 90, 91, 128, 131, 135, 136, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 195, 196, 197, 198, 199, 200, 202, 218, 219, 220, 221, 222, 225, 246, 307, 312 }

C grade: { 113, 114, 115, 118, 119, 120, 123, 124, 125, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 305, 306, 310, 311 }

F grade: { 2, 3, 4, 31, 32, 33, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 193, 263, 264, 265, 266, 267, 269, 270, 277, 278, 279, 301, 302, 303, 304 }

2.1.6 Sympy

A grade: { 6, 9, 10, 11, 12, 13, 14, 19, 23, 24, 25, 26, 34, 35, 39, 55, 60, 61, 62, 63, 64, 66, 67, 68, 69, 74, 75, 81, 85, 86, 105, 117, 122, 127, 129, 130, 132, 137, 138, 139, 141, 142, 144, 145, 146, 147, 148, 153, 160, 182, 184, 187, 188, 190, 194, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 272, 273, 274, 275, 276, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 308, 309, 313, 314 }

B grade: { 131, 133, 140, 189, 192, 238, 240, 246 }

C grade: { 268, 270 }

F grade: { 1, 2, 3, 4, 5, 7, 8, 15, 16, 17, 18, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 70, 71, 72, 73, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 135, 136, 143, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 185, 186, 191, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 234, 263, 264, 265, 266, 267, 269, 271, 277, 278, 279, 280, 281, 284, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

2.1.7 Giac

A grade: { 5, 6, 7, 8, 11, 12, 13, 14, 15, 19, 25, 26, 27, 34, 35, 36, 37, 39, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 117, 122, 127, 130, 132, 133, 135, 138, 139, 140, 142, 143, 144, 145, 146, 148, 150, 151, 152, 153, 160, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 251, 253, 254, 255, 257, 262, 265, 272, 273, 276, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 297, 298, 301, 302, 304, 308, 309, 313, 314 }

B grade: { 9, 10, 23, 24, 28, 89, 90, 91, 129, 131, 136, 141, 149, 180, 221, 222, 241, 250, 252, 256, 274, 275, 296, 303 }

C grade: { 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 16, 17, 18, 20, 21, 22, 29, 30, 31, 32, 33, 38, 40, 41, 42, 43, 44, 45, 52, 53, 54, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 110, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 137, 147, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 249, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 271, 277, 278, 279, 280, 281, 293, 299, 300, 305, 306, 307, 310, 311, 312 }

2.1.8 Mupad

A grade: { 1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314 }

B grade: { 5, 12, 17, 18, 19, 23, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 194, 207, 211, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

C grade: { }

F grade: { 2, 3, 4, 9, 10, 11, 16, 20, 21, 22, 29, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 123, 124, 125, 126, 128, 154, 155, 156, 157, 158, 159, 161, 162, 163, 170, 171, 176, 177, 178, 179, 189, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 263, 264, 265, 266, 267, 270, 277, 278, 279, 280, 281, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	1.264	0.852	0.000	0.485	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	223	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.767	20.380	0.000	0.459	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	149	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.274	0.441	13.005	0.000	0.451	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.174	21.042	0.000	0.453	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	25	0	16	15
normalized size	1	1.00	1.00	1.07	1.00	1.67	0.00	1.07	1.00
time (sec)	N/A	0.022	0.002	0.065	0.588	0.453	0.000	0.164	0.284

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.136	29.522	0.000	0.472	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.797	50.166	0.000	0.484	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	1.629	121.280	0.000	1.078	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	230	61910	1115	655	762	766	-1
normalized size	1	1.00	0.85	227.61	4.10	2.41	2.80	2.82	-0.00
time (sec)	N/A	0.305	0.231	9.064	0.939	0.458	103.522	0.256	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	115	14983	530	267	298	286	-1
normalized size	1	1.00	0.92	119.86	4.24	2.14	2.38	2.29	-0.01
time (sec)	N/A	0.166	0.109	3.162	0.661	0.445	37.589	0.189	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	2146	186	81	68	73	-1
normalized size	1	1.00	1.00	52.34	4.54	1.98	1.66	1.78	-0.02
time (sec)	N/A	0.077	0.036	1.047	0.640	0.436	12.423	0.203	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	51	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	3.40	0.87	0.87
time (sec)	N/A	0.007	0.001	0.066	0.733	0.433	1.703	0.156	0.254

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.463	7.099	0.000	0.442	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.605	107.741	0.000	0.464	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	3.015	127.671	0.000	0.601	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	42	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	-0.04
time (sec)	N/A	0.171	0.094	0.967	0.000	0.442	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	204	0	65	0	0	20
normalized size	1	1.00	1.00	9.27	0.00	2.95	0.00	0.00	0.91
time (sec)	N/A	0.173	0.039	0.662	0.000	0.442	0.000	0.000	0.705

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	135	0	42	0	0	20
normalized size	1	1.00	1.00	6.14	0.00	1.91	0.00	0.00	0.91
time (sec)	N/A	0.109	0.033	0.674	0.000	0.441	0.000	0.000	0.680

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	58	17	16
normalized size	1	1.00	1.00	1.06	1.00	1.75	3.62	1.06	1.00
time (sec)	N/A	0.035	0.017	0.078	0.453	0.441	30.634	0.197	0.304

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	212	22	18	0	0	-1
normalized size	1	1.00	1.00	12.47	1.29	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.185	0.238	0.583	1.031	0.462	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	21	21	0	0	-1
normalized size	1	1.00	1.00	3.40	1.05	1.05	0.00	0.00	-0.05
time (sec)	N/A	0.182	0.040	0.538	1.222	0.443	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	68	49	45	0	0	-1
normalized size	1	1.00	1.00	3.09	2.23	2.05	0.00	0.00	-0.05
time (sec)	N/A	0.180	0.041	5.440	1.674	0.463	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16321	211	195	221	198	52
normalized size	1	1.00	1.00	816.05	10.55	9.75	11.05	9.90	2.60
time (sec)	N/A	0.152	0.018	2.886	0.488	0.460	12.320	0.200	0.330

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	63	74	89	117	90	18
normalized size	1	1.00	1.90	3.15	3.70	4.45	5.85	4.50	0.90
time (sec)	N/A	0.093	0.005	0.091	0.461	0.437	8.069	0.182	0.293

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	60	20	14
normalized size	1	1.00	1.00	1.07	1.00	1.50	4.29	1.43	1.00
time (sec)	N/A	0.009	0.002	0.067	0.445	0.431	1.717	0.171	0.257

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	432	32	28	48	28	16
normalized size	1	1.00	1.00	28.80	2.13	1.87	3.20	1.87	1.07
time (sec)	N/A	0.082	0.098	0.288	0.614	0.459	2.335	0.162	0.348

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	451	31	31	0	31	18
normalized size	1	1.00	1.00	25.06	1.72	1.72	0.00	1.72	1.00
time (sec)	N/A	0.133	0.014	0.306	0.652	0.439	0.000	0.182	0.255

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	447	95	101	0	306	39
normalized size	1	1.00	1.00	22.35	4.75	5.05	0.00	15.30	1.95
time (sec)	N/A	0.160	0.015	0.317	0.853	0.454	0.000	0.197	0.275

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	215	26	23	0	0	-1
normalized size	1	1.00	1.21	11.32	1.37	1.21	0.00	0.00	-0.05
time (sec)	N/A	0.321	0.475	0.602	1.035	0.450	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	1.896	0.896	0.000	0.464	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	445	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	1.623	52.591	0.000	0.477	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	298	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.950	111.813	0.000	0.485	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	157	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.494	73.293	0.000	0.486	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	42	53	27	25
normalized size	1	1.00	1.00	1.04	1.00	1.68	2.12	1.08	1.00
time (sec)	N/A	0.034	0.026	0.082	0.452	0.469	30.674	0.182	0.316

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	5.108	114.838	0.000	0.445	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	7.528	51.280	0.000	0.495	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	71.222	148.516	0.000	1.043	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	158	31	30	0	0	26
normalized size	1	1.00	1.00	6.08	1.19	1.15	0.00	0.00	1.00
time (sec)	N/A	0.245	0.367	0.512	1.245	0.452	0.000	0.000	0.325

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.153	80.035	3.260	0.000	0.469	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	625	0	0	44	0	0	-1
normalized size	1	1.00	12.76	0.00	0.00	0.90	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.484	180.000	0.000	0.437	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	642	0	0	45	0	0	-1
normalized size	1	1.00	12.84	0.00	0.00	0.90	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.375	180.000	0.000	0.450	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	320	0	0	44	0	0	-1
normalized size	1	1.00	6.04	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.096	0.218	180.000	0.000	0.444	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	316	0	0	43	0	0	-1
normalized size	1	1.00	6.08	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.195	180.000	0.000	0.429	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	641	0	0	43	0	0	-1
normalized size	1	1.00	13.08	0.00	0.00	0.88	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.355	180.000	0.000	0.437	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	645	0	0	44	0	0	-1
normalized size	1	1.00	12.90	0.00	0.00	0.88	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.257	180.000	0.000	0.433	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	83	0	83	-1
normalized size	1	1.00	0.84	0.00	0.00	1.24	0.00	1.24	-0.01
time (sec)	N/A	0.057	0.137	180.000	0.000	0.446	0.000	0.243	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	90	0	111	-1
normalized size	1	1.00	0.75	0.00	0.00	1.14	0.00	1.41	-0.01
time (sec)	N/A	0.061	0.169	2.589	0.000	0.454	0.000	0.206	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	-1
normalized size	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	-0.02
time (sec)	N/A	0.049	0.062	0.464	0.000	0.421	0.000	0.176	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	-1
normalized size	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	-0.02
time (sec)	N/A	0.035	0.059	0.464	0.000	0.458	0.000	0.195	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	53	0	42	-1
normalized size	1	1.00	0.96	0.00	0.00	1.18	0.00	0.93	-0.02
time (sec)	N/A	0.030	0.050	0.513	0.000	0.451	0.000	0.188	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	48	64	45	0	54	32
normalized size	1	1.00	1.25	1.50	2.00	1.41	0.00	1.69	1.00
time (sec)	N/A	0.017	0.012	0.077	0.460	0.466	0.000	0.173	0.323

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	46	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.044	0.461	0.000	0.441	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.045	0.437	0.000	0.470	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.047	0.441	0.000	0.457	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	23	26	19	26	22
normalized size	1	1.00	1.00	1.18	1.05	1.18	0.86	1.18	1.00
time (sec)	N/A	0.007	0.016	0.086	0.618	0.442	1.190	0.191	0.310

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	22	24	0	32	20
normalized size	1	1.00	1.10	1.15	1.10	1.20	0.00	1.60	1.00
time (sec)	N/A	0.020	0.007	0.070	0.442	0.449	0.000	0.160	0.284

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	0	0	45	0	36	-1
normalized size	1	1.00	0.98	0.00	0.00	1.12	0.00	0.90	-0.02
time (sec)	N/A	0.022	0.021	0.424	0.000	0.450	0.000	0.163	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	35	55	37	0	43	27
normalized size	1	1.00	1.26	1.30	2.04	1.37	0.00	1.59	1.00
time (sec)	N/A	0.021	0.010	0.074	0.449	0.455	0.000	0.185	0.295

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.021	180.000	0.000	0.451	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	87	98	134	89	85
normalized size	1	1.00	0.86	0.00	0.88	0.99	1.35	0.90	0.86
time (sec)	N/A	0.073	0.051	0.171	0.448	0.441	17.668	0.164	0.324

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	74	0	75	86	119	75	73
normalized size	1	1.00	0.87	0.00	0.88	1.01	1.40	0.88	0.86
time (sec)	N/A	0.062	0.038	0.084	0.445	0.435	10.088	0.186	0.338

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	65	74	107	65	61
normalized size	1	1.00	0.89	0.00	0.92	1.04	1.51	0.92	0.86
time (sec)	N/A	0.053	0.031	0.086	0.444	0.457	5.354	0.175	0.316

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	0	51	59	92	51	49
normalized size	1	1.00	0.86	0.00	0.89	1.04	1.61	0.89	0.86
time (sec)	N/A	0.041	0.022	0.107	0.440	0.430	2.686	0.180	0.368

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	34	36	38	56	37	33
normalized size	1	1.00	0.94	1.03	1.09	1.15	1.70	1.12	1.00
time (sec)	N/A	0.016	0.008	0.074	0.447	0.451	1.383	0.163	0.357

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	0	80	0	0	0	-1
normalized size	1	1.00	0.94	0.00	1.51	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.126	0.021	0.109	0.452	0.442	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	46	46	76	47	43
normalized size	1	1.00	0.96	0.00	0.98	0.98	1.62	1.00	0.91
time (sec)	N/A	0.044	0.012	0.090	0.445	0.469	3.409	0.178	0.796

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	62	70	110	65	54
normalized size	1	1.00	0.90	0.00	0.86	0.97	1.53	0.90	0.75
time (sec)	N/A	0.050	0.036	0.082	0.698	0.482	6.952	0.169	0.476

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	0	75	82	133	80	68
normalized size	1	1.00	0.90	0.00	0.87	0.95	1.55	0.93	0.79
time (sec)	N/A	0.057	0.036	0.082	0.685	0.457	13.292	0.163	0.456

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	86	94	141	92	79
normalized size	1	1.00	0.87	0.00	0.86	0.94	1.41	0.92	0.79
time (sec)	N/A	0.063	0.048	0.086	0.617	0.461	24.246	0.163	0.487

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	137	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.194	1.399	0.000	0.437	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	190	1621	0	444	0	221	395
normalized size	1	1.00	0.92	7.83	0.00	2.14	0.00	1.07	1.91
time (sec)	N/A	0.229	0.220	0.811	0.000	0.470	0.000	0.204	0.592

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	151	1146	0	364	0	176	288
normalized size	1	1.00	0.90	6.86	0.00	2.18	0.00	1.05	1.72
time (sec)	N/A	0.189	0.164	0.611	0.000	0.478	0.000	0.205	0.544

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	870	0	299	0	146	229
normalized size	1	1.00	0.90	6.40	0.00	2.20	0.00	1.07	1.68
time (sec)	N/A	0.149	0.140	0.638	0.000	0.472	0.000	0.269	0.504

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	510	0	245	369	113	166
normalized size	1	1.00	0.86	4.68	0.00	2.25	3.39	1.04	1.52
time (sec)	N/A	0.112	0.096	0.680	0.000	0.478	165.455	0.270	0.554

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	118	0	190	275	92	120
normalized size	1	1.00	0.99	1.49	0.00	2.41	3.48	1.16	1.52
time (sec)	N/A	0.061	0.063	0.078	0.000	0.482	69.124	0.189	0.342

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	156	315	0	0	0	0	-1
normalized size	1	1.00	1.21	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.206	0.341	0.000	0.440	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	87	261	0	199	0	99	262
normalized size	1	1.00	1.01	3.03	0.00	2.31	0.00	1.15	3.05
time (sec)	N/A	0.113	0.118	0.684	0.000	0.503	0.000	0.195	0.932

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	105	1178	0	261	0	129	474
normalized size	1	1.00	0.87	9.74	0.00	2.16	0.00	1.07	3.92
time (sec)	N/A	0.154	0.245	0.460	0.000	0.467	0.000	0.202	0.760

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	423	0	318	0	164	505
normalized size	1	1.00	0.89	2.84	0.00	2.13	0.00	1.10	3.39
time (sec)	N/A	0.200	0.371	0.714	0.000	0.526	0.000	0.249	0.933

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	172	505	0	404	0	210	627
normalized size	1	1.00	0.91	2.66	0.00	2.13	0.00	1.11	3.30
time (sec)	N/A	0.223	0.463	0.878	0.000	0.550	0.000	0.207	1.065

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	38	37	33	46	37	39
normalized size	1	1.00	0.83	0.90	0.88	0.79	1.10	0.88	0.93
time (sec)	N/A	0.026	0.014	0.069	1.341	0.437	0.154	0.166	0.064

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	468	31895	0	1270	0	817	1240
normalized size	1	1.00	0.96	65.76	0.00	2.62	0.00	1.68	2.56
time (sec)	N/A	2.058	1.966	1.088	0.000	0.566	0.000	0.257	1.030

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	324	16059	0	880	0	553	775
normalized size	1	1.00	0.96	47.51	0.00	2.60	0.00	1.64	2.29
time (sec)	N/A	0.516	0.534	0.873	0.000	0.476	0.000	0.245	0.837

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	204	7155	0	567	0	349	457
normalized size	1	1.00	0.90	31.66	0.00	2.51	0.00	1.54	2.02
time (sec)	N/A	0.319	0.446	0.700	0.000	0.480	0.000	0.214	0.672

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	123	1706	0	336	394	188	242
normalized size	1	1.00	0.80	11.08	0.00	2.18	2.56	1.22	1.57
time (sec)	N/A	0.186	0.212	0.695	0.000	0.448	158.892	0.198	0.590

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	118	0	190	275	92	120
normalized size	1	1.00	0.99	1.49	0.00	2.41	3.48	1.16	1.52
time (sec)	N/A	0.062	0.062	0.076	0.000	0.436	65.131	0.190	0.002

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	226	493	0	0	0	0	-1
normalized size	1	1.00	0.99	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.296	0.369	0.000	0.420	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	6540	0	429	0	284	590
normalized size	1	1.00	1.01	39.64	0.00	2.60	0.00	1.72	3.58
time (sec)	N/A	0.245	0.316	0.716	0.000	0.571	0.000	0.239	3.336

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	215	14679	0	1341	0	887	1715
normalized size	1	1.00	0.83	56.68	0.00	5.18	0.00	3.42	6.62
time (sec)	N/A	0.406	0.557	1.190	0.000	2.201	0.000	0.353	4.741

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	310	306209	0	3013	0	1963	2707
normalized size	1	1.00	0.87	860.14	0.00	8.46	0.00	5.51	7.60
time (sec)	N/A	0.621	1.208	1.038	0.000	15.088	0.000	0.606	11.302

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	469	1137077	0	5824	0	3759	4334
normalized size	1	1.00	0.90	2190.90	0.00	11.22	0.00	7.24	8.35
time (sec)	N/A	1.006	2.026	1.297	0.000	92.032	0.000	1.174	18.948

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.051	1.046	0.000	0.970	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	339	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.187	1.709	0.000	1.223	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	626	610	0	0	0	0	-1
normalized size	1	1.00	0.82	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.452	1.005	1.197	0.000	0.685	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	782	663	764	0	0	0	0	-1
normalized size	1	1.00	0.85	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.514	0.851	1.170	0.000	0.930	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	111	0	123	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.064	0.161	0.700	0.694	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	478	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.949	0.850	1.126	0.000	0.963	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	290	279	0	0	0	0	-1
normalized size	1	1.00	0.93	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	0.204	0.093	0.000	0.776	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	323	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.158	0.096	0.000	1.056	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	826	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.684	0.816	0.130	0.000	0.709	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	0	0	134	0	134	-1
normalized size	1	1.00	0.68	0.00	0.00	0.78	0.00	0.78	-0.01
time (sec)	N/A	0.381	0.514	0.073	0.000	1.006	0.000	0.326	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	107	0	0	124	0	124	-1
normalized size	1	1.00	0.72	0.00	0.00	0.83	0.00	0.83	-0.01
time (sec)	N/A	0.292	0.348	0.072	0.000	0.987	0.000	0.314	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	0	0	114	0	114	-1
normalized size	1	1.00	0.80	0.00	0.00	0.90	0.00	0.90	-0.01
time (sec)	N/A	0.245	0.291	0.079	0.000	1.010	0.000	0.348	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	80	0	101	0	101	-1
normalized size	1	1.00	0.89	0.84	0.00	1.06	0.00	1.06	-0.01
time (sec)	N/A	0.162	0.030	0.104	0.000	0.816	0.000	0.272	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.479	0.083	0.000	0.511	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	115	0	92	-1
normalized size	1	1.00	0.89	0.00	0.00	1.51	0.00	1.21	-0.01
time (sec)	N/A	0.262	0.252	0.072	0.000	1.011	0.000	0.304	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	0	0	138	0	130	-1
normalized size	1	1.00	0.81	0.00	0.00	1.37	0.00	1.29	-0.01
time (sec)	N/A	0.291	0.298	0.076	0.000	0.570	0.000	0.312	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	232	0	0	110	0	132	-1
normalized size	1	1.00	1.24	0.00	0.00	0.59	0.00	0.71	-0.01
time (sec)	N/A	0.541	0.874	0.075	0.000	0.963	0.000	0.365	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	209	0	0	100	0	122	-1
normalized size	1	1.00	1.32	0.00	0.00	0.63	0.00	0.77	-0.01
time (sec)	N/A	0.431	0.649	0.072	0.000	0.810	0.000	0.367	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	186	0	0	84	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.574	0.076	0.000	0.979	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	84	0	166	-1
normalized size	1	1.00	1.55	0.00	0.00	0.74	0.00	1.46	-0.01
time (sec)	N/A	0.309	0.455	0.076	0.000	0.809	0.000	0.478	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	204	0	0	108	0	181	-1
normalized size	1	1.00	1.35	0.00	0.00	0.72	0.00	1.20	-0.01
time (sec)	N/A	0.478	0.730	0.076	0.000	0.927	0.000	0.497	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	82	88	0	0	-1
normalized size	1	1.00	1.00	0.90	0.88	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.008	0.079	0.777	1.144	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	69	67	73	0	0	-1
normalized size	1	1.00	1.00	0.90	0.87	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.006	0.071	0.766	1.125	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	52	50	56	0	0	-1
normalized size	1	1.00	1.00	0.88	0.85	0.95	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.005	0.078	0.601	0.600	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	28	34	40	0	0	35
normalized size	1	1.00	1.00	0.74	0.89	1.05	0.00	0.00	0.92
time (sec)	N/A	0.048	0.002	0.077	0.692	1.158	0.000	0.000	0.381

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.032	0.051	0.103	0.000	0.703	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	601	189	128	0	0	-1
normalized size	1	1.00	1.00	4.55	1.43	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.014	0.426	0.778	0.490	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	430	153	93	0	0	-1
normalized size	1	1.00	1.00	4.39	1.56	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.006	0.349	0.810	0.796	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	262	117	58	0	0	-1
normalized size	1	1.00	1.00	4.16	1.86	0.92	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.005	0.346	0.864	1.315	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	76	31	0	0	-1
normalized size	1	1.00	1.00	1.03	2.45	1.00	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.001	0.072	0.897	0.798	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	0.308	0.245	0.000	0.558	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	193	1276	204	245	0	0	-1
normalized size	1	1.00	1.00	6.61	1.06	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.009	0.363	0.806	0.756	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	916	165	205	0	0	-1
normalized size	1	1.00	1.00	5.87	1.06	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.007	0.352	0.866	0.586	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	558	126	169	0	0	-1
normalized size	1	1.00	1.00	4.73	1.07	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.006	0.344	0.778	1.215	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	82	82	106	0	0	-1
normalized size	1	1.00	1.00	1.09	1.09	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.003	0.080	0.829	0.675	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	0.298	0.240	0.000	0.701	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	138	0	106	0	0	-1
normalized size	1	1.00	1.00	3.54	0.00	2.72	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.009	0.153	0.000	0.730	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	22	5
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.00	4.40	1.00
time (sec)	N/A	0.018	0.009	0.059	0.620	0.863	0.096	0.160	0.420

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	13
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	1.08
time (sec)	N/A	0.026	0.004	0.066	0.718	1.948	0.802	0.160	0.386

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	33	8	31	39	31	26
normalized size	1	1.00	1.00	3.30	0.80	3.10	3.90	3.10	2.60
time (sec)	N/A	0.017	0.003	0.060	0.652	0.540	0.144	0.168	0.390

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.014	0.002	0.060	0.768	0.736	0.398	0.168	0.071

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.020	0.012	0.061	1.511	0.725	0.137	0.158	0.525

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	16	16	0	0	12
normalized size	1	1.00	3.00	0.93	1.14	1.14	0.00	0.00	0.86
time (sec)	N/A	0.032	0.016	0.067	0.513	0.722	0.000	0.000	0.467

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	21	0	7	7
normalized size	1	1.00	1.00	0.73	0.64	1.91	0.00	0.64	0.64
time (sec)	N/A	0.035	0.020	0.070	1.223	1.510	0.000	0.229	0.399

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	16	0	16	5
normalized size	1	1.00	1.00	0.86	0.71	2.29	0.00	2.29	0.71
time (sec)	N/A	0.029	0.017	0.066	1.364	1.077	0.000	0.196	0.393

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	87	97	71	17	0	120
normalized size	1	1.00	0.95	0.78	0.87	0.64	0.15	0.00	1.08
time (sec)	N/A	0.100	0.077	0.066	1.354	1.320	0.173	0.000	4.694

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.028	0.004	0.063	0.604	0.564	0.250	0.185	0.521

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	9	9
normalized size	1	1.00	1.00	1.11	1.00	1.22	0.78	1.00	1.00
time (sec)	N/A	0.015	0.007	0.073	0.583	0.845	0.384	0.180	0.403

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	14	12	13	156	12	12
normalized size	1	1.00	0.94	0.82	0.71	0.76	9.18	0.71	0.71
time (sec)	N/A	0.020	0.015	0.307	0.666	0.750	3.369	0.168	0.376

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	10	27	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.83	2.25	1.00
time (sec)	N/A	0.036	0.028	0.066	0.632	0.856	0.108	0.156	0.375

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	24	23	23	27	23	22
normalized size	1	1.00	1.38	1.14	1.10	1.10	1.29	1.10	1.05
time (sec)	N/A	0.036	0.015	0.063	0.506	0.551	0.135	0.191	0.370

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	31	30	36	0	37	26
normalized size	1	1.00	0.74	0.74	0.71	0.86	0.00	0.88	0.62
time (sec)	N/A	0.067	0.022	0.067	1.487	0.634	0.000	0.174	0.395

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	21	20	26	17	34	18
normalized size	1	1.00	0.83	0.88	0.83	1.08	0.71	1.42	0.75
time (sec)	N/A	0.041	0.041	0.068	0.868	0.774	0.125	0.173	0.412

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
normalized size	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.043	0.018	0.066	0.583	1.073	4.820	0.160	0.428

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	20	22	21	16	22	21	15
normalized size	1	1.00	0.69	0.76	0.72	0.55	0.76	0.72	0.52
time (sec)	N/A	0.051	0.022	0.069	0.754	1.637	5.430	0.153	0.439

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	0	18
normalized size	1	1.00	1.00	1.36	1.32	1.32	1.45	0.00	0.82
time (sec)	N/A	0.060	0.020	0.062	0.552	1.144	2.086	0.000	0.407

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	32	32	32	20	18
normalized size	1	1.00	0.81	0.89	1.19	1.19	1.19	0.74	0.67
time (sec)	N/A	0.049	0.044	0.069	0.549	2.125	0.129	0.195	0.424

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	38	0	68	27
normalized size	1	1.00	1.00	0.00	0.00	1.41	0.00	2.52	1.00
time (sec)	N/A	0.026	0.012	3.556	0.000	0.918	0.000	0.360	0.401

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	71	0	27	0	24	27
normalized size	1	1.00	1.00	2.63	0.00	1.00	0.00	0.89	1.00
time (sec)	N/A	0.035	0.006	2.000	0.000	0.787	0.000	0.178	0.384

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	13	0	27	21
normalized size	1	1.00	1.00	0.84	0.80	0.52	0.00	1.08	0.84
time (sec)	N/A	0.020	0.008	0.353	0.626	1.548	0.000	0.203	0.404

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	33	0	35	29
normalized size	1	1.00	1.00	0.00	0.00	1.14	0.00	1.21	1.00
time (sec)	N/A	0.038	0.006	0.711	0.000	0.659	0.000	0.326	0.357

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	5	5	3	5	5
normalized size	1	1.00	0.68	0.94	0.16	0.16	0.10	0.16	0.16
time (sec)	N/A	0.016	0.009	0.070	0.705	0.533	0.072	0.155	0.363

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	80	57	37	0	102	-1
normalized size	1	1.00	0.86	2.29	1.63	1.06	0.00	2.91	-0.03
time (sec)	N/A	0.078	0.052	0.977	0.849	0.905	0.000	0.172	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	79	59	0	123	-1
normalized size	1	1.00	0.76	2.00	1.20	0.89	0.00	1.86	-0.02
time (sec)	N/A	0.114	0.086	1.648	1.238	0.727	0.000	0.212	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	162	110	76	0	454	-1
normalized size	1	1.00	0.74	1.82	1.24	0.85	0.00	5.10	-0.01
time (sec)	N/A	0.518	0.118	1.405	1.112	0.720	0.000	0.237	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	79	55	38	0	108	-1
normalized size	1	1.00	0.86	2.26	1.57	1.09	0.00	3.09	-0.03
time (sec)	N/A	0.061	0.049	1.111	1.018	0.702	0.000	0.191	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	76	60	0	122	-1
normalized size	1	1.00	0.76	2.00	1.15	0.91	0.00	1.85	-0.02
time (sec)	N/A	0.121	0.114	1.334	1.120	0.548	0.000	0.212	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	162	108	76	0	495	-1
normalized size	1	1.00	0.75	1.84	1.23	0.86	0.00	5.62	-0.01
time (sec)	N/A	0.468	0.160	1.727	1.034	1.022	0.000	0.254	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	19	5	5	5	5	5
normalized size	1	1.00	1.00	3.80	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.033	0.024	0.934	0.928	0.625	13.686	0.171	0.496

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	76	87	104	0	0	-1
normalized size	1	1.00	0.89	1.62	1.85	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.018	0.413	2.242	0.780	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	88	89	109	0	0	-1
normalized size	1	1.00	0.96	1.96	1.98	2.42	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.018	0.454	2.323	0.898	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	91	115	0	0	-1
normalized size	1	1.00	1.00	0.00	1.75	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.025	2.075	2.395	1.030	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	107	60	104	0	0	37
normalized size	1	1.00	1.00	2.28	1.28	2.21	0.00	0.00	0.79
time (sec)	N/A	0.053	0.008	0.333	2.223	0.534	0.000	0.000	0.390

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	118	62	106	0	0	39
normalized size	1	1.00	0.96	2.62	1.38	2.36	0.00	0.00	0.87
time (sec)	N/A	0.056	0.022	0.445	2.547	0.529	0.000	0.000	0.081

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	65	115	0	0	41
normalized size	1	1.00	1.00	0.00	1.25	2.21	0.00	0.00	0.79
time (sec)	N/A	0.058	0.024	1.611	3.000	0.519	0.000	0.000	0.369

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	82	42	184	0	0	39
normalized size	1	1.00	1.47	1.61	0.82	3.61	0.00	0.00	0.76
time (sec)	N/A	0.044	0.011	0.168	1.617	0.464	0.000	0.000	0.094

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	184	0	0	41
normalized size	1	1.00	1.53	1.67	0.90	3.76	0.00	0.00	0.84
time (sec)	N/A	0.048	0.012	0.566	1.514	0.482	0.000	0.000	0.057

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	81	0	48	195	0	0	44
normalized size	1	1.00	1.45	0.00	0.86	3.48	0.00	0.00	0.79
time (sec)	N/A	0.048	0.012	180.000	1.507	0.524	0.000	0.000	0.067

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	82	43	147	0	0	-1
normalized size	1	1.00	1.47	1.61	0.84	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.011	0.157	1.473	0.495	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	148	0	0	-1
normalized size	1	1.00	1.53	1.67	0.90	3.02	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.012	0.461	1.366	0.535	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	81	0	49	158	0	0	44
normalized size	1	1.00	1.45	0.00	0.88	2.82	0.00	0.00	0.79
time (sec)	N/A	0.049	0.016	180.000	1.544	0.514	0.000	0.000	0.067

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	118	60	106	0	0	39
normalized size	1	1.00	1.00	2.57	1.30	2.30	0.00	0.00	0.85
time (sec)	N/A	0.052	0.007	0.536	2.275	0.527	0.000	0.000	0.089

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	118	61	102	0	0	39
normalized size	1	1.00	0.96	2.62	1.36	2.27	0.00	0.00	0.87
time (sec)	N/A	0.054	0.021	0.491	2.170	0.503	0.000	0.000	0.381

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	65	117	0	0	43
normalized size	1	1.00	1.00	0.00	1.27	2.29	0.00	0.00	0.84
time (sec)	N/A	0.054	0.025	1.614	3.316	0.520	0.000	0.000	0.383

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	89	87	106	0	0	-1
normalized size	1	1.00	0.89	1.93	1.89	2.30	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.013	0.492	2.260	0.507	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	88	87	107	0	0	-1
normalized size	1	1.00	0.93	1.96	1.93	2.38	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.019	0.553	2.310	0.543	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	91	117	0	0	-1
normalized size	1	1.00	1.00	0.00	1.78	2.29	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.023	2.099	1.751	0.521	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	13	72	17	17	0	13	-1
normalized size	1	1.00	0.62	3.43	0.81	0.81	0.00	0.62	-0.05
time (sec)	N/A	0.021	0.005	0.603	0.452	0.488	0.000	0.181	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	0	32	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.00	5.33	1.00
time (sec)	N/A	0.016	0.014	0.401	0.444	0.441	0.000	0.171	0.501

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	35	6	6	0	6	6
normalized size	1	1.00	0.68	0.95	0.16	0.16	0.00	0.16	0.16
time (sec)	N/A	0.028	0.036	0.450	0.451	0.445	0.000	0.154	0.367

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	61	94	22	15	12	35
normalized size	1	1.00	1.00	5.08	7.83	1.83	1.25	1.00	2.92
time (sec)	N/A	0.023	0.017	0.582	0.999	0.451	18.328	0.181	0.579

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	0	7	7
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.00	0.78	0.78
time (sec)	N/A	0.016	0.004	0.190	0.447	0.468	0.000	0.186	0.381

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	11
normalized size	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.55
time (sec)	N/A	0.039	0.015	0.073	0.447	0.463	4.925	0.186	0.430

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	32	42	65	0	42	29
normalized size	1	1.00	0.66	0.64	0.84	1.30	0.00	0.84	0.58
time (sec)	N/A	0.026	0.011	0.737	0.541	0.493	0.000	0.930	0.521

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	0	7	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.00	1.17	1.00
time (sec)	N/A	0.022	0.009	0.305	0.442	0.429	0.000	0.159	0.399

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.015	0.003	0.189	0.443	0.492	3.532	0.155	0.433

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	10	10	9
normalized size	1	1.00	1.00	1.10	1.00	1.00	1.00	1.00	0.90
time (sec)	N/A	0.008	0.008	0.182	0.441	0.466	0.869	0.166	0.379

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	43	73	108	27	223	27	-1
normalized size	1	1.00	3.07	5.21	7.71	1.93	15.93	1.93	-0.07
time (sec)	N/A	0.020	0.016	0.313	0.453	0.491	2.361	0.185	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8
normalized size	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.009	0.003	0.186	0.442	0.476	0.869	0.158	0.415

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	59	146	104	120	0	0	-1
normalized size	1	1.00	0.80	1.97	1.41	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.058	0.330	1.942	0.495	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	47	134	179	43	439	41	-1
normalized size	1	1.00	1.18	3.35	4.48	1.08	10.98	1.02	-0.02
time (sec)	N/A	0.067	0.043	0.342	0.459	0.504	7.744	0.173	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	88	0	139	0	0	0	-1
normalized size	1	1.00	1.11	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.033	0.299	0.507	0.494	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	72	81	19	17	19	57
normalized size	1	1.00	1.00	4.80	5.40	1.27	1.13	1.27	3.80
time (sec)	N/A	0.021	0.015	0.724	0.991	0.502	16.683	0.196	0.576

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	36	134	0	52	-1
normalized size	1	1.00	0.86	1.66	1.03	3.83	0.00	1.49	-0.03
time (sec)	N/A	0.079	0.054	0.704	0.687	0.476	0.000	0.163	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	67	313	0	67	-1
normalized size	1	1.00	0.76	1.47	1.02	4.74	0.00	1.02	-0.02
time (sec)	N/A	0.143	0.120	1.303	0.689	0.473	0.000	0.192	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	67	116	110	587	0	102	-1
normalized size	1	1.00	0.75	1.30	1.24	6.60	0.00	1.15	-0.01
time (sec)	N/A	0.517	0.115	1.043	0.772	0.481	0.000	0.206	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	37	134	0	54	-1
normalized size	1	1.00	0.86	1.66	1.06	3.83	0.00	1.54	-0.03
time (sec)	N/A	0.070	0.040	0.935	1.225	0.453	0.000	0.172	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	67	305	0	67	-1
normalized size	1	1.00	0.76	1.47	1.02	4.62	0.00	1.02	-0.02
time (sec)	N/A	0.132	0.074	1.239	0.684	0.485	0.000	0.231	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	116	111	587	0	104	-1
normalized size	1	1.00	0.75	1.32	1.26	6.67	0.00	1.18	-0.01
time (sec)	N/A	0.483	0.144	1.690	0.771	0.485	0.000	0.200	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	295	43	57	0	0	-1
normalized size	1	1.00	0.92	7.56	1.10	1.46	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.022	0.990	0.646	0.484	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	454	43	69	0	0	-1
normalized size	1	1.00	0.94	12.97	1.23	1.97	0.00	0.00	-0.03
time (sec)	N/A	0.060	0.018	2.237	0.647	0.477	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	47	65	0	0	-1
normalized size	1	1.00	0.98	0.00	1.07	1.48	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.029	2.091	0.650	0.480	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	321	32	65	0	0	-1
normalized size	1	1.00	0.92	8.23	0.82	1.67	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.021	1.042	1.146	0.479	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	478	32	77	0	0	-1
normalized size	1	1.00	0.94	13.66	0.91	2.20	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.018	2.355	1.151	0.496	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	36	73	0	0	-1
normalized size	1	1.00	0.98	0.00	0.82	1.66	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.028	1.979	1.151	0.508	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	54	101	0	0	20
normalized size	1	1.00	0.90	0.62	1.38	2.59	0.00	0.00	0.51
time (sec)	N/A	0.044	0.007	0.144	1.021	0.477	0.000	0.000	0.470

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	70	56	102	0	0	-1
normalized size	1	1.00	1.20	1.71	1.37	2.49	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.008	0.158	1.024	0.500	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	57	129	0	0	-1
normalized size	1	1.00	1.27	1.27	1.54	3.49	0.00	0.00	-0.03
time (sec)	N/A	0.047	0.011	0.343	1.023	0.486	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	-1
normalized size	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.011	0.230	1.027	0.495	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	49	101	0	0	22
normalized size	1	1.00	0.90	0.62	1.26	2.59	0.00	0.00	0.56
time (sec)	N/A	0.046	0.007	0.145	1.022	0.511	0.000	0.000	0.414

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	70	51	102	0	0	-1
normalized size	1	1.00	1.20	1.71	1.24	2.49	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.008	0.151	1.010	0.504	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	59	127	0	0	-1
normalized size	1	1.00	1.27	1.27	1.59	3.43	0.00	0.00	-0.03
time (sec)	N/A	0.048	0.010	0.273	1.024	0.511	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	-1
normalized size	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.012	0.237	1.025	0.486	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	314	31	84	0	0	-1
normalized size	1	1.00	0.97	8.26	0.82	2.21	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.016	1.149	1.150	0.500	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	480	32	106	0	0	-1
normalized size	1	1.00	0.94	13.71	0.91	3.03	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.018	1.412	1.153	0.477	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	36	92	0	0	-1
normalized size	1	1.00	1.00	0.00	0.84	2.14	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.023	1.857	1.152	0.464	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	293	37	76	0	0	-1
normalized size	1	1.00	0.97	7.71	0.97	2.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.016	0.949	0.649	0.475	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	456	45	97	0	0	-1
normalized size	1	1.00	0.94	13.03	1.29	2.77	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.018	1.344	0.646	0.553	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	47	84	0	0	-1
normalized size	1	1.00	1.00	0.00	1.09	1.95	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.023	2.080	0.647	0.492	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	32	112	258	0	94	31
normalized size	1	1.00	0.66	0.64	2.24	5.16	0.00	1.88	0.62
time (sec)	N/A	0.029	0.011	0.889	0.512	0.483	0.000	0.467	0.509

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	12	62	14	31	9
normalized size	1	1.00	1.00	1.08	0.92	4.77	1.08	2.38	0.69
time (sec)	N/A	0.019	0.009	0.288	0.440	0.463	0.908	0.179	0.379

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	11	14	13	9	60	13	9
normalized size	1	1.00	0.65	0.82	0.76	0.53	3.53	0.76	0.53
time (sec)	N/A	0.007	0.002	0.072	0.444	0.456	1.739	0.158	0.034

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	24	23	27	24	25
normalized size	1	1.00	0.96	0.93	0.89	0.85	1.00	0.89	0.93
time (sec)	N/A	0.021	0.004	0.063	0.444	0.473	0.130	0.196	0.121

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	20	0	3	3
normalized size	1	1.00	1.00	1.33	1.00	6.67	0.00	1.00	1.00
time (sec)	N/A	0.034	0.019	0.064	0.982	0.434	0.000	0.164	0.388

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	17	22	22	22	17
normalized size	1	1.00	1.25	0.96	0.71	0.92	0.92	0.92	0.71
time (sec)	N/A	0.023	0.001	0.063	0.443	0.460	0.107	0.159	0.039

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	32	23	22	29	23	25
normalized size	1	1.00	0.92	1.28	0.92	0.88	1.16	0.92	1.00
time (sec)	N/A	0.010	0.005	0.066	0.443	0.439	0.158	0.158	0.064

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	24	20	22	30	20
normalized size	1	1.00	0.88	0.91	0.71	0.59	0.65	0.88	0.59
time (sec)	N/A	0.014	0.007	0.065	0.444	0.475	0.113	0.155	0.051

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	28	24	27	42	22
normalized size	1	1.00	1.00	0.98	0.70	0.60	0.68	1.05	0.55
time (sec)	N/A	0.016	0.009	0.068	0.441	0.439	0.115	0.161	0.377

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	25	25
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.029	0.008	0.065	0.988	0.462	0.143	0.162	0.405

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
normalized size	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.004	0.017	0.066	0.000	0.449	7.481	0.183	0.079

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
normalized size	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.004	0.017	0.073	0.000	0.450	5.784	0.162	0.555

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
normalized size	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.005	0.018	0.067	0.000	0.457	6.924	0.160	0.486

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	0	28	0	40	37
normalized size	1	1.00	1.00	1.21	0.00	0.65	0.00	0.93	0.86
time (sec)	N/A	0.012	0.024	0.071	0.000	0.518	0.000	0.178	1.079

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	66	13	9
normalized size	1	1.00	0.71	0.67	0.62	0.67	3.14	0.62	0.43
time (sec)	N/A	0.007	0.003	0.066	0.445	0.458	2.610	0.158	0.340

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	24	14	10	14	13
normalized size	1	1.00	0.92	1.00	1.85	1.08	0.77	1.08	1.00
time (sec)	N/A	0.005	0.003	0.065	0.525	0.435	0.391	0.161	0.362

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	14	6	3	6	6
normalized size	1	1.00	1.00	1.17	2.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.010	0.001	0.065	0.763	0.432	0.092	0.158	0.355

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	38	34	33	38	184	33	26
normalized size	1	1.00	1.19	1.06	1.03	1.19	5.75	1.03	0.81
time (sec)	N/A	0.205	0.015	0.072	0.630	0.459	0.888	0.183	0.112

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16
normalized size	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.007	0.003	0.070	1.547	0.434	0.141	0.161	0.050

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	29	23	34	23	19
normalized size	1	1.00	1.00	1.10	1.45	1.15	1.70	1.15	0.95
time (sec)	N/A	0.005	0.007	0.067	0.590	0.439	0.486	0.178	0.387

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	44	33	33	32	139	29
normalized size	1	1.00	0.89	1.26	0.94	0.94	0.91	3.97	0.83
time (sec)	N/A	0.006	0.005	0.166	0.611	0.503	0.168	0.187	0.095

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	12	12	15	12	12
normalized size	1	1.00	1.00	1.17	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.003	0.002	0.073	0.608	0.429	0.106	0.172	0.051

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	39	28	28	27	29	40
normalized size	1	1.00	0.75	1.08	0.78	0.78	0.75	0.81	1.11
time (sec)	N/A	0.015	0.010	0.085	0.774	0.426	0.146	0.168	0.486

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	42	42	48	43	56
normalized size	1	1.00	1.00	0.98	0.78	0.78	0.89	0.80	1.04
time (sec)	N/A	0.039	0.015	0.084	0.683	0.441	0.168	0.174	0.585

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	55	52	42	41	31	46
normalized size	1	1.00	0.94	1.57	1.49	1.20	1.17	0.89	1.31
time (sec)	N/A	0.015	0.021	0.066	0.570	0.415	0.192	0.160	0.473

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	82	74	64	63	31	73
normalized size	1	1.00	1.26	2.34	2.11	1.83	1.80	0.89	2.09
time (sec)	N/A	0.025	0.017	0.069	0.599	0.462	0.227	0.221	0.461

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.018	0.002	0.063	0.697	0.520	0.258	0.162	0.546

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	32	31	21	26	32	25
normalized size	1	1.00	0.68	1.03	1.00	0.68	0.84	1.03	0.81
time (sec)	N/A	0.022	0.006	0.079	0.671	0.499	0.199	0.175	0.492

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	96	44	47	185	0	52
normalized size	1	1.00	0.68	2.18	1.00	1.07	4.20	0.00	1.18
time (sec)	N/A	0.030	0.014	0.073	0.533	0.451	1.470	0.000	0.460

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	19	34	45	18
normalized size	1	1.00	1.00	1.06	1.33	1.06	1.89	2.50	1.00
time (sec)	N/A	0.011	0.018	0.067	0.660	0.437	1.149	0.159	0.379

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	121	126	26	71
normalized size	1	1.00	1.00	0.75	0.00	3.78	3.94	0.81	2.22
time (sec)	N/A	0.018	0.025	0.071	0.000	0.436	5.978	0.158	0.469

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	120	0	480	330	239	153
normalized size	1	1.00	0.78	0.83	0.00	3.33	2.29	1.66	1.06
time (sec)	N/A	0.094	0.051	0.073	0.000	0.491	52.756	0.209	2.283

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	167	168	0	195	246	170	95
normalized size	1	1.00	0.74	0.74	0.00	0.86	1.08	0.75	0.42
time (sec)	N/A	0.162	0.065	0.069	0.000	0.460	36.006	0.215	2.239

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	35	33	28	99	53	27
normalized size	1	1.00	1.26	1.30	1.22	1.04	3.67	1.96	1.00
time (sec)	N/A	0.023	0.016	0.070	0.702	0.436	2.703	0.166	0.365

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	47	43	0	143	177	38	45
normalized size	1	1.00	1.18	1.08	0.00	3.58	4.42	0.95	1.12
time (sec)	N/A	0.027	0.026	0.071	0.000	0.487	7.893	0.164	0.375

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	136	0	149	367	257	174
normalized size	1	1.00	0.89	0.91	0.00	1.00	2.46	1.72	1.17
time (sec)	N/A	0.109	0.053	0.070	0.000	0.470	54.105	0.223	2.401

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	211	181	0	192	270	178	176
normalized size	1	1.00	0.91	0.78	0.00	0.82	1.16	0.76	0.76
time (sec)	N/A	0.183	0.067	0.108	0.000	0.469	36.989	0.213	2.212

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	0	21	22	0	19
normalized size	1	1.00	1.00	0.91	0.00	0.95	1.00	0.00	0.86
time (sec)	N/A	0.018	0.030	0.071	0.000	0.417	0.172	0.000	0.431

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	37	39	22	0	37
normalized size	1	1.00	1.02	0.93	0.90	0.95	0.54	0.00	0.90
time (sec)	N/A	0.058	0.092	0.060	1.495	0.461	0.184	0.000	0.574

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	0	39	22	0	37
normalized size	1	1.00	1.02	0.93	0.00	0.95	0.54	0.00	0.90
time (sec)	N/A	0.033	0.076	0.069	0.000	0.433	0.194	0.000	0.354

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	40	0	41	19	0	37
normalized size	1	1.00	1.05	0.95	0.00	0.98	0.45	0.00	0.88
time (sec)	N/A	0.046	0.094	0.072	0.000	0.432	0.185	0.000	0.598

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	17	21	27	22	21
normalized size	1	1.00	1.00	0.84	0.53	0.66	0.84	0.69	0.66
time (sec)	N/A	0.021	0.002	0.070	0.680	0.441	0.129	0.162	0.410

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	0	0	0	0	0	-1
normalized size	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.011	0.113	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	60	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.013	0.111	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	0	0	0	0	32	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.74	-0.02
time (sec)	N/A	0.031	0.010	1.320	0.000	0.000	0.000	0.202	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	102	0	0	0	0	-1
normalized size	1	1.00	0.99	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.078	0.071	0.000	0.423	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	114	111	0	0	0	-1
normalized size	1	1.00	0.90	1.41	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.032	0.077	0.682	0.418	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	5	12	8	10	0	4
normalized size	1	1.00	1.00	0.56	1.33	0.89	1.11	0.00	0.44
time (sec)	N/A	0.009	0.002	0.063	0.638	0.423	1.724	0.000	0.018

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	77	82	0	0	0	59
normalized size	1	1.00	0.81	1.79	1.91	0.00	0.00	0.00	1.37
time (sec)	N/A	0.073	0.019	0.066	0.637	0.460	0.000	0.000	0.315

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	31	49	0	192	0	-1
normalized size	1	1.00	1.03	1.03	1.63	0.00	6.40	0.00	-0.03
time (sec)	N/A	0.099	0.011	0.073	0.551	0.438	117.576	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	0	0	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.027	0.112	0.079	1.093	0.426	0.000	0.000	0.417

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	57	57	58	46	59	59
normalized size	1	1.00	0.93	0.84	0.84	0.85	0.68	0.87	0.87
time (sec)	N/A	0.251	0.032	0.083	1.292	0.421	0.249	0.216	0.456

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	31	24	23	23
normalized size	1	1.00	1.00	0.83	0.79	1.07	0.83	0.79	0.79
time (sec)	N/A	0.018	0.012	0.064	0.624	0.432	7.630	0.163	0.076

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	18	18	14	80	18
normalized size	1	1.00	1.00	1.56	1.00	1.00	0.78	4.44	1.00
time (sec)	N/A	0.004	0.002	0.201	0.635	0.450	0.108	0.165	0.362

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	35	25	21	15	103	21
normalized size	1	1.00	0.78	1.30	0.93	0.78	0.56	3.81	0.78
time (sec)	N/A	0.005	0.003	0.166	0.581	0.425	0.121	0.226	0.083

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	60	112	54	54	41	56	55
normalized size	1	1.00	1.05	1.96	0.95	0.95	0.72	0.98	0.96
time (sec)	N/A	0.057	0.050	0.280	1.452	0.425	0.200	0.212	0.417

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	249	0	0	0	0	-1
normalized size	1	1.00	1.03	4.15	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.007	0.618	0.000	0.429	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	239	158	0	0	0	0	-1
normalized size	1	1.00	3.92	2.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	0.046	0.191	0.000	0.407	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	373	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.196	180.000	0.000	0.427	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	37	0	0	-1
normalized size	1	1.00	4.62	0.00	0.00	1.28	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.648	0.502	0.000	0.437	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	36	0	0	-1
normalized size	1	1.00	4.62	0.00	0.00	1.24	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.574	0.507	0.000	0.442	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	10	10	8	10	17
normalized size	1	1.00	1.00	0.88	0.59	0.59	0.47	0.59	1.00
time (sec)	N/A	0.003	0.003	0.063	0.624	0.394	0.079	0.169	0.065

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	27	16	20	34	16	49
normalized size	1	1.00	0.93	1.00	0.59	0.74	1.26	0.59	1.81
time (sec)	N/A	0.009	0.026	0.158	0.527	0.456	2.550	0.229	0.598

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	31	25	34	26	25	0	26	27
normalized size	1	1.24	1.00	1.36	1.04	1.00	0.00	1.04	1.08
time (sec)	N/A	0.053	0.024	0.062	0.742	0.401	0.000	0.156	0.661

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	26	24	23	26	24	20
normalized size	1	1.00	0.85	1.00	0.92	0.88	1.00	0.92	0.77
time (sec)	N/A	0.035	0.043	0.623	0.857	0.425	7.376	0.173	0.366

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.036	7.029	0.077	0.000	0.400	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.022	5.719	0.070	0.000	0.412	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.008	0.006	0.069	0.000	0.408	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.032	0.038	0.072	0.000	0.441	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.034	6.660	0.075	0.000	0.405	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	9	11	10	11	11
normalized size	1	1.00	1.00	0.92	0.69	0.85	0.77	0.85	0.85
time (sec)	N/A	0.024	0.009	0.066	0.788	0.430	0.118	0.161	0.464

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	10	11	10	10	8	14	10
normalized size	1	1.00	1.11	1.22	1.11	1.11	0.89	1.56	1.11
time (sec)	N/A	0.072	0.051	0.073	0.896	0.421	0.141	0.172	0.374

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	13	15	0	13
normalized size	1	1.00	1.00	0.00	0.00	1.00	1.15	0.00	1.00
time (sec)	N/A	0.142	0.041	0.074	0.000	0.398	4.295	0.000	0.395

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	108	67	48	53	88	63
normalized size	1	1.00	0.79	1.61	1.00	0.72	0.79	1.31	0.94
time (sec)	N/A	0.074	0.033	0.145	0.558	0.455	40.984	0.246	0.644

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	68	49	53	53	38
normalized size	1	1.00	1.06	0.00	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.057	0.039	180.000	0.590	0.454	45.788	0.228	0.448

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	18	18	17	47	18
normalized size	1	1.00	0.90	1.05	0.86	0.86	0.81	2.24	0.86
time (sec)	N/A	0.008	0.004	0.172	0.592	0.399	0.111	0.157	0.057

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	68	49	53	53	38
normalized size	1	1.00	1.06	0.00	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.049	0.040	180.000	0.631	0.428	46.257	0.227	0.431

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	107	67	48	53	88	63
normalized size	1	1.00	0.93	1.55	0.97	0.70	0.77	1.28	0.91
time (sec)	N/A	0.052	0.029	0.137	0.607	0.433	41.295	0.269	0.324

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	11	9	9	10	0	9
normalized size	1	1.00	1.00	1.22	1.00	1.00	1.11	0.00	1.00
time (sec)	N/A	0.022	0.014	0.072	0.823	0.441	0.263	0.000	0.366

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	16	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.014	0.275	0.000	0.457	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	0	108	0	0	89	-1
normalized size	1	1.00	1.20	0.00	1.80	0.00	0.00	1.48	-0.02
time (sec)	N/A	0.069	0.127	0.176	0.704	0.000	0.000	0.284	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	94	0	0	74	-1
normalized size	1	1.00	1.11	0.00	1.47	0.00	0.00	1.16	-0.02
time (sec)	N/A	0.068	0.097	0.172	0.647	0.000	0.000	0.338	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	156	0	0	129	-1
normalized size	1	1.00	1.16	0.00	2.26	0.00	0.00	1.87	-0.01
time (sec)	N/A	0.069	0.148	0.148	0.742	0.000	0.000	0.305	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	79	0	130	0	0	106	-1
normalized size	1	1.00	1.11	0.00	1.83	0.00	0.00	1.49	-0.01
time (sec)	N/A	0.079	0.130	0.149	0.664	0.000	0.000	0.306	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	0	94	234	0	0	-1
normalized size	1	1.00	0.97	0.00	0.96	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.062	1.438	2.057	0.509	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	70	174	0	0	-1
normalized size	1	1.00	0.99	0.00	0.88	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.040	1.333	1.922	0.469	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	47	368	43	109	0	0	-1
normalized size	1	1.00	0.90	7.08	0.83	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.031	0.840	1.764	0.456	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.019	2.368	3.178	0.000	0.458	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.345	2.014	3.264	0.000	0.461	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	0	94	241	0	0	-1
normalized size	1	1.00	0.97	0.00	0.91	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.069	2.473	2.417	0.491	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	70	181	0	0	-1
normalized size	1	1.00	0.96	0.00	0.82	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.067	2.354	1.428	0.513	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	583	43	116	0	0	-1
normalized size	1	1.00	0.98	10.23	0.75	2.04	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.030	1.045	1.291	0.492	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.022	0.876	4.235	0.000	0.457	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	2.465	4.358	0.000	0.483	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [1.667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	0	0	0.00	0	0.000
2	A	10	5	1.00	32	0.156
3	A	8	5	1.00	32	0.156
4	A	6	5	1.00	30	0.167
5	A	2	2	1.00	14	0.143
6	A	0	0	0.00	0	0.000
7	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	0	0	0.00	0	0.000
9	A	13	5	1.00	28	0.179
10	A	8	5	1.00	28	0.179
11	A	5	4	1.00	26	0.154
12	A	1	1	1.00	10	0.100
13	A	0	0	0.00	0	0.000
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	1	1	1.00	43	0.023
17	A	1	1	1.00	43	0.023
18	A	1	1	1.00	41	0.024
19	A	4	3	1.00	25	0.120
20	A	1	1	1.00	43	0.023
21	A	1	1	1.00	43	0.023
22	A	1	1	1.00	43	0.023
23	A	3	2	1.00	39	0.051
24	A	3	2	1.00	37	0.054
25	A	2	1	1.00	15	0.067
26	A	2	2	1.00	34	0.059
27	A	3	2	1.00	37	0.054
28	A	3	2	1.00	39	0.051
29	A	2	2	1.00	45	0.044
30	A	0	0	0.00	0	0.000
31	A	9	4	1.00	40	0.100
32	A	7	4	1.00	40	0.100
33	A	5	4	1.00	38	0.105
34	A	4	3	1.00	22	0.136
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	1	1	1.00	60	0.017
39	A	0	0	0.00	0	0.000
40	A	1	1	1.00	39	0.026
41	A	1	1	1.00	40	0.025
42	A	1	1	1.00	41	0.024
43	A	1	1	1.00	42	0.024

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	1	1	1.00	40	0.025
45	A	1	1	1.00	41	0.024
46	A	3	3	1.00	19	0.158
47	A	3	3	1.00	21	0.143
48	A	3	3	1.00	19	0.158
49	A	3	3	1.00	17	0.176
50	A	4	3	1.00	15	0.200
51	A	1	1	1.00	19	0.053
52	A	3	3	1.00	19	0.158
53	A	3	3	1.00	19	0.158
54	A	3	3	1.00	19	0.158
55	A	2	2	1.00	9	0.222
56	A	1	1	1.00	13	0.077
57	A	3	3	1.00	11	0.273
58	A	1	1	1.00	15	0.067
59	A	3	3	1.00	18	0.167
60	A	3	2	1.00	18	0.111
61	A	3	2	1.00	18	0.111
62	A	3	2	1.00	18	0.111
63	A	3	2	1.00	16	0.125
64	A	3	2	1.00	14	0.143
65	A	7	6	1.00	18	0.333
66	A	3	2	1.00	18	0.111
67	A	3	2	1.00	18	0.111
68	A	3	2	1.00	18	0.111
69	A	3	2	1.00	18	0.111
70	A	5	3	1.00	19	0.158
71	A	7	6	1.00	19	0.316
72	A	7	6	1.00	19	0.316
73	A	7	6	1.00	19	0.316
74	A	7	6	1.00	17	0.353
75	A	6	6	1.00	15	0.400
76	A	7	4	1.00	19	0.210
77	A	7	6	1.00	19	0.316
78	A	7	6	1.00	19	0.316
79	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	7	6	1.00	19	0.316
81	A	6	6	1.00	7	0.857
82	A	7	6	1.00	23	0.261
83	A	7	6	1.00	23	0.261
84	A	7	6	1.00	23	0.261
85	A	7	6	1.00	21	0.286
86	A	6	6	1.00	15	0.400
87	A	9	5	1.00	23	0.217
88	A	7	6	1.00	23	0.261
89	A	7	6	1.00	23	0.261
90	A	7	6	1.00	23	0.261
91	A	7	6	1.00	23	0.261
92	A	6	7	1.00	25	0.280
93	A	8	9	1.00	32	0.281
94	A	20	6	1.00	25	0.240
95	A	20	6	1.00	28	0.214
96	A	14	10	1.00	16	0.625
97	A	27	14	1.00	17	0.824
98	A	28	12	1.00	21	0.571
99	A	27	14	1.00	9	1.556
100	A	34	16	1.00	13	1.231
101	A	25	11	1.00	21	0.524
102	A	20	10	1.00	21	0.476
103	A	16	10	1.00	19	0.526
104	A	13	9	1.00	17	0.529
105	A	0	0	0.00	0	0.000
106	A	19	11	1.00	21	0.524
107	A	20	12	1.00	21	0.571
108	A	15	12	1.00	23	0.522
109	A	13	12	1.00	23	0.522
110	A	12	10	1.00	23	0.435
111	A	15	13	1.00	23	0.565
112	A	18	13	1.00	23	0.565
113	A	6	5	1.00	12	0.417
114	A	5	5	1.00	12	0.417
115	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	4	4	1.00	8	0.500
117	A	0	0	0.00	0	0.000
118	A	5	4	1.00	20	0.200
119	A	4	4	1.00	20	0.200
120	A	3	3	1.00	18	0.167
121	A	2	2	1.00	16	0.125
122	A	0	0	0.00	0	0.000
123	A	6	5	1.00	20	0.250
124	A	5	5	1.00	20	0.250
125	A	4	4	1.00	18	0.222
126	A	4	4	1.00	16	0.250
127	A	0	0	0.00	0	0.000
128	A	3	3	1.00	16	0.188
129	A	2	2	1.00	10	0.200
130	A	2	2	1.00	12	0.167
131	A	2	2	1.00	10	0.200
132	A	2	2	1.00	10	0.200
133	A	2	1	1.00	12	0.083
134	A	3	2	1.00	14	0.143
135	A	2	1	1.00	16	0.062
136	A	2	1	1.00	14	0.071
137	A	7	6	1.00	16	0.375
138	A	2	1	1.00	15	0.067
139	A	2	2	1.00	8	0.250
140	A	3	2	1.00	9	0.222
141	A	3	2	1.00	16	0.125
142	A	3	2	1.00	16	0.125
143	A	4	3	1.00	18	0.167
144	A	3	2	1.00	16	0.125
145	A	3	2	1.00	14	0.143
146	A	3	2	1.00	16	0.125
147	A	4	4	1.00	16	0.250
148	A	3	1	1.00	20	0.050
149	A	3	2	1.00	14	0.143
150	A	3	2	1.00	14	0.143
151	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
152	A	3	2	1.00	16	0.125
153	A	4	3	1.00	10	0.300
154	A	5	6	1.00	9	0.667
155	A	5	6	1.00	11	0.546
156	A	15	8	1.00	11	0.727
157	A	5	6	1.00	9	0.667
158	A	7	8	1.00	11	0.727
159	A	15	8	1.00	11	0.727
160	A	4	3	1.00	12	0.250
161	A	5	5	1.00	5	1.000
162	A	6	6	1.00	7	0.857
163	A	6	6	1.00	7	0.857
164	A	5	5	1.00	5	1.000
165	A	6	6	1.00	7	0.857
166	A	6	6	1.00	7	0.857
167	A	7	5	1.00	5	1.000
168	A	8	6	1.00	7	0.857
169	A	8	6	1.00	7	0.857
170	A	7	5	1.00	5	1.000
171	A	8	6	1.00	7	0.857
172	A	8	6	1.00	7	0.857
173	A	5	5	1.00	5	1.000
174	A	6	6	1.00	7	0.857
175	A	6	6	1.00	7	0.857
176	A	5	5	1.00	5	1.000
177	A	6	6	1.00	7	0.857
178	A	6	6	1.00	7	0.857
179	A	3	3	1.00	16	0.188
180	A	3	2	1.00	10	0.200
181	A	5	4	1.00	10	0.400
182	A	3	4	1.00	8	0.500
183	A	2	3	1.00	6	0.500
184	A	4	4	1.00	10	0.400
185	A	2	2	1.00	35	0.057
186	A	3	3	1.00	8	0.375
187	A	2	3	1.00	6	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	2	2	1.00	6	0.333
189	A	4	5	1.00	6	0.833
190	A	2	2	1.00	6	0.333
191	A	10	9	1.00	8	1.125
192	A	7	7	1.00	8	0.875
193	A	8	8	1.00	7	1.143
194	A	3	4	1.00	8	0.500
195	A	5	6	1.00	9	0.667
196	A	7	8	1.00	11	0.727
197	A	15	8	1.00	11	0.727
198	A	5	6	1.00	9	0.667
199	A	7	8	1.00	11	0.727
200	A	15	8	1.00	11	0.727
201	A	5	5	1.00	5	1.000
202	A	6	6	1.00	7	0.857
203	A	6	6	1.00	7	0.857
204	A	5	5	1.00	5	1.000
205	A	6	6	1.00	7	0.857
206	A	6	6	1.00	7	0.857
207	A	7	5	1.00	3	1.667
208	A	7	5	1.00	5	1.000
209	A	8	6	1.00	7	0.857
210	A	8	6	1.00	7	0.857
211	A	7	5	1.00	3	1.667
212	A	7	5	1.00	5	1.000
213	A	8	6	1.00	7	0.857
214	A	8	6	1.00	7	0.857
215	A	5	5	1.00	5	1.000
216	A	6	6	1.00	7	0.857
217	A	6	6	1.00	7	0.857
218	A	5	5	1.00	5	1.000
219	A	6	6	1.00	7	0.857
220	A	6	6	1.00	7	0.857
221	A	2	2	1.00	35	0.057
222	A	3	3	1.00	8	0.375
223	A	1	1	1.00	8	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
224	A	3	3	1.00	10	0.300
225	A	2	1	1.00	16	0.062
226	A	3	3	1.00	9	0.333
227	A	2	2	1.00	10	0.200
228	A	3	2	1.00	10	0.200
229	A	3	2	1.00	12	0.167
230	A	4	3	1.00	8	0.375
231	A	2	2	1.00	12	0.167
232	A	2	2	1.00	12	0.167
233	A	2	2	1.00	14	0.143
234	A	6	6	1.00	14	0.429
235	A	1	1	1.00	8	0.125
236	A	2	2	1.00	4	0.500
237	A	1	1	1.00	10	0.100
238	A	3	1	1.00	11	0.091
239	A	3	3	1.00	6	0.500
240	A	2	2	1.00	8	0.250
241	A	2	2	1.00	14	0.143
242	A	2	2	1.00	6	0.333
243	A	4	3	1.00	10	0.300
244	A	4	3	1.00	14	0.214
245	A	2	2	1.00	12	0.167
246	A	2	2	1.00	14	0.143
247	A	2	2	1.00	14	0.143
248	A	2	2	1.00	14	0.143
249	A	2	2	1.00	14	0.143
250	A	2	1	1.00	15	0.067
251	A	2	1	1.00	17	0.059
252	A	7	6	1.00	17	0.353
253	A	10	6	1.00	17	0.353
254	A	3	1	1.00	17	0.059
255	A	3	2	1.00	17	0.118
256	A	8	7	1.00	17	0.412
257	A	11	7	1.00	17	0.412
258	A	3	2	1.00	18	0.111
259	A	5	4	1.00	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	5	4	1.00	21	0.190
261	A	7	5	1.00	27	0.185
262	A	2	2	1.00	10	0.200
263	A	3	3	1.00	12	0.250
264	A	3	3	1.00	12	0.250
265	A	3	3	1.00	12	0.250
266	A	8	7	1.00	15	0.467
267	A	7	7	1.00	15	0.467
268	A	1	1	1.00	8	0.125
269	A	6	6	1.00	19	0.316
270	A	5	4	1.00	19	0.210
271	A	2	1	1.00	14	0.071
272	A	8	7	1.00	24	0.292
273	A	4	3	1.00	8	0.375
274	A	2	2	1.00	9	0.222
275	A	2	2	1.00	10	0.200
276	A	8	6	1.00	22	0.273
277	A	5	6	1.00	18	0.333
278	A	5	6	1.00	20	0.300
279	A	5	6	1.00	25	0.240
280	A	1	1	1.00	39	0.026
281	A	1	1	1.00	39	0.026
282	A	2	2	1.00	8	0.250
283	A	3	3	1.00	10	0.300
284	A	5	5	1.24	12	0.417
285	A	3	4	1.00	10	0.400
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	2	2	1.00	14	0.143
292	A	2	2	1.00	16	0.125
293	A	8	6	1.00	14	0.429
294	A	5	3	1.00	14	0.214
295	A	6	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
296	A	4	4	1.00	12	0.333
297	A	5	3	1.00	14	0.214
298	A	7	4	1.00	14	0.286
299	A	2	1	1.00	14	0.071
300	A	3	3	1.00	6	0.500
301	A	4	4	1.00	13	0.308
302	A	4	4	1.00	14	0.286
303	A	4	4	1.00	17	0.235
304	A	4	4	1.00	18	0.222
305	A	13	13	1.00	10	1.300
306	A	12	12	1.00	8	1.500
307	A	7	6	1.00	6	1.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	14	12	1.00	13	0.923
311	A	13	11	1.00	11	1.000
312	A	7	6	1.00	9	0.667
313	A	0	0	0.00	0	0.000
314	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=76

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{bn(p+1)q}-\frac{am\text{Int}\left(x^{m-1}\left(ax^m+b\log^q(cx^n)\right)^p,x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)/b/n/q+(a*x^m+b*ln(
c*x^n)^q)^(1+p)/b/n/(1+p)/q
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(b*n*(1 + p)*q) - (a*m*Defer[Int][x^(-1 +
m)*(a*x^m + b*Log[c*x^n]^q)^p, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{\left(ax^m+b\log^q(cx^n)\right)^{1+p}}{bn(1+p)q}-\frac{(am)\int x^{-1+m}\left(ax^m+b\log^q(cx^n)\right)^p\,dx}{bnq}$$

Mathematica [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^p \log\left(cx^n\right)^{q-1}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="fricas
")
```

```
[Out] integral((a*x^m + b*log(c*x^n)^q)^p*log(c*x^n)^(q - 1)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to check sign: (4*pi/(pi*sign(n*t_nostep+ln(abs(c)
))*sign(pi*sign(c)-pi)-pi*sign(pi*sign(c)-pi)-2*atan((pi*sign(c)-pi)/(2*n*t
_nostep+2*ln(abs(c)))))/2)>(-4*pi/(pi*sign(n*t_nostep+ln(abs(c)))*sign(pi*s
ign(c)-pi)-pi*sign(pi*sign(c)-pi)-2*atan((pi*sign(c)-pi)/(2*n*t_nostep+2*ln
(abs(c)))))/2)Unable to check sign: (4*pi/(pi*sign(n*t_nostep+ln(abs(c)))*s
ign(pi*sign(c)-pi)-pi*sign(pi*sign(c)-pi)-2*atan((pi*sign(c)-pi)/(2*n*t_nos
tep+2*ln(abs(c)))))/2)>(-4*pi/(pi*sign(n*t_nostep+ln(abs(c)))*sign(pi*sign(
c)-pi)-pi*sign(pi*sign(c)-pi)-2*atan((pi*sign(c)-pi)/(2*n*t_nostep+2*ln(abs
(c)))))/2)Evaluation time: 2.46Unable to divide, perhaps due to rounding er
ror%%%{1,[0,0,2,5,2,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2]%%%}
+%%%{5,[0,0,2,4,2,0,4,1,2,1,2,2]%%%}+%%%{1,[0,0,2,3,2,2,5,0,0,1,2,2]%%%}+%%
%{-8,[0,0,2,3,2,1,4,1,1,1,2,2]%%%}+%%%{10,[0,0,2,3,2,0,3,2,2,1,2,2]%%%}+%%%
{3,[0,0,2,2,2,2,4,1,0,1,2,2]%%%}+%%%{-12,[0,0,2,2,2,1,3,2,1,1,2,2]%%%}+%%%{
10,[0,0,2,2,2,0,2,3,2,1,2,2]%%%}+%%%{3,[0,0,2,1,2,2,3,2,0,1,2,2]%%%}+%%%{-8
,[0,0,2,1,2,1,2,3,1,1,2,2]%%%}+%%%{5,[0,0,2,1,2,0,1,4,2,1,2,2]%%%}+%%%{1,[0
,0,2,0,2,2,2,3,0,1,2,2]%%%}+%%%{-2,[0,0,2,0,2,1,1,4,1,1,2,2]%%%}+%%%{1,[0,0
,2,0,2,0,0,5,2,1,2,2]%%%} / %%%{1,[0,0,2,5,3,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0
,2,4,3,1,5,0,1,1,2,2]%%%}+%%%{5,[0,0,2,4,3,0,4,1,2,1,2,2]%%%}+%%%{1,[0,0,2,
3,3,2,5,0,0,1,2,2]%%%}+%%%{-8,[0,0,2,3,3,1,4,1,1,1,2,2]%%%}+%%%{10,[0,0,2,3
,3,0,3,2,2,1,2,2]%%%}+%%%{3,[0,0,2,2,3,2,4,1,0,1,2,2]%%%}+%%%{-12,[0,0,2,2,
3,1,3,2,1,1,2,2]%%%}+%%%{10,[0,0,2,2,3,0,2,3,2,1,2,2]%%%}+%%%{3,[0,0,2,1,3,
2,3,2,0,1,2,2]%%%}+%%%{-8,[0,0,2,1,3,1,2,3,1,1,2,2]%%%}+%%%{5,[0,0,2,1,3,0,
1,4,2,1,2,2]%%%}+%%%{1,[0,0,2,0,3,2,2,3,0,1,2,2]%%%}+%%%{-2,[0,0,2,0,3,1,1,
4,1,1,2,2]%%%}+%%%{1,[0,0,2,0,3,0,0,5,2,1,2,2]%%%} Error: Bad Argument Valu
e
```

maple [A] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^p \ln\left(c\,x^n\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Timed out
```

3.2 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=231

$$-\frac{a^3 3^{-q} x^{3m} (cx^n)^{-\frac{3m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)}{n}-\frac{3a^2b4^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\Gamma}{n}$$

[Out] 1/4*b^3*ln(c*x^n)^(4*q)/n/q-3*a*b^2*x^m*GAMMA(3*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(3*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(3*q))-3*a^2*b*x^(2*m)*GAMMA(2*q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/(4^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^3*x^(3*m)*GAMMA(q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/n/((c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)

Rubi [A] time = 0.35, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2539, 2310, 2181, 2302, 30}

$$-\frac{3a^2b4^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\text{Gamma}\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{a^3 3^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log}{}\right.}{n}$$

Antiderivative was successfully verified.

[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]

[Out] (b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = \int \left(a^3x^{-1+3m}\log^{-1+q}(cx^n)+3a^2bx^{-1+2m}\log^{-1+2q}(cx^n)+3ab^2x^{-1}\right.$$

$$= a^3\int x^{-1+3m}\log^{-1+q}(cx^n)\,dx+\left(3a^2b\right)\int x^{-1+2m}\log^{-1+2q}(cx^n)\,d$$

$$= \frac{b^3\,\mathrm{Subst}\left(\int x^{-1+4q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\right)\mathrm{Subst}\left(\int e\right.}{n}$$

$$= \frac{b^3\log^{4q}(cx^n)}{4nq}-\frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m}{}\right.}{n}$$

Mathematica [A] time = 0.77, size = 223, normalized size = 0.97

$$\frac{\log^q(cx^n)\left(-4a^33^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)-3a^2b4^{1-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right.\right.}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^3*Log[c*x^n]^(3*q))/q - (12*a*b^2*x^m*Gamma[3*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q))/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q))
- (3*4^(1 - q)*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/
((c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (4*a^3*x^(3*m)*Gamma[q, (
-3*m*Log[c*x^n])/n])/(3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)))/(4*n
)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{3\,ab^2x^m\log(cx^n)^{2\,q}\log(cx^n)^{q-1}+3\,a^2bx^{2\,m}\log(cx^n)^{q-1}\log(cx^n)^q+a^3x^{3\,m}\log(cx^n)^{q-1}+b^3\log(cx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="fricas
")
```

```
[Out] integral((3*a*b^2*x^m*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1) + 3*a^2*b*x^(2*m)
*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^3*x^(3*m)*log(c*x^n)^(q - 1) + b^3*log
(c*x^n)^(3*q)*log(c*x^n)^(q - 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m+b\log(cx^n)^q\right)^3\log(cx^n)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*log(c*x^n)^(q - 1)/x, x)
```

maple [F] time = 20.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a x^{m}+b \ln \left(c x^{n}\right)^{q}\right)^{3} \ln \left(c x^{n}\right)^{q-1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c x^{n}\right)^{q-1} \left(a x^{m}+b \ln \left(c x^{n}\right)^{q}\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Timed out
```

3.3 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=156

$$-\frac{a^2 2^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{2abx^m (cx^n)^{-\frac{m}{n}} \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-2q} \Gamma\left(2\right.}{n}$$

```
[Out] 1/3*b^2*ln(c*x^n)^(3*q)/n/q-2*a*b*x^m*GAMMA(2*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(
2*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*x^(2*m)*GAMMA(q,-2*m*ln
(c*x^n)/n)*ln(c*x^n)^q/(2^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2539, 2310, 2181, 2302, 30}

$$-\frac{a^2 2^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \text{Gamma}\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{2abx^m (cx^n)^{-\frac{m}{n}} \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)]
*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x^(
2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n)*(
-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
```

```
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = \int\left(a^2x^{-1+2m}\log^{-1+q}(cx^n)+2abx^{-1+m}\log^{-1+2q}(cx^n)+\frac{b^2\log^{-1+3q}(\ldots}{x}\right.$$
$$= a^2\int x^{-1+2m}\log^{-1+q}(cx^n)\,dx+(2ab)\int x^{-1+m}\log^{-1+2q}(cx^n)\,dx+b^{\ldots}$$
$$= \frac{b^2\operatorname{Subst}\left(\int x^{-1+3q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\right)\operatorname{Subst}\left(\int e^{\frac{2m\ldots}{n}}\ldots\right.}{n}$$
$$= \frac{b^2\log^{3q}(cx^n)}{3nq}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(\ldots}{n}\right.}{n}$$

Mathematica [A] time = 0.44, size = 149, normalized size = 0.96

$$\frac{\log^q(cx^n)\left(-3a^2 2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)-6abx^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\Gamma\left(2\ldots\right.\right.}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^2*Log[c*x^n]^(2*q))/q - (6*a*b*x^m*Gamma[2*q, -((m*Log[c*
x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (3*a^
2*x^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n])/(2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q)))/(3*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{2\,abx^m\log(cx^n)^{q-1}\log(cx^n)^q+a^2x^{2\,m}\log(cx^n)^{q-1}+b^2\log(cx^n)^{2\,q}\log(cx^n)^{q-1}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*x^m*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^2*x^(2*m)*log(c*x^n
)^(q - 1) + b^2*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m+b\log(cx^n)^q\right)^2\log(cx^n)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*log(c*x^n)^(q - 1)/x, x)
```

maple [F] time = 13.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^m+b\ln(c\,x^n)^q\right)^2\ln(c\,x^n)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}\,{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{m} + b \log{\left(cx^{n} \right)}^{q}\right)^{2} \log{\left(cx^{n} \right)}^{q - 1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)**2*log(c*x**n)**(q - 1)/x, x)
```

3.4 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=81

$$\frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

```
[Out] 1/2*b*ln(c*x^n)^(2*q)/n/q-a*x^m*GAMMA(q,-m*ln(c*x^n)/n)*ln(c*x^n)^q/n/((c*x
^n)^(m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2539, 2310, 2181, 2302, 30}

$$\frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\text{Gamma}\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x
^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \int \left(ax^{-1+m}\log^{-1+q}(cx^n)+\frac{b\log^{-1+2q}(cx^n)}{x}\right)dx$$
$$= a\int x^{-1+m}\log^{-1+q}(cx^n)\,dx + b\int \frac{\log^{-1+2q}(cx^n)}{x}\,dx$$
$$= \frac{b\,\mathrm{Subst}\left(\int x^{-1+2q}\,dx, x, \log(cx^n)\right)}{n} + \frac{\left(ax^m(cx^n)^{-\frac{m}{n}}\right)\mathrm{Subst}\left(\int e^{\frac{mx}{n}}x\right.}{n}$$
$$= \frac{b\log^{2q}(cx^n)}{2nq} - \frac{ax^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right.}{n}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.95

$$\frac{\log^q(cx^n)\left(\frac{b\log^q(cx^n)}{q}-2ax^m(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
[Out] (Log[c*x^n]^q*((b*Log[c*x^n]^q)/q - (2*a*x^m*Gamma[q, -((m*Log[c*x^n])/n)])
/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)))/(2*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ax^m\log(cx^n)^{q-1}+b\log(cx^n)^{q-1}\log(cx^n)^q}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="fricas")
[Out] integral((a*x^m*log(c*x^n)^(q - 1) + b*log(c*x^n)^(q - 1)*log(c*x^n)^q)/x,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m+b\log(cx^n)^q\right)\log(cx^n)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="giac")
[Out] integrate((a*x^m + b*log(c*x^n)^q)*log(c*x^n)^(q - 1)/x, x)
```

maple [F] time = 21.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^m+b\ln(c\,x^n)^q\right)\ln(c\,x^n)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)/x,x)
[Out] int(ln(c*x^n)^(q-1)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{m} + b\log{\left(cx^{n} \right)}^{q}\right)\log{\left(cx^{n} \right)}^{q - 1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)*log(c*x**n)**(q - 1)/x, x)
```

3.5 $\int \frac{\log^{-1+q}(cx^n)}{x} dx$

Optimal. Leaf size=15

$$\frac{\log^q(cx^n)}{nq}$$

```
[Out] ln(c*x^n)^q/n/q
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 30}

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^{-1+q}(cx^n)}{x} dx &= \frac{\text{Subst}\left(\int x^{-1+q} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log^q(cx^n)}{nq} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

fricas [A] time = 0.45, size = 25, normalized size = 1.67

$$\frac{\left(n\log(x) + \log(c)\right)\left(n\log(x) + \log(c)\right)^{q-1}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="fricas")
[Out] (n*log(x) + log(c))*(n*log(x) + log(c))^(q - 1)/(n*q)
```

giac [A] time = 0.16, size = 16, normalized size = 1.07

$$\frac{\left(n\log(x) + \log(c)\right)^{q}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="giac")
[Out] (n*log(x) + log(c))^q/(n*q)
```

maple [A] time = 0.06, size = 16, normalized size = 1.07

$$\frac{\ln\left(c\,x^{n}\right)^{q}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)/x,x)
[Out] ln(c*x^n)^q/n/q
```

maxima [A] time = 0.59, size = 15, normalized size = 1.00

$$\frac{\log\left(cx^{n}\right)^{q}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="maxima")
[Out] log(c*x^n)^q/(n*q)
```

mupad [B] time = 0.28, size = 15, normalized size = 1.00

$$\frac{\ln\left(c\,x^{n}\right)^{q}}{n\,q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/x,x)
[Out] log(c*x^n)^q/(n*q)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^{n}\right)^{q-1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x,x)
[Out] Integral(log(c*x**n)**(q - 1)/x, x)
```

3.6 $\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=68

$$\frac{\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\frac{am\text{Int}\left(\frac{x^{m-1}}{ax^m+b\log^q(cx^n)},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)/b/n/q+ln(a*x^m+b*ln(
c*x^n)^q)/b/n/q
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^q), x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\frac{\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx}{bnq}$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(cx^n)^{q-1}}{axx^m+bx\log(cx^n)^q},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)^{q-1}}{\left(ax^m+b\log(cx^n)^q\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

maple [A] time = 29.52, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\,x^{n}\right)^{q -1}}{\left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \int \frac{x^{m}}{abxx^{m} \log(c) + abxx^{m} \log\left(x^{n}\right) + \left(b^{2} x \log(c) + b^{2} x \log\left(x^{n}\right)\right)\left(\log(c) + \log\left(x^{n}\right)\right)^{q}}\, dx + \frac{\log\left(\log(c) + \log\left(x^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="maxima")
```

```
[Out] -a*integrate(x^m/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b
^2*x*log(x^n))*(log(c) + log(x^n))^q), x) + log(log(c) + log(x^n))/(b*n)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\ln\left(c\,x^n\right)}^{q-1}}{x\,\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^q\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

```
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(cx^{n} \right)}^{q - 1}}{x \left(ax^{m} + b \log{\left(cx^{n} \right)}^{q}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Integral(log(c*x**n)**(q - 1)/(x*(a*x**m + b*log(c*x**n)**q)), x)
```

3.7 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{am\text{Int}\left(\frac{x^{m-1}}{(ax^m+b\log^q(cx^n))^2},x\right)}{bnq}-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)/b/n/q-1/b/n/q/(a*x
^m+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(1/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^q)^2, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^2}\,dx}{bnq}$$

Mathematica [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(cx^n)^{q-1}}{2\,abxx^m\log(cx^n)^q+a^2xx^{2\,m}+b^2x\log(cx^n)^{2\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x^(2*m) + b^2
*x*log(c*x^n)^(2*q)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

maple [A] time = 50.17, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{q-1}}{\left(a\,x^m + b\ln(c\,x^n)^q\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
[Out] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{abmx^m\log(x^n) - \left(nq - m\log(c)\right)abx^m + \left(b^2m\log(x^n) - \left(nq - m\log(c)\right)b^2\right)\left(\log(c) + \log(x^n)\right)^q} + \int -\frac{}{abm^2xx^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="maxima
")
[Out] 1/(a*b*m*x^m*log(x^n) - (n*q - m*log(c))*a*b*x^m + (b^2*m*log(x^n) - (n*q -
 m*log(c))*b^2)*(log(c) + log(x^n))^q) + integrate(-(m*n*(q - 1) - m^2*log(
c) - m^2*log(x^n))/(a*b*m^2*x*x^m*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*a*b*x
*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b*x*x^m + (b^2*
m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n
*q*log(c) + m^2*log(c)^2)*b^2*x)*(log(c) + log(x^n))^q), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(c\,x^n)^{q-1}}{x\left(a\,x^m + b\ln(c\,x^n)^q\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
[Out] Timed out
```

3.8 $\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{am\text{Int}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^3},x\right)}{bnq}-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)/b/n/q-1/2/b/n/q/(a
*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m
 + b*Log[c*x^n]^q)^3, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^3}\,dx}{bnq}$$

Mathematica [A] time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3), x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(cx^n)^{q-1}}{3\,ab^2xx^m\log(cx^n)^{2\,q}+3\,a^2bxx^{2\,m}\log(cx^n)^q+a^3xx^{3\,m}+b^3x\log(cx^n)^{3\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3*a^2*b*x*x^(
2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

maple [A] time = 121.28, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{q-1}}{\left(a\,x^m + b\ln(c\,x^n)^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int(ln(c*x^n)^(q-1)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a*m^2*x^m*log(x^n)^2 + (2*m^2*log(c) + m*n)*a*x^m*log(x^n) - (n^2*q^2
 - m^2*log(c)^2 - m*n*log(c))*a*x^m + (2*b*m^2*log(x^n)^2 - (m*n*(2*q - 1)
- 4*m^2*log(c))*b*log(x^n) - (m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*b)*(lo
g(c) + log(x^n))^q)/(a^3*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c)
)*a^3*b*x^(3*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2
)*a^3*b*x^(3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)
^2 - m^3*log(c)^3)*a^3*b*x^(3*m) + (a*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q -
 m^3*log(c))*a*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*l
og(c)^2)*a*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log
(c)^2 - m^3*log(c)^3)*a*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^2*b^2*m^3
*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^2*x^(2*m)*log(x^n)^2 +
 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^2*x^(2*m)*log(x^n) -
 (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^2*b^2
*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(m^3*n*(2*q - 3)*log(c)^2
 - 2*m^4*log(c)^3 - 2*m^4*log(x^n)^3 + 2*(q^2 - 1)*m^2*n^2*log(c) - (2*q^3
- 3*q^2 + q)*m*n^3 + (m^3*n*(2*q - 3) - 6*m^4*log(c))*log(x^n)^2 + 2*(m^3*n
*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 - 1)*m^2*n^2)*log(x^n))/(a^2*b*m^
4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b*x*x^(2*m)*log(x^n)^
3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b*x*x^(2*m)*log(x
^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(
c)^3)*a^2*b*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*
q^2*log(c)^2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b*x*x^(2*m) + (a*b^2*
m^4*x*x^m*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a*b^2*x*x^m*log(x^n)^3 + 6*
(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a*b^2*x*x^m*log(x^n)^2 - 4*
(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a*b^
2*x*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 -
 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c\, x^{n}\right)^{q-1}}{x \left(a\, x^{m} + b \ln \left(c\, x^{n}\right)^{q}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
[Out] Timed out
```

3.9 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=272

$$\frac{a^3x^{3m}\log(cx^n)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3}-\frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2}+\frac{3a^2bx^{2m}\log^3(cx^n)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{360ab^2n}{}$$

```
[Out] -360*a*b^2*n^5*x^m/m^6-9/8*a^2*b*n^3*x^(2*m)/m^4-1/9*a^3*n*x^(3*m)/m^2+360*
a*b^2*n^4*x^m*ln(c*x^n)/m^5+9/4*a^2*b*n^2*x^(2*m)*ln(c*x^n)/m^3+1/3*a^3*x^(
3*m)*ln(c*x^n)/m-180*a*b^2*n^3*x^m*ln(c*x^n)^2/m^4-9/4*a^2*b*n*x^(2*m)*ln(c
*x^n)^2/m^2+60*a*b^2*n^2*x^m*ln(c*x^n)^3/m^3+3/2*a^2*b*x^(2*m)*ln(c*x^n)^3/
m-15*a*b^2*n*x^m*ln(c*x^n)^4/m^2+3*a*b^2*x^m*ln(c*x^n)^5/m+1/8*b^3*ln(c*x^n
)^8/n
```

Rubi [A] time = 0.31, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2539, 2304, 2305, 2302, 30}

$$\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3}-\frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2}+\frac{3a^2bx^{2m}\log^3(cx^n)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{a^3x^{3m}\log(cx^n)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{60ab^2n^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] (-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))/
(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c*x
^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c*x^
n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^m*Lo
g[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x^m*Lo
g[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
```

```
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^3}{x}\,dx = \int \left(a^3x^{-1+3m}\log(cx^n) + 3a^2bx^{-1+2m}\log^3(cx^n) + 3ab^2x^{-1+m}\log^5(cx^n\right.$$

$$= a^3\int x^{-1+3m}\log(cx^n)\,dx + \left(3a^2b\right)\int x^{-1+2m}\log^3(cx^n)\,dx + \left(3ab^2\right)\int$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} + \frac{3ab^2x^m\log^5(cx^n}{m}$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{3a^2bx^{2m}\log^3(c}{2m}$$

$$= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} - 9a$$

$$= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} - 18$$

$$= -\frac{360ab^2n^5x^m}{m^6} - \frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{360ab^2n^4x^m\log(cx^n)}{m^5} + 9a^2$$

Mathematica [A] time = 0.23, size = 230, normalized size = 0.85

$$\frac{ax^m\log(cx^n)\left(4a^2m^4x^{2m} + 27abm^2n^2x^m + 4320b^2n^4\right)}{12m^5} - \frac{anx^m\left(8a^2m^4x^{2m} + 81abm^2n^2x^m + 25920b^2n^4\right)}{72m^6} - 15ab$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] -1/72*(a*n*x^m*(25920*b^2*n^4 + 81*a*b*m^2*n^2*x^m + 8*a^2*m^4*x^(2*m)))/m^
6 + (a*x^m*(4320*b^2*n^4 + 27*a*b*m^2*n^2*x^m + 4*a^2*m^4*x^(2*m))*Log[c*x^
n])/(12*m^5) - (9*a*b*n*x^m*(80*b*n^2 + a*m^2*x^m)*Log[c*x^n]^2)/(4*m^4) +
(3*a*b*x^m*(40*b*n^2 + a*m^2*x^m)*Log[c*x^n]^3)/(2*m^3) - (15*a*b^2*n*x^m*L
og[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

fricas [B] time = 0.46, size = 655, normalized size = 2.41

$$9\,b^3m^6n^7\log(x)^8 + 72\,b^3m^6n^6\log(c)\log(x)^7 + 252\,b^3m^6n^5\log(c)^2\log(x)^6 + 504\,b^3m^6n^4\log(c)^3\log(x)^5 + 63$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/72*(9*b^3*m^6*n^7*log(x)^8 + 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m^6
*n^5*log(c)^2*log(x)^6 + 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*n^
3*log(c)^4*log(x)^4 + 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*log
(c)^6*log(x)^2 + 72*b^3*m^6*log(c)^7*log(x) + 8*(3*a^3*m^5*n*log(x) + 3*a^3
*m^5*log(c) - a^3*m^4*n)*x^(3*m) + 27*(4*a^2*b*m^5*n^3*log(x)^3 + 4*a^2*b*m
^5*log(c)^3 - 6*a^2*b*m^4*n*log(c)^2 + 6*a^2*b*m^3*n^2*log(c) - 3*a^2*b*m^2
*n^3 + 6*(2*a^2*b*m^5*n^2*log(c) - a^2*b*m^4*n^3)*log(x)^2 + 6*(2*a^2*b*m^5
*n*log(c)^2 - 2*a^2*b*m^4*n^2*log(c) + a^2*b*m^3*n^3)*log(x))*x^(2*m) + 216
*(a*b^2*m^5*n^5*log(x)^5 + a*b^2*m^5*log(c)^5 - 5*a*b^2*m^4*n*log(c)^4 + 20
*a*b^2*m^3*n^2*log(c)^3 - 60*a*b^2*m^2*n^3*log(c)^2 + 120*a*b^2*m*n^4*log(c
) - 120*a*b^2*n^5 + 5*(a*b^2*m^5*n^4*log(c) - a*b^2*m^4*n^5)*log(x)^4 + 10*
```

```
(a*b^2*m^5*n^3*log(c)^2 - 2*a*b^2*m^4*n^4*log(c) + 2*a*b^2*m^3*n^5)*log(x)^
3 + 10*(a*b^2*m^5*n^2*log(c)^3 - 3*a*b^2*m^4*n^3*log(c)^2 + 6*a*b^2*m^3*n^4
*log(c) - 6*a*b^2*m^2*n^5)*log(x)^2 + 5*(a*b^2*m^5*n*log(c)^4 - 4*a*b^2*m^4
*n^2*log(c)^3 + 12*a*b^2*m^3*n^3*log(c)^2 - 24*a*b^2*m^2*n^4*log(c) + 24*a*
b^2*m*n^5)*log(x))*x^m)/m^6
```

giac [B] time = 0.26, size = 766, normalized size = 2.82

$$\frac{1}{8}\,b^3n^7\log(x)^8+b^3n^6\log(c)\log(x)^7+\frac{7}{2}\,b^3n^5\log(c)^2\log(x)^6+7\,b^3n^4\log(c)^3\log(x)^5+\frac{35}{4}\,b^3n^3\log(c)^4\log(x)^4+7\,b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/8*b^3*n^7*log(x)^8 + b^3*n^6*log(c)*log(x)^7 + 7/2*b^3*n^5*log(c)^2*log(x
)^6 + 7*b^3*n^4*log(c)^3*log(x)^5 + 35/4*b^3*n^3*log(c)^4*log(x)^4 + 7*b^3*
n^2*log(c)^5*log(x)^3 + 3*a*b^2*n^5*x^m*log(x)^5/m + 7/2*b^3*n*log(c)^6*log
(x)^2 + 15*a*b^2*n^4*x^m*log(c)*log(x)^4/m + b^3*log(c)^7*log(x) + 30*a*b^2
*n^3*x^m*log(c)^2*log(x)^3/m - 15*a*b^2*n^5*x^m*log(x)^4/m^2 + 30*a*b^2*n^2
*x^m*log(c)^3*log(x)^2/m - 60*a*b^2*n^4*x^m*log(c)*log(x)^3/m^2 + 15*a*b^2*
n*x^m*log(c)^4*log(x)/m - 90*a*b^2*n^3*x^m*log(c)^2*log(x)^2/m^2 + 3/2*a^2*
b*n^3*x^(2*m)*log(x)^3/m + 60*a*b^2*n^5*x^m*log(x)^3/m^3 + 3*a*b^2*x^m*log(
c)^5/m - 60*a*b^2*n^2*x^m*log(c)^3*log(x)/m^2 + 9/2*a^2*b*n^2*x^(2*m)*log(c
)*log(x)^2/m + 180*a*b^2*n^4*x^m*log(c)*log(x)^2/m^3 - 15*a*b^2*n*x^m*log(c
)^4/m^2 + 9/2*a^2*b*n*x^(2*m)*log(c)^2*log(x)/m + 180*a*b^2*n^3*x^m*log(c)^
2*log(x)/m^3 - 9/4*a^2*b*n^3*x^(2*m)*log(x)^2/m^2 - 180*a*b^2*n^5*x^m*log(x
)^2/m^4 + 3/2*a^2*b*x^(2*m)*log(c)^3/m + 60*a*b^2*n^2*x^m*log(c)^3/m^3 - 9/
2*a^2*b*n^2*x^(2*m)*log(c)*log(x)/m^2 - 360*a*b^2*n^4*x^m*log(c)*log(x)/m^4
 - 9/4*a^2*b*n*x^(2*m)*log(c)^2/m^2 - 180*a*b^2*n^3*x^m*log(c)^2/m^4 + 1/3*
a^3*n*x^(3*m)*log(x)/m + 9/4*a^2*b*n^3*x^(2*m)*log(x)/m^3 + 360*a*b^2*n^5*x
^m*log(x)/m^5 + 1/3*a^3*x^(3*m)*log(c)/m + 9/4*a^2*b*n^2*x^(2*m)*log(c)/m^3
 + 360*a*b^2*n^4*x^m*log(c)/m^5 - 1/9*a^3*n*x^(3*m)/m^2 - 9/8*a^2*b*n^3*x^(
2*m)/m^4 - 360*a*b^2*n^5*x^m/m^6
```

maple [C] time = 9.06, size = 61910, normalized size = 227.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^3/x,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.94, size = 1115, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/84*(12*b^3*log(c*x^n)^7/n + 252*a*b^2*x^m*log(c*x^n)^4/m + 126*a^2*b*x^(2
*m)*log(c*x^n)^2/m - 1008*(n*x^m*log(c*x^n)^3/m^2 - 3*(n*x^m*log(c*x^n)^2/m
^2 - 2*n*(n*x^m*log(c*x^n)/m^2 - n^2*x^m/m^3)/m)*n/m)*a*b^2 - 63*a^2*b*(2*n
*x^(2*m)*log(c*x^n)/m^2 - n^2*x^(2*m)/m^3) + 28*a^3*x^(3*m)/m)*log(c*x^n) +
 1/504*(9*b^3*m^6*n^7*log(x)^8 - 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m
^6*n^5*log(c)^2*log(x)^6 - 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*
n^3*log(c)^4*log(x)^4 - 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*l
og(c)^6*log(x)^2 - 72*b^3*m^6*log(c)^7*log(x) - 72*b^3*m^6*log(x)*log(x^n)^
7 - 56*a^3*m^4*n*x^(3*m) + 252*(b^3*m^6*n*log(x)^2 - 2*b^3*m^6*log(c)*log(x
```

```
))*log(x^n)^6 - 504*(b^3*m^6*n^2*log(x)^3 - 3*b^3*m^6*n*log(c)*log(x)^2 + 3
*b^3*m^6*log(c)^2*log(x))*log(x^n)^5 - 189*(2*m^4*n*log(c)^2 - 4*m^3*n^2*lo
g(c) + 3*m^2*n^3)*a^2*b*x^(2*m) - 1512*(m^4*n*log(c)^4 - 8*m^3*n^2*log(c)^3
 + 36*m^2*n^3*log(c)^2 - 96*m*n^4*log(c) + 120*n^5)*a*b^2*x^m + 126*(5*b^3*
m^6*n^3*log(x)^4 - 20*b^3*m^6*n^2*log(c)*log(x)^3 + 30*b^3*m^6*n*log(c)^2*l
og(x)^2 - 20*b^3*m^6*log(c)^3*log(x) - 12*a*b^2*m^4*n*x^m)*log(x^n)^4 - 504
*(b^3*m^6*n^4*log(x)^5 - 5*b^3*m^6*n^3*log(c)*log(x)^4 + 10*b^3*m^6*n^2*log
(c)^2*log(x)^3 - 10*b^3*m^6*n*log(c)^3*log(x)^2 + 5*b^3*m^6*log(c)^4*log(x)
 + 12*(m^4*n*log(c) - 2*m^3*n^2)*a*b^2*x^m)*log(x^n)^3 + 126*(2*b^3*m^6*n^5
*log(x)^6 - 12*b^3*m^6*n^4*log(c)*log(x)^5 + 30*b^3*m^6*n^3*log(c)^2*log(x)
^4 - 40*b^3*m^6*n^2*log(c)^3*log(x)^3 + 30*b^3*m^6*n*log(c)^4*log(x)^2 - 12
*b^3*m^6*log(c)^5*log(x) - 3*a^2*b*m^4*n*x^(2*m) - 72*(m^4*n*log(c)^2 - 4*m
^3*n^2*log(c) + 6*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 - 36*(2*b^3*m^6*n^6*log(x)
^7 - 14*b^3*m^6*n^5*log(c)*log(x)^6 + 42*b^3*m^6*n^4*log(c)^2*log(x)^5 - 70
*b^3*m^6*n^3*log(c)^3*log(x)^4 + 70*b^3*m^6*n^2*log(c)^4*log(x)^3 - 42*b^3*
m^6*n*log(c)^5*log(x)^2 + 14*b^3*m^6*log(c)^6*log(x) + 21*(m^4*n*log(c) - m
^3*n^2)*a^2*b*x^(2*m) + 168*(m^4*n*log(c)^3 - 6*m^3*n^2*log(c)^2 + 18*m^2*n
^3*log(c) - 24*m*n^4)*a*b^2*x^m)*log(x^n))/m^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,x^n\right)\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x, x)
```

sympy [A] time = 103.52, size = 762, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**3/x,x)
```

```
[Out] -a**3*n*Piecewise((Piecewise((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))/(3
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**3*Piecewis
e((x**(3*m)/(3*m), Ne(3*m - 1, -1)), (log(x), True))*log(c*x**n) + 3*a**2*b
*Piecewise((n**3*x**(2*m)*log(x)**3/(2*m) + 3*n**2*x**(2*m)*log(c)*log(x)**
2/(2*m) + 3*n*x**(2*m)*log(c)**2*log(x)/(2*m) + x**(2*m)*log(c)**3/(2*m) -
3*n**3*x**(2*m)*log(x)**2/(4*m**2) - 3*n**2*x**(2*m)*log(c)*log(x)/(2*m**2)
 - 3*n*x**(2*m)*log(c)**2/(4*m**2) + 3*n**3*x**(2*m)*log(x)/(4*m**3) + 3*n*
*2*x**(2*m)*log(c)/(4*m**3) - 3*n**3*x**(2*m)/(8*m**4), Ne(m, 0)), (Piecewi
se((log(c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log(x**(-n)/c)**4/(4*n), 1/Abs
(c*x**n) < 1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x
**n)/n + 6*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n,
 True)), True)) + 3*a*b**2*Piecewise((n**5*x**m*log(x)**5/m + 5*n**4*x**m*l
og(c)*log(x)**4/m + 10*n**3*x**m*log(c)**2*log(x)**3/m + 10*n**2*x**m*log(c
)**3*log(x)**2/m + 5*n*x**m*log(c)**4*log(x)/m + x**m*log(c)**5/m - 5*n**5*
x**m*log(x)**4/m**2 - 20*n**4*x**m*log(c)*log(x)**3/m**2 - 30*n**3*x**m*log
(c)**2*log(x)**2/m**2 - 20*n**2*x**m*log(c)**3*log(x)/m**2 - 5*n*x**m*log(c
)**4/m**2 + 20*n**5*x**m*log(x)**3/m**3 + 60*n**4*x**m*log(c)*log(x)**2/m**
3 + 60*n**3*x**m*log(c)**2*log(x)/m**3 + 20*n**2*x**m*log(c)**3/m**3 - 60*n
**5*x**m*log(x)**2/m**4 - 120*n**4*x**m*log(c)*log(x)/m**4 - 60*n**3*x**m*l
og(c)**2/m**4 + 120*n**5*x**m*log(x)/m**5 + 120*n**4*x**m*log(c)/m**5 - 120
*n**5*x**m/m**6, Ne(m, 0)), (Piecewise((log(c*x**n)**6/(6*n), Abs(c*x**n) <
 1), (log(x**(-n)/c)**6/(6*n), 1/Abs(c*x**n) < 1), (120*meijerg(((), (1, 1,
 1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 120*meijerg(((1,
```

```
 1, 1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0, 0, 0)), c*x**n)/n, True)), Tr
ue)) - b**3*Piecewise((-log(c)**7*log(x), Eq(n, 0)), (-log(c*x**n)**8/(8*n)
, True))
```

3.10 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=125

$$\frac{a^2x^{2m}\log\left(cx^n\right)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log\left(cx^n\right)}{m^3}-\frac{6abnx^m\log^2\left(cx^n\right)}{m^2}+\frac{2abx^m\log^3\left(cx^n\right)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6\left(cx\right.}{6n}$$

```
[Out] -12*a*b*n^3*x^m/m^4-1/4*a^2*n*x^(2*m)/m^2+12*a*b*n^2*x^m*ln(c*x^n)/m^3+1/2*
a^2*x^(2*m)*ln(c*x^n)/m-6*a*b*n*x^m*ln(c*x^n)^2/m^2+2*a*b*x^m*ln(c*x^n)^3/m
+1/6*b^2*ln(c*x^n)^6/n
```

Rubi [A] time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2539, 2304, 2305, 2302, 30}

$$\frac{a^2x^{2m}\log\left(cx^n\right)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log\left(cx^n\right)}{m^3}-\frac{6abnx^m\log^2\left(cx^n\right)}{m^2}+\frac{2abx^m\log^3\left(cx^n\right)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6\left(cx\right.}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] (-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^n
])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 +
(2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = \int\left(a^2x^{-1+2m}\log(cx^n)+2abx^{-1+m}\log^3(cx^n)+\frac{b^2\log^5(cx^n)}{x}\right)dx$$

$$= a^2\int x^{-1+2m}\log(cx^n)\,dx+(2ab)\int x^{-1+m}\log^3(cx^n)\,dx+b^2\int\frac{\log^5(cx}{x}$$

$$= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}+\frac{2abx^m\log^3(cx^n)}{m}+\frac{b^2\,\mathrm{Subst}\left(\int x^5\,dx,x,\mathrm{lo}\right.}{n}$$

$$= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}+$$

$$= -\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx}{}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.92

$$-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{ax^m\log(cx^n)\left(am^2x^m+24bn^2\right)}{2m^3}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{anx^m\left(am^2x^m+48bn^2\right)}{4m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] -1/4*(a*n*x^m*(48*b*n^2 + a*m^2*x^m))/m^4 + (a*x^m*(24*b*n^2 + a*m^2*x^m)*L
og[c*x^n])/(2*m^3) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 + (2*a*b*x^m*Log[c*x^n]
^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

fricas [B] time = 0.44, size = 267, normalized size = 2.14

$$\frac{2\,b^2m^4n^5\log(x)^6+12\,b^2m^4n^4\log(c)\log(x)^5+30\,b^2m^4n^3\log(c)^2\log(x)^4+40\,b^2m^4n^2\log(c)^3\log(x)^3+30\,b^2m^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*m^4*n^5*log(x)^6 + 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*
n^3*log(c)^2*log(x)^4 + 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log
(c)^4*log(x)^2 + 12*b^2*m^4*log(c)^5*log(x) + 3*(2*a^2*m^3*n*log(x) + 2*a^2
*m^3*log(c) - a^2*m^2*n)*x^(2*m) + 24*(a*b*m^3*n^3*log(x)^3 + a*b*m^3*log(c
)^3 - 3*a*b*m^2*n*log(c)^2 + 6*a*b*m*n^2*log(c) - 6*a*b*n^3 + 3*(a*b*m^3*n^
2*log(c) - a*b*m^2*n^3)*log(x)^2 + 3*(a*b*m^3*n*log(c)^2 - 2*a*b*m^2*n^2*lo
g(c) + 2*a*b*m*n^3)*log(x))*x^m)/m^4
```

giac [B] time = 0.19, size = 286, normalized size = 2.29

$$\frac{1}{6}\,b^2n^5\log(x)^6+b^2n^4\log(c)\log(x)^5+\frac{5}{2}\,b^2n^3\log(c)^2\log(x)^4+\frac{10}{3}\,b^2n^2\log(c)^3\log(x)^3+\frac{5}{2}\,b^2n\log(c)^4\log(x)^2+b^2\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*n^5*log(x)^6 + b^2*n^4*log(c)*log(x)^5 + 5/2*b^2*n^3*log(c)^2*log(x
)^4 + 10/3*b^2*n^2*log(c)^3*log(x)^3 + 5/2*b^2*n*log(c)^4*log(x)^2 + b^2*lo
g(c)^5*log(x) + 2*a*b*n^3*x^m*log(x)^3/m + 6*a*b*n^2*x^m*log(c)*log(x)^2/m
+ 6*a*b*n*x^m*log(c)^2*log(x)/m - 6*a*b*n^3*x^m*log(x)^2/m^2 + 2*a*b*x^m*lo
g(c)^3/m - 12*a*b*n^2*x^m*log(c)*log(x)/m^2 - 6*a*b*n*x^m*log(c)^2/m^2 + 1/
2*a^2*n*x^(2*m)*log(x)/m + 12*a*b*n^3*x^m*log(x)/m^3 + 1/2*a^2*x^(2*m)*log(
```

```
c)/m + 12*a*b*n^2*x^m*log(c)/m^3 - 1/4*a^2*n*x^(2*m)/m^2 - 12*a*b*n^3*x^m/m
^4
```

maple [C] time = 3.16, size = 14983, normalized size = 119.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^2/x,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.66, size = 530, normalized size = 4.24

$$\frac{1}{10}\left(\frac{2\,b^2\log\left(cx^n\right)^5}{n}+\frac{20\,abx^m\log\left(cx^n\right)^2}{m}-40\,ab\left(\frac{nx^m\log\left(cx^n\right)}{m^2}-\frac{n^2x^m}{m^3}\right)+\frac{5\,a^2x^{2\,m}}{m}\right)\log\left(cx^n\right)+\frac{2\,b^2m^4n^5\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*(2*b^2*log(c*x^n)^5/n + 20*a*b*x^m*log(c*x^n)^2/m - 40*a*b*(n*x^m*log(
c*x^n)/m^2 - n^2*x^m/m^3) + 5*a^2*x^(2*m)/m)*log(c*x^n) + 1/60*(2*b^2*m^4*n
^5*log(x)^6 - 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*n^3*log(c)^2*log(
x)^4 - 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log(c)^4*log(x)^2 -
12*b^2*m^4*log(c)^5*log(x) - 12*b^2*m^4*log(x)*log(x^n)^5 - 15*a^2*m^2*n*x^
(2*m) + 30*(b^2*m^4*n*log(x)^2 - 2*b^2*m^4*log(c)*log(x))*log(x^n)^4 - 120*
(m^2*n*log(c)^2 - 4*m*n^2*log(c) + 6*n^3)*a*b*x^m - 40*(b^2*m^4*n^2*log(x)^
3 - 3*b^2*m^4*n*log(c)*log(x)^2 + 3*b^2*m^4*log(c)^2*log(x))*log(x^n)^3 + 3
0*(b^2*m^4*n^3*log(x)^4 - 4*b^2*m^4*n^2*log(c)*log(x)^3 + 6*b^2*m^4*n*log(c
)^2*log(x)^2 - 4*b^2*m^4*log(c)^3*log(x) - 4*a*b*m^2*n*x^m)*log(x^n)^2 - 12
*(b^2*m^4*n^4*log(x)^5 - 5*b^2*m^4*n^3*log(c)*log(x)^4 + 10*b^2*m^4*n^2*log
(c)^2*log(x)^3 - 10*b^2*m^4*n*log(c)^3*log(x)^2 + 5*b^2*m^4*log(c)^4*log(x)
 + 20*(m^2*n*log(c) - 2*m*n^2)*a*b*x^m)*log(x^n))/m^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x, x)
```

sympy [A] time = 37.59, size = 298, normalized size = 2.38

$$-a^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2m}}{2m} & \text{for}\ m\neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}}{2m} & \text{for}\ m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log\left(x\right)^2}{2} & \text{otherwise}\end{cases}\right)+a^2\left(\begin{cases}\frac{x^{2m}}{2m} & \text{for}\ 2m-1\neq -1\\ \log\left(x\right) & \text{otherwise}\end{cases}\right)\log\left(cx^n\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**2/x,x)
```

```
[Out] -a**2*n*Piecewise((Piecewise((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))/(2
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**2*Piecewis
e((x**(2*m)/(2*m), Ne(2*m - 1, -1)), (log(x), True))*log(c*x**n) + 2*a*b*Pi
ecewise((n**3*x**m*log(x)**3/m + 3*n**2*x**m*log(c)*log(x)**2/m + 3*n*x**m*
log(c)**2*log(x)/m + x**m*log(c)**3/m - 3*n**3*x**m*log(x)**2/m**2 - 6*n**2
*x**m*log(c)*log(x)/m**2 - 3*n*x**m*log(c)**2/m**2 + 6*n**3*x**m*log(x)/m**
3 + 6*n**2*x**m*log(c)/m**3 - 6*n**3*x**m/m**4, Ne(m, 0)), (Piecewise((log(
c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log(x**(-n)/c)**4/(4*n), 1/Abs(c*x**n)
 < 1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n +
 6*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True)),
 True)) - b**2*Piecewise((-log(c)**5*log(x), Eq(n, 0)), (-log(c*x**n)**6/(6
*n), True))
```

3.11 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=41

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

```
[Out] -a*n*x^m/m^2+a*x^m*ln(c*x^n)/m+1/4*b*ln(c*x^n)^4/n
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2539, 2304, 2302, 30}

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx&=\int\left(ax^{-1+m}\log(cx^n)+\frac{b\log^3(cx^n)}{x}\right)dx\\&=a\int x^{-1+m}\log(cx^n)\,dx+b\int\frac{\log^3(cx^n)}{x}\,dx\\&=-\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\,\mathrm{Subst}\left(\int x^3\,dx,x,\log(cx^n)\right)}{n}\\&=-\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\log^4(cx^n)}{4n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 1.00

$$\frac{ax^m \log(cx^n)}{m} - \frac{anx^m}{m^2} + \frac{b\log^4(cx^n)}{4n}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]

[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)

fricas [B] time = 0.44, size = 81, normalized size = 1.98

$$\frac{bm^2n^3\log(x)^4 + 4\,bm^2n^2\log(c)\log(x)^3 + 6\,bm^2n\log(c)^2\log(x)^2 + 4\,bm^2\log(c)^3\log(x) + 4\left(amn\log(x) + am\log(c) - an\right)x^m}{4\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="fricas")

[Out] 1/4*(b*m^2*n^3*log(x)^4 + 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*log(x)^2 + 4*b*m^2*log(c)^3*log(x) + 4*(a*m*n*log(x) + a*m*log(c) - a*n)*x^m)/m^2

giac [A] time = 0.20, size = 73, normalized size = 1.78

$$\frac{1}{4}\,bn^3\log(x)^4 + bn^2\log(c)\log(x)^3 + \frac{3}{2}\,bn\log(c)^2\log(x)^2 + b\log(c)^3\log(x) + \frac{anx^m\log(x)}{m} + \frac{ax^m\log(c)}{m} - \frac{anx^m}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="giac")

[Out] 1/4*b*n^3*log(x)^4 + b*n^2*log(c)*log(x)^3 + 3/2*b*n*log(c)^2*log(x)^2 + b*log(c)^3*log(x) + a*n*x^m*log(x)/m + a*x^m*log(c)/m - a*n*x^m/m^2

maple [C] time = 1.05, size = 2146, normalized size = 52.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)/x,x)

[Out] 1/4*(-3*Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4*ln(x)*m+6*Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)*ln(x)*m-3*Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2*ln(x)*m+6*Pi^2*b*csgn(I*x^n)*csgn(I*c*x^n)^5*ln(x)*m-12*Pi^2*b*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)*ln(x)*m+6*Pi^2*b*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2*ln(x)*m-3*Pi^2*b*csgn(I*c*x^n)^6*ln(x)*m+6*Pi^2*b*csgn(I*c*x^n)^5*csgn(I*c)*ln(x)*m-3*Pi^2*b*csgn(I*c*x^n)^4*csgn(I*c)^2*ln(x)*m+6*I*ln(x)^2*Pi*b*n*csgn(I*c*x^n)^3*m+6*I*ln(x)^2*Pi*b*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*m-6*I*ln(x)^2*Pi*b*n*csgn(I*x^n)*csgn(I*c*x^n)^2*m+12*I*ln(c)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)*m-12*I*ln(c)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*ln(x)*m+12*I*ln(c)*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*ln(x)*m-6*I*ln(x)^2*Pi*b*n*csgn(I*c*x^n)^2*csgn(I*c)*m-12*I*ln(c)*Pi*b*csgn(I*c*x^n)^3*ln(x)*m+4*b*n^2*ln(x)^3*m-12*ln(x)^2*ln(c)*b*n*m+12*ln(c)^2*b*ln(x)*m+4*a*x^m)/m*ln(x^n)+b*ln(x)*ln(x^n)^3+ln(c)^3*ln(x)*b-1/4*b*n^3*ln(x)^4-a*n*x^m/m^2+1/m*ln(c)*a*x^m+ln(x)^3*ln(c)*b*n^2-3/2*ln(x)^2*ln(c)^2*b*n-3/4*csgn(I*c)*csgn(I*c*x^n)^5*n*b*Pi^2*ln(x)^2+3/8*csgn(I*c)^2*csgn(I*c*x^n)^4*n*b*Pi^2*ln(x)^2-3/4*ln(x)*csgn(I*c*x^n)^4*csgn(I*x^n)^2*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c*x^n)^5*csgn(I*x^n)*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c)*csgn(I*c*x^n)^5*b*Pi^2*ln(c)-3/4*ln(x)*csgn(I*c)^2*csgn(I*c*x^n)^4*b*Pi^2*ln(c)-3/4*ln(x)*csgn(I*c*x^n)^6*b*Pi^2*ln(c)+3/8*csgn(I*c*x^n)^6*n*b*Pi^2*ln(x)^2+3/2*

```
I*ln(x)^2*ln(c)*Pi*b*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3/8*I*Pi^3*b*csg
n(I*c*x^n)^8*csgn(I*c)*ln(x)-3/8*I*Pi^3*b*csgn(I*x^n)*csgn(I*c*x^n)^8*ln(x)
+3/8*I*Pi^3*b*csgn(I*c*x^n)^7*csgn(I*c)^2*ln(x)+3/8*I*Pi^3*b*csgn(I*x^n)^2*
csgn(I*c*x^n)^7*ln(x)-1/8*I*Pi^3*b*csgn(I*c*x^n)^6*csgn(I*c)^3*ln(x)-1/8*I*
Pi^3*b*csgn(I*x^n)^3*csgn(I*c*x^n)^6*ln(x)-1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*c*
x^n)^3-3/2*I*ln(c)^2*Pi*b*csgn(I*c*x^n)^3*ln(x)-1/2*I/m*Pi*a*csgn(I*c*x^n)^
3*x^m+3/8*csgn(I*c*x^n)^4*csgn(I*x^n)^2*n*b*Pi^2*ln(x)^2-3/4*csgn(I*c*x^n)^
5*csgn(I*x^n)*n*b*Pi^2*ln(x)^2-3*ln(x)*csgn(I*c)*csgn(I*c*x^n)^4*csgn(I*x^n
)*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c)^2*csgn(I*c*x^n)^3*csgn(I*x^n)*b*Pi^2*ln(
c)+3/8*csgn(I*c)^2*csgn(I*c*x^n)^2*csgn(I*x^n)^2*n*b*Pi^2*ln(x)^2+3/2*csgn(
I*c)*csgn(I*c*x^n)^4*csgn(I*x^n)*n*b*Pi^2*ln(x)^2-3/4*csgn(I*c)^2*csgn(I*c*
x^n)^3*csgn(I*x^n)*n*b*Pi^2*ln(x)^2+3/2*ln(x)*csgn(I*c)*csgn(I*c*x^n)^3*csg
n(I*x^n)^2*b*Pi^2*ln(c)-3/4*ln(x)*csgn(I*c)^2*csgn(I*c*x^n)^2*csgn(I*x^n)^2
*b*Pi^2*ln(c)-3/4*csgn(I*c)*csgn(I*c*x^n)^3*csgn(I*x^n)^2*n*b*Pi^2*ln(x)^2+
1/8*I*ln(x)*csgn(I*c*x^n)^9*b*Pi^3-3/2*I*ln(x)^2*ln(c)*Pi*b*n*csgn(I*c*x^n)
^2*csgn(I*c)+1/2*I/m*Pi*a*csgn(I*c*x^n)^2*csgn(I*c)*x^m+1/2*I/m*Pi*a*csgn(I
*x^n)*csgn(I*c*x^n)^2*x^m+9/8*I*Pi^3*b*csgn(I*x^n)*csgn(I*c*x^n)^7*csgn(I*c
)*ln(x)-9/8*I*Pi^3*b*csgn(I*x^n)*csgn(I*c*x^n)^6*csgn(I*c)^2*ln(x)-9/8*I*Pi
^3*b*csgn(I*x^n)^2*csgn(I*c*x^n)^6*csgn(I*c)*ln(x)+3/8*I*Pi^3*b*csgn(I*x^n)
*csgn(I*c*x^n)^5*csgn(I*c)^3*ln(x)+9/8*I*Pi^3*b*csgn(I*x^n)^2*csgn(I*c*x^n)
^5*csgn(I*c)^2*ln(x)+3/8*I*Pi^3*b*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)*l
n(x)-3/8*I*Pi^3*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4*csgn(I*c)^3*ln(x)-3/8*I*Pi^
3*b*csgn(I*x^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2*ln(x)+1/8*I*ln(x)*csgn(I*c)^3
*csgn(I*c*x^n)^3*csgn(I*x^n)^3*b*Pi^3+1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*c*x^n)^
2*csgn(I*c)+1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+3/2*I*ln(x)^
2*ln(c)*Pi*b*n*csgn(I*c*x^n)^3+3/2*I*ln(c)^2*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)
*ln(x)+3/2*I*ln(c)^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)+(-3/2*b*n*ln(x)
^2+3/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)-3/2*I*Pi*b*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)*ln(x)-3/2*I*Pi*b*csgn(I*c*x^n)^3*ln(x)+3/2*I*Pi*b*csgn(
I*c*x^n)^2*csgn(I*c)*ln(x)+3*b*ln(c)*ln(x))*ln(x^n)^2-3/2*I*ln(x)^2*ln(c)*P
i*b*n*csgn(I*x^n)*csgn(I*c*x^n)^2-3/2*I*ln(c)^2*Pi*b*csgn(I*x^n)*csgn(I*c*x
^n)*csgn(I*c)*ln(x)-1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I
*c)-1/2*I/m*Pi*a*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*x^m
```

maxima [B] time = 0.64, size = 186, normalized size = 4.54

$$\frac{1}{3}\left(\frac{b\log\left(cx^{n}\right)^{3}}{n}+\frac{3\,ax^{m}}{m}\right)\log\left(cx^{n}\right)+\frac{bm^{2}n^{3}\log(x)^{4}-4\,bm^{2}n^{2}\log(c)\log(x)^{3}+6\,bm^{2}n\log(c)^{2}\log(x)^{2}-4\,bm^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*log(c*x^n)^3/n + 3*a*x^m/m)*log(c*x^n) + 1/12*(b*m^2*n^3*log(x)^4 -
4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*log(x)^2 - 4*b*m^2*log(c)^
3*log(x) - 4*b*m^2*log(x)*log(x^n)^3 - 12*a*n*x^m + 6*(b*m^2*n*log(x)^2 - 2
*b*m^2*log(c)*log(x))*log(x^n)^2 - 4*(b*m^2*n^2*log(x)^3 - 3*b*m^2*n*log(c)
*log(x)^2 + 3*b*m^2*log(c)^2*log(x))*log(x^n))/m^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x, x)
```

sympy [A] time = 12.42, size = 68, normalized size = 1.66

$$-an\left(\begin{cases}\frac{\begin{cases}\frac{x^m}{m} & \text{for } m \neq 0\\ \log(x) & \text{otherwise}\end{cases}}{m} & \text{for } m > -\infty \wedge m < \infty \wedge m \neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{x^m}{m} & \text{for } m-1 \neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)-b\left(\begin{cases}- \\ - \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)/x,x)
```

```
[Out] -a*n*Piecewise((Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/m, (m > -oo)
& (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a*Piecewise((x**m/m, Ne(m -
1, -1)), (log(x), True))*log(c*x**n) - b*Piecewise((-log(c)**3*log(x), Eq(n
, 0)), (-log(c*x**n)**4/(4*n), True))
```

3.12 $\int \frac{\log(cx^n)}{x} dx$

Optimal. Leaf size=15

$$\frac{\log^2(cx^n)}{2n}$$

```
[Out] 1/2*ln(c*x^n)^2/n
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2301}

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*x^n]/x,x]
[Out] Log[c*x^n]^2/(2*n)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\log(cx^n)}{x} dx = \frac{\log^2(cx^n)}{2n}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*x^n]/x,x]
[Out] Log[c*x^n]^2/(2*n)
```

fricas [A] time = 0.43, size = 13, normalized size = 0.87

$$\frac{1}{2}n\log(x)^2 + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="fricas")
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.87

$$\frac{1}{2}n\log(x)^2 + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="giac")
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.93

$$\frac{\ln(c\,x^n)^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x,x)
[Out] 1/2*ln(c*x^n)^2/n
```

maxima [A] time = 0.73, size = 13, normalized size = 0.87

$$\frac{\log(cx^n)^2}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="maxima")
[Out] 1/2*log(c*x^n)^2/n
```

mupad [B] time = 0.25, size = 13, normalized size = 0.87

$$\frac{\ln(c\,x^n)^2}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/x,x)
[Out] log(c*x^n)^2/(2*n)
```

sympy [A] time = 1.70, size = 51, normalized size = 3.40

$$\begin{cases} \frac{\log(cx^n)^2}{2n} & \text{for}\ |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for}\ \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x,x)
[Out] Piecewise((log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(x**(-n)/c)**2/(2*n)
, 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n
+ meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

3.13 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$

Optimal. Leaf size=67

$$\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{am\text{Int}\left(\frac{x^{m-1}}{ax^m+b\log^2(cx^n)},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2),x)/b/n+1/2*ln(a*x^m
+b*ln(c*x^n)^2)/b/n
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^2]/(2*b*n) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2), x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^2(cx^n)}\,dx}{2bn}$$

Mathematica [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(cx^n)}{bx\log(cx^n)^2+axx^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b*x*log(c*x^n)^2 + a*x*x^m), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2+ax^m\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

maple [A] time = 7.10, size = 0, normalized size = 0.00

$$\int \frac{\ln (c\, x^{n})}{\left(b \ln (c\, x^{n})^{2}+a\, x^{m}\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log (cx^{n})}{\left(b \log (cx^{n})^{2}+ax^{m}\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (c\, x^{n})}{x\, \left(a\, x^{m}+b \ln (c\, x^{n})^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)),x)
```

```
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log (cx^{n})}{x \left(ax^{m}+b \log (cx^{n})^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2),x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)), x)
```

3.14 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{am\text{Int}\left(\frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^2},x\right)}{2bn}-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^2,x)/b/n-1/2/b/n/(a
*x^m+b*ln(c*x^n)^2)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] -1/(2*b*n*(a*x^m + b*Log[c*x^n]^2)) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2)^2, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx=-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2}\,dx}{2bn}$$

Mathematica [A] time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(cx^n)}{b^2x\log(cx^n)^4+2\,abxx^m\log(cx^n)^2+a^2xx^{2\,m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^2*x*log(c*x^n)^4 + 2*a*b*x*x^m*log(c*x^n)^2 + a^2*x*
x^(2*m)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2+ax^m\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^2*x), x)
```

maple [A] time = 107.74, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\,x^{n}\right)}{\left(b \ln \left(c\,x^{n}\right)^{2}+a\,x^{m}\right)^{2} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(b*ln(c*x^n)^2+a*x^m)^2,x)
```

```
[Out] int(ln(c*x^n)/x/(b*ln(c*x^n)^2+a*x^m)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{m \log(c) + m \log\left(x^{n}\right) + 2\,n}{4\,b^{2}n^{2}\log(c)^{2} + a^{2}m^{2}x^{2\,m} + \left(m^{2}\log(c)^{2} + 4\,n^{2}\right)abx^{m} + \left(abm^{2}x^{m} + 4\,b^{2}n^{2}\right)\log\left(x^{n}\right)^{2} + 2\left(abm^{2}x^{m}\log(c) + 4\,b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="maxima")
```

```
[Out] -(m*log(c) + m*log(x^n) + 2*n)/(4*b^2*n^2*log(c)^2 + a^2*m^2*x^(2*m) + (m^2
*log(c)^2 + 4*n^2)*a*b*x^m + (a*b*m^2*x^m + 4*b^2*n^2)*log(x^n)^2 + 2*(a*b*
m^2*x^m*log(c) + 4*b^2*n^2*log(c))*log(x^n)) - integrate((a*m^4*x^m*log(x^n
) + 4*b*m*n^3 + (m^4*log(c) + 3*m^3*n)*a*x^m)/(16*b^3*n^4*x*log(c)^2 + a^3*
m^4*x*x^(3*m) + (m^4*log(c)^2 + 8*m^2*n^2)*a^2*b*x*x^(2*m) + 8*(m^2*n^2*log
(c)^2 + 2*n^4)*a*b^2*x*x^m + (a^2*b*m^4*x*x^(2*m) + 8*a*b^2*m^2*n^2*x*x^m +
 16*b^3*n^4*x)*log(x^n)^2 + 2*(a^2*b*m^4*x*x^(2*m)*log(c) + 8*a*b^2*m^2*n^2
*x*x^m*log(c) + 16*b^3*n^4*x*log(c))*log(x^n)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c\,x^{n}\right)}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2),x)
```

```
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(cx^{n}\right)}{x \left(ax^{m}+b \log \left(cx^{n}\right)^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**2,x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**2), x)
```

3.15 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{am\text{Int}\left(\frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^3},x\right)}{2bn}-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^3,x)/b/n-1/4/b/n/(a
*x^m+b*ln(c*x^n)^2)^2
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] -1/(4*b*n*(a*x^m + b*Log[c*x^n]^2)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^2)^3, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx=-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^3}\,dx}{2bn}$$

Mathematica [A] time = 3.01, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(cx^n\right)}{b^3x\log\left(cx^n\right)^6+3\,ab^2xx^m\log\left(cx^n\right)^4+3\,a^2bxx^{2\,m}\log\left(cx^n\right)^2+a^3xx^{3\,m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^3*x*log(c*x^n)^6 + 3*a*b^2*x*x^m*log(c*x^n)^4 + 3*a^
2*b*x*x^(2*m)*log(c*x^n)^2 + a^3*x*x^(3*m)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2+ax^m\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^3*x), x)
```

maple [A] time = 127.67, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)}{\left(b\ln(c\,x^n)^2+a\,x^m\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(b*ln(c*x^n)^2+a*x^m)^3,x)
```

```
[Out] int(ln(c*x^n)/x/(b*ln(c*x^n)^2+a*x^m)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(24*b^3*m*n^4*log(c)^3 - 5*a^3*m^4*n*x^(3*m) - (m^5*log(c)^3 + 7*m^4*n
*log(c)^2 - 18*m^3*n^2*log(c) - 4*m^2*n^3)*a^2*b*x^(2*m) + 2*(5*m^3*n^2*log
(c)^3 - 6*m^2*n^3*log(c)^2 + 20*m*n^4*log(c) + 16*n^5)*a*b^2*x^m - (a^2*b*m
^5*x^(2*m) - 10*a*b^2*m^3*n^2*x^m - 24*b^3*m*n^4)*log(x^n)^3 + (72*b^3*m*n^
4*log(c) - (3*m^5*log(c) + 7*m^4*n)*a^2*b*x^(2*m) + 6*(5*m^3*n^2*log(c) - 2
*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 + (72*b^3*m*n^4*log(c)^2 - (3*m^5*log(c)^2
+ 14*m^4*n*log(c) - 18*m^3*n^2)*a^2*b*x^(2*m) + 2*(15*m^3*n^2*log(c)^2 - 12
*m^2*n^3*log(c) + 20*m*n^4)*a*b^2*x^m)*log(x^n))/(64*a*b^5*n^6*x^m*log(c)^4
 + a^6*m^6*x^(6*m) + 2*(m^6*log(c)^2 + 6*m^4*n^2)*a^5*b*x^(5*m) + (m^6*log(
c)^4 + 24*m^4*n^2*log(c)^2 + 48*m^2*n^4)*a^4*b^2*x^(4*m) + 4*(3*m^4*n^2*log
(c)^4 + 24*m^2*n^4*log(c)^2 + 16*n^6)*a^3*b^3*x^(3*m) + 16*(3*m^2*n^4*log(c
)^4 + 8*n^6*log(c)^2)*a^2*b^4*x^(2*m) + (a^4*b^2*m^6*x^(4*m) + 12*a^3*b^3*m
^4*n^2*x^(3*m) + 48*a^2*b^4*m^2*n^4*x^(2*m) + 64*a*b^5*n^6*x^m)*log(x^n)^4
+ 4*(a^4*b^2*m^6*x^(4*m)*log(c) + 12*a^3*b^3*m^4*n^2*x^(3*m)*log(c) + 48*a^
2*b^4*m^2*n^4*x^(2*m)*log(c) + 64*a*b^5*n^6*x^m*log(c))*log(x^n)^3 + 2*(192
*a*b^5*n^6*x^m*log(c)^2 + a^5*b*m^6*x^(5*m) + 3*(m^6*log(c)^2 + 4*m^4*n^2)*
a^4*b^2*x^(4*m) + 12*(3*m^4*n^2*log(c)^2 + 4*m^2*n^4)*a^3*b^3*x^(3*m) + 16*
(9*m^2*n^4*log(c)^2 + 4*n^6)*a^2*b^4*x^(2*m))*log(x^n)^2 + 4*(64*a*b^5*n^6*
x^m*log(c)^3 + a^5*b*m^6*x^(5*m)*log(c) + (m^6*log(c)^3 + 12*m^4*n^2*log(c)
)*a^4*b^2*x^(4*m) + 12*(m^4*n^2*log(c)^3 + 4*m^2*n^4*log(c))*a^3*b^3*x^(3*m
) + 16*(3*m^2*n^4*log(c)^3 + 4*n^6*log(c))*a^2*b^4*x^(2*m))*log(x^n)) + int
egrate(1/2*((2*m^8*log(c) + 15*m^7*n)*a^3*x^(3*m) - 2*(17*m^6*n^2*log(c) -
m^5*n^3)*a^2*b*x^(2*m) - 32*(3*m^4*n^4*log(c) + 2*m^3*n^5)*a*b^2*x^m - 96*(
m^2*n^6*log(c) + m*n^7)*b^3 + 2*(a^3*m^8*x^(3*m) - 17*a^2*b*m^6*n^2*x^(2*m)
 - 48*a*b^2*m^4*n^4*x^m - 48*b^3*m^2*n^6)*log(x^n))/(256*a*b^5*n^8*x*x^m*lo
g(c)^2 + a^6*m^8*x*x^(6*m) + (m^8*log(c)^2 + 16*m^6*n^2)*a^5*b*x*x^(5*m) +
16*(m^6*n^2*log(c)^2 + 6*m^4*n^4)*a^4*b^2*x*x^(4*m) + 32*(3*m^4*n^4*log(c)^
2 + 8*m^2*n^6)*a^3*b^3*x*x^(3*m) + 256*(m^2*n^6*log(c)^2 + n^8)*a^2*b^4*x*x
^(2*m) + (a^5*b*m^8*x*x^(5*m) + 16*a^4*b^2*m^6*n^2*x*x^(4*m) + 96*a^3*b^3*m
^4*n^4*x*x^(3*m) + 256*a^2*b^4*m^2*n^6*x*x^(2*m) + 256*a*b^5*n^8*x*x^m)*log
(x^n)^2 + 2*(a^5*b*m^8*x*x^(5*m)*log(c) + 16*a^4*b^2*m^6*n^2*x*x^(4*m)*log(
```

```
c) + 96*a^3*b^3*m^4*n^4*x*x^(3*m)*log(c) + 256*a^2*b^4*m^2*n^6*x*x^(2*m)*lo
g(c) + 256*a*b^5*n^8*x*x^m*log(c))*log(x^n)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)}{x\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3),x)
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**3,x)
[Out] Timed out
```

3.16 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{p+1}$$

```
[Out] (a*x^m+b*ln(c*x^n)^q)^(1+p)/(1+p)
```

Rubi [A] time = 0.17, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{\left(ax^m+b\log^q(cx^n)\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.09, size = 26, normalized size = 1.00

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

fricas [A] time = 0.44, size = 42, normalized size = 1.62

$$\frac{\left((n\log(x)+\log(c))^q b+ax^m\right)\left((n\log(x)+\log(c))^q b+ax^m\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="fricas")
```

```
[Out] ((n*log(x) + log(c))^q*b + a*x^m)*((n*log(x) + log(c))^q*b + a*x^m)^p/(p +
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,3,1,2,3]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,2,1,2,3]%%%}+%%%{5,[0,0,2
,4,2,0,4,1,3,1,2,3]%%%}+%%%{1,[0,0,2,3,2,2,5,0,1,1,2,3]%%%}+%%%{-8,[0,0,2,3
,2,1,4,1,2,1,2,3]%%%}+%%%{10,[0,0,2,3,2,0,3,2,3,1,2,3]%%%}+%%%{3,[0,0,2,2,2
,2,4,1,1,1,2,3]%%%}+%%%{-12,[0,0,2,2,2,1,3,2,2,1,2,3]%%%}+%%%{10,[0,0,2,2,2
,0,2,3,3,1,2,3]%%%}+%%%{3,[0,0,2,1,2,2,3,2,1,1,2,3]%%%}+%%%{-8,[0,0,2,1,2,1
,2,3,2,1,2,3]%%%}+%%%{5,[0,0,2,1,2,0,1,4,3,1,2,3]%%%}+%%%{1,[0,0,2,0,2,2,2,
3,1,1,2,3]%%%}+%%%{-2,[0,0,2,0,2,1,1,4,2,1,2,3]%%%}+%%%{1,[0,0,2,0,2,0,0,5,
3,1,2,3]%%%} / %%%{1,[0,0,2,5,3,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0,2,4,3,1,5,0,
1,1,2,2]%%%}+%%%{5,[0,0,2,4,3,0,4,1,2,1,2,2]%%%}+%%%{1,[0,0,2,3,3,2,5,0,0,1
,2,2]%%%}+%%%{-8,[0,0,2,3,3,1,4,1,1,1,2,2]%%%}+%%%{10,[0,0,2,3,3,0,3,2,2,1,
2,2]%%%}+%%%{3,[0,0,2,2,3,2,4,1,0,1,2,2]%%%}+%%%{-12,[0,0,2,2,3,1,3,2,1,1,2
,2]%%%}+%%%{10,[0,0,2,2,3,0,2,3,2,1,2,2]%%%}+%%%{3,[0,0,2,1,3,2,3,2,0,1,2,2
]%%%}+%%%{-8,[0,0,2,1,3,1,2,3,1,1,2,2]%%%}+%%%{5,[0,0,2,1,3,0,1,4,2,1,2,2]%
%%}+%%%{1,[0,0,2,0,3,2,2,3,0,1,2,2]%%%}+%%%{-2,[0,0,2,0,3,1,1,4,1,1,2,2]%%%
}+%%%{1,[0,0,2,0,3,0,0,5,2,1,2,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\left(bnq \ln \left(c\,x^{n}\right)^{q-1}+am\,x^{m}\right) \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((a*m*x^m+b*n*q*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a\,m\,x^m+b\,n\,q\,{\ln \left(c\,x^n\right)}^{q-1}\right)\,{\left(a\,x^m+b\,{\ln \left(c\,x^n\right)}^q\right)}^p}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
```

```
[Out] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x
,x)
```

```
[Out] Timed out
```

3.17 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

```
[Out] 1/3*(a*x^m+b*ln(c*x^n)^q)^3
```

Rubi [A] time = 0.17, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx=\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

fricas [B] time = 0.44, size = 65, normalized size = 2.95

$$\left(n\log(x)+\log(c)\right)^q a^2bx^{2m}+\left(n\log(x)+\log(c)\right)^{2q}ab^2x^m+\frac{1}{3}\left(n\log(x)+\log(c)\right)^{3q}b^3+\frac{1}{3}a^3x^{3m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a^2*b*x^(2*m) + (n*log(x) + log(c))^(2*q)*a*b^2*x^m +
 1/3*(n*log(x) + log(c))^(3*q)*b^3 + 1/3*a^3*x^(3*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bnq\log\left(cx^{n}\right)^{q-1} + amx^{m}\right)\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)^2/x
, x)
```

maple [C] time = 0.66, size = 204, normalized size = 9.27

$$a^{2}b\,x^{2m}\left(-\frac{i\pi\left(\operatorname{csgn}\left(ic\right)-\operatorname{csgn}\left(ic\,x^{n}\right)\right)\left(\operatorname{csgn}\left(ix^{n}\right)-\operatorname{csgn}\left(ic\,x^{n}\right)\right)\operatorname{csgn}\left(ic\,x^{n}\right)}{2}+\ln(c)+\ln\left(x^{n}\right)\right)^{q}+a\,b^{2}x^{m}\left(-\frac{i\pi\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] 1/3*a^3*(x^m)^3+1/3*b^3*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^3+a*b^2*x^m*((ln(c)+ln(x^n)
-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x
^n)))^q)^2+a^2*b*(x^m)^2*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [B] time = 0.70, size = 20, normalized size = 0.91

$$\frac{\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
```

```
[Out] (a*x^m + b*log(c*x^n)^q)^3/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x
,x)
```

```
[Out] Timed out
```

3.18 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

```
[Out] 1/2*(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] time = 0.11, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2544}

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx=\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/
x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

fricas [B] time = 0.44, size = 42, normalized size = 1.91

$$\left(n\log(x)+\log(c)\right)^q abx^m+\frac{1}{2}\left(n\log(x)+\log(c)\right)^{2q}b^2+\frac{1}{2}a^2x^{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a*b*x^m + 1/2*(n*log(x) + log(c))^(2*q)*b^2 + 1/2*a^2
*x^(2*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bnq\log\left(cx^{n}\right)^{q-1} + amx^{m}\right)\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)/x,
x)
```

maple [C] time = 0.67, size = 135, normalized size = 6.14

$$ab\,x^{m}\left(-\frac{i\pi\left(\operatorname{csgn}\left(ic\right)-\operatorname{csgn}\left(ic\,x^{n}\right)\right)\left(\operatorname{csgn}\left(ix^{n}\right)-\operatorname{csgn}\left(ic\,x^{n}\right)\right)\operatorname{csgn}\left(ic\,x^{n}\right)}{2}+\ln(c)+\ln\left(x^{n}\right)\right)^{q}+\frac{a^{2}x^{2m}}{2}+\frac{b^{2}\left(-\frac{i\pi(c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] 1/2*a^2*(x^m)^2+1/2*b^2*((-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-
csgn(I*c*x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))^q)^2+a*b*x^m*(-1/2*I*Pi*(csgn(I
*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))
^q
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [B] time = 0.68, size = 20, normalized size = 0.91

$$\frac{\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q))/x,x)
```

```
[Out] (a*x^m + b*log(c*x^n)^q)^2/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Timed out
```

3.19 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=16

$$ax^m + b\log^q(cx^n)$$

```
[Out] a*x^m+b*ln(c*x^n)^q
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {14, 2302, 30}

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx &= \int \left(amx^{-1+m} + \frac{bnq\log^{-1+q}(cx^n)}{x}\right)dx \\
&= ax^m + (bnq)\int \frac{\log^{-1+q}(cx^n)}{x}\,dx \\
&= ax^m + (bq)\,\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right) \\
&= ax^m + b\log^q(cx^n)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

fricas [A] time = 0.44, size = 28, normalized size = 1.75

$$\left(bn\log(x) + b\log(c)\right)\left(n\log(x) + \log(c)\right)^{q-1} + ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
[Out] (b*n*log(x) + b*log(c))*(n*log(x) + log(c))^(q - 1) + a*x^m
```

giac [A] time = 0.20, size = 17, normalized size = 1.06

$$\left(n\log(x) + \log(c)\right)^q b + ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
[Out] (n*log(x) + log(c))^q*b + a*x^m
```

maple [A] time = 0.08, size = 17, normalized size = 1.06

$$a\,x^m + b\ln\left(c\,x^n\right)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)/x,x)
[Out] a*x^m+b*ln(c*x^n)^q
```

maxima [A] time = 0.45, size = 16, normalized size = 1.00

$$ax^m + b\log\left(cx^n\right)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
[Out] a*x^m + b*log(c*x^n)^q
```

mupad [B] time = 0.30, size = 16, normalized size = 1.00

$$a\,x^m + b\ln\left(c\,x^n\right)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/x,x)
[Out] a*x^m + b*log(c*x^n)^q
```

sympy [A] time = 30.63, size = 58, normalized size = 3.62

$$am\left(\begin{cases} \frac{x^m}{m} & \text{for}\ m \neq 0 \\ \log(x) & \text{otherwise} \end{cases}\right) + bnq\left(\begin{cases} \frac{\log(x)}{\log(c)} & \text{for}\ n = 0 \wedge q = 0 \\ \frac{\log(c)^q \log(x)}{\log(c)} & \text{for}\ n = 0 \\ \frac{\log\left(n\log(x) + \log(c)\right)}{n} & \text{for}\ q = 0 \\ \frac{\left(n\log(x) + \log(c)\right)^q}{nq} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] a*m*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + b*n*q*Piecewise((log(x)
/log(c), Eq(n, 0) & Eq(q, 0)), (log(c)**q*log(x)/log(c), Eq(n, 0)), (log(n*
log(x) + log(c))/n, Eq(q, 0)), ((n*log(x) + log(c))**q/(n*q), True))
```

3.20 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=17

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

```
[Out] ln(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.19, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2541}

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Mathematica [A] time = 0.24, size = 17, normalized size = 1.00

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

fricas [A] time = 0.46, size = 18, normalized size = 1.06

$$\log\left(\left(n\log(x)+\log(c)\right)^q b+ax^m\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="fricas")
```

```
[Out] log((n*log(x) + log(c))^q*b + a*x^m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnq\log\left(cx^n\right)^{q-1}+amx^m}{\left(ax^m+b\log\left(cx^n\right)^q\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)*x)
, x)
```

maple [C] time = 0.58, size = 212, normalized size = 12.47

$$-q\ln\left(-\frac{i\pi\left(\operatorname{csgn}(ic)-\operatorname{csgn}(ic\,x^{n})\right)\left(\operatorname{csgn}(ix^{n})-\operatorname{csgn}(ic\,x^{n})\right)\operatorname{csgn}(ic\,x^{n})}{2}+\ln(c)+\ln(x^{n})\right)+q\ln\left(-\frac{i\pi\operatorname{csgn}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*c*x^n)^2*csgn(I
*c)+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*csgn(I*c*x^n)*csgn(I*c)*c
sgn(I*x^n))-q*ln(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*
x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))+ln((-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*
(csgn(I*x^n)-csgn(I*c*x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))^q+1/b*a*x^m)
```

maxima [A] time = 1.03, size = 22, normalized size = 1.29

$$\log\left(\frac{ax^{m}+b\left(\log(c)+\log\left(x^{n}\right)\right)^{q}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="maxima")
```

```
[Out] log((a*x^m + b*(log(c) + log(x^n))^q)/b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{a\,m\,x^{m}+b\,n\,q\,\ln\left(c\,x^{n}\right)^{q-1}}{x\,\left(a\,x^{m}+b\,\ln\left(c\,x^{n}\right)^{q}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Timed out
```

3.21 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

```
[Out] -1/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.18, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=-\frac{1}{ax^m+b\log^q(cx^n)}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

fricas [A] time = 0.44, size = 21, normalized size = 1.05

$$-\frac{1}{\left(n\log(x)+\log(c)\right)^q b+ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="fricas")
```

[Out] -1/((n*log(x) + log(c))^q*b + a*x^m)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnq\log\left(cx^{n}\right)^{q-1} + amx^{m}}{\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^2*
x), x)
```

maple [C] time = 0.54, size = 68, normalized size = 3.40

$$-\frac{1}{a\,x^{m}+b\left(-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n})\right)\left(\mathrm{csgn}(ix^{n})-\mathrm{csgn}(ic\,x^{n})\right)\mathrm{csgn}(ic\,x^{n})}{2}+\ln(c)+\ln\left(x^{n}\right)\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] -1/(a*x^m+b*(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*x^n)
)*csgn(I*c*x^n)+ln(c)+ln(x^n))^q)
```

maxima [A] time = 1.22, size = 21, normalized size = 1.05

$$-\frac{1}{ax^{m} + b\left(\log(c) + \log\left(x^{n}\right)\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="maxima")
```

```
[Out] -1/(a*x^m + b*(log(c) + log(x^n))^q)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,m\,x^{m} + b\,n\,q\,\ln\left(c\,x^{n}\right)^{q-1}}{x\,\left(a\,x^{m} + b\,\ln\left(c\,x^{n}\right)^{q}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2
,x)
```

```
[Out] Timed out
```

3.22 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] -1/2/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] time = 0.18, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/(2*(a*x^m + b*Log[c*x^n]^q)^2)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^3),x]
```

```
[Out] -1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

fricas [B] time = 0.46, size = 45, normalized size = 2.05

$$-\frac{1}{2\left(2\left(n\log(x)+\log(c)\right)^q abx^m+\left(n\log(x)+\log(c)\right)^{2q}b^2+a^2x^{2m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/2/(2*(n*log(x) + log(c))^q*a*b*x^m + (n*log(x) + log(c))^(2*q)*b^2 + a^2
*x^(2*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnq\log(cx^n)^{q-1} + amx^m}{\left(ax^m + b\log(cx^n)^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^3*
x), x)
```

maple [C] time = 5.44, size = 68, normalized size = 3.09

$$-\frac{1}{2\left(a\,x^m + b\left(-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(\mathrm{csgn}(ix^n)-\mathrm{csgn}(ic\,x^n)\right)\mathrm{csgn}(ic\,x^n)}{2} + \ln(c) + \ln(x^n)\right)^q\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*n*q*ln(c*x^n)^(q-1)+a*m*x^m)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] -1/2/(a*x^m+b*(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*x^
n))*csgn(I*c*x^n)+ln(c)+ln(x^n))^q)^2
```

maxima [B] time = 1.67, size = 49, normalized size = 2.23

$$-\frac{1}{2\left(a^2x^{2m} + b^2\left(\log(c) + \log(x^n)\right)^{2q} + 2\,abe^{\left(m\log(x)+q\log\left(\log(c)+\log(x^n)\right)\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2/(a^2*x^(2*m) + b^2*(log(c) + log(x^n))^(2*q) + 2*a*b*e^(m*log(x) + q*l
og(log(c) + log(x^n))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,m\,x^m + b\,n\,q\ln(c\,x^n)^{q-1}}{x\left(a\,x^m + b\ln(c\,x^n)^q\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3
,x)
```

```
[Out] Timed out
```

3.23 $\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

```
[Out] 1/3*(a*x+b*ln(c*x^n)^2)^3
```

Rubi [A] time = 0.15, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2561, 2544}

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx &= \int \frac{\left(ax+2bn\log(cx^n)\right)\left(ax^2+bx\log^2(cx^n)\right)^2}{x^3}\,dx\\ &= \int \frac{\left(ax+2bn\log(cx^n)\right)\left(ax+b\log^2(cx^n)\right)^2}{x}\,dx\\ &= \frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

fricas [B] time = 0.46, size = 195, normalized size = 9.75

$$\frac{1}{3}\,b^3n^6\log(x)^6+2\,b^3n^5\log(c)\log(x)^5+ab^2x\log(c)^4+a^2bx^2\log(c)^2+\frac{1}{3}\,a^3x^3+\left(5\,b^3n^4\log(c)^2+ab^2n^4x\right)\log(x)^4+\frac{4}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="fricas")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + a*b^2*x*log(c)^4 + a^2*b
*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2 + a*b^2*n^4*x)*log(x)^4 +
 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*log(x)^3 + (5*b^3*n^2*log(
c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*log(x)^2 + 2*(b^3*n*log(c)^5
 + 2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

giac [B] time = 0.20, size = 198, normalized size = 9.90

$$\frac{1}{3}b^3n^6\log(x)^6+2b^3n^5\log(c)\log(x)^5+2b^3n\log(c)^5\log(x)+ab^2x\log(c)^4+a^2bx^2\log(c)^2+\frac{1}{3}a^3x^3+\left(5b^3n^4\log(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="giac")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + 2*b^3*n*log(c)^5*log(x)
+ a*b^2*x*log(c)^4 + a^2*b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2
 + a*b^2*n^4*x)*log(x)^4 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*
log(x)^3 + (5*b^3*n^2*log(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*lo
g(x)^2 + 2*(2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

maple [C] time = 2.89, size = 16321, normalized size = 816.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+2*b*n*ln(c*x^n)/x^3)*(a*x^2+b*x*ln(c*x^n)^2)^2,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.49, size = 211, normalized size = 10.55

$$\frac{1}{3}b^3\log\left(cx^n\right)^6+4ab^2nx\log\left(cx^n\right)^3+ab^2x\log\left(cx^n\right)^4-\frac{1}{2}a^2bn^2x^2+a^2bnx^2\log\left(cx^n\right)+a^2bx^2\log\left(cx^n\right)^2+\frac{1}{3}a^3x^3-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="maxima")
```

```
[Out] 1/3*b^3*log(c*x^n)^6 + 4*a*b^2*n*x*log(c*x^n)^3 + a*b^2*x*log(c*x^n)^4 - 1/
2*a^2*b*n^2*x^2 + a^2*b*n*x^2*log(c*x^n) + a^2*b*x^2*log(c*x^n)^2 + 1/3*a^3
*x^3 - 12*(n*x*log(c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*a*b^2*n + 1/2*(
n^2*x^2 - 2*n*x^2*log(c*x^n))*a^2*b - 4*(n*x*log(c*x^n)^3 - 3*(n*x*log(c*x^
n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*n)*a*b^2
```

mupad [B] time = 0.33, size = 52, normalized size = 2.60

$$\frac{a^3\,x^3}{3}+a^2\,b\,x^2\,\ln\left(c\,x^n\right)^2+a\,b^2\,x\,\ln\left(c\,x^n\right)^4+\frac{b^3\,\ln\left(c\,x^n\right)^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)^2*(a/x^2 + (2*b*n*log(c*x^n))/x^3),x)
```

```
[Out] (b^3*log(c*x^n)^6)/3 + (a^3*x^3)/3 + a^2*b*x^2*log(c*x^n)^2 + a*b^2*x*log(c
*x^n)^4
```

sympy [A] time = 12.32, size = 221, normalized size = 11.05

$$\frac{a^3x^3}{3}+a^2bn^2x^2\log(x)^2-a^2bn^2x^2\log(x)+2a^2bnx^2\log(c)\log(x)-a^2bnx^2\log(c)+a^2bnx^2\log(cx^n)+a^2bx^2\log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+2*b*n*ln(c*x**n)/x**3)*(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] a**3*x**3/3 + a**2*b*n**2*x**2*log(x)**2 - a**2*b*n**2*x**2*log(x) + 2*a**2
*b*n*x**2*log(c)*log(x) - a**2*b*n*x**2*log(c) + a**2*b*n*x**2*log(c*x**n)
+ a**2*b*x**2*log(c)**2 + a*b**2*n**4*x*log(x)**4 + 4*a*b**2*n**3*x*log(c)*
log(x)**3 + 6*a*b**2*n**2*x*log(c)**2*log(x)**2 + 4*a*b**2*n*x*log(c)**3*lo
g(x) + a*b**2*x*log(c)**4 - 2*b**3*n*Piecewise((-log(c)**5*log(x), Eq(n, 0)
), (-log(c*x**n)**6/(6*n), True))
```

3.24 $\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx$

Optimal. Leaf size=20

$$\frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2$$

```
[Out] 1/2*(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2561, 2544}

$$\frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^2/2
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx &= \int \frac{\left(ax+2bn\log(cx^n)\right)\left(ax^2+bx\log^2(cx^n)\right)}{x^2}\,dx\\
&= \int \frac{\left(ax+2bn\log(cx^n)\right)\left(ax+b\log^2(cx^n)\right)}{x}\,dx\\
&= \frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2
\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.90

$$\frac{a^2x^2}{2}+abx\log^2(cx^n)+\frac{1}{2}b^2\log^4(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a^2*x^2)/2 + a*b*x*Log[c*x^n]^2 + (b^2*Log[c*x^n]^4)/2
```

fricas [B] time = 0.44, size = 89, normalized size = 4.45

$$\frac{1}{2}b^2n^4\log(x)^4+2b^2n^3\log(c)\log(x)^3+abx\log(c)^2+\frac{1}{2}a^2x^2+\left(3b^2n^2\log(c)^2+abn^2x\right)\log(x)^2+2\left(b^2n\log(c)^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"fricas")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + a*b*x*log(c)^2 + 1/2*a^2
*x^2 + (3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 2*(b^2*n*log(c)^3 + a*b*
n*x*log(c))*log(x)
```

giac [B] time = 0.18, size = 90, normalized size = 4.50

$$\frac{1}{2}\,b^2n^4\log(x)^4+2\,b^2n^3\log(c)\log(x)^3+2\,b^2n\log(c)^3\log(x)+2\,abnx\log(c)\log(x)+abx\log(c)^2+\frac{1}{2}\,a^2x^2+\left(3\,b^2n^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"giac")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + 2*b^2*n*log(c)^3*log(x)
+ 2*a*b*n*x*log(c)*log(x) + a*b*x*log(c)^2 + 1/2*a^2*x^2 + (3*b^2*n^2*log(c
)^2 + a*b*n^2*x)*log(x)^2
```

maple [B] time = 0.09, size = 63, normalized size = 3.15

$$\frac{b^2\ln\left(c\,x^n\right)^4}{2}+2abnx\ln\left(c\,x^n\right)-2abnx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+abx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+2*b*n*ln(c*x^n)/x^2)*(a*x^2+b*x*ln(c*x^n)^2),x)
```

```
[Out] 1/2*a^2*x^2+a*b*x*ln(c*exp(n*ln(x)))^2-2*a*b*n*x*ln(c*exp(n*ln(x)))+1/2*b^2
*ln(c*x^n)^4+2*ln(c*x^n)*a*b*n*x
```

maxima [B] time = 0.46, size = 74, normalized size = 3.70

$$\frac{1}{2}\,b^2\log\left(cx^n\right)^4-2\,abn^2x+2\,abnx\log\left(cx^n\right)+abx\log\left(cx^n\right)^2+\frac{1}{2}\,a^2x^2+2\left(n^2x-nx\log\left(cx^n\right)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"maxima")
```

```
[Out] 1/2*b^2*log(c*x^n)^4 - 2*a*b*n^2*x + 2*a*b*n*x*log(c*x^n) + a*b*x*log(c*x^n
)^2 + 1/2*a^2*x^2 + 2*(n^2*x - n*x*log(c*x^n))*a*b
```

mupad [B] time = 0.29, size = 18, normalized size = 0.90

$$\frac{\left(b\ln\left(c\,x^n\right)^2+a\,x\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)*(a/x + (2*b*n*log(c*x^n))/x^2),x)
```

```
[Out] (a*x + b*log(c*x^n)^2)^2/2
```

sympy [A] time = 8.07, size = 117, normalized size = 5.85

$$\frac{a^2x^2}{2}+abn^2x\log\left(x\right)^2-2abn^2x\log\left(x\right)+2abnx\log\left(c\right)\log\left(x\right)-2abnx\log\left(c\right)+2abnx\log\left(cx^n\right)+abx\log\left(c\right)^2-2b^2n\left(\left\{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*ln(c*x**n)/x**2)*(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] a**2*x**2/2 + a*b*n**2*x*log(x)**2 - 2*a*b*n**2*x*log(x) + 2*a*b*n*x*log(c)
*log(x) - 2*a*b*n*x*log(c) + 2*a*b*n*x*log(c*x**n) + a*b*x*log(c)**2 - 2*b*
*2*n*Piecewise((-log(c)**3*log(x), Eq(n, 0)), (-log(c*x**n)**4/(4*n), True)
)
```

3.25 $\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx$

Optimal. Leaf size=14

$$ax + b\log^2(cx^n)$$

```
[Out] a*x+b*ln(c*x^n)^2
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2301}

$$ax + b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx &= ax + (2bn)\int \frac{\log(cx^n)}{x}\,dx\\ &= ax + b\log^2(cx^n)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$ax + b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

fricas [A] time = 0.43, size = 21, normalized size = 1.50

$$bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="fricas")
```

```
[Out] b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + a*x
```

giac [A] time = 0.17, size = 20, normalized size = 1.43

$$\left(n\log(x)^2 + 2\,\log(c)\log(x)\right)bn + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="giac")
```

```
[Out] (n*log(x)^2 + 2*log(c)*log(x))*b*n + a*x
```

maple [A] time = 0.07, size = 15, normalized size = 1.07

$$b\ln\left(c\,x^{n}\right)^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+2*b*n*ln(c*x^n)/x,x)
[Out] a*x+b*ln(c*x^n)^2
```

maxima [A] time = 0.45, size = 14, normalized size = 1.00

$$b\log\left(cx^{n}\right)^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="maxima")
[Out] b*log(c*x^n)^2 + a*x
```

mupad [B] time = 0.26, size = 14, normalized size = 1.00

$$b\ln\left(c\,x^{n}\right)^{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + (2*b*n*log(c*x^n))/x,x)
[Out] a*x + b*log(c*x^n)^2
```

sympy [A] time = 1.72, size = 60, normalized size = 4.29

$$ax+2bn\left(\begin{cases}\frac{\log\left(cx^{n}\right)^{2}}{2n} & \text{for}\ \left|cx^{n}\right|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^{2}}{2n} & \text{for}\ \frac{1}{\left|cx^{n}\right|}<1\\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1\\ 0,0,0 & \end{matrix}\middle|cx^{n}\right)}{n}+\frac{G_{3,3}^{0,3}\left(\begin{matrix}1,1,1 & \\ & 0,0,0\end{matrix}\middle|cx^{n}\right)}{n} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*ln(c*x**n)/x,x)
[Out] a*x + 2*b*n*Piecewise((log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(x**(-n)
/c)**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ())
, c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

3.26 $\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=15

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

```
[Out] ln(a*x+b*ln(c*x^n)^2)
```

Rubi [A] time = 0.08, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2561, 2541}

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+2bn\log\left(cx^n\right)}{ax^2+bx\log^2\left(cx^n\right)}\,dx &= \int \frac{ax+2bn\log\left(cx^n\right)}{x\left(ax+b\log^2\left(cx^n\right)\right)}\,dx\\ &= \log\left(ax+b\log^2\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 15, normalized size = 1.00

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

fricas [A] time = 0.46, size = 28, normalized size = 1.87

$$\log\left(bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="fri
cas")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

giac [A] time = 0.16, size = 28, normalized size = 1.87

$$\log\left(bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="gia
c")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

maple [C] time = 0.29, size = 432, normalized size = 28.80

$$\ln\left(\ln\left(x^n\right)^2 + \left(-i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right) + i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\,x^n\right)^2 + i\pi\,\mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)^2 - i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+2*b*n*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2),x)
```

```
[Out] ln(1/4*(-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*b*Pi^2*csgn(I*x^n)^2*csgn(I
*c*x^n)^3*csgn(I*c)-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+2*b*Pi
^2*csgn(I*x^n)*csgn(I*c*x^n)^5-4*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*
c)+2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-b*Pi^2*csgn(I*c*x^n)^6+
2*b*Pi^2*csgn(I*c*x^n)^5*csgn(I*c)-b*Pi^2*csgn(I*c*x^n)^4*csgn(I*c)^2+4*I*b
*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x
^n)*csgn(I*c)-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^3+4*I*b*ln(c)*Pi*csgn(I*c*x^n)^2
*csgn(I*c)+4*b*ln(c)^2+4*a*x)/b+(I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn
(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*csgn(I*c*x^n)^3+I*Pi*csgn(I*c*x^n)^2*c
sgn(I*c)+2*ln(c))*ln(x^n)+ln(x^n)^2)
```

maxima [B] time = 0.61, size = 32, normalized size = 2.13

$$\log\left(\frac{b\log(c)^2 + 2\,b\log(c)\log\left(x^n\right) + b\log\left(x^n\right)^2 + ax}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="max
ima")
```

```
[Out] log((b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)/b)
```

mupad [B] time = 0.35, size = 16, normalized size = 1.07

$$\ln\left(\ln\left(c\,x^n\right)^2 + \frac{a\,x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 2*b*n*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2),x)
```

```
[Out] log(log(c*x^n)^2 + (a*x)/b)
```

sympy [A] time = 2.33, size = 48, normalized size = 3.20

$$\begin{cases} \log\left(x + \frac{bn^2\log\left(x\right)^2}{a} + \frac{2bn\log\left(c\right)\log\left(x\right)}{a} + \frac{b\log\left(c\right)^2}{a}\right) & \text{for}\: a \neq 0 \\ 2\log\left(n\log\left(x\right) + \log\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((log(x + b*n**2*log(x)**2/a + 2*b*n*log(c)*log(x)/a + b*log(c)**2
/a), Ne(a, 0)), (2*log(n*log(x) + log(c)), True))
```

3.27 $\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{ax+b\log^2(cx^n)}$$

```
[Out] -1/(a*x+b*ln(c*x^n)^2)
```

Rubi [A] time = 0.13, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2561, 2544}

$$-\frac{1}{ax+b\log^2(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx &= \int \frac{x\left(ax+2bn\log(cx^n)\right)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx\\
&= \int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^2}\,dx\\
&= -\frac{1}{ax+b\log^2(cx^n)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{ax+b\log^2(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

fricas [A] time = 0.44, size = 31, normalized size = 1.72

$$-\frac{1}{bn^2 \log(x)^2 + 2\,bn \log(c)\log(x) + b\log(c)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="fricas")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

giac [A] time = 0.18, size = 31, normalized size = 1.72

$$-\frac{1}{bn^2 \log(x)^2 + 2\,bn \log(c)\log(x) + b\log(c)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="giac")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

maple [C] time = 0.31, size = 451, normalized size = 25.06

$$-\frac{}{-\pi^2 b\,\mathrm{csgn}\,(ic)^2\,\mathrm{csgn}\,(ix^n)^2\,\mathrm{csgn}\,(ic\,x^n)^2 + 2\pi^2 b\,\mathrm{csgn}\,(ic)^2\,\mathrm{csgn}\,(ix^n)\,\mathrm{csgn}\,(ic\,x^n)^3 - \pi^2 b\,\mathrm{csgn}\,(ic)^2\,\mathrm{csgn}\,(ic\,x^n)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b*n*x*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^2,x)
```

```
[Out] -4/(-Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b*csgn(I*c)*csgn(I*x^n)^2*
csgn(I*c*x^n)^3-Pi^2*b*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+2*Pi^2*b*c
sgn(I*x^n)*csgn(I*c*x^n)^5-4*Pi^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+2
*Pi^2*b*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-Pi^2*b*csgn(I*c*x^n)^6+2*Pi
^2*b*csgn(I*c)*csgn(I*c*x^n)^5-Pi^2*b*csgn(I*c)^2*csgn(I*c*x^n)^4+4*I*b*ln(
x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^2*csgn(I
*c)-4*I*Pi*b*csgn(I*c*x^n)^3*ln(c)-4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3-4*I*b*l
n(x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*I*Pi*b*csgn(I*x^n)*csgn(I*c
*x^n)^2*ln(c)+4*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(c)-4*I*Pi*b*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)*ln(c)+4*b*ln(c)^2+8*b*ln(c)*ln(x^n)+4*b*ln(x^n)^2+
4*a*x)
```

maxima [A] time = 0.65, size = 31, normalized size = 1.72

$$-\frac{1}{b\log(c)^2 + 2\,b\log(c)\log\left(x^n\right) + b\log\left(x^n\right)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="maxima")
```

```
[Out] -1/(b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)
```

mupad [B] time = 0.26, size = 18, normalized size = 1.00

$$-\frac{1}{b\ln\left(c\,x^n\right)^2 + a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + 2*b*n*x*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^2,x)
```

```
[Out] -1/(a*x + b*log(c*x^n)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b*n*x*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] Timed out
```

3.28 $\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

```
[Out] -1/2/(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A] time = 0.16, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2561, 2544}

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/(2*(a*x + b*Log[c*x^n]^2)^2)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{ax^3+2bnx^2\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx &= \int \frac{x^2\left(ax+2bn\log\left(cx^n\right)\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx\\
&= \int \frac{ax+2bn\log\left(cx^n\right)}{x\left(ax+b\log^2\left(cx^n\right)\right)^3}\,dx\\
&= -\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

fricas [B] time = 0.45, size = 101, normalized size = 5.05

$$-\frac{1}{2\left(b^2n^4\log(x)^4 + 4\,b^2n^3\log(c)\log(x)^3 + b^2\log(c)^4 + 2\,abx\log(c)^2 + a^2x^2 + 2\left(3\,b^2n^2\log(c)^2 + abn^2x\right)\log(x)^2 + \cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="fricas")
```

```
[Out] -1/2/(b^2*n^4*log(x)^4 + 4*b^2*n^3*log(c)*log(x)^3 + b^2*log(c)^4 + 2*a*b*x
*log(c)^2 + a^2*x^2 + 2*(3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 4*(b^2*
n*log(c)^3 + a*b*n*x*log(c))*log(x))
```

giac [B] time = 0.20, size = 306, normalized size = 15.30

$$-\frac{\cdots}{2\left(4\,ab^3n^6x\log(x)^4 + 16\,ab^3n^5x\log(c)\log(x)^3 + a^2b^2n^4x^2\log(x)^4 + 24\,ab^3n^4x\log(c)^2\log(x)^2 + 4\,a^2b^2n^3x^2\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="giac")
```

```
[Out] -1/2*(4*a*b*n^2*x + a^2*x^2)/(4*a*b^3*n^6*x*log(x)^4 + 16*a*b^3*n^5*x*log(c
)*log(x)^3 + a^2*b^2*n^4*x^2*log(x)^4 + 24*a*b^3*n^4*x*log(c)^2*log(x)^2 +
4*a^2*b^2*n^3*x^2*log(c)*log(x)^3 + 16*a*b^3*n^3*x*log(c)^3*log(x) + 8*a^2*
b^2*n^4*x^2*log(x)^2 + 6*a^2*b^2*n^2*x^2*log(c)^2*log(x)^2 + 4*a*b^3*n^2*x*
log(c)^4 + 16*a^2*b^2*n^3*x^2*log(c)*log(x) + 4*a^2*b^2*n*x^2*log(c)^3*log(
x) + 2*a^3*b*n^2*x^3*log(x)^2 + 8*a^2*b^2*n^2*x^2*log(c)^2 + a^2*b^2*x^2*lo
g(c)^4 + 4*a^3*b*n*x^3*log(c)*log(x) + 4*a^3*b*n^2*x^3 + 2*a^3*b*x^3*log(c)
^2 + a^4*x^4)
```

maple [C] time = 0.32, size = 447, normalized size = 22.35

$$-\frac{\cdots}{\left(\pi^2 b\,\mathrm{csgn}\left(ic\right)^2\mathrm{csgn}\left(ix^n\right)^2\mathrm{csgn}\left(ic\,x^n\right)^2 - 2\pi^2 b\,\mathrm{csgn}\left(ic\right)^2\mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)^3 + \pi^2 b\,\mathrm{csgn}\left(ic\right)^2\mathrm{csgn}\left(ic\,x^n\right)\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b*n*x^2*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^3,x)
```

```
[Out] -8/(Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4-2*Pi^2*b*csgn(I*c)*csgn(I*x^n)^2*c
sgn(I*c*x^n)^3+Pi^2*b*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-2*Pi^2*b*cs
gn(I*x^n)*csgn(I*c*x^n)^5+4*Pi^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-2*
Pi^2*b*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+Pi^2*b*csgn(I*c*x^n)^6-2*Pi^
2*b*csgn(I*c)*csgn(I*c*x^n)^5+Pi^2*b*csgn(I*c)^2*csgn(I*c*x^n)^4+4*I*b*ln(x
^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3
+4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*b*ln(x^n)*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^2*csgn(I*c)-4*I*b*ln(c)*
Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^2*csgn(I*c)+4
*I*b*ln(c)*Pi*csgn(I*c*x^n)^3-4*b*ln(c)^2-8*b*ln(c)*ln(x^n)-4*b*ln(x^n)^2-4
*a*x)^2
```

maxima [B] time = 0.85, size = 95, normalized size = 4.75

$$-\frac{1}{2\left(b^2\log(c)^4 + 4\,b^2\log(c)\log\left(x^n\right)^3 + b^2\log\left(x^n\right)^4 + 2\,abx\log(c)^2 + a^2x^2 + 2\left(3\,b^2\log(c)^2 + abx\right)\log\left(x^n\right)^2 + \cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="maxima")
```

```
[Out] -1/2/(b^2*log(c)^4 + 4*b^2*log(c)*log(x^n)^3 + b^2*log(x^n)^4 + 2*a*b*x*log
(c)^2 + a^2*x^2 + 2*(3*b^2*log(c)^2 + a*b*x)*log(x^n)^2 + 4*(b^2*log(c)^3 +
 a*b*x*log(c))*log(x^n))
```

mupad [**B**] time = 0.27, size = 39, normalized size = 1.95

$$-\frac{1}{2\,a^2\,x^2+4\,a\,b\,x\,\ln\left(c\,x^n\right)^2+2\,b^2\,\ln\left(c\,x^n\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + 2*b*n*x^2*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^3,x)
```

```
[Out] -1/(2*b^2*log(c*x^n)^4 + 2*a^2*x^2 + 4*a*b*x*log(c*x^n)^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b*n*x**2*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**3,x)
```

```
[Out] Timed out
```

3.29 $\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx$

Optimal. Leaf size=19

$$\log\left(ax^{m-1}+b\log^q\left(cx^n\right)\right)$$

```
[Out] ln(a*x^(-1+m)+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.32, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2561, 2541}

$$\log\left(ax^{m-1}+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*Log[c*
x^n]^q),x]
```

```
[Out] Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{ax^m+bx\log^q\left(cx^n\right)}\,dx &= \int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)}\,dx\\ &= \log\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A] time = 0.48, size = 23, normalized size = 1.21

$$\log\left(ax^m+bx\log^q\left(cx^n\right)\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*
Log[c*x^n]^q),x]
```

```
[Out] -Log[x] + Log[a*x^m + b*x*Log[c*x^n]^q]
```

fricas [A] time = 0.45, size = 23, normalized size = 1.21

$$\log\left(\frac{\left(n\log(x)+\log(c)\right)^q bx+ax^m}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="fricas")
```

```
[Out] log(((n*log(x) + log(c))^q*b*x + a*x^m)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnq \log\left(cx^n\right)^{q-1} + a(m-1)x^{m-1}}{bx \log\left(cx^n\right)^q + ax^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*(m - 1)*x^(m - 1))/(b*x*log(c*x^n)^
q + a*x^m), x)
```

maple [C] time = 0.60, size = 215, normalized size = 11.32

$$-q\ln\left(-\frac{i\pi\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ix^{n}\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)\mathrm{csgn}\left(ic\,x^{n}\right)}{2}+\ln(c)+\ln\left(x^{n}\right)\right)+q\ln\left(-\frac{i\pi\,\mathrm{csgn}\left(ic\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(m-1)*x^(m-1)+b*n*q*ln(c*x^n)^(q-1))/(a*x^m+b*x*ln(c*x^n)^q),x)
```

```
[Out] q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*c*x^n)^2*csgn(I
*c)+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*csgn(I*c*x^n)*csgn(I*c)*c
sgn(I*x^n))-q*ln(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*
x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))+ln((-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*
(csgn(I*x^n)-csgn(I*c*x^n))*csgn(I*c*x^n)+ln(c)+ln(x^n))^q+1/x*a*x^m/b)
```

maxima [A] time = 1.03, size = 26, normalized size = 1.37

$$\log\left(\frac{bx\left(\log(c)+\log\left(x^{n}\right)\right)^{q}+ax^{m}}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="maxima")
```

```
[Out] log((b*x*(log(c) + log(x^n))^q + a*x^m)/(b*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,x^{m-1}\,\left(m-1\right)+b\,n\,q\,\ln\left(c\,x^n\right)^{q-1}}{a\,x^m+b\,x\,\ln\left(c\,x^n\right)^q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q),x)
```

```
[Out] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x**(-1+m)+b*n*q*ln(c*x**n)**(-1+q))/(a*x**m+b*x*ln(c*x*
*n)**q),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=81

$$\left(d-\frac{aem}{bnq}\right)\text{Int}\left(x^{m-1}\left(ax^m+b\log^q\left(cx^n\right)\right)^p,x\right)+\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^{p+1}}{bn(p+1)q}$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)+e*(a*x^
m+b*ln(c*x^n)^q)^(1+p)/b/n/(1+p)/q
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (e*(a*x^m + b*Log[c*x^n]^q)^(1 + p))/(b*n*(1 + p)*q) + (d - (a*e*m)/(b*n*q)
)*Defer[Int][x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p, x]
```

Rubi steps

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx=\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^{1+p}}{bn(1+p)q}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m+b\right.$$

Mathematica [A] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x, x
]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^m+e\log\left(cx^n\right)^{q-1}\right)\left(ax^m+b\log\left(cx^n\right)^q\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,2,1,2,2,1]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2,1]%%%}+%%%{5,[0
,0,2,4,2,0,4,1,2,1,2,2,1]%%%}+%%%{1,[0,0,2,3,2,2,5,0,0,1,2,2,1]%%%}+%%%{-8,
[0,0,2,3,2,1,4,1,1,1,2,2,1]%%%}+%%%{10,[0,0,2,3,2,0,3,2,2,1,2,2,1]%%%}+%%%{
3,[0,0,2,2,2,2,4,1,0,1,2,2,1]%%%}+%%%{-12,[0,0,2,2,2,1,3,2,1,1,2,2,1]%%%}+%
%%{10,[0,0,2,2,2,0,2,3,2,1,2,2,1]%%%}+%%%{3,[0,0,2,1,2,2,3,2,0,1,2,2,1]%%%}
+%%%{-8,[0,0,2,1,2,1,2,3,1,1,2,2,1]%%%}+%%%{5,[0,0,2,1,2,0,1,4,2,1,2,2,1]%%
%}+%%%{1,[0,0,2,0,2,2,2,3,0,1,2,2,1]%%%}+%%%{-2,[0,0,2,0,2,1,1,4,1,1,2,2,1]
%%%}+%%%{1,[0,0,2,0,2,0,0,5,2,1,2,2,1]%%%} / %%%{1,[0,0,2,5,3,0,5,0,2,1,2,2
,0]%%%}+%%%{-2,[0,0,2,4,3,1,5,0,1,1,2,2,0]%%%}+%%%{5,[0,0,2,4,3,0,4,1,2,1,2
,2,0]%%%}+%%%{1,[0,0,2,3,3,2,5,0,0,1,2,2,0]%%%}+%%%{-8,[0,0,2,3,3,1,4,1,1,1
,2,2,0]%%%}+%%%{10,[0,0,2,3,3,0,3,2,2,1,2,2,0]%%%}+%%%{3,[0,0,2,2,3,2,4,1,0
,1,2,2,0]%%%}+%%%{-12,[0,0,2,2,3,1,3,2,1,1,2,2,0]%%%}+%%%{10,[0,0,2,2,3,0,2
,3,2,1,2,2,0]%%%}+%%%{3,[0,0,2,1,3,2,3,2,0,1,2,2,0]%%%}+%%%{-8,[0,0,2,1,3,1
,2,3,1,1,2,2,0]%%%}+%%%{5,[0,0,2,1,3,0,1,4,2,1,2,2,0]%%%}+%%%{1,[0,0,2,0,3,
2,2,3,0,1,2,2,0]%%%}+%%%{-2,[0,0,2,0,3,1,1,4,1,1,2,2,0]%%%}+%%%{1,[0,0,2,0,
3,0,0,5,2,1,2,2,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{q-1}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^q\right)}^p\,\left(d\,x^m+e\,{\ln\left(c\,x^n\right)}^{q-1}\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
[Out] Timed out
```

3.31 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=331

$$-\frac{a^3x^{4m}(aem-bdnq)}{4bmnq}-\frac{a^2 3^{-q}x^{3m}\,(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\Gamma\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq}-\frac{b^2x^m\,(c$$

```
[Out] -1/4*a^3*(-b*d*n*q+a*e*m)*x^(4*m)/b/m/n/q-b^2*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+
3*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(3*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)
^(3*q))-3*2^(-1-2*q)*a*b*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+2*q,-2*m*ln(c*x^n
)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*(
-b*d*n*q+a*e*m)*x^(3*m)*GAMMA(1+q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/m/n/q
/((c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)+1/4*e*(a*x^m+b*ln(c*x^n)^q)^4/b/n/q
```

Rubi [A] time = 0.50, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2545, 6742, 2310, 2181}

$$-\frac{a^2 3^{-q}x^{3m}\,(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\text{Gamma}\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq}-\frac{b^2x^m\,(cx^n)^{-\frac{m}{n}}\log^{3q}(c$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] -(a^3*(a*e*m - b*d*n*q)*x^(4*m))/(4*b*m*n*q) - (b^2*(a*e*m - b*d*n*q)*x^m*G
amma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(m*n*q*(c*x^n)^(m/n)*(
-((m*Log[c*x^n])/n))^(3*q)) - (3*2^(-1 - 2*q)*a*b*(a*e*m - b*d*n*q)*x^(2*m)
*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^((2*m
)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*(a*e*m - b*d*n*q)*x^(3*m)*Gamma[1
+ q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*
Log[c*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = \frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m+b\log\right.$$

$$= \frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}-\left(-d+\frac{aem}{bnq}\right)\int\left(a^3x^{-1+4m}+3a^2b\right.$$

$$= \frac{a^3\left(d-\frac{aem}{bnq}\right)x^{4m}}{4m}+\frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}-\left(3a^2b\left(-d+\right.\right.$$

$$= \frac{a^3\left(d-\frac{aem}{bnq}\right)x^{4m}}{4m}+\frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}-\frac{\left(3a^2b\left(-d+\right.\right.}{}$$

$$= \frac{a^3\left(d-\frac{aem}{bnq}\right)x^{4m}}{4m}-\frac{b^2(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+3q,-\right.}{mn}$$

Mathematica [A] time = 1.62, size = 445, normalized size = 1.34

$$\frac{3^{-q}4^{-q-1}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\left(\left(-\frac{m\log(cx^n)}{n}\right)^q\left(-4a^2bem3^{q+1}qx^{2m}(cx^n)^{m/n}\log^{2q}(cx^n)\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)+4^q\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (4^(-1 - q)*(-(12^(1 + q)*a*b^2*e*m*q*x^m*(c*x^n)^((2*m)/n)*Gamma[3*q, -((m
*Log[c*x^n])/n)]*Log[c*x^n]^(3*q)) + 3^q*4^(1 + q)*b^3*d*n*q*x^m*(c*x^n)^((
2*m)/n)*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q) + (-((m*Log[c*
x^n])/n))^q*(-4*3^(1 + q)*a^2*b*e*m*q*x^(2*m)*(c*x^n)^(m/n)*Gamma[2*q, (-2*
m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 2*3^(1 + q)*a*b^2*d*n*q*x^(2*m)*(c*x^n)
^(m/n)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 4^q*(-((m*Log
[c*x^n])/n))^q*(-4*a^3*e*m*q*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*Log[c*x^
n]^q + 4*a^2*b*d*n*q*x^(3*m)*Gamma[1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q
 + 3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q*(a^3*d*n*q*x^(4*m) + b^3*e
*m*Log[c*x^n]^(4*q))))))/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^(3*q))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3ex^{3m}\log(cx^n)^{q-1}+a^3dx^{4m}+\left(b^3dx^m+b^3e\log(cx^n)^{q-1}\right)\log(cx^n)^{3q}+3\left(ab^2ex^m\log(cx^n)^{q-1}+ab^2d\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^3*e*x^(3*m)*log(c*x^n)^(q - 1) + a^3*d*x^(4*m) + (b^3*d*x^m + b
^3*e*log(c*x^n)^(q - 1))*log(c*x^n)^(3*q) + 3*(a*b^2*e*x^m*log(c*x^n)^(q -
1) + a*b^2*d*x^(2*m))*log(c*x^n)^(2*q) + 3*(a^2*b*e*x^(2*m)*log(c*x^n)^(q -
 1) + a^2*b*d*x^(3*m))*log(c*x^n)^q)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)^{3}\left(dx^{m} + e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

maple [F] time = 52.59, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{m} + e\ln\left(c\,x^{n}\right)^{q-1}\right)\left(a\,x^{m} + b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^{m} + b\ln\left(c\,x^{n}\right)^{q}\right)^{3}\,\left(d\,x^{m} + e\ln\left(c\,x^{n}\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Timed out
```

3.32 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=235

$$-\frac{a^2x^{3m}(aem-bdnq)}{3bmnq}-\frac{a2^{-q}x^{2m}\,(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\Gamma\left(q+1,-\frac{2m\log(cx^n)}{n}\right)}{mnq}-\frac{bx^m\,(cx^n)^{-}}{}$$

```
[Out] -1/3*a^2*(-b*d*n*q+a*e*m)*x^(3*m)/b/m/n/q-b*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+2*
q,-m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(
2*q))-a*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^q/(2
^q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)+1/3*e*(a*x^m+b*ln(c*x^n)^q
)^3/b/n/q
```

Rubi [A] time = 0.40, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2545, 6742, 2310, 2181}

$$-\frac{bx^m\,(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\text{Gamma}\left(2q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}-\frac{a2^{-q}x^{2m}\,(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] -(a^2*(a*e*m - b*d*n*q)*x^(3*m))/(3*b*m*n*q) - (b*(a*e*m - b*d*n*q)*x^m*Gam
ma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^(m/n)*(-(
(m*Log[c*x^n])/n))^(2*q)) - (a*(a*e*m - b*d*n*q)*x^(2*m)*Gamma[1 + q, (-2*m
*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n]
)/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx = \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m + \right.$$

$$= \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(-d + \frac{aem}{bnq}\right)\int \left(a^2x^{-1+3m} + 2\right.$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(2ab\left(-d + \right.\right.$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \frac{\left(2ab\left(-d \right.\right.}{}$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} - \frac{b(aem - bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1 + 2q,\right.}{n}$$

Mathematica [A] time = 0.95, size = 298, normalized size = 1.27

$$\frac{2^{-q}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\left(\left(-\frac{m\log(cx^n)}{n}\right)^q\left(2^q(cx^n)^{\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^q\left(a^2dnqx^{3m} + b^2em\log^{3q}(cx^n)\right) - 3a^2emqx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (-3*2^(1 + q)*a*b*e*m*q*x^m*(c*x^n)^(m/n)*Gamma[2*q, -((m*Log[c*x^n])/n)]*L
og[c*x^n]^(2*q) + 3*2^q*b^2*d*n*q*x^m*(c*x^n)^(m/n)*Gamma[1 + 2*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q) + (-((m*Log[c*x^n])/n))^q*(-3*a^2*e*m*q*x^(2*
m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 3*a*b*d*n*q*x^(2*m)*Gamma[1
 + q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q*(a^2*d*n*q*x^(3*m) + b^2*e*m*Log[c*x^n]^(3*q))))/(3*2^q*m*n*q*
(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2ex^{2m}\log(cx^n)^{q-1} + a^2dx^{3m} + \left(b^2dx^m + b^2e\log(cx^n)^{q-1}\right)\log(cx^n)^{2q} + 2\left(abex^m\log(cx^n)^{q-1} + ab\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^2*e*x^(2*m)*log(c*x^n)^(q - 1) + a^2*d*x^(3*m) + (b^2*d*x^m + b
^2*e*log(c*x^n)^(q - 1))*log(c*x^n)^(2*q) + 2*(a*b*e*x^m*log(c*x^n)^(q - 1)
 + a*b*d*x^(2*m))*log(c*x^n)^q)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m + b\log(cx^n)^q\right)^2\left(dx^m + e\log(cx^n)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

maple [F] time = 111.81, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{q-1}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}\,\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Timed out
```

3.33 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=139

$$x^m (cx^n)^{-\frac{m}{n}} \log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\Gamma\left(q+1,-\frac{m\log(cx^n)}{n}\right)+\frac{e\left(ax^m+b\log^q(cx^n)\right)^2}{2bnq}-\frac{ax^{2m}(aem}{2bn}$$

```
[Out] -1/2*a*(-b*d*n*q+a*e*m)*x^(2*m)/b/m/n/q+(b*d/m-a*e/n/q)*x^m*GAMMA(1+q,-m*ln
(c*x^n)/n)*ln(c*x^n)^q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^q)+1/2*e*(a*x^m+b*
ln(c*x^n)^q)^2/b/n/q
```

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2545, 14, 2310, 2181}

$$x^m (cx^n)^{-\frac{m}{n}} \log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\text{Gamma}\left(q+1,-\frac{m\log(cx^n)}{n}\right)+\frac{e\left(ax^m+b\log^q(cx^n)\right)^2}{2bnq}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] -(a*(a*e*m - b*d*n*q)*x^(2*m))/(2*b*m*n*q) + (((b*d)/m - (a*e)/(n*q))*x^m*G
amma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x
^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rubi steps

$$\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)}{x}\,dx = \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m + b\log\right.$$
$$= \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(-d + \frac{aem}{bnq}\right)\int \left(ax^{-1+2m} + bx^{-1+n}\right.$$
$$= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(b\left(-d + \frac{aem}{bnq}\right)\right.$$
$$= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \frac{\left(b\left(-d + \frac{aem}{bnq}\right)x\right.}{}$$
$$= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \left(\frac{bd}{m} - \frac{ae}{nq}\right)x^m (cx^n)^{-\frac{m}{n}}\Gamma\left(1 + q, -\frac{m\log(c}{n}\right.$$

Mathematica [A] time = 0.49, size = 157, normalized size = 1.13

$$\frac{(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left((cx^n)^{m/n}\left(-\frac{m\log(cx^n)}{n}\right)^q\left(adnqx^{2m} + bem\log^{2q}(cx^n)\right) - 2aemqx^m\log^q(cx^n)\Gamma\left(q, -\frac{m\log(cx^n)}{n}\right.\right.}{2mnq}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (-2*a*e*m*q*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + 2*b*d*n*q*x^m*
Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + (c*x^n)^(m/n)*(-((m*Log[c*
x^n])/n))^q*(a*d*n*q*x^(2*m) + b*e*m*Log[c*x^n]^(2*q)))/(2*m*n*q*(c*x^n)^(m
/n)*(-((m*Log[c*x^n])/n))^q)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^m\log(cx^n)^{q-1} + adx^{2m} + \left(bdx^m + be\log(cx^n)^{q-1}\right)\log(cx^n)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="fricas")
```

```
[Out] integral((a*e*x^m*log(c*x^n)^(q - 1) + a*d*x^(2*m) + (b*d*x^m + b*e*log(c*x
^n)^(q - 1))*log(c*x^n)^q)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m + b\log(cx^n)^q\right)\left(dx^m + e\log(cx^n)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

maple [F] time = 73.29, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^m + e\ln(c\,x^n)^{q-1}\right)\left(a\,x^m + b\ln(c\,x^n)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)\left(dx^m+\frac{e\log\left(cx^n\right)^q}{\log\left(cx^n\right)}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)*(d*x**m + e*log(c*x**n)**q/log(c*x**n)
)/x, x)
```

3.34 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

```
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {14, 2302, 30}

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx &= \int \left(dx^{-1+m}+\frac{e\log^{-1+q}(cx^n)}{x}\right)dx\\ &= \frac{dx^m}{m}+e\int \frac{\log^{-1+q}(cx^n)}{x}\,dx\\ &= \frac{dx^m}{m}+\frac{e\,\text{Subst}\left(\int x^{-1+q}\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 1.00

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

fricas [A] time = 0.47, size = 42, normalized size = 1.68

$$\frac{dnqx^m + \left(emn\log(x) + em\log(c)\right)\left(n\log(x) + \log(c)\right)^{q-1}}{mnq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
[Out] (d*n*q*x^m + (e*m*n*log(x) + e*m*log(c))*(n*log(x) + log(c))^(q - 1))/(m*n*
q)
```

giac [A] time = 0.18, size = 27, normalized size = 1.08

$$\frac{dx^m}{m} + \frac{\left(n\log(x) + \log(c)\right)^q e}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
[Out] d*x^m/m + (n*log(x) + log(c))^q*e/(n*q)
```

maple [A] time = 0.08, size = 26, normalized size = 1.04

$$\frac{d\,x^m}{m} + \frac{e\ln\left(c\,x^n\right)^q}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))/x,x)
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

maxima [A] time = 0.45, size = 25, normalized size = 1.00

$$\frac{dx^m}{m} + \frac{e\log\left(cx^n\right)^q}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
[Out] d*x^m/m + e*log(c*x^n)^q/(n*q)
```

mupad [B] time = 0.32, size = 25, normalized size = 1.00

$$\frac{d\,x^m}{m} + \frac{e\ln\left(c\,x^n\right)^q}{n\,q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/x,x)
[Out] (d*x^m)/m + (e*log(c*x^n)^q)/(n*q)
```

sympy [A] time = 30.67, size = 53, normalized size = 2.12

$$d\left(\begin{cases}\frac{x^m}{m} & \text{for } m \neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\frac{\log\left(x\right)}{\log\left(c\right)} & \text{for } n=0 \wedge q=0\\ \frac{\log\left(c\right)^q \log\left(x\right)}{\log\left(c\right)} & \text{for } n=0\\ \frac{\log\left(n\log\left(x\right)+\log\left(c\right)\right)}{n} & \text{for } q=0\\ \frac{\left(n\log\left(x\right)+\log\left(c\right)\right)^q}{nq} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] d*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + e*Piecewise((log(x)/log(c
), Eq(n, 0) & Eq(q, 0)), (log(c)**q*log(x)/log(c), Eq(n, 0)), (log(n*log(x)
 + log(c))/n, Eq(q, 0)), ((n*log(x) + log(c))**q/(n*q), True))
```

3.35 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=73

$$\left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{m-1}}{ax^m+b\log^q(cx^n)},x\right)+\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)+e*ln(a*x^
m+b*ln(c*x^n)^q)/b/n/q
```

Rubi [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] (e*Log[a*x^m + b*Log[c*x^n]^q])/(b*n*q) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q), x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx$$

Mathematica [A] time = 5.11, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^m+e\log(cx^n)^{q-1}}{axx^m+bx\log(cx^n)^q},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log(cx^n)^{q-1}}{\left(ax^m+b\log(cx^n)^q\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)*x), x)
```

maple [A] time = 114.84, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{q-1}}{\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e\log\left(\log(c)+\log\left(x^{n}\right)\right)}{bn}+\int \frac{bdx^{m}\log\left(x^{n}\right)+\left(bd\log(c)-ae\right)x^{m}}{abxx^{m}\log(c)+abxx^{m}\log\left(x^{n}\right)+\left(b^{2}x\log(c)+b^{2}x\log\left(x^{n}\right)\right)\left(\log(c)+\log\left(x^{n}\right)\right)^{q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="maxima")
```

```
[Out] e*log(log(c) + log(x^n))/(b*n) + integrate((b*d*x^m*log(x^n) + (b*d*log(c)
- a*e)*x^m)/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b^2*x*
log(x^n))*(log(c) + log(x^n))^q), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{q-1}}{x\,\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

```
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{m}+\frac{e\log\left(cx^{n}\right)^{q}}{\log\left(cx^{n}\right)}}{x\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Integral((d*x**m + e*log(c*x**n)**q/log(c*x**n))/(x*(a*x**m + b*log(c*x**n)
**q)), x)
```

3.36 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=75

$$\left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2},x\right)-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)-e/b/n/q
/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(e/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) + (d - (a*e*m)/(b*n*q))*Defer[Int][x^
(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A] time = 7.53, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2), x
]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^m+e\log\left(cx^n\right)^{q-1}}{2\,abxx^m\log\left(cx^n\right)^q+a^2xx^{2\,m}+b^2x\log\left(cx^n\right)^{2\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x
^(2*m) + b^2*x*log(c*x^n)^(2*q)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

maple [A] time = 51.28, size = 0, normalized size = 0.00

$$\int \frac{d\,x^m + e\ln(c\,x^n)^{q-1}}{\left(a\,x^m + b\ln(c\,x^n)^q\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\log(c) + bd\log(x^n) - ae}{a^2bmx^m\log(x^n) - \left(nq - m\log(c)\right)a^2bx^m + \left(ab^2m\log(x^n) - \left(nq - m\log(c)\right)ab^2\right)\left(\log(c) + \log(x^n)\right)^q} + \int -\frac{}{a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="maxima")
```

```
[Out] -(b*d*log(c) + b*d*log(x^n) - a*e)/(a^2*b*m*x^m*log(x^n) - (n*q - m*log(c))
*a^2*b*x^m + (a*b^2*m*log(x^n) - (n*q - m*log(c))*a*b^2)*(log(c) + log(x^n)
)^q) + integrate(-((e*m*n*(q - 1) - e*m^2*log(c))*a + (d*m*n*q*log(c) - (q^
2 - q)*d*n^2)*b + (b*d*m*n*q - a*e*m^2)*log(x^n))/(a^2*b*m^2*x*x^m*log(x^n)
^2 - 2*(m*n*q - m^2*log(c))*a^2*b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c
) + m^2*log(c)^2)*a^2*b*x*x^m + (a*b^2*m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*lo
g(c))*a*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b^2*x)
*(log(c) + log(x^n))^q), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^m + e\ln(c\,x^n)^{q-1}}{x\left(a\,x^m + b\ln(c\,x^n)^q\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

3.37 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=77

$$\left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3},x\right)-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)-1/2*e/b
/n/q/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -e/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Mathematica [A] time = 71.22, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3), x
]
```

fricas [A] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^m+e\log\left(cx^n\right)^{q-1}}{3\,ab^2xx^m\log\left(cx^n\right)^{2\,q}+3\,a^2bxx^{2\,m}\log\left(cx^n\right)^q+a^3xx^{3\,m}+b^3x\log\left(cx^n\right)^{3\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3
*a^2*b*x*x^(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

maple [A] time = 148.52, size = 0, normalized size = 0.00

$$\int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(q-1))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="maxima")
```

```
[Out] -1/2*(a*b*d*m^2*x^m*log(x^n)^3 + (a^2*e*m^2 - (4*d*m*n*q - 3*d*m^2*log(c))*
a*b)*x^m*log(x^n)^2 + ((2*e*m^2*log(c) + e*m*n)*a^2 - (8*d*m*n*q*log(c) - 3
*d*m^2*log(c)^2 - (3*q^2 - q)*d*n^2)*a*b)*x^m*log(x^n) - ((e*n^2*q^2 - e*m^
2*log(c)^2 - e*m*n*log(c))*a^2 + (4*d*m*n*q*log(c)^2 - d*m^2*log(c)^3 - (3*
q^2 - q)*d*n^2*log(c))*a*b)*x^m - ((e*m*n*(2*q - 1)*log(c) - 2*e*m^2*log(c)
^2)*a*b + (2*d*m*n*q*log(c)^2 - (2*q^2 - q)*d*n^2*log(c))*b^2 + 2*(b^2*d*m*
n*q - a*b*e*m^2)*log(x^n)^2 + ((e*m*n*(2*q - 1) - 4*e*m^2*log(c))*a*b + (4*
d*m*n*q*log(c) - (2*q^2 - q)*d*n^2)*b^2)*log(x^n))*(log(c) + log(x^n))^q)/(
a^4*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^4*b*x^(3*m)*log(x
^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^4*b*x^(3*m)*log(x
^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^
4*b*x^(3*m) + (a^2*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^
3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^3*
x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log
(c)^3)*a^2*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^3*b^2*m^3*x^(2*m)*log(
x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^3*b^2*x^(2*m)*log(x^n)^2 + 3*(m*n^2*q^2
 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^3*b^2*x^(2*m)*log(x^n) - (n^3*q^3 - 3
*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^3*b^2*x^(2*m))*(lo
g(c) + log(x^n))^q) - integrate(-1/2*(2*(b*d*m^3*n*q - a*e*m^4)*log(x^n)^3
+ ((e*m^3*n*(2*q - 3) - 6*e*m^4*log(c))*a + (6*d*m^3*n*q*log(c) - (2*q^2 -
3*q)*d*m^2*n^2)*b)*log(x^n)^2 + (e*m^3*n*(2*q - 3)*log(c)^2 - 2*e*m^4*log(c
)^3 + 2*(q^2 - 1)*e*m^2*n^2*log(c) - (2*q^3 - 3*q^2 + q)*e*m*n^3)*a + (2*d*
m^3*n*q*log(c)^3 - (2*q^2 - 3*q)*d*m^2*n^2*log(c)^2 - 2*(q^3 - q)*d*m*n^3*l
og(c) + (2*q^4 - 3*q^3 + q^2)*d*n^4)*b + 2*((e*m^3*n*(2*q - 3)*log(c) - 3*e
*m^4*log(c)^2 + (q^2 - 1)*e*m^2*n^2)*a + (3*d*m^3*n*q*log(c)^2 - (2*q^2 - 3
*q)*d*m^2*n^2*log(c) - (q^3 - q)*d*m*n^3)*b)*log(x^n))/(a^3*b*m^4*x*x^(2*m)
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^3*b*x*x^(2*m)*log(x^n)^3 + 6*(m^2*
n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^3*b*x*x^(2*m)*log(x^n)^2 - 4*(
```

```
m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^3*b
*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^
2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^3*b*x*x^(2*m) + (a^2*b^2*m^4*x*x^m
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b^2*x*x^m*log(x^n)^3 + 6*(m^2*n^
2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b^2*x*x^m*log(x^n)^2 - 4*(m*n^
3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^2*b^2*x
*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 - 4*
m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^m+e\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
[Out] Timed out
```

3.38 $\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2} dx$

Optimal. Leaf size=26

$$\frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

```
[Out] d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] time = 0.25, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {2546}

$$\frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/(x*(a*
x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rule 2546

```
Int[(Log[(c_.)*(x_)^(n_.)]^(q_.)*(f_.) + (d_.)*(x_)^(m_.) + Log[(c_.)*(x_)^
(n_.)]*(e_.)*(x_)^(m_.))/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x
_)^(m_.))^2), x_Symbol] :> Simp[(d*Log[c*x^n])/(a*n*(a*x^m + b*Log[c*x^n]^q
)), x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[e*n + d*m, 0] && EqQ
[a*f + b*d*(q - 1), 0]
```

Rubi steps

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2} dx = \frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

Mathematica [A] time = 0.37, size = 26, normalized size = 1.00

$$\frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/
(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

fricas [A] time = 0.45, size = 30, normalized size = 1.15

$$\frac{dn\log(x) + d\log(c)}{\left(n\log(x) + \log(c)\right)^q b + ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] (d*n*log(x) + d*log(c))/((n*log(x) + log(c))^q*b + a*x^m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bdn\left(q-1\right)\log\left(cx^{n}\right)^{q} + admx^{m}\log\left(cx^{n}\right) - adnx^{m}}{\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(-(b*d*n*(q - 1)*log(c*x^n)^q + a*d*m*x^m*log(c*x^n) - a*d*n*x^m)/
((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

maple [C] time = 0.51, size = 158, normalized size = 6.08

$$\frac{\left(-i\pi\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ix^{n}\right)\operatorname{csgn}\left(icx^{n}\right) + i\pi\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(icx^{n}\right)^{2} + i\pi\operatorname{csgn}\left(ix^{n}\right)\operatorname{csgn}\left(icx^{n}\right)^{2} - i\pi\operatorname{csgn}\left(icx^{n}\right)^{3}}{2ax^{m} + 2b\left(-\frac{i\pi\left(\operatorname{csgn}(ic)-\operatorname{csgn}(icx^{n})\right)\left(\operatorname{csgn}(ix^{n})-\operatorname{csgn}(icx^{n})\right)\operatorname{csgn}(icx^{n})}{2} + \ln(c) + \ln\left(x^{n}\right)\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*n*x^m-a*d*m*x^m*ln(c*x^n)-b*d*n*(q-1)*ln(c*x^n)^q)/x/(a*x^m+b*ln(c
*x^n)^q)^2,x)
```

```
[Out] 1/2*(2*ln(c)+2*ln(x^n)+I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)*csgn(I*c)-I*Pi*csgn(I*c*x^n)^3+I*Pi*csgn(I*c*x^n)^2*csgn(I*c))*
d/(a*x^m+b*(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(csgn(I*x^n)-csgn(I*c*x^n))
*csgn(I*c*x^n)+ln(c)+ln(x^n))^q)
```

maxima [A] time = 1.25, size = 31, normalized size = 1.19

$$\frac{d\log(c) + d\log\left(x^{n}\right)}{ax^{m} + b\left(\log(c) + \log\left(x^{n}\right)\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] (d*log(c) + d*log(x^n))/(a*x^m + b*(log(c) + log(x^n))^q)
```

mupad [B] time = 0.33, size = 26, normalized size = 1.00

$$\frac{d\,\ln\left(c\,x^{n}\right)}{a\,x^{m} + b\ln\left(c\,x^{n}\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d*m*x^m*log(c*x^n) - a*d*n*x^m + b*d*n*log(c*x^n)^q*(q - 1))/(x*(a*
x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] (d*log(c*x^n))/(a*x^m + b*log(c*x^n)^q)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x**m-a*d*m*x**m*ln(c*x**n)-b*d*n*(-1+q)*ln(c*x**n)**q)/x/(
a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

3.39 $\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{\log(cx^n)}{a\left(ax+b\log^q(cx^n)\right)}-\frac{n(1-q)\text{Int}\left(\frac{1}{x\left(ax+b\log^q(cx^n)\right)},x\right)}{a}$$

```
[Out] -n*(1-q)*CannotIntegrate(1/x/(a*x+b*ln(c*x^n)^q),x)/a+ln(c*x^n)/a/(a*x+b*ln
(c*x^n)^q)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^q)) - (n*(1 - q)*Defer[Int][1/(x*(a*x + b
*Log[c*x^n]^q)), x])/a
```

Rubi steps

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx=\frac{\log(cx^n)}{a\left(ax+b\log^q(cx^n)\right)}-\frac{(n(1-q))\int \frac{1}{x\left(ax+b\log^q(cx^n)\right)}\,dx}{a}$$

Mathematica [A] time = 80.03, size = 0, normalized size = 0.00

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{nq-\log(cx^n)}{a^2x^2+2\,abx\log(cx^n)^q+b^2\log(cx^n)^{2q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] integral((n*q - log(c*x^n))/(a^2*x^2 + 2*a*b*x*log(c*x^n)^q + b^2*log(c*x^n
)^(2*q)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log(cx^n)^q\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="giac")
[Out] integrate((n*q - log(c*x^n))/(a*x + b*log(c*x^n)^q)^2, x)
```

maple [A] time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{nq - \ln\left(c\,x^{n}\right)}{\left(ax + b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
[Out] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$n\left(q-1\right)\int \frac{1}{a^{2}x^{2} + abx\left(\log(c) + \log\left(x^{n}\right)\right)^{q}}\,dx + \frac{\log(c) + \log\left(x^{n}\right)}{a^{2}x + ab\left(\log(c) + \log\left(x^{n}\right)\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="maxima")
[Out] n*(q - 1)*integrate(1/(a^2*x^2 + a*b*x*(log(c) + log(x^n))^q), x) + (log(c)
 + log(x^n))/(a^2*x + a*b*(log(c) + log(x^n))^q)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\ln\left(c\,x^{n}\right) - n\,q}{\left(b\ln\left(c\,x^{n}\right)^{q} + a\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2,x)
[Out] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{nq - \log\left(cx^{n}\right)}{\left(ax + b\log\left(cx^{n}\right)^{q}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-ln(c*x**n))/(a*x+b*ln(c*x**n)**q)**2,x)
[Out] Integral((n*q - log(c*x**n))/(a*x + b*log(c*x**n)**q)**2, x)
```

3.40 $\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(\sqrt{-\frac{e}{d}}\,d+ex\right)}{ex^2+d}\right)}{2e}$$

```
[Out] -1/2*polylog(2,1-2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2447}

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)])/(2
*e)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] time = 0.48, size = 625, normalized size = 12.76

$$2\text{Li}_2\left(\frac{\sqrt{e}x+\sqrt{-d}}{\sqrt{-d}+\frac{\sqrt{e}}{\sqrt{-\frac{e}{d}}}}\right)+2\text{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{Li}_2\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)-2\text{Li}_2\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\text{Li}_2\left(\frac{\sqrt{-d}\sqrt{e}-e}{\sqrt{-\frac{e}{d}}d+\sqrt{-d}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
```

```
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d*Sqrt[-(e/d)] + e*x)/(Sqrt[-d]*
Sqrt[e] + d*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(e + d*(-(e/d)
)^(3/2)*x)/(e + Sqrt[-d]*Sqrt[e]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] - Sqrt[e]*
x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] + Sqrt[e]
*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] + 2*PolyLog[2, (Sqrt[-d]
+ Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e]/Sqrt[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]
*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2,
 (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2
)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[-d]*Sqrt[e] + d*Sqrt[-(e/d
)])])/(4*Sqrt[-d]*Sqrt[e])
```

fricas [**A**] time = 0.44, size = 44, normalized size = 0.90

$$-\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(-e/d)*dilog(-2*(e*x^2 + d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{2\left(ex+d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="g
iac")
```

```
[Out] integrate(log(2*(e*x + d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{2\left(ex+\sqrt{-\frac{e}{d}}\,d\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: TypeError
```

3.41 $\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{ex^2+d}+1\right)}{2e}$$

```
[Out] 1/2*polylog(2,1+2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2447}

$$\frac{\sqrt{-\frac{e}{d}}\,\mathrm{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2*
e)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] time = 0.38, size = 642, normalized size = 12.84

$$-2\mathrm{Li}_2\left(\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}+\sqrt{-d}\sqrt{-\frac{e}{d}}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{-d}\sqrt{-\frac{e}{d}}-\sqrt{e}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\mathrm{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)-2\mathrm{I}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
```

```
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[e]*(1 + Sqrt[-(e/d)]*x))/(S
qrt[e] - Sqrt[-d]*Sqrt[-(e/d)])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[e]
*(1 + Sqrt[-(e/d)]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d]
- Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Log[Sqrt[-d]
 + Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*PolyLog[2,
(Sqrt[-(e/d)]*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] +
2*PolyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] + Sqrt[e]*x))/(-Sqrt[e] + Sqrt[-d]*Sqr
t[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sq
rt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2
*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e])
```

fricas [A] time = 0.45, size = 45, normalized size = 0.90

$$\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(-e/d)*dilog(-2*(e*x^2 - d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{2\left(ex-d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"giac")
```

```
[Out] integrate(log(2*(e*x - d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2\left(-ex+\sqrt{-\frac{e}{d}}\,d\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+(-1/d*e)^(1/2)*d)/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+(-1/d*e)^(1/2)*d)/(e*x^2+d))/(e*x^2+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: TypeError
```

3.42 $\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\text{Li}_2\left(\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{ex^2+d}+1\right)}{2\sqrt{-d}\,\sqrt{e}}$$

```
[Out] -1/2*polylog(2,1+2*x*e^(1/2)*((-d)^(1/2)-x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e
^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{d+ex^2}+1\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-
d]*Sqrt[e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\text{Li}_2\left(1+\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [B] time = 0.22, size = 320, normalized size = 6.04

$$2\text{Li}_2\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}+1\right)+2\text{Li}_2\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\text{Li}_2\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}+1\right)-2\log\left(\frac{2\left(ex^2-\sqrt{-d}\,\sqrt{e}\,x\right)}{d+ex^2}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)+2\log\left(\sqrt{-d}\right. -$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x
]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + 2*Log[(d*Sqrt[e]*
x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*
Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
```

```
-d] - Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2)] - 2*L
og[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2
)] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*PolyLog[2, (d + Sqrt[-d]*Sq
rt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*S
qrt[e])
```

fricas [A] time = 0.44, size = 44, normalized size = 0.83

$$\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-\sqrt{-d}\,\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-d)*dilog(-2*(e*x^2 - sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqrt
(e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] sage2
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{2\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: AttributeError
```

3.43 $\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=52

$$\frac{\mathrm{Li}_2\left(1-\frac{2\sqrt{e}\,x\left(\sqrt{e}\,x+\sqrt{-d}\right)}{ex^2+d}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

```
[Out] 1/2*polylog(2,1-2*x*e^(1/2)*((-d)^(1/2)+x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}\,x\left(\sqrt{-d}+\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-d
]*Sqrt[e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\mathrm{Li}_2\left(1-\frac{2\sqrt{e}\,x\left(\sqrt{-d}+\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [B] time = 0.20, size = 316, normalized size = 6.08

$$2\mathrm{Li}_2\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}+1\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\mathrm{Li}_2\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}+1\right)+2\log\left(\frac{2\left(\sqrt{-d}\,\sqrt{e}\,x+ex^2\right)}{d+ex^2}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)-2\log\left(\sqrt{-
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),
x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - 2*
Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
```

```
-d] - Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] - 2*Log[
Sqrt[-d] + Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] + 2
*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*
x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e]
)
```

fricas [A] time = 0.43, size = 43, normalized size = 0.83

$$-\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{-d}\,\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="fricas")
```

```
[Out] -1/2*sqrt(-d)*dilog(-2*(e*x^2 + sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqr
t(e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="giac")
```

```
[Out] sage2
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2\left(-ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+1/(-d)^(1/2)*d*e^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+1/(-d)^(1/2)*d*e^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: AttributeError
```

3.44 $\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$\frac{\text{Li}_2\left(1-\frac{2x\left(ex+\sqrt{d}\sqrt{-e}\right)}{ex^2+d}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] 1/2*polylog(2,1-2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt[
-e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\text{Li}_2\left(1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] time = 0.35, size = 641, normalized size = 13.08

$$2\text{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{Li}_2\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)-2\text{Li}_2\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\text{Li}_2\left(\frac{\sqrt{-d}\sqrt{e}-ex}{\sqrt{d}\sqrt{-e}+\sqrt{-d}\sqrt{e}}\right)+2\text{Li}_2\left(\frac{ex+\cdots}{\sqrt{-d}\sqrt{e}\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] + e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] + e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
```

```
rt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] +
 Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] + 2*PolyLog[2,
1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] +
2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*
x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[d]*Sqrt[-e] +
Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(-(Sqrt[d]*Sqrt[
-e]) + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

fricas [**A**] time = 0.44, size = 43, normalized size = 0.88

$$-\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{d}\,\sqrt{-e}\,x\right)}{ex^2+d}+1\right)}{2\,\sqrt{d}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="fricas")
```

```
[Out] -1/2*sqrt(-e)*dilog(-2*(e*x^2 + sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(
d)*e)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{2\left(ex+\sqrt{d}\,\sqrt{-e}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="giac")
```

```
[Out] integrate(log(2*(e*x + sqrt(d)*sqrt(-e))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{2\left(ex+\sqrt{-e}\,\sqrt{d}\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: AttributeError
```

3.45 $\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\text{Li}_2\left(\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{ex^2+d}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] -1/2*polylog(2,1+2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2
)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$-\frac{\text{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt
[-e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\text{Li}_2\left(1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] time = 0.26, size = 645, normalized size = 12.90

$$2\text{Li}_2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{Li}_2\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)-2\text{Li}_2\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\text{Li}_2\left(\frac{ex-\sqrt{-d}\sqrt{e}}{\sqrt{d}\sqrt{-e}-\sqrt{-d}\sqrt{e}}\right)+2\text{Li}_2\left(\frac{e}{\sqrt{d}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] - e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*S
```

```
qrt[-e] - e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d
] + Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] + 2*PolyL
og[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*
d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sq
rt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (-(Sqrt[-d]*Sqrt[e]) + e*x)/(Sqrt[d]*Sq
rt[-e] - Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(Sqrt[d
]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

fricas [A] time = 0.43, size = 44, normalized size = 0.88

$$\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-\sqrt{d}\sqrt{-e}\,x\right)}{ex^2+d}+1\right)}{2\sqrt{d}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-e)*dilog(-2*(e*x^2 - sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(d
)*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{2\left(ex-\sqrt{d}\sqrt{-e}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] integrate(log(2*(e*x - sqrt(d)*sqrt(-e))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2\left(-ex+\sqrt{-e}\,\sqrt{d}\right)x}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+(-e)^(1/2)*d^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+(-e)^(1/2)*d^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

```
[Out] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError
```

3.46 $\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx$

Optimal. Leaf size=67

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\text{Ei}((m+1)\log(dx))}{e(m+1)}$$

```
[Out] -b*p*(d*x)^(-1-m)*(e*x)^(1+m)*Ei((1+m)*ln(d*x))/e/(1+m)+(e*x)^(1+m)*(a+b*ln
(c*ln(d*x)^p))/e/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\text{Ei}((m+1)\log(dx))}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] -((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
+ m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ex)^m\left(a+b\log\left(c\log^p(dx)\right)\right)\,dx&=\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{(bp)\int\frac{(ex)^m}{\log(dx)}\,dx}{1+m}\\&=\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{\left(bp(dx)^{-1-m}(ex)^{1+m}\right)\text{Subst}\left(\int\frac{e^{(1+m)x}}{x}\,dx\right.}{e(1+m)}\\&=-\frac{bp(dx)^{-1-m}(ex)^{1+m}\text{Ei}((1+m)\log(dx))}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.84

$$\frac{(dx)^{-m}(ex)^m\left(dx(dx)^m\left(a+b\log\left(c\log^p(dx)\right)\right)-bp\text{Ei}((m+1)\log(dx))\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] ((e*x)^m*(-(b*p*ExpIntegralEi[(1 + m)*Log[d*x]]) + d*x*(d*x)^m*(a + b*Log[c
*Log[d*x]^p])))/(d*(1 + m)*(d*x)^m)
```

fricas [A] time = 0.45, size = 83, normalized size = 1.24

$$\frac{bdpxe^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}\log\left(\log\left(dx\right)\right)-bp\left(\frac{e}{d}\right)^{m}\mathrm{Ei}\left(\left(m+1\right)\log\left(dx\right)\right)+\left(bdx\log(c)+adx\right)e^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="fricas")
```

```
[Out] (b*d*p*x*e^(m*log(d*x) + m*log(e/d))*log(log(d*x)) - b*p*(e/d)^m*Ei((m + 1)
*log(d*x)) + (b*d*x*log(c) + a*d*x)*e^(m*log(d*x) + m*log(e/d)))/(d*m + d)
```

giac [A] time = 0.24, size = 83, normalized size = 1.24

$$\frac{bpxx^{m}e^{m}\log\left(\log(d)+\log(x)\right)}{m+1}+\frac{bxx^{m}e^{m}\log(c)}{m+1}+\frac{axx^{m}e^{m}}{m+1}-\frac{bp\mathrm{Ei}\left(m\log(d)+m\log(x)+\log(d)+\log(x)\right)e^{m}}{dd^{m}m+dd^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="giac")
```

```
[Out] b*p*x*x^m*e^m*log(log(d) + log(x))/(m + 1) + b*x*x^m*e^m*log(c)/(m + 1) + a
*x*x^m*e^m/(m + 1) - b*p*Ei(m*log(d) + m*log(x) + log(d) + log(x))*e^m/(d*d
^m*m + d*d^m)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\ln\left(dx\right)^{p}\right)+a\right)\left(ex\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(e^{m}p\int\frac{x^{m}}{\left(m^{2}+2\,m+1\right)\log(d)^{2}+2\left(m^{2}+2\,m+1\right)\log(d)\log(x)+\left(m^{2}+2\,m+1\right)\log(x)^{2}}\,dx-\frac{\left(e^{m}(m+1)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="maxima")
```

```
[Out] -(e^m*p*integrate(x^m/((m^2 + 2*m + 1)*log(d)^2 + 2*(m^2 + 2*m + 1)*log(d)*
log(x) + (m^2 + 2*m + 1)*log(x)^2), x) - ((e^m*(m + 1)*x*log(d) + e^m*(m +
1)*x*log(x))*x^m*log((log(d) + log(x))^p) + (e^m*(m + 1)*x*log(c)*log(x) +
(e^m*(m + 1)*log(c)*log(d) - e^m*p)*x)*x^m)/((m^2 + 2*m + 1)*log(d) + (m^2
+ 2*m + 1)*log(x)))*b + (e*x)^(m + 1)*a/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\ln\left(c\ln\left(d\,x\right)^{p}\right)\right)\left(e\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x)^p))*(e*x)^m,x)
[Out] int((a + b*log(c*log(d*x)^p))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a + b\log\left(c\log\left(dx\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x)**p)),x)
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x)**p)), x)
```

3.47 $\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx$

Optimal. Leaf size=79

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1}\left(dx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m+1)\log\left(dx^n\right)}{n}\right)}{e(m+1)}$$

```
[Out] -b*p*(e*x)^(1+m)*Ei((1+m)*ln(d*x^n)/n)/e/(1+m)/((d*x^n)^((1+m)/n))+(e*x)^(1
+m)*(a+b*ln(c*ln(d*x^n)^p))/e/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2522, 2310, 2178}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1}\left(dx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m+1)\log\left(dx^n\right)}{n}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*x
^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ex)^m\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)dx &= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}-\frac{(bnp)\int\frac{(ex)^m}{\log\left(dx^n\right)}dx}{1+m}\\ &= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}-\frac{\left(bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{e}{}\right.}{e(1+m)}\\ &= -\frac{bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log\left(dx^n\right)}{n}\right)}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(d\right.\right.\right.}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.17, size = 59, normalized size = 0.75

$$\frac{x(ex)^m\left(a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m+1)\log(dx^n)}{n}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x*(e*x)^m*(a - (b*p*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(d*x^n)^((1 + m
)/n) + b*Log[c*Log[d*x^n]^p]))/(1 + m)
```

fricas [A] time = 0.45, size = 90, normalized size = 1.14

$$\frac{bpxe^{(m\log(e)+m\log(x))}\log\left(n\log(x)+\log(d)\right)-bp\text{Ei}\left(\frac{(m+1)n\log(x)+(m+1)\log(d)}{n}\right)e^{\left(\frac{mn\log(e)-(m+1)\log(d)}{n}\right)}+\left(bx\log(c)+ax\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(m*log(e) + m*log(x))*log(n*log(x) + log(d)) - b*p*Ei(((m + 1)*n*l
og(x) + (m + 1)*log(d))/n)*e^((m*n*log(e) - (m + 1)*log(d))/n) + (b*x*log(c
) + a*x)*e^(m*log(e) + m*log(x)))/(m + 1)
```

giac [A] time = 0.21, size = 111, normalized size = 1.41

$$\frac{bpxx^me^m\log\left(n\log(x)+\log(d)\right)}{m+1}-\frac{bnp\text{Ei}\left(m\log(x)+\frac{m\log(d)}{n}+\frac{\log(d)}{n}+\log(x)\right)e^m}{d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}mn+d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}n}+\frac{bxx^me^m\log(c)}{m+1}+\frac{axx^me^m}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] b*p*x*x^m*e^m*log(n*log(x) + log(d))/(m + 1) - b*n*p*Ei(m*log(x) + m*log(d)
/n + log(d)/n + log(x))*e^m/(d^(m/n)*d^(1/n)*m*n + d^(m/n)*d^(1/n)*n) + b*x
*x^m*e^m*log(c)/(m + 1) + a*x*x^m*e^m/(m + 1)
```

maple [F] time = 2.59, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\ln\left(d\,x^n\right)^p\right)+a\right)(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(e^mnp\int\frac{x^m}{(m+1)\log(d)+(m+1)\log\left(x^n\right)}\,dx-\frac{e^mxx^m\log(c)+e^mxx^m\log\left(\left(\log(d)+\log\left(x^n\right)\right)^p\right)}{m+1}\right)b+\frac{(ex)^{m+1}a}{e(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] -(e^m*n*p*integrate(x^m/((m + 1)*log(d) + (m + 1)*log(x^n)), x) - (e^m*x*x^
m*log(c) + e^m*x*x^m*log((log(d) + log(x^n))^p))/(m + 1))*b + (e*x)^(m + 1)
*a/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\left(a+b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x**n)**p)), x)
```

3.48 $\int x^2 \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right) dx$

Optimal. Leaf size=55

$$\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}bpx^3\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)$$

```
[Out] -1/3*b*p*x^3*Ei(3*ln(d*x^n)/n)/((d*x^n)^(3/n))+1/3*x^3*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}bpx^3\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(3*(d*x^n)^(3/n)) + (x^3*(a + b*
Log[c*Log[d*x^n]^p]))/3
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)dx&=\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}(bnp)\int\frac{x^2}{\log\left(dx^n\right)}dx\\&=\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}\left(bpx^3\left(dx^n\right)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{x}dx,x,\log(\right.\\&=-\frac{1}{3}bpx^3\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)+\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.89

$$\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x^3*(a - (b*p*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + b*Log[c*Log
[d*x^n]^p]))/3
```

fricas [A] time = 0.42, size = 70, normalized size = 1.27

$$\frac{bd^{\frac{3}{n}}px^3\log\left(n\log(x)+\log(d)\right)-bp\,\text{log\_integral}\left(d^{\frac{3}{n}}x^3\right)+\left(bx^3\log(c)+ax^3\right)d^{\frac{3}{n}}}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log(n*log(x) + log(d)) - b*p*log_integral(d^(3/n)*x^3)
 + (b*x^3*log(c) + a*x^3)*d^(3/n))/d^(3/n)
```

giac [A] time = 0.18, size = 56, normalized size = 1.02

$$\frac{1}{3}\,bpx^3\log\left(n\log(x)+\log(d)\right)+\frac{1}{3}\,bx^3\log(c)+\frac{1}{3}\,ax^3-\frac{bp\text{Ei}\left(\frac{3\,\log(d)}{n}+3\,\log(x)\right)}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/3*b*p*x^3*log(n*log(x) + log(d)) + 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/3*b*p
*Ei(3*log(d)/n + 3*log(x))/d^(3/n)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\ln\left(d\,x^n\right)^p\right)+a\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*ln(d*x^n)^p)+a),x)
```

```
[Out] int(x^2*(b*ln(c*ln(d*x^n)^p)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,ax^3+\frac{1}{3}\left(x^3\log(c)+x^3\log\left(\left(\log(d)+\log\left(x^n\right)\right)^p\right)-3\,np\int\frac{x^2}{3\left(\log(d)+\log\left(x^n\right)\right)}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/3*(x^3*log(c) + x^3*log((log(d) + log(x^n))^p) - 3*n*p*integr
ate(1/3*x^2/(log(d) + log(x^n)), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int(x^2*(a + b*log(c*log(d*x^n)^p)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*log(d*x**n)**p)), x)
```

3.49 $\int x \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{2} b p x^2 \left(d x^n\right)^{-2/n} \text{Ei}\left(\frac{2 \log \left(d x^n\right)}{n}\right)$$

```
[Out] -1/2*b*p*x^2*Ei(2*ln(d*x^n)/n)/((d*x^n)^(2/n))+1/2*x^2*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2522, 2310, 2178}

$$\frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{2} b p x^2 \left(d x^n\right)^{-2/n} \text{Ei}\left(\frac{2 \log \left(d x^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(2*(d*x^n)^(2/n)) + (x^2*(a + b*
Log[c*Log[d*x^n]^p]))/2
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right) \, dx &= \frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{2} (b n p) \int \frac{x}{\log \left(d x^n\right)} \, dx\\
&= \frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{2} \left(b p x^2 \left(d x^n\right)^{-2/n}\right) \text{Subst}\left(\int \frac{e^{\frac{2 x}{n}}}{x} \, dx,x,\log
\end{aligned}$$

$$= -\frac{1}{2} b p x^2 \left(d x^n\right)^{-2/n} \text{Ei}\left(\frac{2 \log \left(d x^n\right)}{n}\right)+\frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.89

$$\frac{1}{2} x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)-b p \left(d x^n\right)^{-2/n} \text{Ei}\left(\frac{2 \log \left(d x^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x^2*(a - (b*p*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + b*Log[c*Log
[d*x^n]^p]))/2
```

fricas [A] time = 0.46, size = 70, normalized size = 1.27

$$\frac{bd^{\frac{2}{n}}px^2\log\left(n\log(x)+\log(d)\right)-bp\,\text{log\_integral}\left(d^{\frac{2}{n}}x^2\right)+\left(bx^2\log(c)+ax^2\right)d^{\frac{2}{n}}}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log(n*log(x) + log(d)) - b*p*log_integral(d^(2/n)*x^2)
 + (b*x^2*log(c) + a*x^2)*d^(2/n))/d^(2/n)
```

giac [A] time = 0.20, size = 56, normalized size = 1.02

$$\frac{1}{2}\,bpx^2\log\left(n\log(x)+\log(d)\right)+\frac{1}{2}\,bx^2\log(c)+\frac{1}{2}\,ax^2-\frac{bp\text{Ei}\left(\frac{2\,\log(d)}{n}+2\,\log(x)\right)}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/2*b*p*x^2*log(n*log(x) + log(d)) + 1/2*b*x^2*log(c) + 1/2*a*x^2 - 1/2*b*p
*Ei(2*log(d)/n + 2*log(x))/d^(2/n)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\ln\left(d\,x^n\right)^p\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*ln(d*x^n)^p)+a),x)
```

```
[Out] int(x*(b*ln(c*ln(d*x^n)^p)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ax^2-\frac{1}{2}\left(2\,np\int\frac{x}{2\left(\log(d)+\log\left(x^n\right)\right)}\,dx-x^2\log(c)-x^2\log\left(\left(\log(d)+\log\left(x^n\right)\right)^p\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 1/2*(2*n*p*integrate(1/2*x/(log(d) + log(x^n)), x) - x^2*log(c)
 - x^2*log((log(d) + log(x^n))^p))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int(x*(a + b*log(c*log(d*x^n)^p)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x*(a + b*log(c*log(d*x**n)**p)), x)
```

3.50 $\int \left(a+b \log \left(c \log^p\left(d x^n\right)\right)\right) \, dx$

Optimal. Leaf size=45

$$ax+b x \log \left(c \log^p\left(d x^n\right)\right)-b p x \left(d x^n\right)^{-1/n} \text{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)$$

```
[Out] a*x-b*p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+b*x*ln(c*ln(d*x^n)^p)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2520, 2300, 2178}

$$ax+b x \log \left(c \log^p\left(d x^n\right)\right)-b p x \left(d x^n\right)^{-1/n} \text{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d*
x^n]^p]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b \log \left(c \log^p\left(d x^n\right)\right)\right) \, dx &= a x+b \int \log \left(c \log^p\left(d x^n\right)\right) \, dx\\
&= a x+b x \log \left(c \log^p\left(d x^n\right)\right)-(b n p) \int \frac{1}{\log \left(d x^n\right)} \, dx\\
&= a x+b x \log \left(c \log^p\left(d x^n\right)\right)-\left(b p x \left(d x^n\right)^{-1/n}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x} \, dx,x,\log \left(d x^n\right)\right)\\
&= a x-b p x \left(d x^n\right)^{-1/n} \text{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)+b x \log \left(c \log^p\left(d x^n\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.96

$$x \left(a+b \log \left(c \log^p\left(d x^n\right)\right)-b p \left(d x^n\right)^{-1/n} \text{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] x*(a - (b*p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*Log[c*Log[d*x^n
]^p])
```

fricas [A] time = 0.45, size = 53, normalized size = 1.18

$$\frac{bd^{\left(\frac{1}{n}\right)}px\log\left(n\log(x)+\log(d)\right)-bp\,\text{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)+\left(bx\log(c)+ax\right)d^{\left(\frac{1}{n}\right)}}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log(n*log(x) + log(d)) - b*p*log_integral(d^(1/n)*x) + (b*x*
log(c) + a*x)*d^(1/n))/d^(1/n)
```

giac [A] time = 0.19, size = 42, normalized size = 0.93

$$\left(px\log\left(n\log(x)+\log(d)\right)+x\log(c)-\frac{p\mathrm{Ei}\left(\frac{\log(d)}{n}+\log(x)\right)}{d^{\left(\frac{1}{n}\right)}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="giac")
```

```
[Out] (p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n))*b
 + a*x
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)+a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*ln(d*x^n)^p)+a,x)
```

```
[Out] int(b*ln(c*ln(d*x^n)^p)+a,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(np\int\frac{1}{\log(d)+\log\left(x^{n}\right)}\,dx-x\log(c)-x\log\left(\left(\log(d)+\log\left(x^{n}\right)\right)^{p}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="maxima")
```

```
[Out] -(n*p*integrate(1/(log(d) + log(x^n)), x) - x*log(c) - x*log((log(d) + log(
x^n))^p))*b + a*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int a+b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*log(d*x^n)^p),x)
```

```
[Out] int(a + b*log(c*log(d*x^n)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\log\left(dx^{n}\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*ln(d*x**n)**p),x)
```

```
[Out] Integral(a + b*log(c*log(d*x**n)**p), x)
```

3.51 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}-bp\log(x)$$

```
[Out] -b*p*ln(x)+ln(d*x^n)*(a+b*ln(c*ln(d*x^n)^p))/n
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2521}

$$\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}-bp\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

```
[Out] -(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x}\,dx=-bp\log(x)+\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.25

$$a\log(x)+\frac{b\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}-\frac{bp\log(dx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

```
[Out] a*Log[x] - (b*p*Log[d*x^n])/n + (b*Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

fricas [A] time = 0.47, size = 45, normalized size = 1.41

$$\frac{\left(bnp\log(x)+bp\log(d)\right)\log\left(n\log(x)+\log(d)\right)-\left(bnp-bn\log(c)-an\right)\log(x)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="fricas")
```

```
[Out] ((b*n*p*log(x) + b*p*log(d))*log(n*log(x) + log(d)) - (b*n*p - b*n*log(c) -
 a*n)*log(x))/n
```

giac [A] time = 0.17, size = 54, normalized size = 1.69

$$\frac{\left(\left(n\log(x)+\log(d)\right)\log\left(n\log(x)+\log(d)\right)-n\log(x)-\log(d)\right)bp+\left(n\log(x)+\log(d)\right)b\log(c)+\left(n\log(x)+\log(d)\right)a}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="giac")
```

```
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*b*p + (n*
log(x) + log(d))*b*log(c) + (n*log(x) + log(d))*a)/n
```

maple [A] time = 0.08, size = 48, normalized size = 1.50

$$-\frac{bp\ln\left(d\,x^{n}\right)}{n}+\frac{b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\ln\left(d\,x^{n}\right)}{n}+\frac{a\ln\left(d\,x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*ln(d*x^n)^p)+a)/x,x)
```

```
[Out] 1/n*ln(c*ln(d*x^n)^p)*ln(d*x^n)*b-1/n*ln(d*x^n)*b*p+1/n*a*ln(d*x^n)
```

maxima [A] time = 0.46, size = 64, normalized size = 2.00

$$b\log\left(c\log\left(dx^{n}\right)^{p}\right)\log(x)-\left(p\log(x)\log\left(\log\left(dx^{n}\right)\right)-\frac{\left(\log\left(dx^{n}\right)\log\left(\log\left(dx^{n}\right)\right)-\log\left(dx^{n}\right)\right)p}{n}\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="maxima")
```

```
[Out] b*log(c*log(d*x^n)^p)*log(x) - (p*log(x)*log(log(d*x^n)) - (log(d*x^n)*log(
log(d*x^n)) - log(d*x^n))*p/n)*b + a*log(x)
```

mupad [B] time = 0.32, size = 32, normalized size = 1.00

$$\ln(x)\,\left(a-b\,p\right)+\frac{b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\,\ln\left(d\,x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x,x)
```

```
[Out] log(x)*(a - b*p) + (b*log(c*log(d*x^n)^p)*log(d*x^n))/n
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x, x)
```

3.52 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

```
[Out] b*p*(d*x^n)^(1/n)*Ei(-ln(d*x^n)/n)/x+(-a-b*ln(c*ln(d*x^n)^p))/x
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

```
[Out] (b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d*
x^n]^p])/x
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}+(bnp)\int \frac{1}{x^2\log\left(dx^n\right)}\,dx\\
&= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}+\frac{\left(bp\,(dx^n)^{\frac{1}{n}}\right)\mathrm{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{x}\\
&= \frac{bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.94

$$-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

```
[Out] -((a - b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)] + b*Log[c*Log[d*x^
n]^p])/x)
```

fricas [A] time = 0.44, size = 46, normalized size = 0.96

$$\frac{bd^{\left(\frac{1}{n}\right)}px\,\mathrm{log\_integral}\left(\frac{1}{d^{\left(\frac{1}{n}\right)}x}\right) - bp\log\left(n\log(x) + \log(d)\right) - b\log(c) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log_integral(1/(d^(1/n)*x)) - b*p*log(n*log(x) + log(d)) - b
*log(c) - a)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(c\log\left(dx^n\right)^p\right) + a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^2, x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\ln\left(d\,x^n\right)^p\right) + a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*ln(d*x^n)^p)+a)/x^2,x)
```

```
[Out] int((b*ln(c*ln(d*x^n)^p)+a)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(np\int \frac{1}{x^2\log(d) + x^2\log\left(x^n\right)}\,dx - \frac{\log(c) + \log\left(\left(\log(d) + \log\left(x^n\right)\right)^p\right)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="maxima")
```

```
[Out] (n*p*integrate(1/(x^2*log(d) + x^2*log(x^n)), x) - (log(c) + log((log(d) +
log(x^n))^p))/x)*b - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^2,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log \left(c \log \left(dx^{n}\right)^{p}\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**2,x)
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**2, x)
```

3.53 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{bp\,(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{2x^2}$$

```
[Out] 1/2*b*p*(d*x^n)^(2/n)*Ei(-2*ln(d*x^n)/n)/x^2+1/2*(-a-b*ln(c*ln(d*x^n)^p))/x
^2
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] (b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[c
*Log[d*x^n]^p])/(2*x^2)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^3}\,dx&=-\frac{a+b\log\left(c\log^p(dx^n)\right)}{2x^2}+\frac{1}{2}(bnp)\int\frac{1}{x^3\log(dx^n)}\,dx\\&=-\frac{a+b\log\left(c\log^p(dx^n)\right)}{2x^2}+\frac{\left(bp\,(dx^n)^{2/n}\right)\text{Subst}\left(\int\frac{e^{-\frac{2x}{n}}}{x}\,dx,x,\log(dx^n)\right)}{2x^2}\\&=\frac{bp\,(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.89

$$-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{2/n}\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] -1/2*(a - b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^2
```

fricas [A] time = 0.47, size = 53, normalized size = 0.96

$$\frac{bd^{\frac{2}{n}}px^2\,\text{log\_integral}\left(\frac{1}{d^{\frac{2}{n}}x^2}\right)-bp\log\left(n\log(x)+\log(d)\right)-b\log(c)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log_integral(1/(d^(2/n)*x^2)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(c\log\left(dx^n\right)^p\right)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^3, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\ln\left(d\,x^n\right)^p\right)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*ln(d*x^n)^p)+a)/x^3,x)
```

```
[Out] int((b*ln(c*ln(d*x^n)^p)+a)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,np\int\frac{1}{2\left(x^3\log(d)+x^3\log\left(x^n\right)\right)}\,dx-\frac{\log(c)+\log\left(\left(\log(d)+\log\left(x^n\right)\right)^p\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*n*p*integrate(1/2/(x^3*log(d) + x^3*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^2)*b - 1/2*a/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^3,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log \left(c \log \left(dx^n\right)^p\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**3,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**3, x)
```

3.54 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^4}\,dx$

Optimal. Leaf size=55

$$\frac{bp\,(dx^n)^{3/n}\,\mathrm{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

```
[Out] 1/3*b*p*(d*x^n)^(3/n)*Ei(-3*ln(d*x^n)/n)/x^3+1/3*(-a-b*ln(c*ln(d*x^n)^p))/x
^3
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{3/n}\,\mathrm{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] (b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[c
*Log[d*x^n]^p])/(3*x^3)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^4}\,dx &= -\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}+\frac{1}{3}(bnp)\int \frac{1}{x^4\log(dx^n)}\,dx\\ &= -\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}+\frac{\left(bp\,(dx^n)^{3/n}\right)\mathrm{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{x}\,dx,x,\log(dx^n)\right)}{3x^3}\\ &= \frac{bp\,(dx^n)^{3/n}\,\mathrm{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.89

$$-\frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)-bp\left(dx^{n}\right)^{3/n}\operatorname{Ei}\left(-\frac{3\log(dx^{n})}{n}\right)}{3x^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] -1/3*(a - b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^3
```

fricas [A] time = 0.46, size = 53, normalized size = 0.96

$$\frac{bd^{\frac{3}{n}}px^{3}\operatorname{log\_integral}\left(\frac{1}{d^{\frac{3}{n}}x^{3}}\right)-bp\log\left(n\log(x)+\log(d)\right)-b\log(c)-a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log_integral(1/(d^(3/n)*x^3)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right)+a}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^4, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)+a}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*ln(d*x^n)^p)+a)/x^4,x)
```

```
[Out] int((b*ln(c*ln(d*x^n)^p)+a)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(3\,np\int\frac{1}{3\left(x^{4}\log(d)+x^{4}\log\left(x^{n}\right)\right)}\,dx-\frac{\log(c)+\log\left(\left(\log(d)+\log\left(x^{n}\right)\right)^{p}\right)}{x^{3}}\right)b-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*n*p*integrate(1/3/(x^4*log(d) + x^4*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^3)*b - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^4,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^4, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\log\left(dx^n\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**4,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**4, x)
```

3.55 $\int \log \left(c \log ^p(dx)\right) \, dx$

Optimal. Leaf size=22

$$x \log \left(c \log ^p(dx)\right)-\frac{p \text{li}(dx)}{d}$$

```
[Out] -p*Li(d*x)/d+x*ln(c*ln(d*x)^p)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2520, 2298}

$$x \log \left(c \log ^p(dx)\right)-\frac{p \text{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p],x]
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \log \left(c \log ^p(dx)\right) \, dx &= x \log \left(c \log ^p(dx)\right)-p \int \frac{1}{\log (dx)} \, dx\\ &= x \log \left(c \log ^p(dx)\right)-\frac{p \text{li}(dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$x \log \left(c \log ^p(dx)\right)-\frac{p \text{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p],x]
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

fricas [A] time = 0.44, size = 26, normalized size = 1.18

$$\frac{dpx \log \left(\log \left(dx\right)\right) + dx \log(c) - p \text{log\_integral}\left(dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="fricas")
[Out] (d*p*x*log(log(d*x)) + d*x*log(c) - p*log_integral(d*x))/d
```

giac [A] time = 0.19, size = 26, normalized size = 1.18

$$px\log\left(\log(d)+\log(x)\right)+x\log(c)-\frac{p\text{Ei}\left(\log(d)+\log(x)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="giac")
[Out] p*x*log(log(d) + log(x)) + x*log(c) - p*Ei(log(d) + log(x))/d
```

maple [A] time = 0.09, size = 26, normalized size = 1.18

$$x\ln\left(c\ln(dx)^{p}\right)+\frac{p\,\text{Ei}\left(1,-\ln(dx)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p),x)
[Out] x*ln(c*ln(d*x)^p)+p/d*Ei(1,-ln(d*x))
```

maxima [A] time = 0.62, size = 23, normalized size = 1.05

$$x\log\left(c\log(dx)^{p}\right)-\frac{p\text{Ei}\left(\log(dx)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="maxima")
[Out] x*log(c*log(d*x)^p) - p*Ei(log(d*x))/d
```

mupad [B] time = 0.31, size = 22, normalized size = 1.00

$$x\ \ln\left(c\ln(d\,x)^{p}\right)-\frac{p\,\text{logint}(d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x)^p),x)
[Out] x*log(c*log(d*x)^p) - (p*logint(d*x))/d
```

sympy [A] time = 1.19, size = 19, normalized size = 0.86

$$x\log\left(c\log(dx)^{p}\right)-\frac{p\,\text{li}(dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p),x)
[Out] x*log(c*log(d*x)**p) - p*li(d*x)/d
```

3.56 $\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx$

Optimal. Leaf size=20

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(x)$$

```
[Out] -p*ln(x)+ln(d*x)*ln(c*ln(d*x)^p)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2521}

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p]/x,x]
[Out] -(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx=-p\log(x)+\log(dx)\log\left(c\log^p(dx)\right)$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.10

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p]/x,x]
[Out] -(p*Log[d*x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

fricas [A] time = 0.45, size = 24, normalized size = 1.20

$$p\log(dx)\log\left(\log(dx)\right)-\left(p-\log(c)\right)\log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="fricas")
[Out] p*log(d*x)*log(log(d*x)) - (p - log(c))*log(d*x)
```

giac [A] time = 0.16, size = 32, normalized size = 1.60

$$\left(\left(\log(d)+\log(x)\right)\log\left(\log(d)+\log(x)\right)-\log(d)-\log(x)\right)p+\left(\log(d)+\log(x)\right)\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="giac")
```

```
[Out] ((log(d) + log(x))*log(log(d) + log(x)) - log(d) - log(x))*p + (log(d) + lo
g(x))*log(c)
```

maple [A] time = 0.07, size = 23, normalized size = 1.15

$$-p\ln(dx) + \ln\left(c\ln(dx)^p\right)\ln(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p)/x,x)
[Out] ln(d*x)*ln(c*ln(d*x)^p)-p*ln(d*x)
```

maxima [A] time = 0.44, size = 22, normalized size = 1.10

$$-p\log(dx) + \log(dx)\log\left(c\log(dx)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="maxima")
[Out] -p*log(d*x) + log(d*x)*log(c*log(d*x)^p)
```

mupad [B] time = 0.28, size = 20, normalized size = 1.00

$$\ln\left(c\ln(d\,x)^p\right)\,\ln(d\,x) - p\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x)^p)/x,x)
[Out] log(c*log(d*x)^p)*log(d*x) - p*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\log(dx)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p)/x,x)
[Out] Integral(log(c*log(d*x)**p)/x, x)
```

3.57 $\int \log \left(c \log^p\left(d x^n\right)\right) \, dx$

Optimal. Leaf size=40

$$x \log \left(c \log^p\left(d x^n\right)\right)-p x \left(d x^n\right)^{-1/n} \operatorname{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)$$

```
[Out] -p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+x*ln(c*ln(d*x^n)^p)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2520, 2300, 2178}

$$x \log \left(c \log^p\left(d x^n\right)\right)-p x \left(d x^n\right)^{-1/n} \operatorname{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p],x]
```

```
[Out] -((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \log^p\left(d x^n\right)\right) \, dx &= x \log \left(c \log^p\left(d x^n\right)\right)-(n p) \int \frac{1}{\log \left(d x^n\right)} \, dx\\
&= x \log \left(c \log^p\left(d x^n\right)\right)-\left(p x \left(d x^n\right)^{-1/n}\right) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x} \, dx,x,\log \left(d x^n\right)\right)\\
&= -p x \left(d x^n\right)^{-1/n} \operatorname{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)+x \log \left(c \log^p\left(d x^n\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.98

$$x \left(\log \left(c \log^p\left(d x^n\right)\right)-p \left(d x^n\right)^{-1/n} \operatorname{Ei}\left(\frac{\log \left(d x^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p],x]
```

```
[Out] x*(-((p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + Log[c*Log[d*x^n]^p])
```

fricas [A] time = 0.45, size = 45, normalized size = 1.12

$$\frac{d^{\left(\frac{1}{n}\right)} p x \log\left(n \log(x) + \log(d)\right) + d^{\left(\frac{1}{n}\right)} x \log(c) - p \operatorname{log\_integral}\left(d^{\left(\frac{1}{n}\right)} x\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="fricas")
```

```
[Out] (d^(1/n)*p*x*log(n*log(x) + log(d)) + d^(1/n)*x*log(c) - p*log_integral(d^(
1/n)*x))/d^(1/n)
```

giac [A] time = 0.16, size = 36, normalized size = 0.90

$$px \log\left(n \log(x) + \log(d)\right) + x \log(c) - \frac{p \operatorname{Ei}\left(\frac{\log(d)}{n} + \log(x)\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="giac")
```

```
[Out] p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p),x)
```

```
[Out] int(ln(c*ln(d*x^n)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-np \int \frac{1}{\log(d) + \log\left(x^{n}\right)}\, dx + x \log(c) + x \log\left(\left(\log(d) + \log\left(x^{n}\right)\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="maxima")
```

```
[Out] -n*p*integrate(1/(log(d) + log(x^n)), x) + x*log(c) + x*log((log(d) + log(x
^n))^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(c \ln\left(d\,x^{n}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x^n)^p),x)
```

```
[Out] int(log(c*log(d*x^n)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(c \log\left(dx^{n}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p),x)
```

```
[Out] Integral(log(c*log(d*x**n)**p), x)
```

3.58 $\int \frac{\log\left(c\log^p(dx^n)\right)}{x}\,dx$

Optimal. Leaf size=27

$$\frac{\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}-p\log(x)$$

```
[Out] -p*ln(x)+ln(d*x^n)*ln(c*ln(d*x^n)^p)/n
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2521}

$$\frac{\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p]/x,x]
[Out] -(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log\left(c\log^p(dx^n)\right)}{x}\,dx=-p\log(x)+\frac{\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.26

$$\frac{\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}-\frac{p\log(dx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p]/x,x]
[Out] -((p*Log[d*x^n])/n) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

fricas [A] time = 0.45, size = 37, normalized size = 1.37

$$\frac{\left(np\log(x)+p\log(d)\right)\log\left(n\log(x)+\log(d)\right)-\left(np-n\log(c)\right)\log(x)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="fricas")
[Out] ((n*p*log(x) + p*log(d))*log(n*log(x) + log(d)) - (n*p - n*log(c))*log(x))/
n
```

giac [A] time = 0.19, size = 43, normalized size = 1.59

$$\frac{\left(\left(n\log(x)+\log(d)\right)\log\left(n\log(x)+\log(d)\right)-n\log(x)-\log(d)\right)p+\left(n\log(x)+\log(d)\right)\log(c)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*p + (n*lo
g(x) + log(d))*log(c))/n
```

maple [A] time = 0.07, size = 35, normalized size = 1.30

$$-\frac{p\ln\left(d\,x^{n}\right)}{n}+\frac{\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\ln\left(d\,x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p)/x,x)
```

```
[Out] ln(d*x^n)*ln(c*ln(d*x^n)^p)/n-1/n*p*ln(d*x^n)
```

maxima [B] time = 0.45, size = 55, normalized size = 2.04

$$-p\log(x)\log\left(\log\left(dx^{n}\right)\right)+\log\left(c\log\left(dx^{n}\right)^{p}\right)\log(x)+\frac{\left(\log\left(dx^{n}\right)\log\left(\log\left(dx^{n}\right)\right)-\log\left(dx^{n}\right)\right)p}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -p*log(x)*log(log(d*x^n)) + log(c*log(d*x^n)^p)*log(x) + (log(d*x^n)*log(lo
g(d*x^n)) - log(d*x^n))*p/n
```

mupad [B] time = 0.30, size = 27, normalized size = 1.00

$$\frac{\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\,\ln\left(d\,x^{n}\right)}{n}-p\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x^n)^p)/x,x)
```

```
[Out] (log(c*log(d*x^n)^p)*log(d*x^n))/n - p*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*log(d*x**n)**p)/x, x)
```

3.59 $\int x^m \log \left(d \left(bx + cx^2\right)^n\right) dx$

Optimal. Leaf size=66

$$\frac{x^{m+1} \log \left(d \left(bx + cx^2\right)^n\right)}{m+1} + \frac{n x^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{cx}{b}\right)}{(m+1)^2} - \frac{2n x^{m+1}}{(m+1)^2}$$

```
[Out] -2*n*x^(1+m)/(1+m)^2+n*x^(1+m)*hypergeom([1, 1+m],[2+m],-c*x/b)/(1+m)^2+x^(
1+m)*ln(d*(c*x^2+b*x)^n)/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2525, 80, 64}

$$\frac{x^{m+1} \log \left(d \left(bx + cx^2\right)^n\right)}{m+1} + \frac{n x^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{cx}{b}\right)}{(m+1)^2} - \frac{2n x^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (-2*n*x^(1 + m))/(1 + m)^2 + (n*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)^2 + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)dx = \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^m(b+2cx)}{b+cx}dx}{1+m}$$
$$= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} + \frac{(bn)\int \frac{x^m}{b+cx}dx}{1+m}$$
$$= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.73

$$\frac{x^{m+1}\left((m+1)\log\left(d(x(b+cx))^n\right)+n\,{}_2F_1\left(1,m+1;m+2;-\frac{cx}{b}\right)-2n\right)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (x^(1 + m)*(-2*n + n*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*x)/b)] + (1 + m)*Log[d*(x*(b + c*x))^n]))/(1 + m)^2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \log\left(\left(cx^2+bx\right)^n d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*log((c*x^2 + b*x)^n*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log\left(\left(cx^2+bx\right)^n d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*log((c*x^2 + b*x)^n*d), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^m \ln\left(d\left(c\,x^2+bx\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^m\log\left((cx+b)^n\right)+xx^m\log\left(x^n\right)}{m+1}+\int \frac{\left(\left((m+1)\log(d)-2\,n\right)cx+\left((m+1)\log(d)-n\right)b\right)x^m}{c(m+1)x+b(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] (x*x^m*log((c*x + b)^n) + x*x^m*log(x^n))/(m + 1) + integrate((((m + 1)*log
(d) - 2*n)*c*x + ((m + 1)*log(d) - n)*b)*x^m/(c*(m + 1)*x + b*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \ln\left(d\left(c\,x^2+b\,x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] int(x^m*log(d*(b*x + c*x^2)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Integral(x**m*log(d*(b*x + c*x**2)**n), x)
```

3.60 $\int x^4 \log \left(d \left(bx+cx^2\right)^n\right) \, dx$

Optimal. Leaf size=99

$$\frac{b^5 n \log(b+cx)}{5c^5}-\frac{b^4 n x}{5c^4}+\frac{b^3 n x^2}{10c^3}-\frac{b^2 n x^3}{15c^2}+\frac{1}{5}x^5 \log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

```
[Out] -1/5*b^4*n*x/c^4+1/10*b^3*n*x^2/c^3-1/15*b^2*n*x^3/c^2+1/20*b*n*x^4/c-2/25*
n*x^5+1/5*b^5*n*ln(c*x+b)/c^5+1/5*x^5*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{b^3 n x^2}{10c^3}-\frac{b^2 n x^3}{15c^2}-\frac{b^4 n x}{5c^4}+\frac{b^5 n \log(b+cx)}{5c^5}+\frac{1}{5}x^5 \log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -(b^4*n*x)/(5*c^4) + (b^3*n*x^2)/(10*c^3) - (b^2*n*x^3)/(15*c^2) + (b*n*x^4
)/(20*c) - (2*n*x^5)/25 + (b^5*n*Log[b + c*x])/(5*c^5) + (x^5*Log[d*(b*x +
c*x^2)^n])/5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4 \log\left(d\left(bx+cx^2\right)^n\right) dx &= \frac{1}{5}x^5 \log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int \frac{x^4(b+2cx)}{b+cx}\,dx\\ &= \frac{1}{5}x^5 \log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int \left(\frac{b^4}{c^4}-\frac{b^3x}{c^3}+\frac{b^2x^2}{c^2}-\frac{bx^3}{c}+2x^4-\frac{b^5}{c^4(b+cx)}\right) dx\\ &= -\frac{b^4 n x}{5c^4}+\frac{b^3 n x^2}{10c^3}-\frac{b^2 n x^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{b^5 n \log(b+cx)}{5c^5}+\frac{1}{5}x^5 \log\left(d\left(bx+cx\right.\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.86

$$\frac{60b^5 n \log(b+cx)+cnx\left(-60b^4+30b^3cx-20b^2c^2x^2+15bc^3x^3-24c^4x^4\right)+60c^5x^5\log\left(d(x(b+cx))^n\right)}{300c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c^2*x^2 + 15*b*c^3*x^3 - 24*c^4*x^4)
+ 60*b^5*n*Log[b + c*x] + 60*c^5*x^5*Log[d*(x*(b + c*x))^n])/(300*c^5)
```

fricas [A] time = 0.44, size = 98, normalized size = 0.99

$$\frac{60\,c^5nx^5\log\left(cx^2+bx\right)-24\,c^5nx^5+60\,c^5x^5\log(d)+15\,bc^4nx^4-20\,b^2c^3nx^3+30\,b^3c^2nx^2-60\,b^4cnx+60\,b^5}{300\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/300*(60*c^5*n*x^5*log(c*x^2 + b*x) - 24*c^5*n*x^5 + 60*c^5*x^5*log(d) + 1
5*b*c^4*n*x^4 - 20*b^2*c^3*n*x^3 + 30*b^3*c^2*n*x^2 - 60*b^4*c*n*x + 60*b^5
*n*log(c*x + b))/c^5
```

giac [A] time = 0.16, size = 89, normalized size = 0.90

$$\frac{1}{5}\,nx^5\log\left(cx^2+bx\right)-\frac{1}{25}\left(2\,n-5\,\log(d)\right)x^5+\frac{bnx^4}{20\,c}-\frac{b^2nx^3}{15\,c^2}+\frac{b^3nx^2}{10\,c^3}-\frac{b^4nx}{5\,c^4}+\frac{b^5n\log\left(cx+b\right)}{5\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c - 1
/15*b^2*n*x^3/c^2 + 1/10*b^3*n*x^2/c^3 - 1/5*b^4*n*x/c^4 + 1/5*b^5*n*log(c*
x + b)/c^5
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^4\ln\left(d\left(c\,x^2+bx\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

maxima [A] time = 0.45, size = 87, normalized size = 0.88

$$\frac{1}{5}\,x^5\log\left(\left(cx^2+bx\right)^n d\right)+\frac{1}{300}\,n\left(\frac{60\,b^5\log\left(cx+b\right)}{c^5}-\frac{24\,c^4x^5-15\,bc^3x^4+20\,b^2c^2x^3-30\,b^3cx^2+60\,b^4x}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((c*x^2 + b*x)^n*d) + 1/300*n*(60*b^5*log(c*x + b)/c^5 - (24*c^4
*x^5 - 15*b*c^3*x^4 + 20*b^2*c^2*x^3 - 30*b^3*c*x^2 + 60*b^4*x)/c^4)
```

mupad [B] time = 0.32, size = 85, normalized size = 0.86

$$\frac{x^5\,\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{5}-\frac{2\,n\,x^5}{25}-\frac{b^2\,n\,x^3}{15\,c^2}+\frac{b^3\,n\,x^2}{10\,c^3}+\frac{b^5\,n\,\ln\left(b+c\,x\right)}{5\,c^5}+\frac{b\,n\,x^4}{20\,c}-\frac{b^4\,n\,x}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^5*log(d*(b*x + c*x^2)^n))/5 - (2*n*x^5)/25 - (b^2*n*x^3)/(15*c^2) + (b^3
*n*x^2)/(10*c^3) + (b^5*n*log(b + c*x))/(5*c^5) + (b*n*x^4)/(20*c) - (b^4*n
*x)/(5*c^4)
```

sympy [**A**] time = 17.67, size = 134, normalized size = 1.35

$$\begin{cases} \frac{b^5 n \log{(b+cx)}}{5c^5} - \frac{b^4 nx}{5c^4} + \frac{b^3 nx^2}{10c^3} - \frac{b^2 nx^3}{15c^2} + \frac{bnx^4}{20c} + \frac{nx^5 \log{\left(bx+cx^2\right)}}{5} - \frac{2nx^5}{25} + \frac{x^5 \log{(d)}}{5} & \text{for } c \neq 0 \\ \frac{nx^5 \log{(b)}}{5} + \frac{nx^5 \log{(x)}}{5} - \frac{nx^5}{25} + \frac{x^5 \log{(d)}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**5*n*log(b + c*x)/(5*c**5) - b**4*n*x/(5*c**4) + b**3*n*x**2/(
10*c**3) - b**2*n*x**3/(15*c**2) + b*n*x**4/(20*c) + n*x**5*log(b*x + c*x**
2)/5 - 2*n*x**5/25 + x**5*log(d)/5, Ne(c, 0)), (n*x**5*log(b)/5 + n*x**5*lo
g(x)/5 - n*x**5/25 + x**5*log(d)/5, True))
```

3.61 $\int x^3 \log \left(d \left(bx+cx^2\right)^n\right) \, dx$

Optimal. Leaf size=85

$$-\frac{b^4 n \log(b+cx)}{4c^4}+\frac{b^3 n x}{4c^3}-\frac{b^2 n x^2}{8c^2}+\frac{1}{4}x^4 \log \left(d \left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

```
[Out] 1/4*b^3*n*x/c^3-1/8*b^2*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*x^4-1/4*b^4*n*ln(c*x
+b)/c^4+1/4*x^4*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{b^2 n x^2}{8c^2}+\frac{b^3 n x}{4c^3}-\frac{b^4 n \log(b+cx)}{4c^4}+\frac{1}{4}x^4 \log \left(d \left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b^3*n*x)/(4*c^3) - (b^2*n*x^2)/(8*c^2) + (b*n*x^3)/(12*c) - (n*x^4)/8 - (b
^4*n*Log[b + c*x])/(4*c^4) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \log \left(d \left(bx+cx^2\right)^n\right) \, dx &= \frac{1}{4}x^4 \log \left(d \left(bx+cx^2\right)^n\right)-\frac{1}{4}n \int \frac{x^3(b+2cx)}{b+cx} \, dx\\
&= \frac{1}{4}x^4 \log \left(d \left(bx+cx^2\right)^n\right)-\frac{1}{4}n \int \left(-\frac{b^3}{c^3}+\frac{b^2 x}{c^2}-\frac{bx^2}{c}+2x^3+\frac{b^4}{c^3(b+cx)}\right) \, dx\\
&= \frac{b^3 n x}{4c^3}-\frac{b^2 n x^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b^4 n \log(b+cx)}{4c^4}+\frac{1}{4}x^4 \log \left(d \left(bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.87

$$\frac{-6b^4 n \log(b+cx)+cnx\left(6b^3-3b^2 cx+2bc^2x^2-3c^3x^3\right)+6c^4x^4 \log \left(d(x(b+cx))^n\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*x^3) - 6*b^4*n*Log[b + c*x]
 + 6*c^4*x^4*Log[d*(x*(b + c*x))^n])/(24*c^4)
```

fricas [A] time = 0.44, size = 86, normalized size = 1.01

$$\frac{6\,c^4nx^4\log\left(cx^2+bx\right)-3\,c^4nx^4+6\,c^4x^4\log(d)+2\,bc^3nx^3-3\,b^2c^2nx^2+6\,b^3cnx-6\,b^4n\log\left(cx+b\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^4*n*x^4*log(c*x^2 + b*x) - 3*c^4*n*x^4 + 6*c^4*x^4*log(d) + 2*b*c
^3*n*x^3 - 3*b^2*c^2*n*x^2 + 6*b^3*c*n*x - 6*b^4*n*log(c*x + b))/c^4
```

giac [A] time = 0.19, size = 75, normalized size = 0.88

$$\frac{1}{4}\,nx^4\log\left(cx^2+bx\right)-\frac{1}{8}\left(n-2\,\log(d)\right)x^4+\frac{bnx^3}{12\,c}-\frac{b^2nx^2}{8\,c^2}+\frac{b^3nx}{4\,c^3}-\frac{b^4n\log\left(cx+b\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c - 1/8*
b^2*n*x^2/c^2 + 1/4*b^3*n*x/c^3 - 1/4*b^4*n*log(c*x + b)/c^4
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^3\ln\left(d\left(c\,x^2+bx\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^3*ln(d*(c*x^2+b*x)^n),x)
```

maxima [A] time = 0.44, size = 75, normalized size = 0.88

$$\frac{1}{4}\,x^4\log\left(\left(cx^2+bx\right)^nd\right)-\frac{1}{24}\,n\left(\frac{6\,b^4\log\left(cx+b\right)}{c^4}+\frac{3\,c^3x^4-2\,bc^2x^3+3\,b^2cx^2-6\,b^3x}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((c*x^2 + b*x)^n*d) - 1/24*n*(6*b^4*log(c*x + b)/c^4 + (3*c^3*x^
4 - 2*b*c^2*x^3 + 3*b^2*c*x^2 - 6*b^3*x)/c^3)
```

mupad [B] time = 0.34, size = 73, normalized size = 0.86

$$\frac{x^4\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{4}-\frac{n\,x^4}{8}-\frac{b^2\,n\,x^2}{8\,c^2}-\frac{b^4\,n\,\ln\left(b+c\,x\right)}{4\,c^4}+\frac{b\,n\,x^3}{12\,c}+\frac{b^3\,n\,x}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^4*log(d*(b*x + c*x^2)^n))/4 - (n*x^4)/8 - (b^2*n*x^2)/(8*c^2) - (b^4*n*l
og(b + c*x))/(4*c^4) + (b*n*x^3)/(12*c) + (b^3*n*x)/(4*c^3)
```

sympy [A] time = 10.09, size = 119, normalized size = 1.40

$$\begin{cases} -\frac{b^4 n \log(b+cx)}{4c^4} + \frac{b^3 nx}{4c^3} - \frac{b^2 nx^2}{8c^2} + \frac{bnx^3}{12c} + \frac{nx^4 \log(bx+cx^2)}{4} - \frac{nx^4}{8} + \frac{x^4 \log(d)}{4} & \text{for } c \neq 0 \\ \frac{nx^4 \log(b)}{4} + \frac{nx^4 \log(x)}{4} - \frac{nx^4}{16} + \frac{x^4 \log(d)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**4*n*log(b + c*x)/(4*c**4) + b**3*n*x/(4*c**3) - b**2*n*x**2/
(8*c**2) + b*n*x**3/(12*c) + n*x**4*log(b*x + c*x**2)/4 - n*x**4/8 + x**4*l
og(d)/4, Ne(c, 0)), (n*x**4*log(b)/4 + n*x**4*log(x)/4 - n*x**4/16 + x**4*l
og(d)/4, True))
```

3.62 $\int x^2 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=71

$$\frac{b^3 n\log(b+cx)}{3c^3}-\frac{b^2nx}{3c^2}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

```
[Out] -1/3*b^2*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/3*b^3*n*ln(c*x+b)/c^3+1/3*x^3*ln
(d*(c*x^2+b*x)^n)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{b^2nx}{3c^2}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -(b^2*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (b^3*n*Log[b + c*x])/(
3*c^3) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(d\left(bx+cx^2\right)^n\right)\,dx&=\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\frac{x^2(b+2cx)}{b+cx}\,dx\\&=\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\left(\frac{b^2}{c^2}-\frac{bx}{c}+2x^2-\frac{b^3}{c^2(b+cx)}\right)dx\\&=-\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.89

$$\frac{6b^3n\log(b+cx)+cnx\left(-6b^2+3bcx-4c^2x^2\right)+6c^3x^3\log\left(d(x(b+cx))^n\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c^2*x^2) + 6*b^3*n*Log[b + c*x] + 6*c^3*x^3*Lo
g[d*(x*(b + c*x))^n])/(18*c^3)
```

fricas [A] time = 0.46, size = 74, normalized size = 1.04

$$\frac{6\,c^3nx^3\log\left(cx^2+bx\right)-4\,c^3nx^3+6\,c^3x^3\log(d)+3\,bc^2nx^2-6\,b^2cnx+6\,b^3n\log\left(cx+b\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/18*(6*c^3*n*x^3*log(c*x^2 + b*x) - 4*c^3*n*x^3 + 6*c^3*x^3*log(d) + 3*b*c
^2*n*x^2 - 6*b^2*c*n*x + 6*b^3*n*log(c*x + b))/c^3
```

giac [A] time = 0.17, size = 65, normalized size = 0.92

$$\frac{1}{3}\,nx^3\log\left(cx^2+bx\right)-\frac{1}{9}\left(2\,n-3\,\log(d)\right)x^3+\frac{bnx^2}{6\,c}-\frac{b^2nx}{3\,c^2}+\frac{b^3n\log\left(cx+b\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c - 1/3
*b^2*n*x/c^2 + 1/3*b^3*n*log(c*x + b)/c^3
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^2\ln\left(d\left(c\,x^2+bx\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

maxima [A] time = 0.44, size = 65, normalized size = 0.92

$$\frac{1}{3}\,x^3\log\left(\left(cx^2+bx\right)^n d\right)+\frac{1}{18}\,n\left(\frac{6\,b^3\log\left(cx+b\right)}{c^3}-\frac{4\,c^2x^3-3\,bcx^2+6\,b^2x}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((c*x^2 + b*x)^n*d) + 1/18*n*(6*b^3*log(c*x + b)/c^3 - (4*c^2*x^
3 - 3*b*c*x^2 + 6*b^2*x)/c^2)
```

mupad [B] time = 0.32, size = 61, normalized size = 0.86

$$\frac{x^3\,\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{3}-\frac{2\,n\,x^3}{9}+\frac{b^3\,n\,\ln\left(b+c\,x\right)}{3\,c^3}+\frac{b\,n\,x^2}{6\,c}-\frac{b^2\,n\,x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^3*log(d*(b*x + c*x^2)^n))/3 - (2*n*x^3)/9 + (b^3*n*log(b + c*x))/(3*c^3)
 + (b*n*x^2)/(6*c) - (b^2*n*x)/(3*c^2)
```

sympy [A] time = 5.35, size = 107, normalized size = 1.51

$$\begin{cases} \frac{b^3 n \log(b+cx)}{3c^3} - \frac{b^2 nx}{3c^2} + \frac{bnx^2}{6c} + \frac{nx^3 \log(bx+cx^2)}{3} - \frac{2nx^3}{9} + \frac{x^3 \log(d)}{3} & \text{for } c \neq 0 \\ \frac{nx^3 \log(b)}{3} + \frac{nx^3 \log(x)}{3} - \frac{nx^3}{9} + \frac{x^3 \log(d)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**3*n*log(b + c*x)/(3*c**3) - b**2*n*x/(3*c**2) + b*n*x**2/(6*c
) + n*x**3*log(b*x + c*x**2)/3 - 2*n*x**3/9 + x**3*log(d)/3, Ne(c, 0)), (n*
x**3*log(b)/3 + n*x**3*log(x)/3 - n*x**3/9 + x**3*log(d)/3, True))
```

3.63 $\int x \log \left(d \left(bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=57

$$-\frac{b^2 n \log(b+cx)}{2c^2}+\frac{1}{2}x^2 \log \left(d \left(bx+cx^2\right)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

```
[Out] 1/2*b*n*x/c-1/2*n*x^2-1/2*b^2*n*ln(c*x+b)/c^2+1/2*x^2*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2525, 77}

$$-\frac{b^2 n \log(b+cx)}{2c^2}+\frac{1}{2}x^2 \log \left(d \left(bx+cx^2\right)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b^2*n*Log[b + c*x])/(2*c^2) + (x^2*Log[d*(b*x
+ c*x^2)^n])/2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \log \left(d \left(bx+cx^2\right)^n\right) dx &= \frac{1}{2}x^2 \log \left(d \left(bx+cx^2\right)^n\right)-\frac{1}{2}n \int \frac{x(b+2cx)}{b+cx} dx\\
&= \frac{1}{2}x^2 \log \left(d \left(bx+cx^2\right)^n\right)-\frac{1}{2}n \int \left(-\frac{b}{c}+2x+\frac{b^2}{c(b+cx)}\right) dx\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b^2 n \log(b+cx)}{2c^2}+\frac{1}{2}x^2 \log \left(d \left(bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{1}{2}x^2 \log \left(d(x(b+cx))^n\right)-\frac{1}{2}n \left(\frac{b^2 \log(b+cx)}{c^2}-\frac{bx}{c}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/2*(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2)) + (x^2*Log[d*(x*(b + c
*x))^n])/2
```

fricas [A] time = 0.43, size = 59, normalized size = 1.04

$$\frac{c^2nx^2\log\left(cx^2+bx\right)-c^2nx^2+c^2x^2\log(d)+bcnx-b^2n\log\left(cx+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*n*x^2*log(c*x^2 + b*x) - c^2*n*x^2 + c^2*x^2*log(d) + b*c*n*x - b^
2*n*log(c*x + b))/c^2
```

giac [A] time = 0.18, size = 51, normalized size = 0.89

$$\frac{1}{2}\,nx^2\log\left(cx^2+bx\right)-\frac{1}{2}\left(n-\log(d)\right)x^2+\frac{bnx}{2\,c}-\frac{b^2n\log\left(cx+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/2*b^2*n
*log(c*x + b)/c^2
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x\ln\left(d\left(c\,x^2+bx\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x*ln(d*(c*x^2+b*x)^n),x)
```

maxima [A] time = 0.44, size = 51, normalized size = 0.89

$$\frac{1}{2}\,x^2\log\left(\left(cx^2+bx\right)^nd\right)-\frac{1}{2}\,n\left(\frac{b^2\log\left(cx+b\right)}{c^2}+\frac{cx^2-bx}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((c*x^2 + b*x)^n*d) - 1/2*n*(b^2*log(c*x + b)/c^2 + (c*x^2 - b*x
)/c)
```

mupad [B] time = 0.37, size = 49, normalized size = 0.86

$$\frac{x^2\,\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{2}-\frac{n\,x^2}{2}+\frac{b\,n\,x}{2\,c}-\frac{b^2\,n\,\ln\left(b+c\,x\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(b*x + c*x^2)^n))/2 - (n*x^2)/2 + (b*n*x)/(2*c) - (b^2*n*log(b +
 c*x))/(2*c^2)
```

sympy [A] time = 2.69, size = 92, normalized size = 1.61

$$\begin{cases} -\frac{b^2n\log(b+cx)}{2c^2} + \frac{bnx}{2c} + \frac{nx^2\log(bx+cx^2)}{2} - \frac{nx^2}{2} + \frac{x^2\log(d)}{2} & \text{for } c \neq 0 \\ \frac{nx^2\log(b)}{2} + \frac{nx^2\log(x)}{2} - \frac{nx^2}{4} + \frac{x^2\log(d)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**2*n*log(b + c*x)/(2*c**2) + b*n*x/(2*c) + n*x**2*log(b*x + c
*x**2)/2 - n*x**2/2 + x**2*log(d)/2, Ne(c, 0)), (n*x**2*log(b)/2 + n*x**2*l
og(x)/2 - n*x**2/4 + x**2*log(d)/2, True))
```

3.64 $\int \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=33

$$x\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bn\log(b+cx)}{c}-2nx$$

```
[Out] -2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2523, 43}

$$x\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bn\log(b+cx)}{c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(b*x + c*x^2)^n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(d\left(bx+cx^2\right)^n\right)\,dx &= x\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{b+2cx}{b+cx}\,dx\\ &= x\log\left(d\left(bx+cx^2\right)^n\right)-n\int \left(2-\frac{b}{b+cx}\right)\,dx\\ &= -2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$x\log\left(d(x(b+cx))^n\right)+\frac{bn\log(b+cx)}{c}-2nx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(x*(b + c*x))^n]
```

fricas [A] time = 0.45, size = 38, normalized size = 1.15

$$\frac{cnx\log\left(cx^2+bx\right)-2\,cnx+bn\log\left(cx+b\right)+cx\log(d)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] (c*n*x*log(c*x^2 + b*x) - 2*c*n*x + b*n*log(c*x + b) + c*x*log(d))/c
```

giac [A] time = 0.16, size = 37, normalized size = 1.12

$$nx\log\left(cx^2+bx\right)-\left(2n-\log(d)\right)x+\frac{bn\log\left(cx+b\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x) - (2*n - log(d))*x + b*n*log(c*x + b)/c
```

maple [A] time = 0.07, size = 34, normalized size = 1.03

$$\frac{bn\ln\left(cx+b\right)}{c}-2nx+x\ln\left(d\left(cx^2+bx\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] -2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

maxima [A] time = 0.45, size = 36, normalized size = 1.09

$$-n\left(2x-\frac{b\log\left(cx+b\right)}{c}\right)+x\log\left(\left(cx^2+bx\right)^n d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] -n*(2*x - b*log(c*x + b)/c) + x*log((c*x^2 + b*x)^n*d)
```

mupad [B] time = 0.36, size = 33, normalized size = 1.00

$$x\ln\left(d\left(cx^2+bx\right)^n\right)-2nx+\frac{bn\ln\left(b+cx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(b*x + c*x^2)^n) - 2*n*x + (b*n*log(b + c*x))/c
```

sympy [A] time = 1.38, size = 56, normalized size = 1.70

$$\begin{cases}\frac{bn\log\left(b+cx\right)}{c}+nx\log\left(bx+cx^2\right)-2nx+x\log\left(d\right) & \text{for } c\neq 0\\ nx\log\left(b\right)+nx\log\left(x\right)-nx+x\log\left(d\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b*n*log(b + c*x)/c + n*x*log(b*x + c*x**2) - 2*n*x + x*log(d), N
e(c, 0)), (n*x*log(b) + n*x*log(x) - n*x + x*log(d), True))
```

3.65 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x} dx$

Optimal. Leaf size=53

$$\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\mathrm{Li}_2\left(-\frac{cx}{b}\right)-n\log(x)\log\left(\frac{cx}{b}+1\right)-\frac{1}{2}n\log^2(x)$$

```
[Out] -1/2*n*ln(x)^2-n*ln(x)*ln(c*x/b+1)+ln(x)*ln(d*(c*x^2+b*x)^n)-n*polylog(2,-c
*x/b)
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2524, 1593, 2357, 2301, 2317, 2391}

$$-n\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\log(x)\log\left(\frac{cx}{b}+1\right)-\frac{1}{2}n\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]^2)/2 - n*Log[x]*Log[1 + (c*x)/b] + Log[x]*Log[d*(b*x + c*x^2)^n]
 - n*PolyLog[2, -((c*x)/b)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{bx+cx^2}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{\log(x)}{x}\,dx-(cn)\int \frac{\log(x)}{b+cx}\,dx\\
&= -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)+n\int \frac{\log\left(1+\right.}{x}\\
&= -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\text{Li}_2\left(-\frac{cx}{b}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.94

$$\log(x)\log\left(d(x(b+cx))^n\right)-n\left(\text{Li}_2\left(-\frac{cx}{b}\right)+\log(x)\log\left(\frac{b+cx}{b}\right)+\frac{\log^2(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] Log[x]*Log[d*(x*(b + c*x))^n] - n*(Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + P
olyLog[2, -((c*x)/b)])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx\right)^n d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x)^n*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x)^n*d)/x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x,x)
```

maxima [A] time = 0.45, size = 80, normalized size = 1.51

$$-n\log\left(cx^2+bx\right)\log(x)+\frac{1}{2}\left(2\,\log\left(cx^2+bx\right)\log(x)-2\,\log\left(\frac{cx}{b}+1\right)\log(x)-\log(x)^2-2\,\mathrm{Li}_2\left(-\frac{cx}{b}\right)\right)n+\log\left(\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="maxima")
```

```
[Out] -n*log(c*x^2 + b*x)*log(x) + 1/2*(2*log(c*x^2 + b*x)*log(x) - 2*log(c*x/b +
 1)*log(x) - log(x)^2 - 2*dilog(-c*x/b))*n + log((c*x^2 + b*x)^n*d)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x,x)
```

```
[Out] int(log(d*(b*x + c*x^2)^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x,x)
```

```
[Out] Integral(log(d*(b*x + c*x**2)**n)/x, x)
```

3.66 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2} dx$

Optimal. Leaf size=47

$$-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

```
[Out] -n/x+c*n*ln(x)/b-c*n*ln(c*x+b)/b-ln(d*(c*x^2+b*x)^n)/x
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(b*x + c*x^2)^n]/x
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+n\int \frac{b+2cx}{x^2(b+cx)}\,dx\\ &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+n\int \left(\frac{1}{x^2}+\frac{c}{bx}-\frac{c^2}{b(b+cx)}\right)dx\\ &= -\frac{n}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.96

$$-\frac{\log\left(d(x(b+cx))^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(x*(b + c*x))^n]/x
```

fricas [A] time = 0.47, size = 46, normalized size = 0.98

$$-\frac{cnx\log\left(cx+b\right)-cnx\log\left(x\right)+bn\log\left(cx^{2}+bx\right)+bn+b\log\left(d\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="fricas")
```

```
[Out] -(c*n*x*log(c*x + b) - c*n*x*log(x) + b*n*log(c*x^2 + b*x) + b*n + b*log(d)
)/(b*x)
```

giac [A] time = 0.18, size = 47, normalized size = 1.00

$$-\frac{cn\log\left(cx+b\right)}{b}+\frac{cn\log\left(x\right)}{b}-\frac{n\log\left(cx^{2}+bx\right)}{x}-\frac{n+\log\left(d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="giac")
```

```
[Out] -c*n*log(c*x + b)/b + c*n*log(x)/b - n*log(c*x^2 + b*x)/x - (n + log(d))/x
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
```

maxima [A] time = 0.45, size = 46, normalized size = 0.98

$$-n\left(\frac{c\log\left(cx+b\right)}{b}-\frac{c\log\left(x\right)}{b}+\frac{1}{x}\right)-\frac{\log\left(\left(cx^{2}+bx\right)^{n}d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="maxima")
```

```
[Out] -n*(c*log(c*x + b)/b - c*log(x)/b + 1/x) - log((c*x^2 + b*x)^n*d)/x
```

mupad [B] time = 0.80, size = 43, normalized size = 0.91

$$-\frac{\ln\left(d\left(c\,x^{2}+b\,x\right)^{n}\right)}{x}-\frac{n}{x}-\frac{2\,c\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^2,x)
```

```
[Out] - log(d*(b*x + c*x^2)^n)/x - n/x - (2*c*n*atanh((2*c*x)/b + 1))/b
```

<u>**sympy**</u> **[A]** time = 3.41, size = 76, normalized size = 1.62

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{x} - \frac{n}{x} - \frac{\log\left(d\right)}{x} - \frac{2cn\log\left(b+cx\right)}{b} + \frac{cn\log\left(bx+cx^2\right)}{b} & \text{for } b \neq 0 \\ -\frac{n\log\left(c\right)}{x} - \frac{2n\log\left(x\right)}{x} - \frac{2n}{x} - \frac{\log\left(d\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**2,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/x - n/x - log(d)/x - 2*c*n*log(b + c*x)/b +
 c*n*log(b*x + c*x**2)/b, Ne(b, 0)), (-n*log(c)/x - 2*n*log(x)/x - 2*n/x -
log(d)/x, True))
```

3.67 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3} dx$

Optimal. Leaf size=72

$$-\frac{c^2 n \log(x)}{2b^2}+\frac{c^2 n \log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}-\frac{cn}{2bx}-\frac{n}{4x^2}$$

```
[Out] -1/4*n/x^2-1/2*c*n/b/x-1/2*c^2*n*ln(x)/b^2+1/2*c^2*n*ln(c*x+b)/b^2-1/2*ln(d
*(c*x^2+b*x)^n)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{c^2 n \log(x)}{2b^2}+\frac{c^2 n \log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}-\frac{cn}{2bx}-\frac{n}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -n/(4*x^2) - (c*n)/(2*b*x) - (c^2*n*Log[x])/(2*b^2) + (c^2*n*Log[b + c*x])/
(2*b^2) - Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3} dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}+\frac{1}{2}n\int \frac{b+2cx}{x^3(b+cx)} dx\\
&= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}+\frac{1}{2}n\int \left(\frac{1}{x^3}+\frac{c}{bx^2}-\frac{c^2}{b^2x}+\frac{c^3}{b^2(b+cx)}\right) dx\\
&= -\frac{n}{4x^2}-\frac{cn}{2bx}-\frac{c^2 n \log(x)}{2b^2}+\frac{c^2 n \log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.90

$$\frac{1}{2}n\left(-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}-\frac{c}{bx}-\frac{1}{2x^2}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] (n*(-1/2*1/x^2 - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
Log[d*(x*(b + c*x))^n]/(2*x^2)
```

fricas [A] time = 0.48, size = 70, normalized size = 0.97

$$\frac{2\,c^2nx^2\log\left(cx+b\right)-2\,c^2nx^2\log(x)-2\,bcnx-2\,b^2n\log\left(cx^2+bx\right)-b^2n-2\,b^2\log(d)}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^2*n*x^2*log(c*x + b) - 2*c^2*n*x^2*log(x) - 2*b*c*n*x - 2*b^2*n*lo
g(c*x^2 + b*x) - b^2*n - 2*b^2*log(d))/(b^2*x^2)
```

giac [A] time = 0.17, size = 65, normalized size = 0.90

$$\frac{c^2n\log\left(cx+b\right)}{2\,b^2}-\frac{c^2n\log(x)}{2\,b^2}-\frac{n\log\left(cx^2+bx\right)}{2\,x^2}-\frac{2\,cnx+bn+2\,b\log(d)}{4\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c^2*n*log(c*x + b)/b^2 - 1/2*c^2*n*log(x)/b^2 - 1/2*n*log(c*x^2 + b*x)/
x^2 - 1/4*(2*c*n*x + b*n + 2*b*log(d))/(b*x^2)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

maxima [A] time = 0.70, size = 62, normalized size = 0.86

$$\frac{1}{4}\,n\left(\frac{2\,c^2\log\left(cx+b\right)}{b^2}-\frac{2\,c^2\log(x)}{b^2}-\frac{2\,cx+b}{bx^2}\right)-\frac{\log\left(\left(cx^2+bx\right)^n d\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*n*(2*c^2*log(c*x + b)/b^2 - 2*c^2*log(x)/b^2 - (2*c*x + b)/(b*x^2)) - 1
/2*log((c*x^2 + b*x)^n*d)/x^2
```

mupad [B] time = 0.48, size = 54, normalized size = 0.75

$$\frac{c^2\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2}-\frac{\frac{n}{2}+\frac{c\,n\,x}{b}}{2\,x^2}-\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (c^2*n*atanh((2*c*x)/b + 1))/b^2 - (n/2 + (c*n*x)/b)/(2*x^2) - log(d*(b*x +
 c*x^2)^n)/(2*x^2)
```

sympy [A] time = 6.95, size = 110, normalized size = 1.53

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{2x^2} - \frac{n}{4x^2} - \frac{\log(d)}{2x^2} - \frac{cn}{2bx} + \frac{c^2n\log(b+cx)}{b^2} - \frac{c^2n\log\left(bx+cx^2\right)}{2b^2} & \text{for } b \neq 0 \\ -\frac{n\log(c)}{2x^2} - \frac{n\log(x)}{x^2} - \frac{n}{2x^2} - \frac{\log(d)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**3,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(2*x**2) - n/(4*x**2) - log(d)/(2*x**2) - c
*n/(2*b*x) + c**2*n*log(b + c*x)/b**2 - c**2*n*log(b*x + c*x**2)/(2*b**2),
Ne(b, 0)), (-n*log(c)/(2*x**2) - n*log(x)/x**2 - n/(2*x**2) - log(d)/(2*x**
2), True))
```

3.68 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4} dx$

Optimal. Leaf size=86

$$\frac{c^3 n \log(x)}{3b^3} - \frac{c^3 n \log(b+cx)}{3b^3} + \frac{c^2 n}{3b^2 x} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3} - \frac{cn}{6bx^2} - \frac{n}{9x^3}$$

```
[Out] -1/9*n/x^3-1/6*c*n/b/x^2+1/3*c^2*n/b^2/x+1/3*c^3*n*ln(x)/b^3-1/3*c^3*n*ln(c
*x+b)/b^3-1/3*ln(d*(c*x^2+b*x)^n)/x^3
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{c^2 n}{3b^2 x} + \frac{c^3 n \log(x)}{3b^3} - \frac{c^3 n \log(b+cx)}{3b^3} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3} - \frac{cn}{6bx^2} - \frac{n}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -n/(9*x^3) - (c*n)/(6*b*x^2) + (c^2*n)/(3*b^2*x) + (c^3*n*Log[x])/(3*b^3) -
 (c^3*n*Log[b + c*x])/(3*b^3) - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4} dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3} + \frac{1}{3}n \int \frac{b+2cx}{x^4(b+cx)} dx \\
&= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3} + \frac{1}{3}n \int \left(\frac{1}{x^4} + \frac{c}{bx^3} - \frac{c^2}{b^2x^2} + \frac{c^3}{b^3x} - \frac{c^4}{b^3(b+cx)}\right) dx \\
&= -\frac{n}{9x^3} - \frac{cn}{6bx^2} + \frac{c^2 n}{3b^2 x} + \frac{c^3 n \log(x)}{3b^3} - \frac{c^3 n \log(b+cx)}{3b^3} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.90

$$\frac{1}{3}n\left(\frac{c^3 \log(x)}{b^3} - \frac{c^3 \log(b+cx)}{b^3} + \frac{c^2}{b^2 x} - \frac{c}{2bx^2} - \frac{1}{3x^3}\right) - \frac{\log\left(d(x(b+cx))^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] (n*(-1/3*1/x^3 - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
+ c*x])/b^3))/3 - Log[d*(x*(b + c*x))^n]/(3*x^3)
```

fricas [A] time = 0.46, size = 82, normalized size = 0.95

$$-\frac{6\,c^3nx^3\log\left(cx+b\right)-6\,c^3nx^3\log(x)-6\,bc^2nx^2+3\,b^2cnx+6\,b^3n\log\left(cx^2+bx\right)+2\,b^3n+6\,b^3\log(d)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*c^3*n*x^3*log(c*x + b) - 6*c^3*n*x^3*log(x) - 6*b*c^2*n*x^2 + 3*b^
2*c*n*x + 6*b^3*n*log(c*x^2 + b*x) + 2*b^3*n + 6*b^3*log(d))/(b^3*x^3)
```

giac [A] time = 0.16, size = 80, normalized size = 0.93

$$-\frac{c^3n\log\left(cx+b\right)}{3\,b^3}+\frac{c^3n\log(x)}{3\,b^3}-\frac{n\log\left(cx^2+bx\right)}{3\,x^3}+\frac{6\,c^2nx^2-3\,bcnx-2\,b^2n-6\,b^2\log(d)}{18\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*c^3*n*log(c*x + b)/b^3 + 1/3*c^3*n*log(x)/b^3 - 1/3*n*log(c*x^2 + b*x)
/x^3 + 1/18*(6*c^2*n*x^2 - 3*b*c*n*x - 2*b^2*n - 6*b^2*log(d))/(b^2*x^3)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

maxima [A] time = 0.68, size = 75, normalized size = 0.87

$$-\frac{1}{18}\,n\left(\frac{6\,c^3\log\left(cx+b\right)}{b^3}-\frac{6\,c^3\log(x)}{b^3}-\frac{6\,c^2x^2-3\,bcx-2\,b^2}{b^2x^3}\right)-\frac{\log\left(\left(cx^2+bx\right)^nd\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*n*(6*c^3*log(c*x + b)/b^3 - 6*c^3*log(x)/b^3 - (6*c^2*x^2 - 3*b*c*x -
 2*b^2)/(b^2*x^3)) - 1/3*log((c*x^2 + b*x)^n*d)/x^3
```

mupad [B] time = 0.46, size = 68, normalized size = 0.79

$$-\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{3\,x^3}-\frac{\frac{n}{3}-\frac{c^2\,n\,x^2}{b^2}+\frac{c\,n\,x}{2\,b}}{3\,x^3}-\frac{2\,c^3\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^4,x)
```

```
[Out] - log(d*(b*x + c*x^2)^n)/(3*x^3) - (n/3 - (c^2*n*x^2)/b^2 + (c*n*x)/(2*b))/
(3*x^3) - (2*c^3*n*atanh((2*c*x)/b + 1))/(3*b^3)
```

sympy [A] time = 13.29, size = 133, normalized size = 1.55

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{3x^3} - \frac{n}{9x^3} - \frac{\log\left(d\right)}{3x^3} - \frac{cn}{6bx^2} + \frac{c^2n}{3b^2x} - \frac{2c^3n\log\left(b+cx\right)}{3b^3} + \frac{c^3n\log\left(bx+cx^2\right)}{3b^3} & \text{for } b \neq 0 \\ -\frac{n\log\left(c\right)}{3x^3} - \frac{2n\log\left(x\right)}{3x^3} - \frac{2n}{9x^3} - \frac{\log\left(d\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**4,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(3*x**3) - n/(9*x**3) - log(d)/(3*x**3) - c
*n/(6*b*x**2) + c**2*n/(3*b**2*x) - 2*c**3*n*log(b + c*x)/(3*b**3) + c**3*n
*log(b*x + c*x**2)/(3*b**3), Ne(b, 0)), (-n*log(c)/(3*x**3) - 2*n*log(x)/(3
*x**3) - 2*n/(9*x**3) - log(d)/(3*x**3), True))
```

3.69 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal. Leaf size=100

$$-\frac{c^4 n \log(x)}{4b^4}+\frac{c^4 n \log(b+cx)}{4b^4}-\frac{c^3 n}{4b^3 x}+\frac{c^2 n}{8b^2 x^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}-\frac{cn}{12bx^3}-\frac{n}{16x^4}$$

```
[Out] -1/16*n/x^4-1/12*c*n/b/x^3+1/8*c^2*n/b^2/x^2-1/4*c^3*n/b^3/x-1/4*c^4*n*ln(x
)/b^4+1/4*c^4*n*ln(c*x+b)/b^4-1/4*ln(d*(c*x^2+b*x)^n)/x^4
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{c^2 n}{8b^2 x^2}-\frac{c^3 n}{4b^3 x}-\frac{c^4 n \log(x)}{4b^4}+\frac{c^4 n \log(b+cx)}{4b^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}-\frac{cn}{12bx^3}-\frac{n}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -n/(16*x^4) - (c*n)/(12*b*x^3) + (c^2*n)/(8*b^2*x^2) - (c^3*n)/(4*b^3*x) -
(c^4*n*Log[x])/(4*b^4) + (c^4*n*Log[b + c*x])/(4*b^4) - Log[d*(b*x + c*x^2)
^n]/(4*x^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \frac{b+2cx}{x^5(b+cx)}\,dx\\
&= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \left(\frac{1}{x^5}+\frac{c}{bx^4}-\frac{c^2}{b^2x^3}+\frac{c^3}{b^3x^2}-\frac{c^4}{b^4x}+\frac{c^5}{b^4(b+cx)}\right)dx\\
&= -\frac{n}{16x^4}-\frac{cn}{12bx^3}+\frac{c^2 n}{8b^2x^2}-\frac{c^3 n}{4b^3 x}-\frac{c^4 n\log(x)}{4b^4}+\frac{c^4 n\log(b+cx)}{4b^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 87, normalized size = 0.87

$$-\frac{12b^4 \log\left(d(x(b+cx))^n\right) + bn\left(3b^3+4b^2cx-6bc^2x^2+12c^3x^3\right) - 12c^4nx^4\log(b+cx) + 12c^4nx^4\log(x)}{48b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/48*(b*n*(3*b^3 + 4*b^2*c*x - 6*b*c^2*x^2 + 12*c^3*x^3) + 12*c^4*n*x^4*Lo
g[x] - 12*c^4*n*x^4*Log[b + c*x] + 12*b^4*Log[d*(x*(b + c*x))^n])/(b^4*x^4)
```

fricas [A] time = 0.46, size = 94, normalized size = 0.94

$$\frac{12\,c^4nx^4\log(cx+b) - 12\,c^4nx^4\log(x) - 12\,bc^3nx^3 + 6\,b^2c^2nx^2 - 4\,b^3cnx - 12\,b^4n\log\left(cx^2+bx\right) - 3\,b^4n - 12}{48\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(12*c^4*n*x^4*log(c*x + b) - 12*c^4*n*x^4*log(x) - 12*b*c^3*n*x^3 + 6*
b^2*c^2*n*x^2 - 4*b^3*c*n*x - 12*b^4*n*log(c*x^2 + b*x) - 3*b^4*n - 12*b^4*
log(d))/(b^4*x^4)
```

giac [A] time = 0.16, size = 92, normalized size = 0.92

$$\frac{c^4n\log(cx+b)}{4\,b^4} - \frac{c^4n\log(x)}{4\,b^4} - \frac{n\log\left(cx^2+bx\right)}{4\,x^4} - \frac{12\,c^3nx^3 - 6\,bc^2nx^2 + 4\,b^2cnx + 3\,b^3n + 12\,b^3\log(d)}{48\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^4*n*log(c*x + b)/b^4 - 1/4*c^4*n*log(x)/b^4 - 1/4*n*log(c*x^2 + b*x)/
x^4 - 1/48*(12*c^3*n*x^3 - 6*b*c^2*n*x^2 + 4*b^2*c*n*x + 3*b^3*n + 12*b^3*l
og(d))/(b^3*x^4)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

maxima [A] time = 0.62, size = 86, normalized size = 0.86

$$\frac{1}{48}\,n\left(\frac{12\,c^4\log(cx+b)}{b^4} - \frac{12\,c^4\log(x)}{b^4} - \frac{12\,c^3x^3 - 6\,bc^2x^2 + 4\,b^2cx + 3\,b^3}{b^3x^4}\right) - \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*n*(12*c^4*log(c*x + b)/b^4 - 12*c^4*log(x)/b^4 - (12*c^3*x^3 - 6*b*c^2
*x^2 + 4*b^2*c*x + 3*b^3)/(b^3*x^4)) - 1/4*log((c*x^2 + b*x)^n*d)/x^4
```

mupad [B] time = 0.49, size = 79, normalized size = 0.79

$$\frac{c^4\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{2\,b^4}-\frac{\ln\left(d\,\left(c\,x^2+b\,x\right)^n\right)}{4\,x^4}-\frac{\frac{n}{4}-\frac{c^2\,n\,x^2}{2\,b^2}+\frac{c^3\,n\,x^3}{b^3}+\frac{c\,n\,x}{3\,b}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (c^4*n*atanh((2*c*x)/b + 1))/(2*b^4) - log(d*(b*x + c*x^2)^n)/(4*x^4) - (n/
4 - (c^2*n*x^2)/(2*b^2) + (c^3*n*x^3)/b^3 + (c*n*x)/(3*b))/(4*x^4)
```

sympy [A] time = 24.25, size = 141, normalized size = 1.41

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{4x^4}-\frac{n}{16x^4}-\frac{\log\left(d\right)}{4x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}+\frac{c^4n\log\left(b+cx\right)}{2b^4}-\frac{c^4n\log\left(bx+cx^2\right)}{4b^4} & \text{for } b\neq 0 \\ -\frac{n\log\left(c\right)}{4x^4}-\frac{n\log\left(x\right)}{2x^4}-\frac{n}{8x^4}-\frac{\log\left(d\right)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**5,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(4*x**4) - n/(16*x**4) - log(d)/(4*x**4) -
c*n/(12*b*x**3) + c**2*n/(8*b**2*x**2) - c**3*n/(4*b**3*x) + c**4*n*log(b +
 c*x)/(2*b**4) - c**4*n*log(b*x + c*x**2)/(4*b**4), Ne(b, 0)), (-n*log(c)/(
4*x**4) - n*log(x)/(2*x**4) - n/(8*x**4) - log(d)/(4*x**4), True))
```

3.70 $\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)dx$

Optimal. Leaf size=157

$$-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x^{m+1}\log\left(d\left(a+bx\right.\right.}{m+1}$$

```
[Out] x^(1+m)*ln(d*(c*x^2+b*x+a)^n)/(1+m)-2*c*n*x^(2+m)*hypergeom([1, 2+m],[3+m],
-2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b-(-4*a*c+b^2)^(1/2))-2*c*n*x^(
2+m)*hypergeom([1, 2+m],[3+m],-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b
+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2525, 830, 64}

$$-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x^{m+1}\log\left(d\left(a+bx\right.\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*c*n*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) - (2*c*n*x^(2 + m)*H
ypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b +
Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^{1+m}(b+2cx)}{a+bx+cx^2}\,dx}{1+m}$$
$$= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \left(\frac{2cx^{1+m}}{b-\sqrt{b^2-4ac}+2cx} + \frac{2cx^{1+m}}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{1+m}$$
$$= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{(2cn)\int \frac{x^{1+m}}{b-\sqrt{b^2-4ac}+2cx}\,dx}{1+m} - \frac{(2cn)\int \frac{x^{1+m}}{b+\sqrt{b^2-4ac}+}}{1+m}$$
$$= -\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} - \frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;\right.}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(}$$

Mathematica [A] time = 0.19, size = 137, normalized size = 0.87

$$-\frac{x^{m+1}\left(nx\left(\sqrt{b^2-4ac}+b\right){}_2F_1\left(1,m+2;m+3;\frac{2cx}{\sqrt{b^2-4ac}-b}\right)+nx\left(b-\sqrt{b^2-4ac}\right){}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-}}\right.}{2a\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/2*(x^(1 + m)*((b + Sqrt[b^2 - 4*a*c])*n*x*Hypergeometric2F1[1, 2 + m, 3
+ m, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*n*x*Hyperg
eometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])] - 2*a*(2 + m
)*Log[d*(a + x*(b + c*x))^n]))/(a*(2 + 3*m + m^2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\log\left(\left(cx^2+bx+a\right)^n d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\log\left(\left(cx^2+bx+a\right)^n d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int x^m\ln\left(d\left(c\,x^2+bx+a\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^m\log\left(\left(cx^2+bx+a\right)^n\right)}{m+1}+\int\frac{\left(\left((m+1)\log(d)-2\,n\right)cx^2+\left((m+1)\log(d)-n\right)bx+a(m+1)\log(d)\right)x^m}{c(m+1)x^2+b(m+1)x+a(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] x*x^m*log((c*x^2 + b*x + a)^n)/(m + 1) + integrate((((m + 1)*log(d) - 2*n)*
c*x^2 + ((m + 1)*log(d) - n)*b*x + a*(m + 1)*log(d))*x^m/(c*(m + 1)*x^2 + b
*(m + 1)*x + a*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] int(x^m*log(d*(a + b*x + c*x^2)^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.71 $\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=207

$$\frac{bn\left(5a^2c^2-5ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{10c^5}+\frac{n\sqrt{b^2-4ac}\,\left(a^2c^2-3ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5}-\frac{nx\left(2a^2c^2-4ab^2\right.}{5c^4}$$

```
[Out] -1/5*(2*a^2*c^2-4*a*b^2*c+b^4)*n*x/c^4+1/10*b*(-3*a*c+b^2)*n*x^2/c^3-1/15*(
-2*a*c+b^2)*n*x^3/c^2+1/20*b*n*x^4/c-2/25*n*x^5+1/10*b*(5*a^2*c^2-5*a*b^2*c
+b^4)*n*ln(c*x^2+b*x+a)/c^5+1/5*x^5*ln(d*(c*x^2+b*x+a)^n)+1/5*(a^2*c^2-3*a*
b^2*c+b^4)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A] time = 0.23, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{bn\left(5a^2c^2-5ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{10c^5}-\frac{nx\left(2a^2c^2-4ab^2c+b^4\right)}{5c^4}+\frac{n\sqrt{b^2-4ac}\,\left(a^2c^2-3ab^2c+b^4\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{5c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x)/(5*c^4) + (b*(b^2 - 3*a*c)*n*x^2)/(10*
c^3) - ((b^2 - 2*a*c)*n*x^3)/(15*c^2) + (b*n*x^4)/(20*c) - (2*n*x^5)/25 + (
Sqrt[b^2 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(5*c^5) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + b*x + c*x
^2])/(10*c^5) + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{5}n\int \frac{x^5(b+2cx)}{a+bx+cx^2}\,dx$$

$$= \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{5}n\int\left(\frac{b^4-4ab^2c+2a^2c^2}{c^4} - \frac{b\left(b^2-3ac\right)x}{c^3}\right.$$

$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - $$

$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - $$

$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - $$

$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - $$

Mathematica [A] time = 0.22, size = 190, normalized size = 0.92

$$\frac{30bn\left(5a^2c^2-5ab^2c+b^4\right)\log(a+x(b+cx)) + 60n\sqrt{b^2-4ac}\,\left(a^2c^2-3ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + cnx\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c*(-12*a + c*x^2) + 15*b*c^2*x*(-6*a
+ c*x^2) - 8*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)) + 60*Sqrt[b^2 - 4*a*c]*(
b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 30*b*
(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + x*(b + c*x)] + 60*c^5*x^5*Log[d*(a
+ x*(b + c*x))^n])/(300*c^5)
```

fricas [A] time = 0.47, size = 444, normalized size = 2.14

$$\left[-\frac{24\,c^5nx^5 - 60\,c^5x^5\log(d) - 15\,bc^4nx^4 + 20\left(b^2c^3-2\,ac^4\right)nx^3 - 30\left(b^3c^2-3\,abc^3\right)nx^2 - 30\left(b^4-3\,ab^2c+a^2c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4 + 20*(b^2*c^3 -
2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 30*(b^4 - 3*a*b^2*c + a^2
*c^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*n*x - 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2
+ b*x + a))/c^5, -1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4
+ 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 60*(b^4 -
 3*a*b^2*c + a^2*c^2)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x - 30*(2*c^
5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2 + b*x + a))/c^5]
```

giac [A] time = 0.20, size = 221, normalized size = 1.07

$$\frac{1}{5}\,nx^5\log\left(cx^2+bx+a\right)-\frac{1}{25}\left(2\,n-5\,\log(d)\right)x^5+\frac{bnx^4}{20\,c}-\frac{\left(b^2n-2\,acn\right)x^3}{15\,c^2}+\frac{\left(b^3n-3\,abcn\right)x^2}{10\,c^3}-\frac{\left(b^4n-4\,ab^2cn+2\,\right.}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x + a) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c
 - 1/15*(b^2*n - 2*a*c*n)*x^3/c^2 + 1/10*(b^3*n - 3*a*b*c*n)*x^2/c^3 - 1/5*
(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*x/c^4 + 1/10*(b^5*n - 5*a*b^3*c*n + 5*a
^2*b*c^2*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n - 7*a*b^4*c*n + 13*a^2*b^
2*c^2*n - 4*a^3*c^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^5)
```

maple [C] time = 0.81, size = 1621, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/5*x^5*ln((c*x^2+b*x+a)^n)+1/10*I*Pi*x^5*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^
n)^2+1/10*I*Pi*x^5*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/10
*I*Pi*x^5*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/10*I*Pi*x^5*csgn(I*d)*csgn(I*(c*x^2
+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/5*ln(d)*x^5-2/25*n*x^5+1/20*b*n*x^4/
c+2/15/c*a*n*x^3-1/15*b^2*n*x^3/c^2-3/10/c^2*a*b*n*x^2+1/10*b^3*n*x^2/c^3+1
/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b^2
*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5+2
5*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a^2*b
-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b
^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5
+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a*b
^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
b^5+1/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
a^2*b-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*
a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^
5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b
)*a*b^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+
25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4
*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2
)*b)*b^5-2/5/c^2*a^2*n*x+4/5/c^3*a*b^2*n*x-1/5*b^4*n*x/c^4-1/10/c^5*n*ln(4*
a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^
```

```
4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5+25*a^4*b^2*c^4-
50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^4*
b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)+1/10/c^5*n*ln(
4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*
b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*c^5+25*a^4*b^2*c^
4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 0.59, size = 395, normalized size = 1.91

$$x^2\left(\frac{b\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{2c}-\frac{abn}{10c^2}\right)-\frac{2nx^5}{25}+x\left(\frac{a\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{b\left(\frac{b\left(\frac{b^2 n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{abn}{5c^2}\right)}{c}\right)+\frac{x^5\ln\left(d\left(cx^2+bx+a\right)^n\right)}{5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x^2*((b*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/(2*c) - (a*b*n)/(10*c^2)) - (2*n
*x^5)/25 + x*((a*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/c - (b*((b*((b^2*n)/(5*
c^2) - (2*a*n)/(5*c)))/c - (a*b*n)/(5*c^2)))/c) + (x^5*log(d*(a + b*x + c*x
^2)^n))/5 - x^3*((b^2*n)/(15*c^2) - (2*a*n)/(15*c)) + (log(b*(b^2 - 4*a*c)^
(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^5*n)/10 + c^2*((a^2*n*
(b^2 - 4*a*c)^(1/2))/10 + (a^2*b*n)/2) - c*((a*b^3*n)/2 + (3*a*b^2*n*(b^2 -
 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 - (log(4*a*c + b*
(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c^2*((a^2*n*(b^2 -
4*a*c)^(1/2))/10 - (a^2*b*n)/2) - (b^5*n)/10 + c*((a*b^3*n)/2 - (3*a*b^2*n*
(b^2 - 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 + (b*n*x^4)
/(20*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.72 $\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=167

$$-\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8c^4}-\frac{bn\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}+\frac{bnx\left(b^2-3ac\right)}{4c^3}-\frac{nx^2\left(b^2\right.}{8c}$$

```
[Out] 1/4*b*(-3*a*c+b^2)*n*x/c^3-1/8*(-2*a*c+b^2)*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*
x^4-1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x^2+b*x+a)/c^4+1/4*x^4*ln(d*(c*x^2
+b*x+a)^n)-1/4*b*(-2*a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a
*c+b^2)^(1/2)/c^4
```

Rubi [A] time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8c^4}-\frac{nx^2\left(b^2-2ac\right)}{8c^2}+\frac{bnx\left(b^2-3ac\right)}{4c^3}-\frac{bn\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b}{\sqrt{b}}\right.}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*(b^2 - 3*a*c)*n*x)/(4*c^3) - ((b^2 - 2*a*c)*n*x^2)/(8*c^2) + (b*n*x^3)/(
12*c) - (n*x^4)/8 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*
x + c*x^2])/(8*c^4) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int \frac{x^4(b+2cx)}{a+bx+cx^2}\,dx$$

$$= \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int\left(-\frac{b\left(b^2-3ac\right)}{c^3} + \frac{\left(b^2-2ac\right)x}{c^2} - \frac{bx^2}{c}\right.$$

$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right.\right.$$

$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right.\right.$$

$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)n\,\mathrm{lo}}{8c^4}$$

$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{b\sqrt{b^2-4ac}\,\left(b^2-2ac\right)n}{4c^4}$$

Mathematica [A] time = 0.16, size = 151, normalized size = 0.90

$$\frac{-3n\left(2a^2c^2-4ab^2c+b^4\right)\log(a+x(b+cx)) - 6bn\sqrt{b^2-4ac}\,\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + cnx\left(2bc\left(cx^2-9a\right.\right.}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c*(-9*a + c*x^2) - 3*c^2*x*(-2*a + c*x^2))
- 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + x*(b + c*x)] + 6*c^4*x^4*Lo
g[d*(a + x*(b + c*x))^n])/(24*c^4)
```

fricas [A] time = 0.48, size = 364, normalized size = 2.18

$$\left[-\frac{3\,c^4nx^4 - 6\,c^4x^4\log(d) - 2\,bc^3nx^3 + 3\left(b^2c^2-2\,ac^3\right)nx^2 + 3\left(b^3-2\,abc\right)\sqrt{b^2-4\,ac}\,n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+}{cx^2+bx}\right.}{24\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c
^3)*n*x^2 + 3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x
```

```
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*(b^3*
c - 3*a*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(
c*x^2 + b*x + a))/c^4, -1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^
3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 + 6*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 6*(b^3*c - 3*a*b*c^2)
*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(c*x^2 + b*x +
a))/c^4]
```

giac [A] time = 0.20, size = 176, normalized size = 1.05

$$\frac{1}{4}nx^4\log\left(cx^2+bx+a\right)-\frac{1}{8}\left(n-2\log(d)\right)x^4+\frac{bnx^3}{12c}-\frac{\left(b^2n-2acn\right)x^2}{8c^2}+\frac{\left(b^3n-3abcn\right)x}{4c^3}-\frac{\left(b^4n-4ab^2cn+2a^2c^2\right.}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x + a) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c -
1/8*(b^2*n - 2*a*c*n)*x^2/c^2 + 1/4*(b^3*n - 3*a*b*c*n)*x/c^3 - 1/8*(b^4*n
- 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n - 6*a*b^
3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^4)
```

maple [C] time = 0.61, size = 1146, normalized size = 6.86

$$-\frac{i\pi x^4\mathrm{csgn}(id)\,\mathrm{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)}{8}+\frac{i\pi x^4\mathrm{csgn}(id)\,\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)^2}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/4*x^4*ln((c*x^2+b*x+a)^n)+1/8*I*Pi*x^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n
)^2-1/8*I*Pi*x^4*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/8*I*Pi*x^4*csgn(I*d)*csgn(I*
(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/8*I*Pi*x^4*csgn(I*(c*x^2+b*x+a
)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/4*ln(d)*x^4-1/8*n*x^4+1/12*b*n*x^3/c+1/4
/c*a*n*x^2-1/8*b^2*n*x^2/c^2-1/4/c^2*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5+2*(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^3+20*a^2
*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5
+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^
3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/8/c^4*n*ln(8*a^2*b*c^2-6*a
*b^3*c+b^5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*b^4-1/4/c^2*n*ln(8*a^2*b
*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c
*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln
(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)
^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/
8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*
b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b
)*b^4-3/4/c^2*a*b*n*x+1/4*b^3*n*x/c^3-1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^
5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8
*a*b^6*c+b^8)^(1/2)+1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-
8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.54, size = 288, normalized size = 1.72

$$x\left(\frac{b\left(\frac{b^2 n}{4c^2}-\frac{a n}{2c}\right)}{c}-\frac{a b n}{4c^2}\right)-\frac{n x^4}{8}+\frac{x^4 \ln\left(d\left(c x^2+b x+a\right)^n\right)}{4}-x^2\left(\frac{b^2 n}{8c^2}-\frac{a n}{4c}\right)+\frac{\ln\left(4 a c+b\sqrt{b^2-4 a c}-b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*((b*((b^2*n)/(4*c^2) - (a*n)/(2*c)))/c - (a*b*n)/(4*c^2)) - (n*x^4)/8 + (
x^4*log(d*(a + b*x + c*x^2)^n))/4 - x^2*((b^2*n)/(8*c^2) - (a*n)/(4*c)) + (
log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c*((a
*b^2*n)/2 - (a*b*n*(b^2 - 4*a*c)^(1/2))/4) - (b^4*n)/8 + (b^3*n*(b^2 - 4*a*
c)^(1/2))/8 - (a^2*c^2*n)/4))/c^4 - (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^
2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - c*((a*b^2*n)/2 + (a*b*n*(b^2 -
4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4))/c^4 + (b
*n*x^3)/(12*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.73 $\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=136

$$\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6c^3}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}-\frac{nx\left(b^2-2ac\right)}{3c^2}+\frac{1}{3}x^3\log\left(d\left(a+bx+c\right.\right.$$

```
[Out] -1/3*(-2*a*c+b^2)*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/6*b*(-3*a*c+b^2)*n*ln(c
*x^2+b*x+a)/c^3+1/3*x^3*ln(d*(c*x^2+b*x+a)^n)+1/3*(-a*c+b^2)*n*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^3
```

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6c^3}-\frac{nx\left(b^2-2ac\right)}{3c^2}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(a+bx+c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((b^2 - 2*a*c)*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (Sqrt[b^2 -
4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) + (b*(
b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*c^3) + (x^3*Log[d*(a + b*x + c*x^2)
^n])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int\frac{x^3(b+2cx)}{a+bx+cx^2}dx$$

$$= \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int\left(\frac{b^2-2ac}{c^2} - \frac{bx}{c} + 2x^2 - \frac{a\left(b^2-2ac\right)+}{c^2\left(a+b\right.}\right.$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{n\int\frac{a\left(b^2-2a\right.}{a}}{}$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b\left(b^2-3\right.\right.}{}$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{b\left(b^2-3ac\right)n\log\left(a+bx+cx^2\right)}{6c^3} + \frac{1}{3}x^3\,\mathrm{l}$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{\sqrt{b^2-4ac}\left(b^2-ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}$$

Mathematica [A] time = 0.14, size = 122, normalized size = 0.90

$$\frac{cnx\left(-4c\left(cx^2-3a\right)-6b^2+3bcx\right)+3bn\left(b^2-3ac\right)\log(a+x(b+cx))+6n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-4}}\right.}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c*(-3*a + c*x^2)) + 6*Sqrt[b^2 - 4*a*c]*(b^2 -
 a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*b*(b^2 - 3*a*c)*n*Log[a
+ x*(b + c*x)] + 6*c^3*x^3*Log[d*(a + x*(b + c*x))^n])/(18*c^3)
```

fricas [A] time = 0.47, size = 299, normalized size = 2.20

$$\left[-\frac{4c^3nx^3-6c^3x^3\log(d)-3bc^2nx^2+3\left(b^2-ac\right)\sqrt{b^2-4ac}\,n\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+6\left(b^2c\right.}{18c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*n*x^3 - 6*c^3*x^3*log(d) - 3*b*c^2*n*x^2 + 3*(b^2 - a*c)*sqrt
(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2*c^3*n*x^3
+ (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3, -1/18*(4*c^3*n*x^3 - 6*c^3*
x^3*log(d) - 3*b*c^2*n*x^2 - 6*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqr
```

```
t(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2
*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3]
```

giac [A] time = 0.27, size = 146, normalized size = 1.07

$$\frac{1}{3}nx^3\log\left(cx^2+bx+a\right)-\frac{1}{9}\left(2n-3\log(d)\right)x^3+\frac{bnx^2}{6c}-\frac{\left(b^2n-2acn\right)x}{3c^2}+\frac{\left(b^3n-3abcn\right)\log\left(cx^2+bx+a\right)}{6c^3}-\frac{\left(b^4n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x + a) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c -
 1/3*(b^2*n - 2*a*c*n)*x/c^2 + 1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)
/c^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [C] time = 0.64, size = 870, normalized size = 6.40

$$-\frac{i\pi x^3\mathrm{csgn}\left(id\right)\mathrm{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)}{6}+\frac{i\pi x^3\mathrm{csgn}\left(id\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)^2}{6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/3*x^3*ln((c*x^2+b*x+a)^n)+1/6*I*Pi*x^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(
c*x^2+b*x+a)^n)^2-1/6*I*Pi*x^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(
c*x^2+b*x+a)^n)-1/6*I*Pi*x^3*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/6*I*Pi*x^3*csgn(
I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/3*ln(d)*x^3-2/9*n*x^3+1/6*b*n*x^2/c-1/2/
c^2*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6
)^(1/2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n
*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/
2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3-1/2/c^2*n*ln(-
4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*
x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n*ln(-4*a^2
*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x-(-4
*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3+2/3/c*a*n*x-1/3*b^2*n*x/
c^2-1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b
^4*c+b^6)^(1/2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a
^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)+1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c
-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x-(-4*a^3*c^3+9*a^2
*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.50, size = 229, normalized size = 1.68

$$\frac{x^3\ln\left(d\left(cx^2+bx+a\right)^n\right)}{3}-\frac{2nx^3}{9}-x\left(\frac{b^2n}{3c^2}-\frac{2an}{3c}\right)-\frac{\ln\left(4ac+b\sqrt{b^2-4ac}-b^2+2cx\sqrt{b^2-4ac}\right)\left(c\left(\frac{abn}{2}-\right.\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] (x^3*log(d*(a + b*x + c*x^2)^n))/3 - (2*n*x^3)/9 - x*((b^2*n)/(3*c^2) - (2*
a*n)/(3*c)) - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c
)^(1/2))*(c*((a*b*n)/2 - (a*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*
(b^2 - 4*a*c)^(1/2))/6))/c^3 + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2
*c*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - c*((a*b*n)/2 + (a*n*(b^2 - 4*a*c)^(1
/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (b*n*x^2)/(6*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.74 $\int x \log \left(d \left(a+bx+cx^2\right)^n\right) \, dx$

Optimal. Leaf size=109

$$-\frac{n \left(b^2-2ac\right) \log \left(a+bx+cx^2\right)}{4c^2}-\frac{bn\sqrt{b^2-4ac} \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{1}{2}x^2 \log \left(d \left(a+bx+cx^2\right)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

```
[Out] 1/2*b*n*x/c-1/2*n*x^2-1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a)/c^2+1/2*x^2*ln(d*(
c*x^2+b*x+a)^n)-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^
(1/2)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n \left(b^2-2ac\right) \log \left(a+bx+cx^2\right)}{4c^2}-\frac{bn\sqrt{b^2-4ac} \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{1}{2}x^2 \log \left(d \left(a+bx+cx^2\right)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(2*c^2) - ((b^2 - 2*a*c)*n*Log[a + b*x + c*x^2])/(4*c^2) +
(x^2*Log[d*(a + b*x + c*x^2)^n])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(d\left(a+bx+cx^2\right)^n\right)dx &= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x^2(b+2cx)}{a+bx+cx^2}dx\\ &= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{ab+\left(b^2-2ac\right)x}{c\left(a+bx+cx^2\right)}\right)dx\\ &= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{ab+\left(b^2-2ac\right)x}{a+bx+cx^2}dx}{2c}\\ &= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(b\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}dx}{4c^2}-\\ &= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\\ &= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+c\right.}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 94, normalized size = 0.86

$$-\frac{n\left(b^2-2ac\right)\log(a+x(b+cx))+2bn\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2cx\left(cx\log\left(d(a+x(b+cx))^n\right)+n(b-c\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/4*(2*b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + (b^2
 - 2*a*c)*n*Log[a + x*(b + c*x)] - 2*c*x*(n*(b - c*x) + c*x*Log[d*(a + x*(b
 + c*x))^n]))/c^2
```

fricas [A] time = 0.48, size = 245, normalized size = 2.25

$$\left[-\frac{2\,c^2nx^2-2\,c^2x^2\log(d)-2\,bcnx-\sqrt{b^2-4\,ac}\,bn\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-\left(2\,c^2nx^2-\left(b^2-2\,\right.\right.}{4\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x - sqrt(b^2 - 4*a*c)*b*n*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x
^2 + b*x + a)) - (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2,
 -1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x + 2*sqrt(-b^2 + 4*a*c)*b*
n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c^2*n*x^2 - (b
^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2]
```

giac [A] time = 0.27, size = 113, normalized size = 1.04

$$\frac{1}{2}nx^2\log\left(cx^2+bx+a\right)-\frac{1}{2}\left(n-\log(d)\right)x^2+\frac{bnx}{2c}-\frac{\left(b^2n-2acn\right)\log\left(cx^2+bx+a\right)}{4c^2}+\frac{\left(b^3n-4abcn\right)\arctan\left(\frac{2cx}{\sqrt{-b^2}}\right.}{2\sqrt{-b^2+4ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x + a) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/4*(
b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/c^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [C] time = 0.68, size = 510, normalized size = 4.68

$$-\frac{i\pi x^2\operatorname{csgn}(id)\operatorname{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\operatorname{csgn}\left(id\left(cx^2+bx+a\right)^n\right)}{4}+\frac{i\pi x^2\operatorname{csgn}(id)\operatorname{csgn}\left(id\left(cx^2+bx+a\right)^n\right)^2}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/2*x^2*ln((c*x^2+b*x+a)^n)-1/4*I*Pi*x^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*
csgn(I*d*(c*x^2+b*x+a)^n)+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*d)*x^2*P
i+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*x^2+b*x+a)^n)*x^2*Pi-1/4*I*Pi
*x^2*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/2*ln(d)*x^2-1/2*n*x^2+1/2/c*n*ln(-2*(-4*
a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*a-1/4/c^2*n*ln
(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*b^2+1/
2/c*n*ln(2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)
*a-1/4/c^2*n*ln(2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(
1/2)*b)*b^2+1/2*b*n*x/c+1/4/c^2*n*ln(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+
b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^(1/2)-1/4/c^2*n*ln(2*(-4*a*b
^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^
(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.55, size = 166, normalized size = 1.52

$$\frac{x^2\ln\left(d\left(cx^2+bx+a\right)^n\right)}{2}-\frac{nx^2}{2}-\frac{\ln\left(b\sqrt{b^2-4ac}-4ac+b^2+2cx\sqrt{b^2-4ac}\right)\left(b^2n-2acn+bn\sqrt{b^2-4a\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(a + b*x + c*x^2)^n))/2 - (n*x^2)/2 - (log(b*(b^2 - 4*a*c)^(1/2)
 - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4
*a*c)^(1/2)))/(4*c^2) + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b
```

```
^2 - 4*a*c)^(1/2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*c^2) + (
b*n*x)/(2*c)
```

sympy [A] time = 165.46, size = 369, normalized size = 3.39

$$\begin{cases} -\frac{b^2 n \log\left(\frac{b^2}{4c}+bx+cx^2\right)}{8c^2} + \frac{bnx}{2c} + \frac{nx^2 \log\left(\frac{b^2}{4c}+bx+cx^2\right)}{2} - \frac{nx^2}{2} + \frac{x^2 \log(d)}{2} \\ -\frac{a^2 n \log(a+bx)}{2b^2} + \frac{anx}{2b} + \frac{nx^2 \log(a+bx)}{2} - \frac{nx^2}{4} + \frac{x^2 \log(d)}{2} \\ -\frac{abn \log\left(a+bx+cx^2\right)}{c\sqrt{-4ac+b^2}} + \frac{2abn \log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} + \frac{an \log\left(a+bx+cx^2\right)}{2c} + \frac{b^3 n \log\left(a+bx+cx^2\right)}{4c^2\sqrt{-4ac+b^2}} - \frac{b^3 n \log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2\sqrt{-4ac+b^2}} - \frac{b^2 n \log}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((-b**2*n*log(b**2/(4*c) + b*x + c*x**2)/(8*c**2) + b*n*x/(2*c) +
n*x**2*log(b**2/(4*c) + b*x + c*x**2)/2 - n*x**2/2 + x**2*log(d)/2, Eq(a, b
**2/(4*c))), (-a**2*n*log(a + b*x)/(2*b**2) + a*n*x/(2*b) + n*x**2*log(a +
b*x)/2 - n*x**2/4 + x**2*log(d)/2, Eq(c, 0)), (-a*b*n*log(a + b*x + c*x**2)
/(c*sqrt(-4*a*c + b**2)) + 2*a*b*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2
*c))/(c*sqrt(-4*a*c + b**2)) + a*n*log(a + b*x + c*x**2)/(2*c) + b**3*n*log
(a + b*x + c*x**2)/(4*c**2*sqrt(-4*a*c + b**2)) - b**3*n*log(b/(2*c) + x +
sqrt(-4*a*c + b**2)/(2*c))/(2*c**2*sqrt(-4*a*c + b**2)) - b**2*n*log(a + b*
x + c*x**2)/(4*c**2) + b*n*x/(2*c) + n*x**2*log(a + b*x + c*x**2)/2 - n*x**
2/2 + x**2*log(d)/2, True))
```

3.75 $\int \log\left(d\left(a+bx+cx^2\right)^n\right)dx$

Optimal. Leaf size=79

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2523, 773, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(d\left(a+bx+cx^2\right)^n\right)dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right) - n\int \frac{x(b+2cx)}{a+bx+cx^2}dx \\
&= -2nx + x\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{n\int \frac{-2ac-bcx}{a+bx+cx^2}dx}{c} \\
&= -2nx + x\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}dx}{2c} - \frac{\left(\left(b^2-4ac\right)n\right)\int \frac{}{a+bx}
\\
&= -2nx + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + x\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(\left(b^2-4ac\right)n\right)\text{Su}
\\
&= -2nx + \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + x\log\left(d\left(a-\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.99

$$\frac{2n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2cx\left(\log\left(d(a+x(b+cx))^n\right)-2n\right)+bn\log(a+x(b+cx))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

fricas [A] time = 0.48, size = 190, normalized size = 2.41

$$\left[-\frac{4\,cnx-2\,cx\log(d)-\sqrt{b^2-4\,ac}\,n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

giac [A] time = 0.19, size = 92, normalized size = 1.16

$$nx\log\left(cx^2+bx+a\right)-\left(2\,n-\log(d)\right)x+\frac{bn\log\left(cx^2+bx+a\right)}{2\,c}-\frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

maple [A] time = 0.08, size = 118, normalized size = 1.49

$$\frac{4an\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{b^2n\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{bn\ln\left(cx^2+bx+a\right)}{2c}-2nx+x\ln\left(d\left(cx^2+bx+a\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+4*n/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a-n/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^2/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.34, size = 120, normalized size = 1.52

$$x\,\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)-2\,n\,x-\frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)}-\frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\sqrt{4\,a\,c-b^2}}{c}+\frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

sympy [A] time = 69.12, size = 275, normalized size = 3.48

$$\begin{cases} \frac{bn\log\left(\frac{b^2}{4c}+bx+cx^2\right)}{2c}+nx\log\left(\frac{b^2}{4c}+bx+cx^2\right)-2nx+x\log\left(d\right) \\ \frac{an\log\left(a+bx\right)}{b}+nx\log\left(a+bx\right)-nx+x\log\left(d\right) \\ \frac{2an\log\left(a+bx+cx^2\right)}{\sqrt{-4ac+b^2}}-\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}}-\frac{b^2n\log\left(a+bx+cx^2\right)}{2c\sqrt{-4ac+b^2}}+\frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+nx\log\left(a+\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((b*n*log(b**2/(4*c) + b*x + c*x**2)/(2*c) + n*x*log(b**2/(4*c) +
b*x + c*x**2) - 2*n*x + x*log(d), Eq(a, b**2/(4*c))), (a*n*log(a + b*x)/b +
 n*x*log(a + b*x) - n*x + x*log(d), Eq(c, 0)), (2*a*n*log(a + b*x + c*x**2)
/sqrt(-4*a*c + b**2) - 4*a*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/s
qrt(-4*a*c + b**2) - b**2*n*log(a + b*x + c*x**2)/(2*c*sqrt(-4*a*c + b**2))
 + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-4*a*c + b**
2)) + b*n*log(a + b*x + c*x**2)/(2*c) + n*x*log(a + b*x + c*x**2) - 2*n*x +
 x*log(d), True))
```

3.76 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$

Optimal. Leaf size=129

$$-n\text{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\text{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)-n\log(x)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+\ldots}\right.$$

```
[Out] ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))-n*ln
(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))-n*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(
1/2)))-n*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2524, 2357, 2317, 2391}

$$-n\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\text{PolyLog}\left(2,-\frac{2cx}{\sqrt{b^2-4ac}+b}\right)-n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)-n\log(x)\log\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) - n*Log[x]*Log[1 + (2*
c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*PolyL
og[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx = \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - n\int \frac{(b+2cx)\log(x)}{a+bx+cx^2}\,dx$$
$$= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - n\int \left(\frac{2c\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log(x)}{b+\sqrt{b^2-4ac}$$
$$= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - (2cn)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx - (2cn)\int$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) + \log$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) + \log$$

Mathematica [A] time = 0.21, size = 156, normalized size = 1.21

$$-n\mathrm{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\mathrm{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(\frac{\sqrt{b^2-}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) -
n*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log
[x]*Log[d*(a + x*(b + c*x))^n] - n*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*
c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

maple [C] time = 0.34, size = 315, normalized size = 2.44

$$-\frac{i\pi\,\mathrm{csgn}(id)\,\mathrm{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)\ln(x)}{2}+\frac{i\pi\,\mathrm{csgn}(id)\,\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x,x)
```

```
[Out] ln(x)*ln((c*x^2+b*x+a)^n)-ln(x)*ln((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*
c+b^2)^(1/2)))*n-ln(x)*ln((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2
)))*n-dilog((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2)))*n-dilog(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))*n-1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/2*I*ln(x)*Pi*csgn(I*(c*x^2+b*x+a)^n)*csg
n(I*d*(c*x^2+b*x+a)^n)^2-1/2*I*ln(x)*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(x)*l
n(d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x,x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)/x, x)
```

3.77 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

```
[Out] b*n*ln(x)/a-1/2*b*n*ln(c*x^2+b*x+a)/a-ln(d*(c*x^2+b*x+a)^n)/x+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*n*Log[x
])/a - (b*n*Log[a + b*x + c*x^2])/(2*a) - Log[d*(a + b*x + c*x^2)^n]/x
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+n\int \frac{b+2cx}{x\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+n\int \left(\frac{b}{ax}+\frac{-b^2+2ac-bcx}{a\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{bn\log(x)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+\frac{n\int \frac{-b^2+2ac-bcx}{a+bx+cx^2}\,dx}{a}$$

$$= \frac{bn\log(x)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{\left(\left(b^2-4ac\right)n\right)\int \frac{}{a+bx}$$

$$= \frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+\frac{\left(\left(b^2-4ac\right)n\right)\mathrm{Su}}{}$$

$$= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}+\frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d\left(a+\right.\right.}{}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 1.01

$$\frac{2n\sqrt{4ac-b^2}\,\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)-\frac{2a\log\left(d(a+x(b+cx))^n\right)}{x}-bn\log(a+x(b+cx))+2bn\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*b*n*Log[
x] - b*n*Log[a + x*(b + c*x)] - (2*a*Log[d*(a + x*(b + c*x))^n])/x)/(2*a)
```

fricas [A] time = 0.50, size = 199, normalized size = 2.31

$$\left[\frac{2\,bnx\log(x)+\sqrt{b^2-4\,ac}\,nx\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(bnx+2\,an)\log\left(cx^2+bx+a\right)-2\,a\log(d}{2\,ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*n*x*log(x) + sqrt(b^2 - 4*a*c)*n*x*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b*n*x + 2*a
*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x), 1/2*(2*b*n*x*log(x) + 2*sqrt(
-b^2 + 4*a*c)*n*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x)]
```

giac [A] time = 0.19, size = 99, normalized size = 1.15

$$-\frac{bn\log\left(cx^2+bx+a\right)}{2\,a}+\frac{bn\log(x)}{a}-\frac{n\log\left(cx^2+bx+a\right)}{x}-\frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a}-\frac{\log(d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*n*log(c*x^2 + b*x + a)/a + b*n*log(x)/a - n*log(c*x^2 + b*x + a)/x -
 (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - log(d)/x
```

maple [C] time = 0.68, size = 261, normalized size = 3.03

$$-\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{x}-\frac{-i\pi a\,\mathrm{csgn}\,(id)\,\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)+i\pi a\,\mathrm{csgn}\,(id)\,\mathrm{csg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^2,x)
```

```
[Out] -1/x*ln((c*x^2+b*x+a)^n)-1/2*(-I*Pi*a*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csg
n(I*d*(c*x^2+b*x+a)^n)+I*Pi*a*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+I*Pi*a*
csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a*csgn(I*d*(c*x^2+
b*x+a)^n)^3-2*b*n*ln(x)*x-2*sum(_R*ln(((6*a*c-2*b^2)*_R^2+_R*b*c*n+4*c^2*n^
2)*x-a*b*_R^2+(-2*a*c*n+b^2*n)*_R+2*b*c*n^2),_R=RootOf(_Z^2*a+_Z*b*n+c*n^2)
)*a*x+2*ln(d)*a)/a/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.93, size = 262, normalized size = 3.05

$$\frac{b\,n\,\ln(x)}{a}-\frac{\ln\left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\left(b\,n-n\,\sqrt{b^2-\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^2,x)
```

```
[Out] (b*n*log(x))/a - (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b - (b^2 - 4*a*c)^(1/
2))*(b^2*c*n - 2*a*c^2*n + b*c^2*n*x + (c*n*(b - (b^2 - 4*a*c)^(1/2))*(a*b
+ 2*b^2*x - 6*a*c*x))/(2*a)))/(2*a))*(b*n - n*(b^2 - 4*a*c)^(1/2)))/(2*a) -
 (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b + (b^2 - 4*a*c)^(1/2))*(b^2*c*n - 2
*a*c^2*n + b*c^2*n*x + (c*n*(b + (b^2 - 4*a*c)^(1/2))*(a*b + 2*b^2*x - 6*a*
c*x))/(2*a)))/(2*a))*(b*n + n*(b^2 - 4*a*c)^(1/2)))/(2*a) - log(d*(a + b*x
+ c*x^2)^n)/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**2,x)
[Out] Timed out
```

3.78 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=121

$$\frac{n\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{bn\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

```
[Out] -1/2*b*n/a/x-1/2*(-2*a*c+b^2)*n*ln(x)/a^2+1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a
)/a^2-1/2*ln(d*(c*x^2+b*x+a)^n)/x^2-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))*(-4*a*c+b^2)^(1/2)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{bn\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -(b*n)/(2*a*x) - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(2*a^2) - ((b^2 - 2*a*c)*n*Log[x])/(2*a^2) + ((b^2 - 2*a*c)*n*Log[a +
b*x + c*x^2])/(4*a^2) - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{1}{2}n\int \frac{b+2cx}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{1}{2}n\int\left(\frac{b}{ax^2} + \frac{-b^2+2ac}{a^2x} + \frac{b\left(b^2-3ac\right)+c\left(b^2-2ac\right.}{a^2\left(a+bx+cx^2\right)}\right.$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{n\int\frac{b\left(b^2-3ac\right)+c\left(b^2-2ac\right)x}{a+bx+cx^2}\,d}{2a^2}$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{\left(b\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+}}{4a^2}$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} + \frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4a^2} - \frac{\log\left(d\left(a+bx+\right.\right.}{2x^2}$$

$$= -\frac{bn}{2ax} - \frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} + \frac{\left(b^2-2ac\right)n\log}{4a}$$

Mathematica [A] time = 0.24, size = 105, normalized size = 0.87

$$-\frac{\frac{nx\left(2x\log(x)\left(b^2-2ac\right)-x\left(b^2-2ac\right)\log(a+x(b+cx))+2bx\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2ab\right)}{a^2}+2\log\left(d(a+x(b+cx))^n\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/4*((n*x*(2*a*b + 2*b*Sqrt[b^2 - 4*a*c]*x*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]] + 2*(b^2 - 2*a*c)*x*Log[x] - (b^2 - 2*a*c)*x*Log[a + x*(b + c*x)]))
/a^2 + 2*Log[d*(a + x*(b + c*x))^n])/x^2
```

fricas [A] time = 0.47, size = 261, normalized size = 2.16

$$\left[\frac{\sqrt{b^2-4\,ac}\,bnx^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2-2\,ac\right)nx^2\log(x)-2\,abnx-2\,a^2\log(d)+\left(\left(b^2-\right.\right.}{4\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*n*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2 - 2*a*c)*n*x^2*log
(x) - 2*a*b*n*x - 2*a^2*log(d) + ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2
```

```
+ b*x + a))/(a^2*x^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*n*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2 - 2*a*c)*n*x^2*log(x) + 2*a*b
*n*x + 2*a^2*log(d) - ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2 + b*x + a))
/(a^2*x^2)]
```

giac [A] time = 0.20, size = 129, normalized size = 1.07

$$\frac{\left(b^2n - 2\,acn\right)\log\left(cx^2 + bx + a\right)}{4\,a^2} - \frac{n\log\left(cx^2 + bx + a\right)}{2\,x^2} - \frac{\left(b^2n - 2\,acn\right)\log(x)}{2\,a^2} + \frac{\left(b^3n - 4\,abcn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,a}}\right.}{2\,\sqrt{-b^2 + 4\,ac}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/a^2 - 1/2*n*log(c*x^2 + b*x + a)
/x^2 - 1/2*(b^2*n - 2*a*c*n)*log(x)/a^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan((2
*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*n*x + a*log
(d))/(a*x^2)
```

maple [C] time = 0.46, size = 1178, normalized size = 9.74

$$-\frac{\ln\left(\left(c\,x^2 + bx + a\right)^n\right)}{2x^2} - \frac{-4acn\,x^2\ln(x) + 2acn\,x^2\ln\left(-12a^3b\,c^2 + 11a^2b^3c - 2a\,b^5 + 5\sqrt{-4a\,b^2c + b^4}\;a^2bc - 2\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^3,x)
```

```
[Out] -1/2/x^2*ln((c*x^2+b*x+a)^n)-1/4*(-I*Pi*a^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^2*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^2*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^2*csgn
(I*d*(c*x^2+b*x+a)^n)^3+2*n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^
2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^
(1/2)*b^4)*x-12*a^3*b*c^2+11*a^2*b^3*c-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2
*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4
*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-
2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*a^2*b^3*c-2*a*b^5+5*(-4*a*b
^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*b^2*x^2+2*n*ln((-16
*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*
c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*a^2*b^
3*c-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3
)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a
^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*
a^3*b*c^2+11*a^2*b^3*c-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2
*c+b^4)^(1/2)*a*b^3)*b^2*x^2-4*n*ln(x)*c*a*x^2+2*n*ln(x)*b^2*x^2-n*ln((-16*
a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c
+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*a^2*b^3
*c-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)
*(-4*a*b^2*c+b^4)^(1/2)*x^2+n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*
b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4
)^(1/2)*b^4)*x-12*a^3*b*c^2+11*a^2*b^3*c-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a
^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*(-4*a*b^2*c+b^4)^(1/2)*x^2+2*a*b*n*x
+2*ln(d)*a^2)/a^2/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.76, size = 474, normalized size = 3.92

$$\frac{\ln\left(\frac{b^3c^2n^2-2abc^3n^2}{4a^2}+\frac{\left(b^2n-2acn+bn\sqrt{b^2-4ac}\right)\left(\frac{\left(\frac{x\left(24a^3c^2-8a^2b^2c\right)}{4a^2}-abc\right)\left(b^2n-2acn+bn\sqrt{b^2-4ac}\right)}{4a^2}-\frac{2ab^3cn-6a^2bc^2n}{4a^2}+\frac{x\left(12a^2c^3n-4ab^2c^2n\right)}{4a^2}\right)}{4a^2}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (log((b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((b^2*n - 2*a*c*n + b*n*(b^2 -
 4*a*c)^(1/2))*((((x*(24*a^3*c^2 - 8*a^2*b^2*c))/(4*a^2) - a*b*c)*(b^2*n -
2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (2*a*b^3*c*n - 6*a^2*b*c^2*n)
/(4*a^2) + (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*
n^2*x)/(4*a^2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (log
(x)*(b^2*n - 2*a*c*n))/(2*a^2) - log(d*(a + b*x + c*x^2)^n)/(2*x^2) - (log(
(b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((2*a*c*n - b^2*n + b*n*(b^2 - 4*a*
c)^(1/2))*((2*a*b^3*c*n - 6*a^2*b*c^2*n)/(4*a^2) + (((x*(24*a^3*c^2 - 8*a^2
*b^2*c))/(4*a^2) - a*b*c)*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a
^2) - (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*n^2*x
)/(4*a^2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (b*n)/(2*
a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**3,x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4} dx$

Optimal. Leaf size=149

$$-\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{n\left(b^2-2ac\right)}{3a^2x}$$

```
[Out] -1/6*b*n/a/x^2+1/3*(-2*a*c+b^2)*n/a^2/x+1/3*b*(-3*a*c+b^2)*n*ln(x)/a^3-1/6*
b*(-3*a*c+b^2)*n*ln(c*x^2+b*x+a)/a^3-1/3*ln(d*(c*x^2+b*x+a)^n)/x^3+1/3*(-a*
c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^3
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{n\left(b^2-2ac\right)}{3a^2x}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -(b*n)/(6*a*x^2) + ((b^2 - 2*a*c)*n)/(3*a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2 -
a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*a^3) + (b*(b^2 - 3*a*c)*n
*Log[x])/(3*a^3) - (b*(b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*a^3) - Log[d
*(a + b*x + c*x^2)^n]/(3*x^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}+\frac{1}{3}n\int \frac{b+2cx}{x^3\left(a+bx+cx^2\right)}dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}+\frac{1}{3}n\int\left(\frac{b}{ax^3}+\frac{-b^2+2ac}{a^2x^2}+\frac{b^3-3abc}{a^3x}+\frac{-b^4+4ab^2c}{a^3}\right.$$

$$= -\frac{bn}{6ax^2}+\frac{\left(b^2-2ac\right)n}{3a^2x}+\frac{b\left(b^2-3ac\right)n\log(x)}{3a^3}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}+\frac{n\int -}{}$$

$$= -\frac{bn}{6ax^2}+\frac{\left(b^2-2ac\right)n}{3a^2x}+\frac{b\left(b^2-3ac\right)n\log(x)}{3a^3}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}-\frac{\left(b\left(b^2\right.\right.}{}$$

$$= -\frac{bn}{6ax^2}+\frac{\left(b^2-2ac\right)n}{3a^2x}+\frac{b\left(b^2-3ac\right)n\log(x)}{3a^3}-\frac{b\left(b^2-3ac\right)n\log\left(a+bx+cx^2\right)}{6a^3}$$

$$= -\frac{bn}{6ax^2}+\frac{\left(b^2-2ac\right)n}{3a^2x}+\frac{\sqrt{b^2-4ac}\left(b^2-ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{b\left(b^2-3ac\right)}{3a^3}$$

Mathematica [A] time = 0.37, size = 132, normalized size = 0.89

$$-\frac{\frac{nx\left(a^2b-2bx^2\log(x)\left(b^2-3ac\right)+bx^2\left(b^2-3ac\right)\log(a+x(b+cx))-2x^2\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2ax\left(b^2-2ac\right)\right)}{a^3}+2\log\left(d(a+x(b+cx)\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/6*((n*x*(a^2*b - 2*a*(b^2 - 2*a*c)*x - 2*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*x
^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*b*(b^2 - 3*a*c)*x^2*Log[x] +
b*(b^2 - 3*a*c)*x^2*Log[a + x*(b + c*x)]))/a^3 + 2*Log[d*(a + x*(b + c*x))^
n])/x^3
```

fricas [A] time = 0.53, size = 318, normalized size = 2.13

$$\left[-\frac{\left(b^2-ac\right)\sqrt{b^2-4\,ac}\,nx^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^3-3\,abc\right)nx^3\log(x)+a^2bnx-2\left(ab^2-2\,a\right.}{6\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3 - 3*a*b
*c)*n*x^3*log(x) + a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2 + 2*a^3*log(d) + (
(b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 + b*x + a))/(a^3*x^3), 1/6*(2*(b
^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + 2*(b^3 - 3*a*b*c)*n*x^3*log(x) - a^2*b*n*x + 2*(a*b^2 - 2*a
^2*c)*n*x^2 - 2*a^3*log(d) - ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 +
b*x + a))/(a^3*x^3)]
```

giac [A] time = 0.25, size = 164, normalized size = 1.10

$$-\frac{\left(b^3n-3\,abcn\right)\log\left(cx^2+bx+a\right)}{6\,a^3}-\frac{n\log\left(cx^2+bx+a\right)}{3\,x^3}+\frac{\left(b^3n-3\,abcn\right)\log(x)}{3\,a^3}-\frac{\left(b^4n-5\,ab^2cn+4\,a^2c^2n\right)\mathrm{a}}{3\,\sqrt{-b^2+4\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)/a^3 - 1/3*n*log(c*x^2 + b*x +
 a)/x^3 + 1/3*(b^3*n - 3*a*b*c*n)*log(x)/a^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4
*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3)
 + 1/6*(2*b^2*n*x^2 - 4*a*c*n*x^2 - a*b*n*x - 2*a^2*log(d))/(a^2*x^3)
```

maple [C] time = 0.71, size = 423, normalized size = 2.84

$$-\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{3x^3}-\frac{6abcn\,x^3\ln(x)-2b^3n\,x^3\ln(x)-2a^3x^3\,\mathrm{RootOf}\left(\_Z^2a^3+c^3n^2+\left(-3cbna+b^3n\right)\_Z\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^4,x)
```

```
[Out] -1/3/x^3*ln((c*x^2+b*x+a)^n)-1/6*(-I*Pi*a^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^3*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^3*csgn
(I*d*(c*x^2+b*x+a)^n)^3+6*ln(x)*a*b*c*n*x^3-2*ln(x)*b^3*n*x^3-2*sum(_R*ln((
(6*a^5*c-2*a^4*b^2)*_R^2+(-7*a^3*b*c^2*n+2*a^2*b^3*c*n)*_R+4*a^2*c^4*n^2-4*
a*b^2*c^3*n^2+b^4*c^2*n^2)*x-a^5*b*_R^2+(2*a^4*c^2*n-4*a^3*b^2*c*n+a^2*b^4*
n)*_R+6*a^2*b*c^3*n^2-5*a*b^3*c^2*n^2+b^5*c*n^2),_R=RootOf(a^3*_Z^2+(-3*a*b
*c*n+b^3*n)*_Z+c^3*n^2))*a^3*x^3+4*a^2*c*n*x^2-2*a*b^2*n*x^2+a^2*b*n*x+2*ln
(d)*a^3)/a^3/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.93, size = 505, normalized size = 3.39

$$\frac{\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+13\,a^2\,b^3\,c-20\,a^3\,b\,c^2+4\,a^3\,c^3\,x+2\,a^3\,c^2\,\sqrt{b^2-4\,}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^4,x)
```

```
[Out] (log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 13*a^2*b^3*c - 20*a^3*b*c^2 + 4*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*
c)^(1/2) - 25*a^2*b^2*c^2*x + 14*a*b^4*c*x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*
(a*((b*c*n)/2 - (c*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*(b^2 - 4*
a*c)^(1/2))/6))/a^3 - log(d*(a + b*x + c*x^2)^n)/(3*x^3) - ((b*n)/(2*a) + (
n*x*(2*a*c - b^2))/a^2)/(3*x^2) - (log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 - 4
*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c + 20*a^3*b*c^2 - 4
*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*c)^(1/2) + 25*a^2*b^2*c^2*x - 14*a*b^4*c*
x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11
*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - a*((b*c*n)/2 + (c*n*(b^2 - 4
*a*c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/a^3 + (log(x)*(b^3*n - 3*
a*b*c*n))/(3*a^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**4,x)
```

```
[Out] Timed out
```

3.80 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5} \, dx$

Optimal. Leaf size=190

$$-\frac{bn\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4}-\frac{bn\left(b^2-3ac\right)}{4a^3x}+\frac{n\left(b^2-2ac\right)}{8a^2x^2}+\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx\right.}{8a^4}$$

```
[Out] -1/12*b*n/a/x^3+1/8*(-2*a*c+b^2)*n/a^2/x^2-1/4*b*(-3*a*c+b^2)*n/a^3/x-1/4*(
2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(x)/a^4+1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x
^2+b*x+a)/a^4-1/4*ln(d*(c*x^2+b*x+a)^n)/x^4-1/4*b*(-2*a*c+b^2)*n*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^4
```

Rubi [A] time = 0.22, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8a^4}-\frac{n\log(x)\left(2a^2c^2-4ab^2c+b^4\right)}{4a^4}+\frac{n\left(b^2-2ac\right)}{8a^2x^2}-\frac{bn\left(b^2-3ac\right)}{4a^3x}-\frac{bn\sqrt{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -(b*n)/(12*a*x^3) + ((b^2 - 2*a*c)*n)/(8*a^2*x^2) - (b*(b^2 - 3*a*c)*n)/(4*
a^3*x) - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(4*a^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[x])/(4*a^4) + ((b
^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*x + c*x^2])/(8*a^4) - Log[d*(a + b*
x + c*x^2)^n]/(4*x^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \frac{b+2cx}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \left(\frac{b}{ax^4}+\frac{-b^2+2ac}{a^2x^3}+\frac{b^3-3abc}{a^3x^2}+\frac{-b^4+4ab^2c}{a^4x}\right.$$

$$= -\frac{bn}{12ax^3}+\frac{\left(b^2-2ac\right)n}{8a^2x^2}-\frac{b\left(b^2-3ac\right)n}{4a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4}-\frac{\log\left(\right.}{}$$

$$= -\frac{bn}{12ax^3}+\frac{\left(b^2-2ac\right)n}{8a^2x^2}-\frac{b\left(b^2-3ac\right)n}{4a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4}-\frac{\log\left(\right.}{}$$

$$= -\frac{bn}{12ax^3}+\frac{\left(b^2-2ac\right)n}{8a^2x^2}-\frac{b\left(b^2-3ac\right)n}{4a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4}+\frac{\left(b^4-\right.}{}$$

$$= -\frac{bn}{12ax^3}+\frac{\left(b^2-2ac\right)n}{8a^2x^2}-\frac{b\left(b^2-3ac\right)n}{4a^3x}-\frac{b\sqrt{b^2-4ac}\left(b^2-2ac\right)n\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{4a^4}$$

Mathematica [A] time = 0.46, size = 172, normalized size = 0.91

$$-\frac{\frac{nx\left(2a^3b-3a^2x\left(b^2-2ac\right)+6x^3\log(x)\left(2a^2c^2-4ab^2c+b^4\right)-3x^3\left(2a^2c^2-4ab^2c+b^4\right)\log(a+x(b+cx))+6bx^3\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+6abx^2\left(b^2-\right.}{a^4}}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/24*((n*x*(2*a^3*b - 3*a^2*(b^2 - 2*a*c)*x + 6*a*b*(b^2 - 3*a*c)*x^2 + 6*
b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*x^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
] + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[x] - 3*(b^4 - 4*a*b^2*c + 2*a^2
*c^2)*x^3*Log[a + x*(b + c*x)]))/a^4 + 6*Log[d*(a + x*(b + c*x))^n])/x^4
```

fricas [A] time = 0.55, size = 404, normalized size = 2.13

$$\left[-\frac{3\left(b^3-2\,abc\right)\sqrt{b^2-4\,ac}\,nx^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+6\left(b^4-4\,ab^2c+2\,a^2c^2\right)nx^4\log(x)+2\,a^3b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*x^4*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4
- 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)
*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^2 - 3*((b^4 - 4*a*b^2*c +
 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))/(a^4*x^4), -1/24*(6*(b^3
 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*
x + 6*(a*b^3 - 3*a^2*b*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^
2 - 3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))
/(a^4*x^4)]
```

giac [A] time = 0.21, size = 210, normalized size = 1.11

$$\frac{\left(b^4n-4\,ab^2cn+2\,a^2c^2n\right)\log\left(cx^2+bx+a\right)}{8\,a^4}-\frac{n\log\left(cx^2+bx+a\right)}{4\,x^4}-\frac{\left(b^4n-4\,ab^2cn+2\,a^2c^2n\right)\log(x)}{4\,a^4}+\frac{\left(b^5n-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/a^4 - 1/4*n*lo
g(c*x^2 + b*x + a)/x^4 - 1/4*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(x)/a^4
 + 1/4*(b^5*n - 6*a*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/24*(6*b^3*n*x^3 - 18*a*b*c*n*x^3 - 3*
a*b^2*n*x^2 + 6*a^2*c*n*x^2 + 2*a^2*b*n*x + 6*a^3*log(d))/(a^3*x^4)
```

maple [C] time = 0.88, size = 505, normalized size = 2.66

$$-\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{4x^4}-\frac{12a^2c^2n\,x^4\ln(x)-24a\,b^2cn\,x^4\ln(x)+6b^4n\,x^4\ln(x)-6a^4x^4\,\mathrm{RootOf}\left(\_Z^2a^4+c^4n^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^5,x)
```

```
[Out] -1/4/x^4*ln((c*x^2+b*x+a)^n)-1/24*(12*ln(x)*a^2*c^2*n*x^4-24*ln(x)*a*b^2*c*
n*x^4+6*ln(x)*b^4*n*x^4-3*I*Pi*a^4*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*d*(c*x^2+b*x+a)^n)+3*I*Pi*a^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+3*I*Pi
*a^4*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-3*I*Pi*a^4*csgn(I*
d*(c*x^2+b*x+a)^n)^3-6*sum(_R*ln(((6*a^7*c-2*a^6*b^2)*_R^2+(-6*a^5*c^3*n+9*
a^4*b^2*c^2*n-2*a^3*b^4*c*n)*_R+9*a^2*b^2*c^4*n^2-6*a*b^4*c^3*n^2+b^6*c^2*n
^2)*x-a^7*b*_R^2+(-5*a^5*b*c^2*n+5*a^4*b^3*c*n-a^3*b^5*n)*_R-6*a^3*b*c^4*n^
2+14*a^2*b^3*c^3*n^2-7*a*b^5*c^2*n^2+b^7*c*n^2),_R=RootOf(a^4*_Z^2+(-2*a^2*
c^2*n+4*a*b^2*c*n-b^4*n)*_Z+c^4*n^2))*a^4*x^4-18*a^2*b*c*n*x^3+6*a*b^3*n*x^
3+6*a^3*c*n*x^2-3*a^2*b^2*n*x^2+2*a^3*b*n*x+6*ln(d)*a^4)/a^4/x^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.07, size = 627, normalized size = 3.30

$$\ln\Big(2\,a\,b^6+2\,b^7\,x-12\,a^4\,c^3+2\,a\,b^5\,\sqrt{b^2-4\,a\,c}+2\,b^6\,x\,\sqrt{b^2-4\,a\,c}-15\,a^2\,b^4\,c+31\,a^3\,b^2\,c^2+37\,a^2\,b^3\,c^2\,x-16\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (log(2*a*b^6 + 2*b^7*x - 12*a^4*c^3 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x
*(b^2 - 4*a*c)^(1/2) - 15*a^2*b^4*c + 31*a^3*b^2*c^2 + 37*a^2*b^3*c^2*x - 1
6*a*b^5*c*x - 20*a^3*b*c^3*x - 9*a^2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^
2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2
 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - a*((b^
2*c*n)/2 + (b*c*n*(b^2 - 4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 +
 (a^2*c^2*n)/4))/a^4 - log(d*(a + b*x + c*x^2)^n)/(4*x^4) - (log(x)*(b^4*n
+ 2*a^2*c^2*n - 4*a*b^2*c*n))/(4*a^4) - (log(12*a^4*c^3 - 2*b^7*x - 2*a*b^6
 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x*(b^2 - 4*a*c)^(1/2) + 15*a^2*b^4*c
 - 31*a^3*b^2*c^2 - 37*a^2*b^3*c^2*x + 16*a*b^5*c*x + 20*a^3*b*c^3*x - 9*a^
2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x
*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*
(b^2 - 4*a*c)^(1/2))*(a*((b^2*c*n)/2 - (b*c*n*(b^2 - 4*a*c)^(1/2))/4) - (b^
4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 - (a^2*c^2*n)/4))/a^4 - ((b*n)/(3*a)
 + (n*x*(2*a*c - b^2))/(2*a^2) - (b*n*x^2*(3*a*c - b^2))/a^3)/(4*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**5,x)
```

```
[Out] Timed out
```

3.81 $\int \log\left(1+x+x^2\right)\,dx$

Optimal. Leaf size=42

$$x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2523, 773, 634, 618, 204, 628}

$$x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2],x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 + x
 + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
```

```
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(1+x+x^2\right)\,dx &= x\log\left(1+x+x^2\right)-\int \frac{x(1+2x)}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)-\int \frac{-2-x}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -2x+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.83

$$\left(x+\frac{1}{2}\right)\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + x^2],x]
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (1/2 + x)*Log[1 + x + x^2]
```

fricas [A] time = 0.44, size = 33, normalized size = 0.79

$$\frac{1}{2}(2x+1)\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*
x
```

giac [A] time = 0.17, size = 37, normalized size = 0.88

$$x\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="giac")
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

maple [A] time = 0.07, size = 38, normalized size = 0.90

$$x\ln\left(x^2+x+1\right)-2x+\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1),x)
```

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [**A**] time = 1.34, size = 37, normalized size = 0.88

$$x \log\left(x^{2} + x + 1\right) + \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x + 1)\right) - 2 x + \frac{1}{2} \log\left(x^{2} + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

mupad [**B**] time = 0.06, size = 39, normalized size = 0.93

$$\frac{\ln\left(x^{2} + x + 1\right)}{2} - 2 x + \sqrt{3} \operatorname{atan}\left(\frac{2 \sqrt{3} x}{3} + \frac{\sqrt{3}}{3}\right) + x \ln\left(x^{2} + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 + 1),x)
```

```
[Out] log(x + x^2 + 1)/2 - 2*x + 3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3) + x*lo
g(x + x^2 + 1)
```

sympy [**A**] time = 0.15, size = 46, normalized size = 1.10

$$x \log\left(x^{2} + x + 1\right) - 2x + \frac{\log\left(x^{2} + x + 1\right)}{2} + \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1),x)
```

```
[Out] x*log(x**2 + x + 1) - 2*x + log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/
3 + sqrt(3)/3)
```

3.82 $\int (d+ex)^4 \log \left(d \left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=485

$$-\frac{nx\left(c^2e^2\left(2a^2e^2+15abde+10b^2d^2\right)-b^2ce^3(4ae+5bd)-10c^3d^2e(2ae+bd)+b^4e^4+10c^4d^4\right)}{5c^4}-\frac{n(2cd-be)\left(c^2e^2\right.}{}$$

```
[Out] -1/5*(10*c^4*d^4+b^4*e^4-10*c^3*d^2*e*(2*a*e+b*d)-b^2*c*e^3*(4*a*e+5*b*d)+c
^2*e^2*(2*a^2*e^2+15*a*b*d*e+10*b^2*d^2))*n*x/c^4-1/10*e*(20*c^3*d^3-b^3*e^
3-10*c^2*d*e*(a*e+b*d)+b*c*e^2*(3*a*e+5*b*d))*n*x^2/c^3-1/15*e^2*(20*c^2*d^
2+b^2*e^2-c*e*(2*a*e+5*b*d))*n*x^3/c^2-1/20*e^3*(-b*e+10*c*d)*n*x^4/c-2/25*
e^4*n*x^5-1/10*(-b*e+2*c*d)*(c^4*d^4+b^4*e^4-2*c^3*d^2*e*(5*a*e+b*d)-b^2*c*
e^3*(5*a*e+3*b*d)+c^2*e^2*(5*a^2*e^2+10*a*b*d*e+4*b^2*d^2))*n*ln(c*x^2+b*x+
a)/c^5/e+1/5*(e*x+d)^5*ln(d*(c*x^2+b*x+a)^n)/e+1/5*(5*c^4*d^4+b^4*e^4-10*c^
3*d^2*e*(a*e+b*d)-b^2*c*e^3*(3*a*e+5*b*d)+c^2*e^2*(a^2*e^2+10*a*b*d*e+10*b^
2*d^2))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A] time = 2.06, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n(2cd-be)\left(c^2e^2\left(5a^2e^2+10abde+4b^2d^2\right)-b^2ce^3(5ae+3bd)-2c^3d^2e(5ae+bd)+b^4e^4+c^4d^4\right)\log\left(a+bx+\right.}{10c^5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3*(5*b*d + 4
*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*n*x)/(5*c^4) - (e*(2
0*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a*e))*n*x
^2)/(10*c^3) - (e^2*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*n*x^3)/(15
*c^2) - (e^3*(10*c*d - b*e)*n*x^4)/(20*c) - (2*e^4*n*x^5)/25 + (Sqrt[b^2 -
4*a*c]*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(5*c^5) - ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d
^2*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2))*n*Log[a + b*x + c*x^2])/(10*c^5*e) + ((d + e*x)^5*Log[d
*(a + b*x + c*x^2)^n])/(5*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right) dx = \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \frac{(b+2cx)(d+ex)^5}{a+bx+cx^2}\,dx}{5e}$$

$$= \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \left(\frac{e\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2 \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2 \cdots\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2 \cdots\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2 \cdots\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2 \cdots\right.\right.}{5c^4}$$

Mathematica [A] time = 1.97, size = 468, normalized size = 0.96

$$\frac{(d+ex)^5 \log\left(d(a+x(b+cx))^n\right) - \frac{n\left(60cex\left(c^2e^2\left(2a^2e^2+15abde+10b^2d^2\right)-b^2ce^3(4ae+5bd)-10c^3d^2e(2ae+bd)+b^4e^4+10c^4d^4\right)+30(2cd-be \cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/60*(n*(60*c*e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*
c*e^3*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x +
30*c^2*e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d
+ 3*a*e))*x^2 + 20*c^3*e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*x^3
```

```
 + 15*c^4*e^4*(10*c*d - b*e)*x^4 + 24*c^5*e^5*x^5 - 60*Sqrt[b^2 - 4*a*c]*e*
(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d + 3*a*e)
 + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]] + 30*(2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2*e*(b*d + 5*a*
e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b*d*e + 5*a^2*e^
2))*Log[a + x*(b + c*x)]))/c^5 + (d + e*x)^5*Log[d*(a + x*(b + c*x))^n])/(5
*e)
```

fricas [A] time = 0.57, size = 1270, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*e^4*n*x^5 + 15*(10*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^
5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3
*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)
*e^4)*n*x^2 - 30*(5*c^4*d^4 - 10*b*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2
 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(b^2
- 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2*c^3
- 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c^5*d^
2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*(b^2*
c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log(c*x^
2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3 + 10*
c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5, -1/300*(24*c^5*e^4*n*x^5 + 15*(10
*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c
^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3
 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)*e^4)*n*x^2 - 60*(5*c^4*d^4 - 10*b
*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (
b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*
a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2
*c^3 - 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*
c^2 + 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c
^5*d^2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*
(b^2*c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4
*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log
(c*x^2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3
+ 10*c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5]
```

giac [A] time = 0.26, size = 817, normalized size = 1.68

$$\frac{60\, c^4 n x^5 e^4 \log\left(cx^2 + bx + a\right) + 300\, c^4 d n x^4 e^3 \log\left(cx^2 + bx + a\right) + 600\, c^4 d^2 n x^3 e^2 \log\left(cx^2 + bx + a\right) + 600\, c^4 d^3 n x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/300*(60*c^4*n*x^5*e^4*log(c*x^2 + b*x + a) + 300*c^4*d*n*x^4*e^3*log(c*x^
2 + b*x + a) + 600*c^4*d^2*n*x^3*e^2*log(c*x^2 + b*x + a) + 600*c^4*d^3*n*x
^2*e*log(c*x^2 + b*x + a) - 24*c^4*n*x^5*e^4 - 150*c^4*d*n*x^4*e^3 - 400*c^
4*d^2*n*x^3*e^2 - 600*c^4*d^3*n*x^2*e + 300*c^4*d^4*n*x*log(c*x^2 + b*x + a
) + 60*c^4*x^5*e^4*log(d) + 300*c^4*d*x^4*e^3*log(d) + 600*c^4*d^2*x^3*e^2*
log(d) + 600*c^4*d^3*x^2*e*log(d) - 600*c^4*d^4*n*x + 15*b*c^3*n*x^4*e^4 +
100*b*c^3*d*n*x^3*e^3 + 300*b*c^3*d^2*n*x^2*e^2 + 600*b*c^3*d^3*n*x*e + 300
*c^4*d^4*x*log(d) - 20*b^2*c^2*n*x^3*e^4 + 40*a*c^3*n*x^3*e^4 - 150*b^2*c^2
```

```
*d*n*x^2*e^3 + 300*a*c^3*d*n*x^2*e^3 - 600*b^2*c^2*d^2*n*x*e^2 + 1200*a*c^3
*d^2*n*x*e^2 + 30*b^3*c*n*x^2*e^4 - 90*a*b*c^2*n*x^2*e^4 + 300*b^3*c*d*n*x*
e^3 - 900*a*b*c^2*d*n*x*e^3 - 60*b^4*n*x*e^4 + 240*a*b^2*c*n*x*e^4 - 120*a^
2*c^2*n*x*e^4)/c^4 + 1/10*(5*b*c^4*d^4*n - 10*b^2*c^3*d^3*n*e + 20*a*c^4*d^
3*n*e + 10*b^3*c^2*d^2*n*e^2 - 30*a*b*c^3*d^2*n*e^2 - 5*b^4*c*d*n*e^3 + 20*
a*b^2*c^2*d*n*e^3 - 10*a^2*c^3*d*n*e^3 + b^5*n*e^4 - 5*a*b^3*c*n*e^4 + 5*a^
2*b*c^2*n*e^4)*log(c*x^2 + b*x + a)/c^5 - 1/5*(5*b^2*c^4*d^4*n - 20*a*c^5*d
^4*n - 10*b^3*c^3*d^3*n*e + 40*a*b*c^4*d^3*n*e + 10*b^4*c^2*d^2*n*e^2 - 50*
a*b^2*c^3*d^2*n*e^2 + 40*a^2*c^4*d^2*n*e^2 - 5*b^5*c*d*n*e^3 + 30*a*b^3*c^2
*d*n*e^3 - 40*a^2*b*c^3*d*n*e^3 + b^6*n*e^4 - 7*a*b^4*c*n*e^4 + 13*a^2*b^2*
c^2*n*e^4 - 4*a^3*c^3*n*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-
b^2 + 4*a*c)*c^5)
```

maple [C] time = 1.09, size = 31895, normalized size = 65.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.03, size = 1240, normalized size = 2.56

$$x^3\left(\frac{b\left(\frac{e^3\,n\,\left(b\,e+10\,c\,d\right)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{3\,c}+\frac{2\,a\,e^4\,n}{15\,c}-\frac{d\,e^2\,n\,\left(b\,e+4\,c\,d\right)}{3\,c}\right)-x\left(\frac{a\left(\frac{b\left(\frac{e^3\,n\,\left(b\,e+10\,c\,d\right)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{c}+\frac{2\,a\,e^4\,n}{5\,c}-\frac{d\,e^2\,n\,\left(b\,e+4\,c\right.}{c}\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^4,x)
```

```
[Out] x^3*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(3*c) + (2*a*e^
4*n)/(15*c) - (d*e^2*n*(b*e + 4*c*d))/(3*c)) - x*((a*((b*((e^3*n*(b*e + 10*
c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4
*c*d))/c))/c - (b*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)
))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/c - (a*((e^3*n*(b*e
+ 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*d^2*e*n*(b*e + 2*c*d))/c))/c
+ (2*d^3*n*(b*e + c*d))/c) - x^2*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2
*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/(2*c)
```

```
 - (a*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(2*c) + (d^2*e*n*
(b*e + 2*c*d))/c) - x^4*((e^3*n*(b*e + 10*c*d))/(20*c) - (b*e^4*n)/(10*c))
+ log(d*(a + b*x + c*x^2)^n)*(d^4*x + (e^4*x^5)/5 + 2*d^3*e*x^2 + d*e^3*x^4
 + 2*d^2*e^2*x^3) + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 -
 4*a*c)^(1/2))*(b^5*e^4*n + 5*b*c^4*d^4*n + b^4*e^4*n*(b^2 - 4*a*c)^(1/2) +
 5*c^4*d^4*n*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b
^4*c*d*e^3*n + 5*a^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n
+ a^2*c^2*e^4*n*(b^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n - 10*a*c^3*d^2*e
^2*n*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) - 3*a*b
^2*c*e^4*n*(b^2 - 4*a*c)^(1/2) - 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) - 5*b
^3*c*d*e^3*n*(b^2 - 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^
3*n + 10*a*b*c^2*d*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5) - (2*e^4*n*x^5)/25
+ (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^
5*e^4*n + 5*b*c^4*d^4*n - b^4*e^4*n*(b^2 - 4*a*c)^(1/2) - 5*c^4*d^4*n*(b^2
- 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b^4*c*d*e^3*n + 5*a
^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n - a^2*c^2*e^4*n*(b
^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n + 10*a*c^3*d^2*e^2*n*(b^2 - 4*a*c)
^(1/2) - 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c*e^4*n*(b^2 -
4*a*c)^(1/2) + 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) + 5*b^3*c*d*e^3*n*(b^2
- 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^3*n - 10*a*b*c^2*d
*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*ln(d*(c*x**2+b*x+a)**n),x)
[Out] Timed out
```

3.83 $\int (d+ex)^3 \log\left(d\left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=338

$$-\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8c^4e}-\frac{nx}{}$$

```
[Out] -1/4*(8*c^3*d^3-b^3*e^3+b*c*e^2*(3*a*e+4*b*d)-2*c^2*d*e*(4*a*e+3*b*d))*n*x/
c^3-1/8*e*(12*c^2*d^2+b^2*e^2-2*c*e*(a*e+2*b*d))*n*x^2/c^2-1/12*e^2*(-b*e+8
*c*d)*n*x^3/c-1/8*e^3*n*x^4-1/8*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*
c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b
*x+a)/c^4/e+1/4*(e*x+d)^4*ln(d*(c*x^2+b*x+a)^n)/e+1/4*(-b*e+2*c*d)*(2*c^2*d
^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c
+b^2)^(1/2)/c^4
```

Rubi [A] time = 0.52, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8c^4e}-\frac{en}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b*d + 4*a*e
))*n*x)/(4*c^3) - (e*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*n*x^2)/(8
*c^2) - (e^2*(8*c*d - b*e)*n*x^3)/(12*c) - (e^3*n*x^4)/8 + (Sqrt[b^2 - 4*a*
c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*n*Log[a + b*x + c*x^2])/(8*c^4*e) + ((d + e*x)^4*Log[d*(a + b*x +
c*x^2)^n])/(4*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(d\left(a+bx+cx^2\right)^n\right) dx = \frac{(d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)}{4e} - \frac{n\int \frac{(b+2cx)(d+ex)^4}{a+bx+cx^2}\,dx}{4e}$$

$$= \frac{(d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)}{4e} - \frac{n\int \left(\frac{e\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd\right.}{c^3}\right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+\right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+\right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+\right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+\right.}{}$$

Mathematica [A] time = 0.53, size = 324, normalized size = 0.96

$$(d+ex)^4 \log\left(d(a+x(b+cx))^n\right) - \frac{n\left(3\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log(a+x(b+cx))+6cex\left(-2c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/6*(n*(6*c*e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*
(3*b*d + 4*a*e))*x + 3*c^2*e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))
*x^2 + 2*c^3*e^3*(8*c*d - b*e)*x^3 + 3*c^4*e^4*x^4 - 6*Sqrt[b^2 - 4*a*c]*e*
(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*
c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*Log[
a + x*(b + c*x)]))/c^4 + (d + e*x)^4*Log[d*(a + x*(b + c*x))^n])/(4*e)
```

fricas [A] time = 0.48, size = 880, normalized size = 2.60

$$\left[-\frac{3\,c^4e^3nx^4 + 2\left(8\,c^4de^2 - bc^3e^3\right)nx^3 + 3\left(12\,c^4d^2e - 4\,bc^3de^2 + \left(b^2c^2 - 2\,ac^3\right)e^3\right)nx^2 - 3\left(4\,c^3d^3 - 6\,bc^2d^2e + 4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c^3*e^3)*n*x^3 + 3*(12*c^4*d^2
*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*n*x^2 - 3*(4*c^3*d^3 - 6*b*c^
2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a*b*c)*e^3)*sqrt(b^2 - 4*a*c)*
n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*
e^2 - (b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3
 + 12*c^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3
)*d^2*e + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*
n)*log(c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^
2 + 4*c^4*d^3*x)*log(d))/c^4, -1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c
^3*e^3)*n*x^3 + 3*(12*c^4*d^2*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*
n*x^2 - 6*(4*c^3*d^3 - 6*b*c^2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a
*b*c)*e^3)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*e^2 - (
b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3 + 12*c
^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3)*d^2*e
 + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*n)*log(
c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^2 + 4*c
^4*d^3*x)*log(d))/c^4]
```

giac [A] time = 0.25, size = 553, normalized size = 1.64

$$\frac{6\,c^3nx^4e^3\log\left(cx^2 + bx + a\right) + 24\,c^3dnx^3e^2\log\left(cx^2 + bx + a\right) + 36\,c^3d^2nx^2e\log\left(cx^2 + bx + a\right) - 3\,c^3nx^4e^3 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/24*(6*c^3*n*x^4*e^3*log(c*x^2 + b*x + a) + 24*c^3*d*n*x^3*e^2*log(c*x^2 +
 b*x + a) + 36*c^3*d^2*n*x^2*e*log(c*x^2 + b*x + a) - 3*c^3*n*x^4*e^3 - 16*
c^3*d*n*x^3*e^2 - 36*c^3*d^2*n*x^2*e + 24*c^3*d^3*n*x*log(c*x^2 + b*x + a)
+ 6*c^3*x^4*e^3*log(d) + 24*c^3*d*x^3*e^2*log(d) + 36*c^3*d^2*x^2*e*log(d)
- 48*c^3*d^3*n*x + 2*b*c^2*n*x^3*e^3 + 12*b*c^2*d*n*x^2*e^2 + 36*b*c^2*d^2*
n*x*e + 24*c^3*d^3*x*log(d) - 3*b^2*c*n*x^2*e^3 + 6*a*c^2*n*x^2*e^3 - 24*b^
2*c*d*n*x*e^2 + 48*a*c^2*d*n*x*e^2 + 6*b^3*n*x*e^3 - 18*a*b*c*n*x*e^3)/c^3
+ 1/8*(4*b*c^3*d^3*n - 6*b^2*c^2*d^2*n*e + 12*a*c^3*d^2*n*e + 4*b^3*c*d*n*e
^2 - 12*a*b*c^2*d*n*e^2 - b^4*n*e^3 + 4*a*b^2*c*n*e^3 - 2*a^2*c^2*n*e^3)*lo
g(c*x^2 + b*x + a)/c^4 - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*
d^2*n*e + 24*a*b*c^3*d^2*n*e + 4*b^4*c*d*n*e^2 - 20*a*b^2*c^2*d*n*e^2 + 16*
a^2*c^3*d*n*e^2 - b^5*n*e^3 + 6*a*b^3*c*n*e^3 - 8*a^2*b*c^2*n*e^3)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [C] time = 0.87, size = 16059, normalized size = 47.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.84, size = 775, normalized size = 2.29

$$\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)-x^3\,\left(\frac{e^2\,n\,\left(b\,e+8\,c\,d\right)}{12\,c}-\frac{b\,e^3\,n}{6\,c}\right)-x\,\left(\frac{b\,\left(\frac{b\,\left(\frac{e^2\,n\,(b\,e+8\,c\,d)}{4\,c}-\frac{b\,e^3}{2\,c}\right.}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^3,x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x
^3) - x^3*((e^2*n*(b*e + 8*c*d))/(12*c) - (b*e^3*n)/(6*c)) - x*((b*((b*((e^
2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c + (a*e^3*n)/(2*c) - (d*e*n*(
b*e + 3*c*d))/c))/c - (a*((e^2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c
 + (d^2*n*(3*b*e + 4*c*d))/(2*c)) + x^2*((b*((e^2*n*(b*e + 8*c*d))/(4*c) -
(b*e^3*n)/(2*c)))/(2*c) + (a*e^3*n)/(4*c) - (d*e*n*(b*e + 3*c*d))/(2*c)) -
(log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^4*
e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n + b^3*e^3*n*(b^2 - 4*a*c)^(1/2) - 4
*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n - 4*b^3
*c*d*e^2*n + 6*b^2*c^2*d^2*e*n - 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 12*a*b
*c^2*d*e^2*n + 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) + 6*b*c^2*d^2*e*n*(b^2 -
 4*a*c)^(1/2) - 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4) - (e^3*n*x^4)
/8 - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*
(b^4*e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n - b^3*e^3*n*(b^2 - 4*a*c)^(1/2
) + 4*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n -
4*b^3*c*d*e^2*n + 6*b^2*c^2*d^2*e*n + 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 1
2*a*b*c^2*d*e^2*n - 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) - 6*b*c^2*d^2*e*n*(
b^2 - 4*a*c)^(1/2) + 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.84 $\int (d+ex)^2 \log \left(d \left(a+bx+cx^2\right)^n\right) \, dx$

Optimal. Leaf size=226

$$-\frac{nx\left(-ce(2ae+3bd)+b^2e^2+6c^2d^2\right)}{3c^2}-\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e}+\frac{n\sqrt{b^2-4}}{}$$

```
[Out] -1/3*(6*c^2*d^2+b^2*e^2-c*e*(2*a*e+3*b*d))*n*x/c^2-1/6*e*(-b*e+6*c*d)*n*x^2
/c-2/9*e^2*n*x^3-1/6*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*
x^2+b*x+a)/c^3/e+1/3*(e*x+d)^3*ln(d*(c*x^2+b*x+a)^n)/e+1/3*(3*c^2*d^2+b^2*e
^2-c*e*(a*e+3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1
/2)/c^3
```

Rubi [A] time = 0.32, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e}-\frac{nx\left(-ce(2ae+3bd)+b^2e^2+6c^2d^2\right)}{3c^2}+\frac{n\sqrt{b^2-4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*n*x)/(3*c^2) - (e*(6*c*d - b*
e)*n*x^2)/(6*c) - (2*e^2*n*x^3)/9 + (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2
 - c*e*(3*b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) - (
(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[a + b*x + c*x^2
])/(6*c^3*e) + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(d\left(a+bx+cx^2\right)^n\right) dx = \frac{(d+ex)^3 \log\left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n\int \frac{(b+2cx)(d+ex)^3}{a+bx+cx^2}\,dx}{3e}$$

$$= \frac{(d+ex)^3 \log\left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n\int \left(\frac{e\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)}{c^2} + \frac{e^2(6cd-\cdots}{c}\right.}{\cdots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+\cdots}{\cdots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+\cdots}{\cdots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 - \frac{(2cd\cdots}{\cdots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{\sqrt{b^2-\cdots}}{\cdots}$$

Mathematica [A] time = 0.45, size = 204, normalized size = 0.90

$$\frac{(d+ex)^3 \log\left(d(a+x(b+cx))^n\right) - \frac{n\left(cex\left(-3ce(4ae+6bd+bex)+6b^2e^2+2c^2\left(18d^2+9dex+2e^2x^2\right)\right)+3(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log(a+\cdots\right.}{6c^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/6*(n*(c*e*x*(6*b^2*e^2 - 3*c*e*(6*b*d + 4*a*e + b*e*x) + 2*c^2*(18*d^2
+ 9*d*e*x + 2*e^2*x^2)) - 6*Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*
(3*b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c*d - b*e)*(c^
2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + x*(b + c*x)]))/c^3 + (d + e*x)
^3*Log[d*(a + x*(b + c*x))^n])/(3*e)
```

fricas [A] time = 0.48, size = 567, normalized size = 2.51

$$\left[-\frac{4c^3e^2nx^3 + 3\left(6c^3de - bc^2e^2\right)nx^2 + 3\left(3c^2d^2 - 3bcde + \left(b^2-ac\right)e^2\right)\sqrt{b^2-4ac}\,n\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}\cdots}{cx^2+bx+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^2)*n*x^2 + 3*(3*c^2*d^2 -
3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(6*c^3
*d^2 - 3*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^
3*d*e*n*x^2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3
 - 3*a*b*c)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 +
 3*c^3*d^2*x)*log(d))/c^3, -1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^
2)*n*x^2 - 6*(3*c^2*d^2 - 3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(-b^2 + 4*a*c)*n
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(6*c^3*d^2 - 3*b
*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^3*d*e*n*x^
2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c
)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 + 3*c^3*d^2
*x)*log(d))/c^3]
```

giac [A] time = 0.21, size = 349, normalized size = 1.54

$$\frac{6\,c^2nx^3e^2\log\left(cx^2+bx+a\right)+18\,c^2dnx^2e\log\left(cx^2+bx+a\right)-4\,c^2nx^3e^2-18\,c^2dnx^2e+18\,c^2d^2nx\log\left(cx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/18*(6*c^2*n*x^3*e^2*log(c*x^2 + b*x + a) + 18*c^2*d*n*x^2*e*log(c*x^2 + b
*x + a) - 4*c^2*n*x^3*e^2 - 18*c^2*d*n*x^2*e + 18*c^2*d^2*n*x*log(c*x^2 + b
*x + a) + 6*c^2*x^3*e^2*log(d) + 18*c^2*d*x^2*e*log(d) - 36*c^2*d^2*n*x + 3
*b*c*n*x^2*e^2 + 18*b*c*d*n*x*e + 18*c^2*d^2*x*log(d) - 6*b^2*n*x*e^2 + 12*
a*c*n*x*e^2)/c^2 + 1/6*(3*b*c^2*d^2*n - 3*b^2*c*d*n*e + 6*a*c^2*d*n*e + b^3
*n*e^2 - 3*a*b*c*n*e^2)*log(c*x^2 + b*x + a)/c^3 - 1/3*(3*b^2*c^2*d^2*n - 1
2*a*c^3*d^2*n - 3*b^3*c*d*n*e + 12*a*b*c^2*d*n*e + b^4*n*e^2 - 5*a*b^2*c*n*
e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^3)
```

maple [C] time = 0.70, size = 7155, normalized size = 31.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.67, size = 457, normalized size = 2.02

$$\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{\frac{d^2\,n\,\sqrt{b^2-4\,a\,c}}{2}+\frac{b\,d^2\,n}{2}+a\,d\,e\,n}{c}-\frac{\frac{a\,b\,e^2\,n}{2}+\frac{b^2\,d\,e\,n}{2}+\frac{a\,e^2\,n\,\sqrt{b^2-4\,a}}{6}}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^2,x)
```

```
[Out] log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(((d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + (b*d^2*n)/2 + a*d*e*n)/c - ((a*b*e^2*n)/2 + (b^
2*d*e*n)/2 + (a*e^2*n*(b^2 - 4*a*c)^(1/2))/6 + (b*d*e*n*(b^2 - 4*a*c)^(1/2)
)/2)/c^2 + (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(b^2 - 4*a*c)^(1/2))/(6*c^3)) +
 x*((b*((e*n*(b*e + 6*c*d))/(3*c) - (2*b*e^2*n)/(3*c)))/c - (d*n*(b*e + 2*c
*d))/c + (2*a*e^2*n)/(3*c)) - log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c
*x*(b^2 - 4*a*c)^(1/2))*(((a*b*e^2*n)/2 + (b^2*d*e*n)/2 - (a*e^2*n*(b^2 - 4
*a*c)^(1/2))/6 - (b*d*e*n*(b^2 - 4*a*c)^(1/2))/2)/c^2 - ((b*d^2*n)/2 - (d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + a*d*e*n)/c - (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(
b^2 - 4*a*c)^(1/2))/(6*c^3)) + log(d*(a + b*x + c*x^2)^n)*(d^2*x + (e^2*x^3
)/3 + d*e*x^2) - x^2*((e*n*(b*e + 6*c*d))/(6*c) - (b*e^2*n)/(3*c)) - (2*e^2
*n*x^3)/9
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

3.85 $\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=154

$$-\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4c^2e}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{(d+ex)^2\log}{}$$

```
[Out] -1/2*(4*d-b*e/c)*n*x-1/2*e*n*x^2-1/4*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*
ln(c*x^2+b*x+a)/c^2/e+1/2*(e*x+d)^2*ln(d*(c*x^2+b*x+a)^n)/e+1/2*(-b*e+2*c*d
)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^2
```

Rubi [A] time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4c^2e}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{(d+ex)^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((4*d - (b*e)/c)*n*x)/2 - (e*n*x^2)/2 + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((2*c^2*d^2 + b^2*e^2 -
2*c*e*(b*d + a*e))*n*Log[a + b*x + c*x^2])/(4*c^2*e) + ((d + e*x)^2*Log[d*(
a + b*x + c*x^2)^n])/(2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int\frac{(b+2cx)(d+ex)^2}{a+bx+cx^2}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int\left(e\left(4d-\frac{be}{c}\right)+2e^2x+\frac{bcd^2-4acde+a}{\ldots}\right.}{2e}$$
$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int\frac{bcd^2\ldots}{\ldots}}{\ldots}$$
$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{\left(\left(b^2-4\ldots\right.\right.}{\ldots}$$
$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx\ldots\right.}{4c^2e}$$
$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \ldots$$

Mathematica [A] time = 0.21, size = 123, normalized size = 0.80

$$\frac{n\left(2ace+b^2(-e)+2bcd\right)\log(a+x(b+cx)) - 2n\sqrt{b^2-4ac}\,(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + 2cx\left(c(2d+ex)\log\left(d(a\ldots\right.\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]] + (2*b*c*d - b^2*e + 2*a*c*e)*n*Log[a + x*(b + c*x)] + 2*c*x*(b*e*n - c*
n*(4*d + e*x) + c*(2*d + e*x)*Log[d*(a + x*(b + c*x))^n]))/(4*c^2)
```

fricas [A] time = 0.45, size = 336, normalized size = 2.18

$$\left[-\frac{2\,c^2enx^2 + \sqrt{b^2-4\,ac}\,(2\,cd-be)n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right) + 2\left(4\,c^2d-bce\right)nx - \left(2\,c^2enx^2+4\,c^2\ldots\right.}{4\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*e*n*x^2 + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*n*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 -
2*a*c)*e)*n)*log(c*x^2 + b*x + a) - 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2,
-1/4*(2*c^2*e*n*x^2 - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*n*arctan(-sqrt(-b^
2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*
n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 - 2*a*c)*e)*n)*log(c*x^2 + b*x + a) -
 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2]
```

giac [A] time = 0.20, size = 188, normalized size = 1.22

$$\frac{cnx^2e\log\left(cx^2+bx+a\right)-cnx^2e+2\,cdnx\log\left(cx^2+bx+a\right)+cx^2e\log(d)-4\,cdnx+bnxe+2\,cdx\log(d)}{2\,c}+\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*(c*n*x^2*e*log(c*x^2 + b*x + a) - c*n*x^2*e + 2*c*d*n*x*log(c*x^2 + b*x
 + a) + c*x^2*e*log(d) - 4*c*d*n*x + b*n*x*e + 2*c*d*x*log(d))/c + 1/4*(2*b
*c*d*n - b^2*n*e + 2*a*c*n*e)*log(c*x^2 + b*x + a)/c^2 - 1/2*(2*b^2*c*d*n -
 8*a*c^2*d*n - b^3*n*e + 4*a*b*c*n*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [C] time = 0.70, size = 1706, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] (1/2*e*x^2+d*x)*ln((c*x^2+b*x+a)^n)-1/4*I*Pi*e*x^2*csgn(I*d*(c*x^2+b*x+a)^n
)^3-1/2*I*Pi*d*x*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n
)+1/2*I*Pi*d*x*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/4*I*csgn(I*d*(c*x^2+
b*x+a)^n)^2*csgn(I*d)*x^2*e*Pi-1/4*I*Pi*e*x^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a
)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*
x^2+b*x+a)^n)*x^2*e*Pi-1/2*I*Pi*d*x*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/2*I*Pi*d*
x*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/2*ln(d)*e*x^2-1/2*e
*n*x^2+ln(d)*d*x+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^
2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2
)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2
*c^2*d^2)^(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*
(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d
^2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d
*e+4*b^2*c^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^
2*c*d-2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b
^2*c^2*d^2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4
*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b*d+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*
e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d
*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4
*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^
2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4
*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3
*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*
e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b
^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-
16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b*d+1/2/c*b*e*n*x-
2*d*n*x+1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*c*e^2
+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x-(
```

```
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e
+4*b^2*c^2*d^2)^(1/2)-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*
e+4*b^2*c^2*d^2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e
^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.59, size = 242, normalized size = 1.57

$$\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)\left(\frac{e\,x^2}{2}+d\,x\right)-x\left(\frac{n\,\left(b\,e+4\,c\,d\right)}{2\,c}-\frac{b\,e\,n}{c}\right)-\frac{e\,n\,x^2}{2}+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x),x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d*x + (e*x^2)/2) - x*((n*(b*e + 4*c*d))/(2*c) -
 (b*e*n)/c) - (e*n*x^2)/2 + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*
x*(b^2 - 4*a*c)^(1/2))*(c*((a*e*n)/2 + (b*d*n)/2 - (d*n*(b^2 - 4*a*c)^(1/2)
)/2) - (b^2*e*n)/4 + (b*e*n*(b^2 - 4*a*c)^(1/2))/4))/c^2 - (log(b*(b^2 - 4*
a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^2*e*n)/4 - c*((a*
e*n)/2 + (b*d*n)/2 + (d*n*(b^2 - 4*a*c)^(1/2))/2) + (b*e*n*(b^2 - 4*a*c)^(1
/2))/4))/c^2
```

sympy [A] time = 158.89, size = 394, normalized size = 2.56

$$\begin{cases} \frac{aen\log\left(a+bx+cx^2\right)}{2c}-\frac{b^2en\log\left(a+bx+cx^2\right)}{4c^2}+\frac{bdn\log\left(a+bx+cx^2\right)}{2c}+\frac{benx}{2c}+\frac{ben\sqrt{-4ac+b^2}\log\left(a+bx+cx^2\right)}{4c^2}-\frac{ben\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4}}{}\right.}{2c^2} \\ -\frac{a^2en\log\left(a+bx\right)}{2b^2}+\frac{adn\log\left(a+bx\right)}{b}+\frac{aenx}{2b}+dnx\log\left(a+bx\right)-dnx+dx\log\left(d\right)+\frac{enx^2\log\left(a+bx\right)}{2}-\frac{enx^2}{4}+\frac{ex^2\log\left(d\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*e*n*log(a + b*x + c*x**2)/(2*c) - b**2*e*n*log(a + b*x + c*x**
2)/(4*c**2) + b*d*n*log(a + b*x + c*x**2)/(2*c) + b*e*n*x/(2*c) + b*e*n*sqr
t(-4*a*c + b**2)*log(a + b*x + c*x**2)/(4*c**2) - b*e*n*sqrt(-4*a*c + b**2)
*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(2*c**2) + d*n*x*log(a + b*x
+ c*x**2) - 2*d*n*x + d*x*log(d) + e*n*x**2*log(a + b*x + c*x**2)/2 - e*n*x
**2/2 + e*x**2*log(d)/2 - d*n*sqrt(-4*a*c + b**2)*log(a + b*x + c*x**2)/(2*
c) + d*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/c
, Ne(c, 0)), (-a**2*e*n*log(a + b*x)/(2*b**2) + a*d*n*log(a + b*x)/b + a*e*
n*x/(2*b) + d*n*x*log(a + b*x) - d*n*x + d*x*log(d) + e*n*x**2*log(a + b*x)
/2 - e*n*x**2/4 + e*x**2*log(d)/2, True))
```

3.86 $\int \log \left(d \left(a+b x+c x^2\right)^n\right) dx$

Optimal. Leaf size=79

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2523, 773, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(d\left(a+bx+cx^2\right)^n\right)dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{x(b+2cx)}{a+bx+cx^2}\,dx\\ &= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\ &= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx}\end{aligned}$$

$$\begin{aligned}&= -2nx+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(\left(b^2-4ac\right)n\right)\text{Subs}}{}\\ &= -2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+b\right.\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.99

$$\frac{2n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2cx\left(\log\left(d(a+x(b+cx))^n\right)-2n\right)+bn\log(a+x(b+cx))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

fricas [A] time = 0.44, size = 190, normalized size = 2.41

$$\left[-\frac{4\,cnx-2\,cx\log(d)-\sqrt{b^2-4\,ac}\,n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c},-\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

giac [A] time = 0.19, size = 92, normalized size = 1.16

$$nx\log\left(cx^2+bx+a\right)-\left(2\,n-\log(d)\right)x+\frac{bn\log\left(cx^2+bx+a\right)}{2\,c}-\frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

maple [A] time = 0.08, size = 118, normalized size = 1.49

$$\frac{4an\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{b^2n\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{bn\ln\left(cx^2+bx+a\right)}{2c}-2nx+x\ln\left(d\left(cx^2+bx+a\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 4/(4*a*c-b^2)^(1/2)*a*n*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))-1/(4*a*c-b^2)^(
1/2)*b^2/c*n*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))+1/2*b/c*n*ln(c*x^2+b*x+a)-
2*n*x+x*ln(d*(c*x^2+b*x+a)^n)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.00, size = 120, normalized size = 1.52

$$x\,\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)-2\,n\,x-\frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)}-\frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\sqrt{4\,a\,c-b^2}}{c}+\frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

sympy [A] time = 65.13, size = 275, normalized size = 3.48

$$\begin{cases}\frac{bn\log\left(\frac{b^2}{4c}+bx+cx^2\right)}{2c}+nx\log\left(\frac{b^2}{4c}+bx+cx^2\right)-2nx+x\log\left(d\right) \\ \frac{an\log\left(a+bx\right)}{b}+nx\log\left(a+bx\right)-nx+x\log\left(d\right) \\ \frac{2an\log\left(a+bx+cx^2\right)}{\sqrt{-4ac+b^2}}-\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}}-\frac{b^2n\log\left(a+bx+cx^2\right)}{2c\sqrt{-4ac+b^2}}+\frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+nx\log\left(\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((b*n*log(b**2/(4*c) + b*x + c*x**2)/(2*c) + n*x*log(b**2/(4*c) +
b*x + c*x**2) - 2*n*x + x*log(d), Eq(a, b**2/(4*c))), (a*n*log(a + b*x)/b +
 n*x*log(a + b*x) - n*x + x*log(d), Eq(c, 0)), (2*a*n*log(a + b*x + c*x**2)
/sqrt(-4*a*c + b**2) - 4*a*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/s
qrt(-4*a*c + b**2) - b**2*n*log(a + b*x + c*x**2)/(2*c*sqrt(-4*a*c + b**2))
 + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-4*a*c + b**
2)) + b*n*log(a + b*x + c*x**2)/(2*c) + n*x*log(a + b*x + c*x**2) - 2*n*x +
 x*log(d), True))
```

3.87 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$

Optimal. Leaf size=228

$$-\frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{e}-\frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(\cdots\right.}{e}$$

```
[Out] ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-n*ln(e*x+d)*ln(-e*(b+2*c*x-(-4*a*c+b^2)^(
1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*ln(e*x+d)*ln(-e*(b+2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(2*c*d-e*(b+(-4*
a*c+b^2)^(1/2))))/e
```

Rubi [A] time = 0.41, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\,\text{l}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + b*x + c*x^2)^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
```

```
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{n\int \frac{(b+2cx)\log(d+ex)}{a+bx+cx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{n\int \left(\frac{2c\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{e}$$

$$= \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{(2cn)\int \frac{\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{e} - \frac{(2cn)\int \frac{\log(\ldots}{b+\sqrt{b^2-\ldots}}}{e}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \ldots$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \ldots$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \ldots$$

Mathematica [A] time = 0.30, size = 226, normalized size = 0.99

$$-\frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{e} - \frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e} - \frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e} - \frac{n\log(d+ex)\log\left(-\frac{e(\ldots}{2c\ldots}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + x*(b + c*x))^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2 + bx + a\right)^n d\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

maple [C] time = 0.37, size = 493, normalized size = 2.16

$$-\frac{i\pi\,\mathrm{csgn}\,(id)\,\mathrm{csgn}\left(i\left(c\,x^2 + bx + a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2 + bx + a\right)^n\right)\ln\,(ex + d)}{2e} + \frac{i\pi\,\mathrm{csgn}\,(id)\,\mathrm{csgn}\left(id\left(c\,x^2 + bx\right.\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((c*x^2+b*x+a)^n)-1/e*n*ln(e*x+d)*ln((2*(e*x+d)*c+b*e-2*c*d+(
-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*ln
(e*x+d)*ln((-2*(e*x+d)*c-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((2*(e*x+d)*c+b*e-2*c*d+(-4*a*c*e^2
+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((-2*(e
*x+d)*c-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e
^2)^(1/2)))-1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d
*(c*x^2+b*x+a)^n)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^
2+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-
1/2*I*ln(e*x+d)/e*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(e*x+d)/e*ln(d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2 + bx + a\right)^n d\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(c\,x^2 + b\,x + a\right)^n\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x),x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d),x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)/(d + e*x), x)
```

3.88 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=165

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)\right)}{e(d+ex)}$$

```
[Out] -(-b*e+2*c*d)*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/2*(-b*e+2*c*d)*n*ln(c*x^2
+b*x+a)/e/(a*e^2-b*d*e+c*d^2)-ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e
 + a*e^2) - ((2*c*d - b*e)*n*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^2)) + ((
2*c*d - b*e)*n*Log[a + b*x + c*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - Log[d*
(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \frac{b+2cx}{(d+ex)\left(a+bx+cx^2\right)}\,dx}{e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right)dx}{e}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \frac{bcd-b^2e+2ace+c(2cd-be)x}{a+bx+cx^2}\,d}{e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} - \frac{\left(\left(b^2-4ac\right)n\right)\int \frac{1}{a+bx+cx^2}\,d}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx\right.\right.}{e(d+ex)}$$

$$= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+\right.}{2e\left(cd^2-bde+\right.}$$

Mathematica [A] time = 0.32, size = 166, normalized size = 1.01

$$-\frac{n\sqrt{4ac-b^2}\,\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{e(bd-ae)-cd^2}+\frac{n(be-2cd)\log(d+ex)}{e\left(e(ae-bd)+cd^2\right)}-\frac{n(be-2cd)\log(a+x(b+cx))}{2e\left(e(ae-bd)+cd^2\right)}-\frac{\log\left(d(a+x(b+cx))^n\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-(c*d^2) +
 e*(b*d - a*e))) + ((-2*c*d + b*e)*n*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d) +
a*e))) - ((-2*c*d + b*e)*n*Log[a + x*(b + c*x)])/(2*e*(c*d^2 + e*(-(b*d) +
a*e))) - Log[d*(a + x*(b + c*x))^n]/(e*(d + e*x))
```

fricas [A] time = 0.57, size = 429, normalized size = 2.60

$$\left[\frac{\left(e^2nx+den\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(\left(2\,cde-be^2\right)nx+\left(bde-2\,ae^2\right)n\right)\log\left(cx^2+b\right.}{2\left(cd^3e-bd^2e^2+ade^3+\left(cd^2e^2-bde\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((e^2*n*x + d*e*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((2*c*d*e - b*e
^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 + b*x + a) - 2*((2*c*d*e - b*e^2)*
n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) - 2*(c*d^2 - b*d*e + a*e^2)*log(d))
/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e^2 - b*d*e^3 + a*e^4)*x), 1/2*(2*
(e^2*n*x + d*e*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + ((2*c*d*e - b*e^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 +
 b*x + a) - 2*((2*c*d*e - b*e^2)*n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) -
2*(c*d^2 - b*d*e + a*e^2)*log(d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e
^2 - b*d*e^3 + a*e^4)*x)]
```

giac [A] time = 0.24, size = 284, normalized size = 1.72

$$\frac{(2\,cdn - bne)\log\left(cx^2 + bx + a\right)}{2\left(cd^2e - bde^2 + ae^3\right)} - \frac{\left(b^2n - 4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(cd^2 - bde + ae^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,cdnxe\log\left(xe + d\right) + cd^2n\log\left(cx^2 + bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*c*d*n - b*n*e)*log(c*x^2 + b*x + a)/(c*d^2*e - b*d*e^2 + a*e^3) - (b
^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*
e^2)*sqrt(-b^2 + 4*a*c)) - (2*c*d*n*x*e*log(x*e + d) + c*d^2*n*log(c*x^2 +
b*x + a) - b*d*n*e*log(c*x^2 + b*x + a) + 2*c*d^2*n*log(x*e + d) - b*n*x*e^
2*log(x*e + d) - b*d*n*e*log(x*e + d) + a*n*e^2*log(c*x^2 + b*x + a) + c*d^
2*log(d) - b*d*e*log(d) + a*e^2*log(d))/(c*d^2*x*e^2 + c*d^3*e - b*d*x*e^3
- b*d^2*e^2 + a*x*e^4 + a*d*e^3)
```

maple [C] time = 0.72, size = 6540, normalized size = 39.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.34, size = 590, normalized size = 3.58

$$\frac{\ln\left(d + e\,x\right)\,\left(b\,e\,n - 2\,c\,d\,n\right)}{c\,d^2\,e - b\,d\,e^2 + a\,e^3} - \frac{\ln\left(d\,\left(c\,x^2 + b\,x + a\right)^n\right)}{e\,\left(d + e\,x\right)} - \frac{\ln\left(\frac{2\,b\,c^2\,n^2}{e} + \frac{4\,c^3\,n^2\,x}{e} - \frac{n\,\left(b\,e - 2\,c\,d + e\,\sqrt{b^2 - 4\,a\,c}\right)\,\left(c^2\,n\,x\,\left(b\,e - 2\,c\,d\right) - c\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^2,x)
```

```
[Out] (log(d + e*x)*(b*e*n - 2*c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e) - log(d*(a + b
*x + c*x^2)^n)/(e*(d + e*x)) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*
(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(c^2*n*x*(b*e - 2*c*d) - c*n*(2*a*c*e
 - b^2*e + b*c*d) + (c*e*n*(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2
*x + 2*c^2*d^2*x + a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e -
2*b*c*d*e*x))/(2*(a*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2
*e)))*(e*((b*n)/2 + (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 +
 c*d^2*e) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*(2*c*d - b*e + e*(b
^2 - 4*a*c)^(1/2))*(c*n*(2*a*c*e - b^2*e + b*c*d) - c^2*n*x*(b*e - 2*c*d) +
 (c*e*n*(2*c*d - b*e + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2*x + 2*c^2*d^2*x +
a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*b*c*d*e*x))/(2*(a
*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)))*(e*((b*n)/2 -
 (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.89 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3} dx$

Optimal. Leaf size=259

$$\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2}-\frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4}}{}$$

```
[Out] 1/2*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)-1/2*(2*c^2*d^2+b^2*e^2-2*c
*e*(a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^2+1/4*(2*c^2*d^2+b^2*e^2-2*
c*e*(a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^2-1/2*ln(d*(c*x^2+b*
x+a)^n)/e/(e*x+d)^2+1/2*(-b*e+2*c*d)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A] time = 0.41, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2}-\frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((2*c*d - b*e)*n)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*(c*d^2 - b*d
*e + a*e^2)^2) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*Log[d + e*x])
/(2*e*(c*d^2 - b*d*e + a*e^2)^2) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e
))*n*Log[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)^2) - Log[d*(a + b*x
 + c*x^2)^n]/(2*e*(d + e*x)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2} + \frac{n\int \frac{b+2cx}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx}{2e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2} + \frac{n\int\left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{-2\cdots}{\cdots}\right.}{2\cdots}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} - \frac{\log\cdots}{\cdots}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} - \frac{\log\cdots}{\cdots}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} + \frac{(2\cdots}{\cdots}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2} - \frac{\left(2c^2d^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.56, size = 215, normalized size = 0.83

$$\frac{\frac{n(d+ex)\left(-2(d+ex)\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)+(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log(a+x(b+cx))-2e\sqrt{b^2-4ac}\,(d+ex)(be-2cd)\tanh^{-1}\left(\frac{b+2\cdots}{\sqrt{b^2-\cdots}}\right.\cdots}{\left(e(ae-bd)+cd^2\right)^2}\cdots}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)) - 2*Sqrt[b^2 - 4*
a*c]*e*(-2*c*d + b*e)*(d + e*x)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[d + e*x] + (2*c^2*d
^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[a + x*(b + c*x)]))/(c*d^2 +
 e*(-(b*d) + a*e))^2 - 2*Log[d*(a + x*(b + c*x))^n])/(4*e*(d + e*x)^2)
```

fricas [B] time = 2.20, size = 1341, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*n*x -
 ((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 - b*d*e^3)*n*x + (2*c*d^3*e -
b*d^2*e^2)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*c^2*d^4 - 3*b*c*d^3*
e - a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 +
(b^2 - 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d
*e^3)*n*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)
*n)*log(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*
e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2
*c^2*d^4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^
4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c
^2*d^6*e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*
e^3 + (c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*
d^2*e^5)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 +
 (b^2 + 2*a*c)*d^3*e^4)*x), 1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 +
 (b^2 + 2*a*c)*d*e^3)*n*x + 2*((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 -
 b*d*e^3)*n*x + (2*c*d^3*e - b*d^2*e^2)*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(
-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(2*c^2*d^4 - 3*b*c*d^3*e - a*b
*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 -
2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n
*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)*n)*log
(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*e^4)*n*
x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2*c^2*d^
4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^4 - 2*b
*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c^2*d^6*
e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*e^3 + (
c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*d^2*e^5
)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 + (b^2 +
 2*a*c)*d^3*e^4)*x)]
```

giac [B] time = 0.35, size = 887, normalized size = 3.42

$$\frac{\left(2\,c^2d^2n - 2\,bcdne + b^2ne^2 - 2\,acne^2\right)\log\left(cx^2 + bx + a\right)}{4\left(c^2d^4e - 2\,bcd^3e^2 + b^2d^2e^3 + 2\,acd^2e^3 - 2\,abde^4 + a^2e^5\right)} - \frac{\left(2\,b^2cdn - 8\,ac^2dn - b^3ne + 4\,abcne\right)\arctan\left(\right.}{2\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^2*d^2*n - 2*b*c*d*n*e + b^2*n*e^2 - 2*a*c*n*e^2)*log(c*x^2 + b*x +
 a)/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4
+ a^2*e^5) - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*n*e + 4*a*b*c*n*e)*arctan
((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*
a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d^2*n
*x^2*e^2*log(x*e + d) + 4*c^2*d^3*n*x*e*log(x*e + d) - 2*c^2*d^3*n*x*e + c^
2*d^4*n*log(c*x^2 + b*x + a) - 2*b*c*d^3*n*e*log(c*x^2 + b*x + a) + 2*c^2*d
^4*n*log(x*e + d) - 2*b*c*d*n*x^2*e^3*log(x*e + d) - 4*b*c*d^2*n*x*e^2*log(
x*e + d) - 2*b*c*d^3*n*e*log(x*e + d) - 2*c^2*d^4*n + 3*b*c*d^2*n*x*e^2 + 3
*b*c*d^3*n*e + b^2*d^2*n*e^2*log(c*x^2 + b*x + a) + 2*a*c*d^2*n*e^2*log(c*x
^2 + b*x + a) + b^2*n*x^2*e^4*log(x*e + d) - 2*a*c*n*x^2*e^4*log(x*e + d) +
 2*b^2*d*n*x*e^3*log(x*e + d) - 4*a*c*d*n*x*e^3*log(x*e + d) + b^2*d^2*n*e^
2*log(x*e + d) - 2*a*c*d^2*n*e^2*log(x*e + d) + c^2*d^4*log(d) - 2*b*c*d^3*
e*log(d) - b^2*d*n*x*e^3 - 2*a*c*d*n*x*e^3 - b^2*d^2*n*e^2 - 2*a*c*d^2*n*e^
2 - 2*a*b*d*n*e^3*log(c*x^2 + b*x + a) + b^2*d^2*e^2*log(d) + 2*a*c*d^2*e^2
*log(d) + a*b*n*x*e^4 + a*b*d*n*e^3 + a^2*n*e^4*log(c*x^2 + b*x + a) - 2*a*
b*d*e^3*log(d) + a^2*e^4*log(d))/(c^2*d^4*x^2*e^3 + 2*c^2*d^5*x*e^2 + c^2*d
```

```
^6*e - 2*b*c*d^3*x^2*e^4 - 4*b*c*d^4*x*e^3 - 2*b*c*d^5*e^2 + b^2*d^2*x^2*e^
5 + 2*a*c*d^2*x^2*e^5 + 2*b^2*d^3*x*e^4 + 4*a*c*d^3*x*e^4 + b^2*d^4*e^3 + 2
*a*c*d^4*e^3 - 2*a*b*d*x^2*e^6 - 4*a*b*d^2*x*e^5 - 2*a*b*d^3*e^4 + a^2*x^2*
e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

maple [C] time = 1.19, size = 14679, normalized size = 56.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.74, size = 1715, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^3,x)
```

```
[Out] (log(3*b^2*c^3*d^4 - 12*a*c^4*d^4 - 2*b^5*e^4*x - 12*a^3*c^2*e^4 - 2*a*b^4*
e^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) + 1
1*a^2*b^2*c*e^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2 + 40*a^2*c^3*d^2*e^2 + 2*
a*b^3*e^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^3
*d^3*e + 6*a*b^3*c*d*e^3 + 12*a*b^3*c*e^4*x - 32*a*c^4*d^3*e*x + 8*b^4*c*d*
e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1
/2) - 24*a^2*b*c^2*d*e^3 - 16*a^2*b*c^2*e^4*x + 32*a^2*c^3*d*e^3*x + 8*b^2*
c^3*d^3*e*x + 16*a^2*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 -
 4*a*c)^(1/2) + b^3*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 -
4*a*c)^(1/2) - 14*a*b^2*c^2*d^2*e^2 - 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2
*e^2*(b^2 - 4*a*c)^(1/2) - 20*a*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*
c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) -
8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2)
- 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 48*a*b*c^3*d^2*e^2*x - 40*a*b^2*c^2
*d*e^3*x + 20*a*b*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2))*(e*((c*d*n*(b^2 - 4*a*c)
^(1/2))/2 - (b*c*d*n)/2) - e^2*((a*c*n)/2 - (b^2*n)/4 + (b*n*(b^2 - 4*a*c)^
(1/2))/4) + (c^2*d^2*n)/2))/(a^2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^
4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e^2) - (log(d + e*x)*(e^2*(b^2*n - 2*a*c*n) +
 2*c^2*d^2*n - 2*b*c*d*e*n))/(2*a^2*e^5 + 2*c^2*d^4*e + 2*b^2*d^2*e^3 - 4*a
*b*d*e^4 + 4*a*c*d^2*e^3 - 4*b*c*d^3*e^2) + (log(2*a*b^4*e^4 + 12*a*c^4*d^4
 + 2*b^5*e^4*x + 12*a^3*c^2*e^4 - 3*b^2*c^3*d^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)
^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^2*c*e^4 + 2*b^3*c^2*d^3
*e - b^4*c*d^2*e^2 - 40*a^2*c^3*d^2*e^2 + 2*a*b^3*e^4*(b^2 - 4*a*c)^(1/2) +
 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) - 8*a*b*c^3*d^3*e - 6*a*b^3*c*d*e^3 - 12*a
*b^3*c*e^4*x + 32*a*c^4*d^3*e*x - 8*b^4*c*d*e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*
a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1/2) + 24*a^2*b*c^2*d*e^3 + 16*a
```

```
^2*b*c^2*e^4*x - 32*a^2*c^3*d*e^3*x - 8*b^2*c^3*d^3*e*x + 16*a^2*c^2*d*e^3*
(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 - 4*a*c)^(1/2) + b^3*c*d^2*e^2*(
b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 14*a*b^2*c^2*d^2
*e^2 + 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 20*a
*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/
2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(
1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)
^(1/2) - 48*a*b*c^3*d^2*e^2*x + 40*a*b^2*c^2*d*e^3*x + 20*a*b*c^2*d*e^3*x*(
b^2 - 4*a*c)^(1/2))*(e^2*((b^2*n)/4 - (a*c*n)/2 + (b*n*(b^2 - 4*a*c)^(1/2))
/4) - e*((c*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b*c*d*n)/2) + (c^2*d^2*n)/2))/(a^
2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e
^2) - log(d*(a + b*x + c*x^2)^n)/(2*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (n*(b*e
- 2*c*d))/((2*d*e + 2*e^2*x)*(a*e^2 + c*d^2 - b*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.90 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx$

Optimal. Leaf size=356

$$\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{n(2cd-be)\log}{}$$

```
[Out] 1/6*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+1/3*(2*c^2*d^2+b^2*e^2-2
*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-1/3*(-b*e+2*c*d)*(c^2*d^2
+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^3+1/6*(-b*e+2*c
*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^
2)^3-1/3*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^3+1/3*(3*c^2*d^2+b^2*e^2-c*e*(a*e+
3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b
*d*e+c*d^2)^3
```

Rubi [A] time = 0.62, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{n(2cd-be)\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((2*c*d - b*e)*n)/(6*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((2*c^2*d^2 +
 b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(3*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
+ (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*n*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*(c*d^2 - b*d*e + a*e^2)^3) - ((2*c*d - b*e
)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[d + e*x])/(3*e*(c*d^2 - b*d
*e + a*e^2)^3) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*L
og[a + b*x + c*x^2])/(6*e*(c*d^2 - b*d*e + a*e^2)^3) - Log[d*(a + b*x + c*x
^2)^n]/(3*e*(d + e*x)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}+\frac{n\int\frac{b+2cx}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx}{3e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}+\frac{n\int\left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\ldots$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{(2cd-be)\ldots}{\ldots}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{(2cd-be)\ldots}{\ldots}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{(2cd-be)\ldots}{\ldots}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\sqrt{b^2-4ac}\ldots}{\ldots}$$

Mathematica [A] time = 1.21, size = 310, normalized size = 0.87

$$\frac{\frac{n(d+ex)\left(2(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\left(e(ae-bd)+cd^2\right)-2(d+ex)^2(2cd-be)\log(d+ex)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)+(d+ex)^2(2cd-be)\left(-ce(3ae+bd)\ldots\right.\right.}{\left(e(ae-bd)+cd^2\right)^3}}{6\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((n*(d + e*x)*((2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2 + 2*(c*d^2 + e*(-
(b*d) + a*e))*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 2*Sqrt[
b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*(d + e*x)^2*ArcTan
```

```
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e
*(b*d + 3*a*e))*(d + e*x)^2*Log[d + e*x] + (2*c*d - b*e)*(c^2*d^2 + b^2*e^2
 - c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[a + x*(b + c*x)]))/(c*d^2 + e*(-(b*d)
 + a*e))^3 - 2*Log[d*(a + x*(b + c*x))^n])/(6*e*(d + e*x)^3)
```

fricas [B] time = 15.09, size = 3013, normalized size = 8.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*c^3*d^4*e^2 - 4*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a
*b^2 - 2*a^2*c)*e^6)*n*x^2 + (10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 +
 4*(4*b^2*c + a*c^2)*d^3*e^3 - (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c
)*d*e^5)*n*x - ((3*c^2*d^2*e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(
3*c^2*d^3*e^3 - 3*b*c*d^2*e^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2
 - 3*b*c*d^3*e^3 + (b^2 - a*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2
+ (b^2 - a*c)*d^3*e^3)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (6*c^3*d^6 -
13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 - 3*(b^3 + 2*a
*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^3 - 3*b*c^2*
d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3
*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*
d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e
^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*e^5 - 2*a^3*
e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 - 6*(a*b^2 + a
^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*b*c^2*d^2*e^
4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3*d^4*e
^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*d*e^5)
*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e^3 - (
b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b^2*c - 2*a*c
^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3*d^6 - 3*b*c
^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a
*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e - 3*b*c^2*d^8
*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^3 - (b^3 + 6
*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*b*c^2*d^5*e^
5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3 + 6*a*b*c)*
d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b*c^2*d^6*e^4
 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b^3 + 6*a*b*c
)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3*b*c^2*d^7*e
^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4 - (b^3 + 6*a
*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x), 1/6*(2*(2*c^3*d^4*e^2 - 4*b*
c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a*b^2 - 2*a^2*c)*e^6)*n*x^2 +
(10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 + 4*(4*b^2*c + a*c^2)*d^3*e^3
- (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c)*d*e^5)*n*x + 2*((3*c^2*d^2*
e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(3*c^2*d^3*e^3 - 3*b*c*d^2*e
^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2 - 3*b*c*d^3*e^3 + (b^2 - a
*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2 + (b^2 - a*c)*d^3*e^3)*n)*s
qrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (
6*c^3*d^6 - 13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 -
3*(b^3 + 2*a*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^
3 - 3*b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^
3 + 3*(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3
 - 3*a*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*
a*c^2)*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*
e^5 - 2*a^3*e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 -
6*(a*b^2 + a^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*
b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*
(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a
```

```
*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)
*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b
^2*c - 2*a*c^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3
*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2
- (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e -
 3*b*c^2*d^8*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^
3 - (b^3 + 6*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*
b*c^2*d^5*e^5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3
 + 6*a*b*c)*d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b
*c^2*d^6*e^4 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b
^3 + 6*a*b*c)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3
*b*c^2*d^7*e^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4
- (b^3 + 6*a*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x)]
```

giac [B] time = 0.61, size = 1963, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*c^3*d^3*n - 3*b*c^2*d^2*n*e + 3*b^2*c*d*n*e^2 - 6*a*c^2*d*n*e^2 - b^
3*n*e^3 + 3*a*b*c*n*e^3)*log(c*x^2 + b*x + a)/(c^3*d^6*e - 3*b*c^2*d^5*e^2
+ 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b
^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*(3*b^2*c^2*d^
2*n - 12*a*c^3*d^2*n - 3*b^3*c*d*n*e + 12*a*b*c^2*d*n*e + b^4*n*e^2 - 5*a*b
^2*c*n*e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*
d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a
*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)
*sqrt(-b^2 + 4*a*c)) - 1/6*(4*c^3*d^3*n*x^3*e^3*log(x*e + d) + 12*c^3*d^4*n
*x^2*e^2*log(x*e + d) + 12*c^3*d^5*n*x*e*log(x*e + d) - 4*c^3*d^4*n*x^2*e^2
 - 10*c^3*d^5*n*x*e + 2*c^3*d^6*n*log(c*x^2 + b*x + a) - 6*b*c^2*d^5*n*e*lo
g(c*x^2 + b*x + a) + 4*c^3*d^6*n*log(x*e + d) - 6*b*c^2*d^2*n*x^3*e^4*log(x
*e + d) - 18*b*c^2*d^3*n*x^2*e^3*log(x*e + d) - 18*b*c^2*d^4*n*x*e^2*log(x*
e + d) - 6*b*c^2*d^5*n*e*log(x*e + d) - 6*c^3*d^6*n + 8*b*c^2*d^3*n*x^2*e^3
 + 21*b*c^2*d^4*n*x*e^2 + 13*b*c^2*d^5*n*e + 6*b^2*c*d^4*n*e^2*log(c*x^2 +
b*x + a) + 6*a*c^2*d^4*n*e^2*log(c*x^2 + b*x + a) + 6*b^2*c*d*n*x^3*e^5*log
(x*e + d) - 12*a*c^2*d*n*x^3*e^5*log(x*e + d) + 18*b^2*c*d^2*n*x^2*e^4*log(
x*e + d) - 36*a*c^2*d^2*n*x^2*e^4*log(x*e + d) + 18*b^2*c*d^3*n*x*e^3*log(x
*e + d) - 36*a*c^2*d^3*n*x*e^3*log(x*e + d) + 6*b^2*c*d^4*n*e^2*log(x*e + d
) - 12*a*c^2*d^4*n*e^2*log(x*e + d) + 2*c^3*d^6*log(d) - 6*b*c^2*d^5*e*log(
d) - 6*b^2*c*d^2*n*x^2*e^4 - 16*b^2*c*d^3*n*x*e^3 - 4*a*c^2*d^3*n*x*e^3 - 1
0*b^2*c*d^4*n*e^2 - 4*a*c^2*d^4*n*e^2 - 2*b^3*d^3*n*e^3*log(c*x^2 + b*x + a
) - 12*a*b*c*d^3*n*e^3*log(c*x^2 + b*x + a) - 2*b^3*n*x^3*e^6*log(x*e + d)
+ 6*a*b*c*n*x^3*e^6*log(x*e + d) - 6*b^3*d*n*x^2*e^5*log(x*e + d) + 18*a*b*
c*d*n*x^2*e^5*log(x*e + d) - 6*b^3*d^2*n*x*e^4*log(x*e + d) + 18*a*b*c*d^2*
n*x*e^4*log(x*e + d) - 2*b^3*d^3*n*e^3*log(x*e + d) + 6*a*b*c*d^3*n*e^3*log
(x*e + d) + 6*b^2*c*d^4*e^2*log(d) + 6*a*c^2*d^4*e^2*log(d) + 2*b^3*d*n*x^2
*e^5 + 5*b^3*d^2*n*x*e^4 + 6*a*b*c*d^2*n*x*e^4 + 3*b^3*d^3*n*e^3 + 6*a*b*c*
d^3*n*e^3 + 6*a*b^2*d^2*n*e^4*log(c*x^2 + b*x + a) + 6*a^2*c*d^2*n*e^4*log(
c*x^2 + b*x + a) - 2*b^3*d^3*e^3*log(d) - 12*a*b*c*d^3*e^3*log(d) - 2*a*b^2
*n*x^2*e^6 + 4*a^2*c*n*x^2*e^6 - 6*a*b^2*d*n*x*e^5 + 6*a^2*c*d*n*x*e^5 - 4*
a*b^2*d^2*n*e^4 + 2*a^2*c*d^2*n*e^4 - 6*a^2*b*d*n*e^5*log(c*x^2 + b*x + a)
+ 6*a*b^2*d^2*e^4*log(d) + 6*a^2*c*d^2*e^4*log(d) + a^2*b*n*x*e^6 + a^2*b*d
*n*e^5 + 2*a^3*n*e^6*log(c*x^2 + b*x + a) - 6*a^2*b*d*e^5*log(d) + 2*a^3*e^
6*log(d))/(c^3*d^6*x^3*e^4 + 3*c^3*d^7*x^2*e^3 + 3*c^3*d^8*x*e^2 + c^3*d^9*
e - 3*b*c^2*d^5*x^3*e^5 - 9*b*c^2*d^6*x^2*e^4 - 9*b*c^2*d^7*x*e^3 - 3*b*c^2
*d^8*e^2 + 3*b^2*c*d^4*x^3*e^6 + 3*a*c^2*d^4*x^3*e^6 + 9*b^2*c*d^5*x^2*e^5
+ 9*a*c^2*d^5*x^2*e^5 + 9*b^2*c*d^6*x*e^4 + 9*a*c^2*d^6*x*e^4 + 3*b^2*c*d^7
*e^3 + 3*a*c^2*d^7*e^3 - b^3*d^3*x^3*e^7 - 6*a*b*c*d^3*x^3*e^7 - 3*b^3*d^4*
```

```
x^2*e^6 - 18*a*b*c*d^4*x^2*e^6 - 3*b^3*d^5*x*e^5 - 18*a*b*c*d^5*x*e^5 - b^3
*d^6*e^4 - 6*a*b*c*d^6*e^4 + 3*a*b^2*d^2*x^3*e^8 + 3*a^2*c*d^2*x^3*e^8 + 9*
a*b^2*d^3*x^2*e^7 + 9*a^2*c*d^3*x^2*e^7 + 9*a*b^2*d^4*x*e^6 + 9*a^2*c*d^4*x
*e^6 + 3*a*b^2*d^5*e^5 + 3*a^2*c*d^5*e^5 - 3*a^2*b*d*x^3*e^9 - 9*a^2*b*d^2*
x^2*e^8 - 9*a^2*b*d^3*x*e^7 - 3*a^2*b*d^4*e^6 + a^3*x^3*e^10 + 3*a^3*d*x^2*
e^9 + 3*a^3*d^2*x*e^8 + a^3*d^3*e^7)
```

maple [C] time = 1.04, size = 306209, normalized size = 860.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.30, size = 2707, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^4,x)
```

```
[Out] (log(d + e*x)*(e^3*(b^3*n - 3*a*b*c*n) + e^2*(6*a*c^2*d*n - 3*b^2*c*d*n) -
2*c^3*d^3*n + 3*b*c^2*d^2*e*n))/(3*a^3*e^7 + 3*c^3*d^6*e - 3*b^3*d^3*e^4 +
9*a*b^2*d^2*e^5 + 9*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - 9*b*c^2*d^5*e^2 + 9*b
^2*c*d^4*e^3 - 9*a^2*b*d*e^6 - 18*a*b*c*d^3*e^4) - (log(32*a*b^5*e^5 - 2*a*
e^5*(b^2 - 4*a*c)^(5/2) - 192*a*c^5*d^5 + 32*b^6*e^5*x + 48*b^2*c^4*d^5 - 1
8*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) - 3*b^5*e^5*x*(b^2 - 4*a*c)^(1/2) + 96*c^5*
d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320*a^3*b*c^2*e^5 - 704*a^3
*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 - 64*a^3*c^3*e^5*x + 1152*
a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 - 33*b*d*e^4*(b^2 - 4*a*c)^(5/2) - 11*
b*e^5*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^2*e^5*(b^2 - 4*a*c)^(3/2) - 6*a*b^4*e^
5*(b^2 - 4*a*c)^(1/2) + 48*b*c^4*d^5*(b^2 - 4*a*c)^(1/2) + 18*b^3*d*e^4*(b^
2 - 4*a*c)^(3/2) + 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2) + 44*c*d^2*e^3*(b^2 - 4
*a*c)^(5/2) + 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) + 22*c*d*e^4*x*(b^2 - 4*a*c)
^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 + 120*b^3*c^2*d^3*e^2*(b^2 -
 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e*x - 160*b^5*c*d*e^4*x +
 144*b^2*c^4*d^4*e*x - 72*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 120*b^2*c^3*d
^4*e*(b^2 - 4*a*c)^(1/2) - 60*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 144*c^3*d
^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^2 + 320*a*b^3*c^2*d^2*e^
3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4 + 400*a^2*b^2*c^2*e^5*x
+ 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x + 304*b^4*c^2*d^2*e^3*x -
216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^2*c^3*d^2*e^3*x + 240*b^
2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 120*b^3*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(
1/2) - 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 108*b^2*c*d*e^4*x*(b^2 - 4*a
*c)^(3/2) + 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 1152*a*b*c^4*d^3*e^2*x +
```

```
 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e^3*((b^3*n)/6 - (b^2*n*(
b^2 - 4*a*c)^(1/2))/6 + (a*c*n*(b^2 - 4*a*c)^(1/2))/6 - (a*b*c*n)/2) + e^2*
(a*c^2*d*n - (b^2*c*d*n)/2 + (b*c*d*n*(b^2 - 4*a*c)^(1/2))/2) + e*((b*c^2*d
^2*n)/2 - (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3*d^3*n)/3))/(a^3*e^7 + c
^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^
5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e^6 - 6*a*b*c*d^3*e^4) -
(log(2*a*e^5*(b^2 - 4*a*c)^(5/2) + 32*a*b^5*e^5 - 192*a*c^5*d^5 + 32*b^6*e^
5*x + 48*b^2*c^4*d^5 + 18*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) + 3*b^5*e^5*x*(b^2
- 4*a*c)^(1/2) - 96*c^5*d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320
*a^3*b*c^2*e^5 - 704*a^3*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 -
64*a^3*c^3*e^5*x + 1152*a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 + 33*b*d*e^4*(
b^2 - 4*a*c)^(5/2) + 11*b*e^5*x*(b^2 - 4*a*c)^(5/2) + 24*a*b^2*e^5*(b^2 - 4
*a*c)^(3/2) + 6*a*b^4*e^5*(b^2 - 4*a*c)^(1/2) - 48*b*c^4*d^5*(b^2 - 4*a*c)^
(1/2) - 18*b^3*d*e^4*(b^2 - 4*a*c)^(3/2) - 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2)
 - 44*c*d^2*e^3*(b^2 - 4*a*c)^(5/2) - 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) - 22
*c*d*e^4*x*(b^2 - 4*a*c)^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 - 12
0*b^3*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e
*x - 160*b^5*c*d*e^4*x + 144*b^2*c^4*d^4*e*x + 72*b*c^2*d^3*e^2*(b^2 - 4*a*
c)^(3/2) + 120*b^2*c^3*d^4*e*(b^2 - 4*a*c)^(1/2) + 60*b^4*c*d^2*e^3*(b^2 -
4*a*c)^(1/2) - 144*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^
2 + 320*a*b^3*c^2*d^2*e^3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4
+ 400*a^2*b^2*c^2*e^5*x + 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x +
304*b^4*c^2*d^2*e^3*x + 216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^
2*c^3*d^2*e^3*x - 240*b^2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 120*b^3*c^2*d
^2*e^3*x*(b^2 - 4*a*c)^(1/2) + 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 108*
b^2*c*d*e^4*x*(b^2 - 4*a*c)^(3/2) - 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) +
1152*a*b*c^4*d^3*e^2*x + 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e
^3*((b^3*n)/6 + (b^2*n*(b^2 - 4*a*c)^(1/2))/6 - (a*c*n*(b^2 - 4*a*c)^(1/2))
/6 - (a*b*c*n)/2) - e^2*((b^2*c*d*n)/2 - a*c^2*d*n + (b*c*d*n*(b^2 - 4*a*c)
^(1/2))/2) + e*((b*c^2*d^2*n)/2 + (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3
*d^3*n)/3))/(a^3*e^7 + c^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*
d^4*e^3 + 3*a^2*c*d^2*e^5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e
^6 - 6*a*b*c*d^3*e^4) - ((a*b*e^3*n - 6*c^2*d^3*n - 3*b^2*d*e^2*n + 2*a*c*d
*e^2*n + 7*b*c*d^2*e*n)/(2*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 -
 2*b*c*d^3*e + 2*a*c*d^2*e^2)) - (n*x*(b^2*e^3 + 2*c^2*d^2*e - 2*a*c*e^3 -
2*b*c*d*e^2))/(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e
+ 2*a*c*d^2*e^2))/(3*d^2*e + 3*e^3*x^2 + 6*d*e^2*x) - log(d*(a + b*x + c*x^
2)^n)/(3*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx$

Optimal. Leaf size=519

$$\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8e\left(ae^2-bde+cd^2\right)^4}-\frac{n\log(\cdots}{\cdots}$$

```
[Out] 1/12*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+1/8*(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2+1/4*(-b*e+2*c*d)*(c^2*
d^2+b^2*e^2-c*e*(3*a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)-1/4*(2*c^4*d
^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2
+6*a*b*d*e+3*b^2*d^2))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^4+1/8*(2*c^4*d^4+b
^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a
*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^4-1/4*ln(d*(c*x^
2+b*x+a)^n)/e/(e*x+d)^4+1/4*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d)
)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c
*d^2)^4
```

Rubi [A] time = 1.01, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8e\left(ae^2-bde+cd^2\right)^4}-\frac{n\log(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((2*c*d - b*e)*n)/(12*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + ((2*c^2*d^2
+ b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(8*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^
2) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n)/(4*e*(c*d^2
- b*d*e + a*e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(
4*(c*d^2 - b*d*e + a*e^2)^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a
*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^
2))*n*Log[d + e*x])/(4*e*(c*d^2 - b*d*e + a*e^2)^4) + ((2*c^4*d^4 + b^4*e^4
 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d
^2 + 6*a*b*d*e + a^2*e^2))*n*Log[a + b*x + c*x^2])/(8*e*(c*d^2 - b*d*e + a*
e^2)^4) - Log[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4} + \frac{n\int \frac{b+2cx}{(d+ex)^4\left(a+bx+cx^2\right)}\,dx}{4e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4} + \frac{n\int\left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^4} + \frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)^3} + \right.}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd-be}{4e(}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd-be}{4e(}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd-be}{4e(}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd-be}{4e(}$$

Mathematica [A] time = 2.03, size = 469, normalized size = 0.90

$$\frac{n(d+ex)\left(-6(d+ex)^3\log(d+ex)\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)+3(d+ex)^3\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3 + 3*(c*d^2 + e*
(-(b*d) + a*e))^2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 6*(
2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a
*e))*(d + e*x)^2 + 6*Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
 - 2*c*e*(b*d + a*e))*(d + e*x)^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] -
6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e
) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[d + e*x] +
 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*
e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[a + x*(b
+ c*x)]))/(c*d^2 + e*(-(b*d) + a*e))^4 - 6*Log[d*(a + x*(b + c*x))^n])/(24*
e*(d + e*x)^4)
```

fricas [B]   time = 92.03, size = 5824, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [1/24*(6*(2*c^4*d^5*e^3 - 5*b*c^3*d^4*e^4 + 2*(3*b^2*c^2 - 2*a*c^3)*d^3*e^5
 - 2*(2*b^3*c - 3*a*b*c^2)*d^2*e^6 + (b^4 - 6*a^2*c^2)*d*e^7 - (a*b^3 - 3*a
^2*b*c)*e^8)*n*x^3 + 3*(14*c^4*d^6*e^2 - 36*b*c^3*d^5*e^3 + (43*b^2*c^2 - 2
2*a*c^3)*d^4*e^4 - 4*(7*b^3*c - 8*a*b*c^2)*d^3*e^5 + (7*b^4 + 4*a*b^2*c - 3
8*a^2*c^2)*d^2*e^6 - 4*(2*a*b^3 - 5*a^2*b*c)*d*e^7 + (a^2*b^2 - 2*a^3*c)*e^
8)*n*x^2 + 2*(26*c^4*d^7*e - 70*b*c^3*d^6*e^2 - a^3*b*e^8 + 12*(7*b^2*c^2 -
 2*a*c^3)*d^5*e^3 - (53*b^3*c - 27*a*b*c^2)*d^4*e^4 + (13*b^4 + 24*a*b^2*c
- 54*a^2*c^2)*d^3*e^5 - 6*(3*a*b^3 - 4*a^2*b*c)*d^2*e^6 + 2*(3*a^2*b^2 - 2*
a^3*c)*d*e^7)*n*x + 3*((4*c^3*d^3*e^5 - 6*b*c^2*d^2*e^6 + 4*(b^2*c - a*c^2)
*d*e^7 - (b^3 - 2*a*b*c)*e^8)*n*x^4 + 4*(4*c^3*d^4*e^4 - 6*b*c^2*d^3*e^5 +
4*(b^2*c - a*c^2)*d^2*e^6 - (b^3 - 2*a*b*c)*d*e^7)*n*x^3 + 6*(4*c^3*d^5*e^3
 - 6*b*c^2*d^4*e^4 + 4*(b^2*c - a*c^2)*d^3*e^5 - (b^3 - 2*a*b*c)*d^2*e^6)*n
*x^2 + 4*(4*c^3*d^6*e^2 - 6*b*c^2*d^5*e^3 + 4*(b^2*c - a*c^2)*d^4*e^4 - (b^
3 - 2*a*b*c)*d^3*e^5)*n*x + (4*c^3*d^7*e - 6*b*c^2*d^6*e^2 + 4*(b^2*c - a*c
^2)*d^5*e^3 - (b^3 - 2*a*b*c)*d^4*e^4)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 + (22*c^4*d^8 - 62*b*c^3*d^7*e - 2*a^3*b*d*e^7 + 3*(25*b^2*c^2 - 2*a*c^3)*
d^6*e^2 - 2*(23*b^3*c + 3*a*b*c^2)*d^5*e^3 + (11*b^4 + 36*a*b^2*c - 30*a^2*
c^2)*d^4*e^4 - 6*(3*a*b^3 - a^2*b*c)*d^3*e^5 + (9*a^2*b^2 - 2*a^3*c)*d^2*e^
6)*n + 3*((2*c^4*d^4*e^4 - 4*b*c^3*d^3*e^5 + 6*(b^2*c^2 - 2*a*c^3)*d^2*e^6
 - 4*(b^3*c - 3*a*b*c^2)*d*e^7 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^8)*n*x^4 +
4*(2*c^4*d^5*e^3 - 4*b*c^3*d^4*e^4 + 6*(b^2*c^2 - 2*a*c^3)*d^3*e^5 - 4*(b^3
*c - 3*a*b*c^2)*d^2*e^6 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^7)*n*x^3 + 6*(2
*c^4*d^6*e^2 - 4*b*c^3*d^5*e^3 + 6*(b^2*c^2 - 2*a*c^3)*d^4*e^4 - 4*(b^3*c -
 3*a*b*c^2)*d^3*e^5 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^6)*n*x^2 + 4*(2*c
^4*d^7*e - 4*b*c^3*d^6*e^2 + 6*(b^2*c^2 - 2*a*c^3)*d^5*e^3 - 4*(b^3*c - 3*a
*b*c^2)*d^4*e^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^5)*n*x + (4*b*c^3*d^7
*e + 8*a^3*b*d*e^7 - 2*a^4*e^8 - 2*(3*b^2*c^2 + 10*a*c^3)*d^6*e^2 + 4*(b^3*
c + 9*a*b*c^2)*d^5*e^3 - (b^4 + 28*a*b^2*c + 10*a^2*c^2)*d^4*e^4 + 8*(a*b^3
 + 3*a^2*b*c)*d^3*e^5 - 4*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)*n)*log(c*x^2 + b*x
 + a) - 6*((2*c^4*d^4*e^4 - 4*b*c^3*d^3*e^5 + 6*(b^2*c^2 - 2*a*c^3)*d^2*e^6
 - 4*(b^3*c - 3*a*b*c^2)*d*e^7 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^8)*n*x^4 +
 4*(2*c^4*d^5*e^3 - 4*b*c^3*d^4*e^4 + 6*(b^2*c^2 - 2*a*c^3)*d^3*e^5 - 4*(b^
3*c - 3*a*b*c^2)*d^2*e^6 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^7)*n*x^3 + 6*(
2*c^4*d^6*e^2 - 4*b*c^3*d^5*e^3 + 6*(b^2*c^2 - 2*a*c^3)*d^4*e^4 - 4*(b^3*c
- 3*a*b*c^2)*d^3*e^5 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^6)*n*x^2 + 4*(2*
c^4*d^7*e - 4*b*c^3*d^6*e^2 + 6*(b^2*c^2 - 2*a*c^3)*d^5*e^3 - 4*(b^3*c - 3*
a*b*c^2)*d^4*e^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^5)*n*x + (2*c^4*d^8
- 4*b*c^3*d^7*e + 6*(b^2*c^2 - 2*a*c^3)*d^6*e^2 - 4*(b^3*c - 3*a*b*c^2)*d^5
```

```
*e^3 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^4*e^4)*n)*log(e*x + d) - 6*(c^4*d^8
- 4*b*c^3*d^7*e - 4*a^3*b*d*e^7 + a^4*e^8 + 2*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2
 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 -
 4*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)*log(d))/(
c^4*d^12*e - 4*b*c^3*d^11*e^2 - 4*a^3*b*d^5*e^8 + a^4*d^4*e^9 + 2*(3*b^2*c^
2 + 2*a*c^3)*d^10*e^3 - 4*(b^3*c + 3*a*b*c^2)*d^9*e^4 + (b^4 + 12*a*b^2*c +
 6*a^2*c^2)*d^8*e^5 - 4*(a*b^3 + 3*a^2*b*c)*d^7*e^6 + 2*(3*a^2*b^2 + 2*a^3*
c)*d^6*e^7 + (c^4*d^8*e^5 - 4*b*c^3*d^7*e^6 - 4*a^3*b*d*e^12 + a^4*e^13 + 2
*(3*b^2*c^2 + 2*a*c^3)*d^6*e^7 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^8 + (b^4 + 12*
a*b^2*c + 6*a^2*c^2)*d^4*e^9 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^10 + 2*(3*a^2*b^
2 + 2*a^3*c)*d^2*e^11)*x^4 + 4*(c^4*d^9*e^4 - 4*b*c^3*d^8*e^5 - 4*a^3*b*d^2
*e^11 + a^4*d*e^12 + 2*(3*b^2*c^2 + 2*a*c^3)*d^7*e^6 - 4*(b^3*c + 3*a*b*c^2
)*d^6*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^5*e^8 - 4*(a*b^3 + 3*a^2*b*c)*
d^4*e^9 + 2*(3*a^2*b^2 + 2*a^3*c)*d^3*e^10)*x^3 + 6*(c^4*d^10*e^3 - 4*b*c^3
*d^9*e^4 - 4*a^3*b*d^3*e^10 + a^4*d^2*e^11 + 2*(3*b^2*c^2 + 2*a*c^3)*d^8*e^
5 - 4*(b^3*c + 3*a*b*c^2)*d^7*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^6*e^7
- 4*(a*b^3 + 3*a^2*b*c)*d^5*e^8 + 2*(3*a^2*b^2 + 2*a^3*c)*d^4*e^9)*x^2 + 4*
(c^4*d^11*e^2 - 4*b*c^3*d^10*e^3 - 4*a^3*b*d^4*e^9 + a^4*d^3*e^10 + 2*(3*b^
2*c^2 + 2*a*c^3)*d^9*e^4 - 4*(b^3*c + 3*a*b*c^2)*d^8*e^5 + (b^4 + 12*a*b^2*
c + 6*a^2*c^2)*d^7*e^6 - 4*(a*b^3 + 3*a^2*b*c)*d^6*e^7 + 2*(3*a^2*b^2 + 2*a
^3*c)*d^5*e^8)*x), 1/24*(6*(2*c^4*d^5*e^3 - 5*b*c^3*d^4*e^4 + 2*(3*b^2*c^2
- 2*a*c^3)*d^3*e^5 - 2*(2*b^3*c - 3*a*b*c^2)*d^2*e^6 + (b^4 - 6*a^2*c^2)*d*
e^7 - (a*b^3 - 3*a^2*b*c)*e^8)*n*x^3 + 3*(14*c^4*d^6*e^2 - 36*b*c^3*d^5*e^3
 + (43*b^2*c^2 - 22*a*c^3)*d^4*e^4 - 4*(7*b^3*c - 8*a*b*c^2)*d^3*e^5 + (7*b
^4 + 4*a*b^2*c - 38*a^2*c^2)*d^2*e^6 - 4*(2*a*b^3 - 5*a^2*b*c)*d*e^7 + (a^2
*b^2 - 2*a^3*c)*e^8)*n*x^2 + 2*(26*c^4*d^7*e - 70*b*c^3*d^6*e^2 - a^3*b*e^8
 + 12*(7*b^2*c^2 - 2*a*c^3)*d^5*e^3 - (53*b^3*c - 27*a*b*c^2)*d^4*e^4 + (13
*b^4 + 24*a*b^2*c - 54*a^2*c^2)*d^3*e^5 - 6*(3*a*b^3 - 4*a^2*b*c)*d^2*e^6 +
 2*(3*a^2*b^2 - 2*a^3*c)*d*e^7)*n*x + 6*((4*c^3*d^3*e^5 - 6*b*c^2*d^2*e^6 +
 4*(b^2*c - a*c^2)*d*e^7 - (b^3 - 2*a*b*c)*e^8)*n*x^4 + 4*(4*c^3*d^4*e^4 -
6*b*c^2*d^3*e^5 + 4*(b^2*c - a*c^2)*d^2*e^6 - (b^3 - 2*a*b*c)*d*e^7)*n*x^3
+ 6*(4*c^3*d^5*e^3 - 6*b*c^2*d^4*e^4 + 4*(b^2*c - a*c^2)*d^3*e^5 - (b^3 - 2
*a*b*c)*d^2*e^6)*n*x^2 + 4*(4*c^3*d^6*e^2 - 6*b*c^2*d^5*e^3 + 4*(b^2*c - a*
c^2)*d^4*e^4 - (b^3 - 2*a*b*c)*d^3*e^5)*n*x + (4*c^3*d^7*e - 6*b*c^2*d^6*e^
2 + 4*(b^2*c - a*c^2)*d^5*e^3 - (b^3 - 2*a*b*c)*d^4*e^4)*n)*sqrt(-b^2 + 4*a
*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (22*c^4*d^8 - 6
2*b*c^3*d^7*e - 2*a^3*b*d*e^7 + 3*(25*b^2*c^2 - 2*a*c^3)*d^6*e^2 - 2*(23*b^
3*c + 3*a*b*c^2)*d^5*e^3 + (11*b^4 + 36*a*b^2*c - 30*a^2*c^2)*d^4*e^4 - 6*(
3*a*b^3 - a^2*b*c)*d^3*e^5 + (9*a^2*b^2 - 2*a^3*c)*d^2*e^6)*n + 3*((2*c^4*d
^4*e^4 - 4*b*c^3*d^3*e^5 + 6*(b^2*c^2 - 2*a*c^3)*d^2*e^6 - 4*(b^3*c - 3*a*b
*c^2)*d*e^7 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^8)*n*x^4 + 4*(2*c^4*d^5*e^3 -
 4*b*c^3*d^4*e^4 + 6*(b^2*c^2 - 2*a*c^3)*d^3*e^5 - 4*(b^3*c - 3*a*b*c^2)*d^
2*e^6 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^7)*n*x^3 + 6*(2*c^4*d^6*e^2 - 4*b
*c^3*d^5*e^3 + 6*(b^2*c^2 - 2*a*c^3)*d^4*e^4 - 4*(b^3*c - 3*a*b*c^2)*d^3*e^
5 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^6)*n*x^2 + 4*(2*c^4*d^7*e - 4*b*c^3
*d^6*e^2 + 6*(b^2*c^2 - 2*a*c^3)*d^5*e^3 - 4*(b^3*c - 3*a*b*c^2)*d^4*e^4 +
(b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^5)*n*x + (4*b*c^3*d^7*e + 8*a^3*b*d*e^7
 - 2*a^4*e^8 - 2*(3*b^2*c^2 + 10*a*c^3)*d^6*e^2 + 4*(b^3*c + 9*a*b*c^2)*d^5
*e^3 - (b^4 + 28*a*b^2*c + 10*a^2*c^2)*d^4*e^4 + 8*(a*b^3 + 3*a^2*b*c)*d^3*
e^5 - 4*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)*n)*log(c*x^2 + b*x + a) - 6*((2*c^4*
d^4*e^4 - 4*b*c^3*d^3*e^5 + 6*(b^2*c^2 - 2*a*c^3)*d^2*e^6 - 4*(b^3*c - 3*a*
b*c^2)*d*e^7 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^8)*n*x^4 + 4*(2*c^4*d^5*e^3
- 4*b*c^3*d^4*e^4 + 6*(b^2*c^2 - 2*a*c^3)*d^3*e^5 - 4*(b^3*c - 3*a*b*c^2)*d
^2*e^6 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^7)*n*x^3 + 6*(2*c^4*d^6*e^2 - 4*
b*c^3*d^5*e^3 + 6*(b^2*c^2 - 2*a*c^3)*d^4*e^4 - 4*(b^3*c - 3*a*b*c^2)*d^3*e
^5 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^6)*n*x^2 + 4*(2*c^4*d^7*e - 4*b*c^
3*d^6*e^2 + 6*(b^2*c^2 - 2*a*c^3)*d^5*e^3 - 4*(b^3*c - 3*a*b*c^2)*d^4*e^4 +
 (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^5)*n*x + (2*c^4*d^8 - 4*b*c^3*d^7*e +
6*(b^2*c^2 - 2*a*c^3)*d^6*e^2 - 4*(b^3*c - 3*a*b*c^2)*d^5*e^3 + (b^4 - 4*a*
```

```
b^2*c + 2*a^2*c^2)*d^4*e^4)*n)*log(e*x + d) - 6*(c^4*d^8 - 4*b*c^3*d^7*e -
4*a^3*b*d*e^7 + a^4*e^8 + 2*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 4*(b^3*c + 3*a*
b*c^2)*d^5*e^3 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 - 4*(a*b^3 + 3*a^2*
b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)*log(d))/(c^4*d^12*e - 4*b*c
^3*d^11*e^2 - 4*a^3*b*d^5*e^8 + a^4*d^4*e^9 + 2*(3*b^2*c^2 + 2*a*c^3)*d^10*
e^3 - 4*(b^3*c + 3*a*b*c^2)*d^9*e^4 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^8*e^
5 - 4*(a*b^3 + 3*a^2*b*c)*d^7*e^6 + 2*(3*a^2*b^2 + 2*a^3*c)*d^6*e^7 + (c^4*
d^8*e^5 - 4*b*c^3*d^7*e^6 - 4*a^3*b*d*e^12 + a^4*e^13 + 2*(3*b^2*c^2 + 2*a*
c^3)*d^6*e^7 - 4*(b^3*c + 3*a*b*c^2)*d^5*e^8 + (b^4 + 12*a*b^2*c + 6*a^2*c^
2)*d^4*e^9 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^10 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e
^11)*x^4 + 4*(c^4*d^9*e^4 - 4*b*c^3*d^8*e^5 - 4*a^3*b*d^2*e^11 + a^4*d*e^12
 + 2*(3*b^2*c^2 + 2*a*c^3)*d^7*e^6 - 4*(b^3*c + 3*a*b*c^2)*d^6*e^7 + (b^4 +
 12*a*b^2*c + 6*a^2*c^2)*d^5*e^8 - 4*(a*b^3 + 3*a^2*b*c)*d^4*e^9 + 2*(3*a^2
*b^2 + 2*a^3*c)*d^3*e^10)*x^3 + 6*(c^4*d^10*e^3 - 4*b*c^3*d^9*e^4 - 4*a^3*b
*d^3*e^10 + a^4*d^2*e^11 + 2*(3*b^2*c^2 + 2*a*c^3)*d^8*e^5 - 4*(b^3*c + 3*a
*b*c^2)*d^7*e^6 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^6*e^7 - 4*(a*b^3 + 3*a^2
*b*c)*d^5*e^8 + 2*(3*a^2*b^2 + 2*a^3*c)*d^4*e^9)*x^2 + 4*(c^4*d^11*e^2 - 4*
b*c^3*d^10*e^3 - 4*a^3*b*d^4*e^9 + a^4*d^3*e^10 + 2*(3*b^2*c^2 + 2*a*c^3)*d
^9*e^4 - 4*(b^3*c + 3*a*b*c^2)*d^8*e^5 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^7
*e^6 - 4*(a*b^3 + 3*a^2*b*c)*d^6*e^7 + 2*(3*a^2*b^2 + 2*a^3*c)*d^5*e^8)*x)]
```

giac [B] time = 1.17, size = 3759, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/8*(2*c^4*d^4*n - 4*b*c^3*d^3*n*e + 6*b^2*c^2*d^2*n*e^2 - 12*a*c^3*d^2*n*e
^2 - 4*b^3*c*d*n*e^3 + 12*a*b*c^2*d*n*e^3 + b^4*n*e^4 - 4*a*b^2*c*n*e^4 + 2
*a^2*c^2*n*e^4)*log(c*x^2 + b*x + a)/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^2*c
^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b^4*d
^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12*a^2*
b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a^4*e^9
) - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*d^2*n*e + 24*a*b*c^3*
d^2*n*e + 4*b^4*c*d*n*e^2 - 20*a*b^2*c^2*d*n*e^2 + 16*a^2*c^3*d*n*e^2 - b^5
*n*e^3 + 6*a*b^3*c*n*e^3 - 8*a^2*b*c^2*n*e^3)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 -
 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 +
6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^
6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-b^2 + 4*a*c)) - 1/24*(
12*c^4*d^4*n*x^4*e^4*log(x*e + d) + 48*c^4*d^5*n*x^3*e^3*log(x*e + d) + 72*
c^4*d^6*n*x^2*e^2*log(x*e + d) + 48*c^4*d^7*n*x*e*log(x*e + d) - 12*c^4*d^5
*n*x^3*e^3 - 42*c^4*d^6*n*x^2*e^2 - 52*c^4*d^7*n*x*e + 6*c^4*d^8*n*log(c*x^
2 + b*x + a) - 24*b*c^3*d^7*n*e*log(c*x^2 + b*x + a) + 12*c^4*d^8*n*log(x*e
 + d) - 24*b*c^3*d^3*n*x^4*e^5*log(x*e + d) - 96*b*c^3*d^4*n*x^3*e^4*log(x*
e + d) - 144*b*c^3*d^5*n*x^2*e^3*log(x*e + d) - 96*b*c^3*d^6*n*x*e^2*log(x*
e + d) - 24*b*c^3*d^7*n*e*log(x*e + d) - 22*c^4*d^8*n + 30*b*c^3*d^4*n*x^3*
e^4 + 108*b*c^3*d^5*n*x^2*e^3 + 140*b*c^3*d^6*n*x*e^2 + 62*b*c^3*d^7*n*e +
36*b^2*c^2*d^6*n*e^2*log(c*x^2 + b*x + a) + 24*a*c^3*d^6*n*e^2*log(c*x^2 +
b*x + a) + 36*b^2*c^2*d^2*n*x^4*e^6*log(x*e + d) - 72*a*c^3*d^2*n*x^4*e^6*l
og(x*e + d) + 144*b^2*c^2*d^3*n*x^3*e^5*log(x*e + d) - 288*a*c^3*d^3*n*x^3*
e^5*log(x*e + d) + 216*b^2*c^2*d^4*n*x^2*e^4*log(x*e + d) - 432*a*c^3*d^4*n
*x^2*e^4*log(x*e + d) + 144*b^2*c^2*d^5*n*x*e^3*log(x*e + d) - 288*a*c^3*d^
5*n*x*e^3*log(x*e + d) + 36*b^2*c^2*d^6*n*e^2*log(x*e + d) - 72*a*c^3*d^6*n
*e^2*log(x*e + d) + 6*c^4*d^8*log(d) - 24*b*c^3*d^7*e*log(d) - 36*b^2*c^2*d
^3*n*x^3*e^5 + 24*a*c^3*d^3*n*x^3*e^5 - 129*b^2*c^2*d^4*n*x^2*e^4 + 66*a*c^
3*d^4*n*x^2*e^4 - 168*b^2*c^2*d^5*n*x*e^3 + 48*a*c^3*d^5*n*x*e^3 - 75*b^2*c
^2*d^6*n*e^2 + 6*a*c^3*d^6*n*e^2 - 24*b^3*c*d^5*n*e^3*log(c*x^2 + b*x + a)
- 72*a*b*c^2*d^5*n*e^3*log(c*x^2 + b*x + a) - 24*b^3*c*d*n*x^4*e^7*log(x*e
```

```
+ d) + 72*a*b*c^2*d*n*x^4*e^7*log(x*e + d) - 96*b^3*c*d^2*n*x^3*e^6*log(x*e
 + d) + 288*a*b*c^2*d^2*n*x^3*e^6*log(x*e + d) - 144*b^3*c*d^3*n*x^2*e^5*lo
g(x*e + d) + 432*a*b*c^2*d^3*n*x^2*e^5*log(x*e + d) - 96*b^3*c*d^4*n*x*e^4*
log(x*e + d) + 288*a*b*c^2*d^4*n*x*e^4*log(x*e + d) - 24*b^3*c*d^5*n*e^3*lo
g(x*e + d) + 72*a*b*c^2*d^5*n*e^3*log(x*e + d) + 36*b^2*c^2*d^6*e^2*log(d)
+ 24*a*c^3*d^6*e^2*log(d) + 24*b^3*c*d^2*n*x^3*e^6 - 36*a*b*c^2*d^2*n*x^3*e
^6 + 84*b^3*c*d^3*n*x^2*e^5 - 96*a*b*c^2*d^3*n*x^2*e^5 + 106*b^3*c*d^4*n*x*
e^4 - 54*a*b*c^2*d^4*n*x*e^4 + 46*b^3*c*d^5*n*e^3 + 6*a*b*c^2*d^5*n*e^3 + 6
*b^4*d^4*n*e^4*log(c*x^2 + b*x + a) + 72*a*b^2*c*d^4*n*e^4*log(c*x^2 + b*x
+ a) + 36*a^2*c^2*d^4*n*e^4*log(c*x^2 + b*x + a) + 6*b^4*n*x^4*e^8*log(x*e
+ d) - 24*a*b^2*c*n*x^4*e^8*log(x*e + d) + 12*a^2*c^2*n*x^4*e^8*log(x*e + d
) + 24*b^4*d*n*x^3*e^7*log(x*e + d) - 96*a*b^2*c*d*n*x^3*e^7*log(x*e + d) +
 48*a^2*c^2*d*n*x^3*e^7*log(x*e + d) + 36*b^4*d^2*n*x^2*e^6*log(x*e + d) -
144*a*b^2*c*d^2*n*x^2*e^6*log(x*e + d) + 72*a^2*c^2*d^2*n*x^2*e^6*log(x*e +
 d) + 24*b^4*d^3*n*x*e^5*log(x*e + d) - 96*a*b^2*c*d^3*n*x*e^5*log(x*e + d)
 + 48*a^2*c^2*d^3*n*x*e^5*log(x*e + d) + 6*b^4*d^4*n*e^4*log(x*e + d) - 24*
a*b^2*c*d^4*n*e^4*log(x*e + d) + 12*a^2*c^2*d^4*n*e^4*log(x*e + d) - 24*b^3
*c*d^5*e^3*log(d) - 72*a*b*c^2*d^5*e^3*log(d) - 6*b^4*d*n*x^3*e^7 + 36*a^2*
c^2*d*n*x^3*e^7 - 21*b^4*d^2*n*x^2*e^6 - 12*a*b^2*c*d^2*n*x^2*e^6 + 114*a^2
*c^2*d^2*n*x^2*e^6 - 26*b^4*d^3*n*x*e^5 - 48*a*b^2*c*d^3*n*x*e^5 + 108*a^2*
c^2*d^3*n*x*e^5 - 11*b^4*d^4*n*e^4 - 36*a*b^2*c*d^4*n*e^4 + 30*a^2*c^2*d^4*
n*e^4 - 24*a*b^3*d^3*n*e^5*log(c*x^2 + b*x + a) - 72*a^2*b*c*d^3*n*e^5*log(
c*x^2 + b*x + a) + 6*b^4*d^4*e^4*log(d) + 72*a*b^2*c*d^4*e^4*log(d) + 36*a^
2*c^2*d^4*e^4*log(d) + 6*a*b^3*n*x^3*e^8 - 18*a^2*b*c*n*x^3*e^8 + 24*a*b^3*
d*n*x^2*e^7 - 60*a^2*b*c*d*n*x^2*e^7 + 36*a*b^3*d^2*n*x*e^6 - 48*a^2*b*c*d^
2*n*x*e^6 + 18*a*b^3*d^3*n*e^5 - 6*a^2*b*c*d^3*n*e^5 + 36*a^2*b^2*d^2*n*e^6
*log(c*x^2 + b*x + a) + 24*a^3*c*d^2*n*e^6*log(c*x^2 + b*x + a) - 24*a*b^3*
d^3*e^5*log(d) - 72*a^2*b*c*d^3*e^5*log(d) - 3*a^2*b^2*n*x^2*e^8 + 6*a^3*c*
n*x^2*e^8 - 12*a^2*b^2*d*n*x*e^7 + 8*a^3*c*d*n*x*e^7 - 9*a^2*b^2*d^2*n*e^6
+ 2*a^3*c*d^2*n*e^6 - 24*a^3*b*d*n*e^7*log(c*x^2 + b*x + a) + 36*a^2*b^2*d^
2*e^6*log(d) + 24*a^3*c*d^2*e^6*log(d) + 2*a^3*b*n*x*e^8 + 2*a^3*b*d*n*e^7
+ 6*a^4*n*e^8*log(c*x^2 + b*x + a) - 24*a^3*b*d*e^7*log(d) + 6*a^4*e^8*log(
d))/(c^4*d^8*x^4*e^5 + 4*c^4*d^9*x^3*e^4 + 6*c^4*d^10*x^2*e^3 + 4*c^4*d^11*
x*e^2 + c^4*d^12*e - 4*b*c^3*d^7*x^4*e^6 - 16*b*c^3*d^8*x^3*e^5 - 24*b*c^3*
d^9*x^2*e^4 - 16*b*c^3*d^10*x*e^3 - 4*b*c^3*d^11*e^2 + 6*b^2*c^2*d^6*x^4*e^
7 + 4*a*c^3*d^6*x^4*e^7 + 24*b^2*c^2*d^7*x^3*e^6 + 16*a*c^3*d^7*x^3*e^6 + 3
6*b^2*c^2*d^8*x^2*e^5 + 24*a*c^3*d^8*x^2*e^5 + 24*b^2*c^2*d^9*x*e^4 + 16*a*
c^3*d^9*x*e^4 + 6*b^2*c^2*d^10*e^3 + 4*a*c^3*d^10*e^3 - 4*b^3*c*d^5*x^4*e^8
 - 12*a*b*c^2*d^5*x^4*e^8 - 16*b^3*c*d^6*x^3*e^7 - 48*a*b*c^2*d^6*x^3*e^7 -
 24*b^3*c*d^7*x^2*e^6 - 72*a*b*c^2*d^7*x^2*e^6 - 16*b^3*c*d^8*x*e^5 - 48*a*
b*c^2*d^8*x*e^5 - 4*b^3*c*d^9*e^4 - 12*a*b*c^2*d^9*e^4 + b^4*d^4*x^4*e^9 +
12*a*b^2*c*d^4*x^4*e^9 + 6*a^2*c^2*d^4*x^4*e^9 + 4*b^4*d^5*x^3*e^8 + 48*a*b
^2*c*d^5*x^3*e^8 + 24*a^2*c^2*d^5*x^3*e^8 + 6*b^4*d^6*x^2*e^7 + 72*a*b^2*c*
d^6*x^2*e^7 + 36*a^2*c^2*d^6*x^2*e^7 + 4*b^4*d^7*x*e^6 + 48*a*b^2*c*d^7*x*e
^6 + 24*a^2*c^2*d^7*x*e^6 + b^4*d^8*e^5 + 12*a*b^2*c*d^8*e^5 + 6*a^2*c^2*d^
8*e^5 - 4*a*b^3*d^3*x^4*e^10 - 12*a^2*b*c*d^3*x^4*e^10 - 16*a*b^3*d^4*x^3*e
^9 - 48*a^2*b*c*d^4*x^3*e^9 - 24*a*b^3*d^5*x^2*e^8 - 72*a^2*b*c*d^5*x^2*e^8
 - 16*a*b^3*d^6*x*e^7 - 48*a^2*b*c*d^6*x*e^7 - 4*a*b^3*d^7*e^6 - 12*a^2*b*c
*d^7*e^6 + 6*a^2*b^2*d^2*x^4*e^11 + 4*a^3*c*d^2*x^4*e^11 + 24*a^2*b^2*d^3*x
^3*e^10 + 16*a^3*c*d^3*x^3*e^10 + 36*a^2*b^2*d^4*x^2*e^9 + 24*a^3*c*d^4*x^2
*e^9 + 24*a^2*b^2*d^5*x*e^8 + 16*a^3*c*d^5*x*e^8 + 6*a^2*b^2*d^6*e^7 + 4*a^
3*c*d^6*e^7 - 4*a^3*b*d*x^4*e^12 - 16*a^3*b*d^2*x^3*e^11 - 24*a^3*b*d^3*x^2
*e^10 - 16*a^3*b*d^4*x*e^9 - 4*a^3*b*d^5*e^8 + a^4*x^4*e^13 + 4*a^4*d*x^3*e
^12 + 6*a^4*d^2*x^2*e^11 + 4*a^4*d^3*x*e^10 + a^4*d^4*e^9)
```

maple [B] time = 1.30, size = 1137077, normalized size = 2190.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 18.95, size = 4334, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^5,x)
```

```
[Out] (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) - 6*a*e^6*(
4*a*c - b^2)^3 + 96*c^5*d^6*(4*a*c - b^2) - 10*b*e^6*x*(4*a*c - b^2)^3 - 10
*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6*x*
(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(b^2
- 4*a*c)^(1/2) + 44*a*b^2*e^6*(4*a*c - b^2)^2 - 16*b^3*d*e^5*(4*a*c - b^2)^
2 + 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2) + 44
*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) - 232*c^3*d^4*e
^2*(4*a*c - b^2)^2 - 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2 - 4*a*c
)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a*c)^(1/2
) - 48*b*d*e^5*(4*a*c - b^2)^3 + 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^2*d*e^5*
(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e^5*(b^2
- 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) + 288*b*c^2*d^3*e^3*(4*a
*c - b^2)^2 - 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 - 40*b^2*c^3*d^4*e^2*(4*a*c
- b^2) + 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^2 - 4*a*
c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 - 128*b*c^4*d^5*e*(4*
a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*c*d*e^5*x*(4*a*c - b^
2)^3 + 320*c^5*d^5*e*x*(4*a*c - b^2) - 110*b^4*c*d^2*e^4*(4*a*c - b^2) - 16
8*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*c)^(1/2)
 + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 - 4*a*c)^(
1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b^2 - 4*a
*c)^(3/2) - 264*b^2*c*d*e^5*x*(4*a*c - b^2)^2 - 800*b*c^4*d^4*e^2*x*(4*a*c
- b^2) - 928*b*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(3/2) - 116*b*c*d*e^5*x*(b^2 - 4
*a*c)^(5/2) + 528*b*c^2*d^2*e^4*x*(4*a*c - b^2)^2 + 800*b^2*c^3*d^3*e^3*x*(
4*a*c - b^2) - 400*b^3*c^2*d^2*e^4*x*(4*a*c - b^2) + 696*b^2*c^2*d^2*e^4*x*
(b^2 - 4*a*c)^(3/2) + 720*b^2*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 480*b^3*c
^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 180*b^4*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2
) + 100*b^4*c*d*e^5*x*(4*a*c - b^2) - 576*b*c^5*d^5*e*x*(b^2 - 4*a*c)^(1/2)
 - 232*b^3*c*d*e^5*x*(b^2 - 4*a*c)^(3/2) - 36*b^5*c*d*e^5*x*(b^2 - 4*a*c)^(
1/2))*(e^4*((b^4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4 - (a*
b^2*c*n)/2 - (a*b*c*n*(b^2 - 4*a*c)^(1/2))/4) - e^3*((b^3*c*d*n)/2 - (3*a*b
*c^2*d*n)/2 - (a*c^2*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b^2*c*d*n*(b^2 - 4*a*c)^
(1/2))/2) + e^2*((3*b^2*c^2*d^2*n)/4 - (3*a*c^3*d^2*n)/2 + (3*b*c^2*d^2*n*(
b^2 - 4*a*c)^(1/2))/4) - e*((b*c^3*d^3*n)/2 + (c^3*d^3*n*(b^2 - 4*a*c)^(1/2
))/2) + (c^4*d^4*n)/4))/(a^4*e^9 + c^4*d^8*e + b^4*d^4*e^5 - 4*a*b^3*d^3*e^
6 + 4*a*c^3*d^6*e^3 + 4*a^3*c*d^2*e^7 - 4*b*c^3*d^7*e^2 - 4*b^3*c*d^5*e^4 +
 6*a^2*b^2*d^2*e^7 + 6*a^2*c^2*d^4*e^5 + 6*b^2*c^2*d^6*e^3 - 4*a^3*b*d*e^8
```

- 12*a*b*c^2*d^5*e^4 + 12*a*b^2*c*d^4*e^5 - 12*a^2*b*c*d^3*e^6) - (log(d + e*x)*(e^2*(6*b^2*c^2*d^2*n - 12*a*c^3*d^2*n) - e^3*(4*b^3*c*d*n - 12*a*b*c^2*d*n) + e^4*(b^4*n + 2*a^2*c^2*n - 4*a*b^2*c*n) + 2*c^4*d^4*n - 4*b*c^3*d^3*e*n))/(4*a^4*e^9 + 4*c^4*d^8*e + 4*b^4*d^4*e^5 - 16*a*b^3*d^3*e^6 + 16*a*c^3*d^6*e^3 + 16*a^3*c*d^2*e^7 - 16*b*c^3*d^7*e^2 - 16*b^3*c*d^5*e^4 + 24*a^2*b^2*d^2*e^7 + 24*a^2*c^2*d^4*e^5 + 24*b^2*c^2*d^6*e^3 - 16*a^3*b*d*e^8 - 48*a*b*c^2*d^5*e^4 + 48*a*b^2*c*d^4*e^5 - 48*a^2*b*c*d^3*e^6) - log(d*(a + b*x + c*x^2)^n)/(4*e*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)) - (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) + 6*a*e^6*(4*a*c - b^2)^3 - 96*c^5*d^6*(4*a*c - b^2) + 10*b*e^6*x*(4*a*c - b^2)^3 + 10*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6*x*(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(b^2 - 4*a*c)^(1/2) - 44*a*b^2*e^6*(4*a*c - b^2)^2 + 16*b^3*d*e^5*(4*a*c - b^2)^2 - 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2) - 44*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) + 232*c^3*d^4*e^2*(4*a*c - b^2)^2 + 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2 - 4*a*c)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a*c)^(1/2) + 48*b*d*e^5*(4*a*c - b^2)^3 - 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e^5*(b^2 - 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) - 288*b*c^2*d^3*e^3*(4*a*c - b^2)^2 + 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 + 40*b^2*c^3*d^4*e^2*(4*a*c - b^2) - 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 + 128*b*c^4*d^5*e*(4*a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) - 20*c*d*e^5*x*(4*a*c - b^2)^3 - 320*c^5*d^5*e*x*(4*a*c - b^2) + 110*b^4*c*d^2*e^4*(4*a*c - b^2) - 168*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*c)^(1/2) + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(3/2) + 264*b^2*c*d*e^5*x*(4*a*c - b^2)^2 + 800*b*c^4*d^4*e^2*x*(4*a*c - b^2) - 928*b*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(3/2) - 116*b*c*d*e^5*x*(b^2 - 4*a*c)^(5/2) - 528*b*c^2*d^2*e^4*x*(4*a*c - b^2)^2 - 800*b^2*c^3*d^3*e^3*x*(4*a*c - b^2) + 400*b^3*c^2*d^2*e^4*x*(4*a*c - b^2) + 696*b^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(3/2) + 720*b^2*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 480*b^3*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 180*b^4*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 100*b^4*c*d*e^5*x*(4*a*c - b^2) - 576*b*c^5*d^5*e*x*(b^2 - 4*a*c)^(1/2) - 232*b^3*c*d*e^5*x*(b^2 - 4*a*c)^(3/2) - 36*b^5*c*d*e^5*x*(b^2 - 4*a*c)^(1/2))*(e^3*((b^3*c*d*n)/2 - (3*a*b*c^2*d*n)/2 + (a*c^2*d*n*(b^2 - 4*a*c)^(1/2))/2 - (b^2*c*d*n*(b^2 - 4*a*c)^(1/2))/2) - e^4*((b^4*n)/8 - (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4 - (a*b^2*c*n)/2 + (a*b*c*n*(b^2 - 4*a*c)^(1/2))/4) + e^2*((3*a*c^3*d^2*n)/2 - (3*b^2*c^2*d^2*n)/4 + (3*b*c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/4) + e*((b*c^3*d^3*n)/2 - (c^3*d^3*n*(b^2 - 4*a*c)^(1/2))/2) - (c^4*d^4*n)/4))/(a^4*e^9 + c^4*d^8*e + b^4*d^4*e^5 - 4*a*b^3*d^3*e^6 + 4*a*c^3*d^6*e^3 + 4*a^3*c*d^2*e^7 - 4*b*c^3*d^7*e^2 - 4*b^3*c*d^5*e^4 + 6*a^2*b^2*d^2*e^7 + 6*a^2*c^2*d^4*e^5 + 6*b^2*c^2*d^6*e^3 - 4*a^3*b*d*e^8 - 12*a*b*c^2*d^5*e^4 + 12*a*b^2*c*d^4*e^5 - 12*a^2*b*c*d^3*e^6) - ((11*b^3*d^2*e^3*n - 22*c^3*d^5*n + 2*a^2*b*e^5*n - 7*a*b^2*d*e^4*n + 2*a^2*c*d*e^4*n + 40*b*c^2*d^4*e*n + 28*a*c^2*d^3*e^2*n - 35*b^2*c*d^3*e^2*n - 6*a*b*c*d^2*e^3*n)/(6*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)) - (x*(a*b^2*e^5*n - 2*a^2*c*e^5*n - 5*b^3*d*e^4*n + 10*c^3*d^4*e*n - 24*a*c^2*d^2*e^3*n - 16*b*c^2*d^3*e^2*n + 15*b^2*c*d^2*e^3*n + 12*a*b*c*d*e^4*n))/(2*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)) + (n*x^2*(b^3*e^5 - 2*c^3*d^3*e^2 + 3*b*c^2*d^2*e^3 - 3*a*b*c*e^5 + 6*a*c^2*d*e^4 - 3*b^2*c*d*e^4))/(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))/(4*d^3*e + 4*e^4*x^3 + 12*d^2*e^2*x + 12*d*e^3*x^2)

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**5,x)
[Out] Timed out
```

3.92 $\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx$

Optimal. Leaf size=175

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{in\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{c}\,x+\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e}$$

```
[Out] I*n*arctan(x*c^(1/2)/a^(1/2))^2/e/a^(1/2)/c^(1/2)+arctan(x*c^(1/2)/a^(1/2))
*ln(d*(c*x^2+a)^n)/e/a^(1/2)/c^(1/2)+2*n*arctan(x*c^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^
(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[c]*e) + (2*n*ArcTan[(Sqrt
[c]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*
e) + (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e)
 + (I*n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[
c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} - (2cn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e\left(a+cx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} - \frac{\left(2\sqrt{c}\,n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a+cx^2}\,dx}{\sqrt{a}\,e}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{i-\frac{\sqrt{c}\,x}{\sqrt{a}}}\,dx}{ae}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} -$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} +$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} +$$

Mathematica [A] time = 0.05, size = 131, normalized size = 0.75

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(\log\left(d\left(a+cx^2\right)^n\right)+2n\log\left(\frac{2i}{-\frac{\sqrt{c}\,x}{\sqrt{a}}+i}\right)+in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\right)+in\text{Li}_2\left(\frac{\sqrt{c}\,x+i\sqrt{a}}{\sqrt{c}\,x-i\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[c]*x)/Sqrt[a])] + Log[d*(a + c*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[c]*x)/((-I)*Sqrt[a] + Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*e)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="fricas")
[Out] integral(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="giac")
[Out] integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^2+a\right)^n\right)}{ce\,x^2+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
[Out] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="maxima")
[Out] integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(c\,x^2+a\right)^n\right)}{c\,e\,x^2+a\,e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2),x)
[Out] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{a+cx^2}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+a)**n)/(c*e*x**2+a*e),x)
```

```
[Out] Integral(log(d*(a + c*x**2)**n)/(a + c*x**2), x)/e
```

3.93 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$

Optimal. Leaf size=258

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}-\frac{2n\text{Li}_2\left(-\frac{\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}+1}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}+\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}}-\frac{4n\log\left(-\ldots\right.}{}$$

```
[Out] 2*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))^2/e/(-4*a*c+b^2)^(1/2)-2*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(d*(c*x^2+b*x+a)^n)/e/(-4*a*c+b^2)^(1/2)-4*
n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(2/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/
(-4*a*c+b^2)^(1/2)))/e/(-4*a*c+b^2)^(1/2)-2*n*polylog(2,(-1-b/(-4*a*c+b^2)^
(1/2)-2*c*x/(-4*a*c+b^2)^(1/2))/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/(-4*a*c+b^2)^
(1/2)))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {618, 206, 2527, 12, 6121, 5984, 5918, 2402, 2315}

$$-\frac{2n\text{PolyLog}\left(2,-\frac{\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}+1}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}+\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}}-\frac{4n\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (2*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2)/(Sqrt[b^2 - 4*a*c]*e) - (4*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2
*c*x)/Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e) - (2*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sqrt[b^2 - 4*a*c]*e) - (2*n*
PolyLog[2, -((1 + b/Sqrt[b^2 - 4*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c])/(1 - b/S
qrt[b^2 - 4*a*c] - (2*c*x)/Sqrt[b^2 - 4*a*c]))])/(Sqrt[b^2 - 4*a*c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2527

```
Int[Log[(c_.)*(Px_)^(n_.)]/(Qx_), x_Symbol] :> With[{u = IntHide[1/Qx, x]},
 Simp[u*Log[c*Px^n], x] - Dist[n, Int[SimplifyIntegrand[(u*D[Px, x])/Px, x]
, x], x]] /; FreeQ[{c, n}, x] && QuadraticQ[{Qx, Px}, x] && EqQ[D[Px/Qx, x]
, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx = -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}\,e} - n\int \frac{2(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-}}\right.}{\sqrt{b^2-4ac}\,e\,(a+b}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2n)\int \frac{(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{}{\sqrt{}}\right.}{a+bx+cx^2}}{\sqrt{b^2-4ac}\,e}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}\,e} + \frac{n\,\text{Subst}\left(\int \frac{\sqrt{b^2-4ac}\,x\tanh^{-1}(x)}{-\frac{b^2-4ac}{4c}+\frac{(b^2-4ac)x^2}{4c}}d\right.}{ce}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}\,e} + \frac{\left(\sqrt{b^2-4ac}\,n\right)\text{Subst}\left(\int \frac{x\,\mathrm{t}}{-\frac{b^2-4a}{4c}}\right.}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}\,e} - \frac{(4n)\,\text{Su}}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e} - \frac{2\tanh}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e} - \frac{2\tanh}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e} - \frac{2\tanh}{}$$

Mathematica [A] time = 0.19, size = 339, normalized size = 1.31

$$\frac{2\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d(a+x(b+cx))^n\right)-2\log\left(\sqrt{b^2-4ac}+b+2cx\right)\log\left(d(a+x(b+cx))^n\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (-(n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2) + 2*n*Log[(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x] + n*Lo
g[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2 - 2*n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*Log[b + Sqrt[b^2 - 4
*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*n*PolyLog[2, (-b + Sqrt[b^2 -
 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*n*PolyLog[2, (b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(2*Sqrt[b^2 - 4*a*c]*e)
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{cex^2+bex+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{cex^2+bex+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

maple [F] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{ce\,x^2+bex+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

```
[Out] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)}{c\,e\,x^2+b\,e\,x+a\,e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2),x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(c*e*x**2+b*e*x+a*e),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx$

Optimal. Leaf size=762

$$-\frac{n\text{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}c+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{n\text{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}c+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\text{Li}_2\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\text{Li}_2\left(\frac{2c\left(\sqrt{e}x+\sqrt{\cdot}\right.}{2c\sqrt{-d}-\left(b+\sqrt{b^2\cdot}\right.}\right.}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*ln(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)-x*e^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*l
n(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)+x*e^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((
-d)^(1/2)+x*e^(1/2))*ln(-(b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/
2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-
x*e^(1/2))*ln((b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b-(-4*a
*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((-d)^(1/2)+x*e^(1/2))*
ln(-(b+2*c*x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)-(b+(-4*a*c+b^2)^(1
/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-x*e^(1/2))*ln((b+2*c*
x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2
)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)
^(1/2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,
2*c*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)^(1
/2)-(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,2*c
*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-
d)^(1/2)/e^(1/2)
```

Rubi [A] time = 1.45, antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2528, 2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4a\cdot}\right.}\right.}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] -(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) -
(n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[
b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) + (
n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) +
 (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b + Sq
rt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e])
 + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sqrt[
e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sq
rt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt
[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sq
rt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d]
- (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (
2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\log\left(g\left(a+bx+cx^2\right)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\log\left(g\left(a+bx+cx^2\right)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g\left(a+bx\right.\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g\left(a+bx\right.\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g\left(a+bx\right.\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{\ }}\right.}{2\sqrt{-d}\ }$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{\ }}\right.}{2\sqrt{-d}\ }$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{\ }}\right.}{2\sqrt{-d}\ }$$

Mathematica [A] time = 1.00, size = 626, normalized size = 0.82

$$\frac{-n\text{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,c+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)-n\text{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,c+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)+n\text{Li}_2\left(\frac{2c\left(\sqrt{e}\,x+\sqrt{-d}\right)}{2\sqrt{-d}\,c+\left(\sqrt{b^2-4ac}-b\right)\sqrt{e}}\right)+n\text{Li}_2\left(\frac{2c\left(\sqrt{e}\,x+\right.}{2c\sqrt{-d}-\left(b+\sqrt{\ }\right.}\right.}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] (-(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x]) - n*Log[(Sqrt[e]*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)]*Log[Sqrt[-d] - Sqrt[e]*x] + n*Log[(Sqrt[e]*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt
[e]*x] + n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*Sqrt[-d] + (
b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt[e]*x] + Log[Sqrt[-d] -
 Sqrt[e]*x]*Log[g*(a + x*(b + c*x))^n] - Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a
 + x*(b + c*x))^n] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d
] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[
e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])] + n*PolyLog[2, (2*
c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])
```

```
] + n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2
 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{ex^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{ex^2+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

maple [C] time = 1.20, size = 610, normalized size = 0.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x)
```

```
[Out] (ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*
x)+1/2*n/e*sum(1/_alpha*(ln(-_alpha+x)*ln(c*x^2+b*x+a)-ln(-_alpha+x)*ln((Ro
otOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1)-x+_alpha)/R
ootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1))-ln(-_alph
a+x)*ln((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)-
x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)
)-dilog((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1)-
x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1)
)-dilog((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)-
x+_alpha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)
)),_alpha=RootOf(_Z^2*e+d))+1/2*I/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*Pi*
csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2-1/2*I/(d*e)^(1/2)*arcta
n(1/(d*e)^(1/2)*e*x)*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)*c
sgn(I*g)-1/2*I/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*Pi*csgn(I*g*(c*x^2+b*x
+a)^n)^3+1/2*I/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*Pi*csgn(I*g*(c*x^2+b*x
+a)^n)^2*csgn(I*g)+1/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*ln(g)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(g\left(c\,x^2+b\,x+a\right)^n\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2),x)
```

```
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(e*x**2+d),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=782

$$-\frac{n\text{Li}_2\left(-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\text{Li}_2\left(-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\text{Li}_2\left(-\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\text{Li}_2}{}$$

```
[Out] ln(g*(c*x^2+b*x+a)^n)*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*l
n(f*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2
)^(1/2))))*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-ln(g*(c*x^2+b*
x+a)^n)*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*ln(e+2*f*x-(-4*
d*f+e^2)^(1/2))*ln(-f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(c*e-b*f+f*(-4*a*c+b^2)^
(1/2)-c*(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(
1/2))*ln(f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*
d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))*ln(f*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1
/2))))/(-4*d*f+e^2)^(1/2)-n*polylog(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b
-(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-n*polylo
g(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+
e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,-c*(e+2*f*x+(-4*d*f+e^2)^(1/2)
)/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n
*polylog(2,-c*(e+2*f*x+(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(
-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 782, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2528, 2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\text{PolyLog}\left(2,-\frac{c\left(\sqrt{e^2\cdots}\right.}{f\left(b-\sqrt{b^2-\cdots}\right.}\right.}{\sqrt{e^2-4d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] -((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a*
c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^2
- 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a
*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sq
rt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqrt[b^2
 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*
x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sq
rt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]
+ 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a
+ b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d
*f])))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2 - 4*d*f] + 2*
f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 -
 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^
2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] + (n*PolyLo
g[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(
e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx = \int \left(\frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\,\left(e-\sqrt{e^2-4df}\,+2fx\right)} - \frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\,\left(e+\sqrt{e^2-4df}\,+2fx\right)}\right) dx$$

$$= \frac{(2f)\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{e-\sqrt{e^2-4df}\,+2fx}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{e+\sqrt{e^2-4df}\,+2fx}\,dx}{\sqrt{e^2-4df}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}\,+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\,+2fx\right.}{\sqrt{e^2-}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}\,+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\,+2fx\right.}{\sqrt{e^2-}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}\,+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\,+2fx\right.}{\sqrt{e^2-}}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}\,+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}\,+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\right.}{\left(b+\sqrt{b^2-4\right.}}\right.}{}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}\,+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}\,+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\right.}{\left(b+\sqrt{b^2-4\right.}}\right.}{}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}\,+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}\,+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\right.}{\left(b+\sqrt{b^2-4\right.}}\right.}{}$$

Mathematica [A] time = 0.85, size = 663, normalized size = 0.85

$$\frac{-n\text{Li}_2\left(\frac{c\left(-e-2fx+\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f+c\left(\sqrt{e^2-4df}-e\right)}\right)-n\text{Li}_2\left(\frac{c\left(-e-2fx+\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f+c\left(\sqrt{e^2-4df}-e\right)}\right)+n\text{Li}_2\left(\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(\sqrt{b^2-4ac}-b\right)f+c\left(e+\sqrt{e^2-4df}\right)}\right)+n\text{Li}_2\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (-(n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(-(c*e) + b*f - Sqrt[b^2 - 4*a
*c]*f + c*Sqrt[e^2 - 4*d*f])]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]) - n*Log[(
f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqr
t[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 - 4*d*
f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e +
Sqrt[e^2 - 4*d*f] + 2*f*x] + Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a +
x*(b + c*x))^n] - Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a + x*(b + c*x)
)^n] - n*PolyLog[2, (c*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/((b - Sqrt[b^2 - 4
```

```
*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (c*(-e + Sqrt[e^2 -
4*d*f] - 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))]
+ n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((-b + Sqrt[b^2 - 4*a*c]
)*f + c*(e + Sqrt[e^2 - 4*d*f]))] + n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f]
+ 2*f*x))/(-((b + Sqrt[b^2 - 4*a*c])*f) + c*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt
[e^2 - 4*d*f]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{fx^2+ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{fx^2+ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

maple [C] time = 1.17, size = 764, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x)
```

```
[Out] 2*(-n*ln(c*x^2+b*x+a)+ln((c*x^2+b*x+a)^n))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))+n*sum((ln(-_alpha+x)*ln(c*x^2+b*x+a)-ln(-_alpha+x)*ln
((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index
=1)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a
*f-c*d,index=1))-ln(-_alpha+x)*ln((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*
_alpha*f-_alpha*c*e+a*f-c*d,index=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*
f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2))-dilog((RootOf(_Z^2*c*f+(2
*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)-x+_alpha)/RootOf
(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1))-dil
og((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,ind
ex=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e
+a*f-c*d,index=2)))/(2*_alpha*f+e),_alpha=RootOf(_Z^2*f+_Z*e+d))+I/(4*d*f-e
^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*cs
gn(I*g*(c*x^2+b*x+a)^n)^2-I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^
(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)*csgn(I*g)-I/(4*
d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*g*(c*x^2+b*x+a
)^n)^3+I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*g*
(c*x^2+b*x+a)^n)^2*csgn(I*g)+2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^
2)^(1/2))*ln(g)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(g\left(c\,x^2+b\,x+a\right)^n\right)}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2),x)
```

```
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.96 $\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=144

$$x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}-\frac{2bn^2\text{Li}_2\left(\frac{cx}{b}+1\right)}{c}-\frac{bn^2\log}{}$$

```
[Out] 8*n^2*x-4*b*n^2*ln(c*x+b)/c-2*b*n^2*ln(-c*x/b)*ln(c*x+b)/c-b*n^2*ln(c*x+b)^
2/c-4*n*x*ln(d*(c*x^2+b*x)^n)+2*b*n*ln(c*x+b)*ln(d*(c*x^2+b*x)^n)/c+x*ln(d*
(c*x^2+b*x)^n)^2-2*b*n^2*polylog(2,c*x/b+1)/c
```

Rubi [A] time = 0.28, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2523, 2528, 43, 2524, 1593, 2418, 2394, 2315, 2390, 2301}

$$-\frac{2bn^2\text{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*b*n^2*Log[b + c*x])/c - (2*b*n^2*Log[-((c*x)/b)]*Log[b + c*x])
/c - (b*n^2*Log[b + c*x]^2)/c - 4*n*x*Log[d*(b*x + c*x^2)^n] + (2*b*n*Log[b
 + c*x]*Log[d*(b*x + c*x^2)^n])/c + x*Log[d*(b*x + c*x^2)^n]^2 - (2*b*n^2*P
olyLog[2, 1 + (c*x)/b])/c
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)dx = x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int\frac{(b+2cx)\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}dx$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int\left(2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{b\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}\right)$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(4n)\int\log\left(d\left(bx+cx^2\right)^n\right)dx+(2bn)\int\frac{\log\left(d\left(bx+c\right.\right.}{b+cx}$$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx\right.\right.$$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx\right.\right.$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx\right.\right.}{c}$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx\right.\right.}{c}$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{bn^2\log^2(b+cx)}{c}-4nx$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.77

$$\frac{cx\left(\log^2\left(d(x(b+cx))^n\right)-4n\log\left(d(x(b+cx))^n\right)+8n^2\right)-2bn\log(b+cx)\left(-\log\left(d(x(b+cx))^n\right)+n\log\left(-\frac{cx}{b}\right)\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] (-(b*n^2*Log[b + c*x]^2) - 2*b*n*Log[b + c*x]*(2*n + n*Log[-((c*x)/b)] - Lo
g[d*(x*(b + c*x))^n]) + c*x*(8*n^2 - 4*n*Log[d*(x*(b + c*x))^n] + Log[d*(x*
(b + c*x))^n]^2) - 2*b*n^2*PolyLog[2, 1 + (c*x)/b])/c
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left(cx^2+bx\right)^n d\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x)^n*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(cx^2+bx\right)^n d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x)^n*d)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)^2,x)
[Out] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

maxima [A] time = 0.70, size = 123, normalized size = 0.85

$$-\left(\frac{2\left(\log\left(cx+b\right)\log\left(-\frac{cx+b}{b}+1\right)+\mathrm{Li}_2\left(\frac{cx+b}{b}\right)\right)b}{c}+\frac{b\log\left(cx+b\right)^{2}-8\,cx+4\,b\log\left(cx+b\right)}{c}\right)n^{2}-2\,n\left(2\,x-\frac{b\log\left(c\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="maxima")
[Out] -(2*(log(c*x + b)*log(-(c*x + b)/b + 1) + dilog((c*x + b)/b))*b/c + (b*log(
c*x + b)^2 - 8*c*x + 4*b*log(c*x + b))/c)*n^2 - 2*n*(2*x - b*log(c*x + b)/c
)*log((c*x^2 + b*x)^n*d) + x*log((c*x^2 + b*x)^n*d)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(c\,x^{2}+b\,x\right)^{n}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)^2,x)
[Out] int(log(d*(b*x + c*x^2)^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(d\left(bx+cx^{2}\right)^{n}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)**2,x)
[Out] Integral(log(d*(b*x + c*x**2)**n)**2, x)
```

3.97 $\int \log^2\left(d\left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=587

$$\frac{n\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c}+\frac{n\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}+\right.}{c}$$

```
[Out] 8*n^2*x-2*b*n^2*ln(c*x^2+b*x+a)/c-4*n*x*ln(d*(c*x^2+b*x+a)^n)+x*ln(d*(c*x^2
+b*x+a)^n)^2+n*ln(d*(c*x^2+b*x+a)^n)*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*(b-(-4*
a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))^2*(b-(-4*a*c+b^2)^
(1/2))/c-n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*ln(1/2*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-4*n^2*a
rctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c+n*ln(d*(c*x^2+b*x
+a)^n)*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c-n^2*ln(1/2*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*ln(b+2*c*x+(-4*a*c+b^2)^(1
/2))*(b+(-4*a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))^2*(b+(
-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c
```

Rubi [A] time = 0.95, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {2523, 2528, 773, 634, 618, 206, 628, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}+\frac{n\left(b-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*Sqrt[b^2 - 4*a*c]*n^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
c - ((b - Sqrt[b^2 - 4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c
) - ((b + Sqrt[b^2 - 4*a*c])*n^2*Log[-(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sq
rt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/c - ((b + Sqrt[b^2 -
4*a*c])*n^2*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c) - ((b - Sqrt[b^2 -
4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - (2*b*n^2*Log[a + b*x + c*x^2])/c - 4*n*
x*Log[d*(a + b*x + c*x^2)^n] + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + ((b + Sqrt[b^2 - 4*a*c])*
n*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + x*Log[
d*(a + b*x + c*x^2)^n]^2 - ((b - Sqrt[b^2 - 4*a*c])*n^2*PolyLog[2, -(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - ((b + Sqrt[b^2 - 4*a*c
])*n^2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)dx = x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int \frac{x(b+2cx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}dx$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int\left(2\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{(2a+bx)\log\left(\right.}{a+}\right.$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int \frac{(2a+bx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}dx - (4n)$$

$$= -4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int\left(\frac{\left(b-\sqrt{b^2-\right.}{}\right.$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + \left(2\left(b-\sqrt{b^2-\right.\right.$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2\right.}{c}$$

$$= 8n^2x - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac\right.}{}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\right.$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+}{2\sqrt{b^2-4ac}}\right.}{c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-\right.}{2c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-\right.}{2c}$$

Mathematica [A] time = 0.85, size = 478, normalized size = 0.81

$$\frac{n\left(2\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d(a+x(b+cx))^n\right)+2\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] x*Log[d*(a + x*(b + c*x))^n]^2 + (n*(4*n*(4*c*x - 2*Sqrt[b^2 - 4*a*c]*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - b*Log[a + x*(b + c*x)]) - 8*c*x*Log[d*(
a + x*(b + c*x))^n] + 2*(b - Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] +
 2*c*x]*Log[d*(a + x*(b + c*x))^n] + 2*(b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + (-b + Sqrt[b^2 - 4*a*c]
)*n*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
```

```
 + 2*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) + 2*PolyLo
g[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b
^2 - 4*a*c])*n*(Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*(2*Log[(-b + Sqrt[b^2 -
4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]
) + 2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])))/
(2*c)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left(cx^2+bx+a\right)^n d\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(cx^2+bx+a\right)^n d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)^2, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \ln\left(d\left(cx^2+bx+a\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)^2,x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(d \left(a + bx + cx^2\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)**2,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)**2, x)
```

3.98 $\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} dx$

Optimal. Leaf size=311

$$-\text{Li}_2\left(\frac{2(x+1)}{1-i\sqrt{3}}\right)-\text{Li}_2\left(\frac{2(x+1)}{1+i\sqrt{3}}\right)+4\text{Li}_2\left(\frac{2(x+2)}{3-i\sqrt{3}}\right)+4\text{Li}_2\left(\frac{2(x+2)}{3+i\sqrt{3}}\right)+x\log\left(x^2+x+1\right)+\log(2x+2)\log\left(x^2+x\right.$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+ln(2+2*x)*ln(x^2+x+1)-4*ln(4+2*x)*ln(x^2
+x+1)-ln(2+2*x)*ln((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))+4*ln(4+2*x)*ln((-1-2*x
+I*3^(1/2))/(3+I*3^(1/2)))-ln(2+2*x)*ln((-1-2*x-I*3^(1/2))/(1-I*3^(1/2)))+4
*ln(4+2*x)*ln((-1-2*x-I*3^(1/2))/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1-I*3^(1
/2)))+4*polylog(2,2*(2+x)/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1+I*3^(1/2)))+4
*polylog(2,2*(2+x)/(3+I*3^(1/2)))+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2528, 2523, 773, 634, 618, 204, 628, 2524, 2418, 2394, 2393, 2391}

$$-\text{PolyLog}\left(2,\frac{2(x+1)}{1-i\sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3-i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right)+x\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[3
] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(3
+ I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]))]
 + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1 + x
 + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log[4 +
2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - PolyLog[2
, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqrt[3])]
+ 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2 \log\left(1+x+x^2\right)}{2+3x+x^2}\,dx = \int \left(\log\left(1+x+x^2\right) - \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\right)dx$$

$$= \int \log\left(1+x+x^2\right)dx - \int \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\,dx$$

$$= x\log\left(1+x+x^2\right) - \int \frac{x(1+2x)}{1+x+x^2}\,dx - \int\left(-\frac{2\log\left(1+x+x^2\right)}{2+2x} + \frac{8\log\left(1+x+x\right.}{4+2x}\right.$$

$$= -2x + x\log\left(1+x+x^2\right) + 2\int \frac{\log\left(1+x+x^2\right)}{2+2x}\,dx - 8\int \frac{\log\left(1+x+x^2\right)}{4+2x}\,dx - \int$$

$$= -2x + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+x\right.$$

$$= -2x + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - 4\,\mathrm{l}$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+2x)$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log$$

Mathematica [A] time = 0.20, size = 290, normalized size = 0.93

$$-\mathrm{Li}_2\left(\frac{2(x+1)}{1+i\sqrt{3}}\right) - \mathrm{Li}_2\left(\frac{2i(x+1)}{i+\sqrt{3}}\right) + 4\left(\mathrm{Li}_2\left(\frac{2(x+2)}{3+i\sqrt{3}}\right) + \mathrm{Li}_2\left(\frac{2i(x+2)}{3i+\sqrt{3}}\right) + \left(\log\left(\frac{-2ix+\sqrt{3}-i}{\sqrt{3}+3i}\right) + \log\left(\frac{2ix+\sqrt{}}{\sqrt{3}-}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[(-I + Sqrt[3] - (2*I)*x)/(I
+ Sqrt[3])]*Log[2*(1 + x)] - Log[(I + Sqrt[3] + (2*I)*x)/(-I + Sqrt[3])]*Lo
g[2*(1 + x)] + Log[1 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2*(1 + x)]*Log
[1 + x + x^2] - 4*Log[2*(2 + x)]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/
(1 + I*Sqrt[3])] - PolyLog[2, ((2*I)*(1 + x))/(I + Sqrt[3])] + 4*((Log[(-I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])] + Log[(I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])])*Log[2*(2 + x)] + PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])] + Poly
Log[2, ((2*I)*(2 + x))/(3*I + Sqrt[3])])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2\log\left(x^2+x+1\right)}{x^2+3x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(x^2+x+1\right)}{x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

maple [A] time = 0.09, size = 279, normalized size = 0.90

$$x\ln\left(x^2+x+1\right)-\ln\left(\frac{-2x-1+i\sqrt{3}}{1+i\sqrt{3}}\right)\ln(x+1)+4\ln\left(\frac{-2x-1+i\sqrt{3}}{3+i\sqrt{3}}\right)\ln(x+2)+4\ln\left(\frac{2x+1+i\sqrt{3}}{-3+i\sqrt{3}}\right)\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x^2+x+1)/(x^2+3*x+2),x)
```

```
[Out] x*ln(x^2+x+1)-2*x+1/2*ln(x^2+x+1)+3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-4*ln(
x+2)*ln(x^2+x+1)+4*ln(x+2)*ln((-1-2*x+I*3^(1/2))/(3+I*3^(1/2)))+4*ln(x+2)*l
n((1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))+4*dilog((-1-2*x+I*3^(1/2))/(3+I*3^(1/2)
))+4*dilog((1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))+ln(x+1)*ln(x^2+x+1)-ln(x+1)*ln
((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-ln(x+1)*ln((1+2*x+I*3^(1/2))/(I*3^(1/2)-
1))-dilog((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-dilog((1+2*x+I*3^(1/2))/(I*3^(1
/2)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(x^2+x+1\right)}{x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(x^2+x+1\right)}{x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2),x)
```

```
[Out] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x**2+x+1)/(x**2+3*x+2),x)
```

```
[Out] Timed out
```

3.99 $\int \log^2\left(1+x+x^2\right) dx$

Optimal. Leaf size=371

$$-\left(1+i\sqrt{3}\right)\text{Li}_2\left(-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{Li}_2\left(\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{3}\right)\log\left(x^2+x+\right.$$

```
[Out] 8*x-2*ln(x^2+x+1)-4*x*ln(x^2+x+1)+x*ln(x^2+x+1)^2+ln(x^2+x+1)*ln(1+2*x-I*3^
(1/2))*(1-I*3^(1/2))-1/2*ln(1+2*x-I*3^(1/2))^2*(1-I*3^(1/2))-ln(1+2*x-I*3^(
1/2))*ln(-1/6*I*(1+2*x+I*3^(1/2))*3^(1/2))*(1-I*3^(1/2))-polylog(2,1/6*(I+2
*I*x+3^(1/2))*3^(1/2))*(1-I*3^(1/2))+ln(x^2+x+1)*ln(1+2*x+I*3^(1/2))*(1+I*3
^(1/2))-1/2*ln(1+2*x+I*3^(1/2))^2*(1+I*3^(1/2))-ln(1+2*x+I*3^(1/2))*ln(1/6*
I*(1+2*x-I*3^(1/2))*3^(1/2))*(1+I*3^(1/2))-polylog(2,1/6*(-I-2*I*x+3^(1/2))
*3^(1/2))*(1+I*3^(1/2))-4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.556, Rules used = {2523, 2528, 773, 634, 618, 204, 628, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt
[3] + 2*x]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*x))/Sqrt[3]
]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/2
 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[((-I/2)*(1 + I*Sqrt[3] + 2*
x))/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3])*
Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[
3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyLog
[2, -(I - Sqrt[3] + (2*I)*x)/(2*Sqrt[3])] - (1 - I*Sqrt[3])*PolyLog[2, (I +
 Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(1+x+x^2\right)dx = x\log^2\left(1+x+x^2\right)-2\int \frac{x(1+2x)\log\left(1+x+x^2\right)}{1+x+x^2}dx$$

$$= x\log^2\left(1+x+x^2\right)-2\int \left(2\log\left(1+x+x^2\right)-\frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\right)dx$$

$$= x\log^2\left(1+x+x^2\right)+2\int \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}dx-4\int \log\left(1+x+x^2\right)dx$$

$$= -4x\log\left(1+x+x^2\right)+x\log^2\left(1+x+x^2\right)+2\int\left(\frac{\left(1-i\sqrt{3}\right)\log\left(1+x+x^2\right)}{1-i\sqrt{3}+2x}+\frac{(1

```
]^2 - (I/2)*(-I + Sqrt[3])*(Log[1 + I*Sqrt[3] + 2*x]*(2*Log[(I + Sqrt[3] +
(2*I)*x)/(2*Sqrt[3])] + Log[1 + I*Sqrt[3] + 2*x]) + 2*PolyLog[2, (-I + Sqrt
[3] - (2*I)*x)/(2*Sqrt[3])]) + (I/2)*(I + Sqrt[3])*(Log[1 - I*Sqrt[3] + 2*x
]*(2*Log[(-I + Sqrt[3] - (2*I)*x)/(2*Sqrt[3])] + Log[1 - I*Sqrt[3] + 2*x])
+ 2*PolyLog[2, (I + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(x^2+x+1\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x + 1)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(x^2+x+1\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x + 1)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \ln\left(x^2+x+1\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1)^2,x)
```

```
[Out] int(ln(x^2+x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(x^2+x+1\right)^2 - \int \frac{2\left(2\,x^2+x\right)\log\left(x^2+x+1\right)}{x^2+x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1)^2 - integrate(2*(2*x^2 + x)*log(x^2 + x + 1)/(x^2 + x +
1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(x^2+x+1\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 + 1)^2,x)
```

```
[Out] int(log(x + x^2 + 1)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

3.100 $\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx$

Optimal. Leaf size=443

$$3\text{Li}_2\left(-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{Li}_2\left(-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{Li}_2\left(\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3\text{Li}_2\left(\frac{2x}{1-\sqrt{5}}+1\right)-\frac{\log^2}{}$$

```
[Out] ln(x)+ln(x^2+x-1)/x-3*ln(x)*ln(x^2+x-1)-1/2*ln(x^2+x-1)^2/x^2+3*ln(1+2*x-5^
(1/2))*ln(1/2*5^(1/2)-1/2)+3*ln(x)*ln(1+2*x/(5^(1/2)+1))-3*polylog(2,1+2*x/
(-5^(1/2)+1))+3*polylog(2,-2*x/(5^(1/2)+1))-1/2*ln(1+2*x+5^(1/2))*(-5^(1/2)
+1)+1/2*ln(x^2+x-1)*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/2*ln(1/10*(-1-2*x+5^(1/
2))*5^(1/2))*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/4*ln(1+2*x+5^(1/2))^2*(3-5^(1/
2))-1/2*polylog(2,1/10*(1+2*x+5^(1/2))*5^(1/2))*(3-5^(1/2))-1/2*ln(1+2*x-5^
(1/2))*(5^(1/2)+1)+1/2*ln(x^2+x-1)*ln(1+2*x-5^(1/2))*(3+5^(1/2))-1/4*ln(1+2
*x-5^(1/2))^2*(3+5^(1/2))-1/2*ln(1+2*x-5^(1/2))*ln(1/10*(1+2*x+5^(1/2))*5^(
1/2))*(3+5^(1/2))-1/2*polylog(2,1/10*(-1-2*x+5^(1/2))*5^(1/2))*(3+5^(1/2))
```

Rubi [A] time = 0.68, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 16, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.231, Rules used = {2525, 2528, 800, 632, 31, 2524, 2357, 2316, 2315, 2317, 2391, 2418, 2390, 2301, 2394, 2393}

$$3\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2]
*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((1
- Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-(1 - Sqrt[5] + 2
*x)/(2*Sqrt[5])]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqrt[5]
 + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5] + 2*x
)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 + x + x^
2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*L
og[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^
2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqrt[5])] -
((3 + Sqrt[5])*PolyLog[2, -(1 - Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - ((3 - Sqrt
[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[2, 1 + (2*x
)/(1 - Sqrt[5])]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = -\frac{\log^2(-1+x+x^2)}{2x^2} + \int \frac{(1+2x)\log(-1+x+x^2)}{x^2(-1+x+x^2)}\,dx$$

$$= -\frac{\log^2(-1+x+x^2)}{2x^2} + \int\left(-\frac{\log(-1+x+x^2)}{x^2} - \frac{3\log(-1+x+x^2)}{x} + \frac{(4+3x)\log}{-1+}\right.$$

$$= -\frac{\log^2(-1+x+x^2)}{2x^2} - 3\int \frac{\log(-1+x+x^2)}{x}\,dx - \int \frac{\log(-1+x+x^2)}{x^2}\,dx + \int \frac{(4+}{}$$

$$= \frac{\log(-1+x+x^2)}{x} - 3\log(x)\log(-1+x+x^2) - \frac{\log^2(-1+x+x^2)}{2x^2} + 3\int \frac{(1+2x)\,\mathrm{l}}{-1+x}$$

$$= \frac{\log(-1+x+x^2)}{x} - 3\log(x)\log(-1+x+x^2) - \frac{\log^2(-1+x+x^2)}{2x^2} + 3\int\left(\frac{2\log(}{1-\sqrt{5}}\right.$$

$$= \log(x) + \frac{\log(-1+x+x^2)}{x} - 3\log(x)\log(-1+x+x^2) + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+\right.$$

$$= \log(x) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) + 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log}{}$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) -$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) -$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) -$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) -$$

Mathematica [A] time = 0.82, size = 826, normalized size = 1.86

$$\frac{x\left(\sqrt{5}\,x\log^2\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right) + 3x\log^2\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right) - 2\sqrt{5}\,x\log\left(-2x+\sqrt{5}-1\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right) - 6x\log\left(-2x\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] (-2*Log[-1 + x + x^2]^2 + x*(4*x*Log[x] - 12*x*Log[(1 + Sqrt[5])/2]*Log[x]
- 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] - 2*Sqrt[5]*x*Log[-1
 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] + 12*x*Log[x]*Log[1/2 - Sqrt[5]/
2 + x] - 12*x*Log[(2*x)/(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + x] + 3*x*Log[
1/2 - Sqrt[5]/2 + x]^2 + Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]^2 - 6*x*Log[-1
+ Sqrt[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] - 2*Sqrt[5]*x*Log[-1 + Sqrt[5] -
2*x]*Log[(1 + Sqrt[5])/2 + x] + 12*x*Log[x]*Log[(1 + Sqrt[5])/2 + x] + 3*x*
Log[(1 + Sqrt[5])/2 + x]^2 - Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]^2 - 2*x*Log
[1 - Sqrt[5] + 2*x] - 2*Sqrt[5]*x*Log[1 - Sqrt[5] + 2*x] + 3*x*Log[5]*Log[1
 - Sqrt[5] + 2*x] + Sqrt[5]*x*Log[5]*Log[1 - Sqrt[5] + 2*x] - 2*x*Log[1 + S
qrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[1/2 - Sqrt[5]/
2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]*Log[1
```

```
+ Sqrt[5] + 2*x] - 6*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*
Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 6*x*Log[1/2 - S
qrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] - 2*Sqrt[5]*x*Log[1/2 -
Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] + 4*Log[-1 + x + x^2] +
 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] + 2*Sqrt[5]*x*Log[-1 + Sqrt[
5] - 2*x]*Log[-1 + x + x^2] - 12*x*Log[x]*Log[-1 + x + x^2] + 6*x*Log[1 + S
qrt[5] + 2*x]*Log[-1 + x + x^2] - 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x]*Log[-1
 + x + x^2] - 4*Sqrt[5]*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(2*Sqrt[5])] - 12
*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(-1 + Sqrt[5])] + 12*x*PolyLog[2, (-2*x)
/(1 + Sqrt[5])]))/(4*x^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(x^2+x-1\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="fricas")
[Out] integral(log(x^2 + x - 1)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x^2+x-1\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="giac")
[Out] integrate(log(x^2 + x - 1)^2/x^3, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(x^2+x-1\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x-1)^2/x^3,x)
[Out] int(ln(x^2+x-1)^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(x^2+x-1\right)^2}{2\,x^2} + \int \frac{(2\,x+1)\log\left(x^2+x-1\right)}{x^4+x^3-x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="maxima")
[Out] -1/2*log(x^2 + x - 1)^2/x^2 + integrate((2*x + 1)*log(x^2 + x - 1)/(x^4 + x
^3 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(x^2+x-1\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 - 1)^2/x^3,x)
```

```
[Out] int(log(x + x^2 - 1)^2/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x-1)**2/x**3,x)
```

```
[Out] Timed out
```

3.101 $\int x^3 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=172

$$-\frac{x^4}{32}+\frac{x^3}{192}-\frac{x^2}{1024}-\frac{1}{32}\left(x^2-x\right)^{3/2}x-\frac{1}{12}\left(x^2-x\right)^{3/2}+\frac{149(1-2x)\sqrt{x^2-x}}{2048}-\frac{683\sqrt{x^2-x}}{4096}+\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{32768}-\frac{153}{}$$

```
[Out] 1/4096*x-1/1024*x^2+1/192*x^3-1/32*x^4-1/12*(x^2-x)^(3/2)-1/32*x*(x^2-x)^(3
/2)+1/32768*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-1537/16384*arctanh(x/(x^2-x
)^(1/2))-1/32768*ln(1+8*x)+1/4*x^4*ln(-1+4*x+4*(x^2-x)^(1/2))-683/4096*(x^2
-x)^(1/2)+149/2048*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724, 670}

$$-\frac{x^4}{32}+\frac{x^3}{192}-\frac{x^2}{1024}-\frac{1}{32}\left(x^2-x\right)^{3/2}x-\frac{1}{12}\left(x^2-x\right)^{3/2}+\frac{149(1-2x)\sqrt{x^2-x}}{2048}-\frac{683\sqrt{x^2-x}}{4096}+\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/4096 - x^2/1024 + x^3/192 - x^4/32 - (683*Sqrt[-x + x^2])/4096 + (149*(1
- 2*x)*Sqrt[-x + x^2])/2048 - (-x + x^2)^(3/2)/12 - (x*(-x + x^2)^(3/2))/32
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32768 - (1537*ArcTanh[x/Sqrt[-x +
 x^2]])/16384 - Log[1 + 8*x]/32768 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])
/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx$$
$$= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \frac{x^4}{-4(1+2x)\sqrt{-x+x^2}+8\left(-\right.}$$
$$= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \left(\frac{1}{8192} - \frac{x}{1024} + \frac{x^2}{128} - \frac{x^3}{16} -\right.$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-}\right.$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{85(1-2x)\sqrt{-x+x^2}}{2048} -$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{129(1-2x)\sqrt{-x+x^2}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048}$$

Mathematica [A] time = 0.51, size = 117, normalized size = 0.68

$$\frac{-3072x^4 + 24576x^4 \log\left(4x+4\sqrt{(x-1)x}-1\right) + 512x^3 - 96x^2 - 8\sqrt{(x-1)x}\left(384x^3+640x^2+764x+1155\right)}{98304}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (24*x - 96*x^2 + 512*x^3 - 3072*x^4 - 8*Sqrt[(-1 + x)*x]*(1155 + 764*x + 64
0*x^2 + 384*x^3) - 6*Log[1 + 8*x] - 4611*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]]
+ 24576*x^4*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + 3*Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/98304
```

fricas [A] time = 1.01, size = 134, normalized size = 0.78

$$-\frac{1}{32}x^4 + \frac{1}{192}x^3 - \frac{1}{1024}x^2 + \frac{1}{4}\left(x^4-1\right)\log\left(4x+4\sqrt{x^2-x}-1\right) - \frac{1}{12288}\left(384x^3+640x^2+764x+1155\right)\sqrt{x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*x^4 + 1/192*x^3 - 1/1024*x^2 + 1/4*(x^4 - 1)*log(4*x + 4*sqrt(x^2 - x
) - 1) - 1/12288*(384*x^3 + 640*x^2 + 764*x + 1155)*sqrt(x^2 - x) + 1/4096*
x + 4095/32768*log(8*x + 1) - 2559/32768*log(-2*x + 2*sqrt(x^2 - x) + 1) +
4095/32768*log(-2*x + 2*sqrt(x^2 - x) - 1) - 4095/32768*log(-4*x + 4*sqrt(x
^2 - x) + 1)
```

giac [A] time = 0.33, size = 134, normalized size = 0.78

$$\frac{1}{4}x^4 \log\left(4x+4\sqrt{(x-1)x}-1\right) - \frac{1}{32}x^4 + \frac{1}{192}x^3 - \frac{1}{1024}x^2 - \frac{1}{12288}\left(4\left(32\left(3x+5\right)x+191\right)x+1155\right)\sqrt{x^2-x} + \frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/32*x^4 + 1/192*x^3 - 1/1024*x^
2 - 1/12288*(4*(32*(3*x + 5)*x + 191)*x + 1155)*sqrt(x^2 - x) + 1/4096*x -
1/32768*log(abs(8*x + 1)) + 1537/32768*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
 - 1/32768*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/32768*log(abs(-4*x + 4*
sqrt(x^2 - x) + 1))
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^3 \ln\left(4x - 1 + 4\sqrt{(x-1)\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(-1+4*x+4*((x-1)*x)^(1/2)),x)
```

```
[Out] int(x^3*ln(-1+4*x+4*((x-1)*x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log\left(4\,x + 4\,\sqrt{(x-1)x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(4\,x + 4\,\sqrt{x\,(x-1)} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

3.102 $\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=149

$$-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{223\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)}{1536}+\frac{1}{3}x^3\log\left(4\right.$$

```
[Out] -1/384*x+1/96*x^2-1/18*x^3-1/18*(x^2-x)^(3/2)-1/3072*arctanh(1/6*(1-10*x)/(
x^2-x)^(1/2))-223/1536*arctanh(x/(x^2-x)^(1/2))+1/3072*ln(1+8*x)+1/3*x^3*ln
(-1+4*x+4*(x^2-x)^(1/2))-85/384*(x^2-x)^(1/2)+5/64*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724}

$$-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -x/384 + x^2/96 - x^3/18 - (85*Sqrt[-x + x^2])/384 + (5*(1 - 2*x)*Sqrt[-x +
 x^2])/64 - (-x + x^2)^(3/2)/18 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/30
72 - (223*ArcTanh[x/Sqrt[-x + x^2]])/1536 + Log[1 + 8*x]/3072 + (x^3*Log[-1
 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx$$
$$= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \frac{x^3}{-4(1+2x)\sqrt{-x+x^2}+8\left(-\right.}$$
$$= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int\left(-\frac{1}{1024}+\frac{x}{128}-\frac{x^2}{16}+\frac{}{1024}\right.$$
$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}+\frac{\log(1+8x)}{3072}+\frac{1}{3}x^3\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$
$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{11}{192}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x\right.$$
$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x\right.$$
$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x\right.$$
$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x\right.$$

Mathematica [A] time = 0.35, size = 107, normalized size = 0.72

$$\frac{-512x^3+3072x^3\log\left(4x+4\sqrt{(x-1)x}-1\right)+96x^2-8\sqrt{(x-1)x}\left(64x^2+116x+165\right)-24x+6\log(8x+1)}{9216}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-24*x + 96*x^2 - 512*x^3 - 8*Sqrt[(-1 + x)*x]*(165 + 116*x + 64*x^2) + 6*L
og[1 + 8*x] - 669*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 3072*x^3*Log[-1 + 4*x
 + 4*Sqrt[(-1 + x)*x]] - 3*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/9216
```

fricas [A] time = 0.99, size = 124, normalized size = 0.83

$$-\frac{1}{18}x^3+\frac{1}{96}x^2+\frac{1}{3}\left(x^3+1\right)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{1152}\left(64x^2+116x+165\right)\sqrt{x^2-x}-\frac{1}{384}x-\frac{511}{3072}\log(8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/18*x^3 + 1/96*x^2 + 1/3*(x^3 + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/115
2*(64*x^2 + 116*x + 165)*sqrt(x^2 - x) - 1/384*x - 511/3072*log(8*x + 1) +
245/1024*log(-2*x + 2*sqrt(x^2 - x) + 1) - 511/3072*log(-2*x + 2*sqrt(x^2 -
 x) - 1) + 511/3072*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

giac [A] time = 0.31, size = 124, normalized size = 0.83

$$\frac{1}{3}x^3\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{18}x^3+\frac{1}{96}x^2-\frac{1}{1152}(4(16x+29)x+165)\sqrt{x^2-x}-\frac{1}{384}x+\frac{1}{3072}\log(|8x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/18*x^3 + 1/96*x^2 - 1/1152*(4*
(16*x + 29)*x + 165)*sqrt(x^2 - x) - 1/384*x + 1/3072*log(abs(8*x + 1)) + 2
23/3072*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/3072*log(abs(-2*x + 2*sqrt
(x^2 - x) - 1)) - 1/3072*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(4x - 1 + 4\sqrt{(x-1)\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
[Out] int(x^2*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(4\,x + 4\,\sqrt{(x-1)x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(4\,x + 4\,\sqrt{x\,\left(x-1\right)} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
[Out] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
[Out] Timed out
```

3.103 $\int x \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=127

$$-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\tanh^{-1}\left(-\right.$$

```
[Out] 1/32*x-1/8*x^2+1/256*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-33/128*arctanh(x/(
x^2-x)^(1/2))-1/256*ln(1+8*x)+1/2*x^2*ln(-1+4*x+4*(x^2-x)^(1/2))-11/32*(x^2
-x)^(1/2)+1/16*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724}

$$-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\tanh^{-1}\left(-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Arc
Tanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/12
8 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \int x\log\left(-1+4x+4\sqrt{-x+x^2}\right)dx$$
$$= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\frac{x^2}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x\right.}$$
$$= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\left(\frac{1}{128}-\frac{x}{16}-\frac{1}{128(1+8x)}-\right.$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{12}$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1+8x)+$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1+8x)+$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{13}{48}\tanh^{-1}\left(\frac{x}{\sqrt{-x+}}\right.$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}+\frac{1}{256}\tanh^{-1}\left(\frac{1-}{6\sqrt{-}}\right.$$

Mathematica [A] time = 0.29, size = 102, normalized size = 0.80

$$\frac{1}{256}\left(-32x^2+128x^2\log\left(4x+4\sqrt{(x-1)x}-1\right)-32\sqrt{(x-1)x}\,x+8x-72\sqrt{(x-1)x}-2\log(8x+1)-33\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (8*x - 32*x^2 - 72*Sqrt[(-1 + x)*x] - 32*x*Sqrt[(-1 + x)*x] - 2*Log[1 + 8*x
] - 33*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 128*x^2*Log[-1 + 4*x + 4*Sqrt[(-
1 + x)*x]] + Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/256
```

fricas [A] time = 1.01, size = 114, normalized size = 0.90

$$-\frac{1}{8}x^2+\frac{1}{2}\left(x^2-1\right)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{32}\sqrt{x^2-x}\,(4x+9)+\frac{1}{32}x+\frac{63}{256}\log(8x+1)-\frac{31}{256}\log\left(-2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*x^2 + 1/2*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/32*sqrt(x^2 - x
)*(4*x + 9) + 1/32*x + 63/256*log(8*x + 1) - 31/256*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 63/256*log(-2*x + 2*sqrt(x^2 - x) - 1) - 63/256*log(-4*x + 4*sqr
t(x^2 - x) + 1)
```

giac [A] time = 0.35, size = 114, normalized size = 0.90

$$\frac{1}{2}x^2\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{8}x^2-\frac{1}{32}\sqrt{x^2-x}\,(4x+9)+\frac{1}{32}x-\frac{1}{256}\log(|8x+1|)+\frac{33}{256}\log\left(\left|-2x+2\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/8*x^2 - 1/32*sqrt(x^2 - x)*(4*
x + 9) + 1/32*x - 1/256*log(abs(8*x + 1)) + 33/256*log(abs(-2*x + 2*sqrt(x^
```

```
2 - x) + 1)) - 1/256*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/256*log(abs(-
4*x + 4*sqrt(x^2 - x) + 1))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x \ln\left(4x - 1 + 4\sqrt{(x-1)\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
[Out] int(x*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log\left(4\,x + 4\,\sqrt{(x-1)x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(4\,x + 4\,\sqrt{x\,(x-1)} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
[Out] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log\left(4x + 4\sqrt{x^2 - x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
[Out] Integral(x*log(4*x + 4*sqrt(x**2 - x) - 1), x)
```

3.104 $\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{x^2-x}}{2}+x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

```
[Out] -1/2*x-1/16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-7/8*arctanh(x/(x^2-x)^(1/2)
)+1/16*ln(1+8*x)+x*ln(-1+4*x+4*(x^2-x)^(1/2))-1/2*(x^2-x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2537, 2533, 6742, 640, 620, 206, 734, 843, 724}

$$-\frac{\sqrt{x^2-x}}{2}+x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -x/2 - Sqrt[-x + x^2]/2 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16 - (7*Ar
cTanh[x/Sqrt[-x + x^2]])/8 + Log[1 + 8*x]/16 + x*Log[-1 + 4*x + 4*Sqrt[-x +
 x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2533

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
, x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] + Dist[(f^
2*(b^2 - 4*a*c))/2, Int[x/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a
*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx &= \int \log\left(-1+4x+4\sqrt{-x+x^2}\right)dx\\
&= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int \frac{x}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,d\\
&= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int\left(-\frac{1}{16}+\frac{1}{16(1+8x)}-\frac{x}{12\sqrt{-x+x^2}}+\right.\\
&= -\frac{x}{2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{2}{3}\int\frac{x}{\sqrt{-x+x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{1}{12}\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{5}{48}\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x)+x\log\left(-1+\right.\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\end{aligned}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.89

$$\frac{1}{16}\left(-8x-8\sqrt{(x-1)x}+16x\log\left(4x+4\sqrt{(x-1)x}-1\right)+2\log(8x+1)-7\log\left(-2x-2\sqrt{(x-1)x}+1\right)-\log\left(-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-8*x - 8*Sqrt[(-1 + x)*x] + 2*Log[1 + 8*x] - 7*Log[1 - 2*x - 2*Sqrt[(-1 +
x)*x]] + 16*x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/16
```

fricas [A] time = 0.82, size = 101, normalized size = 1.06

$$(x+1)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-x}-\frac{7}{16}\log(8x+1)+\frac{15}{16}\log\left(-2x+2\sqrt{x^2-x}+1\right)-\frac{7}{16}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) - 7/16*l
og(8*x + 1) + 15/16*log(-2*x + 2*sqrt(x^2 - x) + 1) - 7/16*log(-2*x + 2*sqr
t(x^2 - x) - 1) + 7/16*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

giac [A] time = 0.27, size = 101, normalized size = 1.06

$$x\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-x}+\frac{1}{16}\log(|8x+1|)+\frac{7}{16}\log\left(\left|-2x+2\sqrt{x^2-x}+1\right|\right)+\frac{1}{16}\log(|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) + 1/16*log(a
bs(8*x + 1)) + 7/16*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/16*log(abs(-2*
x + 2*sqrt(x^2 - x) - 1)) - 1/16*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

maple [A] time = 0.10, size = 80, normalized size = 0.84

$$x\ln\left(4x-1+4\sqrt{(x-1)x}\right)-\frac{x}{2}-\frac{\operatorname{arctanh}\left(\frac{-\frac{40x}{3}+\frac{4}{3}}{\sqrt{-80x+64\left(x+\frac{1}{8}\right)^2-1}}\right)}{16}+\frac{\ln(8x+1)}{16}-\frac{7\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{16}-\frac{\sqrt{x^2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

```
[Out] x*ln(4*x-1+4*((x-1)*x)^(1/2))-7/16*ln(x-1/2+(x^2-x)^(1/2))-1/16*arctanh(32/
3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/2*(x^2-x)^(1/2)-1/2*x+1/16*ln(
1+8*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{1}{2}x+\int\frac{2x^2+x}{2\left(4x^3-5x^2+4\left(x^{\frac{5}{2}}-x^{\frac{3}{2}}\right)\sqrt{x-1}+x\right)}dx-\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 1/2*x + integrate(1/2*(2*x^2 + x)/
(4*x^3 - 5*x^2 + 4*(x^(5/2) - x^(3/2))*sqrt(x - 1) + x), x) - 1/2*log(sqrt(
x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(4x+4\sqrt{x\,(x-1)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(4x+4\sqrt{x\left(x-1\right)}-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Integral(log(4*x + 4*sqrt(x*(x - 1)) - 1), x)
```

3.105 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(-1+4*x+4*(x^2-x)^(1/2))/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
[Out] Defer[Int][Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x, x]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}\,dx$$

Mathematica [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
[Out] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="fricas")
[Out] integral(log(4*x + 4*sqrt(x^2 - x) - 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x,x)
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}\,-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}\,-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x,x)
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4x+4\sqrt{x^2-x}\,-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x,x)
[Out] Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x, x)
```

3.106 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$

Optimal. Leaf size=76

$$\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+4\log(x)-4\log(8x+1)$$

```
[Out] 4*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))+4*ln(x)-4*ln(1+8*x)-ln(-1+4*x+4*(x^2-
x)^(1/2))/x+4*(x^2-x)^(1/2)/x
```

Rubi [A] time = 0.26, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2537, 2535, 6742, 640, 620, 206, 662, 664, 734, 843, 724}

$$\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+4\log(x)-4\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[-x + x^2])/x + 4*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 4*Log[x]
- 4*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^2}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int \frac{1}{x\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+\right.\right.}$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int\left(-\frac{1}{2x}+\frac{4}{1+8x}-\frac{x}{12\sqrt{-x+x^2}}+\right.$$

$$= 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{2}{3}\int\frac{x}{\sqrt{-x+}}$$

$$= \frac{4\sqrt{-x+x^2}}{x} + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

$$= \frac{4\sqrt{-x+x^2}}{x} + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

$$= \frac{4\sqrt{-x+x^2}}{x} - \frac{40}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) + 4\log(x) - 4\log(1+8x) - \frac{\log}{}$$

$$= \frac{4\sqrt{-x+x^2}}{x} + 4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) + 4\log(x) - 4\log(1+8x) - \frac{\log}{}$$

Mathematica [A] time = 0.25, size = 68, normalized size = 0.89

$$\frac{4\sqrt{(x-1)x}}{x}+4\log(x)-8\log(8x+1)-\frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x}+4\log\left(-10x+6\sqrt{(x-1)x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[(-1 + x)*x])/x + 4*Log[x] - 8*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[
(-1 + x)*x]]/x + 4*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]]
```

fricas [A] time = 1.01, size = 115, normalized size = 1.51

$$-\frac{7\,x\log\left(8\,x+1\right)+2\,(x+1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-8\,x\log(x)+x\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)+7\,x\log\left(\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(7*x*log(8*x + 1) + 2*(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 8*x*log
(x) + x*log(-2*x + 2*sqrt(x^2 - x) + 1) + 7*x*log(-2*x + 2*sqrt(x^2 - x) -
1) - 7*x*log(-4*x + 4*sqrt(x^2 - x) + 1) - 8*x - 8*sqrt(x^2 - x))/x
```

giac [A] time = 0.30, size = 92, normalized size = 1.21

$$-\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}+\frac{4}{x-\sqrt{x^2-x}}-4\,\log\left(|8\,x+1|\right)+4\,\log\left(|x|\right)-4\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)+4\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -log(4*x + 4*sqrt((x - 1)*x) - 1)/x + 4/(x - sqrt(x^2 - x)) - 4*log(abs(8*x
 + 1)) + 4*log(abs(x)) - 4*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 4*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^2,x)
```

```
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}\,-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}\,-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4x+4\sqrt{x^2-x}\,-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**2,x)
```

```
[Out] Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x**2, x)
```

3.107 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{2\left(x^2-x\right)^{3/2}}{3x^3}-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

```
[Out] -2/x-2/3*(x^2-x)^(3/2)/x^3-16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-16*ln(x)+
16*ln(1+8*x)-1/2*ln(-1+4*x+4*(x^2-x)^(1/2))/x^2-10*(x^2-x)^(1/2)/x
```

Rubi [A] time = 0.29, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2537, 2535, 6742, 640, 620, 206, 734, 843, 724, 650, 662, 664}

$$-\frac{2\left(x^2-x\right)^{3/2}}{3x^3}-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(1
 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x +
 4*Sqrt[-x + x^2]]/(2*x^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
```

```
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^3}\,dx$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \frac{1}{x^2\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x\right.\right.}$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int\left(-\frac{1}{2x^2}+\frac{4}{x}-\frac{32}{1+8x}-\frac{x}{12\sqrt{-x+}}\right.$$
$$= -\frac{2}{x} - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} + \frac{1}{3}\int$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x) - \frac{\log}{}$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x) - \frac{\log}{}$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} + \frac{160}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) - 16\log$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - 16\log$$

Mathematica [A] time = 0.30, size = 82, normalized size = 0.81

$$-\frac{2\sqrt{(x-1)x}\,(16x-1)}{3x^2} - \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{2x^2} - \frac{2}{x} - 16\log(x) + 32\log(8x+1) - 16\log\left(-10x+6\sqrt{(x-1)x}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (2*Sqrt[(-1 + x)*x]*(-1 + 16*x))/(3*x^2) - 16*Log[x] + 32*Log[1 + 8*
x] - Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/(2*x^2) - 16*Log[1 - 10*x + 6*Sqrt[
(-1 + x)*x]]
```

fricas [A] time = 0.57, size = 138, normalized size = 1.37

$$\frac{189\,x^2\log\left(8\,x+1\right) - 192\,x^2\log(x) + 3\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right) + 189\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right) - 1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(189*x^2*log(8*x + 1) - 192*x^2*log(x) + 3*x^2*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 189*x^2*log(-2*x + 2*sqrt(x^2 - x) - 1) - 189*x^2*log(-4*x + 4*s
qrt(x^2 - x) + 1) - 128*x^2 + 6*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) -
8*sqrt(x^2 - x)*(16*x - 1) - 24*x)/x^2
```

giac [A] time = 0.31, size = 130, normalized size = 1.29

$$-\frac{2}{x} - \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{2\,x^2} - \frac{2\left(18\left(x-\sqrt{x^2-x}\right)^2 - 3\,x + 3\,\sqrt{x^2-x}+1\right)}{3\left(x-\sqrt{x^2-x}\right)^3} + 16\,\log\left(|8\,x+1|\right) - 16\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -2/x - 1/2*log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2 - 2/3*(18*(x - sqrt(x^2 - x
))^2 - 3*x + 3*sqrt(x^2 - x) + 1)/(x - sqrt(x^2 - x))^3 + 16*log(abs(8*x +
1)) - 16*log(abs(x)) + 16*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) - 16*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^3,x)
```

```
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}\,-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}\,-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

3.108 $\int x^{3/2} \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal. Leaf size=187

$$-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2\left(x^2-x\right)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{320\sqrt{2}\sqrt{x-1}\sqrt{x}}-\frac{71\left(x^2-x\right)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}\right.$$

```
[Out] 1/60*x^(3/2)-2/25*x^(5/2)-71/300*(x^2-x)^(3/2)/x^(3/2)+2/5*x^(5/2)*ln(-1+4*
x+4*(x^2-x)^(1/2))+1/640*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2/25*(x^2-x)^(3/
2)/x^(1/2)-1/160*x^(1/2)-17/32/x^(1/2)*(x^2-x)^(1/2)-1/640*arctan(2/3*2^(1/
2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2000, 2016, 1146, 444, 50, 63}

$$-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2\left(x^2-x\right)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{71\left(x^2-x\right)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\right.}{320\sqrt{2}\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -Sqrt[x]/160 + x^(3/2)/60 - (2*x^(5/2))/25 - (17*Sqrt[-x + x^2])/(32*Sqrt[x
]) - (71*(-x + x^2)^(3/2))/(300*x^(3/2)) - (2*(-x + x^2)^(3/2))/(25*Sqrt[x]
) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(320*Sqrt[2]*Sqrt[-
1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(320*Sqrt[2]) + (2*x^(5/2)*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \int x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)dx$$
$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{5}\int\frac{x^{5/2}}{-4(1+2x)\sqrt{-x+x^2}+$$
$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{5}\text{Subst}\left(\int\frac{}{-4\left(1+2x^2\right)\sqrt{-}}\right.$$
$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{5}\text{Subst}\left(\int\left(-\frac{1}{1024}+\frac{x^2}{128}-\right.\right.$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}+\frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{160}\text{Su}$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{11\left(-x+x^2\right)^{3/2}}{60x^{3/2}}-\frac{2\left(-x+x^2\right.}{25\sqrt{x}}$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right.}{25\sqrt{x}}$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x\right.}{25\sqrt{x}}$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x\right.}{25\sqrt{x}}$$
$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x\right.}{25\sqrt{x}}$$

Mathematica [C] time = 0.87, size = 232, normalized size = 1.24

$$-3072x^{5/2}-3072\sqrt{(x-1)x}\,x^{3/2}+640x^{3/2}+15360x^{5/2}\log\left(4x+4\sqrt{(x-1)x}-1\right)-6016\sqrt{(x-1)x}\sqrt{x}-240$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-240*Sqrt[x] + 640*x^(3/2) - 3072*x^(5/2) - (11312*Sqrt[(-1 + x)*x])/Sqrt[
x] - 6016*Sqrt[x]*Sqrt[(-1 + x)*x] - 3072*x^(3/2)*Sqrt[(-1 + x)*x] + 60*Sqr
t[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 60*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)
*x])/(3*Sqrt[x])] - (30*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] + (15*I)*Sqrt[2]*Log[
(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 15360*x^(5/2)*Log[-1 + 4*x + 4
*Sqrt[(-1 + x)*x]] + (15*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 +
x)*x])])/38400
```

fricas [A] time = 0.96, size = 110, normalized size = 0.59

$$\frac{3840\,x^{\frac{7}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)+15\,\sqrt{2}\,x\arctan\left(2\,\sqrt{2}\,\sqrt{x}\right)+15\,\sqrt{2}\,x\arctan\left(\frac{3\,\sqrt{2}\,\sqrt{x}}{4\,\sqrt{x^2-x}}\right)-4\left(192\,x^2+376\,x\right.}{9600\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9600*(3840*x^(7/2)*log(4*x + 4*sqrt(x^2 - x) - 1) + 15*sqrt(2)*x*arctan(2
*sqrt(2)*sqrt(x)) + 15*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x))
- 4*(192*x^2 + 376*x + 707)*sqrt(x^2 - x)*sqrt(x) - 4*(192*x^3 - 40*x^2 + 1
5*x)*sqrt(x))/x
```

giac [A] time = 0.37, size = 132, normalized size = 0.71

$$\frac{2}{5}x^{\frac{5}{2}}\log\left(4x+4\sqrt{(x-1)x}-1\right)+\frac{1}{1280}\sqrt{2}\pi i-\frac{2}{25}x^{\frac{5}{2}}+\frac{1}{1280}\sqrt{2}\left(\pi-2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{1}{24\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) + 1/1280*sqrt(2)*pi*i - 2/25*x
^(5/2) + 1/1280*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))
^2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 1/2400*(8*(24*x + 47)*x + 707)*sqrt(x -
 1) + 1/60*x^(3/2) + 1/640*sqrt(2)*arctan(2/3*sqrt(2)*i) + 1/640*sqrt(2)*ar
ctan(2*sqrt(2)*sqrt(x)) + 707/2400*i - 1/160*sqrt(x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

```
[Out] int(x^(3/2)*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5}x^{\frac{5}{2}}\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{2}{25}\left(2x^{2}+5\right)\sqrt{x}-\frac{2}{15}x^{\frac{3}{2}}+\int\frac{2x^{\frac{5}{2}}+x^{\frac{3}{2}}}{5\left(4x^{2}+4\left(x^{\frac{3}{2}}-\sqrt{x}\right)\sqrt{x-1}-5x+1\right)}dx+\frac{1}{5}\log\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 2/25*(2*x^2 + 5)*sqrt(x)
 - 2/15*x^(3/2) + integrate(1/5*(2*x^(5/2) + x^(3/2))/(4*x^2 + 4*(x^(3/2) -
 sqrt(x))*sqrt(x - 1) - 5*x + 1), x) + 1/5*log(sqrt(x) + 1) - 1/5*log(sqrt(
x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

3.109 $\int \sqrt{x} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal. Leaf size=158

$$-\frac{2x^{3/2}}{9}-\frac{11\sqrt{x^2-x}}{12\sqrt{x}}+\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}}-\frac{2\left(x^2-x\right)^{3/2}}{9x^{3/2}}+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{\sqrt{x}}{12}-\frac{\tan^{-}}{}$$

```
[Out] -2/9*x^(3/2)-2/9*(x^2-x)^(3/2)/x^(3/2)+2/3*x^(3/2)*ln(-1+4*x+4*(x^2-x)^(1/2
))-1/48*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)+1/12*x^(1/2)-11/12/x^(1/2)*(x^2-x
)^(1/2)+1/48*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^
(1/2)/x^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2000, 1146, 444, 50, 63, 204}

$$-\frac{2x^{3/2}}{9}-\frac{11\sqrt{x^2-x}}{12\sqrt{x}}-\frac{2\left(x^2-x\right)^{3/2}}{9x^{3/2}}+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\sqrt{x}}{12}-\frac{\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] Sqrt[x]/12 - (2*x^(3/2))/9 - (11*Sqrt[-x + x^2])/(12*Sqrt[x]) - (2*(-x + x^
2)^(3/2))/(9*x^(3/2)) + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])
/(24*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(24*Sqrt[2])
 + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt{x} \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int \sqrt{x} \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{3}\int \frac{x^{3/2}}{-4(1+2x)\sqrt{-x+x^2}+ \cdots}$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\text{Subst}\left(\int \frac{\cdots}{-4\left(1+2x^2\right)\sqrt{-\cdots}}\right.$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\text{Subst}\left(\int\left(\frac{1}{128}-\frac{x^2}{16}-\frac{\cdots}{128\cdots}\right.\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}+\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{1}{12}\text{Subst}\left(\int \frac{\cdots}{1\cdots}\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}\cdots$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}\cdots$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\cdots$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\cdots$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}+\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\cdots\right.}{24\sqrt{2}\sqrt{-1+\cdots}}$$

Mathematica [C] time = 0.65, size = 209, normalized size = 1.32

$$\frac{1}{576}\left(-128x^{3/2}+384x^{3/2}\log\left(4x+4\sqrt{(x-1)x}-1\right)-128\sqrt{(x-1)x}\sqrt{x}+48\sqrt{x}-\frac{400\sqrt{(x-1)x}}{\sqrt{x}}+6i\sqrt{2}\log\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (48*Sqrt[x] - 128*x^(3/2) - (400*Sqrt[(-1 + x)*x])/Sqrt[x] - 128*Sqrt[x]*Sq
rt[(-1 + x)*x] - 12*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 12*Sqrt[2]*ArcTan[(
2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (6*I)*Sqrt[2]*Log[4*(1 + 8*x)^2]
 - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 384*x^(3/
2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10
*x + 6*Sqrt[(-1 + x)*x])])/576
```

fricas [A] time = 0.81, size = 100, normalized size = 0.63

$$\frac{96x^{\frac{5}{2}}\log\left(4x+4\sqrt{x^2-x}-1\right)-3\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)-3\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4\sqrt{x^2-x}(8x+25)\sqrt{\cdots}}{144x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/144*(96*x^(5/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 3*sqrt(2)*x*arctan(2*sqr
t(2)*sqrt(x)) - 3*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 4*s
qrt(x^2 - x)*(8*x + 25)*sqrt(x) - 4*(8*x^2 - 3*x)*sqrt(x))/x
```

giac [A] time = 0.37, size = 122, normalized size = 0.77

$$-\frac{1}{96}\sqrt{2}\pi i+\frac{2}{3}x^{\frac{3}{2}}\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{96}\sqrt{2}\left(\pi-2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{1}{36}(8x+25)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*pi*i + 2/3*x^(3/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/96*sq
rt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x -
1) - sqrt(x)))) - 1/36*(8*x + 25)*sqrt(x - 1) - 2/9*x^(3/2) - 1/48*sqrt(2)*
arctan(2/3*sqrt(2)*i) - 1/48*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) + 25/36*i +
1/12*sqrt(x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

```
[Out] int(x^(1/2)*ln(4*x-1+4*((x-1)*x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3}x^{\frac{3}{2}}\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{4}{9}x^{\frac{3}{2}}-\frac{2}{3}\sqrt{x}+\int\frac{2x^2+x}{3\left(4x^{\frac{5}{2}}+4\left(x^2-x\right)\sqrt{x-1}-5x^{\frac{3}{2}}+\sqrt{x}\right)}dx+\frac{1}{3}\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4/9*x^(3/2) - 2/3*sqrt(x
) + integrate(1/3*(2*x^2 + x)/(4*x^(5/2) + 4*(x^2 - x)*sqrt(x - 1) - 5*x^(3
/2) + sqrt(x)), x) + 1/3*log(sqrt(x) + 1) - 1/3*log(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\ln\left(4x+4\sqrt{x\,(x-1)}-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=118

$$-\frac{2\sqrt{x^2-x}}{\sqrt{x}}+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}-2\sqrt{x}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*x^(1/2)+2*ln(-1+4*x+4*(x^2-x)^(1/2)
)*x^(1/2)-2/x^(1/2)*(x^2-x)^(1/2)-1/2*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2
-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 1146, 444, 50, 63}

$$-\frac{2\sqrt{x^2-x}}{\sqrt{x}}+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}-2\sqrt{x}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - (2*Sqrt[-x + x^2])/Sqrt[x] - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]
*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x
]]/Sqrt[2] + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\,dx$$
$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+16\int \frac{\sqrt{x}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x\right.}$$
$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+32\,\mathrm{Subst}\left(\int \frac{x^2}{-4\left(1+2x^2\right)\sqrt{-x^2+}}\right.$$
$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+32\,\mathrm{Subst}\left(\int\left(-\frac{1}{16}+\frac{1}{16\left(1+8x^2\right)}\right.\right.$$
$$= -2\sqrt{x}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+2\,\mathrm{Subst}\left(\int \frac{1}{1+8x^2}\,dx,x,\right.$$
$$= -2\sqrt{x}-\frac{8\sqrt{-x+x^2}}{3\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x}\right.$$
$$= -2\sqrt{x}-\frac{8\sqrt{-x+x^2}}{3\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x}\right.$$
$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x}\right.$$
$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x}\right.$$
$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}-\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\right.}{\sqrt{2}}$$

Mathematica [C] time = 0.57, size = 186, normalized size = 1.58

$$\frac{1}{8}\left(-16\sqrt{x}-\frac{16\sqrt{(x-1)x}}{\sqrt{x}}-2i\sqrt{2}\log\left(4(8x+1)^2\right)+i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x-1)x}+1\right)\right)+16\sqrt{x}\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] (-16*Sqrt[x] - (16*Sqrt[(-1 + x)*x])/Sqrt[x] + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*S
qrt[x]] - 4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I
)*Sqrt[2]*Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(
-1 + x)*x])] + 16*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + I*Sqrt[2]*Lo
g[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])])/8
```

fricas [A] time = 0.98, size = 84, normalized size = 0.71

$$\frac{\sqrt{2}\,x\arctan\left(2\sqrt{2}\sqrt{x}\right)+\sqrt{2}\,x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)+4\,x^{\frac{3}{2}}\log\left(4\,x+4\sqrt{x^2-x}-1\right)-4\,x^{\frac{3}{2}}-4\sqrt{x^2-x}\sqrt{x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + sqrt(2)*x*arctan(3/4*sqrt(2)*sqr
t(x)/sqrt(x^2 - x)) + 4*x^(3/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 4*x^(3/2)
- 4*sqrt(x^2 - x)*sqrt(x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(4\,x + 4\,\sqrt{(x-1)x} - 1\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/sqrt(x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x - 1 + 4\sqrt{(x-1)\,x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(1/2),x)
```

```
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,\sqrt{x}\,\log\left(4\,\sqrt{x-1}\,\sqrt{x} + 4\,x - 1\right) - 4\,\sqrt{x} + \int \frac{2\,x^2 + x}{4\,x^{\frac{7}{2}} - 5\,x^{\frac{5}{2}} + 4\left(x^3 - x^2\right)\sqrt{x-1} + x^{\frac{3}{2}}}\,dx + \log\left(\sqrt{x} + 1\right) - \log\left(\sqrt{x} -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4*sqrt(x) + integrate((2*x
^2 + x)/(4*x^(7/2) - 5*x^(5/2) + 4*(x^3 - x^2)*sqrt(x - 1) + x^(3/2)), x) +
 log(sqrt(x) + 1) - log(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x + 4\,\sqrt{x\,(x-1)} - 1\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(1/2),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}-\frac{4\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}-8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)$$

```
[Out] -8*arctan(x^(1/2)/(x^2-x)^(1/2))+4*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*ln(-
1+4*x+4*(x^2-x)^(1/2))/x^(1/2)-4*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(
1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2021, 2008, 1146, 444, 50, 63, 204}

$$-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}-\frac{4\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}-8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x
]*Sqrt[x]) + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 8*ArcTan[Sqrt[x]/Sqrt[-x
 + x^2]] - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{3/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 16\int \frac{1}{\sqrt{x}\left(-4(1+2x)\sqrt{-x+x^2}+8\right.}$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\,\text{Subst}\left(\int \frac{1}{-4\left(1+2x^2\right)\sqrt{-x^2+x}}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\,\text{Subst}\left(\int\left(-\frac{1}{2\left(1+8x^2\right)} - \frac{x}{12\sqrt{-}}\right.\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{8}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x\right.}}{\sqrt{x}}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x\right.}}{\sqrt{x}}$$

$$= -\frac{4\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan$$

Mathematica [C] time = 0.46, size = 177, normalized size = 1.55

$$-2i\sqrt{2}\log\left(4(8x+1)^2\right)+i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x-1)x}+1\right)\right)-\frac{2\log\left(4x+4\sqrt{(x-1)x}-1\right)}{\sqrt{x}}+i\sqrt{2}\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 8*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] -
4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I)*Sqrt[2]*
Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x]
)] - (2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/Sqrt[x] + I*Sqrt[2]*Log[(1 + 8*
x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])]
```

fricas [A] time = 0.81, size = 84, normalized size = 0.74

$$\frac{2\left(2\sqrt{2}\,x\arctan\left(2\sqrt{2}\sqrt{x}\right)+2\sqrt{2}\,x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4\,x\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\sqrt{x}\log\left(4\,x+4\sqrt{x^2-x}-1\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + 2*sqrt(2)*x*arctan(3/4*sqrt(2)*s
qrt(x)/sqrt(x^2 - x)) - 4*x*arctan(sqrt(x)/sqrt(x^2 - x)) - sqrt(x)*log(4*x
 + 4*sqrt(x^2 - x) - 1))/x
```

giac [A] time = 0.48, size = 166, normalized size = 1.46

$$2\sqrt{2}\pi i-4\pi i+4\pi\mathrm{sgn}\left(\sqrt{x-1}-\sqrt{x}\right)-2\sqrt{2}\left(\pi\mathrm{sgn}\left(\sqrt{x-1}-\sqrt{x}\right)+2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*pi*i - 4*pi*i + 4*pi*sgn(sqrt(x - 1) - sqrt(x)) - 2*sqrt(2)*(pi*s
gn(sqrt(x - 1) - sqrt(x)) + 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2
 - 1)/(sqrt(x - 1) - sqrt(x)))) + 4*sqrt(2)*arctan(2/3*sqrt(2)*i) + 4*sqrt(
2)*arctan(2*sqrt(2)*sqrt(x)) - 2*log(4*x + 4*sqrt((x - 1)*x) - 1)/sqrt(x) -
 8*arctan(i) + 8*arctan(1/2*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1) -
sqrt(x)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x-1+4\sqrt{(x-1)\,x}\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(3/2),x)
```

```
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)}{\sqrt{x}}-\frac{2}{\sqrt{x}}-\int\frac{2x^2+x}{4x^{\frac{9}{2}}-5x^{\frac{7}{2}}+x^{\frac{5}{2}}+4\left(x^4-x^3\right)\sqrt{x-1}}\,dx-\log\left(\sqrt{x}+1\right)+\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/sqrt(x) - 2/sqrt(x) - integrate((2*
x^2 + x)/(4*x^(9/2) - 5*x^(7/2) + x^(5/2) + 4*(x^4 - x^3)*sqrt(x - 1)), x)
- log(sqrt(x) + 1) + log(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4x+4\sqrt{x\,(x-1)}-1\right)}{x^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(3/2),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=151

$$\frac{32\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}}+\frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+\frac{4\sqrt{x^2-x}}{3x^{3/2}}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}-\frac{16}{3\sqrt{x}}-\frac{32}{3}\sqrt{2}$$

```
[Out] 44/3*arctan(x^(1/2)/(x^2-x)^(1/2))-2/3*ln(-1+4*x+4*(x^2-x)^(1/2))/x^(3/2)-3
2/3*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-16/3/x^(1/2)+4/3*(x^2-x)^(1/2)/x^(3/2
)+32/3*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/
x^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2020, 2008, 2021, 1146, 444, 50, 63}

$$\frac{4\sqrt{x^2-x}}{3x^{3/2}}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}+\frac{32\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}}+\frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\frac{16}{3\sqrt{x}}-\frac{32}{3}\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] -16/(3*Sqrt[x]) + (4*Sqrt[-x + x^2])/(3*x^(3/2)) + (32*Sqrt[2]*Sqrt[-x + x^
2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(3*Sqrt[-1 + x]*Sqrt[x]) - (32*Sqrt[
2]*ArcTan[2*Sqrt[2]*Sqrt[x]])/3 + (44*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/3 - (
2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{5/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{16}{3}\int \frac{1}{x^{3/2}\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-\right.\right.}$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(-4\left(1+2x^2\right)\sqrt{-x^2+\right.}}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\,\text{Subst}\left(\int \left(-\frac{1}{2x^2}+\frac{4}{1+8x^2}-\frac{}{12\sqrt{-}}\right.\right.$$

$$= -\frac{16}{3\sqrt{x}} - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{8}{9}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \right.$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(\right.}{}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(\right.}{}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) -$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) -$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{32\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}} - \frac{32}{3}\sqrt{2}\,\text{t}$$

Mathematica [C] time = 0.73, size = 204, normalized size = 1.35

$$\frac{2}{3}\left(\frac{2\sqrt{(x-1)x}}{x^{3/2}} - \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x^{3/2}} - \frac{8}{\sqrt{x}} + 8i\sqrt{2}\log\left(4(8x+1)^2\right) - 4i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] (2*(-8/Sqrt[x] + (2*Sqrt[(-1 + x)*x])/x^(3/2) - 16*Sqrt[2]*ArcTan[2*Sqrt[2]
*Sqrt[x]] - 22*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] + 16*Sqrt[2]*ArcTan[(2*Sqrt
[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (8*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] - (4*
I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] - Log[-1 + 4*x +
```

```
4*Sqrt[(-1 + x)*x]]/x^(3/2) - (4*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqr
t[(-1 + x)*x]])])/3
```

fricas [A] time = 0.93, size = 108, normalized size = 0.72

$$-\frac{2\left(16\sqrt{2}x^2\arctan\left(2\sqrt{2}\sqrt{x}\right)+16\sqrt{2}x^2\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-22x^2\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+8x^{\frac{3}{2}}+\sqrt{x}\log\left(4x+4\right.\right.}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*sqrt(2)*x^2*arctan(2*sqrt(2)*sqrt(x)) + 16*sqrt(2)*x^2*arctan(3/4*
sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 22*x^2*arctan(sqrt(x)/sqrt(x^2 - x)) + 8*x
^(3/2) + sqrt(x)*log(4*x + 4*sqrt(x^2 - x) - 1) - 2*sqrt(x^2 - x)*sqrt(x))/
x^2
```

giac [A] time = 0.50, size = 181, normalized size = 1.20

$$\frac{22}{3}\pi-\frac{16}{3}\sqrt{2}\left(\pi-2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{32}{3}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)+\frac{8\left(\sqrt{x-1}-\sqrt{x}-\right.}{3\left(\left(\sqrt{x-1}-\sqrt{x}-\frac{}{\sqrt{x-}}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] 22/3*pi - 16/3*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^
2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 32/3*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) +
 8/3*(sqrt(x - 1) - sqrt(x) - 1/(sqrt(x - 1) - sqrt(x)))/((sqrt(x - 1) - sq
rt(x) - 1/(sqrt(x - 1) - sqrt(x)))^2 + 4) - 16/3/sqrt(x) - 2/3*log(4*x + 4*
sqrt(x^2 - x) - 1)/x^(3/2) - 44/3*arctan(1/2*((sqrt(x - 1) - sqrt(x))^2 - 1
)/(sqrt(x - 1) - sqrt(x)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(4x-1+4\sqrt{(x-1)x}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(5/2),x)
```

```
[Out] int(ln(4*x-1+4*((x-1)*x)^(1/2))/x^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3\sqrt{x}}-\frac{2\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)}{3x^{\frac{3}{2}}}-\frac{2}{9x^{\frac{3}{2}}}-\int\frac{2x^2+x}{3\left(4x^{\frac{11}{2}}-5x^{\frac{9}{2}}+x^{\frac{7}{2}}+4\left(x^5-x^4\right)\sqrt{x-1}\right)}\,dx-\frac{1}{3}\log\left(\sqrt{x}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/sqrt(x) - 2/3*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/x^(3/2) - 2/9/x^(3/2
) - integrate(1/3*(2*x^2 + x)/(4*x^(11/2) - 5*x^(9/2) + x^(7/2) + 4*(x^5 -
x^4)*sqrt(x - 1)), x) - 1/3*log(sqrt(x) + 1) + 1/3*log(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4x+4\sqrt{x\,(x-1)}-1\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(5/2),x)
```

```
[Out] Timed out
```

3.113 $\int x^3 \log(a + be^x)\, dx$

Optimal. Leaf size=93

$$-x^3\text{Li}_2\left(-\frac{be^x}{a}\right)+3x^2\text{Li}_3\left(-\frac{be^x}{a}\right)-6x\text{Li}_4\left(-\frac{be^x}{a}\right)+6\text{Li}_5\left(-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log(a+be^x)-\frac{1}{4}x^4\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2532, 2531, 6609, 2282, 6589}

$$-x^3\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-6x\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\text{PolyLog}\left(5,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log(a+be^x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \log \left(a+be^x\right) \, dx = \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) + \int x^3 \log \left(1+\frac{be^x}{a}\right) \, dx$$

$$= \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3\int x^2\text{Li}_2\left(-\frac{be^x}{a}\right) \, dx$$

$$= \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6\int x\text{Li}_3\left(-\frac{be^x}{a}\right.$$

$$= \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right)$$

$$= \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right)$$

$$= \frac{1}{4}x^4 \log \left(a+be^x\right) - \frac{1}{4}x^4 \log \left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right)$$

Mathematica [A] time = 0.01, size = 93, normalized size = 1.00

$$-x^3\text{Li}_2\left(-\frac{be^x}{a}\right)+3x^2\text{Li}_3\left(-\frac{be^x}{a}\right)-6x\text{Li}_4\left(-\frac{be^x}{a}\right)+6\text{Li}_5\left(-\frac{be^x}{a}\right)+\frac{1}{4}x^4 \log \left(a+be^x\right)-\frac{1}{4}x^4 \log \left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

fricas [C] time = 1.14, size = 88, normalized size = 0.95

$$\frac{1}{4}\,x^4 \log \left(be^x+a\right)-\frac{1}{4}\,x^4 \log \left(\frac{be^x+a}{a}\right)-x^3\text{Li}_2\left(-\frac{be^x+a}{a}+1\right)+3\,x^2\text{polylog}\left(3,-\frac{be^x}{a}\right)-6\,x\text{polylog}\left(4,-\frac{be^x}{a}\right)+6\,\text{pc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log((b*e^x + a)/a) - x^3*dilog(-(b*e^x + a
)/a + 1) + 3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylo
g(5, -b*e^x/a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log \left(be^x+a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(b*e^x + a), x)
```

maple [A] time = 0.08, size = 84, normalized size = 0.90

$$-\frac{x^4 \ln\left(\frac{b\,e^x}{a}+1\right)}{4}+\frac{x^4 \ln\left(b\,e^x+a\right)}{4}-x^3 \operatorname{polylog}\left(2,-\frac{b\,e^x}{a}\right)+3x^2 \operatorname{polylog}\left(3,-\frac{b\,e^x}{a}\right)-6x \operatorname{polylog}\left(4,-\frac{b\,e^x}{a}\right)+6\,\mathrm{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(b*exp(x)+a),x)
```

```
[Out] 1/4*x^4*ln(b*exp(x)+a)-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

maxima [A] time = 0.78, size = 82, normalized size = 0.88

$$\frac{1}{4}\,x^4 \log\left(be^x+a\right)-\frac{1}{4}\,x^4 \log\left(\frac{be^x}{a}+1\right)-x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+3\,x^2\mathrm{Li}_3(-\frac{be^x}{a})-6\,x\mathrm{Li}_4(-\frac{be^x}{a})+6\,\mathrm{Li}_5(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log(b*e^x/a + 1) - x^3*dilog(-b*e^x/a) + 3
*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylog(5, -b*e^x/
a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \ln\left(a+b\,e^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(a + b*exp(x)),x)
```

```
[Out] int(x^3*log(a + b*exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^4e^x}{a+be^x}\,dx}{4}+\frac{x^4\log\left(a+be^x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**4*exp(x)/(a + b*exp(x)), x)/4 + x**4*log(a + b*exp(x))/4
```

3.114 $\int x^2 \log(a + be^x) \, dx$

Optimal. Leaf size=77

$$-x^2 \mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 2\mathrm{Li}_4\left(-\frac{be^x}{a}\right) + \frac{1}{3}x^3 \log(a + be^x) - \frac{1}{3}x^3 \log\left(\frac{be^x}{a} + 1\right)$$

```
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2532, 2531, 6609, 2282, 6589}

$$-x^2 \mathrm{PolyLog}\left(2, -\frac{be^x}{a}\right) + 2x\mathrm{PolyLog}\left(3, -\frac{be^x}{a}\right) - 2\mathrm{PolyLog}\left(4, -\frac{be^x}{a}\right) + \frac{1}{3}x^3 \log(a + be^x) - \frac{1}{3}x^3 \log\left(\frac{be^x}{a} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \log(a+be^x)\,dx = \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) + \int x^2\log\left(1+\frac{be^x}{a}\right)dx$$
$$= \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2\int x\text{Li}_2\left(-\frac{be^x}{a}\right)dx$$
$$= \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\int \text{Li}_3\left(-\frac{be^x}{a}\right.$$
$$= \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\,\text{Subst}\left(\int\right.$$
$$= \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\text{Li}_4\left(-\frac{be^x}{a}\right)$$

Mathematica [A] time = 0.01, size = 77, normalized size = 1.00

$$-x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\text{Li}_4\left(-\frac{be^x}{a}\right) + \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a + b*E^x],x]
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

fricas [C] time = 1.12, size = 73, normalized size = 0.95

$$\frac{1}{3}x^3\log(be^x+a) - \frac{1}{3}x^3\log\left(\frac{be^x+a}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x+a}{a}+1\right) + 2x\text{polylog}\left(3,-\frac{be^x}{a}\right) - 2\,\text{polylog}\left(4,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="fricas")
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log((b*e^x + a)/a) - x^2*dilog(-(b*e^x + a
)/a + 1) + 2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\log(be^x+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x^2*log(b*e^x + a), x)
```

maple [A] time = 0.07, size = 69, normalized size = 0.90

$$-\frac{x^3\ln\left(\frac{b\,\mathrm{e}^x}{a}+1\right)}{3} + \frac{x^3\ln(b\,\mathrm{e}^x+a)}{3} - x^2\,\text{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right) + 2x\,\text{polylog}\left(3,-\frac{b\,\mathrm{e}^x}{a}\right) - 2\,\text{polylog}\left(4,-\frac{b\,\mathrm{e}^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(b*exp(x)+a),x)
```

```
[Out] 1/3*x^3*ln(b*exp(x)+a)-1/3*x^3*ln(1/a*b*exp(x)+1)-x^2*polylog(2,-1/a*b*exp(
x))+2*x*polylog(3,-1/a*b*exp(x))-2*polylog(4,-1/a*b*exp(x))
```

maxima [A] time = 0.77, size = 67, normalized size = 0.87

$$\frac{1}{3}\,x^3\log\left(be^x+a\right)-\frac{1}{3}\,x^3\log\left(\frac{be^x}{a}+1\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2\,x\mathrm{Li}_3(-\frac{be^x}{a})-2\,\mathrm{Li}_4(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log(b*e^x/a + 1) - x^2*dilog(-b*e^x/a) + 2
*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(a + b*exp(x)),x)
```

```
[Out] int(x^2*log(a + b*exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^3e^x}{a+be^x}\,dx}{3}+\frac{x^3\log\left(a+be^x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**3*exp(x)/(a + b*exp(x)), x)/3 + x**3*log(a + b*exp(x))/3
```

3.115 $\int x \log(a + be^x)\, dx$

Optimal. Leaf size=59

$$-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2532, 2531, 2282, 6589}

$$-x\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+\text{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\log(a+be^x)\,dx &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)+\int x\log\left(1+\frac{be^x}{a}\right)dx\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\int \mathrm{Li}_2\left(-\frac{be^x}{a}\right)dx\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{x}dx,x,e^x\right)\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Li}_3\left(-\frac{be^x}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 59, normalized size = 1.00

$$-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Li}_3\left(-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

fricas [C] time = 0.60, size = 56, normalized size = 0.95

$$\frac{1}{2}x^2\log(be^x+a)-\frac{1}{2}x^2\log\left(\frac{be^x+a}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log((b*e^x + a)/a) - x*dilog(-(b*e^x + a)/
a + 1) + polylog(3, -b*e^x/a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log(be^x+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(b*e^x + a), x)
```

maple [A] time = 0.08, size = 52, normalized size = 0.88

$$-\frac{x^2\ln\left(\frac{b\,\mathrm{e}^x}{a}+1\right)}{2}+\frac{x^2\ln(b\,\mathrm{e}^x+a)}{2}-x\,\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)+\mathrm{polylog}\left(3,-\frac{b\,\mathrm{e}^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(b*exp(x)+a),x)
```

```
[Out] 1/2*x^2*ln(b*exp(x)+a)-1/2*x^2*ln(1/a*b*exp(x)+1)-x*polylog(2,-1/a*b*exp(x)
)+polylog(3,-1/a*b*exp(x))
```

maxima [A] time = 0.60, size = 50, normalized size = 0.85

$$\frac{1}{2}x^2\log\left(be^x+a\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log(b*e^x/a + 1) - x*dilog(-b*e^x/a) + pol
ylog(3, -b*e^x/a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a + b*exp(x)),x)
```

```
[Out] int(x*log(a + b*exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^2e^x}{a+be^x}\,dx}{2}+\frac{x^2\log\left(a+be^x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**2*exp(x)/(a + b*exp(x)), x)/2 + x**2*log(a + b*exp(x))/2
```

3.116 $\int \log(a+be^x)\,dx$

Optimal. Leaf size=38

$$-\text{Li}_2\left(-\frac{be^x}{a}\right)+x\log(a+be^x)-x\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] x*ln(a+b*exp(x))-x*ln(1+b*exp(x)/a)-polylog(2,-b*exp(x)/a)
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2280, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+x\log(a+be^x)-x\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*E^x],x]
```

```
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2280

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[(x*
(F^(e*(c + d*x)))^n)/(a + b*(F^(e*(c + d*x)))^n), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a+be^x)\,dx &= x\log(a+be^x) - b\int \frac{e^x x}{a+be^x}\,dx\\ &= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \int \log\left(1+\frac{be^x}{a}\right)dx\\ &= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^x\right)\\ &= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{Li}_2\left(-\frac{be^x}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$-\text{Li}_2\left(-\frac{be^x}{a}\right) + x\log(a+be^x) - x\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*E^x],x]
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

fricas [A] time = 1.16, size = 40, normalized size = 1.05

$$x\log(be^x+a) - x\log\left(\frac{be^x+a}{a}\right) - \text{Li}_2\left(-\frac{be^x+a}{a}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="fricas")
[Out] x*log(b*e^x + a) - x*log((b*e^x + a)/a) - dilog(-(b*e^x + a)/a + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(be^x+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(log(b*e^x + a), x)
```

maple [A] time = 0.08, size = 28, normalized size = 0.74

$$\ln\left(-\frac{b\,e^x}{a}\right)\ln(b\,e^x+a) + \text{dilog}\left(-\frac{b\,e^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*exp(x)+a),x)
[Out] dilog(-1/a*b*exp(x))+ln(b*exp(x)+a)*ln(-1/a*b*exp(x))
```

maxima [A] time = 0.69, size = 34, normalized size = 0.89

$$\log(be^x+a)\log\left(-\frac{be^x+a}{a}+1\right) + \text{Li}_2\left(\frac{be^x+a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="maxima")
[Out] log(b*e^x + a)*log(-(b*e^x + a)/a + 1) + dilog((b*e^x + a)/a)
```

mupad [B] time = 0.38, size = 35, normalized size = 0.92

$$x\ \ln\left(a+b\,\mathrm{e}^{x}\right)-x\ \ln\left(\frac{b\,\mathrm{e}^{x}}{a}+1\right)-\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^{x}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*exp(x)),x)
[Out] x*log(a + b*exp(x)) - x*log((b*exp(x))/a + 1) - polylog(2, -(b*exp(x))/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{xe^{x}}{a+be^{x}}\,dx+x\log\left(a+be^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x)),x)
[Out] -b*Integral(x*exp(x)/(a + b*exp(x)), x) + x*log(a + b*exp(x))
```

3.117 $\int \frac{\log(a+be^x)}{x} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\log(a+be^x)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(a+b*exp(x))/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(a+be^x)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[a + b*E^x]/x,x]
```

```
[Out] Defer[Int][Log[a + b*E^x]/x, x]
```

Rubi steps

$$\int \frac{\log(a+be^x)}{x} \, dx = \int \frac{\log(a+be^x)}{x} \, dx$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\log(a+be^x)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[a + b*E^x]/x,x]
```

```
[Out] Integrate[Log[a + b*E^x]/x, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(be^x+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(b*e^x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(be^x+a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(b*e^x + a)/x, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\ln(b\,e^x+a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*exp(x)+a)/x,x)
```

```
[Out] int(ln(b*exp(x)+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(be^{x} + a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(b*e^x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\ln\left(a + b\,\mathrm{e}^{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*exp(x))/x,x)
```

```
[Out] int(log(a + b*exp(x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(a + be^{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x))/x,x)
```

```
[Out] Integral(log(a + b*exp(x))/x, x)
```

3.118 $\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx$

Optimal. Leaf size=132

$$\frac{6\text{Li}_5\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -x^3*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(
b*x+a)))^n)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3
/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n)/b^4/c^4/n^4/ln(f)^4
```

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{x^3\text{PolyLog}\left(2,\right.}{bcn\,\text{l}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3\int x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right) dx}{bcn\log(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{6\int x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right) dx}{b^2c^2n^2\log^2(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{6 \ldots}{\ldots}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{6S \ldots}{\ldots}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{6L \ldots}{\ldots}$$

Mathematica [A] time = 0.01, size = 132, normalized size = 1.00

$$\frac{6\text{Li}_5\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)} - \frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

fricas [C] time = 0.49, size = 128, normalized size = 0.97

$$-\frac{b^3c^3n^3x^3\text{Li}_2\left(-ef^{bcnx+acn}\right)\log(f)^3 - 3\,b^2c^2n^2x^2\log(f)^2\text{polylog}\left(3,-ef^{bcnx+acn}\right) + 6\,bcnx\log(f)\text{polylog}\left(4,-ef \ldots\right.}{b^4c^4n^4\log(f)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^3 - 3*b^2*c^2*n^2*x^
2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 6*b*c*n*x*log(f)*polylog(4,
 -e*f^(b*c*n*x + a*c*n)) - 6*polylog(5, -e*f^(b*c*n*x + a*c*n)))/(b^4*c^4*n
^4*log(f)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log\left(e\left(f^{(bx+a)c}\right)^n + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + 1), x)
```

maple [B] time = 0.43, size = 601, normalized size = 4.55

$$\frac{x^4 \ln\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{4}-\frac{x^4 \ln\left(e\, f^{bcnx} f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{4}-\frac{x^3 \operatorname{dilog}\left(e\, f^{bcnx} f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{bcn\ln(f)}+\frac{3x^2 \operatorname{dil}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(1+e*(f^((b*x+a)*c))^n),x)
```

```
[Out] 1/4*x^4*ln(1+e*(f^((b*x+a)*c))^n)+6/c^4/ln(f)^4/b^4/n^4*polylog(5,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)+3/c^2/ln(f)^2/b^2/n*dilog(1+e*f^(b*c*
n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))*x^2-3/c^3/ln(f)^3/b^
3/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))
^2*x+3/c^3/ln(f)^3/b^3/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*
c))^n)*ln(f^((b*x+a)*c))^2*x-3/c^2/ln(f)^2/b^2/n*polylog(2,-e*f^(b*c*n*x)*f
^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))*x^2-1/4*ln(1+e*f^(b*c*n*x)
*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x^4-6/c^3/ln(f)^3/b^3/n^3*polylog(4,-e*f^(
b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x+3/c^2/ln(f)^2/b^2/n^2*polylog(3,
-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x^2-1/c^4/ln(f)^4/b^4/n*poly
log(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^3-1/
c/ln(f)/b/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x^3+1/c^4
/ln(f)^4/b^4/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^(
(b*x+a)*c))^3
```

maxima [A] time = 0.78, size = 189, normalized size = 1.43

$$\frac{1}{4}\,x^4\log\left(ef^{(bx+a)cn}+1\right)-\frac{b^4c^4n^4x^4\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)^4+4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log(f)^3-12\,b^2c}{4\,b^4c^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(e*f^((b*x + a)*c*n) + 1) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(a*c*
n))*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)*f^(a*c
*n)) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)) - 24*polylog(
5, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^4*c^4*n^4*log(f)^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^4e^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx}{4}+\frac{x^4\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/4 + x**4*log(e*(f**(c*(a + b*x))
)**n + 1)/4
```

3.119 $\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=98

$$-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -x^2*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*
x+a)))^n)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2531, 6609, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = -\frac{x^2\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{2\int x\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right) dx}{bcn\log(f)}$$
$$= -\frac{x^2\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{2x\mathrm{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{2\int \mathrm{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right) d}{b^2c^2n^2\log^2(f)}$$
$$= -\frac{x^2\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{2x\mathrm{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{2\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_3(-ex^n)}{x} dx, x,\right.}{b^3c^3n^2\log^3(f)}$$
$$= -\frac{x^2\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{2x\mathrm{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{2\mathrm{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{2\mathrm{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{2x\mathrm{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{x^2\mathrm{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

fricas [C] time = 0.80, size = 93, normalized size = 0.95

$$-\frac{b^2c^2n^2x^2\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log(f)^2 - 2\,bcnx\log(f)\mathrm{polylog}\left(3, -ef^{bcnx+acn}\right) + 2\,\mathrm{polylog}\left(4, -ef^{bcnx+acn}\right)}{b^3c^3n^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^2 - 2*b*c*n*x*log(f)
*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 2*polylog(4, -e*f^(b*c*n*x + a*c*n)))
/(b^3*c^3*n^3*log(f)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(e\left(f^{(bx+a)c}\right)^n + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + 1), x)
```

maple [B] time = 0.35, size = 430, normalized size = 4.39

$$\frac{x^3\ln\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{3} - \frac{x^3\ln\left(e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{3} - \frac{x^2\,\mathrm{dilog}\left(e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{bcn\ln(f)} + \frac{2x\,\mathrm{dilo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*(f^((b*x+a)*c))^n+1),x)
```

```
[Out] 1/3*x^3*ln(e*(f^((b*x+a)*c))^n+1)-2/c^3/ln(f)^3/b^3/n^3*polylog(4,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)-1/c/ln(f)/b/n*dilog(e*f^(b*c*n*x)*f^(
-b*c*n*x)*(f^((b*x+a)*c))^n+1)*x^2+2/c^2/ln(f)^2/b^2/n*dilog(e*f^(b*c*n*x)*
f^(-b*c*n*x)*(f^((b*x+a)*c))^n+1)*ln(f^((b*x+a)*c))*x-1/c^3/ln(f)^3/b^3/n*d
ilog(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+1)*ln(f^((b*x+a)*c))^2-2/
c^2/ln(f)^2/b^2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*
ln(f^((b*x+a)*c))*x+1/c^3/ln(f)^3/b^3/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*
x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^2+2/c^2/ln(f)^2/b^2/n^2*polylog(3,-
e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x-1/3*ln(e*f^(b*c*n*x)*f^(-b*
c*n*x)*(f^((b*x+a)*c))^n+1)*x^3
```

maxima [A] time = 0.81, size = 153, normalized size = 1.56

$$\frac{1}{3}x^3\log\left(ef^{(bx+a)cn}+1\right)-\frac{b^3c^3n^3x^3\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)^3+3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log(f)^2-6\,bcnx\,\mathrm{lo}}{3\,b^3c^3n^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(e*f^((b*x + a)*c*n) + 1) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(a*c*
n))*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)) + 6*po
lylog(4, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^3*c^3*n^3*log(f)^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^3e^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx}{3}+\frac{x^3\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/3 + x**3*log(e*(f**(c*(a + b*x))
)**n + 1)/3
```

3.120 $\int x \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=63

$$\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -x*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))
^n)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx = -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\int \text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}$$
$$= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-ex^n)}{x}dx,x,f^{c(a+bx)}\right)}{b^2c^2n\log^2(f)}$$
$$= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

fricas [C] time = 1.32, size = 58, normalized size = 0.92

$$-\frac{bcnx\text{Li}_2\left(-ef^{bcnx+acn}\right)\log(f)-\text{polylog}\left(3,-ef^{bcnx+acn}\right)}{b^2c^2n^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b*c*n*x*dilog(-e*f^(b*c*n*x + a*c*n))*log(f) - polylog(3, -e*f^(b*c*n*x +
 a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + 1), x)
```

maple [B] time = 0.35, size = 262, normalized size = 4.16

$$\frac{x^2\ln\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{2}-\frac{x^2\ln\left(e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{2}-\frac{x\,\text{dilog}\left(e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+1\right)}{bcn\ln(f)}+\frac{\text{dilog}\left(e\,f^{bc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*(f^((b*x+a)*c))^n+1),x)
```

```
[Out] 1/2*x^2*ln(e*(f^((b*x+a)*c))^n+1)-1/2*ln(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*
x+a)*c))^n+1)*x^2-1/c^2/ln(f)^2/b^2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)
*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))+1/c^2/ln(f)^2/b^2/n^2*polylog(3,-e*f^
```

```
(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)-1/c/ln(f)/b/n*dilog(e*f^(b*c*n*x)
*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+1)*x+1/c^2/ln(f)^2/b^2/n*dilog(e*f^(b*c*n*x
)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+1)*ln(f^((b*x+a)*c))
```

maxima [A] time = 0.86, size = 117, normalized size = 1.86

$$\frac{1}{2}\,x^2\log\left(ef^{(bx+a)cn}+1\right)-\frac{b^2c^2n^2x^2\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)^2+2\,bcnx\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log(f)-2\,\mathrm{Li}_3(-ef^{bc}}{2\,b^2c^2n^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(e*f^((b*x + a)*c*n) + 1) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n))*log(
f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(x*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log{(f)}}\log{(f)}\int\frac{x^2e^{bcnx\log{(f)}}}{ee^{acn\log{(f)}}e^{bcnx\log{(f)}}+1}\,dx}{2}+\frac{x^2\log{\left(e\left(f^{c(a+bx)}\right)^n+1\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/2 + x**2*log(e*(f**(c*(a + b*x))
)**n + 1)/2
```

3.121 $\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=31

$$-\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\log(1+ex)}{x}\,dx,x,\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\\ &= -\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

fricas [A] time = 0.80, size = 31, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-ef^{bcnx+acn}\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -dilog(-e*f^(b*c*n*x + a*c*n))/(b*c*n*log(f))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(e\left(f^{(bx+a)c}\right)^{n}+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1), x)
```

maple [A] time = 0.07, size = 32, normalized size = 1.03

$$-\frac{\operatorname{dilog}\left(e\left(f^{(bx+a)c}\right)^{n}+1\right)}{bcn\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f^((b*x+a)*c))^n+1),x)
```

```
[Out] -1/b/c/ln(f)/n*dilog(e*(f^((b*x+a)*c))^n+1)
```

maxima [B] time = 0.90, size = 76, normalized size = 2.45

$$x\log\left(ef^{(bx+a)cn}+1\right)-\frac{bcnx\log\left(ef^{bcnx}f^{acn}+1\right)\log(f)+\operatorname{Li}_2\left(-ef^{bcnx}f^{acn}\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] x*log(e*f^((b*x + a)*c*n) + 1) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n) + 1)*
log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)))/(b*c*n*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(e\left(f^{c\,(a+b\,x)}\right)^{n}+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-bcene^{acn\log(f)}\log(f)\int\frac{xe^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx+x\log\left(e\left(f^{c(a+bx)}\right)^{n}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(e*exp(a*c
*n*log(f))*exp(b*c*n*x*log(f)) + 1), x) + x*log(e*(f**(c*(a + b*x)))**n + 1
)
```

3.122 $\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Defer[Int][Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e\left(f^{bcx+ac}\right)^n+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
[Out] integral(log(e*(f^(b*c*x + a*c))^n + 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1)/x, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f^{(bx+a)c}\right)^{n}+1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f^((b*x+a)*c))^n+1)/x,x)
[Out] int(ln(e*(f^((b*x+a)*c))^n+1)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e f^{(bx+a)cn}+1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
[Out] integrate(log(e*f^((b*x + a)*c*n) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(e\left(f^{c\,(a+b\,x)}\right)^{n}+1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1)/x,x)
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f^{ac} f^{bcx}\right)^{n}+1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n)/x,x)
[Out] Integral(log(e*(f**(a*c)*f**(b*c*x))**n + 1)/x, x)
```

3.123 $\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=193

$$\frac{6\text{Li}_5\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}-\frac{6x\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{3x^2\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^3\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-$$

```
[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^3*p
olylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(b*x+
a)))^n/d)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3
/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n/d)/b^4/c^4/n^4/ln(f)^4
```

Rubi [A] time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2532, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}-\frac{x^3\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int x^3\log\left(1+\frac{e\left(\right.}{}\right.$$

$$= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

Mathematica [A] time = 0.01, size = 193, normalized size = 1.00

$$\frac{6\mathrm{Li}_5\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)} - \frac{6x\mathrm{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)} + \frac{3x^2\mathrm{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{x^3\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

fricas [C] time = 0.76, size = 245, normalized size = 1.27

$$-\frac{4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log(f)^3 - 12\,b^2c^2n^2x^2\log(f)^2\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right) - \left(b^4c^4n^4x^4 - a^4c^4n^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*c^3*n^3*x^3*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^3
- 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^4*c
^4*n^4*x^4 - a^4*c^4*n^4)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^4 + (b^4*c^
4*n^4*x^4 - a^4*c^4*n^4)*log(f)^4*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 24*b
*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x + a*c*n)/d) - 24*polylog(5, -e*f^(b*
c*n*x + a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log\left(e\left(f^{(bx+a)c}\right)^n + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + d), x)
```

maple [B] time = 0.36, size = 1276, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d+e*(f^((b*x+a)*c))^n),x)
```

```
[Out] 1/4*x^4*ln(d+e*(f^((b*x+a)*c))^n)-1/c^4/ln(f)^4/b^4/n*polylog(2,-e*f^(b*c*n
*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*ln(f^((b*x+a)*c))^3-3/4/c^4/ln(f)^4/b
^4*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*ln(f^((b*x+a)*c))^4
-6/c^3/ln(f)^3/b^3/n^3*polylog(4,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c)
)^n/d)*x+1/c^4/ln(f)^4/b^4*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))
^n)/d)*ln(f^((b*x+a)*c))^4-1/c/ln(f)/b/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x
)*(f^((b*x+a)*c))^n)/d)*x^3-3/c^3/ln(f)^3/b^3/n*dilog((d+e*f^(b*c*n*x)*f^(-
b*c*n*x)*(f^((b*x+a)*c))^n)/d)*ln(f^((b*x+a)*c))^2*x+6/c^4/ln(f)^4/b^4/n^4*
polylog(5,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)-3/c^2/ln(f)^2/b^
2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*ln(f^((b*x+a
)*c))*x^2-1/4*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x^4-3/2/c^
2/ln(f)^2/b^2*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*x^2*ln(f
^((b*x+a)*c))^2+2/c^3/ln(f)^3/b^3*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+
a)*c))^n/d)*x*ln(f^((b*x+a)*c))^3-1/c/ln(f)/b*ln((d+e*f^(b*c*n*x)*f^(-b*c*n
*x)*(f^((b*x+a)*c))^n)/d)*x^3*ln(f^((b*x+a)*c))+3/c^2/ln(f)^2/b^2*ln((d+e*f
^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)/d)*x^2*ln(f^((b*x+a)*c))^2-3/c^3
/ln(f)^3/b^3*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)/d)*x*ln(f^
((b*x+a)*c))^3+1/c/ln(f)/b*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^
n)*ln(f^((b*x+a)*c))*x^3-3/2/c^2/ln(f)^2/b^2*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x
)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^2*x^2+1/c^3/ln(f)^3/b^3*ln(d+e*f^(b*
c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^3*x+3/c^2/ln(f)^2/
b^2/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)/d)*ln(f^((b*x+
a)*c))*x^2+1/c^4/ln(f)^4/b^4/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x
+a)*c))^n)/d)*ln(f^((b*x+a)*c))^3+3/c^2/ln(f)^2/b^2/n^2*polylog(3,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*x^2-1/4/c^4/ln(f)^4/b^4*ln(d+e*f^(b
*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^4+3/c^3/ln(f)^3/b
^3/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n/d)*ln(f^((b*x+
a)*c))^2*x
```

maxima [A] time = 0.81, size = 204, normalized size = 1.06

$$\frac{1}{4}\,x^4\log\left(ef^{(bx+a)cn}+d\right)-\frac{b^4c^4n^4x^4\log\left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log(f)^4+4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log(f)^3-12\,b^2c^2n^2x^2}{4\,b^4c^4n^4\log(f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(e*f^((b*x + a)*c*n) + d) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(a*
c*n)/d)*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)*f^
(a*c*n)/d) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d) - 24*
polylog(5, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \ln\left(d + e\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x^3*log(d + e*(f^(c*(a + b*x)))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^4e^{bcnx\log(f)}}{d+ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx}{4}+\frac{x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/4 + x**4*log(d + e*(f**(c*(a + b
*x)))**n)/4
```

3.124 $\int x^2 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=156

$$-\frac{2\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.$$

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^2*p
olylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*x+a)
))^n/d)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2532, 2531, 6609, 2282, 6589\}$

$$\frac{2x\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx = \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int x^2\log\left(1+\frac{e(\right.$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right.}{bcn\log(f)}$$

Mathematica [A] time = 0.01, size = 156, normalized size = 1.00

$$-\frac{2\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+b}\right.}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

fricas [C] time = 0.59, size = 205, normalized size = 1.31

$$-\frac{3b^2c^2n^2x^2\text{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log(f)^2-6bcnx\log(f)\text{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-\left(b^3c^3n^3x^3+a^3c^3n^3\right)\log\left(e\right.}{3b^3c^3n^3\log(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*c^2*n^2*x^2*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^2
- 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^3*c^3*n^3*x^3
+ a^3*c^3*n^3)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^3 + (b^3*c^3*n^3*x^3 +
 a^3*c^3*n^3)*log(f)^3*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 6*polylog(4, -e
*f^(b*c*n*x + a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(e\left(f^{(bx+a)c}\right)^n + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + d), x)
```

maple [B] time = 0.35, size = 916, normalized size = 5.87

$$\frac{x^3 \ln\left(e\left(f^{(bx+a)c}\right)^n + d\right)}{3} - \frac{x^3 \ln\left(e\, f^{bcnx} f^{-bcnx}\left(f^{(bx+a)c}\right)^n + d\right)}{3} - \frac{x^2 \ln\left(f^{(bx+a)c}\right) \ln\left(\frac{e\, f^{bcnx} f^{-bcnx}\left(f^{(bx+a)c}\right)^n + d}{d}\right)}{bc\ln(f)} + \frac{x^2 \ln\left(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*(f^((b*x+a)*c))^n+d),x)
```

```
[Out] 1/3*x^3*ln(e*(f^((b*x+a)*c))^n+d)-2/c^2/ln(f)^2/b^2/n*polylog(2,-1/d*e*f^(b
*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))*x-1/c^2/ln(f)^2/b
^2*ln(1/d*e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+1)*x*ln(f^((b*x+a)*c
))^2-1/c/ln(f)/b*ln((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*x^2
*ln(f^((b*x+a)*c))+2/c^2/ln(f)^2/b^2*ln((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*
x+a)*c))^n+d)/d)*x*ln(f^((b*x+a)*c))^2+2/c^2/ln(f)^2/b^2/n*dilog((e*f^(b*c*
n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*ln(f^((b*x+a)*c))*x+1/c/ln(f)/b*l
n(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*ln(f^((b*x+a)*c))*x^2-1/c
/ln(f)/b/n*dilog((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*x^2-1/
c^2/ln(f)^2/b^2*ln(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*ln(f^((b
*x+a)*c))^2*x-2/c^3/ln(f)^3/b^3/n^3*polylog(4,-1/d*e*f^(b*c*n*x)*f^(-b*c*n*
x)*(f^((b*x+a)*c))^n)-1/c^3/ln(f)^3/b^3/n*dilog((e*f^(b*c*n*x)*f^(-b*c*n*x)
*(f^((b*x+a)*c))^n+d)/d)*ln(f^((b*x+a)*c))^2+1/c^3/ln(f)^3/b^3/n*polylog(2,
-1/d*e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))^2-1/3*
ln(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*x^3+1/3/c^3/ln(f)^3/b^3*
ln(e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*ln(f^((b*x+a)*c))^3-1/c^
3/ln(f)^3/b^3*ln((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*ln(f^(
(b*x+a)*c))^3+2/3/c^3/ln(f)^3/b^3*ln(1/d*e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*
x+a)*c))^n+1)*ln(f^((b*x+a)*c))^3+2/c^2/ln(f)^2/b^2/n^2*polylog(3,-1/d*e*f^
(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*x
```

maxima [A] time = 0.87, size = 165, normalized size = 1.06

$$\frac{1}{3}x^3 \log\left(ef^{(bx+a)cn} + d\right) - \frac{b^3c^3n^3x^3 \log\left(\frac{ef^{bcnx}f^{acn}}{d} + 1\right)\log(f)^3 + 3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log(f)^2 - 6\,bcnx\log(}{3\,b^3c^3n^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(e*f^((b*x + a)*c*n) + d) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(a*
c*n)/d)*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/d)
+ 6*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \ln\left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x^2*log(d + e*(f^(c*(a + b*x)))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^3e^{bcnx\log(f)}}{d+ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx}{3}+\frac{x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/3 + x**3*log(d + e*(f**(c*(a + b
*x)))**n)/3
```

3.125 $\int x \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=118

$$\frac{\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n/d)-x*pol
ylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))^n/d
)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2532, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int x\log\left(1+\frac{e\left(f^{c(\ldots}\right.}{}\right.$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

Mathematica [A] time = 0.01, size = 118, normalized size = 1.00

$$\frac{\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

fricas [C] time = 1.22, size = 169, normalized size = 1.43

$$-\frac{2\,bcnx\text{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log(f)-\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(ef^{bcnx+acn}+d\right)\log(f)^2+\left(b^2c^2n^2x^2-a^2c^2n^2\ldots\right.}{2\,b^2c^2n^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*n*x*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f) - (b^2*c^2
*n^2*x^2 - a^2*c^2*n^2)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^2 + (b^2*c^2*
n^2*x^2 - a^2*c^2*n^2)*log(f)^2*log((e*f^(b*c*n*x + a*c*n) + d)/d) - 2*poly
log(3, -e*f^(b*c*n*x + a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + d), x)
```

maple [B] time = 0.34, size = 558, normalized size = 4.73

$$\frac{x^2\ln\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{2}-\frac{x^2\ln\left(e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+d\right)}{2}-\frac{x\ln\left(f^{(bx+a)c}\right)\ln\left(\frac{e\,f^{bcnx}f^{-bcnx}\left(f^{(bx+a)c}\right)^n+d}{d}\right)}{bc\ln(f)}+\frac{x\ln\left(f^{(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*(f^((b*x+a)*c))^n+d),x)
```

```
[Out] 1/2*x^2*ln(e*(f^((b*x+a)*c))^n+d)+1/c/ln(f)/b*ln(e*f^(b*c*n*x)*f^(-b*c*n*x)
*(f^((b*x+a)*c))^n+d)*ln(f^((b*x+a)*c))*x-1/c/ln(f)/b*ln((e*f^(b*c*n*x)*f^(
-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*x*ln(f^((b*x+a)*c))+1/c^2/ln(f)^2/b^2/n^2
*polylog(3,-1/d*e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)-1/2*ln(e*f^(b
*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*x^2-1/2/c^2/ln(f)^2/b^2*ln(e*f^(b
*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)*ln(f^((b*x+a)*c))^2-1/c/ln(f)/b/n
*dilog((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*x+1/c^2/ln(f)^2/
b^2/n*dilog((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d)/d)*ln(f^((b*x+
a)*c))+1/c^2/ln(f)^2/b^2*ln((e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n+d
)/d)*ln(f^((b*x+a)*c))^2-1/2/c^2/ln(f)^2/b^2*ln(1/d*e*f^(b*c*n*x)*f^(-b*c*n
*x)*(f^((b*x+a)*c))^n+1)*ln(f^((b*x+a)*c))^2-1/c^2/ln(f)^2/b^2/n*polylog(2,
-1/d*e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^((b*x+a)*c))^n)*ln(f^((b*x+a)*c))
```

maxima [A] time = 0.78, size = 126, normalized size = 1.07

$$\frac{1}{2}x^2\log\left(ef^{(bx+a)cn}+d\right)-\frac{b^2c^2n^2x^2\log\left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log(f)^2+2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log(f)-2\,\mathrm{Li}_3(-\frac{ef^{bcnx}f^{acn}}{d})}{2\,b^2c^2n^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(e*f^((b*x + a)*c*n) + d) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d)*
log(f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x*log(d + e*(f^(c*(a + b*x)))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcne e^{acn\log(f)}\log(f)\int\frac{x^2e^{bcnx\log(f)}}{d+ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx}{2}+\frac{x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/2 + x**2*log(d + e*(f**(c*(a + b
*x)))**n)/2
```

3.126 $\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=75

$$-\frac{\operatorname{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[Out] x*ln(d+e*(f^(c*(b*x+a)))^n)-x*ln(1+e*(f^(c*(b*x+a)))^n/d)-polylog(2,-e*(f^(
c*(b*x+a)))^n/d)/b/c/n/ln(f)
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2280, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2280

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[(x*
(F^(e*(c + d*x)))^n)/(a + b*(F^(e*(c + d*x)))^n), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx = x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - (bcen\log(f))\int \frac{\left(f^{c(a+bx)}\right)^n x}{d+e\left(f^{c(a+bx)}\right)^n}dx$$
$$= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx$$
$$= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}dx, x, \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$
$$= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

Mathematica [A] time = 0.00, size = 75, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right) - x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

fricas [A] time = 0.68, size = 106, normalized size = 1.41

$$\frac{(bcnx+acn)\log\left(ef^{bcnx+acn}+d\right)\log(f) - (bcnx+acn)\log(f)\log\left(\frac{ef^{bcnx+acn}+d}{d}\right) - \text{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] ((b*c*n*x + a*c*n)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f) - (b*c*n*x + a*c*n
)*log(f)*log((e*f^(b*c*n*x + a*c*n) + d)/d) - dilog(-(e*f^(b*c*n*x + a*c*n)
 + d)/d + 1))/(b*c*n*log(f))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(e\left(f^{(bx+a)c}\right)^n+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d), x)
```

maple [A] time = 0.08, size = 82, normalized size = 1.09

$$\frac{\ln\left(-\frac{e\left(f^{(bx+a)c}\right)^n}{d}\right)\ln\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{bcn\ln(f)} + \frac{\text{dilog}\left(-\frac{e\left(f^{(bx+a)c}\right)^n}{d}\right)}{bcn\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f^((b*x+a)*c))^n+d),x)
```

```
[Out] 1/b/c/ln(f)/n*ln(e*(f^((b*x+a)*c))^n+d)*ln(-e*(f^((b*x+a)*c))^n/d)+1/b/c/ln
(f)/n*dilog(-e*(f^((b*x+a)*c))^n/d)
```

maxima [A] time = 0.83, size = 82, normalized size = 1.09

$$x \log\left(ef^{(bx+a)cn} + d\right) - \frac{bcnx \log\left(\frac{ef^{bcnx}f^{acn}}{d} + 1\right)\log(f) + \mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] x*log(e*f^((b*x + a)*c*n) + d) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n)/d + 1
)*log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d))/(b*c*n*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d + e\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(log(d + e*(f^(c*(a + b*x)))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-bcne e^{acn\log(f)}\log(f)\int \frac{xe^{bcnx\log(f)}}{d + ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx + x\log\left(d + e\left(f^{c(a+bx)}\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(d + e*exp
(a*c*n*log(f))*exp(b*c*n*x*log(f))), x) + x*log(d + e*(f**(c*(a + b*x)))**n
)
```

3.127 $\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Defer[Int][Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e\left(f^{bcx+ac}\right)^n+d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
[Out] integral(log(e*(f^(b*c*x + a*c))^n + d)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d)/x, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f^{(bx+a)c}\right)^{n}+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f^((b*x+a)*c))^n+d)/x,x)
[Out] int(ln(e*(f^((b*x+a)*c))^n+d)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ef^{(bx+a)cn}+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
[Out] integrate(log(e*f^((b*x + a)*c*n) + d)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(d+e\left(f^{c\,(a+b\,x)}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n)/x,x)
[Out] int(log(d + e*(f^(c*(a + b*x)))^n)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d+e\left(f^{ac}f^{bcx}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n)/x,x)
[Out] Integral(log(d + e*(f**(a*c)*f**(b*c*x))**n)/x, x)
```

3.128 $\int \log \left(b \left(F^{e(c+dx)}\right)^n+\pi \right) \, dx$

Optimal. Leaf size=39

$$x \log(\pi)-\frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

```
[Out] x*ln(Pi)-polylog(2,-b*(F^(e*(d*x+c)))^n/Pi)/d/e/n/ln(F)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2279, 2392, 2391}

$$x \log(\pi)-\frac{\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \log \left(b \left(F^{e(c+dx)}\right)^n+\pi \right) \, dx &= \frac{\text{Subst}\left(\int \frac{\log(\pi+bx)}{x} \, dx,x,\left(F^{e(c+dx)}\right)^n\right)}{den \log(F)}\\
&= x \log(\pi)+\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{\pi}\right)}{x} \, dx,x,\left(F^{e(c+dx)}\right)^n\right)}{den \log(F)}\\
&= x \log(\pi)-\frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$x \log(\pi)-\frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

fricas [B] time = 0.73, size = 106, normalized size = 2.72

$$\frac{(denx + cen)\log\left(\pi + F^{denx+cen}b\right)\log(F) - (denx + cen)\log(F)\log\left(\frac{\pi+F^{denx+cen}b}{\pi}\right) - \mathrm{Li}_2\left(-\frac{\pi+F^{denx+cen}b}{\pi} + 1\right)}{den\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="fricas")
```

```
[Out] ((d*e*n*x + c*e*n)*log(pi + F^(d*e*n*x + c*e*n)*b)*log(F) - (d*e*n*x + c*e*
n)*log(F)*log((pi + F^(d*e*n*x + c*e*n)*b)/pi) - dilog(-(pi + F^(d*e*n*x +
c*e*n)*b)/pi + 1))/(d*e*n*log(F))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\pi + \left(F^{(dx+c)e}\right)^n b\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="giac")
```

```
[Out] integrate(log(pi + (F^((d*x + c)*e))^n*b), x)
```

maple [B] time = 0.15, size = 138, normalized size = 3.54

$$-\frac{\ln\left(\frac{b\left(F^{(dx+c)e}\right)^n+\pi}{\pi}\right)\ln\left(-\frac{b\left(F^{(dx+c)e}\right)^n}{\pi}\right)}{den\ln(F)}+\frac{\ln\left(-\frac{b\left(F^{(dx+c)e}\right)^n}{\pi}\right)\ln\left(b\left(F^{(dx+c)e}\right)^n+\pi\right)}{den\ln(F)}-\frac{\mathrm{dilog}\left(\frac{b\left(F^{(dx+c)e}\right)^n+\pi}{\pi}\right)}{den\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*(F^((d*x+c)*e))^n+Pi),x)
```

```
[Out] -1/d/e/ln(F)/n*ln(-b*(F^((d*x+c)*e))^n/Pi)*ln((b*(F^((d*x+c)*e))^n+Pi)/Pi)+
1/d/e/ln(F)/n*ln(-b*(F^((d*x+c)*e))^n/Pi)*ln(b*(F^((d*x+c)*e))^n+Pi)-1/d/e/
ln(F)/n*dilog((b*(F^((d*x+c)*e))^n+Pi)/Pi)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}denx^2\log(F) + \pi den\int \frac{x}{\pi + \left(F^{dex}\right)^n\left(F^{ce}\right)^n b}\,dx\log(F) + x\log\left(\pi + \left(F^{dex}\right)^n\left(F^{ce}\right)^n b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="maxima")
```

```
[Out] -1/2*d*e*n*x^2*log(F) + pi*d*e*n*integrate(x/(pi + (F^(d*e*x))^n*(F^(c*e))^
n*b), x)*log(F) + x*log(pi + (F^(d*e*x))^n*(F^(c*e))^n*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\Pi + b\left(F^{e\,(c+d\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(Pi + b*(F^(e*(c + d*x)))^n),x)
```

```
[Out] int(log(Pi + b*(F^(e*(c + d*x)))^n), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-bdene^{cen\log(F)}\log(F)\int\frac{xe^{denx\log(F)}}{be^{cen\log(F)}e^{denx\log(F)}+\pi}\,dx+x\log\left(b\left(F^{e(c+dx)}\right)^{n}+\pi\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*(F**(e*(d*x+c)))**n+pi),x)
```

```
[Out] -b*d*e*n*exp(c*e*n*log(F))*log(F)*Integral(x*exp(d*e*n*x*log(F))/(b*exp(c*e
*n*log(F))*exp(d*e*n*x*log(F)) + pi), x) + x*log(b*(F**(e*(c + d*x)))**n +
pi)
```

3.129 $\int \frac{1}{x(3+\log(x))}\,dx$

Optimal. Leaf size=5

$$\log(\log(x)+3)$$

```
[Out] ln(3+ln(x))
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2302, 29}

$$\log(\log(x)+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(3+\log(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, 3+\log(x)\right)\\ &= \log(3+\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$\log(\log(x)+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

fricas [A] time = 0.86, size = 5, normalized size = 1.00

$$\log\left(\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="fricas")
```

```
[Out] log(log(x) + 3)
```

giac [B] time = 0.16, size = 22, normalized size = 4.40

$$\frac{1}{2}\log\left(\frac{1}{4}\pi^2\left(\text{sgn}(x)-1\right)^2+\left(\log\left(|x|\right)+3\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2)
```

maple [A] time = 0.06, size = 6, normalized size = 1.20

$$\ln\left(\ln(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3+ln(x)),x)
[Out] ln(3+ln(x))
```

maxima [A] time = 0.62, size = 5, normalized size = 1.00

$$\log\left(\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="maxima")
[Out] log(log(x) + 3)
```

mupad [B] time = 0.42, size = 5, normalized size = 1.00

$$\ln\left(\ln(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x) + 3)),x)
[Out] log(log(x) + 3)
```

sympy [A] time = 0.10, size = 5, normalized size = 1.00

$$\log\left(\log\left(x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+ln(x)),x)
[Out] log(log(x) + 3)
```

3.130 $\int \frac{\sqrt{1+\log(x)}}{x} \, dx$

Optimal. Leaf size=12

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 30}

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+\log(x)}}{x} \, dx &= \text{Subst}\left(\int \sqrt{x} \, dx, x, 1+\log(x)\right) \\ &= \frac{2}{3}(1+\log(x))^{3/2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

fricas [A] time = 1.95, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

giac [A] time = 0.16, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2)
```

maple [A] time = 0.07, size = 9, normalized size = 0.75

$$\frac{2\left(\ln(x)+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x,x)
[Out] 2/3*(1+ln(x))^(3/2)
```

maxima [A] time = 0.72, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2)
```

mupad [B] time = 0.39, size = 13, normalized size = 1.08

$$\sqrt{\ln(x)+1}\,\left(\frac{2\,\ln(x)}{3}+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^(1/2)/x,x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 + 2/3)
```

sympy [A] time = 0.80, size = 10, normalized size = 0.83

$$\frac{2\left(\log\left(x\right)+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x,x)
[Out] 2*(log(x) + 1)**(3/2)/3
```

3.131 $\int \frac{(1+\log(x))^5}{x} dx$

Optimal. Leaf size=10

$$\frac{1}{6}(\log(x)+1)^6$$

```
[Out] 1/6*(1+ln(x))^6
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2302, 30}

$$\frac{1}{6}(\log(x)+1)^6$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Log[x])^5/x,x]
```

```
[Out] (1 + Log[x])^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+\log(x))^5}{x} dx &= \text{Subst}\left(\int x^5 dx, x, 1+\log(x)\right)\\ &= \frac{1}{6}(1+\log(x))^6\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6}(\log(x)+1)^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])^5/x,x]
```

```
[Out] (1 + Log[x])^6/6
```

fricas [B] time = 0.54, size = 31, normalized size = 3.10

$$\frac{1}{6}\log(x)^6 + \log(x)^5 + \frac{5}{2}\log(x)^4 + \frac{10}{3}\log(x)^3 + \frac{5}{2}\log(x)^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="fricas")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

giac [B] time = 0.17, size = 31, normalized size = 3.10

$$\frac{1}{6}\log(x)^6 + \log(x)^5 + \frac{5}{2}\log(x)^4 + \frac{10}{3}\log(x)^3 + \frac{5}{2}\log(x)^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="giac")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

maple [B] time = 0.06, size = 33, normalized size = 3.30

$$\frac{\ln(x)^6}{6} + \ln(x)^5 + \frac{5\ln(x)^4}{2} + \frac{10\ln(x)^3}{3} + \frac{5\ln(x)^2}{2} + \ln(x) + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)+1)^5/x,x)
```

```
[Out] 1/6*ln(x)^6+ln(x)^5+5/2*ln(x)^4+10/3*ln(x)^3+5/2*ln(x)^2+ln(x)+1/6
```

maxima [A] time = 0.65, size = 8, normalized size = 0.80

$$\frac{1}{6}\left(\log(x) + 1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="maxima")
```

```
[Out] 1/6*(log(x) + 1)^6
```

mupad [B] time = 0.39, size = 26, normalized size = 2.60

$$\frac{\ln(x)\,\left(\ln(x) + 2\right)\,\left(\ln(x)^2 + \ln(x) + 1\right)\,\left(\ln(x)^2 + 3\,\ln(x) + 3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^5/x,x)
```

```
[Out] (log(x)*(log(x) + 2)*(log(x) + log(x)^2 + 1)*(3*log(x) + log(x)^2 + 3))/6
```

sympy [B] time = 0.14, size = 39, normalized size = 3.90

$$\frac{\log{\left(x \right)}^{6}}{6} + \log{\left(x \right)}^{5} + \frac{5\log{\left(x \right)}^{4}}{2} + \frac{10\log{\left(x \right)}^{3}}{3} + \frac{5\log{\left(x \right)}^{2}}{2} + \log{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**5/x,x)
```

```
[Out] log(x)**6/6 + log(x)**5 + 5*log(x)**4/2 + 10*log(x)**3/3 + 5*log(x)**2/2 +
log(x)
```

3.132 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

```
[Out] 2*ln(x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2302, 30}

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\log(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

fricas [A] time = 0.74, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x))
```

giac [A] time = 0.17, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(log(x))
```

maple [A] time = 0.06, size = 7, normalized size = 0.88

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)^(1/2),x)
```

```
[Out] 2*ln(x)^(1/2)
```

maxima [A] time = 0.77, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x))
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)^(1/2)),x)
```

```
[Out] 2*log(x)^(1/2)
```

sympy [A] time = 0.40, size = 7, normalized size = 0.88

$$2\sqrt{\log{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

3.133 $\int \frac{1}{x(1+\log^2(x))}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

fricas [A] time = 0.73, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

giac [A] time = 0.16, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

maple [A] time = 0.06, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
[Out] arctan(ln(x))
```

maxima [A] time = 1.51, size = 3, normalized size = 1.00

$$\arctan\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

mupad [B] time = 0.53, size = 3, normalized size = 1.00

$$\mathrm{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

sympy [B] time = 0.14, size = 15, normalized size = 5.00

$$\mathrm{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

```
[Out] arctanh(ln(x)/(-3+ln(x)^2)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-3+x^2}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)\\ &= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 42, normalized size = 3.00

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] -1/2*Log[1 - Log[x]/Sqrt[-3 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-3 + Log[x]^
2]]/2
```

fricas [A] time = 0.72, size = 16, normalized size = 1.14

$$-\log\left(\sqrt{\log(x)^2 - 3} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(log(x)^2 - 3) - log(x))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.07, size = 13, normalized size = 0.93

$$\ln\left(\ln(x) + \sqrt{\ln(x)^2 - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(-3+ln(x)^2)^(1/2))
```

maxima [A] time = 0.51, size = 16, normalized size = 1.14

$$\log\left(2\sqrt{\log(x)^2 - 3} + 2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(log(x)^2 - 3) + 2*log(x))
```

mupad [B] time = 0.47, size = 12, normalized size = 0.86

$$\ln\left(\ln(x) + \sqrt{\ln(x)^2 - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 - 3)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 - 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\log(x)^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(log(x)**2 - 3)), x)
```

3.135 $\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*ln(x))
```

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
[Out] ArcSin[(3*Log[x])/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{4-9x^2}}\,dx, x, \log(x)\right)$$
$$= \frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
[Out] ArcSin[(3*Log[x])/2]/3
```

fricas [B] time = 1.51, size = 21, normalized size = 1.91

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9\log(x)^2+4}-2}{3\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*log(x)^2 + 4) - 2)/log(x))
```

giac [**A**] time = 0.23, size = 7, normalized size = 0.64

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*arcsin(3/2*log(x))
```

maple [**A**] time = 0.07, size = 8, normalized size = 0.73

$$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4-9*ln(x)^2)^(1/2),x)
[Out] 1/3*arcsin(3/2*ln(x))
```

maxima [**A**] time = 1.22, size = 7, normalized size = 0.64

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*log(x))
```

mupad [**B**] time = 0.40, size = 7, normalized size = 0.64

$$\frac{\operatorname{asin}\left(\frac{3\ln(x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4 - 9*log(x)^2)^(1/2)),x)
[Out] asin((3*log(x))/2)/3
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(3\log\left(x\right)-2\right)\left(3\log\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(3*log(x) - 2)*(3*log(x) + 2))), x)
```

3.136 $\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx$

Optimal. Leaf size=7

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

```
[Out] arcsinh(1/2*ln(x))
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {215}

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 + Log[x]^2]),x]
[Out] ArcSinh[Log[x]/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \log(x)\right)\\ &= \sinh^{-1}\left(\frac{\log(x)}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 7, normalized size = 1.00

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 + Log[x]^2]),x]
[Out] ArcSinh[Log[x]/2]
```

fricas [B] time = 1.08, size = 16, normalized size = 2.29

$$-\log\left(\sqrt{\log(x)^2+4}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

giac [B] time = 0.20, size = 16, normalized size = 2.29

$$-\log\left(\sqrt{\log(x)^2+4}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

maple [A] time = 0.07, size = 6, normalized size = 0.86

$$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+ln(x)^2)^(1/2),x)
[Out] arcsinh(1/2*ln(x))
```

maxima [A] time = 1.36, size = 5, normalized size = 0.71

$$\operatorname{arsinh}\left(\frac{1}{2}\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/2*log(x))
```

mupad [B] time = 0.39, size = 5, normalized size = 0.71

$$\operatorname{asinh}\left(\frac{\ln(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 4)^(1/2)),x)
[Out] asinh(log(x)/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\log(x)^2 + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(log(x)**2 + 4)), x)
```

3.137 $\int \frac{1}{x(2+3\log^3(6x))}\,dx$

Optimal. Leaf size=111

$$-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}$$

```
[Out] 1/6*arctan(1/3*2^(2/3)*ln(6*x)*3^(5/6)-1/3*3^(1/2))*2^(1/3)*3^(1/6)+1/18*ln
(2^(1/3)+3^(1/3)*ln(6*x))*2^(1/3)*3^(2/3)-1/36*ln(2^(2/3)-6^(1/3)*ln(6*x)+3
^(2/3)*ln(6*x)^2)*2^(1/3)*3^(2/3)
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2^(2/3)*Log[6*x])/3^(1/6)]/(2^(2/3)*3^(5/6))) + Log[2
^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) - 6^(1/3)*Log[
6*x] + 3^(2/3)*Log[6*x]^2]/(6*2^(2/3)*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \text{Subst}\left(\int \frac{1}{2+3x^3}\,dx, x, \log(6x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}+\sqrt[3]{3}x}\,dx, x, \log(6x)\right)}{3\,2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}x}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{3\,2^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\,2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{6}+}{2^{2/3}-\sqrt[3]{6}}\right.}{6}$$

$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\,2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\,2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\,2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log\right.}{6\,2^{2/3}\sqrt[3]{3}}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.95

$$\frac{\sqrt{3}\left(2\log\left(2^{2/3}\sqrt[3]{3}\log(6x)+2\right)-\log\left(\sqrt[3]{2}\,3^{2/3}\log^2(6x)-2^{2/3}\sqrt[3]{3}\log(6x)+2\right)\right)+6\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{3}\log(6x)-1}{\sqrt{3}}\right)}{6\,2^{2/3}3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] (6*ArcTan[(-1 + 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]] + Sqrt[3]*(2*Log[2 + 2^(
2/3)*3^(1/3)*Log[6*x]] - Log[2 - 2^(2/3)*3^(1/3)*Log[6*x] + 2^(1/3)*3^(2/3)
*Log[6*x]^2]))/(6*2^(2/3)*3^(5/6))
```

fricas [A] time = 1.32, size = 71, normalized size = 0.64

$$-\frac{1}{72}\cdot 12^{\frac{2}{3}}\log\left(6\log(6x)^2-12^{\frac{2}{3}}\log(6x)+2\cdot 12^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 12^{\frac{2}{3}}\log\left(12^{\frac{2}{3}}+6\log(6x)\right)+\frac{1}{6}\cdot 12^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{1}{6}}\left(12^{\frac{2}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="fricas")
```

```
[Out] -1/72*12^(2/3)*log(6*log(6*x)^2 - 12^(2/3)*log(6*x) + 2*12^(1/3)) + 1/36*12
^(2/3)*log(12^(2/3) + 6*log(6*x)) + 1/6*12^(1/6)*arctan(1/6*12^(1/6)*(12^(2
/3)*log(6*x) - 12^(1/3)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\log(6x)^3+2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*log(6*x)^3 + 2)*x), x)
```

maple [A] time = 0.07, size = 87, normalized size = 0.78

$$\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*ln(6*x)^3),x)
```

```
[Out] 1/18*2^(1/3)*3^(2/3)*ln(ln(6*x)+1/3*2^(1/3)*3^(2/3))-1/36*2^(1/3)*3^(2/3)*l
n(ln(6*x)^2-1/3*2^(1/3)*3^(2/3)*ln(6*x)+1/3*2^(2/3)*3^(1/3))+1/6*2^(1/3)*3^
(1/6)*arctan(1/3*3^(1/2)*(2^(2/3)*3^(1/3)*ln(6*x)-1))
```

maxima [A] time = 1.35, size = 97, normalized size = 0.87

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}\log(6x)^2-3^{\frac{1}{3}}2^{\frac{1}{3}}\log(6x)+2^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\log(6x)+2^{\frac{1}{3}}\right)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{1}{3}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*2^(1/3)*log(3^(2/3)*log(6*x)^2 - 3^(1/3)*2^(1/3)*log(6*x) + 2
^(2/3)) + 1/18*3^(2/3)*2^(1/3)*log(1/3*3^(2/3)*(3^(1/3)*log(6*x) + 2^(1/3))
) + 1/6*3^(1/6)*2^(1/3)*arctan(1/6*3^(1/6)*2^(2/3)*(2*3^(2/3)*log(6*x) - 3^
(1/3)*2^(1/3)))
```

mupad [B] time = 4.69, size = 120, normalized size = 1.08

$$\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}}{3\,x^2}\right)}{18}+\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}-\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}-\frac{2^{1/3}\,3^{2/}}{}\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*log(6*x)^3 + 2)),x)
```

```
[Out] (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3))/(3*x^2)))/18 + (2^(1/
3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3)*((3^(1/2)*1i)/2 - 1/2))/(3*x
^2))*((3^(1/2)*1i)/2 - 1/2))/18 - (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 - (2^(1
/3)*3^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(3*x^2))*((3^(1/2)*1i)/2 + 1/2))/18
```

sympy [A] time = 0.17, size = 17, normalized size = 0.15

$$\operatorname{RootSum}\left(324z^3-1,\left(i\mapsto i\log\left(6i+\log(6x)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*ln(6*x)**3),x)
```

```
[Out] RootSum(324*_z**3 - 1, Lambda(_i, _i*log(6*_i + log(6*x))))
```

3.138 $\int \frac{\log(\log(6x))}{x\log(6x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log^2(\log(6x))$$

[Out] 1/2*ln(ln(6*x))^2

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2301}

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

[In] Int[Log[Log[6*x]]/(x*Log[6*x]),x]

[Out] Log[Log[6*x]]^2/2

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(\log(6x))}{x\log(6x)}\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \log(6x)\right)\\ &= \frac{1}{2}\log^2(\log(6x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

[In] Integrate[Log[Log[6*x]]/(x*Log[6*x]),x]

[Out] Log[Log[6*x]]^2/2

fricas [A] time = 0.56, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\log\left(6x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="fricas")

[Out] 1/2*log(log(6*x))^2

giac [A] time = 0.19, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\log\left(6x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="giac")
```

```
[Out] 1/2*log(log(6*x))^2
```

maple [A] time = 0.06, size = 10, normalized size = 0.91

$$\frac{\ln\left(\ln\left(6x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(6*x))/x/ln(6*x),x)
```

```
[Out] 1/2*ln(ln(6*x))^2
```

maxima [A] time = 0.60, size = 9, normalized size = 0.82

$$\frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(log(6*x))^2
```

mupad [B] time = 0.52, size = 9, normalized size = 0.82

$$\frac{\ln\left(\ln\left(6\,x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(6*x))/(x*log(6*x)),x)
```

```
[Out] log(log(6*x))^2/2
```

sympy [A] time = 0.25, size = 8, normalized size = 0.73

$$\frac{\log\left(\log\left(6x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(6*x))/x/ln(6*x),x)
```

```
[Out] log(log(6*x))**2/2
```

3.139 $\int \frac{2^{\log(x)}}{x} \, dx$

Optimal. Leaf size=9

$$\frac{2^{\log(x)}}{\log(2)}$$

```
[Out] 2^ln(x)/ln(2)
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2274, 30}

$$\frac{x^{\log(2)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x]/x,x]
[Out] x^Log[2]/Log[2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{\log(x)}}{x} \, dx &= \int x^{-1+\log(2)} \, dx\\ &= \frac{x^{\log(2)}}{\log(2)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x]/x,x]
[Out] 2^Log[x]/Log[2]
```

fricas [A] time = 0.85, size = 11, normalized size = 1.22

$$\frac{e^{(\log(2)\log(x))}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="fricas")
[Out] e^(log(2)*log(x))/log(2)
```

giac [A]   time = 0.18, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="giac")
[Out] 2^log(x)/log(2)
```

maple [A]   time = 0.07, size = 10, normalized size = 1.11

$$\frac{2^{\ln(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x)/x,x)
[Out] 2^ln(x)/ln(2)
```

maxima [A]   time = 0.58, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="maxima")
[Out] 2^log(x)/log(2)
```

mupad [B]   time = 0.40, size = 9, normalized size = 1.00

$$\frac{x^{\ln(2)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(x)/x,x)
[Out] x^log(2)/log(2)
```

sympy [A]   time = 0.38, size = 7, normalized size = 0.78

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x)/x,x)
[Out] 2**log(x)/log(2)
```

3.140 $\int \frac{\sin^2(\log(x))}{x} dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2} - \frac{1}{2}\sin(\log(x))\cos(\log(x))$$

```
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$\frac{\log(x)}{2} - \frac{1}{2}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]]^2/x,x]
```

```
[Out] Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(\log(x))}{x} dx &= \text{Subst}\left(\int \sin^2(x)\,dx, x, \log(x)\right)\\ &= -\frac{1}{2}\cos(\log(x))\sin(\log(x)) + \frac{1}{2}\text{Subst}(\int 1\,dx, x, \log(x))\\ &= \frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.94

$$\frac{\log(x)}{2} - \frac{1}{4}\sin(2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]]^2/x,x]
```

```
[Out] Log[x]/2 - Sin[2*Log[x]]/4
```

fricas [A] time = 0.75, size = 13, normalized size = 0.76

$$-\frac{1}{2}\cos\left(\log(x)\right)\sin\left(\log(x)\right) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*cos(log(x))*sin(log(x)) + 1/2*log(x)
```

giac [A] time = 0.17, size = 12, normalized size = 0.71

$$\frac{1}{2}\log(x) - \frac{1}{4}\sin\left(2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

maple [A] time = 0.31, size = 14, normalized size = 0.82

$$-\frac{\cos(\ln(x))\sin(\ln(x))}{2} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x))^2/x,x)
```

```
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

maxima [A] time = 0.67, size = 12, normalized size = 0.71

$$\frac{1}{2}\log(x) - \frac{1}{4}\sin\left(2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

mupad [B] time = 0.38, size = 12, normalized size = 0.71

$$\frac{\ln(x)}{2} - \frac{\sin(2\ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x))^2/x,x)
```

```
[Out] log(x)/2 - sin(2*log(x))/4
```

sympy [B] time = 3.37, size = 156, normalized size = 9.18

$$\frac{\log(x)\tan^4\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} + \frac{2\log(x)\tan^2\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} + \frac{\log(x)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x))**2/x,x)
```

```
[Out] log(x)*tan(log(x)/2)**4/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*l
og(x)*tan(log(x)/2)**2/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + log(
x)/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*tan(log(x)/2)**3/(2*ta
n(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) - 2*tan(log(x)/2)/(2*tan(log(x)/2)
**4 + 4*tan(log(x)/2)**2 + 2)
```

3.141 $\int \frac{7-\log(x)}{x(3+\log(x))} \, dx$

Optimal. Leaf size=12

$$10\log(\log(x)+3)-\log(x)$$

```
[Out] -ln(x)+10*ln(3+ln(x))
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2365, 43}

$$10\log(\log(x)+3)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{7-\log(x)}{x(3+\log(x))} \, dx &= \text{Subst}\left(\int \frac{7-x}{3+x} \, dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \left(-1+\frac{10}{3+x}\right) \, dx, x, \log(x)\right)\\
&= -\log(x)+10\log(3+\log(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$10\log(\log(x)+3)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

fricas [A] time = 0.86, size = 12, normalized size = 1.00

$$-\log(x)+10\,\log\left(\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="fricas")
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

giac [B] time = 0.16, size = 27, normalized size = 2.25

$$5 \log\left(\frac{1}{4}\pi^2\left(\operatorname{sgn}(x)-1\right)^2+\left(\log\left(|x|\right)+3\right)^2\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="giac")
[Out] 5*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2) - log(x)
```

maple [A] time = 0.07, size = 13, normalized size = 1.08

$$-\ln(x)+10\ln\left(\ln(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7-ln(x))/x/(ln(x)+3),x)
[Out] -ln(x)+10*ln(ln(x)+3)
```

maxima [A] time = 0.63, size = 12, normalized size = 1.00

$$-\log(x)+10\log\left(\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="maxima")
[Out] -log(x) + 10*log(log(x) + 3)
```

mupad [B] time = 0.37, size = 12, normalized size = 1.00

$$10\ln\left(\ln(x)+3\right)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 7)/(x*(log(x) + 3)),x)
[Out] 10*log(log(x) + 3) - log(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.83

$$-\log(x)+10\log\left(\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-ln(x))/x/(3+ln(x)),x)
[Out] -log(x) + 10*log(log(x) + 3)
```

3.142 $\int \frac{(2-\log(x))(3+\log(x))^2}{x} dx$

Optimal. Leaf size=21

$$\frac{5}{3}(\log(x)+3)^3 - \frac{1}{4}(\log(x)+3)^4$$

[Out] 5/3*(3+ln(x))^3-1/4*(3+ln(x))^4

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2365, 43}

$$\frac{5}{3}(\log(x)+3)^3 - \frac{1}{4}(\log(x)+3)^4$$

Antiderivative was successfully verified.

[In] Int[((2 - Log[x])*(3 + Log[x])^2)/x,x]

[Out] (5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-\log(x))(3+\log(x))^2}{x} dx &= \text{Subst}\left(\int (2-x)(3+x)^2 dx, x, \log(x)\right) \\
&= \text{Subst}\left(\int \left(5(3+x)^2-(3+x)^3\right) dx, x, \log(x)\right) \\
&= \frac{5}{3}(3+\log(x))^3 - \frac{1}{4}(3+\log(x))^4
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.38

$$-\frac{1}{4}\log^4(x) - \frac{4\log^3(x)}{3} + \frac{3\log^2(x)}{2} + 18\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((2 - Log[x])*(3 + Log[x])^2)/x,x]

[Out] 18*Log[x] + (3*Log[x]^2)/2 - (4*Log[x]^3)/3 - Log[x]^4/4

fricas [A] time = 0.55, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4 - \frac{4}{3}\log(x)^3 + \frac{3}{2}\log(x)^2 + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="fricas")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

giac [A] time = 0.19, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4 - \frac{4}{3}\log(x)^3 + \frac{3}{2}\log(x)^2 + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="giac")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

maple [A] time = 0.06, size = 24, normalized size = 1.14

$$-\frac{\ln(x)^4}{4} - \frac{4\ln(x)^3}{3} + \frac{3\ln(x)^2}{2} + 18\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-ln(x))*(ln(x)+3)^2/x,x)
[Out] -1/4*ln(x)^4-4/3*ln(x)^3+3/2*ln(x)^2+18*ln(x)
```

maxima [A] time = 0.51, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4 - \frac{4}{3}\log(x)^3 + \frac{3}{2}\log(x)^2 + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="maxima")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

mupad [B] time = 0.37, size = 22, normalized size = 1.05

$$\frac{\ln(x)\left(-3\ln(x)^3 - 16\ln(x)^2 + 18\ln(x) + 216\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((log(x) - 2)*(log(x) + 3)^2)/x,x)
[Out] (log(x)*(18*log(x) - 16*log(x)^2 - 3*log(x)^3 + 216))/12
```

sympy [A] time = 0.14, size = 27, normalized size = 1.29

$$-\frac{\log(x)^4}{4} - \frac{4\log(x)^3}{3} + \frac{3\log(x)^2}{2} + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-ln(x))*(3+ln(x))**2/x,x)
[Out] -log(x)**4/4 - 4*log(x)**3/3 + 3*log(x)**2/2 + 18*log(x)
```

3.143 $\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx$

Optimal. Leaf size=42

$$\frac{1}{8}\sqrt{\log^2(x)+1}\,\log(x)+\frac{1}{4}\sqrt{\log^2(x)+1}\,\log^3(x)-\frac{1}{8}\sinh^{-1}(\log(x))$$

```
[Out] -1/8*arcsinh(ln(x))+1/8*ln(x)*(1+ln(x)^2)^(1/2)+1/4*ln(x)^3*(1+ln(x)^2)^(1/
2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {279, 321, 215}

$$\frac{1}{4}\sqrt{\log^2(x)+1}\,\log^3(x)+\frac{1}{8}\sqrt{\log^2(x)+1}\,\log(x)-\frac{1}{8}\sinh^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

```
[Out] -ArcSinh[Log[x]]/8 + (Log[x]*Sqrt[1 + Log[x]^2])/8 + (Log[x]^3*Sqrt[1 + Log
[x]^2])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx &= \text{Subst}\left(\int x^2\sqrt{1+x^2}\,dx,x,\log(x)\right)\\
&= \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,\log(x)\right)\\
&= \frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\log\right.\\
&= -\frac{1}{8}\sinh^{-1}(\log(x))+\frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.74

$$\frac{1}{8}\left(\log(x)\sqrt{\log^2(x)+1}\left(2\log^2(x)+1\right)-\sinh^{-1}(\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
[Out] (-ArcSinh[Log[x]] + Log[x]*Sqrt[1 + Log[x]^2]*(1 + 2*Log[x]^2))/8
```

fricas [A] time = 0.63, size = 36, normalized size = 0.86

$$\frac{1}{8}\left(2\log(x)^3+\log(x)\right)\sqrt{\log(x)^2+1}+\frac{1}{8}\log\left(\sqrt{\log(x)^2+1}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/8*(2*log(x)^3 + log(x))*sqrt(log(x)^2 + 1) + 1/8*log(sqrt(log(x)^2 + 1) -
 log(x))
```

giac [A] time = 0.17, size = 37, normalized size = 0.88

$$\frac{1}{8}\left(2\log(x)^2+1\right)\sqrt{\log(x)^2+1}\log(x)+\frac{1}{8}\log\left(\sqrt{\log(x)^2+1}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/8*(2*log(x)^2 + 1)*sqrt(log(x)^2 + 1)*log(x) + 1/8*log(sqrt(log(x)^2 + 1)
 - log(x))
```

maple [A] time = 0.07, size = 31, normalized size = 0.74

$$-\frac{\operatorname{arcsinh}(\ln(x))}{8}+\frac{\left(\ln(x)^2+1\right)^{\frac{3}{2}}\ln(x)}{4}-\frac{\sqrt{\ln(x)^2+1}\,\ln(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2*(1+ln(x)^2)^(1/2)/x,x)
[Out] 1/4*ln(x)*(1+ln(x)^2)^(3/2)-1/8*ln(x)*(1+ln(x)^2)^(1/2)-1/8*arcsinh(ln(x))
```

maxima [A] time = 1.49, size = 30, normalized size = 0.71

$$\frac{1}{4}\left(\log(x)^2+1\right)^{\frac{3}{2}}\log(x)-\frac{1}{8}\sqrt{\log(x)^2+1}\log(x)-\frac{1}{8}\operatorname{arsinh}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/4*(log(x)^2 + 1)^(3/2)*log(x) - 1/8*sqrt(log(x)^2 + 1)*log(x) - 1/8*arcsi
nh(log(x))
```

mupad [B] time = 0.40, size = 26, normalized size = 0.62

$$\left(\frac{\ln(x)^3}{4}+\frac{\ln(x)}{8}\right)\sqrt{\ln(x)^2+1}-\frac{\operatorname{asinh}(\ln(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2*(log(x)^2 + 1)^(1/2))/x,x)
[Out] (log(x)/8 + log(x)^3/4)*(log(x)^2 + 1)^(1/2) - asinh(log(x))/8
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(x)^2 + 1}\,\log(x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2*(1+ln(x)**2)**(1/2)/x,x)
[Out] Integral(sqrt(log(x)**2 + 1)*log(x)**2/x, x)
```

3.144 $\int \frac{1+\log(x)}{x(3+2\log(x))^2} \, dx$

Optimal. Leaf size=24

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

```
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
[Out] 1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\log(x)}{x(3+2\log(x))^2} \, dx &= \text{Subst}\left(\int \frac{1+x}{(3+2x)^2} \, dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{2(3+2x)^2}+\frac{1}{2(3+2x)}\right) dx, x, \log(x)\right)\\ &= \frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))\end{aligned}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 0.83

$$\frac{1}{4}\left(\log(2\log(x)+3)+\frac{1}{2\log(x)+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
[Out] ((3 + 2*Log[x])^(-1) + Log[3 + 2*Log[x]])/4
```

fricas [A] time = 0.77, size = 26, normalized size = 1.08

$$\frac{\left(2\,\log(x)+3\right)\log\left(2\,\log(x)+3\right)+1}{4\left(2\,\log(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="fricas")
[Out] 1/4*((2*log(x) + 3)*log(2*log(x) + 3) + 1)/(2*log(x) + 3)
```

giac [A] time = 0.17, size = 34, normalized size = 1.42

$$\frac{1}{4\left(2\,\log(x)+3\right)}+\frac{1}{8}\,\log\left(\pi^2\left(\text{sgn}(x)-1\right)^2+\left(2\,\log\left(|x|\right)+3\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="giac")
[Out] 1/4/(2*log(x) + 3) + 1/8*log(pi^2*(sgn(x) - 1)^2 + (2*log(abs(x)) + 3)^2)
```

maple [A] time = 0.07, size = 21, normalized size = 0.88

$$\frac{\ln\left(2\ln(x)+3\right)}{4}+\frac{1}{8\ln(x)+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)+1)/x/(3+2*ln(x))^2,x)
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

maxima [A] time = 0.87, size = 20, normalized size = 0.83

$$\frac{1}{4\left(2\,\log(x)+3\right)}+\frac{1}{4}\,\log\left(2\,\log(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4/(2*log(x) + 3) + 1/4*log(2*log(x) + 3)
```

mupad [B] time = 0.41, size = 18, normalized size = 0.75

$$\frac{\ln\left(2\,\ln(x)+3\right)}{4}+\frac{1}{4\,\left(2\,\ln(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)/(x*(2*log(x) + 3)^2),x)
[Out] log(2*log(x) + 3)/4 + 1/(4*(2*log(x) + 3))
```

sympy [A] time = 0.12, size = 17, normalized size = 0.71

$$\frac{\log\left(\log\left(x\right)+\frac{3}{2}\right)}{4}+\frac{1}{8\log\left(x\right)+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))/x/(3+2*ln(x))**2,x)
[Out] log(log(x) + 3/2)/4 + 1/(8*log(x) + 12)
```

3.145 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2365, 43}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx, x, \log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.70

$$\frac{2}{3}(\log(x)-2)\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

fricas [A] time = 1.07, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\log(x)+1}\,(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

giac [A] time = 0.16, size = 17, normalized size = 0.74

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

maple [A] time = 0.07, size = 18, normalized size = 0.78

$$\frac{2\left(\ln(x)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\ln(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(ln(x)+1)^(1/2),x)
[Out] 2/3*(ln(x)+1)^(3/2)-2*(ln(x)+1)^(1/2)
```

maxima [A] time = 0.58, size = 17, normalized size = 0.74

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

mupad [B] time = 0.43, size = 13, normalized size = 0.57

$$\sqrt{\ln(x)+1}\,\left(\frac{2\,\ln(x)}{3}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

sympy [A] time = 4.82, size = 20, normalized size = 0.87

$$\frac{2\left(\log\left(x\right)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\log\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

3.146 $\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

```
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

```
[Out] Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{-1+4x}}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{4\sqrt{-1+4x}}+\frac{1}{4}\sqrt{-1+4x}\right)dx,x,\log(x)\right)\\ &= \frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.69

$$\frac{1}{12}(2\log(x)+1)\sqrt{4\log(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

```
[Out] ((1 + 2*Log[x])*Sqrt[-1 + 4*Log[x]])/12
```

fricas [A] time = 1.64, size = 16, normalized size = 0.55

$$\frac{1}{12}\sqrt{4\log(x)-1}\left(2\log(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(4*log(x) - 1)*(2*log(x) + 1)
```

giac [A] time = 0.15, size = 21, normalized size = 0.72

$$\frac{1}{24}\left(4\log(x)-1\right)^{\frac{3}{2}}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="giac")
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

maple [A] time = 0.07, size = 22, normalized size = 0.76

$$\frac{(4\ln(x)-1)^{\frac{3}{2}}}{24}+\frac{\sqrt{4\ln(x)-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(-1+4*ln(x))^(1/2),x)
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

maxima [A] time = 0.75, size = 21, normalized size = 0.72

$$\frac{1}{24}\left(4\log(x)-1\right)^{\frac{3}{2}}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="maxima")
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

mupad [B] time = 0.44, size = 15, normalized size = 0.52

$$\sqrt{4\ln(x)-1}\left(\frac{\ln(x)}{6}+\frac{1}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(4*log(x) - 1)^(1/2)),x)
[Out] (4*log(x) - 1)^(1/2)*(log(x)/6 + 1/12)
```

sympy [A] time = 5.43, size = 22, normalized size = 0.76

$$\frac{\left(4\log{(x)}-1\right)^{\frac{3}{2}}}{24}+\frac{\sqrt{4\log{(x)}-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(-1+4*ln(x))**(1/2),x)
[Out] (4*log(x) - 1)**(3/2)/24 + sqrt(4*log(x) - 1)/8
```

3.147 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

```
[Out] -2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2365, 50, 63, 207}

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)$$
$$= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)$$
$$= 2\sqrt{1+\log(x)} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)$$
$$= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

fricas [A] time = 1.14, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.06, size = 30, normalized size = 1.36

$$-\ln\left(1+\sqrt{\ln(x)+1}\right) + \ln\left(\sqrt{\ln(x)+1}-1\right) + 2\sqrt{\ln(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)+1)^(1/2)/x/ln(x),x)
[Out] 2*(ln(x)+1)^(1/2)+ln((ln(x)+1)^(1/2)-1)-ln(1+(ln(x)+1)^(1/2))
```

maxima [A] time = 0.55, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

mupad [**B**] time = 0.41, size = 18, normalized size = 0.82

$$2\sqrt{\ln(x)+1}-2\,\mathrm{atanh}\left(\sqrt{\ln(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^(1/2)/(x*log(x)),x)
[Out] 2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

sympy [**A**] time = 2.09, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1}+\log\left(\sqrt{\log(x)+1}-1\right)-\log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

3.148 $\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx$

Optimal. Leaf size=27

$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

```
[Out] -2/3/(1-ln(x))^3+1/(1-ln(x))+1/(-1+ln(x))^2
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] -2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx &= \text{Subst}\left(\int \frac{1-4x+x^2}{(-1+x)^4}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{2}{(-1+x)^4}-\frac{2}{(-1+x)^3}+\frac{1}{(-1+x)^2}\right)dx,x,\log(x)\right)\\ &= -\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.81

$$\frac{-3\log^2(x)+9\log(x)-4}{3(\log(x)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] (-4 + 9*Log[x] - 3*Log[x]^2)/(3*(-1 + Log[x])^3)
```

fricas [A] time = 2.13, size = 32, normalized size = 1.19

$$-\frac{3\,\log(x)^2-9\,\log(x)+4}{3\left(\log(x)^3-3\,\log(x)^2+3\,\log(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

giac [A] time = 0.19, size = 20, normalized size = 0.74

$$-\frac{3\log(x)^2 - 9\log(x) + 4}{3\left(\log(x) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x) - 1)^3
```

maple [A] time = 0.07, size = 24, normalized size = 0.89

$$-\frac{1}{\ln(x) - 1} + \frac{1}{(\ln(x) - 1)^2} + \frac{2}{3\,(\ln(x) - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-4*ln(x)+ln(x)^2)/x/(-1+ln(x))^4,x)
```

```
[Out] -1/(-1+ln(x))+1/(-1+ln(x))^2+2/3/(-1+ln(x))^3
```

maxima [A] time = 0.55, size = 32, normalized size = 1.19

$$-\frac{3\log(x)^2 - 9\log(x) + 4}{3\left(\log(x)^3 - 3\log(x)^2 + 3\log(x) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

mupad [B] time = 0.42, size = 18, normalized size = 0.67

$$-\frac{\ln(x)^2 - 3\ln(x) + \frac{4}{3}}{(\ln(x) - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2 - 4*log(x) + 1)/(x*(log(x) - 1)^4),x)
```

```
[Out] -(log(x)^2 - 3*log(x) + 4/3)/(log(x) - 1)^3
```

sympy [A] time = 0.13, size = 32, normalized size = 1.19

$$\frac{-3\log(x)^2 + 9\log(x) - 4}{3\log(x)^3 - 9\log(x)^2 + 9\log(x) - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*ln(x)+ln(x)**2)/x/(-1+ln(x))**4,x)
```

```
[Out] (-3*log(x)**2 + 9*log(x) - 4)/(3*log(x)**3 - 9*log(x)**2 + 9*log(x) - 3)
```

3.149 $\int \frac{\log^2(ax^n)^p}{x} dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

```
[Out] ln(a*x^n)*(ln(a*x^n)^2)^p/n/(1+2*p)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(ax^n)^p}{x} dx &= \frac{\text{Subst}\left(\int \left(x^2\right)^p dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-2p}(ax^n)\log^2(ax^n)^p\right)\text{Subst}\left(\int x^{2p} dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

fricas [A] time = 0.92, size = 38, normalized size = 1.41

$$\frac{\left(n\log(x)+\log(a)\right)\left(n^2\log(x)^2+2\,n\log(a)\log(x)+\log(a)^2\right)^p}{2\,np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)^p/(2*n*p
+ n)
```

giac [B] time = 0.36, size = 68, normalized size = 2.52

$$\frac{\left(n\log(x)\operatorname{sgn}\left(\log\left(ax^n\right)\right)+\log(a)\operatorname{sgn}\left(\log\left(ax^n\right)\right)\right)\left(n\log(x)\operatorname{sgn}\left(\log\left(ax^n\right)\right)+\log(a)\operatorname{sgn}\left(\log\left(ax^n\right)\right)\right)^{2p}}{n\left(2p+1\right)\operatorname{sgn}\left(\log\left(ax^n\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x)*sgn(log(a*x^n)) + log(a)*sgn(log(a*x^n)))*(n*log(x)*sgn(log(a*x^n
)) + log(a)*sgn(log(a*x^n)))^(2*p)/(n*(2*p + 1)*sgn(log(a*x^n)))
```

maple [F] time = 3.56, size = 0, normalized size = 0.00

$$\int \frac{\left(\ln\left(a\,x^n\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^p/x,x)
```

```
[Out] int((ln(a*x^n)^2)^p/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [B] time = 0.40, size = 27, normalized size = 1.00

$$\frac{\ln\left(a\,x^n\right)\,\left(\ln\left(a\,x^n\right)^2\right)^p}{n\,\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^2)^p/x,x)
```

```
[Out] (log(a*x^n)*(log(a*x^n)^2)^p)/(n*(2*p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\log\left(ax^n\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**p/x,x)
```

```
[Out] Integral((log(a*x**n)**2)**p/x, x)
```

3.150 $\int \frac{\log^m(ax^n)^p}{x} dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

```
[Out] ln(a*x^n)*(ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^m(ax^n)^p}{x} dx &= \frac{\text{Subst}\left(\int (x^m)^p\, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-mp}(ax^n)\log^m(ax^n)^p\right)\text{Subst}\left(\int x^{mp}\, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

fricas [A] time = 0.79, size = 27, normalized size = 1.00

$$\frac{\left(n\log(x)+\log(a)\right)\left(n\log(x)+\log(a)\right)^{mp}}{mnp+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*(n*log(x) + log(a))^(m*p)/(m*n*p + n)
```

giac [A] time = 0.18, size = 24, normalized size = 0.89

$$\frac{\left(n\log(x) + \log(a)\right)^{mp+1}}{\left(mp+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(a))^(m*p + 1)/((m*p + 1)*n)
```

maple [C] time = 2.00, size = 71, normalized size = 2.63

$$\frac{\left(-\frac{i\pi\left(\mathrm{csgn}(ia)-\mathrm{csgn}(ia\,x^{n})\right)\left(\mathrm{csgn}(ix^{n})-\mathrm{csgn}(ia\,x^{n})\right)\mathrm{csgn}(ia\,x^{n})}{2} + \ln(a) + \ln\left(x^{n}\right)\right)^{mp+1}}{\left(mp+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^m)^p/x,x)
```

```
[Out] 1/n*(ln(a)+ln(x^n)-1/2*I*Pi*csgn(I*a*x^n)*(-csgn(I*a*x^n)+csgn(I*a))*(-csgn
(I*a*x^n)+csgn(I*x^n)))^(m*p+1)/(m*p+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [B] time = 0.38, size = 27, normalized size = 1.00

$$\frac{\ln\left(a\,x^{n}\right)\,{\left({\ln\left(a\,x^{n}\right)}^{m}\right)}^{p}}{n\,\left(m\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^m)^p/x,x)
```

```
[Out] (log(a*x^n)*(log(a*x^n)^m)^p)/(n*(m*p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\log\left(ax^{n}\right)^{m}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**m)**p/x,x)
```

```
[Out] Integral((log(a*x**n)**m)**p/x, x)
```

3.151 $\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx$

Optimal. Leaf size=25

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

```
[Out] 1/2*ln(a*x^n)*(ln(a*x^n)^2)^(1/2)/n
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Log[a*x^n]^2]/x,x]
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx &= \frac{\text{Subst}\left(\int \sqrt{x^2} \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\sqrt{\log^2(ax^n)} \, \text{Subst}\left(\int x \, dx, x, \log(ax^n)\right)}{n \log(ax^n)} \\
&= \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]^2]/x,x]
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

fricas [A]   time = 1.55, size = 13, normalized size = 0.52

$$\frac{1}{2}\,n\log(x)^2+\log(a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*n*log(x)^2 + log(a)*log(x)
```

giac [A]   time = 0.20, size = 27, normalized size = 1.08

$$\frac{1}{2}\,n\log(x)^2\mathrm{sgn}\left(\log\left(ax^n\right)\right)+\log(a)\log(x)\mathrm{sgn}\left(\log\left(ax^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*n*log(x)^2*sgn(log(a*x^n)) + log(a)*log(x)*sgn(log(a*x^n))
```

maple [C]   time = 0.35, size = 21, normalized size = 0.84

$$\frac{\mathrm{csgn}\left(\ln\left(a\,x^n\right)\right)\ln\left(a\,x^n\right)^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^(1/2)/x,x)
[Out] 1/2/n*csgn(ln(a*x^n))*ln(a*x^n)^2
```

maxima [A]   time = 0.63, size = 20, normalized size = 0.80

$$-\frac{1}{2}\,n\log(x)^2+\log(a)\log(x)+\log(x)\log\left(x^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="maxima")
[Out] -1/2*n*log(x)^2 + log(a)*log(x) + log(x)*log(x^n)
```

mupad [B]   time = 0.40, size = 21, normalized size = 0.84

$$\frac{\ln\left(a\,x^n\right)\,\sqrt{\ln\left(a\,x^n\right)^2}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^2)^(1/2)/x,x)
[Out] (log(a*x^n)*(log(a*x^n)^2)^(1/2))/(2*n)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\log\left(ax^n\right)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**(1/2)/x,x)
[Out] Integral(sqrt(log(a*x**n)**2)/x, x)
```

3.152 $\int \frac{(b\log^m(ax^n))^p}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

```
[Out] ln(a*x^n)*(b*ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Log[a*x^n]^m)^p/x,x]
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(b\log^m(ax^n)\right)^p}{x}\,dx &= \frac{\text{Subst}\left(\int (bx^m)^p\,dx, x, \log(ax^n)\right)}{n}\\
&= \frac{\left(\log^{-mp}(ax^n)\left(b\log^m(ax^n)\right)^p\right)\text{Subst}\left(\int x^{mp}\,dx, x, \log(ax^n)\right)}{n}\\
&= \frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(1+mp)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Log[a*x^n]^m)^p/x,x]
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

fricas [A] time = 0.66, size = 33, normalized size = 1.14

$$\frac{\left(n\log(x)+\log(a)\right)e^{\left(mp\log\left(n\log(x)+\log(a)\right)+p\log(b)\right)}}{mnp+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="fricas")
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/(m*n*p + n)
```

giac [A] time = 0.33, size = 35, normalized size = 1.21

$$\frac{\left(n\log(x)+\log(a)\right)e^{\left(mp\log\left(n\log(x)+\log(a)\right)+p\log(b)\right)}}{\left(mp+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="giac")
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/((m*p + 1)*n)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(a\,x^{n}\right)^{m}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(a*x^n)^m)^p/x,x)
[Out] int((b*ln(a*x^n)^m)^p/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [B] time = 0.36, size = 29, normalized size = 1.00

$$\frac{\ln\left(a\,x^{n}\right)\,\left(b\ln\left(a\,x^{n}\right)^{m}\right)^{p}}{n\,\left(m\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*log(a*x^n)^m)^p/x,x)
[Out] (log(a*x^n)*(b*log(a*x^n)^m)^p)/(n*(m*p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(ax^{n}\right)^{m}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*ln(a*x**n)**m)**p/x,x)
[Out] Integral((b*log(a*x**n)**m)**p/x, x)
```

3.153 $\int \frac{1}{x\log(e^x)}\,dx$

Optimal. Leaf size=31

$$\frac{\log(\log(e^x))}{x-\log(e^x)}-\frac{\log(x)}{x-\log(e^x)}$$

```
[Out] -ln(x)/(x-ln(exp(x)))+ln(ln(exp(x)))/(x-ln(exp(x)))
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2160, 2157, 29}

$$\frac{\log(\log(e^x))}{x-\log(e^x)}-\frac{\log(x)}{x-\log(e^x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[E^x]),x]
```

```
[Out] -(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\log(e^x)}\,dx &= -\frac{\int \frac{1}{x}\,dx}{x-\log(e^x)}+\frac{\int \frac{1}{\log(e^x)}\,dx}{x-\log(e^x)}\\
&= -\frac{\log(x)}{x-\log(e^x)}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,\log(e^x)\right)}{x-\log(e^x)}\\
&= -\frac{\log(x)}{x-\log(e^x)}+\frac{\log(\log(e^x))}{x-\log(e^x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.68

$$\frac{\log(\log(e^x))-\log(x)}{x-\log(e^x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[E^x]),x]
```

```
[Out] (-Log[x] + Log[Log[E^x]])/(x - Log[E^x])
```

fricas [A] time = 0.53, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="fricas")
[Out] -1/x
```

giac [A] time = 0.16, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="giac")
[Out] -1/x
```

maple [A] time = 0.07, size = 29, normalized size = 0.94

$$\frac{\ln(x)}{-x+\ln\left(e^{x}\right)}-\frac{\ln\left(\ln\left(e^{x}\right)\right)}{-x+\ln\left(e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(exp(x)),x)
[Out] -1/(ln(exp(x))-x)*ln(ln(exp(x)))+1/(ln(exp(x))-x)*ln(x)
```

maxima [A] time = 0.70, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="maxima")
[Out] -1/x
```

mupad [B] time = 0.36, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(exp(x))),x)
[Out] -1/x
```

sympy [A] time = 0.07, size = 3, normalized size = 0.10

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(exp(x)),x)
[Out] -1/x
```

3.154 $\int \log(x)\sin(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos(a)\text{Ci}(bx)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

```
[Out] Ci(b*x)*cos(a)/b-cos(b*x+a)*ln(x)/b-Si(b*x)*sin(a)/b
```

Rubi [A] time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2638, 2554, 12, 3303, 3299, 3302}

$$\frac{\cos(a)\text{CosIntegral}(bx)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x])/b - (Cos[a + b*x]*Log[x])/b - (Sin[a]*SinIntegral
[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sin(a+bx)\,dx &= -\frac{\cos(a+bx)\log(x)}{b} + \int \frac{\cos(a+bx)}{bx}\,dx \\ &= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\int \frac{\cos(a+bx)}{x}\,dx}{b} \\ &= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{b} \\ &= \frac{\cos(a)\mathrm{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} - \frac{\sin(a)\mathrm{Si}(bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.86

$$\frac{\cos(a)\mathrm{Ci}(bx) - \sin(a)\mathrm{Si}(bx) - \log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x] - Cos[a + b*x]*Log[x] - Sin[a]*SinIntegral[b*x])/b
```

fricas [A] time = 0.90, size = 37, normalized size = 1.06

$$\frac{(\mathrm{Ci}\,(bx) + \mathrm{Ci}\,(-bx))\cos(a) - 2\,\cos\,(bx+a)\log(x) - 2\,\sin(a)\,\mathrm{Si}\,(bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(b*x) + cos_integral(-b*x))*cos(a) - 2*cos(b*x + a)*log(x
) - 2*sin(a)*sin_integral(b*x))/b
```

giac [C] time = 0.17, size = 102, normalized size = 2.91

$$-\frac{\cos\,(bx+a)\log(x)}{b} - \frac{\Re\,(\mathrm{Ci}\,(bx))\tan\left(\frac{1}{2}\,a\right)^2 + \Re\,(\mathrm{Ci}\,(-bx))\tan\left(\frac{1}{2}\,a\right)^2 + 2\,\Im\,(\mathrm{Ci}\,(bx))\tan\left(\frac{1}{2}\,a\right) - 2\,\Im\,(\mathrm{Ci}\,(-bx)}{2\left(b\tan\left(\frac{1}{2}\,a\right)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*(real_part(cos_integral(b*x))*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(b*x))*
tan(1/2*a) - 2*imag_part(cos_integral(-b*x))*tan(1/2*a) + 4*sin_integral(b*
x)*tan(1/2*a) - real_part(cos_integral(b*x)) - real_part(cos_integral(-b*x)
))/(b*tan(1/2*a)^2 + b)
```

maple [C] time = 0.98, size = 80, normalized size = 2.29

$$-\frac{\mathrm{Ei}\,(1,-ibx)\,\mathrm{e}^{-ia}}{2b} - \frac{\mathrm{Ei}\,(1,-ibx)\,\mathrm{e}^{ia}}{2b} - \frac{i\,\mathrm{Si}\,(bx)\,\mathrm{e}^{-ia}}{b} - \frac{\cos\,(bx+a)\ln(x)}{b} + \frac{i\pi\,\mathrm{csgn}\,(bx)\,\mathrm{e}^{-ia}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a),x)
```

```
[Out] -cos(b*x+a)*ln(x)/b+1/2*I/b*exp(-I*a)*Pi*csgn(b*x)-I/b*exp(-I*a)*Si(b*x)-1/
2/b*exp(-I*a)*Ei(1,-I*b*x)-1/2/b*exp(I*a)*Ei(1,-I*b*x)
```

maxima [C] time = 0.85, size = 57, normalized size = 1.63

$$-\frac{\cos(bx+a)\log(x)}{b}-\frac{(E_1(i\,bx)+E_1(-i\,bx))\cos(a)-(i\,E_1(i\,bx)-i\,E_1(-i\,bx))\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*((exp_integral_e(1, I*b*x) + exp_integral_e(1,
 -I*b*x))*cos(a) - (I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x
))*sin(a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sin(a+b\,x)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*log(x),x)
```

```
[Out] int(sin(a + b*x)*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a),x)
```

```
[Out] Integral(log(x)*sin(a + b*x), x)
```

3.155 $\int \log(x)\sin^2(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{\sin(2a)\mathrm{Ci}(2bx)}{4b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -1/2*x+1/2*x*ln(x)+1/4*cos(2*a)*Si(2*b*x)/b+1/4*Ci(2*b*x)*sin(2*a)/b-1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2635, 8, 2554, 3303, 3299, 3302}

$$\frac{\sin(2a)\mathrm{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] -x/2 + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sin^2(a+bx)\,dx &= \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int \left(\frac{1}{2} - \frac{\sin(2a+2bx)}{4bx}\right)dx \\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\int \frac{\sin(2a+2bx)}{x}dx}{4b} \\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\int \frac{\sin(2bx)}{x}dx}{4b} + \frac{\sin(2a)\int}{4} \\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\text{Ci}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\text{Si}(2bx}{4b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.76

$$\frac{\sin(2a)\text{Ci}(2bx) + \cos(2a)\text{Si}(2bx) - \log(x)\sin(2(a+bx)) - 2bx + 2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] (-2*b*x + 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b*
x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

fricas [A] time = 0.73, size = 59, normalized size = 0.89

$$\frac{4\,bx\log(x) - 4\,\cos(bx+a)\log(x)\sin(bx+a) - 4\,bx + (\text{Ci}(2\,bx) + \text{Ci}(-2\,bx))\sin(2\,a) + 2\,\cos(2\,a)\,\text{Si}(2\,bx)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) - 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x + (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) + 2*cos(2*a)*sin_integral(2*b*x
))/b
```

giac [C] time = 0.21, size = 123, normalized size = 1.86

$$\frac{1}{4}\left(2\,x - \frac{\sin(2\,bx+2\,a)}{b}\right)\log(x) - \frac{4\,bx\tan(a)^2 + \Im(\text{Ci}(2\,bx))\tan(a)^2 - \Im(\text{Ci}(-2\,bx))\tan(a)^2 + 2\,\text{Si}(2\,bx)\tan(a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x - sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 + imag_part(cos
_integral(2*b*x))*tan(a)^2 - imag_part(cos_integral(-2*b*x))*tan(a)^2 + 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x - 2*real_part(cos_integral(2*b*x))*tan(
a) - 2*real_part(cos_integral(-2*b*x))*tan(a) - imag_part(cos_integral(2*b*
x)) + imag_part(cos_integral(-2*b*x)) - 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

maple [C] time = 1.65, size = 132, normalized size = 2.00

$$\frac{x\ln(x)}{2} + \frac{a\ln(ibx)}{2b} - \frac{a\ln(a+i(ibx+ia))}{2b} - \frac{i\,\text{Ei}(1,-2ibx)\,\mathrm{e}^{-2ia}}{8b} + \frac{i\,\text{Ei}(1,-2ibx)\,\mathrm{e}^{2ia}}{8b} + \frac{\text{Si}(2bx)\,\mathrm{e}^{-2ia}}{4b} - \frac{\pi\,\text{csgn}(bx)\,\mathrm{e}^{-2}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*x*ln(x)-1/4*ln(x)/b*sin(2*b*x+2*a)-1/8/b*exp(-2*I*a)*Pi*csgn(b*x)+1/4/b
*exp(-2*I*a)*Si(2*b*x)-1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2*x-1/2*a/b-1/2/b*a*ln(a+I*(I*b*x+I*a))+1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

maxima [C] time = 1.24, size = 79, normalized size = 1.20

$$\frac{(2\,bx+2\,a-\sin(2\,bx+2\,a))\log(x)}{4\,b}-\frac{4\,bx+(i\,\mathrm{Ei}(2i\,bx)-i\,\mathrm{Ei}(-2i\,bx))\cos(2\,a)+4\,a\log(x)-(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (I*Ei(2*I*b*x)
 - I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) - (Ei(2*I*b*x) + Ei(-2*I*b*x))*sin
(2*a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin(a+b\,x)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*log(x),x)
```

```
[Out] int(sin(a + b*x)^2*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**2, x)
```

3.156 $\int \log(x)\sin^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cos(a)\text{Ci}(bx)}{4b}-\frac{\cos(3a)\text{Ci}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

```
[Out] 3/4*Ci(b*x)*cos(a)/b-1/12*Ci(3*b*x)*cos(3*a)/b-cos(b*x+a)*ln(x)/b+1/3*cos(b
*x+a)^3*ln(x)/b-3/4*Si(b*x)*sin(a)/b+1/12*Si(3*b*x)*sin(3*a)/b
```

Rubi [A] time = 0.52, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3299, 3302, 3312}

$$\frac{3\cos(a)\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\text{log}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x])/(4*b) - (Cos[3*a]*CosIntegral[3*b*x])/(12*b) -
(Cos[a + b*x]*Log[x])/b + (Cos[a + b*x]^3*Log[x])/(3*b) - (3*Sin[a]*SinInte
gral[b*x])/(4*b) + (Sin[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sin^3(a+bx)\,dx &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{3bx}\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{x}\,dx}{3b}\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \left(-\frac{3\cos(a+bx)}{x}+\frac{\cos^3(a+bx)}{x}\right)dx}{3b}\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cos^3(a+bx)}{x}\,dx}{3b}+\frac{\int \frac{\cos(a+bx)}{x}\,dx}{b}\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \left(\frac{3\cos(a+bx)}{4x}+\frac{\cos(3a+3bx)}{4x}\right)dx}{3b}+\ \mathrm{cos}\\ &= \frac{\cos(a)\mathrm{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}-\int \frac{\cos(3}{}\\ &= \frac{\cos(a)\mathrm{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}-\frac{\cos(a)}{}\\ &= \frac{3\cos(a)\mathrm{Ci}(bx)}{4b}-\frac{\cos(3a)\mathrm{Ci}(3bx)}{12b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-3\end{aligned}$$

Mathematica [A] time = 0.12, size = 66, normalized size = 0.74

$$\frac{9\cos(a)\mathrm{Ci}(bx)-\cos(3a)\mathrm{Ci}(3bx)-9\sin(a)\mathrm{Si}(bx)+\sin(3a)\mathrm{Si}(3bx)-9\log(x)\cos(a+bx)+\log(x)\cos(3(a+}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^3,x]
[Out] (9*Cos[a]*CosIntegral[b*x] - Cos[3*a]*CosIntegral[3*b*x] - 9*Cos[a + b*x]*L
og[x] + Cos[3*(a + b*x)]*Log[x] - 9*Sin[a]*SinIntegral[b*x] + Sin[3*a]*SinI
ntegral[3*b*x])/(12*b)
```

fricas [A] time = 0.72, size = 76, normalized size = 0.85

$$-\frac{(\mathrm{Ci}(3bx)+\mathrm{Ci}(-3bx))\cos(3a)-9(\mathrm{Ci}(bx)+\mathrm{Ci}(-bx))\cos(a)-8\left(\cos(bx+a)^3-3\cos(bx+a)\right)\log(x)-}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/24*((cos_integral(3*b*x) + cos_integral(-3*b*x))*cos(3*a) - 9*(cos_integ
ral(b*x) + cos_integral(-b*x))*cos(a) - 8*(cos(b*x + a)^3 - 3*cos(b*x + a))
*log(x) - 2*sin(3*a)*sin_integral(3*b*x) + 18*sin(a)*sin_integral(b*x))/b
```

giac [C] time = 0.24, size = 454, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(cos(b*x + a)^3/b - 3*cos(b*x + a)/b)*log(x) + 1/24*(real_part(cos_inte
gral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan
(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1
/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 18*im
ag_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) + 18*imag_part(cos_integ
ral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 36*sin_integral(b*x)*tan(3/2*a)^2*tan(
1/2*a) + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*imag_
part(cos_integral(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + 4*sin_integral(3*b*x)*
tan(3/2*a)*tan(1/2*a)^2 + real_part(cos_integral(3*b*x))*tan(3/2*a)^2 + 9*r
eal_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*real_part(cos_integral(-b*x))*
tan(3/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2 - real_part(cos
_integral(3*b*x))*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan(1/2*a)^
2 - 9*real_part(cos_integral(-b*x))*tan(1/2*a)^2 - real_part(cos_integral(-
3*b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a) - 2*imag
_part(cos_integral(-3*b*x))*tan(3/2*a) + 4*sin_integral(3*b*x)*tan(3/2*a) -
 18*imag_part(cos_integral(b*x))*tan(1/2*a) + 18*imag_part(cos_integral(-b*
x))*tan(1/2*a) - 36*sin_integral(b*x)*tan(1/2*a) - real_part(cos_integral(3
*b*x)) + 9*real_part(cos_integral(b*x)) + 9*real_part(cos_integral(-b*x)) -
 real_part(cos_integral(-3*b*x)))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*
a)^2 + b*tan(1/2*a)^2 + b)
```

maple [C] time = 1.40, size = 162, normalized size = 1.82

$$\frac{\mathrm{Ei}\left(1,-3ibx\right)\mathrm{e}^{-3ia}}{24b}+\frac{\mathrm{Ei}\left(1,-3ibx\right)\mathrm{e}^{3ia}}{24b}-\frac{3\,\mathrm{Ei}\left(1,-ibx\right)\mathrm{e}^{-ia}}{8b}-\frac{3\,\mathrm{Ei}\left(1,-ibx\right)\mathrm{e}^{ia}}{8b}-\frac{3i\,\mathrm{Si}\left(bx\right)\mathrm{e}^{-ia}}{4b}+\frac{i\,\mathrm{Si}\left(3bx\right)\mathrm{e}^{-3ia}}{12b}-\frac{3\cos(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^3,x)
```

```
[Out] -3/4/b*cos(b*x+a)*ln(x)+1/12*ln(x)/b*cos(3*b*x+3*a)-1/24*I/b*exp(-3*I*a)*Pi
*csgn(b*x)+1/12*I/b*exp(-3*I*a)*Si(3*b*x)+1/24/b*exp(-3*I*a)*Ei(1,-3*I*b*x)
+3/8*I/b*exp(-I*a)*Pi*csgn(b*x)-3/4*I/b*exp(-I*a)*Si(b*x)-3/8/b*exp(-I*a)*E
i(1,-I*b*x)-3/8/b*exp(I*a)*Ei(1,-I*b*x)+1/24/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

maxima [C] time = 1.11, size = 110, normalized size = 1.24

$$\frac{\left(\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\log(x)}{3\,b}+\frac{\left(E_1\left(3i\,bx\right)+E_1\left(-3i\,bx\right)\right)\cos\left(3\,a\right)-9\left(E_1\left(i\,bx\right)+E_1\left(-i\,bx\right)\right)\cos(a)-(i\ldots}{24\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))*log(x)/b + 1/24*((exp_integral_e(1, 3
*I*b*x) + exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(exp_integral_e(1, I*b*
x) + exp_integral_e(1, -I*b*x))*cos(a) - (I*exp_integral_e(1, 3*I*b*x) - I*
exp_integral_e(1, -3*I*b*x))*sin(3*a) - (-9*I*exp_integral_e(1, I*b*x) + 9*
I*exp_integral_e(1, -I*b*x))*sin(a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*log(x),x)
```

```
[Out] int(sin(a + b*x)^3*log(x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sin^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**3, x)
```

3.157 $\int \cos(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\sin(a)\text{Ci}(bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}+\frac{\log(x)\sin(a+bx)}{b}$$

```
[Out] -cos(a)*Si(b*x)/b-Ci(b*x)*sin(a)/b+ln(x)*sin(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2637, 2554, 12, 3303, 3299, 3302}

$$-\frac{\sin(a)\text{CosIntegral}(bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}+\frac{\log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a])/b) + (Log[x]*Sin[a + b*x])/b - (Cos[a]*SinInteg
ral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log(x)\sin(a+bx)}{b} - \int \frac{\sin(a+bx)}{bx}\,dx$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\int \frac{\sin(a+bx)}{x}\,dx}{b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\int \frac{\sin(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\cos(bx)}{x}\,dx}{b}$$
$$= -\frac{\text{Ci}(bx)\sin(a)}{b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\text{Si}(bx)}{b}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.86

$$-\frac{\sin(a)\text{Ci}(bx) + \cos(a)\text{Si}(bx) - \log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a] - Log[x]*Sin[a + b*x] + Cos[a]*SinIntegral[b*x])
/b)
```

fricas [A] time = 0.70, size = 38, normalized size = 1.09

$$\frac{2\,\log(x)\sin(bx+a) - (\text{Ci}(bx) + \text{Ci}(-bx))\sin(a) - 2\,\cos(a)\,\text{Si}(bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*log(x)*sin(b*x + a) - (cos_integral(b*x) + cos_integral(-b*x))*sin(a
) - 2*cos(a)*sin_integral(b*x))/b
```

giac [C] time = 0.19, size = 108, normalized size = 3.09

$$\frac{\log(x)\sin(bx+a)}{b} + \frac{\Im(\text{Ci}(bx))\tan\left(\frac{1}{2}a\right)^2 - \Im(\text{Ci}(-bx))\tan\left(\frac{1}{2}a\right)^2 + 2\,\text{Si}(bx)\tan\left(\frac{1}{2}a\right)^2 - 2\,\Re(\text{Ci}(bx))\tan\left(\right.}{2\left(b\tan\left(\frac{1}{2}a\right)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*(imag_part(cos_integral(b*x))*tan(1/2*a)^2 - im
ag_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*sin_integral(b*x)*tan(1/2*a)^2
 - 2*real_part(cos_integral(b*x))*tan(1/2*a) - 2*real_part(cos_integral(-b*
x))*tan(1/2*a) - imag_part(cos_integral(b*x)) + imag_part(cos_integral(-b*x
)) - 2*sin_integral(b*x))/(b*tan(1/2*a)^2 + b)
```

maple [C] time = 1.11, size = 79, normalized size = 2.26

$$\frac{i\,\text{Ei}(1,-ibx)\,e^{-ia}}{2b} - \frac{i\,\text{Ei}(1,-ibx)\,e^{ia}}{2b} - \frac{\text{Si}(bx)\,e^{-ia}}{b} + \frac{\pi\,\text{csgn}(bx)\,e^{-ia}}{2b} + \frac{\ln(x)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(x),x)
```

```
[Out] ln(x)*sin(b*x+a)/b+1/2/b*exp(-I*a)*Pi*csgn(b*x)-1/b*exp(-I*a)*Si(b*x)+1/2*I
/b*exp(-I*a)*Ei(1,-I*b*x)-1/2*I/b*exp(I*a)*Ei(1,-I*b*x)
```

maxima [C] time = 1.02, size = 55, normalized size = 1.57

$$\frac{\log(x)\sin(bx+a)}{b} + \frac{(i\,E_1(i\,bx) - i\,E_1(-i\,bx))\cos(a) + (E_1(i\,bx) + E_1(-i\,bx))\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e
(1, -I*b*x))*cos(a) + (exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x)
)*sin(a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \cos(a + b\,x)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*log(x),x)
```

```
[Out] int(cos(a + b*x)*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cos(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x), x)
```

3.158 $\int \cos^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sin(2a)\text{Ci}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cos(2*a)*Si(2*b*x)/b-1/4*Ci(2*b*x)*sin(2*a)/b+1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 3327, 3303, 3299, 3302}

$$-\frac{\sin(2a)\text{CosIntegral}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -x/2 + (x*Log[x])/2 - (CosIntegral[2*b*x]*Sin[2*a])/(4*b) + (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) - (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3327

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[ExpandToSum[
u, x]^m*(a + b*Sin[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] && L
inearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\int \frac{1}{4}\left(2+\frac{\sin(2(a+bx))}{bx}\right)dx\\ &= \frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{1}{4}\int\left(2+\frac{\sin(2(a+bx))}{bx}\right)dx\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\int\frac{\sin(2(a+bx))}{x}\,dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\int\frac{\sin(2a+2bx)}{x}\,dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\int\frac{\sin(2bx)}{x}\,dx}{4b}-\frac{\sin(2a)\int}{4}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Ci}(2bx)\sin(2a)}{4b}+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\text{Si}(2bx}{4b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.76

$$-\frac{\sin(2a)\text{Ci}(2bx)+\cos(2a)\text{Si}(2bx)-\log(x)\sin(2(a+bx))+2bx-2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a
+ b*x)] + Cos[2*a]*SinIntegral[2*b*x])/b
```

fricas [A] time = 0.55, size = 60, normalized size = 0.91

$$\frac{4\,bx\log(x)+4\,\cos(bx+a)\log(x)\sin(bx+a)-4\,bx-(\text{Ci}(2\,bx)+\text{Ci}(-2\,bx))\sin(2\,a)-2\,\cos(2\,a)\,\text{Si}(2\,bx)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) + 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x - (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) - 2*cos(2*a)*sin_integral(2*b*x
))/b
```

giac [C] time = 0.21, size = 122, normalized size = 1.85

$$\frac{1}{4}\left(2\,x+\frac{\sin(2\,bx+2\,a)}{b}\right)\log(x)-\frac{4\,bx\tan(a)^2-\Im(\text{Ci}(2\,bx))\tan(a)^2+\Im(\text{Ci}(-2\,bx))\tan(a)^2-2\,\text{Si}(2\,bx)\tan(a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 - imag_part(cos
_integral(2*b*x))*tan(a)^2 + imag_part(cos_integral(-2*b*x))*tan(a)^2 - 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x + 2*real_part(cos_integral(2*b*x))*tan(
a) + 2*real_part(cos_integral(-2*b*x))*tan(a) + imag_part(cos_integral(2*b*
x)) - imag_part(cos_integral(-2*b*x)) + 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

maple [C] time = 1.33, size = 132, normalized size = 2.00

$$\frac{x\ln(x)}{2}+\frac{a\ln(ibx)}{2b}-\frac{a\ln(a+i(ibx+ia))}{2b}+\frac{i\,\mathrm{Ei}(1,-2ibx)\,\mathrm{e}^{-2ia}}{8b}-\frac{i\,\mathrm{Ei}(1,-2ibx)\,\mathrm{e}^{2ia}}{8b}-\frac{\mathrm{Si}(2bx)\,\mathrm{e}^{-2ia}}{4b}+\frac{\pi\,\mathrm{csgn}(bx)\,\mathrm{e}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*ln(x),x)
```

```
[Out] 1/2*x*ln(x)+1/4/b*ln(x)*sin(2*b*x+2*a)+1/8*Pi/b*csgn(b*x)*exp(-2*I*a)-1/4/b
*exp(-2*I*a)*Si(2*b*x)+1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2*a/b*ln(I*b*x)
-1/2*x-1/2*a/b-1/2*a/b*ln(a+I*(I*b*x+I*a))-1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

maxima [C] time = 1.12, size = 76, normalized size = 1.15

$$\frac{(2\,bx+2\,a+\sin(2\,bx+2\,a))\log(x)}{4\,b}-\frac{4\,bx+(-i\,\mathrm{Ei}(2i\,bx)+i\,\mathrm{Ei}(-2i\,bx))\cos(2\,a)+4\,a\log(x)+(\mathrm{Ei}(2i\,bx)+}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (-I*Ei(2*I*b*x
) + I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) + (Ei(2*I*b*x) + Ei(-2*I*b*x))*si
n(2*a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(a+b\,x)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*log(x),x)
```

```
[Out] int(cos(a + b*x)^2*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**2, x)
```

3.159 $\int \cos^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\sin(a)\text{Ci}(bx)}{4b}-\frac{\sin(3a)\text{Ci}(3bx)}{12b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

```
[Out] -3/4*cos(a)*Si(b*x)/b-1/12*cos(3*a)*Si(3*b*x)/b-3/4*Ci(b*x)*sin(a)/b-1/12*C
i(3*b*x)*sin(3*a)/b+ln(x)*sin(b*x+a)/b-1/3*ln(x)*sin(b*x+a)^3/b
```

Rubi [A] time = 0.47, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2633, 2554, 12, 6742, 3303, 3299, 3302, 4430}

$$-\frac{3\sin(a)\text{CosIntegral}(bx)}{4b}-\frac{\sin(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CosIntegral[b*x]*Sin[a])/(4*b) - (CosIntegral[3*b*x]*Sin[3*a])/(12*b) +
 (Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - (3*Cos[a]*SinInt
egral[b*x])/(4*b) - (Cos[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4430

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IGtQ[q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\log(x)\,dx &= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{6bx}\,dx \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{x}\,dx}{6b} \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(\frac{5\sin(a+bx)}{x} + \frac{\cos(2a+2bx)\sin(a+bx)}{x}\right)dx}{6b} \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{\cos(2a+2bx)\sin(a+bx)}{x}\,dx}{6b} - \frac{5\int \frac{\sin(a+bx)}{x}}{6b} \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(-\frac{\sin(a+bx)}{2x} + \frac{\sin(3a+3bx)}{2x}\right)dx}{6b} - (5\cos \\
&= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} + \\
&= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} + \\
&= -\frac{3\mathrm{Ci}(bx)\sin(a)}{4b} - \frac{\mathrm{Ci}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} -
\end{aligned}$$

Mathematica [A] time = 0.16, size = 66, normalized size = 0.75

$$-\frac{9\sin(a)\mathrm{Ci}(bx) + \sin(3a)\mathrm{Ci}(3bx) + 9\cos(a)\mathrm{Si}(bx) + \cos(3a)\mathrm{Si}(3bx) - 9\log(x)\sin(a+bx) - \log(x)\sin(3(a+}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] -1/12*(9*CosIntegral[b*x]*Sin[a] + CosIntegral[3*b*x]*Sin[3*a] - 9*Log[x]*S
in[a + b*x] - Log[x]*Sin[3*(a + b*x)] + 9*Cos[a]*SinIntegral[b*x] + Cos[3*a
]*SinIntegral[3*b*x])/b
```

fricas [A] time = 1.02, size = 76, normalized size = 0.86

$$\frac{8\left(\cos(bx+a)^2+2\right)\log(x)\sin(bx+a) - (\mathrm{Ci}(3bx)+\mathrm{Ci}(-3bx))\sin(3a) - 9(\mathrm{Ci}(bx)+\mathrm{Ci}(-bx))\sin(a) - 2}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(cos(b*x + a)^2 + 2)*log(x)*sin(b*x + a) - (cos_integral(3*b*x) + c
os_integral(-3*b*x))*sin(3*a) - 9*(cos_integral(b*x) + cos_integral(-b*x))*
sin(a) - 2*cos(3*a)*sin_integral(3*b*x) - 18*cos(a)*sin_integral(b*x))/b
```

giac [C] time = 0.25, size = 495, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*(imag_part(cos_integ
ral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(
3/2*a)^2*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/
2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 2*sin_
integral(3*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 18*sin_integral(b*x)*tan(3/2*a)
^2*tan(1/2*a)^2 - 18*real_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) -
 18*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 2*real_part(cos
_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*real_part(cos_integral(-3*b*x
))*tan(3/2*a)*tan(1/2*a)^2 + imag_part(cos_integral(3*b*x))*tan(3/2*a)^2 -
9*imag_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*imag_part(cos_integral(-b*x
))*tan(3/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2 + 2*sin_inte
gral(3*b*x)*tan(3/2*a)^2 - 18*sin_integral(b*x)*tan(3/2*a)^2 - imag_part(co
s_integral(3*b*x))*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(1/2*a)
^2 - 9*imag_part(cos_integral(-b*x))*tan(1/2*a)^2 + imag_part(cos_integral(
-3*b*x))*tan(1/2*a)^2 - 2*sin_integral(3*b*x)*tan(1/2*a)^2 + 18*sin_integra
l(b*x)*tan(1/2*a)^2 - 2*real_part(cos_integral(3*b*x))*tan(3/2*a) - 2*real_
part(cos_integral(-3*b*x))*tan(3/2*a) - 18*real_part(cos_integral(b*x))*tan
(1/2*a) - 18*real_part(cos_integral(-b*x))*tan(1/2*a) - imag_part(cos_integ
ral(3*b*x)) - 9*imag_part(cos_integral(b*x)) + 9*imag_part(cos_integral(-b*
x)) + imag_part(cos_integral(-3*b*x)) - 2*sin_integral(3*b*x) - 18*sin_inte
gral(b*x))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*a)^2 + b*tan(1/2*a)^2 +
 b)
```

maple [C] time = 1.73, size = 162, normalized size = 1.84

$$\frac{i\,\mathrm{Ei}\left(1,-3ibx\right)\mathrm{e}^{-3ia}}{24b}-\frac{i\,\mathrm{Ei}\left(1,-3ibx\right)\mathrm{e}^{3ia}}{24b}+\frac{3i\,\mathrm{Ei}\left(1,-ibx\right)\mathrm{e}^{-ia}}{8b}-\frac{3i\,\mathrm{Ei}\left(1,-ibx\right)\mathrm{e}^{ia}}{8b}-\frac{3\,\mathrm{Si}\left(bx\right)\mathrm{e}^{-ia}}{4b}-\frac{\mathrm{Si}\left(3bx\right)\mathrm{e}^{-3ia}}{12b}+\frac{\pi\,\mathrm{csg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*ln(x),x)
```

```
[Out] 3/4/b*ln(x)*sin(b*x+a)+1/12*ln(x)/b*sin(3*b*x+3*a)+1/24/b*exp(-3*I*a)*Pi*cs
gn(b*x)-1/12/b*exp(-3*I*a)*Si(3*b*x)+1/24*I/b*exp(-3*I*a)*Ei(1,-3*I*b*x)+3/
8*Pi/b*csgn(b*x)*exp(-I*a)-3/4/b*exp(-I*a)*Si(b*x)+3/8*I/b*exp(-I*a)*Ei(1,-
I*b*x)-3/8*I/b*exp(I*a)*Ei(1,-I*b*x)-1/24*I/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

maxima [C] time = 1.03, size = 108, normalized size = 1.23

$$-\frac{\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)\log\left(x\right)}{3\,b}+\frac{\left(i\,E_{1}\left(3i\,bx\right)-i\,E_{1}\left(-3i\,bx\right)\right)\cos\left(3\,a\right)+\left(9i\,E_{1}\left(i\,bx\right)-9i\,E_{1}\left(-i\,bx\right)\right)\cos(a}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*((I*exp_integral_e(1
, 3*I*b*x) - I*exp_integral_e(1, -3*I*b*x))*cos(3*a) + (9*I*exp_integral_e(
1, I*b*x) - 9*I*exp_integral_e(1, -I*b*x))*cos(a) + (exp_integral_e(1, 3*I*
b*x) + exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(exp_integral_e(1, I*b*x)
+ exp_integral_e(1, -I*b*x))*sin(a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*log(x),x)
```

```
[Out] int(cos(a + b*x)^3*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(x\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**3, x)
```

3.160 $\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx$

Optimal. Leaf size=5

$$\log(x)\sin(x)$$

```
[Out] ln(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2637, 2554, 3299}

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx &= \int \cos(x)\log(x)\,dx + \int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x) + \text{Si}(x) - \int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

fricas [A] time = 0.63, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
[Out] log(x)*sin(x)
```

giac [A] time = 0.17, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
[Out] log(x)*sin(x)
```

maple [B] time = 0.93, size = 19, normalized size = 3.80

$$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(x)+sin(x)/x,x)
[Out] 2*ln(x)*tan(1/2*x)/(1+tan(1/2*x)^2)
```

maxima [A] time = 0.93, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
[Out] log(x)*sin(x)
```

mupad [B] time = 0.50, size = 5, normalized size = 1.00

$$\ln(x)\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*log(x) + sin(x)/x,x)
[Out] log(x)*sin(x)
```

sympy [A] time = 13.69, size = 5, normalized size = 1.00

$$\log{\left(x \right)}\sin{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(x)+sin(x)/x,x)
[Out] log(x)*sin(x)
```

3.161 $\int \log(a\sin(x))\,dx$

Optimal. Leaf size=47

$$x\log(a\sin(x))+\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)$$

[Out] 1/2*I*x^2-x*ln(1-exp(2*I*x))+x*ln(a*sin(x))+1/2*I*polylog(2,exp(2*I*x))

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3717, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\sin(x))+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Sin[x]],x]

[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]] + (I/2)*PolyLog[2, E^((2*I)*x)]

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log(a\sin(x))\,dx = x\log(a\sin(x)) - \int x\cot(x)\,dx$$
$$= \frac{ix^2}{2} + x\log(a\sin(x)) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \int \log\left(1-e^{2ix}\right)\,dx$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.89

$$x\log(a\sin(x)) + \frac{1}{2}i\left(x^2 + \text{Li}_2\left(e^{2ix}\right)\right) - x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]],x]
```

```
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[a*Sin[x]] + (I/2)*(x^2 + PolyLog[2, E^((2
*I)*x)])
```

fricas [B] time = 0.78, size = 104, normalized size = 2.21

$$x\log\left(a\sin(x)\right) - \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right) - \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right) - \frac{1}{2}x\log\left(-\cos(x) + i\sin(x) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x) - I*s
in(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I*sin(
x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(x)) -
 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)), x)
```

maple [B] time = 0.41, size = 76, normalized size = 1.62

$$-i\ln\left(2a\sin(x)\right)\ln\left(e^{ix}\right) + i\ln\left(e^{ix}+1\right)\ln\left(e^{ix}\right) - \frac{i\ln\left(e^{ix}\right)^2}{2} + i\,\text{dilog}\left(e^{ix}+1\right) - i\,\text{dilog}\left(e^{ix}\right) + i\ln(2)\ln\left(e^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)),x)
```

```
[Out] -I*ln(exp(I*x))*ln(2*a*sin(x))-I*dilog(exp(I*x))+I*ln(exp(I*x))*ln(1+exp(I*
x))+I*dilog(1+exp(I*x))-1/2*I*ln(exp(I*x))^2+I*ln(2)*ln(exp(I*x))
```

maxima [B] time = 2.24, size = 87, normalized size = 1.85

$$\frac{1}{2}i\,x^2-i\,x\arctan\left(\sin(x),\cos(x)+1\right)+i\,x\arctan\left(\sin(x),-\cos(x)+1\right)-\frac{1}{2}\,x\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(x), cos(x) + 1) + I*x*arctan2(sin(x), -cos(x) +
 1) - 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) + x*log(a*sin(x)) + I*dilog(-e^(I*x)) + I*dilog(e^
(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\ \sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)),x)
```

```
[Out] int(log(a*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)),x)
```

```
[Out] Integral(log(a*sin(x)), x)
```

3.162 $\int \log\left(a\sin^2(x)\right)\,dx$

Optimal. Leaf size=45

$$x\log\left(a\sin^2(x)\right) + i\text{Li}_2\left(e^{2ix}\right) + ix^2 - 2x\log\left(1 - e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^2)+I*polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$i\text{PolyLog}\left(2, e^{2ix}\right) + x\log\left(a\sin^2(x)\right) + ix^2 - 2x\log\left(1 - e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^2] + I*PolyLog[2, E^((2*I
)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\sin^2(x)\right)dx &= x\log\left(a\sin^2(x)\right)-\int 2x\cot(x)\,dx\\ &= x\log\left(a\sin^2(x)\right)-2\int x\cot(x)\,dx\\ &= ix^2+x\log\left(a\sin^2(x)\right)+4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+2\int\log\left(1-e^{2ix}\right)dx\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)-i\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.96

$$x\left(\log\left(a\sin^2(x)\right)+ix-2\log\left(1-e^{2ix}\right)\right)+i\text{Li}_2\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 - E^((2*I)*x)] + Log[a*Sin[x]^2]) + I*PolyLog[2, E^((2*I)*
x)]
```

fricas [B] time = 0.90, size = 109, normalized size = 2.42

$$x\log\left(-a\cos(x)^2+a\right)-x\log\left(\cos(x)+i\sin(x)+1\right)-x\log\left(\cos(x)-i\sin(x)+1\right)-x\log\left(-\cos(x)+i\sin(x)+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a*cos(x)^2 + a) - x*log(cos(x) + I*sin(x) + 1) - x*log(cos(x) - I*si
n(x) + 1) - x*log(-cos(x) + I*sin(x) + 1) - x*log(-cos(x) - I*sin(x) + 1) +
 I*dilog(cos(x) + I*sin(x)) - I*dilog(cos(x) - I*sin(x)) - I*dilog(-cos(x)
+ I*sin(x)) + I*dilog(-cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sin(x)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^2), x)
```

maple [B] time = 0.45, size = 88, normalized size = 1.96

$$-i\ln\left(-\left(e^{2ix}-1\right)^2a\,e^{-2ix}\right)\ln\left(e^{ix}\right)+2i\ln\left(e^{ix}+1\right)\ln\left(e^{ix}\right)-i\ln\left(e^{ix}\right)^2+2i\,\text{dilog}\left(e^{ix}+1\right)-2i\,\text{dilog}\left(e^{ix}\right)+2i\ln(2)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^2),x)
```

```
[Out] 2*I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(-a*(exp(2*I*x)-1)
^2*exp(-2*I*x))+2*I*ln(exp(I*x))*ln(exp(I*x)+1)-2*I*dilog(exp(I*x))+2*I*dil
og(exp(I*x)+1)
```

maxima [B] time = 2.32, size = 89, normalized size = 1.98

$$i\,x^2 - 2i\,x\arctan\left(\sin(x), \cos(x) + 1\right) + 2i\,x\arctan\left(\sin(x), -\cos(x) + 1\right) + x\log\left(a\sin(x)^2\right) - x\log\left(\cos(x)^2 + \sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) +
 1) + x*log(a*sin(x)^2) - x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I
*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sin(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)^2),x)
```

```
[Out] int(log(a*sin(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**2),x)
```

```
[Out] Integral(log(a*sin(x)**2), x)
```

3.163 $\int \log(a\sin^n(x))\,dx$

Optimal. Leaf size=52

$$x\log(a\sin^n(x))+\frac{1}{2}in\mathrm{Li}_2\left(e^{2ix}\right)+\frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^n)+1/2*I*n*polylog(2,exp(2*I
*x))
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$\frac{1}{2}in\mathrm{PolyLog}\left(2,e^{2ix}\right)+x\log(a\sin^n(x))+\frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sin^n(x)\right)\,dx &= x\log\left(a\sin^n(x)\right) - \int nx\cot(x)\,dx \\
&= x\log\left(a\sin^n(x)\right) - n\int x\cot(x)\,dx \\
&= \frac{1}{2}inx^2 + x\log\left(a\sin^n(x)\right) + (2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + n\int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) - \frac{1}{2}(in)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 1.00

$$x\log\left(a\sin^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right) + \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

fricas [B] time = 1.03, size = 115, normalized size = 2.21

$$-\frac{1}{2}\,nx\log\left(\cos(x) + i\,\sin(x) + 1\right) - \frac{1}{2}\,nx\log\left(\cos(x) - i\,\sin(x) + 1\right) - \frac{1}{2}\,nx\log\left(-\cos(x) + i\,\sin(x) + 1\right) - \frac{1}{2}\,nx\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(cos(x) + I*sin(x) + 1) - 1/2*n*x*log(cos(x) - I*sin(x) + 1) -
1/2*n*x*log(-cos(x) + I*sin(x) + 1) - 1/2*n*x*log(-cos(x) - I*sin(x) + 1) +
 n*x*log(sin(x)) + 1/2*I*n*dilog(cos(x) + I*sin(x)) - 1/2*I*n*dilog(cos(x)
- I*sin(x)) - 1/2*I*n*dilog(-cos(x) + I*sin(x)) + 1/2*I*n*dilog(-cos(x) - I
*sin(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^n), x)
```

maple [F] time = 2.08, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\sin^n(x)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^n),x)
```

```
[Out] int(ln(a*sin(x)^n),x)
```

maxima [B] time = 2.40, size = 91, normalized size = 1.75

$$-\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin(x),\cos(x)+1\right)-2i\,x\arctan\left(\sin(x),-\cos(x)+1\right)+x\log\left(\cos(x)^2+\sin(x)^2+2\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*l
og(a*sin(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sin(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)^n),x)
```

```
[Out] int(log(a*sin(x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin^n(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**n),x)
```

```
[Out] Integral(log(a*sin(x)**n), x)
```

3.164 $\int \log(a\cos(x))\,dx$

Optimal. Leaf size=47

$$x\log(a\cos(x))+\frac{1}{2}i\mathrm{Li}_2\left(-e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-x*ln(1+exp(2*I*x))+x*ln(a*cos(x))+1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3719, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\cos(x))+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\cos(x))\,dx &= x\log(a\cos(x)) + \int x\tan(x)\,dx\\ &= \frac{ix^2}{2} + x\log(a\cos(x)) - 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\ &= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \int \log\left(1+e^{2ix}\right)\,dx\\ &= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\ &= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$x\log(a\cos(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

fricas [B] time = 0.53, size = 104, normalized size = 2.21

$$x\log\left(a\cos(x)\right) - \frac{1}{2}x\log\left(i\cos(x)+\sin(x)+1\right) - \frac{1}{2}x\log\left(i\cos(x)-\sin(x)+1\right) - \frac{1}{2}x\log\left(-i\cos(x)+\sin(x)+1\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)) - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(
x) + 1) - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)) +
 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cos(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)), x)
```

maple [B] time = 0.33, size = 107, normalized size = 2.28

$$-i\ln\left(2a\cos(x)\right)\ln\left(e^{ix}\right) + i\ln\left(-ie^{ix}+1\right)\ln\left(e^{ix}\right) + i\ln\left(ie^{ix}+1\right)\ln\left(e^{ix}\right) - \frac{i\ln\left(e^{ix}\right)^2}{2} + i\,\text{dilog}\left(-ie^{ix}+1\right) + i\,\text{dilog}\left(ie\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)),x)
```

```
[Out] -I*ln(exp(I*x))*ln(2*a*cos(x))+I*ln(exp(I*x))*ln(1+I*exp(I*x))+I*ln(exp(I*x
))*ln(1-I*exp(I*x))+I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp(I*x))-1/2*I*ln(ex
p(I*x))^2+I*ln(2)*ln(exp(I*x))
```

maxima [A] time = 2.22, size = 60, normalized size = 1.28

$$\frac{1}{2} i x^2 - i x \arctan(\sin(2x), \cos(2x) + 1) - \frac{1}{2} x \log\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right) + x \log(a\cos(x)) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) - 1/2*x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*cos(x)) + 1/2*I*dilog(-e^(2*I*x))
```

mupad [B] time = 0.39, size = 37, normalized size = 0.79

$$x \ln(a \cos(x)) + \frac{\operatorname{polylog}\left(2, -e^{x\,2i}\right)\, 1i}{2} + \frac{x\left(x + \ln\left(e^{x\,2i} + 1\right) 2i\right) 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)),x)
```

```
[Out] (polylog(2, -exp(x*2i))*1i)/2 + (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 + x*lo
g(a*cos(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\cos(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)),x)
```

```
[Out] Integral(log(a*cos(x)), x)
```

3.165 $\int \log\left(a\cos^2(x)\right)\,dx$

Optimal. Leaf size=45

$$x\log\left(a\cos^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)+ix^2-2x\log\left(1+e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^2)+I*polylog(2,-exp(2*I*x))
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^2(x)\right)+ix^2-2x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^2] + I*PolyLog[2, -E^((2*
I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\cos^2(x)\right)dx &= x\log\left(a\cos^2(x)\right) - \int -2x\tan(x)\,dx\\
&= x\log\left(a\cos^2(x)\right) + 2\int x\tan(x)\,dx\\
&= ix^2 + x\log\left(a\cos^2(x)\right) - 4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + 2\int \log\left(1+e^{2ix}\right)dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) - i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.96

$$
x\left(\log\left(a\cos^2(x)\right) + ix - 2\log\left(1+e^{2ix}\right)\right) + i\text{Li}_2\left(-e^{2ix}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 + E^((2*I)*x)] + Log[a*Cos[x]^2]) + I*PolyLog[2, -E^((2*I)
*x)]
```

fricas [B] time = 0.53, size = 106, normalized size = 2.36

$$
x\log\left(a\cos(x)^2\right) - x\log\left(i\cos(x)+\sin(x)+1\right) - x\log\left(i\cos(x)-\sin(x)+1\right) - x\log\left(-i\cos(x)+\sin(x)+1\right) - x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)^2) - x*log(I*cos(x) + sin(x) + 1) - x*log(I*cos(x) - sin(x)
+ 1) - x*log(-I*cos(x) + sin(x) + 1) - x*log(-I*cos(x) - sin(x) + 1) - I*di
log(I*cos(x) + sin(x)) + I*dilog(I*cos(x) - sin(x)) + I*dilog(-I*cos(x) + s
in(x)) - I*dilog(-I*cos(x) - sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log\left(a\cos(x)^2\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^2), x)
```

maple [B] time = 0.44, size = 118, normalized size = 2.62

$$
-i\ln\left(\left(e^{2ix}+1\right)^2 a\,e^{-2ix}\right)\ln\left(e^{ix}\right) + 2i\ln\left(-ie^{ix}+1\right)\ln\left(e^{ix}\right) + 2i\ln\left(ie^{ix}+1\right)\ln\left(e^{ix}\right) - i\ln\left(e^{ix}\right)^2 + 2i\,\text{dilog}\left(-ie^{ix}+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^2),x)
```

```
[Out] 2*I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(a*(1+exp(2*I*x))^
2*exp(-2*I*x))+2*I*ln(exp(I*x))*ln(I*exp(I*x)+1)+2*I*ln(exp(I*x))*ln(-I*exp
(I*x)+1)+2*I*dilog(-I*exp(I*x)+1)+2*I*dilog(I*exp(I*x)+1)
```

maxima [A] time = 2.55, size = 62, normalized size = 1.38

$$i x^2 - 2 i x \arctan(\sin(2x), \cos(2x) + 1) + x \log\left(a \cos(x)^2\right) - x \log\left(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1\right) + i \operatorname{Li}_2($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*cos(x)^2) - x*log(c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + I*dilog(-e^(2*I*x))
```

mupad [B] time = 0.08, size = 39, normalized size = 0.87

$$x \ln\left(a \cos(x)^2\right) + \operatorname{polylog}\left(2, -e^{x\,2i}\right) 1i + x \left(x + \ln\left(e^{x\,2i} + 1\right) 2i\right) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)^2),x)
```

```
[Out] polylog(2, -exp(x*2i))*1i + x*(x + log(exp(x*2i) + 1)*2i)*1i + x*log(a*cos(
x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \cos^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**2),x)
```

```
[Out] Integral(log(a*cos(x)**2), x)
```

3.166 $\int \log \left(a\cos^n(x)\right)\,dx$

Optimal. Leaf size=52

$$x\log\left(a\cos^n(x)\right)+\frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right)+\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^n)+1/2*I*n*polylog(2,-exp(2*
I*x))
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$\frac{1}{2}in\mathrm{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log \left(a \cos^n(x)\right) dx &= x \log \left(a \cos^n(x)\right) + \int n x \tan(x)\, dx \\ &= x \log \left(a \cos^n(x)\right) + n \int x \tan(x)\, dx \\ &= \frac{1}{2} i n x^2 + x \log \left(a \cos^n(x)\right) - (2 i n) \int \frac{e^{2ix} x}{1+e^{2ix}}\, dx \\ &= \frac{1}{2} i n x^2 - n x \log \left(1+e^{2ix}\right) + x \log \left(a \cos^n(x)\right) + n \int \log \left(1+e^{2ix}\right) dx \\ &= \frac{1}{2} i n x^2 - n x \log \left(1+e^{2ix}\right) + x \log \left(a \cos^n(x)\right) - \frac{1}{2}(i n) \operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\, dx, x, e^{2ix}\right) \\ &= \frac{1}{2} i n x^2 - n x \log \left(1+e^{2ix}\right) + x \log \left(a \cos^n(x)\right) + \frac{1}{2} i n \operatorname{Li}_2\left(-e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$x \log \left(a \cos^n(x)\right) + \frac{1}{2} i n \operatorname{Li}_2\left(-e^{2ix}\right) + \frac{1}{2} i n x^2 - n x \log \left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^n],x]
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

fricas [B] time = 0.52, size = 115, normalized size = 2.21

$$-\frac{1}{2} n x \log \left(i \cos(x) + \sin(x) + 1\right) - \frac{1}{2} n x \log \left(i \cos(x) - \sin(x) + 1\right) - \frac{1}{2} n x \log \left(-i \cos(x) + \sin(x) + 1\right) - \frac{1}{2} n x \log (-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="fricas")
[Out] -1/2*n*x*log(I*cos(x) + sin(x) + 1) - 1/2*n*x*log(I*cos(x) - sin(x) + 1) -
1/2*n*x*log(-I*cos(x) + sin(x) + 1) - 1/2*n*x*log(-I*cos(x) - sin(x) + 1) +
 n*x*log(cos(x)) - 1/2*I*n*dilog(I*cos(x) + sin(x)) + 1/2*I*n*dilog(I*cos(x
) - sin(x)) + 1/2*I*n*dilog(-I*cos(x) + sin(x)) - 1/2*I*n*dilog(-I*cos(x) -
 sin(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(a \cos(x)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="giac")
[Out] integrate(log(a*cos(x)^n), x)
```

maple [F] time = 1.61, size = 0, normalized size = 0.00

$$\int \ln \left(a \left(\cos^n(x)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^n),x)
[Out] int(ln(a*cos(x)^n),x)
```

maxima [A] time = 3.00, size = 65, normalized size = 1.25

$$-\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)-i\,\mathrm{Li}_2\left(-e^{(2i\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*cos(x)^n)
```

mupad [B] time = 0.37, size = 41, normalized size = 0.79

$$x\,\ln\left(a\cos(x)^n\right)+\frac{n\,\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+\frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)^n),x)
```

```
[Out] x*log(a*cos(x)^n) + (n*polylog(2, -exp(x*2i))*1i)/2 + (n*x*(x + log(exp(x*2
i) + 1)*2i)*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cos^n\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**n),x)
```

```
[Out] Integral(log(a*cos(x)**n), x)
```

3.167 $\int \log(a\tan(x))\,dx$

Optimal. Leaf size=51

$$x\log(a\tan(x))-\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)+\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)+2x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 2*x*arctanh(exp(2*I*x))+x*ln(a*tan(x))-1/2*I*polylog(2,-exp(2*I*x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 4419, 4183, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\tan(x))+2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]],x]
```

```
[Out] 2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]] - (I/2)*PolyLog[2, -E^((2*I)*x)]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\tan(x))\,dx &= x\log(a\tan(x)) - \int x\csc(x)\sec(x)\,dx \\
&= x\log(a\tan(x)) - 2\int x\csc(2x)\,dx \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) + \int \log\left(1-e^{2ix}\right)\,dx - \int \log\left(1+e^{2ix}\right)\,dx \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{\right. \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.47

$$
-\frac{1}{2}i\log(-i(-\tan(x)+i))\log(a\tan(x))+\frac{1}{2}i\log(-i(\tan(x)+i))\log(a\tan(x))-\frac{1}{2}i\text{Li}_2(-i\tan(x))+\frac{1}{2}i\text{Li}_2(i\tan(x))
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]] + (I/2)*Log[a*Tan[x]]*Log[(-I
)*(I + Tan[x])] - (I/2)*PolyLog[2, (-I)*Tan[x]] + (I/2)*PolyLog[2, I*Tan[x]
]
```

fricas [B] time = 0.46, size = 184, normalized size = 3.61

$$
x\log\left(a\tan(x)\right)-\frac{1}{2}\,x\log\left(\frac{2\left(\tan(x)^2+i\,\tan(x)\right)}{\tan(x)^2+1}\right)-\frac{1}{2}\,x\log\left(\frac{2\left(\tan(x)^2-i\,\tan(x)\right)}{\tan(x)^2+1}\right)+\frac{1}{2}\,x\log\left(-\frac{2\left(i\,\tan(x)-1\right)}{\tan(x)^2+1}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)) - 1/2*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*x
*log(2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*x*log(-2*(I*tan(x) - 1)/
(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/4*I*dilo
g(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(-2*(tan(x)^2 -
 I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1
) + 1) - 1/4*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log\left(a\tan(x)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)), x)
```

maple [B] time = 0.17, size = 82, normalized size = 1.61

$$
-\frac{i\ln\left(a\tan(x)\right)\ln\left(\frac{ia\tan(x)+a}{a}\right)}{2}+\frac{i\ln\left(a\tan(x)\right)\ln\left(-\frac{ia\tan(x)-a}{a}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{ia\tan(x)+a}{a}\right)}{2}+\frac{i\,\text{dilog}\left(-\frac{ia\tan(x)-a}{a}\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)),x)
```

```
[Out] -1/2*I*ln(a*tan(x))*ln((I*a*tan(x)+a)/a)+1/2*I*ln(a*tan(x))*ln(-(I*a*tan(x)
-a)/a)-1/2*I*dilog((I*a*tan(x)+a)/a)+1/2*I*dilog(-(I*a*tan(x)-a)/a)
```

maxima [A] time = 1.62, size = 42, normalized size = 0.82

$$x \log\left(a \tan(x)\right) + \frac{1}{4} \pi \log\left(\tan(x)^2 + 1\right) - x \log\left(\tan(x)\right) + \frac{1}{2} i \operatorname{Li}_2\left(i \tan(x) + 1\right) - \frac{1}{2} i \operatorname{Li}_2\left(-i \tan(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)) + 1/4*pi*log(tan(x)^2 + 1) - x*log(tan(x)) + 1/2*I*dilog(I*
tan(x) + 1) - 1/2*I*dilog(-I*tan(x) + 1)
```

mupad [B] time = 0.09, size = 39, normalized size = 0.76

$$2 x \operatorname{atanh}\left(e^{x 2i}\right) + x \ln\left(a \tan(x)\right) - \frac{\operatorname{polylog}\left(2, -e^{x 2i}\right) 1i}{2} + \frac{\operatorname{polylog}\left(2, e^{x 2i}\right) 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)),x)
```

```
[Out] 2*x*atanh(exp(x*2i)) - (polylog(2, -exp(x*2i))*1i)/2 + (polylog(2, exp(x*2i
))*1i)/2 + x*log(a*tan(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \tan\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)),x)
```

```
[Out] Integral(log(a*tan(x)), x)
```

3.168 $\int \log\left(a\tan^2(x)\right)\,dx$

Optimal. Leaf size=49

$$x\log\left(a\tan^2(x)\right)-i\text{Li}_2\left(-e^{2ix}\right)+i\text{Li}_2\left(e^{2ix}\right)+4x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 4*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^2)-I*polylog(2,-exp(2*I*x))+I*polylog
(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^2(x)\right)+4x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^2] - I*PolyLog[2, -E^((2*I)*x)] +
 I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int \log\left(a\tan^2(x)\right)\,dx = x\log\left(a\tan^2(x)\right) - \int 2x\csc(x)\sec(x)\,dx$$
$$= x\log\left(a\tan^2(x)\right) - 2\int x\csc(x)\sec(x)\,dx$$
$$= x\log\left(a\tan^2(x)\right) - 4\int x\csc(2x)\,dx$$
$$= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) + 2\int \log\left(1-e^{2ix}\right)\,dx - 2\int \log\left(1+e^{2ix}\right)\,dx$$
$$= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + i\,\text{Subst}\left(\int \frac{\log}{}\right.$$
$$= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right) + i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.53

$$-\frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\tan^2(x)\right)+\frac{1}{2}i\log(-i(\tan(x)+i))\log\left(a\tan^2(x)\right)-i\text{Li}_2(-i\tan(x))+i\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^2],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^2] + (I/2)*Log[a*Tan[x]^2]*Log
[(-I)*(I + Tan[x])] - I*PolyLog[2, (-I)*Tan[x]] + I*PolyLog[2, I*Tan[x]]
```

fricas [B] time = 0.48, size = 184, normalized size = 3.76

$$x\log\left(a\tan(x)^2\right)-x\log\left(\frac{2\left(\tan(x)^2+i\,\tan(x)\right)}{\tan(x)^2+1}\right)-x\log\left(\frac{2\left(\tan(x)^2-i\,\tan(x)\right)}{\tan(x)^2+1}\right)+x\log\left(-\frac{2\,(i\,\tan(x)-1)}{\tan(x)^2+1}\right)+x\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)^2) - x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - x*log(2
*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + x*log(-2*(I*tan(x) - 1)/(tan(x)^2
+ 1)) + x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*I*dilog(-2*(tan(x)^2
 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(-2*(tan(x)^2 - I*tan(x))/(ta
n(x)^2 + 1) + 1) + 1/2*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/2*I
*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\tan(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^2), x)
```

maple [A] time = 0.57, size = 82, normalized size = 1.67

$$i\ln(-i\tan(x))\ln(\tan(x)-i)-i\ln(i\tan(x))\ln(\tan(x)+i)-\frac{i\ln\left(a\left(\tan^2(x)\right)\right)\ln(\tan(x)-i)}{2}+\frac{i\ln\left(a\left(\tan^2(x)\right)\right)\ln(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)^2),x)
```

```
[Out] I*ln(tan(x)-I)*ln(-I*tan(x))-1/2*I*ln(tan(x)-I)*ln(a*tan(x)^2)+I*dilog(-I*t
an(x))+1/2*I*ln(a*tan(x)^2)*ln(tan(x)+I)-I*ln(tan(x)+I)*ln(I*tan(x))-I*dilo
g(I*tan(x))
```

maxima [A] time = 1.51, size = 44, normalized size = 0.90

$$x \log\left(a \tan(x)^2\right) + \frac{1}{2}\pi \log\left(\tan(x)^2 + 1\right) - 2x \log\left(\tan(x)\right) + i \operatorname{Li}_2\left(i \tan(x) + 1\right) - i \operatorname{Li}_2\left(-i \tan(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)^2) + 1/2*pi*log(tan(x)^2 + 1) - 2*x*log(tan(x)) + I*dilog(I*
tan(x) + 1) - I*dilog(-I*tan(x) + 1)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.84

$$x \ln\left(a \tan(x)^2\right) - \operatorname{polylog}\left(2, -e^{x\,2i}\right) 1i + 4\,x \operatorname{atanh}\left(e^{x\,2i}\right) + \operatorname{polylog}\left(2, e^{x\,2i}\right) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)^2),x)
```

```
[Out] x*log(a*tan(x)^2) - polylog(2, -exp(x*2i))*1i + 4*x*atanh(exp(x*2i)) + poly
log(2, exp(x*2i))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \tan^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**2),x)
```

```
[Out] Integral(log(a*tan(x)**2), x)
```

3.169 $\int \log(a\tan^n(x))\,dx$

Optimal. Leaf size=56

$$x\log(a\tan^n(x)) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right) + 2nx\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^n)-1/2*I*n*polylog(2,-exp(2*I*x))+1
/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right) + \frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right) + x\log(a\tan^n(x)) + 2nx\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^n] - (I/2)*n*PolyLog[2, -E^((2*
I)*x)] + (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tan^n(x)\right)\,dx &= x\log\left(a\tan^n(x)\right) - \int nx\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^n(x)\right) - n\int x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^n(x)\right) - (2n)\int x\csc(2x)\,dx\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) + n\int \log\left(1-e^{2ix}\right)dx - n\int\log\left(1+e^{2ix}\right)dx\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}(in)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}dx, x, e^{2ix}\right) + \frac{1}{2}(in)\,\text{S}\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.45

$$-\frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\tan^n(x)\right)+\frac{1}{2}i\log(-i(\tan(x)+i))\log\left(a\tan^n(x)\right)-\frac{1}{2}in\text{Li}_2(-i\tan(x))+\frac{1}{2}in\text{Li}_2(i\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^n],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^n] + (I/2)*Log[a*Tan[x]^n]*Log
[(-I)*(I + Tan[x])] - (I/2)*n*PolyLog[2, (-I)*Tan[x]] + (I/2)*n*PolyLog[2,
I*Tan[x]]
```

fricas [B] time = 0.52, size = 195, normalized size = 3.48

$$-\frac{1}{2}nx\log\left(\frac{2\left(\tan(x)^2+i\tan(x)\right)}{\tan(x)^2+1}\right)-\frac{1}{2}nx\log\left(\frac{2\left(\tan(x)^2-i\tan(x)\right)}{\tan(x)^2+1}\right)+\frac{1}{2}nx\log\left(-\frac{2\left(i\tan(x)-1\right)}{\tan(x)^2+1}\right)+\frac{1}{2}nx\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*n*x*log(2*(tan(x
)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*n*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 +
 1)) + 1/2*n*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + n*x*log(tan(x)) - 1
/4*I*n*dilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(-
2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(2*(I*tan(x) - 1
)/(tan(x)^2 + 1) + 1) - 1/4*I*n*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
 + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\tan(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\tan^n(x)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)^n),x)
```

```
[Out] int(ln(a*tan(x)^n),x)
```

maxima [A] time = 1.51, size = 48, normalized size = 0.86

$$-nx\log\left(\tan(x)\right)+\frac{1}{4}\left(\pi\log\left(\tan(x)^2+1\right)+2i\operatorname{Li}_2\left(i\tan(x)+1\right)-2i\operatorname{Li}_2\left(-i\tan(x)+1\right)\right)n+x\log\left(a\tan(x)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="maxima")
```

```
[Out] -n*x*log(tan(x)) + 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*
I*dilog(-I*tan(x) + 1))*n + x*log(a*tan(x)^n)
```

mupad [B] time = 0.07, size = 44, normalized size = 0.79

$$\frac{n\operatorname{polylog}\left(2,\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+x\,\ln\left(a\tan(x)^n\right)-\frac{n\operatorname{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+2\,n\,x\,\operatorname{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)^n),x)
```

```
[Out] (n*polylog(2, exp(x*2i))*1i)/2 + x*log(a*tan(x)^n) - (n*polylog(2, -exp(x*2
i))*1i)/2 + 2*n*x*atanh(exp(x*2i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tan^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**n),x)
```

```
[Out] Integral(log(a*tan(x)**n), x)
```

3.170 $\int \log(a\cot(x))\,dx$

Optimal. Leaf size=51

$$x\log(a\cot(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right) - 2x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] -2*x*arctanh(exp(2*I*x))+x*ln(a*cot(x))+1/2*I*polylog(2,-exp(2*I*x))-1/2*I*
polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 4419, 4183, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right) + x\log(a\cot(x)) - 2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]],x]
```

```
[Out] -2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]] + (I/2)*PolyLog[2, -E^((2*I)*x)
] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int \log(a\cot(x))\,dx = x\log(a\cot(x)) + \int x\csc(x)\sec(x)\,dx$$
$$= x\log(a\cot(x)) + 2\int x\csc(2x)\,dx$$
$$= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) - \int \log\left(1-e^{2ix}\right)dx + \int \log\left(1+e^{2ix}\right)dx$$
$$= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log}{}\right.$$
$$= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.47

$$\frac{1}{2}i\log(-i(-\cot(x)+i))\log(a\cot(x)) - \frac{1}{2}i\log(-i(\cot(x)+i))\log(a\cot(x)) + \frac{1}{2}i\text{Li}_2(-i\cot(x)) - \frac{1}{2}i\text{Li}_2(i\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]],x]
```

```
[Out] (I/2)*Log[(-I)*(I - Cot[x])]*Log[a*Cot[x]] - (I/2)*Log[a*Cot[x]]*Log[(-I)*(
I + Cot[x])] + (I/2)*PolyLog[2, (-I)*Cot[x]] - (I/2)*PolyLog[2, I*Cot[x]]
```

fricas [B] time = 0.50, size = 147, normalized size = 2.88

$$x\log\left(\frac{a\cos(2x)+a}{\sin(2x)}\right) - \frac{1}{2}x\log(\cos(2x)+i\sin(2x)+1) - \frac{1}{2}x\log(\cos(2x)-i\sin(2x)+1) + \frac{1}{2}x\log(-\cos(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="fricas")
```

```
[Out] x*log((a*cos(2*x) + a)/sin(2*x)) - 1/2*x*log(cos(2*x) + I*sin(2*x) + 1) - 1
/2*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) + I*sin(2*x) + 1)
 + 1/2*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*dilog(-cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(-cos(2*x) - I*sin(2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\cot(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)), x)
```

maple [B] time = 0.16, size = 82, normalized size = 1.61

$$\frac{i\ln(a\cot(x))\ln\left(\frac{ia\cot(x)+a}{a}\right)}{2} - \frac{i\ln(a\cot(x))\ln\left(-\frac{ia\cot(x)-a}{a}\right)}{2} + \frac{i\,\text{dilog}\left(\frac{ia\cot(x)+a}{a}\right)}{2} - \frac{i\,\text{dilog}\left(-\frac{ia\cot(x)-a}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)),x)
```

```
[Out] 1/2*I*ln(a*cot(x))*ln((I*a*cot(x)+a)/a)-1/2*I*ln(a*cot(x))*ln(-(I*a*cot(x)-
a)/a)+1/2*I*dilog((I*a*cot(x)+a)/a)-1/2*I*dilog(-(I*a*cot(x)-a)/a)
```

maxima [A] time = 1.47, size = 43, normalized size = 0.84

$$-\frac{1}{4}\pi\log\left(\tan(x)^2+1\right)+x\log\left(\frac{a}{\tan(x)}\right)+x\log(\tan(x))-\frac{1}{2}i\,\mathrm{Li}_2(i\tan(x)+1)+\frac{1}{2}i\,\mathrm{Li}_2(-i\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="maxima")
```

```
[Out] -1/4*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)) + x*log(tan(x)) - 1/2*I*dilog(I
*tan(x) + 1) + 1/2*I*dilog(-I*tan(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(a\cot(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)),x)
```

```
[Out] int(log(a*cot(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\cot(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)),x)
```

```
[Out] Integral(log(a*cot(x)), x)
```

3.171 $\int \log\left(a\cot^2(x)\right)\,dx$

Optimal. Leaf size=49

$$x\log\left(a\cot^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)-i\text{Li}_2\left(e^{2ix}\right)-4x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] -4*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^2)+I*polylog(2,-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-4x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^2] + I*PolyLog[2, -E^((2*I)*x)]
- I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int \log\left(a\cot^2(x)\right)\,dx = x\log\left(a\cot^2(x)\right) - \int -2x\csc(x)\sec(x)\,dx$$
$$= x\log\left(a\cot^2(x)\right) + 2\int x\csc(x)\sec(x)\,dx$$
$$= x\log\left(a\cot^2(x)\right) + 4\int x\csc(2x)\,dx$$
$$= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) - 2\int \log\left(1-e^{2ix}\right)\,dx + 2\int \log\left(1+e^{2ix}\right)\,dx$$
$$= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) + i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) - i\,\text{Subst}\left(\int\right.$$
$$= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) + i\text{Li}_2\left(-e^{2ix}\right) - i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.53

$$-\frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\cot^2(x)\right)+\frac{1}{2}i\log(-i(\tan(x)+i))\log\left(a\cot^2(x)\right)+i\text{Li}_2(-i\tan(x))-i\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^2],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]^2]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^2]*Log
[(-I)*(I + Tan[x])] + I*PolyLog[2, (-I)*Tan[x]] - I*PolyLog[2, I*Tan[x]]
```

fricas [B] time = 0.54, size = 148, normalized size = 3.02

$$x\log\left(-\frac{a\cos(2x)+a}{\cos(2x)-1}\right)-x\log(\cos(2x)+i\sin(2x)+1)-x\log(\cos(2x)-i\sin(2x)+1)+x\log(-\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-(a*cos(2*x) + a)/(cos(2*x) - 1)) - x*log(cos(2*x) + I*sin(2*x) + 1)
- x*log(cos(2*x) - I*sin(2*x) + 1) + x*log(-cos(2*x) + I*sin(2*x) + 1) + x*
log(-cos(2*x) - I*sin(2*x) + 1) - 1/2*I*dilog(cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(cos(2*x) - I*sin(2*x)) - 1/2*I*dilog(-cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(-cos(2*x) - I*sin(2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cot(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^2), x)
```

maple [A] time = 0.46, size = 82, normalized size = 1.67

$$-i\ln(-i\cot(x))\ln(\cot(x)-i)+i\ln(i\cot(x))\ln(\cot(x)+i)+\frac{i\ln\left(a\left(\cot^2(x)\right)\right)\ln(\cot(x)-i)}{2}-\frac{i\ln\left(a\left(\cot^2(x)\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^2),x)
```

```
[Out] 1/2*I*ln(a*cot(x)^2)*ln(cot(x)-I)-I*ln(cot(x)-I)*ln(-I*cot(x))-I*dilog(-I*c
ot(x))-1/2*I*ln(a*cot(x)^2)*ln(cot(x)+I)+I*ln(cot(x)+I)*ln(I*cot(x))+I*dilo
g(I*cot(x))
```

maxima [A] time = 1.37, size = 44, normalized size = 0.90

$$-\frac{1}{2}\,\pi\log\left(\tan(x)^2+1\right)+x\log\left(\frac{a}{\tan(x)^2}\right)+2\,x\log\left(\tan(x)\right)-i\,\mathrm{Li}_2\left(i\,\tan(x)+1\right)+i\,\mathrm{Li}_2\left(-i\,\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)^2) + 2*x*log(tan(x)) - I*dilog(I
*tan(x) + 1) + I*dilog(-I*tan(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\cot(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)^2),x)
```

```
[Out] int(log(a*cot(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cot^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**2),x)
```

```
[Out] Integral(log(a*cot(x)**2), x)
```

3.172 $\int \log(a\cot^n(x))\,dx$

Optimal. Leaf size=56

$$x\log(a\cot^n(x))+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)-2nx\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] -2*n*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^n)+1/2*I*n*polylog(2,-exp(2*I*x))-
1/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\cot^n(x))-2nx\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^n] + (I/2)*n*PolyLog[2, -E^((2
*I)*x)] - (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int \log(a\cot^n(x))\,dx = x\log(a\cot^n(x)) + \int nx\csc(x)\sec(x)\,dx$$
$$= x\log(a\cot^n(x)) + n\int x\csc(x)\sec(x)\,dx$$
$$= x\log(a\cot^n(x)) + (2n)\int x\csc(2x)\,dx$$
$$= -2nx\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot^n(x)) - n\int \log\left(1-e^{2ix}\right)dx + n\int \log\left(1+e^{2ix}\right)dx$$
$$= -2nx\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot^n(x)) + \frac{1}{2}(in)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) - \frac{1}{2}(in)\,\text{Su}$$
$$= -2nx\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot^n(x)) + \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.45

$$-\frac{1}{2}i\log(-i(-\tan(x)+i))\log(a\cot^n(x))+\frac{1}{2}i\log(-i(\tan(x)+i))\log(a\cot^n(x))+\frac{1}{2}in\text{Li}_2(-i\tan(x))-\frac{1}{2}in\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^n],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]^n]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^n]*Log
[(-I)*(I + Tan[x])] + (I/2)*n*PolyLog[2, (-I)*Tan[x]] - (I/2)*n*PolyLog[2,
I*Tan[x]]
```

fricas [B] time = 0.51, size = 158, normalized size = 2.82

$$nx\log\left(\frac{\cos(2x)+1}{\sin(2x)}\right)-\frac{1}{2}nx\log(\cos(2x)+i\sin(2x)+1)-\frac{1}{2}nx\log(\cos(2x)-i\sin(2x)+1)+\frac{1}{2}nx\log(-\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log((cos(2*x) + 1)/sin(2*x)) - 1/2*n*x*log(cos(2*x) + I*sin(2*x) + 1) -
 1/2*n*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*n*x*log(-cos(2*x) + I*sin(2*x
) + 1) + 1/2*n*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*n*dilog(cos(2*x) +
 I*sin(2*x)) + 1/4*I*n*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*n*dilog(-cos(2*
x) + I*sin(2*x)) + 1/4*I*n*dilog(-cos(2*x) - I*sin(2*x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\cot(x)^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \ln(a(\cot^n(x)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^n),x)
```

```
[Out] int(ln(a*cot(x)^n),x)
```

maxima [A] time = 1.54, size = 49, normalized size = 0.88

$$nx\log(\tan(x)) - \frac{1}{4}\left(\pi\log\left(\tan(x)^2+1\right) + 2i\operatorname{Li}_2(i\tan(x)+1) - 2i\operatorname{Li}_2(-i\tan(x)+1)\right)n + x\log\left(a\frac{1}{\tan(x)}^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="maxima")
```

```
[Out] n*x*log(tan(x)) - 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*I
*dilog(-I*tan(x) + 1))*n + x*log(a*(1/tan(x))^n)
```

mupad [B] time = 0.07, size = 44, normalized size = 0.79

$$x\ln\left(a\cot(x)^n\right) - \frac{n\operatorname{polylog}\left(2, e^{x\,2i}\right)\,1i}{2} + \frac{n\operatorname{polylog}\left(2, -e^{x\,2i}\right)\,1i}{2} - 2\,n\,x\operatorname{atanh}\left(e^{x\,2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)^n),x)
```

```
[Out] x*log(a*cot(x)^n) - (n*polylog(2, exp(x*2i))*1i)/2 + (n*polylog(2, -exp(x*2
i))*1i)/2 - 2*n*x*atanh(exp(x*2i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cot^n(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**n),x)
```

```
[Out] Integral(log(a*cot(x)**n), x)
```

3.173 $\int \log(a\sec(x))\,dx$

Optimal. Leaf size=46

$$x\log(a\sec(x))-\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1+exp(2*I*x))+x*ln(a*sec(x))-1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\sec(x))-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]],x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2, -E
^((2*I)*x)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\sec(x))\,dx &= x\log(a\sec(x)) - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} + x\log(a\sec(x)) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \int \log\left(1+e^{2ix}\right)dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) + \frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$
x\log(a\sec(x)) - \frac{1}{2}i\mathrm{Li}_2\left(-e^{2ix}\right) - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
-E^((2*I)*x)]
```

fricas [B] time = 0.53, size = 106, normalized size = 2.30

$$
x\log\left(\frac{a}{\cos(x)}\right)+\frac{1}{2}x\log\left(i\cos(x)+\sin(x)+1\right)+\frac{1}{2}x\log\left(i\cos(x)-\sin(x)+1\right)+\frac{1}{2}x\log\left(-i\cos(x)+\sin(x)+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)) + 1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - s
in(x) + 1) + 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(
x) + 1) + 1/2*I*dilog(I*cos(x) + sin(x)) - 1/2*I*dilog(I*cos(x) - sin(x)) -
 1/2*I*dilog(-I*cos(x) + sin(x)) + 1/2*I*dilog(-I*cos(x) - sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log\left(a\sec(x)\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)), x)
```

maple [B] time = 0.54, size = 118, normalized size = 2.57

$$
-i\ln\left(\frac{a\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}\right)\ln\left(\mathrm{e}^{ix}\right)-i\ln\left(-i\mathrm{e}^{ix}+1\right)\ln\left(\mathrm{e}^{ix}\right)-i\ln\left(i\mathrm{e}^{ix}+1\right)\ln\left(\mathrm{e}^{ix}\right)+\frac{i\ln\left(\mathrm{e}^{ix}\right)^2}{2}-i\,\mathrm{dilog}\left(-i\mathrm{e}^{ix}+1\right)-i\,\mathrm{dilog}\left(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)),x)
```

```
[Out] -I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))*ln(a*exp(I*x)/(exp(2*I*x)+1))-I*ln(exp
(I*x))*ln(I*exp(I*x)+1)-I*ln(exp(I*x))*ln(-I*exp(I*x)+1)-I*dilog(I*exp(I*x)
+1)-I*dilog(-I*exp(I*x)+1)+1/2*I*ln(exp(I*x))^2
```

maxima [A] time = 2.27, size = 60, normalized size = 1.30

$$-\frac{1}{2} i x^2 + i x \arctan\left(\sin\left(2 x\right), \cos\left(2 x\right) + 1\right) + \frac{1}{2} x \log\left(\cos\left(2 x\right)^2 + \sin\left(2 x\right)^2 + 2 \cos\left(2 x\right) + 1\right) + x \log\left(a \sec(x)\right) - \frac{1}{2} i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/2*x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*sec(x)) - 1/2*I*dilog(-e^(2*I*x))
```

mupad [B] time = 0.09, size = 39, normalized size = 0.85

$$x \ln\left(\frac{a}{\cos(x)}\right) - \frac{\operatorname{polylog}\left(2, -\mathrm{e}^{x\,2\mathrm{i}}\right) 1\mathrm{i}}{2} - \frac{x\,\left(x + \ln\left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right) 2\mathrm{i}\right) 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cos(x)),x)
```

```
[Out] x*log(a/cos(x)) - (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 - (polylog(2, -exp(x
*2i))*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \sec\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)),x)
```

```
[Out] Integral(log(a*sec(x)), x)
```

3.174 $\int \log \left(a \sec^2(x)\right) dx$

Optimal. Leaf size=45

$$x \log \left(a \sec^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right) - ix^2 + 2x \log \left(1 + e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^2)-I*polylog(2,-exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2, -e^{2ix}\right) + x \log \left(a \sec^2(x)\right) - ix^2 + 2x \log \left(1 + e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^2] - I*PolyLog[2, -E^(
(2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \log\left(a\sec^2(x)\right)\,dx = x\log\left(a\sec^2(x)\right) - \int 2x\tan(x)\,dx$$
$$= x\log\left(a\sec^2(x)\right) - 2\int x\tan(x)\,dx$$
$$= -ix^2 + x\log\left(a\sec^2(x)\right) + 4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx$$
$$= -ix^2 + 2x\log\left(1+e^{2ix}\right) + x\log\left(a\sec^2(x)\right) - 2\int \log\left(1+e^{2ix}\right)\,dx$$
$$= -ix^2 + 2x\log\left(1+e^{2ix}\right) + x\log\left(a\sec^2(x)\right) + i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)$$
$$= -ix^2 + 2x\log\left(1+e^{2ix}\right) + x\log\left(a\sec^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right)$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.96

$$x\left(\log\left(a\sec^2(x)\right) - ix + 2\log\left(1+e^{2ix}\right)\right) - i\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^2],x]
```

```
[Out] x*((-I)*x + 2*Log[1 + E^((2*I)*x)] + Log[a*Sec[x]^2]) - I*PolyLog[2, -E^((2
*I)*x)]
```

fricas [B] time = 0.50, size = 102, normalized size = 2.27

$$x\log\left(\frac{a}{\cos(x)^2}\right) + x\log\left(i\cos(x) + \sin(x) + 1\right) + x\log\left(i\cos(x) - \sin(x) + 1\right) + x\log\left(-i\cos(x) + \sin(x) + 1\right) + x\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)^2) + x*log(I*cos(x) + sin(x) + 1) + x*log(I*cos(x) - sin(x)
+ 1) + x*log(-I*cos(x) + sin(x) + 1) + x*log(-I*cos(x) - sin(x) + 1) + I*di
log(I*cos(x) + sin(x)) - I*dilog(I*cos(x) - sin(x)) - I*dilog(-I*cos(x) + s
in(x)) + I*dilog(-I*cos(x) - sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sec(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^2), x)
```

maple [B] time = 0.49, size = 118, normalized size = 2.62

$$-i\ln\left(\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}\right)\ln\left(e^{ix}\right) - 2i\ln\left(-ie^{ix}+1\right)\ln\left(e^{ix}\right) - 2i\ln\left(ie^{ix}+1\right)\ln\left(e^{ix}\right) + i\ln\left(e^{ix}\right)^2 - 2i\,\text{dilog}\left(-ie^{ix}+1\right) - 2i\,\text{di}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^2),x)
```

```
[Out] -I*ln(a*exp(2*I*x)/(exp(2*I*x)+1)^2)*ln(exp(I*x))-2*I*ln(I*exp(I*x)+1)*ln(e
xp(I*x))-2*I*ln(-I*exp(I*x)+1)*ln(exp(I*x))+I*ln(exp(I*x))^2-2*I*ln(exp(I*x
))*ln(2)-2*I*dilog(I*exp(I*x)+1)-2*I*dilog(-I*exp(I*x)+1)
```

maxima [A] time = 2.17, size = 61, normalized size = 1.36

$$-i\,x^2+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(a\sec(x)^2\right)+x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)-i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*sec(x)^2) + x*log(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x))
```

mupad [B] time = 0.38, size = 39, normalized size = 0.87

$$x\,\ln\left(\frac{a}{\cos(x)^2}\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}-x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cos(x)^2),x)
```

```
[Out] x*log(a/cos(x)^2) - x*(x + log(exp(x*2i) + 1)*2i)*1i - polylog(2, -exp(x*2i
))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sec^2\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**2),x)
```

```
[Out] Integral(log(a*sec(x)**2), x)
```

3.175 $\int \log(a\sec^n(x))\,dx$

Optimal. Leaf size=51

$$x\log(a\sec^n(x)) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^n)-1/2*I*n*polylog(2,-exp(2
*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right) + x\log(a\sec^n(x)) - \frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*PolyL
og[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int \log(a\sec^n(x))\,dx = x\log(a\sec^n(x)) - \int nx\tan(x)\,dx$$
$$= x\log(a\sec^n(x)) - n\int x\tan(x)\,dx$$
$$= -\frac{1}{2}inx^2 + x\log(a\sec^n(x)) + (2in)\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx$$
$$= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) - n\int \log\left(1+e^{2ix}\right)dx$$
$$= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) + \frac{1}{2}(in)\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)$$
$$= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) - \frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right)$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$x\log(a\sec^n(x)) - \frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Pol
yLog[2, -E^((2*I)*x)]
```

fricas [B] time = 0.52, size = 117, normalized size = 2.29

$$nx\log\left(\frac{1}{\cos(x)}\right)+\frac{1}{2}\,nx\log(i\,\cos(x)+\sin(x)+1)+\frac{1}{2}\,nx\log(i\,\cos(x)-\sin(x)+1)+\frac{1}{2}\,nx\log(-i\,\cos(x)+\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/cos(x)) + 1/2*n*x*log(I*cos(x) + sin(x) + 1) + 1/2*n*x*log(I*cos(
x) - sin(x) + 1) + 1/2*n*x*log(-I*cos(x) + sin(x) + 1) + 1/2*n*x*log(-I*cos
(x) - sin(x) + 1) + 1/2*I*n*dilog(I*cos(x) + sin(x)) - 1/2*I*n*dilog(I*cos(
x) - sin(x)) - 1/2*I*n*dilog(-I*cos(x) + sin(x)) + 1/2*I*n*dilog(-I*cos(x)
- sin(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\sec(x)^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^n), x)
```

maple [F] time = 1.61, size = 0, normalized size = 0.00

$$\int \ln(a(\sec^n(x)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^n),x)
```

```
[Out] int(ln(a*sec(x)^n),x)
```

maxima [A] time = 3.32, size = 65, normalized size = 1.27

$$\frac{1}{2}\left(-i\,x^2 + 2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) - i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)\right)n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*sec(x)^n)
```

mupad [B] time = 0.38, size = 43, normalized size = 0.84

$$x\,\ln\left(a\left(\frac{1}{\cos(x)}\right)^n\right) - \frac{n\,\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} - \frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/cos(x))^n),x)
```

```
[Out] x*log(a*(1/cos(x))^n) - (n*polylog(2, -exp(x*2i))*1i)/2 - (n*x*(x + log(exp
(x*2i) + 1)*2i)*1i)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sec^n\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**n),x)
```

```
[Out] Integral(log(a*sec(x)**n), x)
```

3.176 $\int \log(a\csc(x))\,dx$

Optimal. Leaf size=46

$$x\log(a\csc(x))-\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*x))+x*ln(a*csc(x))-1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\csc(x))-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]],x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*PolyLog[2, E^
((2*I)*x)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log(a\csc(x))\,dx = x\log(a\csc(x)) + \int x\cot(x)\,dx$$
$$= -\frac{ix^2}{2} + x\log(a\csc(x)) - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx$$
$$= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \int \log\left(1-e^{2ix}\right)\,dx$$
$$= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) + \frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)$$
$$= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.89

$$x\log(a\csc(x)) - \frac{1}{2}i\left(x^2 + \mathrm{Li}_2\left(e^{2ix}\right)\right) + x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]],x]
```

```
[Out] x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

fricas [B] time = 0.51, size = 106, normalized size = 2.30

$$x\log\left(\frac{a}{\sin(x)}\right) + \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right) + \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right) + \frac{1}{2}x\log\left(-\cos(x) + i\sin(x) + 1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/sin(x)) + 1/2*x*log(cos(x) + I*sin(x) + 1) + 1/2*x*log(cos(x) - I*s
in(x) + 1) + 1/2*x*log(-cos(x) + I*sin(x) + 1) + 1/2*x*log(-cos(x) - I*sin(
x) + 1) - 1/2*I*dilog(cos(x) + I*sin(x)) + 1/2*I*dilog(cos(x) - I*sin(x)) +
 1/2*I*dilog(-cos(x) + I*sin(x)) - 1/2*I*dilog(-cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)), x)
```

maple [B] time = 0.49, size = 89, normalized size = 1.93

$$-i\ln\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)\ln\left(\mathrm{e}^{ix}\right) - i\ln\left(\mathrm{e}^{ix}+1\right)\ln\left(\mathrm{e}^{ix}\right) + \frac{i\ln\left(\mathrm{e}^{ix}\right)^2}{2} - i\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right) + i\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right) - i\ln(2)\ln\left(\mathrm{e}^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)),x)
```

```
[Out] -I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))*ln(I*a*exp(I*x)/(exp(2*I*x)-1))+I*dilo
g(exp(I*x))-I*ln(exp(I*x))*ln(exp(I*x)+1)-I*dilog(exp(I*x)+1)+1/2*I*ln(exp(
I*x))^2
```

maxima [B] time = 2.26, size = 87, normalized size = 1.89

$$-\frac{1}{2} i x^2 + i x \arctan(\sin(x), \cos(x) + 1) - i x \arctan(\sin(x), -\cos(x) + 1) + \frac{1}{2} x \log\left(\cos(x)^2 + \sin(x)^2 + 2 \cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
+ 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) + x*log(a*csc(x)) - I*dilog(-e^(I*x)) - I*dilog(e
^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\frac{a}{\sin(x)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sin(x)),x)
```

```
[Out] int(log(a/sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a \csc(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)),x)
```

```
[Out] Integral(log(a*csc(x)), x)
```

3.177 $\int \log\left(a\csc^2(x)\right)\,dx$

Optimal. Leaf size=45

$$x\log\left(a\csc^2(x)\right)-i\text{Li}_2\left(e^{2ix}\right)-ix^2+2x\log\left(1-e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^2)-I*polylog(2,exp(2*I*x))
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-ix^2+2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*PolyLog[2, E^((
2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\csc^2(x)\right)\,dx &= x\log\left(a\csc^2(x)\right)-\int -2x\cot(x)\,dx\\
&= x\log\left(a\csc^2(x)\right)+2\int x\cot(x)\,dx\\
&= -ix^2+x\log\left(a\csc^2(x)\right)-4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-2\int \log\left(1-e^{2ix}\right)\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)+i\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-i\mathrm{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.93

$$x\log\left(a\csc^2(x)\right)-i\left(x^2+\mathrm{Li}_2\left(e^{2ix}\right)\right)+2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^2],x]
```

```
[Out] 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

fricas [B] time = 0.54, size = 107, normalized size = 2.38

$$x\log\left(-\frac{a}{\cos(x)^2-1}\right)+x\log\left(\cos(x)+i\,\sin(x)+1\right)+x\log\left(\cos(x)-i\,\sin(x)+1\right)+x\log\left(-\cos(x)+i\,\sin(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a/(cos(x)^2 - 1)) + x*log(cos(x) + I*sin(x) + 1) + x*log(cos(x) - I*
sin(x) + 1) + x*log(-cos(x) + I*sin(x) + 1) + x*log(-cos(x) - I*sin(x) + 1)
 - I*dilog(cos(x) + I*sin(x)) + I*dilog(cos(x) - I*sin(x)) + I*dilog(-cos(x
) + I*sin(x)) - I*dilog(-cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^2), x)
```

maple [B] time = 0.55, size = 88, normalized size = 1.96

$$-i\ln\left(-\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^2}\right)\ln\left(\mathrm{e}^{ix}\right)-2i\ln\left(\mathrm{e}^{ix}+1\right)\ln\left(\mathrm{e}^{ix}\right)+i\ln\left(\mathrm{e}^{ix}\right)^2-2i\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+2i\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right)-2i\ln(2)\ln\left(\mathrm{e}^{i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^2),x)
```

```
[Out] -I*ln(-a*exp(2*I*x)/(exp(2*I*x)-1)^2)*ln(exp(I*x))-2*I*ln(exp(I*x)+1)*ln(ex
p(I*x))+I*ln(exp(I*x))^2-2*I*ln(2)*ln(exp(I*x))+2*I*dilog(exp(I*x))-2*I*dil
og(exp(I*x)+1)
```

maxima [B] time = 2.31, size = 87, normalized size = 1.93

$$-i\,x^2+2i\,x\arctan\left(\sin(x),\cos(x)+1\right)-2i\,x\arctan\left(\sin(x),-\cos(x)+1\right)+x\log\left(a\csc(x)^2\right)+x\log\left(\cos(x)^2+\sin(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x)
+ 1) + x*log(a*csc(x)^2) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(
I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\frac{a}{\sin(x)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sin(x)^2),x)
```

```
[Out] int(log(a/sin(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc^2(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**2),x)
```

```
[Out] Integral(log(a*csc(x)**2), x)
```

3.178 $\int \log(a \csc^n(x))\,dx$

Optimal. Leaf size=51

$$x\log(a\csc^n(x)) - \frac{1}{2}in\mathrm{Li}_2\left(e^{2ix}\right) - \frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^n)-1/2*I*n*polylog(2,exp(2*
I*x))
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}in\mathrm{PolyLog}\left(2,e^{2ix}\right) + x\log(a\csc^n(x)) - \frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*PolyL
og[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\csc^n(x)\right)dx &= x\log\left(a\csc^n(x)\right)+\int nx\cot(x)\,dx\\
&= x\log\left(a\csc^n(x)\right)+n\int x\cot(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\csc^n(x)\right)-(2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-n\int \log\left(1-e^{2ix}\right)dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)+\frac{1}{2}(in)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Pol
yLog[2, E^((2*I)*x)]
```

fricas [B] time = 0.52, size = 117, normalized size = 2.29

$$nx\log\left(\frac{1}{\sin(x)}\right)+\frac{1}{2}nx\log\left(\cos(x)+i\sin(x)+1\right)+\frac{1}{2}nx\log\left(\cos(x)-i\sin(x)+1\right)+\frac{1}{2}nx\log\left(-\cos(x)+i\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/sin(x)) + 1/2*n*x*log(cos(x) + I*sin(x) + 1) + 1/2*n*x*log(cos(x)
 - I*sin(x) + 1) + 1/2*n*x*log(-cos(x) + I*sin(x) + 1) + 1/2*n*x*log(-cos(x
) - I*sin(x) + 1) - 1/2*I*n*dilog(cos(x) + I*sin(x)) + 1/2*I*n*dilog(cos(x)
 - I*sin(x)) + 1/2*I*n*dilog(-cos(x) + I*sin(x)) - 1/2*I*n*dilog(-cos(x) -
I*sin(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^n), x)
```

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\csc^n(x)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^n),x)
```

```
[Out] int(ln(a*csc(x)^n),x)
```

maxima [B] time = 1.75, size = 91, normalized size = 1.78

$$\frac{1}{2}\left(-i\,x^2 + 2i\,x\arctan\left(\sin(x), \cos(x) + 1\right) - 2i\,x\arctan\left(\sin(x), -\cos(x) + 1\right) + x\log\left(\cos(x)^2 + \sin(x)^2 + 2\,\mathrm{cc}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*lo
g(a*csc(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\left(\frac{1}{\sin(x)}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/sin(x))^n),x)
```

```
[Out] int(log(a*(1/sin(x))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc^n(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**n),x)
```

```
[Out] Integral(log(a*csc(x)**n), x)
```

3.179 $\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx$

Optimal. Leaf size=21

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

```
[Out] -2*sin(x)+ln(1/2-1/2*cos(2*x))*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2637, 2554, 12}

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

```
[Out] -2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)dx &= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)-\int 2\cos(x)\,dx\\ &= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)-2\int \cos(x)\,dx\\ &= -2\sin(x)+\log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.62

$$\sin(x)\log\left(\sin^2(x)\right)-2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

```
[Out] -2*Sin[x] + Log[Sin[x]^2]*Sin[x]
```

fricas [A] time = 0.49, size = 17, normalized size = 0.81

$$\log\left(-\cos(x)^2+1\right)\sin(x)-2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="fricas")
[Out] log(-cos(x)^2 + 1)*sin(x) - 2*sin(x)
```

giac [A] time = 0.18, size = 13, normalized size = 0.62

$$\log\left(\sin(x)^2\right)\sin(x)-2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="giac")
[Out] log(sin(x)^2)*sin(x) - 2*sin(x)
```

maple [C] time = 0.60, size = 72, normalized size = 3.43

$$\frac{ie^{-ix}\ln\left(-2\cos\left(2x\right)+2\right)}{2}-\frac{ie^{ix}\ln\left(-2\cos\left(2x\right)+2\right)}{2}-i\ln(2)e^{-ix}-ie^{-ix}+i\ln(2)e^{ix}+ie^{ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(1/2-1/2*cos(2*x)),x)
[Out] I*ln(2)*exp(I*x)-I*ln(2)*exp(-I*x)-1/2*I*exp(I*x)*ln(2-2*cos(2*x))+1/2*I*ex
p(-I*x)*ln(2-2*cos(2*x))+I*exp(I*x)-I*exp(-I*x)
```

maxima [A] time = 0.45, size = 17, normalized size = 0.81

$$\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin(x)-2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="maxima")
[Out] log(-1/2*cos(2*x) + 1/2)*sin(x) - 2*sin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \ln\left(\frac{1}{2}-\frac{\cos\left(2\,x\right)}{2}\right)\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/2 - cos(2*x)/2)*cos(x),x)
[Out] int(log(1/2 - cos(2*x)/2)*cos(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{1}{2}-\frac{\cos\left(2x\right)}{2}\right)\cos\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(1/2-1/2*cos(2*x)),x)
[Out] Integral(log(1/2 - cos(2*x)/2)*cos(x), x)
```

3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$

Optimal. Leaf size=6

$$\log(\log(e\sin(x)))$$

```
[Out] ln(ln(E*sin(x)))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4338, 31}

$$\log(\log(\sin(x))+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[E*Sin[x]],x]
[Out] Log[1 + Log[Sin[x]]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\log(e\sin(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x+x\log(x)}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \log(\sin(x))\right)\\ &= \log(1+\log(\sin(x)))\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$\log(\log(\sin(x))+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E*Sin[x]],x]
[Out] Log[1 + Log[Sin[x]]]
```

fricas [A] time = 0.44, size = 6, normalized size = 1.00

$$\log\left(\log\left(E\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="fricas")
[Out] log(log(E*sin(x)))
```

giac [**B**] time = 0.17, size = 32, normalized size = 5.33

$$\frac{1}{2}\log\left(\frac{1}{4}\left(\pi\left(\operatorname{sgn}(E)-1\right)+\pi\left(\operatorname{sgn}\left(\sin(x)\right)-1\right)\right)^2+\left(\log\left(|E|\right)+\log\left(|\sin(x)|\right)\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*(sgn(E) - 1) + pi*(sgn(sin(x)) - 1))^2 + (log(abs(E)) + log
(abs(sin(x))))^2)
```

maple [**A**] time = 0.40, size = 7, normalized size = 1.17

$$\ln\left(\ln\left(E\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(E*sin(x)),x)
```

```
[Out] ln(ln(E*sin(x)))
```

maxima [**A**] time = 0.44, size = 6, normalized size = 1.00

$$\log\left(\log\left(E\sin(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="maxima")
```

```
[Out] log(log(E*sin(x)))
```

mupad [**B**] time = 0.50, size = 6, normalized size = 1.00

$$\ln\left(\ln\left(\sin(x)\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/log(exp(1)*sin(x)),x)
```

```
[Out] log(log(sin(x)) + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(x\right)}{\log\left(\sin\left(x\right)\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(E*sin(x)),x)
```

```
[Out] Integral(cot(x)/(log(sin(x)) + 1), x)
```

3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx$

Optimal. Leaf size=37

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

```
[Out] ln(ln(exp(sin(x))))/(-ln(exp(sin(x)))+sin(x))-ln(sin(x))/(-ln(exp(sin(x)))+
sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4338, 2160, 2157, 29}

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[E^Sin[x]],x]
```

```
[Out] Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
+ Sin[x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = \text{Subst}\left(\int \frac{1}{x\log(e^x)}\,dx, x, \sin(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{\log(e^x)}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$
$$= \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$
$$= -\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.68

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)-\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E^Sin[x]],x]
[Out] (Log[Log[E^Sin[x]]] - Log[Sin[x]])/(-Log[E^Sin[x]] + Sin[x])
```

fricas [A] time = 0.44, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="fricas")
[Out] -1/sin(x)
```

giac [A] time = 0.15, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="giac")
[Out] -1/sin(x)
```

maple [A] time = 0.45, size = 35, normalized size = 0.95

$$-\frac{\ln\left(\ln\left(e^{\sin(x)}\right)\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\ln(\sin(x))}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(exp(sin(x))),x)
[Out] -1/(ln(exp(sin(x)))-sin(x))*ln(ln(exp(sin(x))))+1/(ln(exp(sin(x)))-sin(x))*
ln(sin(x))
```

maxima [A] time = 0.45, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

mupad [B] time = 0.37, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/log(exp(sin(x))),x)
```

```
[Out] -1/sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(exp(sin(x))),x)
```

```
[Out] Integral(cot(x)/log(exp(sin(x))), x)
```

3.182 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

```
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8, 2554, 3473}

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(\cos(x)) \sec^2(x) \, dx &= \log(\cos(x)) \tan(x) + \int \tan^2(x) \, dx \\
&= \tan(x) + \log(\cos(x)) \tan(x) - \int 1 \, dx \\
&= -x + \tan(x) + \log(\cos(x)) \tan(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

fricas [A] time = 0.45, size = 22, normalized size = 1.83

$$-\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

giac [A] time = 0.18, size = 12, normalized size = 1.00

$$\log(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

maple [C] time = 0.58, size = 61, normalized size = 5.08

$$-\frac{2ie^{2ix}\ln(2\cos(x))}{e^{2ix}+1} + i\ln\left(e^{2ix}+1\right) + \frac{2i}{e^{2ix}+1} - \frac{2i\ln(2)}{e^{2ix}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sec(x)^2,x)
```

```
[Out] -2*I/(exp(2*I*x)+1)*exp(2*I*x)*ln(2*cos(x))+2*I/(exp(2*I*x)+1)+I*ln(exp(2*I
*x)+1)-2*I*ln(2)/(exp(2*I*x)+1)
```

maxima [B] time = 1.00, size = 94, normalized size = 7.83

$$-\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 35, normalized size = 2.92

$$\tan(x) - 2x + \ln(\cos(x))\tan(x) + \ln(\cos(x))\,1i - \ln(\cos(2x) + 1 + \sin(2x)\,1i)\,1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))/cos(x)^2,x)
```

```
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

sympy [A] time = 18.33, size = 15, normalized size = 1.25

$$-x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

3.183 $\int \cot(x) \log(\sin(x)) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\sin(x))$$

[Out] 1/2*ln(sin(x))^2

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475, 4338, 2301}

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Log[Sin[x]],x]

[Out] Log[Sin[x]]^2/2

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot(x)\log(\sin(x))\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\log^2(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]*Log[Sin[x]],x]

[Out] Log[Sin[x]]^2/2

fricas [A] time = 0.47, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x))^2
```

giac [A] time = 0.19, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x))^2
```

maple [A] time = 0.19, size = 8, normalized size = 0.89

$$\frac{\ln\left(\sin(x)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*ln(sin(x)),x)
```

```
[Out] 1/2*ln(sin(x))^2
```

maxima [A] time = 0.45, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x))^2
```

mupad [B] time = 0.38, size = 7, normalized size = 0.78

$$\frac{\ln\left(\sin(x)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cot(x),x)
```

```
[Out] log(sin(x))^2/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*ln(sin(x)),x)
```

```
[Out] Timed out
```

3.184 $\int \cos(x) \log(\sin(x)) \sin^2(x) \, dx$

Optimal. Leaf size=20

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2564, 30, 2554, 12}

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]

[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\log(\sin(x))\sin^2(x)\,dx &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\
&= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.75

$$\frac{1}{9}\sin^3(x)(3\log(\sin(x)) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

fricas [A] time = 0.46, size = 24, normalized size = 1.20

$$-\frac{1}{3}\left(\cos(x)^2-1\right)\log\left(\sin(x)\right)\sin(x)+\frac{1}{9}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

giac [A] time = 0.19, size = 16, normalized size = 0.80

$$\frac{1}{3}\log\left(\sin(x)\right)\sin(x)^3-\frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

maple [A] time = 0.07, size = 17, normalized size = 0.85

$$\frac{\ln\left(\sin(x)\right)\left(\sin^3(x)\right)}{3}-\frac{\left(\sin^3(x)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x)
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

maxima [A] time = 0.45, size = 16, normalized size = 0.80

$$\frac{1}{3}\log\left(\sin(x)\right)\sin(x)^3-\frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

mupad [B] time = 0.43, size = 11, normalized size = 0.55

$$\frac{\sin(x)^3\left(\ln\left(\sin(x)\right)-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x)*sin(x)^2,x)
[Out] (sin(x)^3*(log(sin(x)) - 1/3))/3
```

sympy [A] time = 4.93, size = 17, normalized size = 0.85

$$\frac{\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3}-\frac{\sin^3\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

3.185 $\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$

Optimal. Leaf size=50

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -sin(b*x+a)/b+ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x))*sin(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2637, 2554}

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*x
])/b
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b}-\int \cos(a$$

$$=-\frac{\sin(a+bx)}{b}+\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+}{b}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.66

$$\frac{\sin(a+bx)\log\left(\frac{1}{2}\sin(a+bx)\right)}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Sin[a + b*x]/2]*Sin[a + b*x])/b
```

fricas [A] time = 0.49, size = 65, normalized size = 1.30

$$\frac{2\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="fricas")
```

```
[Out] 2*(cos(1/2*b*x + 1/2*a)*log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(
1/2*b*x + 1/2*a) - cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))/b
```

giac [A] time = 0.93, size = 42, normalized size = 0.84

$$\frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="giac")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

maple [A] time = 0.74, size = 32, normalized size = 0.64

$$\frac{\ln\left(\frac{\sin(bx+a)}{2}\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)),x)
```

```
[Out] ln(1/2*sin(b*x+a))/b*sin(b*x+a)-sin(b*x+a)/b
```

maxima [A] time = 0.54, size = 42, normalized size = 0.84

$$\frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="maxima")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

mupad [B] time = 0.52, size = 29, normalized size = 0.58

$$-\frac{\sin\left(a+b\,x\right)-\ln\left(\frac{\sin(a+b\,x)}{2}\right)\sin\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(a/2 + (b*x)/2)*sin(a/2 + (b*x)/2))*cos(a + b*x),x)
```

```
[Out] -(sin(a + b*x) - log(sin(a + b*x)/2)*sin(a + b*x))/b
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
[Out] Integral(log(sin(a/2 + b*x/2)*cos(a/2 + b*x/2))*cos(a + b*x), x)
```

3.186 $\int \frac{\tan(x)}{\log(\cos(x))} \, dx$

Optimal. Leaf size=6

$$-\log(\log(\cos(x)))$$

```
[Out] -ln(ln(cos(x)))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4339, 2302, 29}

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Log[Cos[x]],x]
```

```
[Out] -Log[Log[Cos[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(x)}{\log(\cos(x))} \, dx &= -\text{Subst}\left(\int \frac{1}{x \log(x)} \, dx, x, \cos(x)\right) \\ &= -\text{Subst}\left(\int \frac{1}{x} \, dx, x, \log(\cos(x))\right) \\ &= -\log(\log(\cos(x))) \end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Log[Cos[x]],x]
```

```
[Out] -Log[Log[Cos[x]]]
```

fricas [A] time = 0.43, size = 6, normalized size = 1.00

$$-\log\left(\log\left(\cos(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="fricas")
[Out] -log(log(cos(x)))
```

giac [A] time = 0.16, size = 7, normalized size = 1.17

$$-\log\left(\left|\log\left(\cos(x)\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="giac")
[Out] -log(abs(log(cos(x))))
```

maple [A] time = 0.30, size = 7, normalized size = 1.17

$$-\ln\left(\ln\left(\cos(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/ln(cos(x)),x)
[Out] -ln(ln(cos(x)))
```

maxima [A] time = 0.44, size = 6, normalized size = 1.00

$$-\log\left(\log\left(\cos(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="maxima")
[Out] -log(log(cos(x)))
```

mupad [B] time = 0.40, size = 6, normalized size = 1.00

$$-\ln\left(\ln\left(\cos(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/log(cos(x)),x)
[Out] -log(log(cos(x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)}{\log\left(\cos\left(x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/ln(cos(x)),x)
[Out] Integral(tan(x)/log(cos(x)), x)
```

3.187 $\int \log(\cos(x))\tan(x)\,dx$

Optimal. Leaf size=9

$$-\frac{1}{2}\log^2(\cos(x))$$

[Out] -1/2*ln(cos(x))^2

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475, 4339, 2301}

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Log[Cos[x]]*Tan[x],x]

[Out] -Log[Cos[x]]^2/2

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \log(\cos(x))\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log^2(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Log[Cos[x]]*Tan[x],x]

[Out] -1/2*Log[Cos[x]]^2

fricas [A] time = 0.49, size = 7, normalized size = 0.78

$$-\frac{1}{2}\log\left(\cos(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(cos(x))^2
```

giac [A] time = 0.16, size = 7, normalized size = 0.78

$$-\frac{1}{2}\log\left(\cos(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x))^2
```

maple [A] time = 0.19, size = 8, normalized size = 0.89

$$-\frac{\ln\left(\cos(x)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*tan(x),x)
```

```
[Out] -1/2*ln(cos(x))^2
```

maxima [A] time = 0.44, size = 7, normalized size = 0.78

$$-\frac{1}{2}\log\left(\cos(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x))^2
```

mupad [B] time = 0.43, size = 7, normalized size = 0.78

$$-\frac{\ln\left(\cos(x)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*tan(x),x)
```

```
[Out] -log(cos(x))^2/2
```

sympy [A] time = 3.53, size = 8, normalized size = 0.89

$$-\frac{\log\left(\cos\left(x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*tan(x),x)
```

```
[Out] -log(cos(x))**2/2
```

3.188 $\int \log(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=10

$$\cos(x) - \cos(x)\log(\cos(x))$$

```
[Out] cos(x)-cos(x)*ln(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2638, 2554}

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \log(\cos(x))\sin(x)\,dx &= -\cos(x)\log(\cos(x)) - \int \sin(x)\,dx \\ &= \cos(x) - \cos(x)\log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

fricas [A] time = 0.47, size = 10, normalized size = 1.00

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)*log(cos(x)) + cos(x)
```

giac [A] time = 0.17, size = 10, normalized size = 1.00

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="giac")
[Out] -cos(x)*log(cos(x)) + cos(x)
```

maple [A] time = 0.18, size = 11, normalized size = 1.10

$$-\cos(x)\ln(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sin(x),x)
[Out] cos(x)-cos(x)*ln(cos(x))
```

maxima [A] time = 0.44, size = 10, normalized size = 1.00

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="maxima")
[Out] -cos(x)*log(cos(x)) + cos(x)
```

mupad [B] time = 0.38, size = 9, normalized size = 0.90

$$-\cos(x)\ (\ln(\cos(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*sin(x),x)
[Out] -cos(x)*(log(cos(x)) - 1)
```

sympy [A] time = 0.87, size = 10, normalized size = 1.00

$$-\log(\cos(x))\cos(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sin(x),x)
[Out] -log(cos(x))*cos(x) + cos(x)
```

3.189 $\int \cos(x)\log(\cos(x))\,dx$

Optimal. Leaf size=14

$$-\sin(x)+\tanh^{-1}(\sin(x))+\sin(x)\log(\cos(x))$$

```
[Out] arctanh(sin(x))-sin(x)+ln(cos(x))*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {2637, 2554, 2592, 321, 206}

$$-\sin(x)+\tanh^{-1}(\sin(x))+\sin(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Cos[x]],x]
```

```
[Out] ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\cos(x))\,dx &= \log(\cos(x))\sin(x) + \int \sin(x)\tan(x)\,dx\\ &= \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)\\ &= -\sin(x) + \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \tanh^{-1}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)\end{aligned}$$

Mathematica [B] time = 0.02, size = 43, normalized size = 3.07

$$-\sin(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + \sin(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Cos[x]],x]
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x] + Log[Cos[x]]
*Sin[x]
```

fricas [A] time = 0.49, size = 27, normalized size = 1.93

$$\log\left(\cos(x)\right)\sin(x) + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(-\sin(x) + 1\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="fricas")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

giac [A] time = 0.18, size = 27, normalized size = 1.93

$$\log\left(\cos(x)\right)\sin(x) + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(-\sin(x) + 1\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="giac")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

maple [C] time = 0.31, size = 73, normalized size = 5.21

$$\frac{ie^{-ix}\ln\left(2\cos(x)\right)}{2} - \frac{ie^{ix}\ln\left(2\cos(x)\right)}{2} - 2i\arctan\left(e^{ix}\right) - \frac{i\ln(2)e^{-ix}}{2} - \frac{ie^{-ix}}{2} + \frac{i\ln(2)e^{ix}}{2} + \frac{ie^{ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(cos(x)),x)
[Out] -1/2*I*ln(2)*exp(-I*x)+1/2*I*ln(2)*exp(I*x)+1/2*I*exp(-I*x)*ln(2*cos(x))-1/
2*I*ln(2*cos(x))*exp(I*x)-1/2*I*exp(-I*x)-2*I*arctan(exp(I*x))+1/2*I*exp(I*
x)
```

maxima [B] time = 0.45, size = 108, normalized size = 7.71

$$\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="maxima")
```

```
[Out] 2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)/
((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
+ 1)^2 + 1)*(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(
x) + 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \ln(\cos(x))\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*cos(x),x)
```

```
[Out] int(log(cos(x))*cos(x), x)
```

sympy [B] time = 2.36, size = 223, normalized size = 15.93

$$-\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}-\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(cos(x)),x)
```

```
[Out] -log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan
(x/2)**2 + 1) + 2*log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))
*tan(x/2)/(tan(x/2)**2 + 1) - log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x
/2)**2 + 1))/(tan(x/2)**2 + 1) + 2*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)*
*2 + 1) + 2*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 + 1) - 2*tan(
x/2)/(tan(x/2)**2 + 1)
```

3.190 $\int \cos(x)\log(\sin(x))\,dx$

Optimal. Leaf size=11

$$\sin(x)\log(\sin(x)) - \sin(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2637, 2554}

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \cos(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x) - \int \cos(x)\,dx \\ &= -\sin(x) + \log(\sin(x))\sin(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

fricas [A] time = 0.48, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

giac [A] time = 0.16, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

maple [A] time = 0.19, size = 12, normalized size = 1.09

$$\ln(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x)),x)
```

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

maxima [A] time = 0.44, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

mupad [B] time = 0.41, size = 8, normalized size = 0.73

$$\sin(x)\,\left(\ln(\sin(x)) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x),x)
```

```
[Out] sin(x)*(log(sin(x)) - 1)
```

sympy [A] time = 0.87, size = 10, normalized size = 0.91

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

3.191 $\int \log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}i\text{Li}_2\left(e^{2ix}\right)+\frac{ix^2}{4}+\frac{x}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}\sin(x)\cos(x)-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

```
[Out] 1/4*x+1/4*I*x^2-1/2*x*ln(1-exp(2*I*x))+1/2*x*ln(sin(x))+1/4*I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)-1/2*cos(x)*ln(sin(x))*sin(x)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2635, 8, 2554, 12, 6742, 3717, 2190, 2279, 2391}

$$\frac{1}{4}i\text{PolyLog}\left(2,e^{2ix}\right)+\frac{ix^2}{4}+\frac{x}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}\sin(x)\cos(x)-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] x/4 + (I/4)*x^2 - (x*Log[1 - E^((2*I)*x)])/2 + (x*Log[Sin[x]])/2 + (I/4)*Po
lyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (Cos[x]*Log[Sin[x]]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \log(\sin(x))\sin^2(x)\,dx &= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \int \frac{1}{2}\cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \left(-\cos^2(x)+x\cot(x)\right)\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + \frac{1}{2}\int \cos^2(x)\,dx - \frac{1}{2}\int x\cot(x)\,dx \\
&= \frac{ix^2}{4} + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x)) \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x)) \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos
\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.80

$$\frac{1}{8}\left(2i\text{Li}_2\left(e^{2ix}\right) + 2x\left(ix - 2\log\left(1-e^{2ix}\right) + 2\log(\sin(x)) + 1\right) + \sin(2x)(1 - 2\log(\sin(x)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] (2*x*(1 + I*x - 2*Log[1 - E^((2*I)*x)] + 2*Log[Sin[x]]) + (2*I)*PolyLog[2,
E^((2*I)*x)] + (1 - 2*Log[Sin[x]])*Sin[2*x])/8
```

fricas [B] time = 0.50, size = 120, normalized size = 1.62

$$-\frac{1}{4}x\log(\cos(x)+i\sin(x)+1) - \frac{1}{4}x\log(\cos(x)-i\sin(x)+1) - \frac{1}{4}x\log(-\cos(x)+i\sin(x)+1) - \frac{1}{4}x\log(-\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*x*log(cos(x) + I*sin(x) + 1) - 1/4*x*log(cos(x) - I*sin(x) + 1) - 1/4*
x*log(-cos(x) + I*sin(x) + 1) - 1/4*x*log(-cos(x) - I*sin(x) + 1) - 1/2*(co
s(x)*sin(x) - x)*log(sin(x)) + 1/4*cos(x)*sin(x) + 1/4*x + 1/4*I*dilog(cos(
x) + I*sin(x)) - 1/4*I*dilog(cos(x) - I*sin(x)) - 1/4*I*dilog(-cos(x) + I*s
in(x)) + 1/4*I*dilog(-cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin(x)\right)\sin(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(sin(x))*sin(x)^2, x)
```

maple [B] time = 0.33, size = 146, normalized size = 1.97

$$-\frac{i e^{-2ix}\ln\left(2\sin(x)\right)}{8}+\frac{i e^{2ix}\ln\left(2\sin(x)\right)}{8}-\frac{i\ln\left(2\sin(x)\right)\ln\left(e^{ix}\right)}{2}+\frac{i\ln\left(e^{ix}+1\right)\ln\left(e^{ix}\right)}{2}-\frac{i\ln\left(e^{ix}\right)^2}{4}+\frac{i\,\mathrm{dilog}\left(e^{ix}+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^2,x)
```

```
[Out] 1/8*I*ln(2*sin(x))*exp(2*I*x)-1/16*I*exp(2*I*x)-1/2*I*ln(exp(I*x))*ln(2*sin
(x))-1/4*I*ln(exp(I*x))^2+1/2*I*ln(exp(I*x))*ln(exp(I*x)+1)-1/2*I*dilog(exp
(I*x))+1/2*I*dilog(exp(I*x)+1)-1/8*I*exp(-2*I*x)*ln(2*sin(x))+1/16*I*exp(-2
*I*x)-1/4*I*ln(exp(I*x))-1/8*I*ln(2)*exp(2*I*x)+1/8*I*ln(2)*exp(-2*I*x)+1/2
*I*ln(2)*ln(exp(I*x))
```

maxima [B] time = 1.94, size = 104, normalized size = 1.41

$$\frac{1}{4}i\,x^2-\frac{1}{2}i\,x\arctan\left(\sin(x),\cos(x)+1\right)+\frac{1}{2}i\,x\arctan\left(\sin(x),-\cos(x)+1\right)-\frac{1}{4}\,x\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*x^2 - 1/2*I*x*arctan2(sin(x), cos(x) + 1) + 1/2*I*x*arctan2(sin(x), -
cos(x) + 1) - 1/4*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/4*x*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 1/4*(2*x - sin(2*x))*log(sin(x)) + 1/4*x
 + 1/2*I*dilog(-e^(I*x)) + 1/2*I*dilog(e^(I*x)) + 1/8*sin(2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\sin(x)\right)\,{\sin(x)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*sin(x)^2,x)
```

```
[Out] int(log(sin(x))*sin(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(x\right)\right)\sin^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**2,x)
```

```
[Out] Integral(log(sin(x))*sin(x)**2, x)
```

3.192 $\int \log(\sin(x))\sin^3(x)\,dx$

Optimal. Leaf size=40

$$-\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}-\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

```
[Out] -2/3*arctanh(cos(x))+2/3*cos(x)-1/9*cos(x)^3-cos(x)*ln(sin(x))+1/3*cos(x)^3
*ln(sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {2633, 2554, 12, 4366, 459, 321, 206}

$$-\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}-\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (-2*ArcTanh[Cos[x]])/3 + (2*Cos[x])/3 - Cos[x]^3/9 - Cos[x]*Log[Sin[x]] + (
Cos[x]^3*Log[Sin[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \log(\sin(x))\sin^3(x)\,dx &= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\int \frac{1}{6}\cos(x)(-5+\cos(2x))\cot(x)\,dx \\
&= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{1}{6}\int \cos(x)(-5+\cos(2x))\cot(x)\,dx \\
&= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))+\frac{1}{6}\text{Subst}\left(\int \frac{2x^2\left(-3+x^2\right)}{1-x^2}\,dx,x,\cos
\end{aligned}$$

$$\begin{aligned}
&= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))+\frac{1}{3}\text{Subst}\left(\int \frac{x^2\left(-3+x^2\right)}{1-x^2}\,dx,x,\cos \right. \\
&= -\frac{1}{9}\cos^3(x)-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{2}{3}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, \right. \\
&= \frac{2\cos(x)}{3}-\frac{\cos^3(x)}{9}-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{2}{3}\text{Subst}\left(\int \frac{}{1-}\right. \\
&= -\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{2\cos(x)}{3}-\frac{\cos^3(x)}{9}-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.18

$$\frac{1}{36}\left(24\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)+\cos(3x)(3\log(\sin(x))-1)-3\cos(x)(9\log(\sin(x))-7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (24*(-Log[Cos[x/2]] + Log[Sin[x/2]]) + Cos[3*x]*(-1 + 3*Log[Sin[x]]) - 3*Co
s[x]*(-7 + 9*Log[Sin[x]]))/36
```

fricas [A] time = 0.50, size = 43, normalized size = 1.08

$$-\frac{1}{9}\cos(x)^3+\frac{1}{3}\left(\cos(x)^3-3\cos(x)\right)\log(\sin(x))+\frac{2}{3}\cos(x)-\frac{1}{3}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{3}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(1/2*cos(x) + 1/2) + 1/3*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.17, size = 41, normalized size = 1.02

$$-\frac{1}{9}\cos(x)^3+\frac{1}{3}\left(\cos(x)^3-3\cos(x)\right)\log(\sin(x))+\frac{2}{3}\cos(x)-\frac{1}{3}\log(\cos(x)+1)+\frac{1}{3}\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(cos(x) + 1) + 1/3*log(-cos(x) + 1)
```

maple [C] time = 0.34, size = 134, normalized size = 3.35

$$\frac{e^{-3ix}\ln(2\sin(x))}{24}-\frac{3\,e^{-ix}\ln(2\sin(x))}{8}-\frac{3\,e^{ix}\ln(2\sin(x))}{8}+\frac{e^{3ix}\ln(2\sin(x))}{24}-\frac{e^{-3ix}}{72}-\frac{\ln(2)e^{-3ix}}{24}+\frac{7\,e^{-ix}}{24}+\frac{3\ln(2)e^{-i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^3,x)
```

```
[Out] 1/24*exp(3*I*x)*ln(2*sin(x))-1/72*exp(3*I*x)+7/24*exp(I*x)+2/3*ln(exp(I*x)-
1)-2/3*ln(exp(I*x)+1)-3/8*exp(I*x)*ln(2*sin(x))-3/8*exp(-I*x)*ln(2*sin(x))+
7/24*exp(-I*x)+1/24*exp(-3*I*x)*ln(2*sin(x))-1/72*exp(-3*I*x)-1/24*ln(2)*ex
p(3*I*x)+3/8*ln(2)*exp(I*x)+3/8*ln(2)*exp(-I*x)-1/24*ln(2)*exp(-3*I*x)
```

maxima [B] time = 0.46, size = 179, normalized size = 4.48

$$-\frac{4\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+1\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{3\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}+\frac{2\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{9\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}-\frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*(3*sin(x)^2/(cos(x) + 1)^2 + 1)*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2
 + 1)*(cos(x) + 1)))/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1)^4
 + sin(x)^6/(cos(x) + 1)^6 + 1) + 2/9*(12*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x
)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1
)^4 + sin(x)^6/(cos(x) + 1)^6 + 1) - 2/3*log(sin(x)^2/(cos(x) + 1)^2 + 1) +
 2/3*log(sin(x)^2/(cos(x) + 1)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(\sin(x))\,\sin(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*sin(x)^3,x)
```

```
[Out] int(log(sin(x))*sin(x)^3, x)
```

sympy [B] time = 7.74, size = 439, normalized size = 10.98

$$-\frac{6\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^6\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9}-\frac{18\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^4\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9}+\frac{18\log\left(\tan^2\left(\frac{x}{2}\right)\right.}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**3,x)
```

```
[Out] -6*log(tan(x/2)**2 + 1)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*ta
n(x/2)**2 + 9) - 18*log(tan(x/2)**2 + 1)*tan(x/2)**4/(9*tan(x/2)**6 + 27*ta
n(x/2)**4 + 27*tan(x/2)**2 + 9) + 18*log(tan(x/2)**2 + 1)*tan(x/2)**2/(9*ta
n(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 + 1)/(
9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*log(tan(x/2))*tan
```

```
(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*log(tan
(x/2))*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) +
12*log(2)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9)
 + 6*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 36
*log(2)*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) +
 24*tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 10/
(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9)
```

3.193 $\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=79

$$i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

```
[Out] 1/3*I*x^(3/2)-x*ln(1-exp(2*I*x^(1/2)))+x*ln(sin(x^(1/2)))-1/2*polylog(3,exp
(2*I*x^(1/2)))+I*polylog(2,exp(2*I*x^(1/2)))*x^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {2548, 12, 3748, 3717, 2190, 2531, 2282, 6589}

$$i\sqrt{x}\,\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/3)*x^(3/2) - x*Log[1 - E^((2*I)*Sqrt[x])] + x*Log[Sin[Sqrt[x]]] + I*Sqrt
[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*I)*Sqrt[x])]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx &= x\log\left(\sin\left(\sqrt{x}\right)\right)-\int \frac{1}{2}\sqrt{x}\cot\left(\sqrt{x}\right)\,dx\\
&= x\log\left(\sin\left(\sqrt{x}\right)\right)-\frac{1}{2}\int \sqrt{x}\cot\left(\sqrt{x}\right)\,dx\\
&= x\log\left(\sin\left(\sqrt{x}\right)\right)-\text{Subst}\left(\int x^2\cot(x)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)+2i\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+2\,\text{Subst}\left(\int x\log\left(1-e^{2ix}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right)-i\,\text{Subst}\left(\int \text{Li}_2\left(e^{2ix}\right)\right.\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\right.\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 1.11

$$-i\sqrt{x}\,\text{Li}_2\left(e^{-2i\sqrt{x}}\right)-\frac{1}{2}\text{Li}_3\left(e^{-2i\sqrt{x}}\right)-\frac{1}{3}ix^{3/2}-x\log\left(1-e^{-2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+\frac{i\pi^3}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/24)*Pi^3 - (I/3)*x^(3/2) - x*Log[1 - E^((-2*I)*Sqrt[x])] + x*Log[Sin[Sqr
t[x]]] - I*Sqrt[x]*PolyLog[2, E^((-2*I)*Sqrt[x])] - PolyLog[3, E^((-2*I)*Sq
rt[x])]/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\sin\left(\sqrt{x}\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(log(sin(sqrt(x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="giac")
[Out] integrate(log(sin(sqrt(x))), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \ln\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x^(1/2))),x)
[Out] int(ln(sin(x^(1/2))),x)
```

maxima [B] time = 0.51, size = 139, normalized size = 1.76

$$-i\,x\arctan\left(\sin\left(\sqrt{x}\right),\cos\left(\sqrt{x}\right)+1\right)+i\,x\arctan\left(\sin\left(\sqrt{x}\right),-\cos\left(\sqrt{x}\right)+1\right)-\frac{1}{2}\,x\log\left(\cos\left(\sqrt{x}\right)^2+\sin\left(\sqrt{x}\right)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="maxima")
[Out] -I*x*arctan2(sin(sqrt(x)), cos(sqrt(x)) + 1) + I*x*arctan2(sin(sqrt(x)), -c
os(sqrt(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 + 2*cos(sqrt(x
)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 - 2*cos(sqrt(x)) + 1) +
 x*log(sin(sqrt(x))) + 1/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(I*sqrt(x))) +
2*I*sqrt(x)*dilog(e^(I*sqrt(x))) - 2*polylog(3, -e^(I*sqrt(x))) - 2*polylog
(3, e^(I*sqrt(x)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x^(1/2))),x)
[Out] int(log(sin(x^(1/2))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x**(1/2))),x)
[Out] Integral(log(sin(sqrt(x))), x)
```

3.194 $\int \csc^2(x) \log(\sin(x)) \, dx$

Optimal. Leaf size=15

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

```
[Out] -x-cot(x)-cot(x)*ln(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3767, 8, 2554, 3473}

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(x) \log(\sin(x)) \, dx &= -\cot(x) \log(\sin(x)) + \int \cot^2(x) \, dx \\
&= -\cot(x) - \cot(x) \log(\sin(x)) - \int 1 \, dx \\
&= -x - \cot(x) - \cot(x) \log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

fricas [A] time = 0.50, size = 19, normalized size = 1.27

$$-\frac{\cos(x)\log(\sin(x)) + x\sin(x) + \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x)*log(sin(x)) + x*sin(x) + cos(x))/sin(x)
```

giac [A] time = 0.20, size = 19, normalized size = 1.27

$$-x - \frac{\log(\sin(x))}{\tan(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="giac")
```

```
[Out] -x - log(sin(x))/tan(x) - 1/tan(x)
```

maple [C] time = 0.72, size = 72, normalized size = 4.80

$$-\frac{2ie^{2ix}\ln(2\sin(x))}{e^{2ix}-1} + i\ln\left(e^{ix}+1\right) + i\ln\left(e^{ix}-1\right) - \frac{2i}{e^{2ix}-1} + \frac{2i\ln(2)}{e^{2ix}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*ln(sin(x)),x)
```

```
[Out] -2*I/(exp(2*I*x)-1)*ln(2*sin(x))*exp(2*I*x)-2*I/(exp(2*I*x)-1)+I*ln(exp(I*x
)-1)+I*ln(exp(I*x)+1)+2*I*ln(2)/(exp(2*I*x)-1)
```

maxima [B] time = 0.99, size = 81, normalized size = 5.40

$$-\frac{1}{2}\left(\frac{\cos(x)+1}{\sin(x)} - \frac{\sin(x)}{\cos(x)+1}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right) - \frac{\cos(x)+1}{2\sin(x)} + \frac{\sin(x)}{2(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(x) + 1)/sin(x) - sin(x)/(cos(x) + 1))*log(2*sin(x)/((sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))) - 1/2*(cos(x) + 1)/sin(x) + 1/2*sin(x)/(co
s(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 57, normalized size = 3.80

$$-2x - \ln\left(e^{x\,2i}-1\right)1i - \frac{\ln\left(\frac{e^{-x\,1i}\,1i}{2} - \frac{e^{x\,1i}\,1i}{2}\right)2i}{e^{x\,2i}-1} - \frac{2i}{e^{x\,2i}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))/sin(x)^2,x)
```

```
[Out] - 2*x - log(exp(x*2i) - 1)*1i - (log((exp(-x*1i)*1i)/2 - (exp(x*1i)*1i)/2)*
2i)/(exp(x*2i) - 1) - 2i/(exp(x*2i) - 1)
```

sympy [A] time = 16.68, size = 17, normalized size = 1.13

$$-x - \log(\sin(x))\cot(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*ln(sin(x)),x)
```

```
[Out] -x - log(sin(x))*cot(x) - cos(x)/sin(x)
```

3.195 $\int \log(x) \sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{\cosh(a)\mathrm{Chi}(bx)}{b}-\frac{\sinh(a)\mathrm{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

```
[Out] -Chi(b*x)*cosh(a)/b+cosh(b*x+a)*ln(x)/b-Shi(b*x)*sinh(a)/b
```

Rubi [A] time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2638, 2554, 12, 3303, 3298, 3301}

$$-\frac{\cosh(a)\mathrm{Chi}(bx)}{b}-\frac{\sinh(a)\mathrm{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x])/b) + (Cosh[a + b*x]*Log[x])/b - (Sinh[a]*Sinh
Integral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \log(x)\sinh(a+bx)\,dx = \frac{\cosh(a+bx)\log(x)}{b} - \int \frac{\cosh(a+bx)}{bx}\,dx$$
$$= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b}$$
$$= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{b}$$
$$= -\frac{\cosh(a)\mathrm{Chi}(bx)}{b} + \frac{\cosh(a+bx)\log(x)}{b} - \frac{\sinh(a)\mathrm{Shi}(bx)}{b}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.86

$$-\frac{\cosh(a)\mathrm{Chi}(bx) + \sinh(a)\mathrm{Shi}(bx) - \log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x] - Cosh[a + b*x]*Log[x] + Sinh[a]*SinhIntegral[
b*x])/b)
```

fricas [B] time = 0.48, size = 134, normalized size = 3.83

$$-\frac{(\mathrm{Ei}(bx) + \mathrm{Ei}(-bx))\cosh(bx+a)\cosh(a) - \log(x)\sinh(bx+a)^2 + (\mathrm{Ei}(bx) - \mathrm{Ei}(-bx))\cosh(bx+a)\sinh(a}{2\,(b\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 + 1)*log(x)
+ ((Ei(b*x) + Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) - Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [A] time = 0.16, size = 52, normalized size = 1.49

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b} + \frac{e^{(-bx-a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}(-bx)\,e^{(-a)} + \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b + e^(-b*x - a)/b)*log(x) - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x
)*e^a)/b
```

maple [A] time = 0.70, size = 58, normalized size = 1.66

$$\frac{\mathrm{Ei}(1,bx)\,\mathrm{e}^{-a}}{2b} + \frac{\mathrm{Ei}(1,-bx)\,\mathrm{e}^{a}}{2b} + \left(\frac{\mathrm{e}^{-bx-a}}{2b} + \frac{\mathrm{e}^{bx+a}}{2b}\right)\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a),x)
```

```
[Out] (1/2/b*exp(b*x+a)+1/2*exp(-b*x-a)/b)*ln(x)+1/2/b*exp(-a)*Ei(1,b*x)+1/2/b*ex
p(a)*Ei(1,-b*x)
```

maxima [A] time = 0.69, size = 36, normalized size = 1.03

$$\frac{\cosh(bx+a)\log(x)}{b} - \frac{\mathrm{Ei}(-bx)e^{(-a)} + \mathrm{Ei}(bx)e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)*log(x)/b - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x)*e^a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh(a+b\,x)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*log(x),x)
```

```
[Out] int(sinh(a + b*x)*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a),x)
```

```
[Out] Integral(log(x)*sinh(a + b*x), x)
```

3.196 $\int \log(x)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

```
[Out] 1/2*x-1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/2
*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A] time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 5274, 3303, 3298, 3301}

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] x/2 - (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x]
*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5274

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sinh^2(a+bx)\,dx &= -\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\int \frac{1}{4}\left(-2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= -\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{1}{4}\int\left(-2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int\frac{\sinh(2(a+bx))}{x}\,dx}{4b}\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int\frac{\sinh(2a+2bx)}{x}\,dx}{4b}\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\int\frac{\sinh(2bx)}{x}\,dx}{4b}-\frac{\sinh\cdots}{}\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a\cdots}{4\cdots}\end{aligned}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.76

$$-\frac{\sinh(2a)\text{Chi}(2bx)+\cosh(2a)\text{Shi}(2bx)-\log(x)\sinh(2(a+bx))-2bx+2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*b*x + 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2
*(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

fricas [B] time = 0.47, size = 313, normalized size = 4.74

$$\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\text{Ei}(2bx)+\text{Ei}(-2bx))\cosh(bx+a)^2\sinh(2a)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) + (4*b*x - (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 + (4*b*x - (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x - 3*cosh(b*x + a)^2)*log(x) - (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 - (4*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 + 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) - (4*b*x - (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) + 2*(2*b*x*cosh(b*x + a) - cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.19, size = 67, normalized size = 1.02

$$-\frac{1}{8}\left(4x-\frac{e^{(2bx+2a)}}{b}+\frac{e^{(-2bx-2a)}}{b}\right)\log(x)+\frac{4bx-\mathrm{Ei}(2bx)e^{(2a)}+\mathrm{Ei}(-2bx)e^{(-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/8*(4*b*x - E
i(2*b*x)*e^(2*a) + Ei(-2*b*x)*e^(-2*a))/b
```

maple [A] time = 1.30, size = 97, normalized size = 1.47

$$-\frac{a\ln(bx)}{2b}+\frac{a\ln(-bx)}{2b}+\frac{\mathrm{Ei}(1,-2bx)\,\mathrm{e}^{2a}}{8b}-\frac{\mathrm{Ei}(1,2bx)\,\mathrm{e}^{-2a}}{8b}+\frac{x}{2}+\left(-\frac{x}{2}-\frac{\mathrm{e}^{-2bx-2a}}{8b}+\frac{\mathrm{e}^{2bx+2a}}{8b}\right)\ln(x)+\frac{a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^2,x)
```

```
[Out] (-1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei
(1,-2*b*x)-1/2/b*a*ln(b*x)+1/2/b*a*ln(-b*x)+1/2*x+1/2*a/b-1/8/b*exp(-2*a)*E
i(1,2*b*x)
```

maxima [A] time = 0.69, size = 67, normalized size = 1.02

$$-\frac{1}{8}\left(4x-\frac{e^{(2bx+2a)}}{b}+\frac{e^{(-2bx-2a)}}{b}\right)\log(x)+\frac{1}{2}x-\frac{\mathrm{Ei}(2bx)e^{(2a)}}{8b}+\frac{\mathrm{Ei}(-2bx)e^{(-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/2*x - 1/8*Ei
(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sinh(a+b\,x)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*log(x),x)
```

```
[Out] int(sinh(a + b*x)^2*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**2, x)
```

3.197 $\int \log(x)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cosh(a)\text{Chi}(bx)}{4b}-\frac{\cosh(3a)\text{Chi}(3bx)}{12b}+\frac{3\sinh(a)\text{Shi}(bx)}{4b}-\frac{\sinh(3a)\text{Shi}(3bx)}{12b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh}{b}$$

```
[Out] 3/4*Chi(b*x)*cosh(a)/b-1/12*Chi(3*b*x)*cosh(3*a)/b-cosh(b*x+a)*ln(x)/b+1/3*
cosh(b*x+a)^3*ln(x)/b+3/4*Shi(b*x)*sinh(a)/b-1/12*Shi(3*b*x)*sinh(3*a)/b
```

Rubi [A] time = 0.52, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3298, 3301, 3312}

$$\frac{3\cosh(a)\text{Chi}(bx)}{4b}-\frac{\cosh(3a)\text{Chi}(3bx)}{12b}+\frac{3\sinh(a)\text{Shi}(bx)}{4b}-\frac{\sinh(3a)\text{Shi}(3bx)}{12b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x])/(4*b) - (Cosh[3*a]*CoshIntegral[3*b*x])/(12*b
) - (Cosh[a + b*x]*Log[x])/b + (Cosh[a + b*x]^3*Log[x])/(3*b) + (3*Sinh[a]*
SinhIntegral[b*x])/(4*b) - (Sinh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log(x)\sinh^3(a+bx)\,dx = -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\int \frac{\cosh(a+bx)\left(-3+\cosh^2(a\right.}{3bx}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{x}\,dx}{3b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \left(-\frac{3\cosh(a+bx)}{x}+\frac{\cosh^3(a+bx)}{x}\right) dx}{3b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cosh^3(a+bx)}{x}\,dx}{3b}+\frac{\int \frac{\cosh(a+bx)}{x}\,d}{b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \left(\frac{3\cosh(a+bx)}{4x}+\frac{\cosh(3a+3bx)}{4x}\right) dx}{3b}$$

$$= \frac{\cosh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}+\frac{\sinh(a)\text{Shi}(bx}{b}$$

$$= \frac{\cosh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}+\frac{\sinh(a)\text{Shi}(bx}{b}$$

$$= \frac{3\cosh(a)\text{Chi}(bx)}{4b}-\frac{\cosh(3a)\text{Chi}(3bx)}{12b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\,l}{3b}$$

Mathematica [A] time = 0.11, size = 67, normalized size = 0.75

$$\frac{9\cosh(a)\text{Chi}(bx)-\cosh(3a)\text{Chi}(3bx)+9\sinh(a)\text{Shi}(bx)-\sinh(3a)\text{Shi}(3bx)-9\log(x)\cosh(a+bx)+\log(}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a]*CoshIntegral[b*x] - Cosh[3*a]*CoshIntegral[3*b*x] - 9*Cosh[a + b
*x]*Log[x] + Cosh[3*(a + b*x)]*Log[x] + 9*Sinh[a]*SinhIntegral[b*x] - Sinh[
3*a]*SinhIntegral[3*b*x])/(12*b)
```

fricas [B] time = 0.48, size = 587, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) - Ei(-3*b*x))*co
```

```
sh(b*x + a)^3*sinh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 - 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) - Ei(-
3*b*x))*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 - 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 + 1)*log(x) - 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) - 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) + Ei(-3*b
*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 - 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

giac [A] time = 0.21, size = 102, normalized size = 1.15

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)} - 9\,\mathrm{Ei}\,(-bx)\,e^{(-a)} + \mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)} - 9\,\mathrm{E}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) + Ei(-3*b*x)*e
^(-3*a) - 9*Ei(b*x)*e^a)/b
```

maple [A] time = 1.04, size = 116, normalized size = 1.30

$$-\frac{3\,\mathrm{Ei}\,(1,bx)\,\mathrm{e}^{-a}}{8b} + \frac{\mathrm{Ei}\,(1,-3bx)\,\mathrm{e}^{3a}}{24b} - \frac{3\,\mathrm{Ei}\,(1,-bx)\,\mathrm{e}^{a}}{8b} + \frac{\mathrm{Ei}\,(1,3bx)\,\mathrm{e}^{-3a}}{24b} + \left(\frac{\mathrm{e}^{-3bx-3a}}{24b} - \frac{3\,\mathrm{e}^{-bx-a}}{8b} - \frac{3\,\mathrm{e}^{bx+a}}{8b} + \frac{\mathrm{e}^{3bx+3a}}{24b}\right)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^3,x)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)-3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)+1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(-3*a)*Ei(1,3*b*x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)-3/8/b*exp(a)*Ei(1,-b*x)
```

maxima [A] time = 0.77, size = 110, normalized size = 1.24

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)}}{24\,b} + \frac{3\,\mathrm{Ei}\,(-bx)\,e^{(-a)}}{8\,b} - \frac{\mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)}}{24\,b} + \frac{3\,\mathrm{Ei}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b - 1/24*Ei
(-3*b*x)*e^(-3*a)/b + 3/8*Ei(b*x)*e^a/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+b\,x\right)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*log(x),x)
```

```
[Out] int(sinh(a + b*x)^3*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**3,x)
[Out] Integral(log(x)*sinh(a + b*x)**3, x)
```

3.198 $\int \cosh(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\sinh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a)\text{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

```
[Out] -cosh(a)*Shi(b*x)/b-Chi(b*x)*sinh(a)/b+ln(x)*sinh(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2637, 2554, 12, 3303, 3298, 3301}

$$-\frac{\sinh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a)\text{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a])/b) + (Log[x]*Sinh[a + b*x])/b - (Cosh[a]*Sinh
Integral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \cosh(a+bx)\log(x)\,dx = \frac{\log(x)\sinh(a+bx)}{b} - \int \frac{\sinh(a+bx)}{bx}\,dx$$
$$= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}$$
$$= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{b}$$
$$= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\text{Shi}(bx)}{b}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.86

$$-\frac{\sinh(a)\text{Chi}(bx) + \cosh(a)\text{Shi}(bx) - \log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a] - Log[x]*Sinh[a + b*x] + Cosh[a]*SinhIntegral[
b*x])/b)
```

fricas [B] time = 0.45, size = 134, normalized size = 3.83

$$-\frac{(\text{Ei}(bx) - \text{Ei}(-bx))\cosh(bx+a)\cosh(a) - \log(x)\sinh(bx+a)^2 + (\text{Ei}(bx) + \text{Ei}(-bx))\cosh(bx+a)\sinh(a}{2\,(b\,\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 - 1)*log(x)
+ ((Ei(b*x) - Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) + Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [A] time = 0.17, size = 54, normalized size = 1.54

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b} - \frac{e^{(-bx-a)}}{b}\right)\log(x) + \frac{\text{Ei}(-bx)\,e^{(-a)} - \text{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(x) + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x
)*e^a)/b
```

maple [A] time = 0.94, size = 58, normalized size = 1.66

$$-\frac{\text{Ei}(1, bx)\,e^{-a}}{2b} + \frac{\text{Ei}(1, -bx)\,e^{a}}{2b} + \left(-\frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}}{2b}\right)\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(x),x)
```

```
[Out] (1/2/b*exp(b*x+a)-1/2/b*exp(-b*x-a))*ln(x)+1/2/b*exp(a)*Ei(1,-b*x)-1/2/b*ex
p(-a)*Ei(1,b*x)
```

maxima [A] time = 1.23, size = 37, normalized size = 1.06

$$\frac{\log(x)\sinh\left(bx+a\right)}{b}+\frac{\mathrm{Ei}\left(-bx\right)e^{(-a)}-\mathrm{Ei}\left(bx\right)e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sinh(b*x + a)/b + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x)*e^a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\cosh\left(a+b\,x\right)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*log(x),x)
```

```
[Out] int(cosh(a + b*x)*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(x\right)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x), x)
```

3.199 $\int \cosh^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/
2*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 5274, 3303, 3298, 3301}

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -x/2 + (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x
]*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5274

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\int\frac{1}{4}\left(2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{1}{4}\int\left(2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int\frac{\sinh(2(a+bx))}{x}\,dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int\frac{\sinh(2a+2bx)}{x}\,dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\int\frac{\sinh(2bx)}{x}\,dx}{4b}-\frac{\text{sir}}{}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2}{}\end{aligned}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.76

$$-\frac{\sinh(2a)\text{Chi}(2bx)+\cosh(2a)\text{Shi}(2bx)-\log(x)\sinh(2(a+bx))+2bx-2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2*
(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

fricas [B] time = 0.48, size = 305, normalized size = 4.62

$$\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\text{Ei}(2bx)+\text{Ei}(-2bx))\cosh(bx+a)^2\sinh(2a)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) - (4*b*x + (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 - (4*b*x + (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x + 3*cosh(b*x + a)^2)*log(x) + (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 + (4*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 - 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) + (4*b*x + (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) - 2*(2*b*x*cosh(b*x + a) + cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.23, size = 67, normalized size = 1.02

$$\frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)-\frac{4\,bx+\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/8*(4*b*x + Ei
(2*b*x)*e^(2*a) - Ei(-2*b*x)*e^(-2*a))/b
```

maple [A] time = 1.24, size = 97, normalized size = 1.47

$$\frac{a\ln\left(bx\right)}{2b}-\frac{a\ln\left(-bx\right)}{2b}+\frac{\mathrm{Ei}\left(1,-2bx\right)\mathrm{e}^{2a}}{8b}-\frac{\mathrm{Ei}\left(1,2bx\right)\mathrm{e}^{-2a}}{8b}-\frac{x}{2}+\left(\frac{x}{2}-\frac{\mathrm{e}^{-2bx-2a}}{8b}+\frac{\mathrm{e}^{2bx+2a}}{8b}\right)\ln(x)-\frac{a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*ln(x),x)
```

```
[Out] (1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei(
1,-2*b*x)+1/2*a/b*ln(b*x)-1/2*a/b*ln(-b*x)-1/2*x-1/2*a/b-1/8/b*exp(-2*a)*Ei
(1,2*b*x)
```

maxima [A] time = 0.68, size = 67, normalized size = 1.02

$$\frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)-\frac{1}{2}\,x-\frac{\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/2*x - 1/8*Ei(
2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh\left(a+b\,x\right)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*log(x),x)
```

```
[Out] int(cosh(a + b*x)^2*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(x\right)\cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**2, x)
```

3.200 $\int \cosh^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\sinh(a)\text{Chi}(bx)}{4b}-\frac{\sinh(3a)\text{Chi}(3bx)}{12b}-\frac{3\cosh(a)\text{Shi}(bx)}{4b}-\frac{\cosh(3a)\text{Shi}(3bx)}{12b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sin}{b}$$

```
[Out] -3/4*cosh(a)*Shi(b*x)/b-1/12*cosh(3*a)*Shi(3*b*x)/b-3/4*Chi(b*x)*sinh(a)/b-
1/12*Chi(3*b*x)*sinh(3*a)/b+ln(x)*sinh(b*x+a)/b+1/3*ln(x)*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.48, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3298, 3301, 3312}

$$-\frac{3\sinh(a)\text{Chi}(bx)}{4b}-\frac{\sinh(3a)\text{Chi}(3bx)}{12b}-\frac{3\cosh(a)\text{Shi}(bx)}{4b}-\frac{\cosh(3a)\text{Shi}(3bx)}{12b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sin}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CoshIntegral[b*x]*Sinh[a])/(4*b) - (CoshIntegral[3*b*x]*Sinh[3*a])/(12*
b) + (Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - (3*Cosh[a]
*SinhIntegral[b*x])/(4*b) - (Cosh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\log(x)\,dx &= \frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx\right.}{3bx}\\ &= \frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{\int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx)\right)}{x}\,dx}{3b}\\ &= \frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{\int \left(\frac{3\sinh(a+bx)}{x}+\frac{\sinh^3(a+bx)}{x}\right)dx}{3b}\\ &= \frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{\int \frac{\sinh^3(a+bx)}{x}\,dx}{3b}-\frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}\\ &= \frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{i\int \left(\frac{3i\sinh(a+bx)}{4x}-\frac{i\sinh(3a+3bx)}{4x}\right)dx}{3b}\\ &= -\frac{\mathrm{Chi}(bx)\sinh(a)}{b}+\frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{\cosh(a)\mathrm{Shi}(}{b}\\ &= -\frac{\mathrm{Chi}(bx)\sinh(a)}{b}+\frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{\cosh(a)\mathrm{Shi}(}{b}\\ &= -\frac{3\mathrm{Chi}(bx)\sinh(a)}{4b}-\frac{\mathrm{Chi}(3bx)\sinh(3a)}{12b}+\frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(}{3b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 66, normalized size = 0.75

$$-\frac{9\sinh(a)\mathrm{Chi}(bx)+\sinh(3a)\mathrm{Chi}(3bx)+9\cosh(a)\mathrm{Shi}(bx)+\cosh(3a)\mathrm{Shi}(3bx)-9\log(x)\sinh(a+bx)-\log}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] -1/12*(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] - 9*Log[
x]*Sinh[a + b*x] - Log[x]*Sinh[3*(a + b*x)] + 9*Cosh[a]*SinhIntegral[b*x] +
 Cosh[3*a]*SinhIntegral[3*b*x])/b
```

fricas [B] time = 0.49, size = 587, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 + 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) + Ei(-3*b*x))*co
```

```
sh(b*x + a)^3*sinh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 + 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) + Ei(-
3*b*x))*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 + 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 - 1)*log(x) - 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) + 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) - Ei(-3*b
*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 + 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

giac [A] time = 0.20, size = 104, normalized size = 1.18

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-9\,\mathrm{Ei}\left(-bx\right)e^{(-a)}-\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+9\,\mathrm{E}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) - Ei(-3*b*x)*e
^(-3*a) + 9*Ei(b*x)*e^a)/b
```

maple [A] time = 1.69, size = 116, normalized size = 1.32

$$-\frac{3\,\mathrm{Ei}\left(1,bx\right)\mathrm{e}^{-a}}{8b}+\frac{\mathrm{Ei}\left(1,-3bx\right)\mathrm{e}^{3a}}{24b}+\frac{3\,\mathrm{Ei}\left(1,-bx\right)\mathrm{e}^{a}}{8b}-\frac{\mathrm{Ei}\left(1,3bx\right)\mathrm{e}^{-3a}}{24b}+\left(-\frac{\mathrm{e}^{-3bx-3a}}{24b}-\frac{3\,\mathrm{e}^{-bx-a}}{8b}+\frac{3\,\mathrm{e}^{bx+a}}{8b}+\frac{\mathrm{e}^{3bx+3a}}{24b}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*ln(x),x)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)+3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)-1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-1/24/b*exp(-3*a)*Ei(1,3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)+3/8/b*exp(a)*Ei(1,-b*x)
```

maxima [A] time = 0.77, size = 111, normalized size = 1.26

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}\left(-bx\right)e^{(-a)}}{8\,b}+\frac{\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}}{24\,b}-\frac{3\,\mathrm{Ei}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b + 1/24*Ei
(-3*b*x)*e^(-3*a)/b - 3/8*Ei(b*x)*e^a/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(a+b\,x\right)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*log(x),x)
```

```
[Out] int(cosh(a + b*x)^3*log(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*ln(x),x)
[Out] Integral(log(x)*cosh(a + b*x)**3, x)
```

3.201 $\int \log(a\sinh(x))\,dx$

Optimal. Leaf size=39

$$x\log(a\sinh(x)) - \frac{\text{Li}_2\left(e^{2x}\right)}{2} + \frac{x^2}{2} - x\log\left(1-e^{2x}\right)$$

[Out] 1/2*x^2-x*ln(1-exp(2*x))+x*ln(a*sinh(x))-1/2*polylog(2,exp(2*x))

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right) + x\log(a\sinh(x)) + \frac{x^2}{2} - x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Sinh[x]],x]

[Out] x^2/2 - x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]] - PolyLog[2, E^(2*x)]/2

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\sinh(x))\,dx &= x\log(a\sinh(x)) - \int x\coth(x)\,dx \\
&= \frac{x^2}{2} + x\log(a\sinh(x)) + 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \frac{1}{2}\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(\text{Li}_2\left(e^{-2x}\right) - x\left(-2\log(a\sinh(x)) + x + 2\log\left(1-e^{-2x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]])) + PolyLog[2, E^(-2*x)])/
2
```

fricas [A] time = 0.48, size = 57, normalized size = 1.46

$$\frac{1}{2}x^2 + x\log\left(a\sinh(x)\right) - x\log\left(\cosh(x) + \sinh(x) + 1\right) - x\log\left(-\cosh(x) - \sinh(x) + 1\right) - \text{Li}_2\left(\cosh(x) + \sinh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(cosh(x) + sinh(x) + 1) - x*log(-cosh(x)
- sinh(x) + 1) - dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sinh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)), x)
```

maple [C] time = 0.99, size = 295, normalized size = 7.56

$$-\frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(i\left(e^{2x}-1\right)e^{-x}\right)\text{csgn}\left(i\left(e^{2x}-1\right)a\,e^{-x}\right)}{2} + \frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(i\left(e^{2x}-1\right)a\,e^{-x}\right)^2}{2} - \frac{i\pi x\,\text{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)),x)
```

```
[Out] -x*ln(exp(x))+1/2*I*Pi*csgn(I*(exp(2*x)-1))*csgn(I*exp(-x)*(exp(2*x)-1))^2*
x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)-1)*exp(-x))^2*x-1/2*I*Pi*csgn(I*exp
(-x)*(exp(2*x)-1))^3*x+1/2*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)-1)
)^2*x-1/2*I*Pi*csgn(I*a*(exp(2*x)-1)*exp(-x))^3*x-1/2*I*Pi*csgn(I*(exp(2*x)
```

```
-1))*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)-1))*x-ln(2)*x+x*ln(a)+1/2*x^2
+1/2*I*Pi*csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x))^2*x-1
/2*I*Pi*csgn(I*a)*csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x
))*x+ln(exp(x))*ln(exp(2*x)-1)+dilog(exp(x))-dilog(exp(x)+1)-ln(exp(x))*ln(
exp(x)+1)
```

maxima [A] time = 0.65, size = 43, normalized size = 1.10

$$\frac{1}{2}x^2 + x\log(a\sinh(x)) - x\log(e^x + 1) - x\log(-e^x + 1) - \mathrm{Li}_2(-e^x) - \mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="maxima")
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(e^x + 1) - x*log(-e^x + 1) - dilog(-e^x)
 - dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln(a\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)),x)
[Out] int(log(a*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(a\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)),x)
[Out] Integral(log(a*sinh(x)), x)
```

3.202 $\int \log\left(a\sinh^2(x)\right)\,dx$

Optimal. Leaf size=35

$$x\log\left(a\sinh^2(x)\right)-\text{Li}_2\left(e^{2x}\right)+x^2-2x\log\left(1-e^{2x}\right)$$

```
[Out] x^2-2*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^2)-polylog(2,exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^2(x)\right)+x^2-2x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]^2],x]

[Out] x^2 - 2*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^2] - PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log\left(a\sinh^2(x)\right)\,dx = x\log\left(a\sinh^2(x)\right) - \int 2x\coth(x)\,dx$$
$$= x\log\left(a\sinh^2(x)\right) - 2\int x\coth(x)\,dx$$
$$= x^2 + x\log\left(a\sinh^2(x)\right) + 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx$$
$$= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + 2\int \log\left(1-e^{2x}\right)\,dx$$
$$= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)$$
$$= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) - \text{Li}_2\left(e^{2x}\right)$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.94

$$x\left(\log\left(a\sinh^2(x)\right) - x - 2\log\left(1-e^{-2x}\right)\right) + \text{Li}_2\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 - E^(-2*x)] + Log[a*Sinh[x]^2]) + PolyLog[2, E^(-2*x)]
```

fricas [B] time = 0.48, size = 69, normalized size = 1.97

$$x^2 + x\log\left(\frac{1}{2}\,a\cosh(x)^2 + \frac{1}{2}\,a\sinh(x)^2 - \frac{1}{2}\,a\right) - 2\,x\log\left(\cosh(x) + \sinh(x) + 1\right) - 2\,x\log\left(-\cosh(x) - \sinh(x) + 1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 - 1/2*a) - 2*x*log(cosh(x) +
sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x) + sinh(x))
 - 2*dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sinh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^2), x)
```

maple [C] time = 2.24, size = 454, normalized size = 12.97

$$-\frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(i\left(e^{2x}-1\right)^2 e^{-2x}\right)\text{csgn}\left(i\left(e^{2x}-1\right)^2 a\,e^{-2x}\right)}{2} + \frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(i\left(e^{2x}-1\right)^2 a\,e^{-2x}\right)^2}{2} - \frac{i\pi x\text{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^2),x)
```

```
[Out] -2*ln(2)*x+1/2*I*Pi*csgn(I*(exp(2*x)-1)^2)*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)
^2*x+1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x+x*ln(a)+x^2+1/2*I*Pi*csgn
(I*exp(-2*x)*(exp(2*x)-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))^2*x-1/2*I*P
```

```
i*csgn(I*(exp(2*x)-1)^2)*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)
*x+1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^2*x+1/2*I*Pi
*csgn(I*exp(2*x))^3*x+2*dilog(exp(x))-2*dilog(exp(x)+1)-2*ln(exp(x)+1)*ln(e
xp(x))-2*x*ln(exp(x))+2*ln(exp(2*x)-1)*ln(exp(x))-1/2*I*Pi*csgn(I*a*(exp(2*
x)-1)^2*exp(-2*x))^3*x-I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x-1/2*I*Pi*cs
gn(I*(exp(2*x)-1)^2)^3*x-1/2*I*Pi*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^3*x-1/2*
I*Pi*csgn(I*a)*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp
(-2*x))*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))^2*x+I*Pi*cs
gn(I*(exp(2*x)-1))*csgn(I*(exp(2*x)-1)^2)^2*x-1/2*I*Pi*csgn(I*(exp(2*x)-1))
^2*csgn(I*(exp(2*x)-1)^2)*x
```

maxima [A] time = 0.65, size = 43, normalized size = 1.23

$$x^2 + x\log\left(a\sinh(x)^2\right) - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2\left(-e^x\right) - 2\,\mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="maxima")
[Out] x^2 + x*log(a*sinh(x)^2) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-
e^x) - 2*dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\sinh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)^2),x)
[Out] int(log(a*sinh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sinh^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**2),x)
[Out] Integral(log(a*sinh(x)**2), x)
```

3.203 $\int \log\left(a\sinh^n(x)\right)\,dx$

Optimal. Leaf size=44

$$x\log\left(a\sinh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)+\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)$$

[Out] 1/2*n*x^2-n*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^n)-1/2*n*polylog(2,exp(2*x))

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Sinh[x]^n],x]

[Out] (n*x^2)/2 - n*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^n] - (n*PolyLog[2, E^(2*x)])/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\sinh^n(x)\right)dx &= x\log\left(a\sinh^n(x)\right)-\int nx\coth(x)\,dx\\ &= x\log\left(a\sinh^n(x)\right)-n\int x\coth(x)\,dx\\ &= \frac{nx^2}{2}+x\log\left(a\sinh^n(x)\right)+(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\ &= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+n\int\log\left(1-e^{2x}\right)dx\\ &= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{1}{2}n\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\ &= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(n\text{Li}_2\left(e^{-2x}\right)-x\left(-2\log\left(a\sinh^n(x)\right)+nx+2n\log\left(1-e^{-2x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^n],x]
[Out] (-(x*(n*x + 2*n*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]^n])) + n*PolyLog[2, E^(
-2*x)])/2
```

fricas [A] time = 0.48, size = 65, normalized size = 1.48

$$\frac{1}{2}nx^2-nx\log\left(\cosh(x)+\sinh(x)+1\right)-nx\log\left(-\cosh(x)-\sinh(x)+1\right)+nx\log\left(\sinh(x)\right)-n\text{Li}_2\left(\cosh(x)+\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="fricas")
[Out] 1/2*n*x^2 - n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(-cosh(x) - sinh(x) + 1
) + n*x*log(sinh(x)) - n*dilog(cosh(x) + sinh(x)) - n*dilog(-cosh(x) - sinh
(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sinh(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="giac")
[Out] integrate(log(a*sinh(x)^n), x)
```

maple [F] time = 2.09, size = 0, normalized size = 0.00

$$\int\ln\left(a\left(\sinh^n(x)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^n),x)
[Out] int(ln(a*sinh(x)^n),x)
```

maxima [A] time = 0.65, size = 47, normalized size = 1.07

$$\frac{1}{2}\left(x^{2}-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)-2\,\mathrm{Li}_{2}\left(-e^{x}\right)-2\,\mathrm{Li}_{2}\left(e^{x}\right)\right)n+x\log\left(a\sinh(x)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(e
^x))*n + x*log(a*sinh(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sinh(x)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)^n),x)
```

```
[Out] int(log(a*sinh(x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sinh^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**n),x)
```

```
[Out] Integral(log(a*sinh(x)**n), x)
```

3.204 $\int \log(a\cosh(x))\,dx$

Optimal. Leaf size=39

$$x\log(a\cosh(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{x^2}{2} - x\log\left(e^{2x}+1\right)$$

```
[Out] 1/2*x^2-x*ln(1+exp(2*x))+x*ln(a*cosh(x))-1/2*polylog(2,-exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + x\log(a\cosh(x)) + \frac{x^2}{2} - x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]] - PolyLog[2, -E^(2*x)]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\cosh(x))\,dx &= x\log(a\cosh(x)) - \int x\tanh(x)\,dx\\ &= \frac{x^2}{2} + x\log(a\cosh(x)) - 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \int \log\left(1+e^{2x}\right)\,dx\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\operatorname{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(\operatorname{Li}_2\left(-e^{-2x}\right) - x\left(-2\log(a\cosh(x)) + x + 2\log\left(e^{-2x}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]])) + PolyLog[2, -E^(-2*x)])
/2
```

fricas [C] time = 0.48, size = 65, normalized size = 1.67

$$\frac{1}{2}x^2 + x\log\left(a\cosh(x)\right) - x\log\left(i\cosh(x) + i\sinh(x) + 1\right) - x\log\left(-i\cosh(x) - i\sinh(x) + 1\right) - \operatorname{Li}_2\left(i\cosh(x) + i\,\text{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(I*cosh(x) + I*sinh(x) + 1) - x*log(-I*co
sh(x) - I*sinh(x) + 1) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I*cosh(x) -
I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)), x)
```

maple [C] time = 1.04, size = 321, normalized size = 8.23

$$-\frac{i\pi x\operatorname{csgn}(ia)\operatorname{csgn}\left(i\left(e^{2x}+1\right)e^{-x}\right)\operatorname{csgn}\left(i\left(e^{2x}+1\right)a\,e^{-x}\right)}{2} + \frac{i\pi x\operatorname{csgn}(ia)\operatorname{csgn}\left(i\left(e^{2x}+1\right)a\,e^{-x}\right)^2}{2} - \frac{i\pi x\operatorname{csgn}\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)),x)
```

```
[Out] -x*ln(exp(x))-1/2*I*Pi*csgn(I*a*(exp(2*x)+1)*exp(-x))^3*x+1/2*I*Pi*csgn(I*(
exp(2*x)+1))*csgn(I*exp(-x)*(exp(2*x)+1))^2*x-1/2*I*Pi*csgn(I*a)*csgn(I*exp
(-x)*(exp(2*x)+1))*csgn(I*a*(exp(2*x)+1)*exp(-x))*x+1/2*I*Pi*csgn(I*exp(-x)
```

```
*(exp(2*x)+1))*csgn(I*a*(exp(2*x)+1)*exp(-x))^2*x+1/2*I*Pi*csgn(I*exp(-x))*
csgn(I*exp(-x)*(exp(2*x)+1))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)+1)*e
xp(-x))^2*x+x*ln(a)-ln(2)*x+1/2*x^2-1/2*I*Pi*csgn(I*exp(-x))*csgn(I*(exp(2*
x)+1))*csgn(I*exp(-x)*(exp(2*x)+1))*x-1/2*I*Pi*csgn(I*exp(-x)*(exp(2*x)+1))
^3*x+ln(exp(x))*ln(exp(2*x)+1)-ln(exp(x))*ln(1+I*exp(x))-ln(exp(x))*ln(1-I*
exp(x))-dilog(1+I*exp(x))-dilog(1-I*exp(x))
```

maxima [A] time = 1.15, size = 32, normalized size = 0.82

$$\frac{1}{2}\,x^2 + x\log\left(a\cosh(x)\right) - x\log\left(e^{(2\,x)} + 1\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="maxima")
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(e^(2*x) + 1) - 1/2*dilog(-e^(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)),x)
[Out] int(log(a*cosh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)),x)
[Out] Integral(log(a*cosh(x)), x)
```

3.205 $\int \log\left(a\cosh^2(x)\right)\,dx$

Optimal. Leaf size=35

$$x\log\left(a\cosh^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)+x^2-2x\log\left(e^{2x}+1\right)$$

```
[Out] x^2-2*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^2)-polylog(2,-exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+x^2-2x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^2] - PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\cosh^2(x)\right)\,dx &= x\log\left(a\cosh^2(x)\right) - \int 2x\tanh(x)\,dx \\
&= x\log\left(a\cosh^2(x)\right) - 2\int x\tanh(x)\,dx \\
&= x^2 + x\log\left(a\cosh^2(x)\right) - 4\int \frac{e^{2x}x}{1+e^{2x}}\,dx \\
&= x^2 - 2x\log\left(1+e^{2x}\right) + x\log\left(a\cosh^2(x)\right) + 2\int \log\left(1+e^{2x}\right)\,dx \\
&= x^2 - 2x\log\left(1+e^{2x}\right) + x\log\left(a\cosh^2(x)\right) + \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= x^2 - 2x\log\left(1+e^{2x}\right) + x\log\left(a\cosh^2(x)\right) - \text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.94

$$
x\left(\log\left(a\cosh^2(x)\right) - x - 2\log\left(e^{-2x}+1\right)\right) + \text{Li}_2\left(-e^{-2x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 + E^(-2*x)] + Log[a*Cosh[x]^2]) + PolyLog[2, -E^(-2*x)]
```

fricas [C] time = 0.50, size = 77, normalized size = 2.20

$$
x^2 + x\log\left(\frac{1}{2}a\cosh(x)^2 + \frac{1}{2}a\sinh(x)^2 + \frac{1}{2}a\right) - 2x\log\left(i\cosh(x) + i\sinh(x) + 1\right) - 2x\log\left(-i\cosh(x) - i\sinh
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 + 1/2*a) - 2*x*log(I*cosh(x)
+ I*sinh(x) + 1) - 2*x*log(-I*cosh(x) - I*sinh(x) + 1) - 2*dilog(I*cosh(x)
+ I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log\left(a\cosh(x)^2\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^2), x)
```

maple [C] time = 2.36, size = 478, normalized size = 13.66

$$
-\frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(i\left(e^{2x}+1\right)^2 e^{-2x}\right)\text{csgn}\left(i\left(e^{2x}+1\right)^2 a\,e^{-2x}\right)}{2} + \frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(i\left(e^{2x}+1\right)^2 a\,e^{-2x}\right)^2}{2} - \frac{i\pi
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^2),x)
```

```
[Out] -2*ln(2)*x+x*ln(a)+x^2+2*ln(exp(2*x)+1)*ln(exp(x))-1/2*I*Pi*csgn(I*a)*csgn(
I*exp(-2*x)*(exp(2*x)+1)^2)*csgn(I*a*(exp(2*x)+1)^2*exp(-2*x))*x-2*ln(I*exp
(x)+1)*ln(exp(x))-2*ln(-I*exp(x)+1)*ln(exp(x))-2*dilog(I*exp(x)+1)-2*dilog(
```

```
-I*exp(x)+1)+1/2*I*Pi*csgn(I*exp(-2*x)*(exp(2*x)+1)^2)*csgn(I*a*(exp(2*x)+1
)^2*exp(-2*x))^2*x+1/2*I*Pi*csgn(I*(exp(2*x)+1)^2)*csgn(I*exp(-2*x)*(exp(2*
x)+1)^2)^2*x-I*Pi*x*csgn(I*exp(x))*csgn(I*exp(2*x))^2-2*x*ln(exp(x))-1/2*I*
Pi*csgn(I*exp(-2*x)*(exp(2*x)+1)^2)^3*x+1/2*I*Pi*x*csgn(I*exp(x))^2*csgn(I*
exp(2*x))+I*Pi*csgn(I*(exp(2*x)+1))*csgn(I*(exp(2*x)+1)^2)^2*x+1/2*I*Pi*x*c
sgn(I*exp(2*x))^3-1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*(exp(2*x)+1)^2)*csgn(I*
exp(-2*x)*(exp(2*x)+1)^2)*x-1/2*I*Pi*csgn(I*(exp(2*x)+1))^2*csgn(I*(exp(2*x
)+1)^2)*x+1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)+1)^2)^2*x+1
/2*I*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)+1)^2*exp(-2*x))^2*x-1/2*I*Pi*csgn(I*(e
xp(2*x)+1)^2)^3*x-1/2*I*Pi*csgn(I*a*(exp(2*x)+1)^2*exp(-2*x))^3*x
```

maxima [A] time = 1.15, size = 32, normalized size = 0.91

$$x^2 + x\log\left(a\cosh(x)^2\right) - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="maxima")
[Out] x^2 + x*log(a*cosh(x)^2) - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\cosh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)^2),x)
[Out] int(log(a*cosh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**2),x)
[Out] Integral(log(a*cosh(x)**2), x)
```

3.206 $\int \log\left(a\cosh^n(x)\right)\,dx$

Optimal. Leaf size=44

$$x\log\left(a\cosh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)+\frac{nx^2}{2}-nx\log\left(e^{2x}+1\right)$$

```
[Out] 1/2*n*x^2-n*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^n)-1/2*n*polylog(2,-exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^n] - (n*PolyLog[2, -E^(2
*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\cosh^n(x)\right)dx &= x\log\left(a\cosh^n(x)\right) - \int nx\tanh(x)\,dx\\ &= x\log\left(a\cosh^n(x)\right) - n\int x\tanh(x)\,dx\\ &= \frac{nx^2}{2} + x\log\left(a\cosh^n(x)\right) - (2n)\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= \frac{nx^2}{2} - nx\log\left(1+e^{2x}\right) + x\log\left(a\cosh^n(x)\right) + n\int \log\left(1+e^{2x}\right)dx\\ &= \frac{nx^2}{2} - nx\log\left(1+e^{2x}\right) + x\log\left(a\cosh^n(x)\right) + \frac{1}{2}n\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= \frac{nx^2}{2} - nx\log\left(1+e^{2x}\right) + x\log\left(a\cosh^n(x)\right) - \frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(n\text{Li}_2\left(-e^{-2x}\right) - x\left(-2\log\left(a\cosh^n(x)\right) + nx + 2n\log\left(e^{-2x}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^n],x]
```

```
[Out] (-(x*(n*x + 2*n*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]^n])) + n*PolyLog[2, -E^
(-2*x)])/2
```

fricas [C] time = 0.51, size = 73, normalized size = 1.66

$$\frac{1}{2}nx^2 - nx\log\left(i\cosh(x) + i\sinh(x) + 1\right) - nx\log\left(-i\cosh(x) - i\sinh(x) + 1\right) + nx\log\left(\cosh(x)\right) - n\text{Li}_2\left(i\cosh(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*n*x^2 - n*x*log(I*cosh(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sin
h(x) + 1) + n*x*log(cosh(x)) - n*dilog(I*cosh(x) + I*sinh(x)) - n*dilog(-I*
cosh(x) - I*sinh(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^n), x)
```

maple [F] time = 1.98, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\cosh^n(x)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^n),x)
```

```
[Out] int(ln(a*cosh(x)^n),x)
```

maxima [A] time = 1.15, size = 36, normalized size = 0.82

$$\frac{1}{2}\left(x^2 - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\cosh(x)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="maxima")
[Out] 1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*cosh(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\cosh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)^n),x)
[Out] int(log(a*cosh(x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh^n(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**n),x)
[Out] Integral(log(a*cosh(x)**n), x)
```

3.207 $\int \log(\tanh(x))\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)-\frac{\text{Li}_2\left(e^{2x}\right)}{2}+2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(2,
exp(2*x))
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2548, 5461, 4182, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[2,
 E^(2*x)]/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \log(\tanh(x))\,dx = x\log(\tanh(x)) - \int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$
$$= x\log(\tanh(x)) - 2\int x\operatorname{csch}(2x)\,dx$$
$$= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \int \log\left(1-e^{2x}\right)dx - \int \log\left(1+e^{2x}\right)dx$$
$$= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log}{}\right.$$
$$= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{1}{2}\text{Li}_2(1-\tanh(x)) + \frac{1}{2}\text{Li}_2(-\tanh(x)) + \frac{1}{2}\log(\tanh(x))\log(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Tanh[x]],x]
```

```
[Out] (Log[Tanh[x]]*Log[1 + Tanh[x]])/2 + PolyLog[2, 1 - Tanh[x]]/2 + PolyLog[2,
-Tanh[x]]/2
```

fricas [C] time = 0.48, size = 101, normalized size = 2.59

$$x\log\left(\frac{\sinh(x)}{\cosh(x)}\right) - x\log(\cosh(x)+\sinh(x)+1) + x\log(i\cosh(x)+i\sinh(x)+1) + x\log(-i\cosh(x)-i\sinh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) + I
*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh(x
) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I
*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\tanh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(tanh(x)), x)
```

maple [A] time = 0.14, size = 24, normalized size = 0.62

$$\frac{\ln(\tanh(x)+1)\ln(\tanh(x))}{2} + \frac{\text{dilog}(\tanh(x)+1)}{2} + \frac{\text{dilog}(\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tanh(x)),x)
```

```
[Out] 1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1)+1/2*ln(tanh(x))*ln(tanh(x)+1)
```

maxima [A] time = 1.02, size = 54, normalized size = 1.38

$$x\log\left(e^{(2x)}+1\right)-x\log\left(e^{x}+1\right)-x\log\left(-e^{x}+1\right)+x\log\left(\tanh(x)\right)+\frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right)-\operatorname{Li}_2\left(-e^{x}\right)-\operatorname{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) + x*log(tanh(x)) + 1/
2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

mupad [B] time = 0.47, size = 20, normalized size = 0.51

$$x\ \ln\left(\tanh(x)\right)-\frac{\operatorname{polylog}\left(2,\tanh(x)\right)}{2}+\frac{\operatorname{polylog}\left(2,-\tanh(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tanh(x)),x)
```

```
[Out] x*log(tanh(x)) - polylog(2, tanh(x))/2 + polylog(2, -tanh(x))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\tanh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tanh(x)),x)
```

```
[Out] Integral(log(tanh(x)), x)
```

3.208 $\int \log(a\tanh(x))\,dx$

Optimal. Leaf size=41

$$x\log(a\tanh(x))+\frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right)-\frac{\mathrm{Li}_2\left(e^{2x}\right)}{2}+2x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(a*tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(
2,exp(2*x))
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 5461, 4182, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\tanh(x))+2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[
2, E^(2*x)]/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\tanh(x))\,dx &= x\log(a\tanh(x)) - \int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log(a\tanh(x)) - 2\int x\text{csch}(2x)\,dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \int \log\left(1-e^{2x}\right)dx - \int \log\left(1+e^{2x}\right)dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log}{}\right. \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.20

$$
-\frac{1}{2}\log(1-\tanh(x))\log(a\tanh(x))+\frac{1}{2}\log(\tanh(x)+1)\log(a\tanh(x))+\frac{1}{2}\text{Li}_2(-\tanh(x))-\frac{\text{Li}_2(\tanh(x))}{2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]],x]
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]]) + (Log[a*Tanh[x]]*Log[1 + Tanh[x]])/
2 + PolyLog[2, -Tanh[x]]/2 - PolyLog[2, Tanh[x]]/2
```

fricas [C] time = 0.50, size = 102, normalized size = 2.49

$$
x\log\left(\frac{a\sinh(x)}{\cosh(x)}\right)-x\log\left(\cosh(x)+\sinh(x)+1\right)+x\log\left(i\cosh(x)+i\sinh(x)+1\right)+x\log\left(-i\cosh(x)-i\sinh(x)\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="fricas")
[Out] x*log(a*sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) +
 I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh
(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(
-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log\left(a\tanh(x)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="giac")
[Out] integrate(log(a*tanh(x)), x)
```

maple [B] time = 0.16, size = 70, normalized size = 1.71

$$
\frac{\ln\left(a\tanh(x)\right)\ln\left(\frac{a\tanh(x)+a}{a}\right)}{2}-\frac{\ln\left(a\tanh(x)\right)\ln\left(-\frac{a\tanh(x)-a}{a}\right)}{2}+\frac{\text{dilog}\left(\frac{a\tanh(x)+a}{a}\right)}{2}-\frac{\text{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)),x)
[Out] -1/2*ln(a*tanh(x))*ln(-(a*tanh(x)-a)/a)-1/2*dilog(-(a*tanh(x)-a)/a)+1/2*ln(
a*tanh(x))*ln((a*tanh(x)+a)/a)+1/2*dilog((a*tanh(x)+a)/a)
```

maxima [A] time = 1.02, size = 56, normalized size = 1.37

$$x \log \left(a \tanh(x)\right) + x \log \left(e^{(2x)} + 1\right) - x \log \left(e^{x} + 1\right) - x \log \left(-e^{x} + 1\right) + \frac{1}{2} \operatorname{Li}_2\left(-e^{(2x)}\right) - \operatorname{Li}_2\left(-e^{x}\right) - \operatorname{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)) + x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) +
1/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a \tanh(x)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)),x)
```

```
[Out] int(log(a*tanh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \tanh\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)),x)
```

```
[Out] Integral(log(a*tanh(x)), x)
```

3.209 $\int \log\left(a\tanh^2(x)\right)\,dx$

Optimal. Leaf size=37

$$x\log\left(a\tanh^2(x)\right)+\text{Li}_2\left(-e^{2x}\right)-\text{Li}_2\left(e^{2x}\right)+4x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 4*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^2)+polylog(2,-exp(2*x))-polylog(2,exp(
2*x))
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$\text{PolyLog}\left(2,-e^{2x}\right)-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+4x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^2] + PolyLog[2, -E^(2*x)] - PolyLog[
2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \log \left(a \tanh^2(x)\right) dx &= x \log \left(a \tanh^2(x)\right) - \int 2x\text{csch}(x)\text{sech}(x)\, dx \\
&= x \log \left(a \tanh^2(x)\right) - 2 \int x\text{csch}(x)\text{sech}(x)\, dx \\
&= x \log \left(a \tanh^2(x)\right) - 4 \int x\text{csch}(2x)\, dx \\
&= 4x \tanh^{-1}\left(e^{2x}\right) + x \log \left(a \tanh^2(x)\right) + 2 \int \log \left(1 - e^{2x}\right) dx - 2 \int \log \left(1 + e^{2x}\right) dx \\
&= 4x \tanh^{-1}\left(e^{2x}\right) + x \log \left(a \tanh^2(x)\right) + \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2x}\right) - \text{Subst}\left(\int \frac{1}{}\right. \\
&= 4x \tanh^{-1}\left(e^{2x}\right) + x \log \left(a \tanh^2(x)\right) + \text{Li}_2\left(-e^{2x}\right) - \text{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.27

$$
-\frac{1}{2} \log(1-\tanh(x)) \log \left(a \tanh^2(x)\right)+\frac{1}{2} \log(\tanh(x)+1) \log \left(a \tanh^2(x)\right)+\text{Li}_2(-\tanh(x))-\text{Li}_2(\tanh(x))
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^2],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^2]) + (Log[a*Tanh[x]^2]*Log[1 + Tanh[x
]])/2 + PolyLog[2, -Tanh[x]] - PolyLog[2, Tanh[x]]
```

fricas [C] time = 0.49, size = 129, normalized size = 3.49

$$
x \log \left(\frac{a \cosh(x)^2 + a \sinh(x)^2 - a}{\cosh(x)^2 + \sinh(x)^2 + 1}\right) - 2 x \log (\cosh(x) + \sinh(x) + 1) + 2 x \log (i \cosh(x) + i \sinh(x) + 1) + 2 x \text{l}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 - a)/(cosh(x)^2 + sinh(x)^2 + 1)) - 2*x*lo
g(cosh(x) + sinh(x) + 1) + 2*x*log(I*cosh(x) + I*sinh(x) + 1) + 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x
) + sinh(x)) + 2*dilog(I*cosh(x) + I*sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh
(x)) - 2*dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \log \left(a \tanh(x)^2\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)^2), x)
```

maple [A] time = 0.34, size = 47, normalized size = 1.27

$$
-\frac{\ln \left(a \left(\tanh^2(x)\right)\right) \ln (\tanh(x) - 1)}{2} + \frac{\ln \left(a \left(\tanh^2(x)\right)\right) \ln (\tanh(x) + 1)}{2} + \ln (\tanh(x) - 1) \ln (\tanh(x)) + \text{dilog} (
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^2),x)
```

```
[Out] -1/2*ln(tanh(x)-1)*ln(a*tanh(x)^2)+dilog(tanh(x))+ln(tanh(x)-1)*ln(tanh(x))
+1/2*ln(tanh(x)+1)*ln(a*tanh(x)^2)+dilog(tanh(x)+1)
```

maxima [A] time = 1.02, size = 57, normalized size = 1.54

$$x \log\left(a \tanh(x)^2\right) + 2\,x \log\left(e^{(2\,x)} + 1\right) - 2\,x \log\left(e^x + 1\right) - 2\,x \log\left(-e^x + 1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right) - 2\,\mathrm{Li}_2\left(-e^x\right) - 2\,\mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)^2) + 2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x
 + 1) + dilog(-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a \tanh(x)^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)^2),x)
```

```
[Out] int(log(a*tanh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \tanh^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**2),x)
```

```
[Out] Integral(log(a*tanh(x)**2), x)
```

3.210 $\int \log\left(a\tanh^n(x)\right)\,dx$

Optimal. Leaf size=46

$$x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)+2nx\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^n)+1/2*n*polylog(2,-exp(2*x))-1/2*n*
polylog(2,exp(2*x))
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+2nx\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^n] + (n*PolyLog[2, -E^(2*x)])/2 -
(n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \log\left(a\tanh^n(x)\right)\,dx = x\log\left(a\tanh^n(x)\right) - \int nx\text{csch}(x)\text{sech}(x)\,dx$$
$$= x\log\left(a\tanh^n(x)\right) - n\int x\text{csch}(x)\text{sech}(x)\,dx$$
$$= x\log\left(a\tanh^n(x)\right) - (2n)\int x\text{csch}(2x)\,dx$$
$$= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + n\int \log\left(1-e^{2x}\right)\,dx - n\int \log\left(1+e^{2x}\right)\,dx$$
$$= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + \frac{1}{2}n\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}n\,\text{Subst}$$
$$= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + \frac{1}{2}n\text{Li}_2\left(-e^{2x}\right) - \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.20

$$-\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^n(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Li}_2(-\tanh(x))-\frac{1}{2}n\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^n],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^n]) + (Log[a*Tanh[x]^n]*Log[1 + Tanh[x
]])/2 + (n*PolyLog[2, -Tanh[x]])/2 - (n*PolyLog[2, Tanh[x]])/2
```

fricas [C] time = 0.49, size = 116, normalized size = 2.52

$$nx\log\left(\frac{\sinh(x)}{\cosh(x)}\right)-nx\log\left(\cosh(x)+\sinh(x)+1\right)+nx\log\left(i\cosh(x)+i\sinh(x)+1\right)+nx\log\left(-i\cosh(x)-i\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(sinh(x)/cosh(x)) - n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(I*cosh(
x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x) - I*sinh(x) + 1) - n*x*log(-cosh(x
) - sinh(x) + 1) - n*dilog(cosh(x) + sinh(x)) + n*dilog(I*cosh(x) + I*sinh(
x)) + n*dilog(-I*cosh(x) - I*sinh(x)) - n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\tanh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)^n), x)
```

maple [A] time = 0.23, size = 43, normalized size = 0.93

$$\frac{n\ln\left(\tanh(x)+1\right)\ln\left(\tanh(x)\right)}{2}+\frac{n\,\text{dilog}\left(\tanh(x)+1\right)}{2}+\frac{n\,\text{dilog}\left(\tanh(x)\right)}{2}+\left(-n\ln\left(\tanh(x)\right)+\ln\left(a\left(\tanh^n(x)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^n),x)
```

```
[Out] (ln(a*tanh(x)^n)-n*ln(tanh(x)))*x+1/2*n*dilog(tanh(x))+1/2*n*dilog(tanh(x)+
1)+1/2*n*ln(tanh(x))*ln(tanh(x)+1)
```

maxima [A] time = 1.03, size = 61, normalized size = 1.33

$$\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2\left(-e^{x}\right)-2\,\mathrm{Li}_2\left(e^{x}\right)\right)n+x\log\left(a\,\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-e
^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*tanh(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\tanh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)^n),x)
```

```
[Out] int(log(a*tanh(x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\tanh^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**n),x)
```

```
[Out] Integral(log(a*tanh(x)**n), x)
```

3.211 $\int \log(\coth(x))\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right)+\frac{\mathrm{Li}_2\left(e^{2x}\right)}{2}-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog(2
,exp(2*x))
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2548, 5461, 4182, 2279, 2391}

$$-\frac{1}{2}\mathrm{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\mathrm{PolyLog}\left(2,e^{2x}\right)-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog[2
, E^(2*x)]/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log(\coth(x))\,dx &= x\log(\coth(x)) + \int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log(\coth(x)) + 2\int x\text{csch}(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \int \log\left(1-e^{2x}\right)dx + \int \log\left(1+e^{2x}\right)dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\text{lo}}{}\right.\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{1}{2}\text{Li}_2(1-\coth(x)) + \frac{1}{2}\text{Li}_2(-\coth(x)) + \frac{1}{2}\log(\coth(x))\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Coth[x]],x]
```

```
[Out] (Log[Coth[x]]*Log[1 + Coth[x]])/2 + PolyLog[2, 1 - Coth[x]]/2 + PolyLog[2,
-Coth[x]]/2
```

fricas [C] time = 0.51, size = 101, normalized size = 2.59

$$x\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + x\log(\cosh(x)+\sinh(x)+1) - x\log(i\cosh(x)+i\sinh(x)+1) - x\log(-i\cosh(x)-i\sinh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) + I
*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh(x
) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I
*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\coth(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(coth(x)), x)
```

maple [A] time = 0.14, size = 24, normalized size = 0.62

$$\frac{\ln(\coth(x)+1)\ln(\coth(x))}{2} + \frac{\text{dilog}(\coth(x)+1)}{2} + \frac{\text{dilog}(\coth(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(coth(x)),x)
```

```
[Out] 1/2*dilog(coth(x))+1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*ln(coth(x)+1)
```

maxima [A] time = 1.02, size = 49, normalized size = 1.26

$$-x\log\left(e^{(2x)}+1\right)+x\log\left(e^{x}+1\right)+x\log\left(-e^{x}+1\right)+x\log\left(\coth(x)\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)+\mathrm{Li}_2\left(-e^{x}\right)+\mathrm{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="maxima")
```

```
[Out] -x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) + x*log(coth(x)) - 1
/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

mupad [B] time = 0.41, size = 22, normalized size = 0.56

$$\frac{\mathrm{polylog}\left(2,-\coth(x)\right)}{2}-\frac{\mathrm{polylog}\left(2,\coth(x)\right)}{2}+\mathrm{atanh}\left(\coth(x)\right)\,\ln\left(\coth(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(coth(x)),x)
```

```
[Out] polylog(2, -coth(x))/2 - polylog(2, coth(x))/2 + atanh(coth(x))*log(coth(x)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\coth\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(coth(x)),x)
```

```
[Out] Integral(log(coth(x)), x)
```

3.212 $\int \log(a\coth(x))\,dx$

Optimal. Leaf size=41

$$x\log(a\coth(x))-\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)+\frac{\text{Li}_2\left(e^{2x}\right)}{2}-2x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(a*coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog
(2,exp(2*x))
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 5461, 4182, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\coth(x))-2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog
[2, E^(2*x)]/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log(a\coth(x))\,dx &= x\log(a\coth(x)) + \int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log(a\coth(x)) + 2\int x\text{csch}(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \int \log\left(1-e^{2x}\right)dx + \int \log\left(1+e^{2x}\right)dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\text{lo}}{}\right.\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.20

$$-\frac{1}{2}\log(1-\coth(x))\log(a\coth(x))+\frac{1}{2}\log(\coth(x)+1)\log(a\coth(x))+\frac{1}{2}\text{Li}_2(-\coth(x))-\frac{\text{Li}_2(\coth(x))}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]],x]
```

```
[Out] -1/2*(Log[1 - Coth[x]]*Log[a*Coth[x]]) + (Log[a*Coth[x]]*Log[1 + Coth[x]])/
2 + PolyLog[2, -Coth[x]]/2 - PolyLog[2, Coth[x]]/2
```

fricas [C] time = 0.50, size = 102, normalized size = 2.49

$$x\log\left(\frac{a\cosh(x)}{\sinh(x)}\right)+x\log\left(\cosh(x)+\sinh(x)+1\right)-x\log\left(i\cosh(x)+i\sinh(x)+1\right)-x\log\left(-i\cosh(x)-i\sinh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) +
 I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh
(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(
-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\coth(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)), x)
```

maple [B] time = 0.15, size = 70, normalized size = 1.71

$$\frac{\ln\left(a\coth(x)\right)\ln\left(\frac{a\coth(x)+a}{a}\right)}{2}-\frac{\ln\left(a\coth(x)\right)\ln\left(-\frac{a\coth(x)-a}{a}\right)}{2}+\frac{\text{dilog}\left(\frac{a\coth(x)+a}{a}\right)}{2}-\frac{\text{dilog}\left(-\frac{a\coth(x)-a}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)),x)
```

```
[Out] -1/2*ln(a*coth(x))*ln(-(a*coth(x)-a)/a)-1/2*dilog(-(a*coth(x)-a)/a)+1/2*ln(
a*coth(x))*ln((a*coth(x)+a)/a)+1/2*dilog((a*coth(x)+a)/a)
```

maxima [A] time = 1.01, size = 51, normalized size = 1.24

$$x \log \left(a \coth(x)\right) - x \log \left(e^{(2x)} + 1\right) + x \log \left(e^{x} + 1\right) + x \log \left(-e^{x} + 1\right) - \frac{1}{2} \operatorname{Li}_2 \left(-e^{(2x)}\right) + \operatorname{Li}_2 \left(-e^{x}\right) + \operatorname{Li}_2 \left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)) - x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) -
1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(a \coth(x)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)),x)
```

```
[Out] int(log(a*coth(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(a \coth \left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)),x)
```

```
[Out] Integral(log(a*coth(x)), x)
```

3.213 $\int \log \left(a \coth^2(x)\right) dx$

Optimal. Leaf size=37

$$x \log \left(a \coth^2(x)\right) - \text{Li}_2\left(-e^{2x}\right) + \text{Li}_2\left(e^{2x}\right) - 4x \tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -4*x*arctanh(exp(2*x))+x*ln(a*coth(x)^2)-polylog(2,-exp(2*x))+polylog(2,exp
(2*x))
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{2x}\right) + \text{PolyLog}\left(2,e^{2x}\right) + x \log \left(a \coth^2(x)\right) - 4x \tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^2] - PolyLog[2, -E^(2*x)] + PolyLog
[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \log\left(a\coth^2(x)\right)\,dx = x\log\left(a\coth^2(x)\right) - \int -2x\text{csch}(x)\text{sech}(x)\,dx$$
$$= x\log\left(a\coth^2(x)\right) + 2\int x\text{csch}(x)\text{sech}(x)\,dx$$
$$= x\log\left(a\coth^2(x)\right) + 4\int x\text{csch}(2x)\,dx$$
$$= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) - 2\int \log\left(1-e^{2x}\right)\,dx + 2\int \log\left(1+e^{2x}\right)\,dx$$
$$= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \text{Subst}\left(\int\right.$$
$$= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) - \text{Li}_2\left(-e^{2x}\right) + \text{Li}_2\left(e^{2x}\right)$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.27

$$-\frac{1}{2}\log(1-\tanh(x))\log\left(a\coth^2(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\coth^2(x)\right)-\text{Li}_2(-\tanh(x))+\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^2],x]
```

```
[Out] -1/2*(Log[a*Coth[x]^2]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^2]*Log[1 + Tanh[x
]])/2 - PolyLog[2, -Tanh[x]] + PolyLog[2, Tanh[x]]
```

fricas [C] time = 0.51, size = 127, normalized size = 3.43

$$x\log\left(\frac{a\cosh(x)^2 + a\sinh(x)^2 + a}{\cosh(x)^2 + \sinh(x)^2 - 1}\right)+2\,x\log\left(\cosh(x) + \sinh(x) + 1\right)-2\,x\log\left(i\,\cosh(x) + i\,\sinh(x) + 1\right)-2\,x\,\text{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 + a)/(cosh(x)^2 + sinh(x)^2 - 1)) + 2*x*lo
g(cosh(x) + sinh(x) + 1) - 2*x*log(I*cosh(x) + I*sinh(x) + 1) - 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x
) + sinh(x)) - 2*dilog(I*cosh(x) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh
(x)) + 2*dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\coth(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)^2), x)
```

maple [A] time = 0.27, size = 47, normalized size = 1.27

$$-\frac{\ln\left(a\left(\coth^2(x)\right)\right)\ln\left(\coth(x)-1\right)}{2}+\frac{\ln\left(a\left(\coth^2(x)\right)\right)\ln\left(\coth(x)+1\right)}{2}+\ln\left(\coth(x)-1\right)\ln\left(\coth(x)\right)+\text{dilog}\left(\text{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^2),x)
```

```
[Out] -1/2*ln(coth(x)-1)*ln(a*coth(x)^2)+dilog(coth(x))+ln(coth(x)-1)*ln(coth(x))
+1/2*ln(coth(x)+1)*ln(a*coth(x)^2)+dilog(coth(x)+1)
```

maxima [A] time = 1.02, size = 59, normalized size = 1.59

$$x \log\left(a \coth(x)^2\right) - 2\, x \log\left(e^{(2\,x)} + 1\right) + 2\, x \log\left(e^x + 1\right) + 2\, x \log\left(-e^x + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right) + 2\, \mathrm{Li}_2\left(-e^x\right) + 2\, \mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)^2) - 2*x*log(e^(2*x) + 1) + 2*x*log(e^x + 1) + 2*x*log(-e^x
 + 1) - dilog(-e^(2*x)) + 2*dilog(-e^x) + 2*dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a \coth(x)^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)^2),x)
```

```
[Out] int(log(a*coth(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \coth^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**2),x)
```

```
[Out] Integral(log(a*coth(x)**2), x)
```

3.214 $\int \log \left(a \coth^n(x)\right) \, dx$

Optimal. Leaf size=46

$$x \log \left(a \coth^n(x)\right) - \frac{1}{2} n \text{Li}_2\left(-e^{2x}\right) + \frac{1}{2} n \text{Li}_2\left(e^{2x}\right) - 2 n x \tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -2*n*x*arctanh(exp(2*x))+x*ln(a*coth(x)^n)-1/2*n*polylog(2,-exp(2*x))+1/2*n
*polylog(2,exp(2*x))
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$-\frac{1}{2} n \text{PolyLog}\left(2, -e^{2x}\right) + \frac{1}{2} n \text{PolyLog}\left(2, e^{2x}\right) + x \log \left(a \coth^n(x)\right) - 2 n x \tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^n] - (n*PolyLog[2, -E^(2*x)])/2 +
 (n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\coth^n(x)\right)dx &= x\log\left(a\coth^n(x)\right) + \int nx\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^n(x)\right) + n\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^n(x)\right) + (2n)\int x\text{csch}(2x)\,dx \\
&= -2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^n(x)\right) - n\int \log\left(1-e^{2x}\right)dx + n\int \log\left(1+e^{2x}\right)dx \\
&= -2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^n(x)\right) - \frac{1}{2}n\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2x}\right) + \frac{1}{2}n\,\text{Subs} \\
&= -2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^n(x)\right) - \frac{1}{2}n\text{Li}_2\left(-e^{2x}\right) + \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.20

$$-\frac{1}{2}\log(1-\tanh(x))\log\left(a\coth^n(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\coth^n(x)\right)-\frac{1}{2}n\text{Li}_2(-\tanh(x))+\frac{1}{2}n\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^n],x]
[Out] -1/2*(Log[a*Coth[x]^n]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^n]*Log[1 + Tanh[x
]])/2 - (n*PolyLog[2, -Tanh[x]])/2 + (n*PolyLog[2, Tanh[x]])/2
```

fricas [C] time = 0.49, size = 116, normalized size = 2.52

$$nx\log\left(\frac{\cosh(x)}{\sinh(x)}\right)+nx\log\left(\cosh(x)+\sinh(x)+1\right)-nx\log\left(i\cosh(x)+i\sinh(x)+1\right)-nx\log\left(-i\cosh(x)-i\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="fricas")
[Out] n*x*log(cosh(x)/sinh(x)) + n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(I*cosh(
x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sinh(x) + 1) + n*x*log(-cosh(x
) - sinh(x) + 1) + n*dilog(cosh(x) + sinh(x)) - n*dilog(I*cosh(x) + I*sinh(
x)) - n*dilog(-I*cosh(x) - I*sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\coth(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="giac")
[Out] integrate(log(a*coth(x)^n), x)
```

maple [A] time = 0.24, size = 43, normalized size = 0.93

$$\frac{n\ln(\coth(x)+1)\ln(\coth(x))}{2}+\frac{n\,\text{dilog}(\coth(x)+1)}{2}+\frac{n\,\text{dilog}(\coth(x))}{2}+\left(-n\ln(\coth(x))+\ln\left(a\left(\coth^n(x)\right)\right)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^n),x)
```

```
[Out] (ln(a*coth(x)^n)-n*ln(coth(x)))*x+1/2*n*dilog(coth(x))+1/2*n*dilog(coth(x)+
1)+1/2*n*ln(coth(x))*ln(coth(x)+1)
```

maxima [A] time = 1.03, size = 61, normalized size = 1.33

$$-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2\left(-e^{x}\right)-2\,\mathrm{Li}_2\left(e^{x}\right)\right)n+x\log\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-
e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*coth(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\,\mathrm{coth}(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)^n),x)
```

```
[Out] int(log(a*coth(x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\coth^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**n),x)
```

```
[Out] Integral(log(a*coth(x)**n), x)
```

3.215 $\int \log(a\,\mathbf{sech}(x))\,dx$

Optimal. Leaf size=38

$$x\log(a\,\mathrm{sech}(x)) + \frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right) - \frac{x^2}{2} + x\log\left(e^{2x}+1\right)$$

[Out] -1/2*x^2+x*ln(1+exp(2*x))+x*ln(a*sech(x))+1/2*polylog(2,-exp(2*x))

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3718, 2190, 2279, 2391}

$$\frac{1}{2}\mathrm{PolyLog}\left(2,-e^{2x}\right) + x\log(a\,\mathrm{sech}(x)) - \frac{x^2}{2} + x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Sech[x]],x]

[Out] -x^2/2 + x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]] + PolyLog[2, -E^(2*x)]/2

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\text{sech}(x))\,dx &= x\log(a\text{sech}(x)) + \int x\tanh(x)\,dx\\ &= -\frac{x^2}{2} + x\log(a\text{sech}(x)) + 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \int \log\left(1+e^{2x}\right)\,dx\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{1}{2}\left(x\left(2\log(a\text{sech}(x)) + x + 2\log\left(e^{-2x}+1\right)\right) - \text{Li}_2\left(-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]],x]
```

```
[Out] (x*(x + 2*Log[1 + E^(-2*x)] + 2*Log[a*Sech[x]]) - PolyLog[2, -E^(-2*x)])/2
```

fricas [C] time = 0.50, size = 84, normalized size = 2.21

$$-\frac{1}{2}x^2 + x\log\left(\frac{2\,(a\cosh(x) + a\sinh(x))}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1}\right) + x\log\left(i\,\cosh(x) + i\,\sinh(x) + 1\right) + x\log\left(-i\,\text{cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + x*log(I*cosh(x) + I*sinh(x) + 1) + x*log(-I*cosh(x) - I*
sinh(x) + 1) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I*cosh(x) - I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\text{sech}(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)), x)
```

maple [C] time = 1.15, size = 314, normalized size = 8.26

$$-\frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(\frac{ie^{x}}{e^{2x}+1}\right)\text{csgn}\left(\frac{ia\,e^{x}}{e^{2x}+1}\right)}{2} + \frac{i\pi x\,\text{csgn}\left(ia\right)\text{csgn}\left(\frac{ia\,e^{x}}{e^{2x}+1}\right)^2}{2} - \frac{i\pi x\,\text{csgn}\left(\frac{i}{e^{2x}+1}\right)\text{csgn}\left(ie^{x}\right)\text{csgn}\left(\frac{ie^{x}}{e^{2x}+\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)),x)
```

```
[Out] x*ln(exp(x))+1/2*I*Pi*csgn(I/(exp(2*x)+1))*csgn(I*exp(x)/(exp(2*x)+1))^2*x-
1/2*I*Pi*csgn(I*a)*csgn(I*exp(x)/(exp(2*x)+1))*csgn(I*a/(exp(2*x)+1)*exp(x)
)*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(exp(2*x)+1))^2*x-1/2*I*Pi*csgn(I
*exp(x))*csgn(I/(exp(2*x)+1))*csgn(I*exp(x)/(exp(2*x)+1))*x-1/2*I*Pi*csgn(I
```

```
*a/(exp(2*x)+1)*exp(x))^3*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(exp(2*x)+1)*exp(x)
)^2*x+ln(2)*x+x*ln(a)-1/2*x^2+1/2*I*Pi*csgn(I*exp(x)/(exp(2*x)+1))*csgn(I*a
/(exp(2*x)+1)*exp(x))^2*x-1/2*I*Pi*csgn(I*exp(x)/(exp(2*x)+1))^3*x-ln(exp(2
*x)+1)*ln(exp(x))+ln(I*exp(x)+1)*ln(exp(x))+ln(-I*exp(x)+1)*ln(exp(x))+dilo
g(I*exp(x)+1)+dilog(-I*exp(x)+1)
```

maxima [A] time = 1.15, size = 31, normalized size = 0.82

$$-\frac{1}{2}x^2 + x\log\left(a\operatorname{sech}(x)\right) + x\log\left(e^{(2x)} + 1\right) + \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="maxima")
[Out] -1/2*x^2 + x*log(a*sech(x)) + x*log(e^(2*x) + 1) + 1/2*dilog(-e^(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \ln\left(\cosh(x)\right) - \ln(a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cosh(x)),x)
[Out] -int(log(cosh(x)) - log(a), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{sech}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)),x)
[Out] Integral(log(a*sech(x)), x)
```

3.216 $\int \log\left(a\,\mathrm{sech}^2(x)\right)\,dx$

Optimal. Leaf size=35

$$x\log\left(a\,\mathrm{sech}^2(x)\right)+\mathrm{Li}_2\left(-e^{2x}\right)-x^2+2x\log\left(e^{2x}+1\right)$$

```
[Out] -x^2+2*x*ln(1+exp(2*x))+x*ln(a*sech(x)^2)+polylog(2,-exp(2*x))
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$\mathrm{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\,\mathrm{sech}^2(x)\right)-x^2+2x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^2] + PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\text{sech}^2(x)\right)\,dx &= x\log\left(a\text{sech}^2(x)\right) - \int -2x\tanh(x)\,dx\\ &= x\log\left(a\text{sech}^2(x)\right) + 2\int x\tanh(x)\,dx\\ &= -x^2 + x\log\left(a\text{sech}^2(x)\right) + 4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\text{sech}^2(x)\right) - 2\int \log\left(1+e^{2x}\right)\,dx\\ &= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\text{sech}^2(x)\right) - \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\text{sech}^2(x)\right) + \text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.94

$$x\left(\log\left(a\text{sech}^2(x)\right) + x + 2\log\left(e^{-2x}+1\right)\right) - \text{Li}_2\left(-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^2],x]
```

```
[Out] x*(x + 2*Log[1 + E^(-2*x)] + Log[a*Sech[x]^2]) - PolyLog[2, -E^(-2*x)]
```

fricas [C] time = 0.48, size = 106, normalized size = 3.03

$$-x^2+x\log\left(\frac{4\left(a\cosh(x)+a\sinh(x)\right)}{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2+1\right)\sinh(x)+3\cosh(x)}\right)+2x\log\left(i\cosh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + 3*cosh(x))) + 2*x*log(I*cosh(x) + I*
sinh(x) + 1) + 2*x*log(-I*cosh(x) - I*sinh(x) + 1) + 2*dilog(I*cosh(x) + I*
sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\text{sech}(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^2), x)
```

maple [C] time = 1.41, size = 480, normalized size = 13.71

$$-\frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(\frac{ie^{2x}}{\left(e^{2x}+1\right)^2}\right)\text{csgn}\left(\frac{ia\,e^{2x}}{\left(e^{2x}+1\right)^2}\right)}{2}+\frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(\frac{ia\,e^{2x}}{\left(e^{2x}+1\right)^2}\right)^2}{2}-\frac{i\pi x\,\text{csgn}\left(\frac{i}{\left(e^{2x}+1\right)^2}\right)\text{csgn}\left(ie^{2x}\right)\text{csg}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^2),x)
```

```
[Out] 2*ln(2)*x+x*ln(a)-x^2-2*ln(exp(2*x)+1)*ln(exp(x))+2*ln(I*exp(x)+1)*ln(exp(x
))+2*ln(-I*exp(x)+1)*ln(exp(x))+2*dilog(I*exp(x)+1)+2*dilog(-I*exp(x)+1)+1/
```

```
2*I*Pi*csgn(I*(exp(2*x)+1)^2)^3*x-1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)+1)^2)^
3*x+I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(e
xp(2*x)+1)^2*exp(2*x))^2*x+1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)+1)^2)*csgn(I*
a/(exp(2*x)+1)^2*exp(2*x))^2*x-1/2*I*Pi*csgn(I*exp(2*x))^3*x+1/2*I*Pi*csgn(
I*(exp(2*x)+1))^2*csgn(I*(exp(2*x)+1)^2)*x-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I
*exp(2*x))*x-I*Pi*csgn(I*(exp(2*x)+1))*csgn(I*(exp(2*x)+1)^2)^2*x-1/2*I*Pi*
csgn(I*a/(exp(2*x)+1)^2*exp(2*x))^3*x+2*x*ln(exp(x))-1/2*I*Pi*csgn(I*a)*csg
n(I*exp(2*x)/(exp(2*x)+1)^2)*csgn(I*a/(exp(2*x)+1)^2*exp(2*x))*x+1/2*I*Pi*c
sgn(I*exp(2*x))*csgn(I*exp(2*x)/(exp(2*x)+1)^2)^2*x-1/2*I*Pi*csgn(I*exp(2*x
))*csgn(I/(exp(2*x)+1)^2)*csgn(I*exp(2*x)/(exp(2*x)+1)^2)*x+1/2*I*Pi*csgn(I
/(exp(2*x)+1)^2)*csgn(I*exp(2*x)/(exp(2*x)+1)^2)^2*x
```

maxima [A] time = 1.15, size = 32, normalized size = 0.91

$$-x^2 + x\log\left(a\,\mathrm{sech}(x)^2\right) + 2\,x\log\left(e^{(2\,x)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="maxima")
[Out] -x^2 + x*log(a*sech(x)^2) + 2*x*log(e^(2*x) + 1) + dilog(-e^(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int 2\,\ln\left(\cosh(x)\right) - \ln(a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cosh(x)^2),x)
[Out] -int(2*log(cosh(x)) - log(a), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\mathrm{sech}^2\,(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**2),x)
[Out] Integral(log(a*sech(x)**2), x)
```

3.217 $\int \log \left(a \mathbf{sech}^n(x)\right) dx$

Optimal. Leaf size=43

$$x \log \left(a \operatorname{sech}^n(x)\right)+\frac{1}{2} n \operatorname{Li}_2\left(-e^{2 x}\right)-\frac{n x^2}{2}+n x \log \left(e^{2 x}+1\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1+exp(2*x))+x*ln(a*sech(x)^n)+1/2*n*polylog(2,-exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$\frac{1}{2} n \operatorname{PolyLog}\left(2,-e^{2 x}\right)+x \log \left(a \operatorname{sech}^n(x)\right)-\frac{n x^2}{2}+n x \log \left(e^{2 x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^n],x]
```

```
[Out] -(n*x^2)/2 + n*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^n] + (n*PolyLog[2, -E^(
2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\operatorname{sech}^n(x)\right)dx &= x\log\left(a\operatorname{sech}^n(x)\right) + \int nx\tanh(x)\,dx\\ &= x\log\left(a\operatorname{sech}^n(x)\right) + n\int x\tanh(x)\,dx\\ &= -\frac{nx^2}{2} + x\log\left(a\operatorname{sech}^n(x)\right) + (2n)\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= -\frac{nx^2}{2} + nx\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^n(x)\right) - n\int \log\left(1+e^{2x}\right)dx\\ &= -\frac{nx^2}{2} + nx\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^n(x)\right) - \frac{1}{2}n\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= -\frac{nx^2}{2} + nx\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^n(x)\right) + \frac{1}{2}n\operatorname{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$x\log\left(a\operatorname{sech}^n(x)\right) - \frac{1}{2}n\operatorname{Li}_2\left(-e^{-2x}\right) + \frac{nx^2}{2} + nx\log\left(e^{-2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^n],x]
```

```
[Out] (n*x^2)/2 + n*x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]^n] - (n*PolyLog[2, -E^(
-2*x)])/2
```

fricas [C] time = 0.46, size = 92, normalized size = 2.14

$$-\frac{1}{2}nx^2 + nx\log\left(\frac{2\left(\cosh(x)+\sinh(x)\right)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}\right) + nx\log\left(i\cosh(x) + i\sinh(x) + 1\right) + nx\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + n*x*log(I*cosh(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x)
- I*sinh(x) + 1) + n*dilog(I*cosh(x) + I*sinh(x)) + n*dilog(-I*cosh(x) - I*
sinh(x)) + x*log(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{sech}(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^n), x)
```

maple [F] time = 1.86, size = 0, normalized size = 0.00

$$\int \ln\left(a\operatorname{sech}(x)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^n),x)
```

```
[Out] int(ln(a*sech(x)^n),x)
```

maxima [A] time = 1.15, size = 36, normalized size = 0.84

$$-\frac{1}{2}\left(x^2 - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\,\mathrm{sech}(x)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="maxima")
[Out] -1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*sech(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\left(\frac{1}{\cosh(x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/cosh(x))^n),x)
[Out] int(log(a*(1/cosh(x))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\mathrm{sech}^n\,(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**n),x)
[Out] Integral(log(a*sech(x)**n), x)
```

3.218 $\int \log(a\,\mathbf{csch}(x))\,dx$

Optimal. Leaf size=38

$$x\log(a\operatorname{csch}(x))+\frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)$$

[Out] -1/2*x^2+x*ln(1-exp(2*x))+x*ln(a*csch(x))+1/2*polylog(2,exp(2*x))

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2548, 3716, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\operatorname{csch}(x))-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Csch[x]],x]

[Out] -x^2/2 + x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]] + PolyLog[2, E^(2*x)]/2

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\operatorname{csch}(x))\,dx &= x\log(a\operatorname{csch}(x)) + \int x\coth(x)\,dx\\ &= -\frac{x^2}{2} + x\log(a\operatorname{csch}(x)) - 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx\\ &= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \int \log\left(1-e^{2x}\right)dx\\ &= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)\\ &= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) + \frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{1}{2}\left(x\left(2\log(a\operatorname{csch}(x)) + x + 2\log\left(1-e^{-2x}\right)\right) - \operatorname{Li}_2\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]],x]
[Out] (x*(x + 2*Log[1 - E^(-2*x)] + 2*Log[a*Csch[x]]) - PolyLog[2, E^(-2*x)])/2
```

fricas [B] time = 0.47, size = 76, normalized size = 2.00

$$-\frac{1}{2}x^2 + x\log\left(\frac{2\,(a\cosh(x) + a\sinh(x))}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}\right) + x\log\left(\cosh(x) + \sinh(x) + 1\right) + x\log\left(-\cosh(x) - s\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="fricas")
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + x*log(cosh(x) + sinh(x) + 1) + x*log(-cosh(x) - sinh(x)
+ 1) + dilog(cosh(x) + sinh(x)) + dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{csch}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="giac")
[Out] integrate(log(a*csch(x)), x)
```

maple [C] time = 0.95, size = 293, normalized size = 7.71

$$-\frac{i\pi x\operatorname{csgn}(ia)\operatorname{csgn}\left(\frac{ie^x}{e^{2x}-1}\right)\operatorname{csgn}\left(\frac{ia\,e^x}{e^{2x}-1}\right)}{2} + \frac{i\pi x\operatorname{csgn}(ia)\operatorname{csgn}\left(\frac{ia\,e^x}{e^{2x}-1}\right)^2}{2} - \frac{i\pi x\operatorname{csgn}\left(\frac{i}{e^{2x}-1}\right)\operatorname{csgn}(ie^x)\operatorname{csgn}\left(\frac{ie^x}{e^{2x}-1}\right)}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)),x)
[Out] x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(exp(2*x)-1))^2*x+1/2*I*
Pi*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x)-1)*exp(x))^2*x-1/2*I*Pi*c
sgn(I*exp(x))*csgn(I/(exp(2*x)-1))*csgn(I*exp(x)/(exp(2*x)-1))*x-1/2*I*Pi*c
```

```
sgn(I*a)*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x)-1)*exp(x))*x-1/2*I*
Pi*csgn(I*exp(x)/(exp(2*x)-1))^3*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(exp(2*x)-1)
*exp(x))^2*x+ln(2)*x+x*ln(a)-1/2*x^2-1/2*I*Pi*csgn(I*a/(exp(2*x)-1)*exp(x))
^3*x+1/2*I*Pi*csgn(I/(exp(2*x)-1))*csgn(I*exp(x)/(exp(2*x)-1))^2*x-ln(exp(2
*x)-1)*ln(exp(x))-dilog(exp(x))+dilog(exp(x)+1)+ln(exp(x)+1)*ln(exp(x))
```

maxima [A] time = 0.65, size = 37, normalized size = 0.97

$$-\frac{1}{2}x^2 + x\log\left(a\operatorname{csch}(x)\right) + x\log\left(e^x + 1\right) + x\log\left(-e^x + 1\right) + \operatorname{Li}_2\left(-e^x\right) + \operatorname{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x*log(a*csch(x)) + x*log(e^x + 1) + x*log(-e^x + 1) + dilog(-e^x
) + dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\frac{a}{\sinh(x)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sinh(x)),x)
```

```
[Out] int(log(a/sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{csch}\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)),x)
```

```
[Out] Integral(log(a*csch(x)), x)
```

3.219 $\int \log \left(a\operatorname{csch}^2(x)\right) dx$

Optimal. Leaf size=35

$$x \log \left(a\operatorname{csch}^2(x)\right)+\text{Li}_2\left(e^{2x}\right)-x^2+2x\log \left(1-e^{2x}\right)$$

```
[Out] -x^2+2*x*ln(1-exp(2*x))+x*ln(a*csch(x)^2)+polylog(2,exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,e^{2x}\right)+x\log \left(a\operatorname{csch}^2(x)\right)-x^2+2x\log \left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^2] + PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log\left(a\text{csch}^2(x)\right)\,dx = x\log\left(a\text{csch}^2(x)\right) - \int -2x\coth(x)\,dx$$
$$= x\log\left(a\text{csch}^2(x)\right) + 2\int x\coth(x)\,dx$$
$$= -x^2 + x\log\left(a\text{csch}^2(x)\right) - 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx$$
$$= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) - 2\int \log\left(1-e^{2x}\right)\,dx$$
$$= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)$$
$$= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) + \text{Li}_2\left(e^{2x}\right)$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.94

$$x\left(\log\left(a\text{csch}^2(x)\right) + x + 2\log\left(1-e^{-2x}\right)\right) - \text{Li}_2\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^2],x]
```

```
[Out] x*(x + 2*Log[1 - E^(-2*x)] + Log[a*Csch[x]^2]) - PolyLog[2, E^(-2*x)]
```

fricas [B] time = 0.55, size = 97, normalized size = 2.77

$$-x^2+x\log\left(\frac{4\,(a\cosh(x)+a\sinh(x))}{\cosh(x)^3+3\,\cosh(x)\sinh(x)^2+\sinh(x)^3+3\left(\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}\right)+2\,x\log\left(\cosh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x) - cosh(x))) + 2*x*log(cosh(x) + sinh(x
) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x) + sinh(x)) + 2*d
ilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\text{csch}(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^2), x)
```

maple [C] time = 1.34, size = 456, normalized size = 13.03

$$-\frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(\frac{ie^{2x}}{\left(e^{2x}-1\right)^2}\right)\text{csgn}\left(\frac{ia\,e^{2x}}{\left(e^{2x}-1\right)^2}\right)}{2}+\frac{i\pi x\,\text{csgn}(ia)\,\text{csgn}\left(\frac{ia\,e^{2x}}{\left(e^{2x}-1\right)^2}\right)^2}{2}-\frac{i\pi x\,\text{csgn}\left(\frac{i}{\left(e^{2x}-1\right)^2}\right)\text{csgn}\left(ie^{2x}\right)\,\text{c}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^2),x)
```

```
[Out] 2*ln(2)*x+x*ln(a)-x^2-2*dilog(exp(x))+2*dilog(exp(x)+1)-I*Pi*csgn(I*(exp(2*
x)-1))*csgn(I*(exp(2*x)-1)^2)^2*x+1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2)*
```

```
csgn(I*a/(exp(2*x)-1)^2*exp(2*x))^2*x-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(
2*x))*x-1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(exp(2*x)-1)^2)*csgn(I*exp(2*x)/(e
xp(2*x)-1)^2)*x+1/2*I*Pi*csgn(I*(exp(2*x)-1)^2)^3*x-1/2*I*Pi*csgn(I*a/(exp(
2*x)-1)^2*exp(2*x))^3*x-1/2*I*Pi*csgn(I*a)*csgn(I*exp(2*x)/(exp(2*x)-1)^2)*
csgn(I*a/(exp(2*x)-1)^2*exp(2*x))*x+2*ln(exp(x)+1)*ln(exp(x))+2*x*ln(exp(x)
)-2*ln(exp(2*x)-1)*ln(exp(x))+1/2*I*Pi*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(ex
p(2*x)-1)^2)^2*x-1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^3*x+1/2*I*Pi*csgn
(I/(exp(2*x)-1)^2)*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^2*x+1/2*I*Pi*csgn(I*(exp
(2*x)-1))^2*csgn(I*(exp(2*x)-1)^2)*x+1/2*I*Pi*csgn(I*a)*csgn(I*a/(exp(2*x)-
1)^2*exp(2*x))^2*x+I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x-1/2*I*Pi*csgn(I
*exp(2*x))^3*x
```

maxima [A] time = 0.65, size = 45, normalized size = 1.29

$$-x^2 + x\log\left(a\operatorname{csch}(x)^2\right) + 2\,x\log\left(e^x+1\right) + 2\,x\log\left(-e^x+1\right) + 2\operatorname{Li}_2\left(-e^x\right) + 2\operatorname{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*csch(x)^2) + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 2*dilog(
-e^x) + 2*dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\frac{a}{\sinh(x)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sinh(x)^2),x)
```

```
[Out] int(log(a/sinh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{csch}^2(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**2),x)
```

```
[Out] Integral(log(a*csch(x)**2), x)
```

3.220 $\int \log\left(a\operatorname{csch}^n(x)\right)\,dx$

Optimal. Leaf size=43

$$x\log\left(a\operatorname{csch}^n(x)\right)+\frac{1}{2}n\operatorname{Li}_2\left(e^{2x}\right)-\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1-exp(2*x))+x*ln(a*csch(x)^n)+1/2*n*polylog(2,exp(2*x))
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^n],x]
```

```
[Out] -(n*x^2)/2 + n*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^n] + (n*PolyLog[2, E^(2
*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\operatorname{csch}^n(x)\right)\,dx &= x\log\left(a\operatorname{csch}^n(x)\right)+\int nx\coth(x)\,dx\\
&= x\log\left(a\operatorname{csch}^n(x)\right)+n\int x\coth(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\operatorname{csch}^n(x)\right)-(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)-\frac{1}{2}n\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)+\frac{1}{2}n\operatorname{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$
x\log\left(a\operatorname{csch}^n(x)\right)-\frac{1}{2}n\operatorname{Li}_2\left(e^{-2x}\right)+\frac{nx^2}{2}+nx\log\left(1-e^{-2x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^n],x]
```

```
[Out] (n*x^2)/2 + n*x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]^n] - (n*PolyLog[2, E^(-
2*x)])/2
```

fricas [B] time = 0.49, size = 84, normalized size = 1.95

$$
-\frac{1}{2}nx^2+nx\log\left(\frac{2\left(\cosh(x)+\sinh(x)\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}\right)+nx\log\left(\cosh(x)+\sinh(x)+1\right)+nx\log\left(-\cosh(\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(-cosh(x) - sinh
(x) + 1) + n*dilog(cosh(x) + sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int\log\left(a\operatorname{csch}(x)^n\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^n), x)
```

maple [F] time = 2.08, size = 0, normalized size = 0.00

$$
\int\ln\left(a\operatorname{csch}(x)^n\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^n),x)
```

```
[Out] int(ln(a*csch(x)^n),x)
```

maxima [A] time = 0.65, size = 47, normalized size = 1.09

$$-\frac{1}{2}\left(x^2 - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2\left(-e^x\right) - 2\,\mathrm{Li}_2\left(e^x\right)\right)n + x\log\left(a\,\mathrm{csch}(x)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(
e^x))*n + x*log(a*csch(x)^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\left(\frac{1}{\sinh(x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/sinh(x))^n),x)
```

```
[Out] int(log(a*(1/sinh(x))^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\mathrm{csch}^n\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**n),x)
```

```
[Out] Integral(log(a*csch(x)**n), x)
```

3.221 $\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx$

Optimal. Leaf size=50

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

```
[Out] -sinh(b*x+a)/b+ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x))*sinh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2637, 2554}

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a +
 b*x])/b
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx=\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+bx)}{b}-\int c$$

$$=-\frac{\sinh(a+bx)}{b}+\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\mathrm{si}}{b}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.66

$$\frac{\sinh(a+bx)\log\left(\frac{1}{2}\sinh(a+bx)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Sinh[a + b*x]/2]*Sinh[a + b*x])/b
```

fricas [B] time = 0.48, size = 258, normalized size = 5.16

$$-\frac{\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+4\,\mathrm{c}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="fricas")
```

```
[Out] -1/2*(cosh(1/2*b*x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/
2*a) + 6*cosh(1/2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x +
 1/2*a)*sinh(1/2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - (cosh(1/2*b*x +
 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/2*a) + 6*cosh(1/2*b*
x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x
 + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - 1)*log(cosh(1/2*b*x + 1/2*a)*sinh(1
/2*b*x + 1/2*a)) - 1)/(b*cosh(1/2*b*x + 1/2*a)^2 + 2*b*cosh(1/2*b*x + 1/2*a
)*sinh(1/2*b*x + 1/2*a) + b*sinh(1/2*b*x + 1/2*a)^2)
```

giac [B] time = 0.47, size = 94, normalized size = 1.88

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(\frac{1}{4}\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\right)-\frac{e^{(bx+a)}-e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(1/4*(e^(1/2*b*x + 1/2*a) + e^(-1/2
*b*x - 1/2*a))*(e^(1/2*b*x + 1/2*a) - e^(-1/2*b*x - 1/2*a))) - 1/2*(e^(b*x
+ a) - e^(-b*x - a))/b
```

maple [A] time = 0.89, size = 32, normalized size = 0.64

$$\frac{\ln\left(\frac{\sinh(bx+a)}{2}\right)\sinh\left(bx+a\right)}{b}-\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)),x)
```

```
[Out] ln(1/2*sinh(b*x+a))/b*sinh(b*x+a)-sinh(b*x+a)/b
```

maxima [B] time = 0.51, size = 112, normalized size = 2.24

$$\frac{\log\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sinh\left(bx+a\right)}{b}-\frac{b\left(\frac{2\,(bx+a)}{b}+\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)-b\left(\frac{2\,(bx+a)}{b}-\frac{e^{(bx+a)}}{b}+\frac{e^{(-b\cdots}}{b}\cdots\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="maxima")
```

```
[Out] log(cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x + 1/2*a))*sinh(b*x + a)/b - 1/4*(b*(
2*(b*x + a)/b + e^(b*x + a)/b - e^(-b*x - a)/b) - b*(2*(b*x + a)/b - e^(b*x
 + a)/b + e^(-b*x - a)/b))/b
```

mupad [B] time = 0.51, size = 31, normalized size = 0.62

$$\frac{\ln\left(\frac{\sinh(a+b\,x)}{2}\right)\sinh\left(a+b\,x\right)}{b}-\frac{\sinh\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(a/2 + (b*x)/2)*sinh(a/2 + (b*x)/2))*cosh(a + b*x),x)
[Out] (log(sinh(a + b*x)/2)*sinh(a + b*x))/b - sinh(a + b*x)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
[Out] Integral(log(sinh(a/2 + b*x/2)*cosh(a/2 + b*x/2))*cosh(a + b*x), x)
```

3.222 $\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx$

Optimal. Leaf size=13

$$\cosh(x)\log\left(\cosh^2(x)\right) - 2\cosh(x)$$

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2638, 2554, 12}

$$\cosh(x)\log\left(\cosh^2(x)\right) - 2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx &= \cosh(x)\log\left(\cosh^2(x)\right) - \int 2\sinh(x)\,dx\\ &= \cosh(x)\log\left(\cosh^2(x)\right) - 2\int \sinh(x)\,dx\\ &= -2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\cosh(x)\log\left(\cosh^2(x)\right) - 2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

fricas [B] time = 0.46, size = 62, normalized size = 4.77

$$-\frac{2\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{1}{2}\cosh(x)^2 + \frac{1}{2}\sinh(x)^2 + \frac{1}{2}\right) + 4\cosh(x}{2\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(1/2
*cosh(x)^2 + 1/2*sinh(x)^2 + 1/2) + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + 2)/(c
osh(x) + sinh(x))
```

giac [B] time = 0.18, size = 31, normalized size = 2.38

$$\left(e^{(-x)} + e^x\right) \log\left(\frac{1}{2} e^{(-x)} + \frac{1}{2} e^x\right) - e^{(-x)} - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="giac")
```

```
[Out] (e^(-x) + e^x)*log(1/2*e^(-x) + 1/2*e^x) - e^(-x) - e^x
```

maple [A] time = 0.29, size = 14, normalized size = 1.08

$$\cosh(x) \ln\left(\cosh^2(x)\right) - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x)^2)*sinh(x),x)
```

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

maxima [A] time = 0.44, size = 12, normalized size = 0.92

$$2\,\cosh(x) \log\left(\cosh(x)\right) - 2\,\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="maxima")
```

```
[Out] 2*cosh(x)*log(cosh(x)) - 2*cosh(x)
```

mupad [B] time = 0.38, size = 9, normalized size = 0.69

$$2\cosh(x)\,\left(\ln\left(\cosh(x)\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(x)^2)*sinh(x),x)
```

```
[Out] 2*cosh(x)*(log(cosh(x)) - 1)
```

sympy [A] time = 0.91, size = 14, normalized size = 1.08

$$2\log\left(\cosh\left(x\right)\right)\cosh\left(x\right) - 2\cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x)**2)*sinh(x),x)
```

```
[Out] 2*log(cosh(x))*cosh(x) - 2*cosh(x)
```

3.223 $\int \frac{\log(x)}{\sqrt{x}} \, dx$

Optimal. Leaf size=17

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[x],x]
```

```
[Out] -4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.65

$$2\sqrt{x}(\log(x) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*(-2 + Log[x])
```

fricas [A] time = 0.46, size = 9, normalized size = 0.53

$$2\sqrt{x}\left(\log(x) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*(log(x) - 2)
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.82

$$2\sqrt{x}\ \ln(x) - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^(1/2),x)
```

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.76

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.53

$$2\sqrt{x}\ (\ln(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)*(log(x) - 2)
```

sympy [A] time = 1.74, size = 60, normalized size = 3.53

$$\begin{cases} 2\sqrt{x}\log(x) - 4\sqrt{x} & \text{for } |x| < 1 \\ -2\sqrt{x}\log\left(\frac{1}{x}\right) - 4\sqrt{x} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{3}{2}, \frac{3}{2} \\ \frac{1}{2}, \frac{1}{2} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{3}{2}, \frac{3}{2}, 1 & \\ & \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(x)*log(x) - 4*sqrt(x), Abs(x) < 1), (-2*sqrt(x)*log(1/x)
- 4*sqrt(x), 1/Abs(x) < 1), (-meijerg(((1,), (3/2, 3/2)), ((1/2, 1/2), (0,)
), x) + meijerg(((3/2, 3/2, 1), ()), ((), (1/2, 1/2, 0)), x), True))
```

3.224 $\int x \log\left(2-3x^2\right) dx$

Optimal. Leaf size=27

$$-\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)$$

```
[Out] -1/2*x^2-1/6*(-3*x^2+2)*ln(-3*x^2+2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2454, 2389, 2295}

$$-\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[2 - 3*x^2],x]
```

```
[Out] -x^2/2 - ((2 - 3*x^2)*Log[2 - 3*x^2])/6
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x \log\left(2-3x^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int \log(2-3x)\,dx, x, x^2\right)\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \log(x)\,dx, x, 2-3x^2\right)\right)\\
&= -\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{6}\left(\left(3x^2-2\right)\log\left(2-3x^2\right)-3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[2 - 3*x^2],x]
```

```
[Out] (-3*x^2 + (-2 + 3*x^2)*Log[2 - 3*x^2])/6
```

fricas [A] time = 0.47, size = 23, normalized size = 0.85

$$-\frac{1}{2}x^2+\frac{1}{6}\left(3x^2-2\right)\log\left(-3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2)
```

giac [A] time = 0.20, size = 24, normalized size = 0.89

$$-\frac{1}{2}x^2+\frac{1}{6}\left(3x^2-2\right)\log\left(-3x^2+2\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="giac")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

maple [A] time = 0.06, size = 25, normalized size = 0.93

$$-\frac{x^2}{2}-\frac{\left(-3x^2+2\right)\ln\left(-3x^2+2\right)}{6}+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-3*x^2+2),x)
[Out] -1/6*(-3*x^2+2)*ln(-3*x^2+2)-1/2*x^2+1/3
```

maxima [A] time = 0.44, size = 24, normalized size = 0.89

$$-\frac{1}{2}x^2+\frac{1}{6}\left(3x^2-2\right)\log\left(-3x^2+2\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="maxima")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

mupad [B] time = 0.12, size = 25, normalized size = 0.93

$$x^2\left(\frac{\ln\left(2-3x^2\right)}{2}-\frac{1}{2}\right)-\frac{\ln\left(x^2-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(2 - 3*x^2),x)
[Out] x^2*(log(2 - 3*x^2)/2 - 1/2) - log(x^2 - 2/3)/3
```

sympy [A] time = 0.13, size = 27, normalized size = 1.00

$$\frac{x^2\log\left(2-3x^2\right)}{2}-\frac{x^2}{2}-\frac{\log\left(3x^2-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-3*x**2+2),x)
[Out] x**2*log(2 - 3*x**2)/2 - x**2/2 - log(3*x**2 - 2)/3
```

3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx$

Optimal. Leaf size=3

$$\sin^{-1}(\log(x))$$

```
[Out] arcsin(ln(x))
```

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {216}

$$\sin^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \log(x)\right)\\ &= \sin^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 3, normalized size = 1.00

$$\sin^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

fricas [B] time = 0.43, size = 20, normalized size = 6.67

$$-2\,\arctan\left(\frac{\sqrt{-\log(x)^2+1}-1}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-log(x)^2 + 1) - 1)/log(x))
```

giac [A] time = 0.16, size = 3, normalized size = 1.00

$$\arcsin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(log(x))
```

maple [A] time = 0.06, size = 4, normalized size = 1.33

$$\arcsin(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1-ln(x)^2)^(1/2),x)
[Out] arcsin(ln(x))
```

maxima [A] time = 0.98, size = 3, normalized size = 1.00

$$\arcsin\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(log(x))
```

mupad [B] time = 0.39, size = 3, normalized size = 1.00

$$\operatorname{asin}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - log(x)^2)^(1/2)),x)
[Out] asin(log(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(\log(x)-1\right)\left(\log(x)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(log(x) - 1)*(log(x) + 1))), x)
```

3.226 $\int 16x^3 \log^2(x)\, dx$

Optimal. Leaf size=24

$$\frac{x^4}{2} + 4x^4\log^2(x) - 2x^4\log(x)$$

```
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 2305, 2304}

$$\frac{x^4}{2} + 4x^4\log^2(x) - 2x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[16*x^3*Log[x]^2,x]
```

```
[Out] x^4/2 - 2*x^4*Log[x] + 4*x^4*Log[x]^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int 16x^3\log^2(x)\,dx &= 16\int x^3\log^2(x)\,dx \\
&= 4x^4\log^2(x) - 8\int x^3\log(x)\,dx \\
&= \frac{x^4}{2} - 2x^4\log(x) + 4x^4\log^2(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.25

$$16\left(\frac{x^4}{32} + \frac{1}{4}x^4\log^2(x) - \frac{1}{8}x^4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*x^3*Log[x]^2,x]
```

```
[Out] 16*(x^4/32 - (x^4*Log[x])/8 + (x^4*Log[x]^2)/4)
```

fricas [A] time = 0.46, size = 22, normalized size = 0.92

$$4\,x^4\log(x)^2 - 2\,x^4\log(x) + \frac{1}{2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="fricas")
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

giac [A] time = 0.16, size = 22, normalized size = 0.92

$$4\,x^4\log(x)^2 - 2\,x^4\log(x) + \frac{1}{2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="giac")
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

maple [A] time = 0.06, size = 23, normalized size = 0.96

$$4x^4\ln(x)^2 - 2x^4\ln(x) + \frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*x^3*ln(x)^2,x)
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

maxima [A] time = 0.44, size = 17, normalized size = 0.71

$$\frac{1}{2}\left(8\,\log(x)^2 - 4\,\log(x) + 1\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="maxima")
[Out] 1/2*(8*log(x)^2 - 4*log(x) + 1)*x^4
```

mupad [B] time = 0.04, size = 17, normalized size = 0.71

$$\frac{x^4\left(8\ln(x)^2 - 4\,\ln(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*x^3*log(x)^2,x)
[Out] (x^4*(8*log(x)^2 - 4*log(x) + 1))/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.92

$$4x^4\log{(x)}^2 - 2x^4\log{(x)} + \frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x**3*ln(x)**2,x)
[Out] 4*x**4*log(x)**2 - 2*x**4*log(x) + x**4/2
```

3.227 $\int \log \left(\sqrt{a+bx}\right) dx$

Optimal. Leaf size=25

$$\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

```
[Out] -1/2*x+1/2*(b*x+a)*ln(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2389, 2295}

$$\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[a + b*x]],x]
[Out] -x/2 + ((a + b*x)*Log[Sqrt[a + b*x]])/b
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\sqrt{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \log\left(\sqrt{x}\right) dx, x, a+bx\right)}{b}\\ &= -\frac{x}{2}+\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\frac{(a+bx)\log(a+bx)}{b}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[a + b*x]],x]
[Out] (-x + ((a + b*x)*Log[a + b*x])/b)/2
```

fricas [A] time = 0.44, size = 22, normalized size = 0.88

$$-\frac{bx-(bx+a)\log(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a))/b
```

giac [A] time = 0.16, size = 23, normalized size = 0.92

$$-\frac{bx - (bx + a)\log(bx + a) + a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="giac")
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

maple [A] time = 0.07, size = 32, normalized size = 1.28

$$\frac{x\ln(bx + a)}{2} + \frac{a\ln(bx + a)}{2b} - \frac{x}{2} - \frac{a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(b*x+a),x)
[Out] 1/2*x*ln(b*x+a)+1/2*a/b*ln(b*x+a)-1/2*x-1/2*a/b
```

maxima [A] time = 0.44, size = 23, normalized size = 0.92

$$-\frac{bx - (bx + a)\log(bx + a) + a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="maxima")
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

mupad [B] time = 0.06, size = 25, normalized size = 1.00

$$\frac{x\,\ln(a + b\,x)}{2} - \frac{x}{2} + \frac{a\,\ln(a + b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/2,x)
[Out] (x*log(a + b*x))/2 - x/2 + (a*log(a + b*x))/(2*b)
```

sympy [A] time = 0.16, size = 29, normalized size = 1.16

$$-b\left(-\frac{a\log(a + bx)}{2b^2} + \frac{x}{2b}\right) + \frac{x\log(a + bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(b*x+a),x)
[Out] -b*(-a*log(a + b*x)/(2*b**2) + x/(2*b)) + x*log(a + b*x)/2
```

3.228 $\int x \log \left(\sqrt{2+x}\right) dx$

Optimal. Leaf size=34

$$-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

```
[Out] 1/2*x-1/8*x^2-ln(2+x)+1/4*x^2*ln(2+x)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2395, 43}

$$-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Sqrt[2 + x]],x]
```

```
[Out] x/2 - x^2/8 + (x^2*Log[Sqrt[2 + x]])/2 - Log[2 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(\sqrt{2+x}\right)dx &= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\frac{1}{4}\int\frac{x^2}{2+x}\,dx\\ &= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\frac{1}{4}\int\left(-2+x+\frac{4}{2+x}\right)dx\\ &= \frac{x}{2}-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{2}\left(-\frac{x^2}{4}+\frac{1}{2}x^2\log(x+2)+x-2\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Sqrt[2 + x]],x]
```

```
[Out] (x - x^2/4 - 2*Log[2 + x] + (x^2*Log[2 + x])/2)/2
```

fricas [A] time = 0.48, size = 20, normalized size = 0.59

$$-\frac{1}{8}x^2 + \frac{1}{4}\left(x^2 - 4\right)\log\left(x + 2\right) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="fricas")
[Out] -1/8*x^2 + 1/4*(x^2 - 4)*log(x + 2) + 1/2*x
```

giac [A] time = 0.15, size = 30, normalized size = 0.88

$$\frac{1}{4}(x + 2)^2\log\left(x + 2\right) - \frac{1}{8}(x + 2)^2 - (x + 2)\log\left(x + 2\right) + x + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="giac")
[Out] 1/4*(x + 2)^2*log(x + 2) - 1/8*(x + 2)^2 - (x + 2)*log(x + 2) + x + 2
```

maple [A] time = 0.06, size = 31, normalized size = 0.91

$$-\frac{x^2}{8} + \frac{x}{2} - (x + 2)\ln\left(x + 2\right) + \frac{(x + 2)^2\ln\left(x + 2\right)}{4} + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x*ln(x+2),x)
[Out] -(x+2)*ln(x+2)+1/2*x+3/2+1/4*(x+2)^2*ln(x+2)-1/8*x^2
```

maxima [A] time = 0.44, size = 24, normalized size = 0.71

$$\frac{1}{4}x^2\log\left(x + 2\right) - \frac{1}{8}x^2 + \frac{1}{2}x - \log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="maxima")
[Out] 1/4*x^2*log(x + 2) - 1/8*x^2 + 1/2*x - log(x + 2)
```

mupad [B] time = 0.05, size = 20, normalized size = 0.59

$$\frac{x}{2} - \frac{x^2}{8} + \frac{\ln\left(x + 2\right)\left(x^2 - 4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x + 2))/2,x)
[Out] x/2 - x^2/8 + (log(x + 2)*(x^2 - 4))/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.65

$$\frac{x^2\log\left(x + 2\right)}{4} - \frac{x^2}{8} + \frac{x}{2} - \log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*ln(2+x),x)
[Out] x**2*log(x + 2)/4 - x**2/8 + x/2 - log(x + 2)
```

3.229 $\int x \log\left(\sqrt[3]{1+3x}\right) dx$

Optimal. Leaf size=40

$$-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{3x+1}\right)+\frac{x}{18}-\frac{1}{54}\log(3x+1)$$

```
[Out] 1/18*x-1/12*x^2+1/6*x^2*ln(1+3*x)-1/54*ln(1+3*x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2395, 43}

$$-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{3x+1}\right)+\frac{x}{18}-\frac{1}{54}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] x/18 - x^2/12 + (x^2*Log[(1 + 3*x)^(1/3)])/2 - Log[1 + 3*x]/54
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(\sqrt[3]{1+3x}\right)dx &= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\frac{x^2}{1+3x}\,dx\\ &= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\left(-\frac{1}{9}+\frac{x}{3}+\frac{1}{9(1+3x)}\right)dx\\ &= \frac{x}{18}-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{54}\log(1+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{1}{3}\left(-\frac{x^2}{4}+\frac{1}{2}x^2\log(3x+1)+\frac{x}{6}-\frac{1}{18}\log(3x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] (x/6 - x^2/4 - Log[1 + 3*x]/18 + (x^2*Log[1 + 3*x])/2)/3
```

fricas [A] time = 0.44, size = 24, normalized size = 0.60

$$-\frac{1}{12}\,x^2+\frac{1}{54}\left(9\,x^2-1\right)\log\left(3\,x+1\right)+\frac{1}{18}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="fricas")
```

```
[Out] -1/12*x^2 + 1/54*(9*x^2 - 1)*log(3*x + 1) + 1/18*x
```

giac [A] time = 0.16, size = 42, normalized size = 1.05

$$\frac{1}{54}\left(3\,x+1\right)^2\log\left(3\,x+1\right)-\frac{1}{108}\left(3\,x+1\right)^2-\frac{1}{27}\left(3\,x+1\right)\log\left(3\,x+1\right)+\frac{1}{9}\,x+\frac{1}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="giac")
```

```
[Out] 1/54*(3*x + 1)^2*log(3*x + 1) - 1/108*(3*x + 1)^2 - 1/27*(3*x + 1)*log(3*x
+ 1) + 1/9*x + 1/27
```

maple [A] time = 0.07, size = 39, normalized size = 0.98

$$-\frac{x^2}{12}+\frac{x}{18}+\frac{(3x+1)^2\ln(3x+1)}{54}-\frac{(3x+1)\ln(3x+1)}{27}+\frac{1}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3*x*ln(1+3*x),x)
```

```
[Out] 1/54*(1+3*x)^2*ln(1+3*x)-1/12*x^2+1/18*x+1/36-1/27*(1+3*x)*ln(1+3*x)
```

maxima [A] time = 0.44, size = 28, normalized size = 0.70

$$\frac{1}{6}\,x^2\log\left(3\,x+1\right)-\frac{1}{12}\,x^2+\frac{1}{18}\,x-\frac{1}{54}\,\log\left(3\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^2*log(3*x + 1) - 1/12*x^2 + 1/18*x - 1/54*log(3*x + 1)
```

mupad [B] time = 0.38, size = 22, normalized size = 0.55

$$\frac{x}{18}+\frac{\ln\left(3\,x+1\right)\,\left(x^2-\frac{1}{9}\right)}{6}-\frac{x^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(3*x + 1))/3,x)
```

```
[Out] x/18 + (log(3*x + 1)*(x^2 - 1/9))/6 - x^2/12
```

sympy [A] time = 0.12, size = 27, normalized size = 0.68

$$\frac{x^2\log(3x+1)}{6}-\frac{x^2}{12}+\frac{x}{18}-\frac{\log(3x+1)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*ln(1+3*x),x)
```

```
[Out] x**2*log(3*x + 1)/6 - x**2/12 + x/18 - log(3*x + 1)/54
```

3.230 $\int x \log\left(x+x^3\right)\,dx$

Optimal. Leaf size=31

$$-\frac{3x^2}{4}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\log\left(x^3+x\right)$$

```
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2525, 444, 43}

$$-\frac{3x^2}{4}+\frac{1}{2}x^2\log\left(x^3+x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(x+x^3\right)\,dx &= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{2}\int\frac{x\left(1+3x^2\right)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1+3x}{1+x}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\text{Subst}\left(\int\left(3-\frac{2}{1+x}\right)\,dx,x,x^2\right)\\
&= -\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{3x^2}{4}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\log\left(x^3+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + x^3],x]
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

fricas [A] time = 0.46, size = 25, normalized size = 0.81

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

giac [A] time = 0.16, size = 25, normalized size = 0.81

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="giac")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

maple [A] time = 0.06, size = 26, normalized size = 0.84

$$\frac{x^2\ln\left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^3+x),x)
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

maxima [A] time = 0.99, size = 25, normalized size = 0.81

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.41, size = 25, normalized size = 0.81

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{x^2\,\ln\left(x^3+x\right)}{2}-\frac{3\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x + x^3),x)
[Out] log(x^2 + 1)/2 + (x^2*log(x + x^3))/2 - (3*x^2)/4
```

<u>**sympy**</u> **[A]** time = 0.14, size = 26, normalized size = 0.84

$$\frac{x^2\log\left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**3+x),x)
```

```
[Out] x**2*log(x**3 + x)/2 - 3*x**2/4 + log(x**2 + 1)/2
```

3.231 $\int \log\left(x+\sqrt{1+x^2}\right)\,dx$

Optimal. Leaf size=26

$$x\log\left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

```
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2534, 261}

$$x\log\left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(x+\sqrt{1+x^2}\right)\,dx &= x\log\left(x+\sqrt{1+x^2}\right)-\int \frac{x}{\sqrt{1+x^2}}\,dx\\ &= -\sqrt{1+x^2}+x\log\left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$x\log\left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

fricas [A] time = 0.45, size = 22, normalized size = 0.85

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

giac [A] time = 0.18, size = 22, normalized size = 0.85

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

maple [A] time = 0.07, size = 23, normalized size = 0.88

$$x\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2+1)^(1/2)),x)
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(x+\sqrt{x^2+1}\right)-x+\arctan(x)-\int\frac{x}{x^3+\left(x^2+1\right)^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x*log(x + sqrt(x^2 + 1)) - x + arctan(x) - integrate(x/(x^3 + (x^2 + 1)^(3/
2) + x), x)
```

mupad [B] time = 0.08, size = 22, normalized size = 0.85

$$x\,\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 + 1)^(1/2)),x)
[Out] x*log(x + (x^2 + 1)^(1/2)) - (x^2 + 1)^(1/2)
```

sympy [A] time = 7.48, size = 20, normalized size = 0.77

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2+1)**(1/2)),x)
[Out] x*log(x + sqrt(x**2 + 1)) - sqrt(x**2 + 1)
```

3.232 $\int \log\left(x+\sqrt{-1+x^2}\right)dx$

Optimal. Leaf size=26

$$x\log\left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

```
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2534, 261}

$$x\log\left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(x+\sqrt{-1+x^2}\right)dx &= x\log\left(x+\sqrt{-1+x^2}\right)-\int \frac{x}{\sqrt{-1+x^2}}\,dx\\ &= -\sqrt{-1+x^2}+x\log\left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$x\log\left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

fricas [A] time = 0.45, size = 22, normalized size = 0.85

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

giac [A] time = 0.16, size = 22, normalized size = 0.85

$$x \log \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

maple [A] time = 0.07, size = 23, normalized size = 0.88

$$x \ln \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2-1)^(1/2)),x)
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \log \left(\sqrt{x+1}\sqrt{x-1} + x\right) - x + \int \frac{x}{x^3 + \left(x^2 - 1\right)e^{\left(\frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)\right)} - x}\, dx + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="maxima")
[Out] x*log(sqrt(x + 1)*sqrt(x - 1) + x) - x + integrate(x/(x^3 + (x^2 - 1)*e^(1/
2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.55, size = 22, normalized size = 0.85

$$x \ln \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 - 1)^(1/2)),x)
[Out] x*log(x + (x^2 - 1)^(1/2)) - (x^2 - 1)^(1/2)
```

sympy [A] time = 5.78, size = 20, normalized size = 0.77

$$x \log \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2-1)**(1/2)),x)
[Out] x*log(x + sqrt(x**2 - 1)) - sqrt(x**2 - 1)
```

3.233 $\int \log\left(x - \sqrt{-1 + x^2}\right) dx$

Optimal. Leaf size=26

$$\sqrt{x^2 - 1} + x \log\left(x - \sqrt{x^2 - 1}\right)$$

```
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2534, 261}

$$\sqrt{x^2 - 1} + x \log\left(x - \sqrt{x^2 - 1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x - Sqrt[-1 + x^2]],x]
```

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(x - \sqrt{-1 + x^2}\right) dx &= x \log\left(x - \sqrt{-1 + x^2}\right) + \int \frac{x}{\sqrt{-1 + x^2}} dx \\
&= \sqrt{-1 + x^2} + x \log\left(x - \sqrt{-1 + x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\sqrt{x^2 - 1} + x \log\left(x - \sqrt{x^2 - 1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x - Sqrt[-1 + x^2]],x]
```

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

fricas [A] time = 0.46, size = 22, normalized size = 0.85

$$x \log\left(x - \sqrt{x^2 - 1}\right) + \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

giac [A] time = 0.16, size = 22, normalized size = 0.85

$$x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

maple [A] time = 0.07, size = 23, normalized size = 0.88

$$x\ln\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2-1)^(1/2)),x)
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(-\sqrt{x+1}\,\sqrt{x-1}\,+x\right)-x-\int-\frac{x}{x^3-\left(x^2-1\right)e^{\left(\frac{1}{2}\,\log(x+1)+\frac{1}{2}\,\log(x-1)\right)}-x}\,dx+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="maxima")
[Out] x*log(-sqrt(x + 1)*sqrt(x - 1) + x) - x - integrate(-x/(x^3 - (x^2 - 1)*e^(
1/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.49, size = 22, normalized size = 0.85

$$x\,\ln\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x - (x^2 - 1)^(1/2)),x)
[Out] x*log(x - (x^2 - 1)^(1/2)) + (x^2 - 1)^(1/2)
```

sympy [A] time = 6.92, size = 20, normalized size = 0.77

$$x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x-(x**2-1)**(1/2)),x)
[Out] x*log(x - sqrt(x**2 - 1)) + sqrt(x**2 - 1)
```

3.234 $\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2548, 12, 1958, 50, 54, 215}

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 + x]
]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(\sqrt{x}+\sqrt{1+x}\right)dx &= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + Sqrt[1 + x]],x]
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 +
x]]
```

fricas [A] time = 0.52, size = 28, normalized size = 0.65

$$\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

giac [A] time = 0.18, size = 40, normalized size = 0.93

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x^2+x}-\frac{1}{4}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

maple [A] time = 0.07, size = 52, normalized size = 1.21

$$x\ln\left(\sqrt{x}+\sqrt{x+1}\right)-\frac{\sqrt{x+1}\,\sqrt{x}}{2}+\frac{\sqrt{(x+1)\,x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{4\sqrt{x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] x*ln(x^(1/2)+(x+1)^(1/2))-1/2*(x+1)^(1/2)*x^(1/2)+1/4*(x*(x+1))^(1/2)/x^(1/
2)/(x+1)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}x-\int\frac{x}{2\left(x^2+\left(x^{\frac{3}{2}}+\sqrt{x}\right)\sqrt{x+1}+x\right)}\,dx+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*x - integrate(1/2*x/(x^2 + (x^(3/2) + sq
rt(x))*sqrt(x + 1) + x), x) + 1/2*log(x + 1)
```

mupad [B] time = 1.08, size = 37, normalized size = 0.86

$$\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\,\ln\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) - (x^(1/2)*(x + 1)^(1/2))/2 + x*log((x +
 1)^(1/2) + x^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\sqrt{x}+\sqrt{x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Integral(log(sqrt(x) + sqrt(x + 1)), x)
```

3.235 $\int \sqrt[3]{x} \log(x) \, dx$

Optimal. Leaf size=21

$$\frac{3}{4}x^{4/3}\log(x) - \frac{9x^{4/3}}{16}$$

```
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{3}{4}x^{4/3}\log(x) - \frac{9x^{4/3}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*Log[x],x]
```

```
[Out] (-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x} \log(x) \, dx = -\frac{9x^{4/3}}{16} + \frac{3}{4}x^{4/3}\log(x)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.71

$$\frac{3}{16}x^{4/3}(4\log(x) - 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*Log[x],x]
```

```
[Out] (3*x^(4/3)*(-3 + 4*Log[x]))/16
```

fricas [A] time = 0.46, size = 14, normalized size = 0.67

$$\frac{3}{16}\left(4\,x\log(x) - 3\,x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="fricas")
```

```
[Out] 3/16*(4*x*log(x) - 3*x)*x^(1/3)
```

giac [A] time = 0.16, size = 13, normalized size = 0.62

$$\frac{3}{4}\,x^{\frac{4}{3}}\log(x) - \frac{9}{16}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

maple [A] time = 0.07, size = 14, normalized size = 0.67

$$\frac{3x^{\frac{4}{3}}\ln(x)}{4}-\frac{9x^{\frac{4}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*ln(x),x)
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.62

$$\frac{3}{4}\,x^{\frac{4}{3}}\log(x)-\frac{9}{16}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="maxima")
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

mupad [B] time = 0.34, size = 9, normalized size = 0.43

$$\frac{3\,x^{4/3}\left(\ln(x)-\frac{3}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*log(x),x)
[Out] (3*x^(4/3)*(log(x) - 3/4))/4
```

sympy [A] time = 2.61, size = 66, normalized size = 3.14

$$\begin{cases} \frac{3x^{\frac{4}{3}}\log(x)}{4}-\frac{9x^{\frac{4}{3}}}{16} & \text{for}\ |x|<1\\ -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4}-\frac{9x^{\frac{4}{3}}}{16} & \text{for}\ \frac{1}{|x|}<1\\ -G_{3,3}^{2,1}\left(\left.\begin{matrix} 1 & \frac{7}{3},\frac{7}{3}\\ \frac{4}{3},\frac{4}{3} & 0\end{matrix}\right|x\right)+G_{3,3}^{0,3}\left(\left.\begin{matrix} \frac{7}{3},\frac{7}{3},1 & \\ & \frac{4}{3},\frac{4}{3},0\end{matrix}\right|x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*ln(x),x)
[Out] Piecewise((3*x**(4/3)*log(x)/4 - 9*x**(4/3)/16, Abs(x) < 1), (-3*x**(4/3)*l
og(1/x)/4 - 9*x**(4/3)/16, 1/Abs(x) < 1), (-meijerg(((1,), (7/3, 7/3)), ((4
/3, 4/3), (0,)), x) + meijerg(((7/3, 7/3, 1), ()), ((), (4/3, 4/3, 0)), x),
 True))
```

3.236 $\int 2^{\log(x)}\,dx$

Optimal. Leaf size=13

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

```
[Out] x^(1+ln(2))/(1+ln(2))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2274, 30}

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x],x]
[Out] x^(1 + Log[2])/(1 + Log[2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(x)}\,dx &= \int x^{\log(2)}\,dx\\ &= \frac{x^{1+\log(2)}}{1+\log(2)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x2^{\log(x)}}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x],x]
[Out] (2^Log[x]*x)/(1 + Log[2])
```

fricas [A] time = 0.43, size = 14, normalized size = 1.08

$$\frac{xe^{\left(\log(2)\log(x)\right)}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="fricas")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

giac [A] time = 0.16, size = 14, normalized size = 1.08

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="giac")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

maple [A] time = 0.06, size = 13, normalized size = 1.00

$$\frac{x2^{\ln(x)}}{1+\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x),x)
[Out] x/(1+ln(2))*2^ln(x)
```

maxima [A] time = 0.52, size = 24, normalized size = 1.85

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="maxima")
[Out] 2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

mupad [B] time = 0.36, size = 13, normalized size = 1.00

$$\frac{x^{\ln(2)+1}}{\ln(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(x),x)
[Out] x^(log(2) + 1)/(log(2) + 1)
```

sympy [A] time = 0.39, size = 10, normalized size = 0.77

$$\frac{2^{\log(x)}x}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x),x)
[Out] 2**log(x)*x/(log(2) + 1)
```

3.237 $\int \frac{1-\log(x)}{x^2} \, dx$

Optimal. Leaf size=6

$$\frac{\log(x)}{x}$$

[Out] ln(x)/x

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2303}

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[(1 - Log[x])/x^2,x]

[Out] Log[x]/x

Rule 2303

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(b*(d*x)^(m + 1)*Log[c*x^n])/(d*(m + 1)), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Rubi steps

$$\int \frac{1-\log(x)}{x^2} \, dx = \frac{\log(x)}{x}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Log[x])/x^2,x]

[Out] Log[x]/x

fricas [A] time = 0.43, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-log(x))/x^2,x, algorithm="fricas")

[Out] log(x)/x

giac [A] time = 0.16, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-log(x))/x^2,x, algorithm="giac")

```
[Out] log(x)/x
```

maple [A] time = 0.06, size = 7, normalized size = 1.17

$$\frac{\ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x^2,x)
[Out] ln(x)/x
```

maxima [B] time = 0.76, size = 14, normalized size = 2.33

$$\frac{\log(x)+1}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="maxima")
[Out] (log(x) + 1)/x - 1/x
```

mupad [B] time = 0.35, size = 6, normalized size = 1.00

$$\frac{\ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 1)/x^2,x)
[Out] log(x)/x
```

sympy [A] time = 0.09, size = 3, normalized size = 0.50

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x**2,x)
[Out] log(x)/x
```

3.238 $\int \log \left(1+x+\sqrt{1+x}\right) dx$

Optimal. Leaf size=32

$$-x+\sqrt{x+1}+x\log \left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log (x+1)$$

```
[Out] -x+1/2*ln(1+x)+x*ln(1+x+(1+x)^(1/2))+(1+x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2548}

$$-x+\sqrt{x+1}+x\log \left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log (x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] + Log[1 + x]/2 + x*Log[1 + x + Sqrt[1 + x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log \left(1+x+\sqrt{1+x}\right) dx &= x\log \left(1+x+\sqrt{1+x}\right)-\int \frac{x\left(1+\frac{1}{2\sqrt{1+x}}\right)}{1+x+\sqrt{1+x}} dx\\ &= x\log \left(1+x+\sqrt{1+x}\right)-2\,\text{Subst}\left(\int \left(-\frac{1}{2}-\frac{1}{2x}+x\right) dx,x,\sqrt{1+x}\right)\\ &= -x+\sqrt{1+x}+\frac{1}{2}\log (1+x)+x\log \left(1+x+\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.19

$$-x+\sqrt{x+1}-\log \left(\sqrt{x+1}+1\right)+(x+1)\log \left(x+\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] - Log[1 + Sqrt[1 + x]] + (1 + x)*Log[1 + x + Sqrt[1 + x]]
```

fricas [A] time = 0.46, size = 38, normalized size = 1.19

$$(x-1)\log \left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}+\log \left(\sqrt{x+1}+1\right)+2\,\log \left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x - 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) + log(sqrt(x + 1) + 1) +
 2*log(sqrt(x + 1))
```

giac [A] time = 0.18, size = 33, normalized size = 1.03

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

maple [A] time = 0.07, size = 34, normalized size = 1.06

$$-x-\ln\left(1+\sqrt{x+1}\right)+(x+1)\ln\left(x+1+\sqrt{x+1}\right)-1+\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+1+(x+1)^(1/2)),x)
```

```
[Out] (x+1)*ln(x+1+(x+1)^(1/2))-x-1+(x+1)^(1/2)-ln(1+(x+1)^(1/2))
```

maxima [A] time = 0.63, size = 33, normalized size = 1.03

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

mupad [B] time = 0.11, size = 26, normalized size = 0.81

$$\ln\left(\sqrt{x+1}\right)-x+\sqrt{x+1}+x\,\ln\left(x+\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x + 1)^(1/2) + 1),x)
```

```
[Out] log((x + 1)^(1/2)) - x + (x + 1)^(1/2) + x*log(x + (x + 1)^(1/2) + 1)
```

sympy [B] time = 0.89, size = 184, normalized size = 5.75

$$\frac{x\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{x\sqrt{x+1}}{\sqrt{x+1}+1}+\frac{x\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}+\frac{\sqrt{x+1}\log\left(x\right.}{\sqrt{x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x+(1+x)**(1/2)),x)
```

```
[Out] x*sqrt(x + 1)*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - x*sqrt(x + 1)/(s
qrt(x + 1) + 1) + x*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - sqrt(x + 1
)*log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)*log(x + sqrt(x + 1)
+ 1)/(sqrt(x + 1) + 1) - sqrt(x + 1)/(sqrt(x + 1) + 1) - log(sqrt(x + 1) +
1)/(sqrt(x + 1) + 1) + log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - 1/(sqrt
(x + 1) + 1)
```

3.239 $\int \log\left(x+x^3\right)\,dx$

Optimal. Leaf size=16

$$x\log\left(x^3+x\right)-3x+2\tan^{-1}(x)$$

```
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2523, 388, 203}

$$x\log\left(x^3+x\right)-3x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(x+x^3\right)\,dx &= x\log\left(x+x^3\right)-\int \frac{1+3x^2}{1+x^2}\,dx\\
&= -3x+x\log\left(x+x^3\right)+2\int \frac{1}{1+x^2}\,dx\\
&= -3x+2\tan^{-1}(x)+x\log\left(x+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$x\log\left(x^3+x\right)-3x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

fricas [A] time = 0.43, size = 16, normalized size = 1.00

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="fricas")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

giac [A] time = 0.16, size = 16, normalized size = 1.00

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="giac")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

maple [A] time = 0.07, size = 17, normalized size = 1.06

$$x \ln \left(x^{3}+x \right)-3 x +2 \arctan \left(x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^3+x),x)
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

maxima [A] time = 1.55, size = 16, normalized size = 1.00

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="maxima")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

mupad [B] time = 0.05, size = 16, normalized size = 1.00

$$2\,\mathrm{atan}\left(x\right)-3\,x+x\,\ln\left(x^3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^3),x)
[Out] 2*atan(x) - 3*x + x*log(x + x^3)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.94

$$x \log {\left(x^{3} + x \right)} - 3 x + 2 \operatorname{atan}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**3+x),x)
[Out] x*log(x**3 + x) - 3*x + 2*atan(x)
```

3.240 $\int 2^{\log(-8+7x)}\,dx$

Optimal. Leaf size=20

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

[Out] 1/7*(-8+7*x)^(1+ln(2))/(1+ln(2))

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2274, 32}

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

Antiderivative was successfully verified.

[In] Int[2^Log[-8 + 7*x],x]

[Out] (-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(-8+7x)}\,dx &= \int (-8+7x)^{\log(2)}\,dx\\ &= \frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(7x-8)2^{\log(7x-8)}}{7+\log(128)}$$

Antiderivative was successfully verified.

[In] Integrate[2^Log[-8 + 7*x],x]

[Out] (2^Log[-8 + 7*x]*(-8 + 7*x))/(7 + Log[128])

fricas [A] time = 0.44, size = 23, normalized size = 1.15

$$\frac{(7\,x-8)e^{\left(\log(2)\log(7\,x-8)\right)}}{7\left(\log(2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^log(-8+7*x),x, algorithm="fricas")

```
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

giac [A] time = 0.18, size = 23, normalized size = 1.15

$$\frac{(7\,x-8)e^{\left(\log(2)\log(7\,x-8)\right)}}{7\left(\log(2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="giac")
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

maple [A] time = 0.07, size = 22, normalized size = 1.10

$$\frac{(7x-8)\,2^{\ln(7x-8)}}{7\ln(2)+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(-8+7*x),x)
[Out] 1/7*(-8+7*x)/(1+ln(2))*2^ln(-8+7*x)
```

maxima [A] time = 0.59, size = 29, normalized size = 1.45

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(7\,x-8)}}{7\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="maxima")
[Out] 1/7*2^((1/log(2) + 1)*log(7*x - 8))/((1/log(2) + 1)*log(2))
```

mupad [B] time = 0.39, size = 19, normalized size = 0.95

$$\frac{(7\,x-8)^{\ln(2)+1}}{7\,\left(\ln(2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(7*x - 8),x)
[Out] (7*x - 8)^(log(2) + 1)/(7*(log(2) + 1))
```

sympy [B] time = 0.49, size = 34, normalized size = 1.70

$$\frac{7\cdot 2^{\log{(7x-8)}}x}{7\log{(2)}+7}-\frac{8\cdot 2^{\log{(7x-8)}}}{7\log{(2)}+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(-8+7*x),x)
[Out] 7*2**log(7*x - 8)*x/(7*log(2) + 7) - 8*2**log(7*x - 8)/(7*log(2) + 7)
```

3.241 $\int \log \left(\frac{-11+5x}{5+76x}\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

```
[Out] -1/5*(11-5*x)*ln((-11+5*x)/(5+76*x))-861/380*ln(5+76*x)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2486, 31}

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] -((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))])/5 - (861*Log[5 + 76*x])/380
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{-11+5x}{5+76x}\right)dx &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{5}\int\frac{1}{5+76x}dx\\ &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.89

$$\left(x-\frac{11}{5}\right)\log\left(\frac{5x-11}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] (-11/5 + x)*Log[(-11 + 5*x)/(5 + 76*x)] - (861*Log[5 + 76*x])/380
```

fricas [A] time = 0.50, size = 33, normalized size = 0.94

$$x\log\left(\frac{5x-11}{76x+5}\right)-\frac{5}{76}\log(76x+5)-\frac{11}{5}\log(5x-11)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="fricas")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

giac [B] time = 0.19, size = 139, normalized size = 3.97

$$-\frac{861 \log\left(\frac{\frac{5\left(\frac{5(5x-11)}{76x+5}+11\right)}{\frac{76(5x-11)}{76x+5}-5}+11}{\frac{76\left(\frac{5(5x-11)}{76x+5}+11\right)}{\frac{76(5x-11)}{76x+5}-5}-5}\right)}{76\left(\frac{76(5x-11)}{76x+5}-5\right)} - \frac{861}{380}\log\left(\frac{|5x-11|}{|76x+5|}\right) + \frac{861}{380}\log\left(\left|\frac{76(5x-11)}{76x+5}-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="giac")
```

```
[Out] -861/76*log((5*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5
) + 11)/(76*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5) -
 5))/(76*(5*x - 11)/(76*x + 5) - 5) - 861/380*log(abs(5*x - 11)/abs(76*x +
5)) + 861/380*log(abs(76*(5*x - 11)/(76*x + 5) - 5))
```

maple [A] time = 0.17, size = 44, normalized size = 1.26

$$\frac{861\ln\left(-\frac{861}{76x+5}\right)}{380} + \frac{\left(\frac{5}{76}-\frac{861}{76(76x+5)}\right)(76x+5)\ln\left(\frac{5}{76}-\frac{861}{76(76x+5)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-11+5*x)/(5+76*x)),x)
```

```
[Out] 861/380*ln(-861/(5+76*x))+1/5*ln(5/76-861/76/(5+76*x))*(5/76-861/76/(5+76*x
))*(5+76*x)
```

maxima [A] time = 0.61, size = 33, normalized size = 0.94

$$x\log\left(\frac{5x-11}{76x+5}\right) - \frac{5}{76}\log(76x+5) - \frac{11}{5}\log(5x-11)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="maxima")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

mupad [B] time = 0.10, size = 29, normalized size = 0.83

$$x\ln\left(\frac{5x-11}{76x+5}\right) - \frac{5\ln\left(x+\frac{5}{76}\right)}{76} - \frac{11\ln\left(x-\frac{11}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((5*x - 11)/(76*x + 5)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - (5*log(x + 5/76))/76 - (11*log(x - 11/5))/5
```

sympy [A] time = 0.17, size = 32, normalized size = 0.91

$$x\log\left(\frac{5x-11}{76x+5}\right) - \frac{11\log\left(x-\frac{11}{5}\right)}{5} - \frac{5\log\left(x+\frac{5}{76}\right)}{76}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-11+5*x)/(5+76*x)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 11*log(x - 11/5)/5 - 5*log(x + 5/76)/76
```

3.242 $\int \log\left(\frac{1}{13+x}\right) dx$

Optimal. Leaf size=12

$$x + (x + 13)\log\left(\frac{1}{x + 13}\right)$$

```
[Out] x+(13+x)*ln(1/(13+x))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2295}

$$x + (x + 13)\log\left(\frac{1}{x + 13}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(13 + x)^(-1)],x]
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{1}{13+x}\right) dx &= \text{Subst}\left(\int \log\left(\frac{1}{x}\right) dx, x, 13+x\right)\\ &= x + (13+x)\log\left(\frac{1}{13+x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + (x + 13)\log\left(\frac{1}{x + 13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(13 + x)^(-1)],x]
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

fricas [A] time = 0.43, size = 12, normalized size = 1.00

$$(x + 13)\log\left(\frac{1}{x + 13}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="fricas")
```

```
[Out] (x + 13)*log(1/(x + 13)) + x
```

giac [A] time = 0.17, size = 12, normalized size = 1.00

$$-(x+13)\log(x+13)+x+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="giac")
[Out] -(x + 13)*log(x + 13) + x + 13
```

maple [A] time = 0.07, size = 14, normalized size = 1.17

$$x+(x+13)\ln\left(\frac{1}{x+13}\right)+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/(13+x)),x)
[Out] (13+x)*ln(1/(13+x))+13+x
```

maxima [A] time = 0.61, size = 12, normalized size = 1.00

$$-(x+13)\log(x+13)+x+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="maxima")
[Out] -(x + 13)*log(x + 13) + x + 13
```

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$\left(\ln\left(\frac{1}{x+13}\right)+1\right)(x+13)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/(x + 13)),x)
[Out] (log(1/(x + 13)) + 1)*(x + 13)
```

sympy [A] time = 0.11, size = 15, normalized size = 1.25

$$x\log\left(\frac{1}{x+13}\right)+x-13\log(x+13)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/(13+x)),x)
[Out] x*log(1/(x + 13)) + x - 13*log(x + 13)
```

3.243 $\int x \log\left(\frac{1+x}{x^2}\right) dx$

Optimal. Leaf size=36

$$\frac{x^2}{4}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

```
[Out] 1/2*x+1/4*x^2-1/2*ln(1+x)+1/2*x^2*ln((1+x)/x^2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2495, 30, 43}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[(1 + x)/x^2],x]
```

```
[Out] x/2 + x^2/4 - Log[1 + x]/2 + (x^2*Log[(1 + x)/x^2])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(\frac{1+x}{x^2}\right)dx &= \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\frac{x^2}{1+x}\,dx+\int x\,dx\\
&= \frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\left(-1+x+\frac{1}{1+x}\right)dx\\
&= \frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\log(1+x)+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{1}{4}\left(x\left(2x\log\left(\frac{x+1}{x^2}\right)+x+2\right)-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + x)/x^2],x]
[Out] (-2*Log[1 + x] + x*(2 + x + 2*x*Log[(1 + x)/x^2]))/4
```

fricas [A] time = 0.43, size = 28, normalized size = 0.78

$$\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="fricas")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

giac [A] time = 0.17, size = 29, normalized size = 0.81

$$\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="giac")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(abs(x + 1))
```

maple [A] time = 0.08, size = 39, normalized size = 1.08

$$\frac{x^2\ln\left(\frac{\frac{1}{x}+1}{x}\right)}{2}+\frac{x^2}{4}+\frac{x}{2}+\frac{\ln\left(\frac{1}{x}\right)}{2}-\frac{\ln\left(\frac{1}{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln((x+1)/x^2),x)
[Out] 1/2*x^2*ln(1/x*(1+1/x))-1/2*ln(1+1/x)+1/4*x^2+1/2*x+1/2*ln(1/x)
```

maxima [A] time = 0.77, size = 28, normalized size = 0.78

$$\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="maxima")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

mupad [B] time = 0.49, size = 40, normalized size = 1.11

$$\frac{x}{2}-\frac{\ln(x\,(x+1))}{3}-\frac{\ln\left(\frac{x+1}{x^2}\right)}{6}+\frac{x^2\,\ln\left(\frac{x+1}{x^2}\right)}{2}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log((x + 1)/x^2),x)
[Out] x/2 - log(x*(x + 1))/3 - log((x + 1)/x^2)/6 + (x^2*log((x + 1)/x^2))/2 + x^
2/4
```

sympy [A] time = 0.15, size = 27, normalized size = 0.75

$$\frac{x^2 \log\left(\frac{x+1}{x^2}\right)}{2} + \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln((1+x)/x**2),x)
```

```
[Out] x**2*log((x + 1)/x**2)/2 + x**2/4 + x/2 - log(x + 1)/2
```

3.244 $\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx$

Optimal. Leaf size=54

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

```
[Out] 343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5*
x)/x^2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2495, 30, 43}

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\log\left(\frac{7+5x}{x^2}\right)dx &= \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)+\frac{\int x^3\,dx}{2}-\frac{5}{4}\int\frac{x^4}{7+5x}\,dx\\
&= \frac{x^4}{8}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)-\frac{5}{4}\int\left(-\frac{343}{625}+\frac{49x}{125}-\frac{7x^2}{25}+\frac{x^3}{5}+\frac{2401}{625(7+5x)}\right)dx\\
&= \frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

fricas [A] time = 0.44, size = 42, normalized size = 0.78

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

giac [A] time = 0.17, size = 43, normalized size = 0.80

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(|5\,x+7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(abs(5*x + 7))
```

maple [A] time = 0.08, size = 53, normalized size = 0.98

$$\frac{x^4\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}+\frac{2401\ln\left(\frac{1}{x}\right)}{2500}-\frac{2401\ln\left(\frac{7}{x}+5\right)}{2500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln((7+5*x)/x^2),x)
```

```
[Out] 1/4*x^4*ln((7/x+5)/x)-2401/2500*ln(7/x+5)+1/16*x^4+7/60*x^3-49/200*x^2+343/
500*x+2401/2500*ln(1/x)
```

maxima [A] time = 0.68, size = 42, normalized size = 0.78

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

mupad [B] time = 0.59, size = 56, normalized size = 1.04

$$\frac{343\,x}{500}-\frac{2401\,\ln\left(x\,\left(5\,x+7\right)\right)}{3750}-\frac{2401\,\ln\left(\frac{5\,x+7}{x^2}\right)}{7500}+\frac{x^4\,\ln\left(\frac{5\,x+7}{x^2}\right)}{4}-\frac{49\,x^2}{200}+\frac{7\,x^3}{60}+\frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log((5*x + 7)/x^2),x)
```

```
[Out] (343*x)/500 - (2401*log(x*(5*x + 7)))/3750 - (2401*log((5*x + 7)/x^2))/7500
 + (x^4*log((5*x + 7)/x^2))/4 - (49*x^2)/200 + (7*x^3)/60 + x^4/16
```

sympy [A] time = 0.17, size = 48, normalized size = 0.89

$$\frac{x^4 \log\left(\frac{5x+7}{x^2}\right)}{4} + \frac{x^4}{16} + \frac{7x^3}{60} - \frac{49x^2}{200} + \frac{343x}{500} - \frac{2401 \log(5x+7)}{2500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln((7+5*x)/x**2),x)
```

```
[Out] x**4*log((5*x + 7)/x**2)/4 + x**4/16 + 7*x**3/60 - 49*x**2/200 + 343*x/500
- 2401*log(5*x + 7)/2500
```

3.245 $\int (a+bx)\log(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/2*(b*x+a)^2*ln(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2390, 2304}

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[a + b*x],x]
```

```
[Out] -(a + b*x)^2/(4*b) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(a+bx)\,dx&=\frac{\text{Subst}\left(\int x\log(x)\,dx,x,a+bx\right)}{b}\\&=-\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.94

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{1}{4}x(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[a + b*x],x]
```

```
[Out] -1/4*(x*(2*a + b*x)) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

fricas [A] time = 0.42, size = 42, normalized size = 1.20

$$-\frac{b^2x^2+2\,abx-2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="fricas")
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/b
```

giac [A] time = 0.16, size = 31, normalized size = 0.89

$$\frac{(bx+a)^2\log(bx+a)}{2b} - \frac{(bx+a)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="giac")
[Out] 1/2*(b*x + a)^2*log(b*x + a)/b - 1/4*(b*x + a)^2/b
```

maple [A] time = 0.07, size = 55, normalized size = 1.57

$$\frac{b\,x^2\ln(bx+a)}{2} + ax\ln(bx+a) - \frac{b\,x^2}{4} + \frac{a^2\ln(bx+a)}{2b} - \frac{ax}{2} - \frac{a^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(b*x+a),x)
[Out] 1/2*b*ln(b*x+a)*x^2+ln(b*x+a)*x*a+1/2/b*ln(b*x+a)*a^2-1/4*b*x^2-1/2*a*x-1/4
/b*a^2
```

maxima [A] time = 0.57, size = 52, normalized size = 1.49

$$\frac{1}{4}\,b\left(\frac{2\,a^2\log(bx+a)}{b^2} - \frac{bx^2+2\,ax}{b}\right) + \frac{1}{2}\left(bx^2+2\,ax\right)\log(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="maxima")
[Out] 1/4*b*(2*a^2*log(b*x + a)/b^2 - (b*x^2 + 2*a*x)/b) + 1/2*(b*x^2 + 2*a*x)*lo
g(b*x + a)
```

mupad [B] time = 0.47, size = 46, normalized size = 1.31

$$\frac{a^2\,\ln(a+b\,x)}{2\,b} - \frac{b\,x^2}{4} - \frac{a\,x}{2} + a\,x\,\ln(a+b\,x) + \frac{b\,x^2\,\ln(a+b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x),x)
[Out] (a^2*log(a + b*x))/(2*b) - (b*x^2)/4 - (a*x)/2 + a*x*log(a + b*x) + (b*x^2*
log(a + b*x))/2
```

sympy [A] time = 0.19, size = 41, normalized size = 1.17

$$\frac{a^2\log(a+bx)}{2b} - \frac{ax}{2} - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log(a+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(b*x+a),x)
[Out] a**2*log(a + b*x)/(2*b) - a*x/2 - b*x**2/4 + (a*x + b*x**2/2)*log(a + b*x)
```

3.246 $\int (a+bx)^2 \log(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

```
[Out] -1/9*(b*x+a)^3/b+1/3*(b*x+a)^3*ln(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2304}

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -(a + b*x)^3/(9*b) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\log(x)\,dx,x,a+bx\right)}{b}\\ &= -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.26

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{1}{9}x\left(3a^2+3abx+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -1/9*(x*(3*a^2 + 3*a*b*x + b^2*x^2)) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

fricas [B] time = 0.46, size = 64, normalized size = 1.83

$$-\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx-3\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log\left(bx+a\right)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/b
```

giac [A] time = 0.22, size = 31, normalized size = 0.89

$$\frac{(bx+a)^3 \log(bx+a)}{3b} - \frac{(bx+a)^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*log(b*x + a)/b - 1/9*(b*x + a)^3/b
```

maple [B] time = 0.07, size = 82, normalized size = 2.34

$$\frac{b^2x^3 \ln(bx+a)}{3} + ab\,x^2 \ln(bx+a) - \frac{b^2x^3}{9} + a^2x \ln(bx+a) - \frac{ab\,x^2}{3} + \frac{a^3 \ln(bx+a)}{3b} - \frac{a^2x}{3} - \frac{a^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(b*x+a),x)
```

```
[Out] 1/3*b^2*ln(b*x+a)*x^3+b*ln(b*x+a)*x^2*a+ln(b*x+a)*x*a^2+1/3/b*ln(b*x+a)*a^3
-1/9*b^2*x^3-1/3*a*b*x^2-1/3*a^2*x-1/9/b*a^3
```

maxima [B] time = 0.60, size = 74, normalized size = 2.11

$$\frac{1}{9}\left(\frac{3\,a^3 \log(bx+a)}{b^2} - \frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{b}\right)b + \frac{1}{3}\left(b^2x^3 + 3\,abx^2 + 3\,a^2x\right)\log(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(3*a^3*log(b*x + a)/b^2 - (b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/b)*b + 1/3*(b
^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(b*x + a)
```

mupad [B] time = 0.46, size = 73, normalized size = 2.09

$$\frac{a^3\,\ln(a+b\,x)}{3\,b} - \frac{b^2\,x^3}{9} - \frac{a^2\,x}{3} + \frac{b^2\,x^3\,\ln(a+b\,x)}{3} - \frac{a\,b\,x^2}{3} + a^2\,x\,\ln(a+b\,x) + a\,b\,x^2\,\ln(a+b\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a^3*log(a + b*x))/(3*b) - (b^2*x^3)/9 - (a^2*x)/3 + (b^2*x^3*log(a + b*x))
/3 - (a*b*x^2)/3 + a^2*x*log(a + b*x) + a*b*x^2*log(a + b*x)
```

sympy [B] time = 0.23, size = 63, normalized size = 1.80

$$\frac{a^3 \log(a+bx)}{3b} - \frac{a^2x}{3} - \frac{abx^2}{3} - \frac{b^2x^3}{9} + \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\log(a+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(b*x+a),x)
```

```
[Out] a**3*log(a + b*x)/(3*b) - a**2*x/3 - a*b*x**2/3 - b**2*x**3/9 + (a**2*x + a
*b*x**2 + b**2*x**3/3)*log(a + b*x)
```

3.247 $\int \frac{\log(a+bx)}{a+bx} \, dx$

Optimal. Leaf size=15

$$\frac{\log^2(a+bx)}{2b}$$

```
[Out] 1/2*ln(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2390, 2301}

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(a+bx)}{a+bx} \, dx &= \frac{\text{Subst}\left(\int \frac{\log(x)}{x} \, dx, x, a+bx\right)}{b} \\ &= \frac{\log^2(a+bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

fricas [A] time = 0.52, size = 13, normalized size = 0.87

$$\frac{\log\left(bx+a\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

giac [A] time = 0.16, size = 13, normalized size = 0.87

$$\frac{\log\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

maple [A] time = 0.06, size = 14, normalized size = 0.93

$$\frac{\ln\left(bx+a\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a),x)
```

```
[Out] 1/2*ln(b*x+a)^2/b
```

maxima [A] time = 0.70, size = 13, normalized size = 0.87

$$\frac{\log\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

mupad [B] time = 0.55, size = 13, normalized size = 0.87

$$\frac{\ln\left(a+b\,x\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/(a + b*x),x)
```

```
[Out] log(a + b*x)^2/(2*b)
```

sympy [A] time = 0.26, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a),x)
```

```
[Out] log(a + b*x)**2/(2*b)
```

3.248 $\int \frac{\log(a+bx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2390, 2304}

$$-\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(1/(b*(a + b*x))) - Log[a + b*x]/(b*(a + b*x))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(a+bx)}{(a+bx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{\log(x)}{x^2} \, dx, x, a+bx\right)}{b} \\ &= -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.68

$$-\frac{\log(a+bx)+1}{ab+b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -((1 + Log[a + b*x])/(a*b + b^2*x))
```

fricas [A] time = 0.50, size = 21, normalized size = 0.68

$$-\frac{\log(bx+a)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(log(b*x + a) + 1)/(b^2*x + a*b)
```

giac [A] time = 0.17, size = 32, normalized size = 1.03

$$-b\left(\frac{\log\left(bx+a\right)}{(bx+a)b^2}+\frac{1}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="giac")
[Out] -b*(log(b*x + a)/((b*x + a)*b^2) + 1/((b*x + a)*b^2))
```

maple [A] time = 0.08, size = 32, normalized size = 1.03

$$-\frac{\ln\left(bx+a\right)}{\left(bx+a\right)b}-\frac{1}{\left(bx+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a)^2,x)
[Out] -1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

maxima [A] time = 0.67, size = 31, normalized size = 1.00

$$-\frac{\log\left(bx+a\right)}{(bx+a)b}-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
[Out] -log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

mupad [B] time = 0.49, size = 25, normalized size = 0.81

$$-\frac{a+a\,\ln\left(a+b\,x\right)}{a\,b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/(a + b*x)^2,x)
[Out] -(a + a*log(a + b*x))/(a*b*(a + b*x))
```

sympy [A] time = 0.20, size = 26, normalized size = 0.84

$$-\frac{\log\left(a+bx\right)}{ab+b^2x}-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a)**2,x)
[Out] -log(a + b*x)/(a*b + b**2*x) - 1/(a*b + b**2*x)
```

3.249 $\int (a+bx)^n \log(a+bx)\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)}-\frac{(a+bx)^{n+1}}{b(n+1)^2}$$

```
[Out] -(b*x+a)^(1+n)/b/(1+n)^2+(b*x+a)^(1+n)*ln(b*x+a)/b/(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2304}

$$\frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)}-\frac{(a+bx)^{n+1}}{b(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] -((a + b*x)^(1 + n)/(b*(1 + n)^2)) + ((a + b*x)^(1 + n)*Log[a + b*x])/(b*(1
 + n))
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^n \log(x)\,dx, x, a+bx\right)}{b}\\ &= -\frac{(a+bx)^{1+n}}{b(1+n)^2}+\frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{(a+bx)^{n+1}((n+1)\log(a+bx)-1)}{b(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] ((a + b*x)^(1 + n)*(-1 + (1 + n)*Log[a + b*x]))/(b*(1 + n)^2)
```

fricas [A] time = 0.45, size = 47, normalized size = 1.07

$$-\frac{\left(bx-(an+(bn+b)x+a)\log(bx+a)+a\right)(bx+a)^n}{bn^2+2\,bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - (a*n + (b*n + b)*x + a)*log(b*x + a) + a)*(b*x + a)^n/(b*n^2 + 2*b*
n + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n \log(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*log(b*x + a), x)
```

maple [B] time = 0.07, size = 96, normalized size = 2.18

$$\frac{x\,e^{n\ln(bx+a)}\ln(bx+a)}{n+1}+\frac{a\,e^{n\ln(bx+a)}\ln(bx+a)}{(n+1)\,b}-\frac{x\,e^{n\ln(bx+a)}}{n^2+2n+1}-\frac{a\,e^{n\ln(bx+a)}}{\left(n^2+2n+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*ln(b*x+a),x)
```

```
[Out] 1/(n+1)*x*ln(b*x+a)*exp(n*ln(b*x+a))+a/b/(n+1)*ln(b*x+a)*exp(n*ln(b*x+a))-1
/(n^2+2*n+1)*x*exp(n*ln(b*x+a))-a/b/(n^2+2*n+1)*exp(n*ln(b*x+a))
```

maxima [A] time = 0.53, size = 44, normalized size = 1.00

$$\frac{(bx+a)^{n+1}\log(bx+a)}{b(n+1)}-\frac{(bx+a)^{n+1}}{b(n+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*log(b*x + a)/(b*(n + 1)) - (b*x + a)^(n + 1)/(b*(n + 1)^2
)
```

mupad [B] time = 0.46, size = 52, normalized size = 1.18

$$\begin{cases} \frac{\ln(a+b\,x)^2}{2\,b} & \text{if } n=-1 \\ \frac{\left(\ln(a+b\,x)-\frac{1}{n+1}\right)(a+b\,x)^{n+1}}{b\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, log(a + b*x)^2/(2*b), n ~= -1, ((log(a + b*x) - 1/(n + 1
))*(a + b*x)^(n + 1))/(b*(n + 1)))
```

sympy [A] time = 1.47, size = 185, normalized size = 4.20

$$\begin{cases} \frac{x\log(a)}{a} & \text{for } b = \\ a^n x\log(a) & \text{for } b = \\ \frac{\log(a+bx)^2}{2b} & \text{for } n = \\ \frac{an(a+bx)^n\log(a+bx)}{bn^2+2bn+b}+\frac{a(a+bx)^n\log(a+bx)}{bn^2+2bn+b}-\frac{a(a+bx)^n}{bn^2+2bn+b}+\frac{bnx(a+bx)^n\log(a+bx)}{bn^2+2bn+b}+\frac{bx(a+bx)^n\log(a+bx)}{bn^2+2bn+b}-\frac{bx(a+bx)^n}{bn^2+2bn+b} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*ln(b*x+a),x)
```

```
[Out] Piecewise((x*log(a)/a, Eq(b, 0) & Eq(n, -1)), (a**n*x*log(a), Eq(b, 0)), (l
og(a + b*x)**2/(2*b), Eq(n, -1)), (a*n*(a + b*x)**n*log(a + b*x)/(b*n**2 +
2*b*n + b) + a*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - a*(a + b*x)
**n/(b*n**2 + 2*b*n + b) + b*n*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n
+ b) + b*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - b*x*(a + b*x)**
n/(b*n**2 + 2*b*n + b), True))
```

3.250 $\int \frac{1}{ax+bx\log(cx^n)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] ln(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n])^(-1),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log\left(cx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n])^(-1),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

fricas [A] time = 0.44, size = 19, normalized size = 1.06

$$\frac{\log\left(bn\log(x)+b\log(c)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] log(b*n*log(x) + b*log(c) + a)/(b*n)
```

giac [B] time = 0.16, size = 45, normalized size = 2.50

$$\frac{\log\left(\frac{1}{4}\left(\pi bn\left(\operatorname{sgn}(x)-1\right)+\pi b\left(\operatorname{sgn}(c)-1\right)\right)^{2}+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)+a\right)^{2}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
+ b*log(abs(c)) + a)^2)/(b*n)
```

maple [A] time = 0.07, size = 19, normalized size = 1.06

$$\frac{\ln\left(b\ln\left(c\,x^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)),x)
```

```
[Out] 1/b/n*ln(b*ln(c*x^n)+a)
```

maxima [A] time = 0.66, size = 24, normalized size = 1.33

$$\frac{\log\left(\frac{b\log(c)+b\log(x^{n})+a}{b}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log((b*log(c) + b*log(x^n) + a)/b)/(b*n)
```

mupad [B] time = 0.38, size = 18, normalized size = 1.00

$$\frac{\ln\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)),x)
```

```
[Out] log(a + b*log(c*x^n))/(b*n)
```

sympy [A] time = 1.15, size = 34, normalized size = 1.89

$$\begin{cases} \frac{\log(x)}{a} & \text{for}\: b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b\log(c)} & \text{for}\: n = 0 \\ \frac{\log(x)}{a} & \text{for}\: b = 0 \\ \frac{\log\left(\frac{a}{b}+n\log(x)+\log(c)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b*log(c)), Eq(n, 0)
), (log(x)/a, Eq(b, 0)), (log(a/b + n*log(x) + log(c))/(b*n), True))
```

3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

[Out] arctan(ln(c*x^n)*b^(1/2)/a^(1/2))/n/a^(1/2)/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x*Log[c*x^n]^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log^2(cx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x*Log[c*x^n]^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)

fricas [A] time = 0.44, size = 121, normalized size = 3.78

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2-2\,\sqrt{-ab}\left(n\log(x)+\log(c)\right)-a}{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+a}\right)}{2\,abn}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\left(n\log(x)+\log(c)\right)}{a}\right)}{abn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 - 2*sqrt(-a*b)*(n*log(x) + log(c)) - a)/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a))/(a*b*n), sqrt(a*b)*arctan(sqrt(a*b)*(n*log(x) + log(c))/a)/(a*b*n)]

giac [A] time = 0.16, size = 26, normalized size = 0.81

$$\frac{\arctan\left(\frac{bn\log(x)+b\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="giac")

[Out] arctan((b*n*log(x) + b*log(c))/sqrt(a*b))/(sqrt(a*b)*n)

maple [A] time = 0.07, size = 24, normalized size = 0.75

$$\frac{\arctan\left(\frac{b\ln(c\,x^n)}{\sqrt{ab}}\right)}{\sqrt{ab}\;n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+b*x*ln(c*x^n)^2),x)

[Out] 1/n/(a*b)^(1/2)*arctan(b*ln(c*x^n)/(a*b)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx\log\left(cx^n\right)^2 + ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="maxima")

[Out] integrate(1/(b*x*log(c*x^n)^2 + a*x), x)

mupad [B] time = 0.47, size = 71, normalized size = 2.22

$$-\frac{\ln\left(\frac{1}{b\,x}+\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)-\ln\left(\frac{1}{b\,x}-\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)}{2\,\sqrt{-a}\,\sqrt{b}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x + b*x*log(c*x^n)^2),x)

[Out] -(log(1/(b*x) + log(c*x^n)/((-a)^(1/2)*b^(1/2)*x)) - log(1/(b*x) - log(c*x^n)/((-a)^(1/2)*b^(1/2)*x)))/(2*(-a)^(1/2)*b^(1/2)*n)

sympy [A] time = 5.98, size = 126, normalized size = 3.94

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b\log(c)^2} & \text{for } n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{1}{b\left(n^2\log(x)+n\log(c)\right)} & \text{for } a = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+n\log(x)+\log(c)\right)}{2\sqrt{a}\,bn\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+n\log(x)+\log(c)\right)}{2\sqrt{a}\,bn\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(
a + b*log(c)**2), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (-1/(b*(n**2*log(x) + n*
log(c))), Eq(a, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + n*log(x) + log(c))/(2*s
qrt(a)*b*n*sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + n*log(x) + log(c))/(2*s
qrt(a)*b*n*sqrt(1/b)), True))
```

3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$

Optimal. Leaf size=144

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}\,n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}\,n}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,n}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*ln(c*x^n))/a^(2/3)/b^(1/3)/n-1/6*ln(a^(2/3)-a^(1/3)*
b^(1/3)*ln(c*x^n)+b^(2/3)*ln(c*x^n)^2)/a^(2/3)/b^(1/3)/n-1/3*arctan(1/3*(a^
(1/3)-2*b^(1/3)*ln(c*x^n))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/n*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}\,n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}\,n}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Log[c*x^n])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/
3)*b^(1/3)*n)) + Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)*n) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(6*a^(2/3)
*b^(1/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{2\sqrt[3]{a}n} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subs}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+\right.}{6a^{2/3}\sqrt[3]{b}n}$$

Mathematica [A] time = 0.05, size = 112, normalized size = 0.78

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{b}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*Log[c*x^n]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n]
+ b^(2/3)*Log[c*x^n]^2])/(a^(2/3)*b^(1/3)*n)
```

fricas [A] time = 0.49, size = 480, normalized size = 3.33

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abn^3\log(x)^3+6abn^2\log(c)\log(x)^2+6abn\log(c)^2\log(x)+2ab\log(c)^3-a^2+3\sqrt{\frac{1}{3}}\left(2abn^2\log(x)^2+4abn\log(c)\log\right.}{bn^3\log(x)^3+3bn^2\log(c)\log(x)^2+3bn\log}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*n^3*log(x)^3 + 6*a*
b*n^2*log(c)*log(x)^2 + 6*a*b*n*log(c)^2*log(x) + 2*a*b*log(c)^3 - a^2 + 3*
sqrt(1/3)*(2*a*b*n^2*log(x)^2 + 4*a*b*n*log(c)*log(x) + 2*a*b*log(c)^2 + (a
^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) -
 3*(a^2*b)^(1/3)*(a*n*log(x) + a*log(c)))/(b*n^3*log(x)^3 + 3*b*n^2*log(c)*
log(x)^2 + 3*b*n*log(c)^2*log(x) + b*log(c)^3 + a)) - (a^2*b)^(2/3)*log(a*b
*n^2*log(x)^2 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log
(x) + log(c)) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*l
og(c) + (a^2*b)^(2/3)))/(a^2*b*n), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/
b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*n^2*log(x)^2 + 2*a*b*n*
log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log(x) + log(c)) + (a^2*b)^
(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*log(c) + (a^2*b)^(2/3)))/
(a^2*b*n)]
```

giac [B] time = 0.21, size = 239, normalized size = 1.66

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\pi b\left(\operatorname{sgn}(c)-1\right)-2\,bn\log(x)-2\,b\log\left(|c|\right)-2\,\left(ab^2\right)^{\frac{1}{3}}}{2\,\sqrt{3}\,bn\log(x)+\pi b\left(\operatorname{sgn}(c)-1\right)+2\,\sqrt{3}\,b\log\left(|c|\right)-2\,\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}}\right)+\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*n^3))^(1/3)*arctan((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n
*log(x) - 2*b*log(abs(c)) - 2*(a*b^2)^(1/3))/(2*sqrt(3)*b*n*log(x) + pi*b*(
sgn(c) - 1) + 2*sqrt(3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))) + 1/6*(1/
(a^2*b*n^3))^(1/3)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b
*n*log(abs(x)) + b*log(abs(c)) + (a*b^2)^(1/3))^2) - 1/6*(1/(a^2*b*n^3))^(1
/3)*log((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n*log(x) - 2*b*log(abs(c)) - 2*(a*
b^2)^(1/3))^2 + (2*sqrt(3)*b*n*log(x) + pi*b*(sgn(c) - 1) + 2*sqrt(3)*b*log
(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))^2)
```

maple [A] time = 0.07, size = 120, normalized size = 0.83

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\ln\left(c\,x^n\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}bn}+\frac{\ln\left(\ln\left(c\,x^n\right)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}bn}-\frac{\ln\left(\ln\left(c\,x^n\right)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(c\,x^n\right)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^3),x)
```

```
[Out] 1/3/n/b/(a/b)^(2/3)*ln(ln(c*x^n)+(a/b)^(1/3))-1/6/n/b/(a/b)^(2/3)*ln(ln(c*x
^n)^2-(a/b)^(1/3)*ln(c*x^n)+(a/b)^(2/3))+1/3/n/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*ln(c*x^n)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx\log\left(cx^n\right)^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^3 + a*x), x)
```

mupad [B] time = 2.28, size = 153, normalized size = 1.06

$$\frac{\ln\left(\frac{3\,a^{1/3}\,n}{b^{4/3}\,x^2}+\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}\right)}{3\,a^{2/3}\,b^{1/3}\,n}+\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}+\frac{3\,a^{1/3}\,n\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}-\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}-\frac{3\,a^{1/3}\,n\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(1+\cdots\right.}{6\,a^{2/3}\,b^{1/3}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)^3),x)
```

```
[Out] log((3*a^(1/3)*n)/(b^(4/3)*x^2) + (3*n*log(c*x^n))/(b*x^2))/(3*a^(2/3)*b^(1
/3)*n) + (log((3*n*log(c*x^n))/(b*x^2) + (3*a^(1/3)*n*(3^(1/2)*1i - 1))/(2*
b^(4/3)*x^2))*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)*n) - (log((3*n*log(c*x^n
))/(b*x^2) - (3*a^(1/3)*n*(3^(1/2)*1i + 1))/(2*b^(4/3)*x^2))*(3^(1/2)*1i +
1))/(6*a^(2/3)*b^(1/3)*n)
```

sympy [A] time = 52.76, size = 330, normalized size = 2.29

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^3} \\ \frac{\log(x)}{a+b\log(c)^3} \\ \frac{\log(x)}{a} \\ -\frac{1}{b\left(2n^3\log(x)^2+4n^2\log(c)\log(x)+2n\log(c)^2\right)} \\ -\frac{\sqrt[3]{-1}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{3a^{\frac{2}{3}}n}+\frac{\sqrt[3]{-1}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\,n\sqrt[3]{\frac{1}{b}}\log(x)+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log(c)+4n^2\log(\cdots\right.}{6a^{\frac{2}{3}}n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(
a + b*log(c)**3), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (-1/(b*(2*n**3*log(x)**2
 + 4*n**2*log(c)*log(x) + 2*n*log(c)**2)), Eq(a, 0)), (-(-1)**(1/3)*(1/b)**
(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + n*log(x) + log(c))/(3*a**(2/
3)*n) + (-1)**(1/3)*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) +
4*(-1)**(1/3)*a**(1/3)*n*(1/b)**(1/3)*log(x) + 4*(-1)**(1/3)*a**(1/3)*(1/b)
**(1/3)*log(c) + 4*n**2*log(x)**2 + 8*n*log(c)*log(x) + 4*log(c)**2)/(6*a**
(2/3)*n) - (-1)**(1/3)*sqrt(3)*(1/b)**(1/3)*atan(-sqrt(3)/3 + 2*(-1)**(2/3)
*sqrt(3)*n*log(x)/(3*a**(1/3)*(1/b)**(1/3)) + 2*(-1)**(2/3)*sqrt(3)*log(c)/
(3*a**(1/3)*(1/b)**(1/3)))/(3*a**(2/3)*n), True))
```

3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}-\frac{\tan^{-1}\left(1\right.}{2\surd}$$

```
[Out] -1/4*arctan(1-b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)+
1/4*arctan(1+b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)-1
/8*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*x^n)^2*b^(1/2))/a^(3/
4)/b^(1/4)/n*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*
x^n)^2*b^(1/2))/a^(3/4)/b^(1/4)/n*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}-\frac{\tan^{-1}\left(1\right.}{2\surd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4
)*n) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
b^(1/4)*n) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*Log
[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\log(cx^n)\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\log(cx^n)\right)}{2\sqrt{a}\,n} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\log(cx^n)\right)}{2\sqrt{a}\,n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\log(cx^n)\right)}{4\sqrt{a}\sqrt{b}\,n} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\log(cx^n)\right)}{4\sqrt{a}\sqrt{b}\,n}$$
$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\,n} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\,n} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\right.}{4\sqrt{2}a^{3/}}$$

Mathematica [A] time = 0.07, size = 167, normalized size = 0.74

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)-2\,\text{ta}}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Log[c*x^n])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

fricas [A] time = 0.46, size = 195, normalized size = 0.86

$$\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2n^2\sqrt{-\frac{1}{a^3bn^4}}+n^2\log(x)^2+2\,n\log(c)\log(x)+\log(c)^2}\,a^2bn^3\left(-\frac{1}{a^3bn^4}\right)^{\frac{3}{4}}-\left(a^2bn^4\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*n^4))^(1/4)*arctan(sqrt(a^2*n^2*sqrt(-1/(a^3*b*n^4)) + n^2*log(x
)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^2*b*n^3*(-1/(a^3*b*n^4))^(3/4) - (a^2
*b*n^4*log(x) + a^2*b*n^3*log(c))*(-1/(a^3*b*n^4))^(3/4)) + 1/4*(-1/(a^3*b*
n^4))^(1/4)*log(a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) - 1/4*(-1/(
a^3*b*n^4))^(1/4)*log(-a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c))
```

giac [A] time = 0.22, size = 170, normalized size = 0.75

$$-\frac{1}{2}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi b\left(\mathrm{sgn}(c)-1\right)+2\left(-ab^3\right)^{\frac{1}{4}}}{2\left(bn\log(x)+b\log\left(|c|\right)\right)}\right)+\frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn\left(\mathrm{sgn}(x)-1\right)+\pi b\left(\mathrm{sgn}(c)-1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*n^4))^(1/4)*arctan(1/2*(pi*b*(sgn(c) - 1) + 2*(-a*b^3)^(1/4
))/(b*n*log(x) + b*log(abs(c)))) + 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b
*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) +
 (-a*b^3)^(1/4))^2) - 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x) -
1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) - (-a*b^3)^(1/
4))^2)
```

maple [A] time = 0.07, size = 168, normalized size = 0.74

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4an}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4an}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{\ln(c\,x^n)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln(c\,x^n)+\sqrt{\frac{a}{b}}}{\ln(c\,x^n)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln(c\,x^n)+\sqrt{\frac{a}{b}}}\right)}{8an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^4),x)
```

```
[Out] 1/8/n*(a/b)^(1/4)/a*2^(1/2)*ln((ln(c*x^n)^2+(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+(
a/b)^(1/2))/(ln(c*x^n)^2-(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+(a/b)^(1/2)))+1/4/n*
(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*ln(c*x^n)+1)-1/4/n*(a/b)^(
1/4)/a*2^(1/2)*arctan(-2^(1/2)/(a/b)^(1/4)*ln(c*x^n)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx\log\left(cx^n\right)^4+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^4 + a*x), x)
```

mupad [B] time = 2.24, size = 95, normalized size = 0.42

$$-\frac{\ln\left((-a)^{1/4}+b^{1/4}\,\ln\left(c\,x^n\right)\right)-\ln\left((-a)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\right)+\ln\left((-a)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\,1\mathrm{i}\right)\,1\mathrm{i}-\ln\left((-a)^{1/4}+b^{1/4}\,\mathrm{l}\right.}{4\,(-a)^{3/4}\,b^{1/4}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)^4),x)
```

```
[Out] -(log((-a)^(1/4) + b^(1/4)*log(c*x^n)) - log((-a)^(1/4) - b^(1/4)*log(c*x^n
)) + log((-a)^(1/4) - b^(1/4)*log(c*x^n)*1i)*1i - log((-a)^(1/4) + b^(1/4)*
log(c*x^n)*1i)*1i)/(4*(-a)^(3/4)*b^(1/4)*n)
```

sympy [A] time = 36.01, size = 246, normalized size = 1.08

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^4} \\ \frac{\log(x)}{a+b\log(c)^4} \\ \frac{\log(x)}{a} \\ -\frac{1}{b\left(3n^4\log(x)^3+9n^3\log(c)\log(x)^2+9n^2\log(c)^2\log(x)+3n\log(c)^3\right)} \\ -\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{4a^{\frac{3}{4}}n}+\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{4a^{\frac{3}{4}}n}-\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}n\log(x)}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\ldots}{\sqrt[4]{a}\ldots}\right.}{2a^{\frac{3}{4}}n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(
a + b*log(c)**4), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (-1/(b*(3*n**4*log(x)**3
 + 9*n**3*log(c)*log(x)**2 + 9*n**2*log(c)**2*log(x) + 3*n*log(c)**3)), Eq(
a, 0)), (-(-1)**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
 n*log(x) + log(c))/(4*a**(3/4)*n) + (-1)**(1/4)*(1/b)**(1/4)*log((-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + n*log(x) + log(c))/(4*a**(3/4)*n) - (-1)**(1/4)*
(1/b)**(1/4)*atan((-1)**(3/4)*n*log(x)/(a**(1/4)*(1/b)**(1/4)) + (-1)**(3/4
)*log(c)/(a**(1/4)*(1/b)**(1/4)))/(2*a**(3/4)*n), True))
```

3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}} dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{a} - \frac{b \log\left(a \log\left(cx^n\right) + b\right)}{a^2 n}$$

```
[Out] ln(x)/a-b*ln(b+a*ln(c*x^n))/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{\log(x)}{a} - \frac{b \log\left(a \log\left(cx^n\right) + b\right)}{a^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[x]/a - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax + \frac{bx}{\log(cx^n)}} dx &= \frac{\text{Subst}\left(\int \frac{x}{b+ax} dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right) dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\log(x)}{a} - \frac{b \log\left(b + a \log\left(cx^n\right)\right)}{a^2 n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.26

$$\frac{\log\left(cx^n\right)}{an} - \frac{b \log\left(a \log\left(cx^n\right) + b\right)}{a^2 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[c*x^n]/(a*n) - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

fricas [A] time = 0.44, size = 28, normalized size = 1.04

$$\frac{an \log(x) - b \log\left(an \log(x) + a \log(c) + b\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (a*n*log(x) - b*log(a*n*log(x) + a*log(c) + b))/(a^2*n)
```

giac [A] time = 0.17, size = 53, normalized size = 1.96

$$\frac{\log(x)}{a} - \frac{b\log\left(\frac{1}{4}\left(\pi an\left(\text{sgn}(x)-1\right)+\pi a\left(\text{sgn}(c)-1\right)\right)^2+\left(an\log\left(|x|\right)+a\log\left(|c|\right)+b\right)^2\right)}{2\,a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/2*b*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n
*log(abs(x)) + a*log(abs(c)) + b)^2)/(a^2*n)
```

maple [A] time = 0.07, size = 35, normalized size = 1.30

$$\frac{\ln\left(c\,x^n\right)}{an} - \frac{b\ln\left(a\ln\left(c\,x^n\right)+b\right)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)),x)
```

```
[Out] 1/n*ln(c*x^n)/a-b*ln(b+a*ln(c*x^n))/a^2/n
```

maxima [A] time = 0.70, size = 33, normalized size = 1.22

$$\frac{\log(x)}{a} - \frac{b\log\left(\frac{a\log(c)+a\log(x^n)+b}{a}\right)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - b*log((a*log(c) + a*log(x^n) + b)/a)/(a^2*n)
```

mupad [B] time = 0.36, size = 27, normalized size = 1.00

$$\frac{\ln(x)}{a} - \frac{b\,\ln\left(b+a\,\ln\left(c\,x^n\right)\right)}{a^2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)),x)
```

```
[Out] log(x)/a - (b*log(b + a*log(c*x^n)))/(a^2*n)
```

sympy [A] time = 2.70, size = 99, normalized size = 3.67

$$\begin{cases} \frac{\log(c)\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ \frac{\log(c)\log(x)}{a\log(c)+b} & \text{for } n = 0 \\ \frac{\begin{cases} \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix} \middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix} \middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a = 0 \\ \frac{\log(x)}{a} - \frac{b\log\left(n\log(x)+\log(c)+\frac{b}{a}\right)}{a^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)),x)
```

```
[Out] Piecewise((log(c)*log(x)/b, Eq(a, 0) & Eq(n, 0)), (log(c)*log(x)/(a*log(c)
+ b), Eq(n, 0)), (Piecewise((log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(x
**(-n)/c)**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0
), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, Tr
ue))/b, Eq(a, 0)), (log(x)/a - b*log(n*log(x) + log(c) + b/a)/(a**2*n), Tru
e))
```

3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$

Optimal. Leaf size=40

$$\frac{\log(x)}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

```
[Out] ln(x)/a-arctan(ln(c*x^n)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/n
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {321, 205}

$$\frac{\log(x)}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\log(x)}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx,x,\log\left(cx^n\right)\right)}{an}\\ &= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}+\frac{\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.18

$$\frac{\log\left(cx^n\right)}{an}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[c*x^n]/
(a*n)
```

fricas [A] time = 0.49, size = 143, normalized size = 3.58

$$\left[\frac{2\,n\log(x)+\sqrt{-\frac{b}{a}}\log\left(\frac{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2-2\left(an\log(x)+a\log(c)\right)\sqrt{-\frac{b}{a}}-b}{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2+b}\right)}{2\,an},\ \frac{n\log(x)-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(an\log(x)\right.}{}\right.}{an}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*n*log(x) + sqrt(-b/a)*log((a*n^2*log(x)^2 + 2*a*n*log(c)*log(x) + a
*log(c)^2 - 2*(a*n*log(x) + a*log(c))*sqrt(-b/a) - b)/(a*n^2*log(x)^2 + 2*a
*n*log(c)*log(x) + a*log(c)^2 + b)))/(a*n), (n*log(x) - sqrt(b/a)*arctan((a
*n*log(x) + a*log(c))*sqrt(b/a)/b))/(a*n)]
```

giac [A] time = 0.16, size = 38, normalized size = 0.95

$$\frac{\log(x)}{a}-\frac{b\arctan\left(\frac{an\log(x)+a\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] log(x)/a - b*arctan((a*n*log(x) + a*log(c))/sqrt(a*b))/(sqrt(a*b)*a*n)
```

maple [A] time = 0.07, size = 43, normalized size = 1.08

$$-\frac{b\arctan\left(\frac{a\ln(c\,x^n)}{\sqrt{ab}}\right)}{\sqrt{ab}\;an}+\frac{\ln\left(c\,x^n\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^2),x)
```

```
[Out] 1/a/n*ln(c*x^n)-1/n/a*b/(a*b)^(1/2)*arctan(a*ln(c*x^n)/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{1}{2\,a^2x\log(c)\log\left(x^n\right)+a^2x\log\left(x^n\right)^2+\left(a^2\log(c)^2+ab\right)x}\,dx+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(2*a^2*x*log(c)*log(x^n) + a^2*x*log(x^n)^2 + (a^2*log(c)^2
+ a*b)*x), x) + log(x)/a
```

mupad [B] time = 0.38, size = 45, normalized size = 1.12

$$\frac{\ln(x)}{a}-\frac{\sqrt{b}\;\mathrm{atan}\left(\frac{a^2\,n\,\ln(c\,x^n)}{\sqrt{b}\,\sqrt{a^3\,n^2}}\right)}{\sqrt{a^3\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^2),x)
```

```
[Out] log(x)/a - (b^(1/2)*atan((a^2*n*log(c*x^n))/(b^(1/2)*(a^3*n^2)^(1/2))))/(a^
3*n^2)^(1/2)
```

sympy [A] time = 7.89, size = 177, normalized size = 4.42

$$\begin{cases} \frac{\log(c)^2\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ \frac{\log(c)^2\log(x)}{a\log(c)^2+b} & \text{for } n = 0 \\ \frac{\begin{cases} \frac{\log(cx^n)^3}{3n} & \text{for } |cx^n| < 1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^3}{3n} & \text{for } \frac{1}{|cx^n|} < 1 \\ -\frac{2G_{4,4}^{4,0}\left(\begin{matrix} & 1,1,1,1 \\ 0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{2G_{4,4}^{0,4}\left(\begin{matrix} 1,1,1,1 & \\ & 0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a = 0 \\ \frac{\log(x)}{a} + \frac{i\sqrt{b}\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+n\log(x)+\log(c)\right)}{2a^2n\sqrt{\frac{1}{a}}} - \frac{i\sqrt{b}\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+n\log(x)+\log(c)\right)}{2a^2n\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**2),x)
```

```
[Out] Piecewise((log(c)**2*log(x)/b, Eq(a, 0) & Eq(n, 0)), (log(c)**2*log(x)/(a*l
og(c)**2 + b), Eq(n, 0)), (Piecewise((log(c*x**n)**3/(3*n), Abs(c*x**n) < 1
), (-log(x**(-n)/c)**3/(3*n), 1/Abs(c*x**n) < 1), (-2*meijerg(((), (1, 1, 1
, 1)), ((0, 0, 0, 0), ()), c*x**n)/n + 2*meijerg(((1, 1, 1, 1), ()), ((), (
0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(x)/a + I*sqrt(b)*log(-I*
sqrt(b)*sqrt(1/a) + n*log(x) + log(c))/(2*a**2*n*sqrt(1/a)) - I*sqrt(b)*log
(I*sqrt(b)*sqrt(1/a) + n*log(x) + log(c))/(2*a**2*n*sqrt(1/a)), True))
```

3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)}{6a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)}{3a^{4/3}n}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n}+\text{l}\cdots$$

```
[Out] ln(x)/a-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*ln(c*x^n))/a^(4/3)/n+1/6*b^(1/3)*ln(
b^(2/3)-a^(1/3)*b^(1/3)*ln(c*x^n)+a^(2/3)*ln(c*x^n)^2)/a^(4/3)/n+1/3*b^(1/3
)*arctan(1/3*(b^(1/3)-2*a^(1/3)*ln(c*x^n))/b^(1/3)*3^(1/2))/a^(4/3)/n*3^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)}{6a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)}{3a^{4/3}n}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n}+\text{l}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Log[c*x^n])/(Sqrt[3]*b^(1/3))])/(Sqrt[
3]*a^(4/3)*n) + Log[x]/a - (b^(1/3)*Log[b^(1/3) + a^(1/3)*Log[c*x^n]])/(3*a
^(4/3)*n) + (b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log
[c*x^n]^2])/(6*a^(4/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \log(cx^n)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, \log(cx^n)\right)}{3an} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x,\right.}{3an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{6a^{4/3}n}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\log(cx^n)+a^{2/3}\log^2(c\right.}{6a^{4/3}n}$$

$$= \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}n} + \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\right.}{}$$

Mathematica [A] time = 0.05, size = 132, normalized size = 0.89

$$\frac{\sqrt[3]{b}\left(\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\,\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)-2\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)\right)+2\sqrt{3}\,\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right.}{6a^{4/3}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]] + 6
*a^(1/3)*Log[c*x^n] + b^(1/3)*(-2*Log[b^(1/3) + a^(1/3)*Log[c*x^n]] + Log[b
^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]))/(6*a^(4/3)*n)
```

fricas [A] time = 0.47, size = 149, normalized size = 1.00

$$\frac{6\,n\log(x) + 2\,\sqrt{3}\,\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\left(\sqrt{3}\,an\log(x)+\sqrt{3}\,a\log(c)\right)\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) - \left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(n^2\log(x)^2 + 2\,n\log(c)\log(x) + \right.}{6\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*n*log(x) + 2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*(sqrt(3)*a*n*log(x)
+ sqrt(3)*a*log(c))*(-b/a)^(2/3) - sqrt(3)*b)/b) - (-b/a)^(1/3)*log(n^2*log
(x)^2 + 2*n*log(c)*log(x) + log(c)^2 + (n*log(x) + log(c))*(-b/a)^(1/3) + (
-b/a)^(2/3)) + 2*(-b/a)^(1/3)*log(n*log(x) - (-b/a)^(1/3) + log(c)))/(a*n)
```

giac [B] time = 0.22, size = 257, normalized size = 1.72

$$\frac{\log(x)}{a} + \frac{2\,\sqrt{3}\,\left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\pi a\left(\operatorname{sgn}(c)-1\right)-2\,an\log(x)-2\,a\log(|c|)+2\,\left(-a^2b\right)^{\frac{1}{3}}}{2\,\sqrt{3}\,an\log(x)+\pi a\left(\operatorname{sgn}(c)-1\right)+2\,\sqrt{3}\,a\log(|c|)+2\,\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}\right) + \left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\left(\pi an\left(\operatorname{sgn}(x) - 1\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] log(x)/a + 1/6*(2*sqrt(3)*(-b*n^6/a)^(1/3)*arctan((sqrt(3)*pi*a*(sgn(c) - 1
) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^(1/3))/(2*sqrt(3)*a*n*log(x
) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/3))
) + (-b*n^6/a)^(1/3)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 +
(a*n*log(abs(x)) + a*log(abs(c)) - (-a^2*b)^(1/3))^2) - (-b*n^6/a)^(1/3)*lo
g((sqrt(3)*pi*a*(sgn(c) - 1) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^
(1/3))^2 + (2*sqrt(3)*a*n*log(x) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(
c)) + 2*sqrt(3)*(-a^2*b)^(1/3))^2))/(a*n^3)
```

maple [A] time = 0.07, size = 136, normalized size = 0.91

$$\frac{\ln\left(c\,x^n\right)}{an} - \frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2\ln\left(c\,x^n\right)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2n} - \frac{b\ln\left(\ln\left(c\,x^n\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2n} + \frac{b\ln\left(\ln\left(c\,x^n\right)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\ln\left(c\,x^n\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^3),x)
```

```
[Out] 1/a/n*ln(c*x^n)-1/3/n/a^2*b/(1/a*b)^(2/3)*ln(ln(c*x^n)+(1/a*b)^(1/3))+1/6/n
/a^2*b/(1/a*b)^(2/3)*ln(ln(c*x^n)^2-(1/a*b)^(1/3)*ln(c*x^n)+(1/a*b)^(2/3))-
1/3/n/a^2*b/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*ln(c*
x^n)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{1}{3\,a^2x\log(c)^2\log\left(x^n\right)+3\,a^2x\log(c)\log\left(x^n\right)^2+a^2x\log\left(x^n\right)^3+\left(a^2\log(c)^3+ab\right)x}\,dx+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(3*a^2*x*log(c)^2*log(x^n) + 3*a^2*x*log(c)*log(x^n)^2 + a^2
*x*log(x^n)^3 + (a^2*log(c)^3 + a*b)*x), x) + log(x)/a
```

mupad [B] time = 2.40, size = 174, normalized size = 1.17

$$\frac{\ln(x)}{a}+\frac{(-b)^{1/3}\ln\left(\frac{3(-b)^{4/3}n}{a^{7/3}x^2}+\frac{3bn\ln(cx^n)}{a^2x^2}\right)}{3a^{4/3}n}+\frac{(-b)^{1/3}\ln\left(\frac{3bn\ln(cx^n)}{a^2x^2}+\frac{3(-b)^{4/3}n\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}x^2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3a^{4/3}n}-\frac{(-b)^{1/3}\ \mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^3),x)
```

```
[Out] log(x)/a + ((-b)^(1/3)*log((3*(-b)^(4/3)*n)/(a^(7/3)*x^2) + (3*b*n*log(c*x^
n))/(a^2*x^2)))/(3*a^(4/3)*n) + ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2
) + (3*(-b)^(4/3)*n*((3^(1/2)*1i)/2 - 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2
- 1/2))/(3*a^(4/3)*n) - ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2) - (3*(
-b)^(4/3)*n*((3^(1/2)*1i)/2 + 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2 + 1/2))/
(3*a^(4/3)*n)
```

sympy [A] time = 54.11, size = 367, normalized size = 2.46

$$\begin{cases} \tilde{\infty}\log(c)^3\log(x) \\ \frac{\log(c)^3\log(x)}{a\log(c)^3+b} \\ \frac{\log(x)}{a} \\ \frac{\begin{cases} \frac{\log(cx^n)^4}{4n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1 \\ 0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{6G_{5,5}^{0,5}\left(\begin{matrix} 1,1,1,1,1 & \\ & 0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} \\ \frac{\sqrt[3]{-1}\sqrt[3]{b}\sqrt[3]{\frac{1}{a}}\log\left(-\sqrt[3]{-1}\sqrt[3]{b}\sqrt[3]{\frac{1}{a}}+n\log(x)+\log(c)\right)}{3an}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sqrt[3]{\frac{1}{a}}\log\left(4(-1)^{\frac{2}{3}}b^{\frac{2}{3}}\left(\frac{1}{a}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{b}n\sqrt[3]{\frac{1}{a}}\log(x)+4\sqrt[3]{-1}\sqrt[3]{b}\sqrt[3]{\frac{1}{a}}\log(c)+4n^2\right.}{6an} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(c)**3*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(c)**
3*log(x)/(a*log(c)**3 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (Piecewise((lo
g(c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log(x**(-n)/c)**4/(4*n), 1/Abs(c*x**
n) < 1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n
 + 6*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True)
)/b, Eq(a, 0)), ((-1)**(1/3)*b**(1/3)*(1/a)**(1/3)*log(-(-1)**(1/3)*b**(1/3
)*(1/a)**(1/3) + n*log(x) + log(c))/(3*a*n) - (-1)**(1/3)*b**(1/3)*(1/a)**(
1/3)*log(4*(-1)**(2/3)*b**(2/3)*(1/a)**(2/3) + 4*(-1)**(1/3)*b**(1/3)*n*(1/
```

```
a)**(1/3)*log(x) + 4*(-1)**(1/3)*b**(1/3)*(1/a)**(1/3)*log(c) + 4*n**2*log(
x)**2 + 8*n*log(c)*log(x) + 4*log(c)**2)/(6*a*n) + (-1)**(1/3)*sqrt(3)*b**(
1/3)*(1/a)**(1/3)*atan(-sqrt(3)/3 + 2*(-1)**(2/3)*sqrt(3)*n*log(x)/(3*b**(1
/3)*(1/a)**(1/3)) + 2*(-1)**(2/3)*sqrt(3)*log(c)/(3*b**(1/3)*(1/a)**(1/3)))
/(3*a*n) + log(x)/a, True))
```

3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$

Optimal. Leaf size=233

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}\log^2\left(cx^n\right)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}\log^2\left(cx^n\right)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}$$

```
[Out] ln(x)/a-1/4*b^(1/4)*arctan(-1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*
2^(1/2)-1/4*b^(1/4)*arctan(1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*2
^(1/2)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+ln(c*x^n)^2*a^(1/2
)+b^(1/2))/a^(5/4)/n*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/
2)+ln(c*x^n)^2*a^(1/2)+b^(1/2))/a^(5/4)/n*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}\log^2\left(cx^n\right)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}\log^2\left(cx^n\right)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[2]*a^(5/
4)*n) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[
2]*a^(5/4)*n) + Log[x]/a + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(5/4)*n) - (b^(1/4)*Log[Sqr
t[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[
2]*a^(5/4)*n)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, \log(cx^n)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}x^2}{b+ax^4}\,dx, x, \log(cx^n)\right)}{2an} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}x^2}{b+ax^4}\,dx, x, \log(cx^n)\right)}{2an}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{2}a^{5/4}n} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x\right.}{4\sqrt{2}a^{5/4}n}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4\sqrt{2}}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} + \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\right.}{}$$

Mathematica [A] time = 0.07, size = 211, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\right.}{8a^{5/4}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] - 2*Sqr
t[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] + 8*a^(1/4)*L
og[c*x^n] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n
] + Sqrt[a]*Log[c*x^n]^2] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b
^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(8*a^(5/4)*n)
```

fricas [A] time = 0.47, size = 192, normalized size = 0.82

$$-\frac{4\,a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2n^2\sqrt{-\frac{b}{a^5n^4}}+n^2\log(x)^2+2\,n\log(c)\log(x)+\log(c)^2}\,a^4n^3\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}-\left(a^4n^4\log(x)+a^4n^3\log(c)\right)\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}}{b}\right)+a\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*(-b/(a^5*n^4))^(1/4)*arctan((sqrt(a^2*n^2*sqrt(-b/(a^5*n^4)) + n^
2*log(x)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^4*n^3*(-b/(a^5*n^4))^(3/4) - (
a^4*n^4*log(x) + a^4*n^3*log(c))*(-b/(a^5*n^4))^(3/4))/b) + a*(-b/(a^5*n^4)
)^(1/4)*log(a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - a*(-b/(a^5*n^4)
)^(1/4)*log(-a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - 4*log(x))/a
```

giac [A] time = 0.21, size = 178, normalized size = 0.76

$$\frac{\log(x)}{a}-\frac{4\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi a\left(\mathrm{sgn}(c)-1\right)-2\left(-a^3b\right)^{\frac{1}{4}}}{2\left(an\log(x)+a\log(|c|)\right)}\right)+\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi an\left(\mathrm{sgn}(x)-1\right)+\pi a\left(\mathrm{sgn}(c)-1\right)\right)^2+\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/8*(4*(-b*n^12/a)^(1/4)*arctan(1/2*(pi*a*(sgn(c) - 1) - 2*(-a^3
*b)^(1/4))/(a*n*log(x) + a*log(abs(c)))) + (-b*n^12/a)^(1/4)*log(1/4*(pi*a*
n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) +
(-a^3*b)^(1/4))^2) - (-b*n^12/a)^(1/4)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*
(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) - (-a^3*b)^(1/4))^2))/(a
*n^4)
```

maple [A] time = 0.11, size = 181, normalized size = 0.78

$$\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)}{4an}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)}{4an}+\frac{\ln(c\,x^n)}{an}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{\ln(c\,x^n)^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\cdots}{\ln(c\,x^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\cdots}\right.}{8an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^4),x)
```

```
[Out] 1/a/n*ln(c*x^n)-1/8/n/a*(1/a*b)^(1/4)*2^(1/2)*ln((ln(c*x^n)^2+(1/a*b)^(1/4)
*ln(c*x^n)*2^(1/2)+(1/a*b)^(1/2))/(ln(c*x^n)^2-(1/a*b)^(1/4)*ln(c*x^n)*2^(1
/2)+(1/a*b)^(1/2)))-1/4/n/a*(1/a*b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/a*b)^(1
/4)*ln(c*x^n)+1)+1/4/n/a*(1/a*b)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(1/a*b)^(1/4
)*ln(c*x^n)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{1}{4\,a^2x\log(c)^3\log\left(x^n\right)+6\,a^2x\log(c)^2\log\left(x^n\right)^2+4\,a^2x\log(c)\log\left(x^n\right)^3+a^2x\log\left(x^n\right)^4+\left(a^2\log(c)^4+ab\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(4*a^2*x*log(c)^3*log(x^n) + 6*a^2*x*log(c)^2*log(x^n)^2 + 4
*a^2*x*log(c)*log(x^n)^3 + a^2*x*log(x^n)^4 + (a^2*log(c)^4 + a*b)*x), x) +
 log(x)/a
```

mupad [B] time = 2.21, size = 176, normalized size = 0.76

$$\frac{\ln(x)}{a}+\frac{(-b)^{1/4}\left(\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}-\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)1\mathrm{i}-\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}+\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)1\mathrm{i}\right)}{4\,a^{5/4}\,n}-\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}+a^{1/4}\,(-b)^{9/4}\,\ln \cdots}{x^3}\right.}{4\,a^{5/4}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^4),x)
```

```
[Out] log(x)/a + ((-b)^(1/4)*(log(- (-b)^(5/2)/(a^(11/2)*x^3) - ((-b)^(9/4)*log(c
*x^n)*1i)/(a^(21/4)*x^3))*1i - log(((-b)^(9/4)*log(c*x^n)*1i)/(a^(21/4)*x^3
) - (-b)^(5/2)/(a^(11/2)*x^3))*1i))/(4*a^(5/4)*n) - ((-b)^(1/4)*log(((-b)^(
5/2) + a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n) + ((-b)^(1/4)*log
(((-b)^(5/2) - a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n)
```

sympy [A] time = 36.99, size = 270, normalized size = 1.16

$$\begin{cases} \tilde{\infty}\log(c)^4\log(x) \\ \frac{\log(c)^4\log(x)}{a\log(c)^4+b} \\ \frac{\begin{cases} \frac{\log(cx^n)^5}{5n} & \text{for}\ |cx^n|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n} & \text{for}\ \frac{1}{|cx^n|}<1 \\ -\frac{24G_{6,6}^{6,0}\left(\begin{matrix} & 1,1,1,1,1,1 \\ 0,0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{24G_{6,6}^{0,6}\left(\begin{matrix} 1,1,1,1,1,1 & \\ & 0,0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} \\ \frac{\log(x)}{a} \\ \frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}+n\log(x)+\log(c)\right)}{4an}-\frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}+n\log(x)+\log(c)\right)}{4an}+\frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}n\log(x)}{\sqrt[4]{b}\sqrt[4]{\frac{1}{a}}}+\cdots\right.}{2an} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(c)**4*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(c)**
4*log(x)/(a*log(c)**4 + b), Eq(n, 0)), (Piecewise((log(c*x**n)**5/(5*n), Ab
s(c*x**n) < 1), (-log(x**(-n)/c)**5/(5*n), 1/Abs(c*x**n) < 1), (-24*meijerg
(((), (1, 1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 24*meijerg
(((1, 1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0, 0)), c*x**n)/n, True))/b, E
q(a, 0)), (log(x)/a, Eq(b, 0)), ((-1)**(1/4)*b**(1/4)*(1/a)**(1/4)*log(-(-1
)**(1/4)*b**(1/4)*(1/a)**(1/4) + n*log(x) + log(c))/(4*a*n) - (-1)**(1/4)*b
**(1/4)*(1/a)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/a)**(1/4) + n*log(x) + log
(c))/(4*a*n) + (-1)**(1/4)*b**(1/4)*(1/a)**(1/4)*atan((-1)**(3/4)*n*log(x)/
(b**(1/4)*(1/a)**(1/4)) + (-1)**(3/4)*log(c)/(b**(1/4)*(1/a)**(1/4)))/(2*a*
n) + log(x)/a, True))
```

3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$

Optimal. Leaf size=22

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(7x)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(7x)\right)\right)\\
&= \frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.42, size = 21, normalized size = 0.95

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} \log\left(7 x\right) + \frac{1}{3} \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(2/3*sqrt(3)*log(7*x) + 1/3*sqrt(3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log\left(7 x\right)^{2} + x \log\left(7 x\right) + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

maple [A] time = 0.07, size = 20, normalized size = 0.91

$$\frac{2\sqrt{3}\, \arctan\left(\frac{\left(2 \ln\left(7 x\right)+1\right) \sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x*ln(7*x)+x*ln(7*x)^2),x)
```

```
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log\left(7 x\right)^{2} + x \log\left(7 x\right) + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

mupad [B] time = 0.43, size = 19, normalized size = 0.86

$$\frac{2 \sqrt{3}\, \mathrm{atan}\left(\frac{\sqrt{3}\, \left(2 \ln\left(7 x\right)+1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x*log(7*x) + x*log(7*x)^2),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*log(7*x) + 1))/3))/3
```

sympy [A] time = 0.17, size = 22, normalized size = 1.00

$$\operatorname{RootSum}\left(3z^{2} + 1, \left(i \mapsto i \log\left(\frac{3i}{2} + \log\left(7x\right) + \frac{1}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*ln(7*x)+x*ln(7*x)**2),x)
```

```
[Out] RootSum(3*_z**2 + 1, Lambda(_i, _i*log(3*_i/2 + log(7*x) + 1/2)))
```

3.259 $\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log$$
$$= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

Mathematica [A] time = 0.09, size = 42, normalized size = 1.02

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right) - \frac{\tan^{-1}\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

fricas [A] time = 0.46, size = 39, normalized size = 0.95

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3x\right)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log\left(3x\right)^2-\log\left(3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(3x\right)-1}{\left(\log\left(3x\right)^2-\log\left(3x\right)+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="giac")
[Out] integrate((log(3*x) - 1)/((log(3*x)^2 - log(3*x) + 1)*x), x)
```

maple [A] time = 0.06, size = 38, normalized size = 0.93

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2\ln(3x)-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(\ln(3x)^2-\ln(3x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)^2),x)
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

maxima [A] time = 1.49, size = 37, normalized size = 0.90

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\log\left(3\,x\right)-1\right)\right)+\frac{1}{2}\,\log\left(\log\left(3\,x\right)^{2}-\log\left(3\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

mupad [B] time = 0.57, size = 37, normalized size = 0.90

$$\frac{\ln\left(\ln\left(3\,x\right)^{2}-\ln\left(3\,x\right)+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln\left(3\,x\right)-1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x) - 1)/(x*(log(3*x)^2 - log(3*x) + 1)),x)
```

```
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

sympy [A] time = 0.18, size = 22, normalized size = 0.54

$$\operatorname{RootSum}\left(3z^{2}-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)**2),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\ &= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)\end{aligned}$$

Mathematica [A] time = 0.08, size = 42, normalized size = 1.02

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right) - \frac{\tan^{-1}\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

fricas [A] time = 0.43, size = 39, normalized size = 0.95

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3x\right) - \frac{1}{3}\sqrt{3}\right) + \frac{1}{2}\log\left(\log\left(3x\right)^2 - \log\left(3x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(3x\right)^2 - 1}{x\log\left(3x\right)^3 + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="giac")
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

maple [A] time = 0.07, size = 38, normalized size = 0.93

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2\ln(3x)-1)\sqrt{3}}{3}\right)}{3} + \frac{\ln\left(\ln\left(3x\right)^2 - \ln\left(3x\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)^3),x)
[Out] -1/3*3^(1/2)*arctan(1/3*(2*ln(3*x)-1)*3^(1/2))+1/2*ln(ln(3*x)^2-ln(3*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(3\,x)^2 - 1}{x\log(3\,x)^3 + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="maxima")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

mupad [B] time = 0.35, size = 37, normalized size = 0.90

$$\frac{\ln\left(\ln(3\,x)^2 - \ln(3\,x) + 1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,\ln(3\,x)-1)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x)^3),x)
```

```
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

sympy [A] time = 0.19, size = 22, normalized size = 0.54

$$\mathrm{RootSum}\left(3z^2 - 3z + 1, \left(i \mapsto i\log\left(-3i + \log(3x) + 1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)**3),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

```
[Out] ln(x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[x] - Log[1 + Log[3*x] + L
og[3*x]^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \text{Subst}\left(\int \frac{-1+x^2}{1+x+x^2}\,dx, x, \log(3x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{2+x}{1+x+x^2}\right)dx, x, \log(3x)\right)$$
$$= \log(x) - \text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, \log(3x)\right)$$
$$= \log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \log(3x)\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,\right.$$
$$= \log(x) - \frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right) + 3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log\right.$$
$$= -\sqrt{3}\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right) + \log(x) - \frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

Mathematica [A] time = 0.09, size = 44, normalized size = 1.05

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(3x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x] +
 Log[3*x]^2]/2
```

fricas [A] time = 0.43, size = 41, normalized size = 0.98

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\log\left(\log(3x)^2+\log(3x)+1\right)+\log(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) + 1/3*sqrt(3)) - 1/2*log(log(3*x)^2 +
log(3*x) + 1) + log(3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(3x)^2-1}{x\log(3x)^2+x\log(3x)+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="giac")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^2 + x*log(3*x) + x), x)
```

maple [A] time = 0.07, size = 40, normalized size = 0.95

$$-\sqrt{3}\ \arctan\left(\frac{(2\ln(3x)+1)\sqrt{3}}{3}\right)+\ln(3x)-\frac{\ln\left(\ln(3x)^2+\ln(3x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)+x*ln(3*x)^2),x)
```

```
[Out] ln(3*x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log(3)+\log(x)+2}{x\left(2\log(3)+1\right)\log(x)+x\log(x)^2+\left(\log(3)^2+\log(3)+1\right)x}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="maxima"
)
```

```
[Out] -integrate((log(3) + log(x) + 2)/(x*(2*log(3) + 1)*log(x) + x*log(x)^2 + (l
og(3)^2 + log(3) + 1)*x), x) + log(x)
```

mupad [B] time = 0.60, size = 37, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(\ln(3\,x)^2+\ln(3\,x)+1\right)}{2}-\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln(3\,x)+1\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x) + x*log(3*x)^2),x)
```

```
[Out] log(x) - log(log(3*x) + log(3*x)^2 + 1)/2 - 3^(1/2)*atan((3^(1/2)*(2*log(3*
x) + 1))/3)
```

sympy [A] time = 0.18, size = 19, normalized size = 0.45

$$\log(x)+\operatorname{RootSum}\left(z^2+z+1,\left(i\mapsto i\log\left(-i+\log(3x)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)+x*ln(3*x)**2),x)
```

```
[Out] log(x) + RootSum(_z**2 + _z + 1, Lambda(_i, _i*log(-_i + log(3*x))))
```

3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2305, 2304}

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/(32*x^4) + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx &= -\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}-\frac{1}{2}\int \frac{\log\left(\frac{1}{x}\right)}{x^5}\,dx\\ &= -\frac{1}{32x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

fricas [A] time = 0.44, size = 21, normalized size = 0.66

$$-\frac{8 \log\left(\frac{1}{x}\right)^2 - 4 \log\left(\frac{1}{x}\right) + 1}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(8*log(1/x)^2 - 4*log(1/x) + 1)/x^4
```

giac [A] time = 0.16, size = 22, normalized size = 0.69

$$-\frac{\log(x)^2}{4\,x^4} - \frac{\log(x)}{8\,x^4} - \frac{1}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(x)^2/x^4 - 1/8*log(x)/x^4 - 1/32/x^4
```

maple [A] time = 0.07, size = 27, normalized size = 0.84

$$-\frac{\ln\left(\frac{1}{x}\right)^2}{4x^4} + \frac{\ln\left(\frac{1}{x}\right)}{8x^4} - \frac{1}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x)^2/x^5,x)
```

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

maxima [A] time = 0.68, size = 17, normalized size = 0.53

$$-\frac{8\,\log(x)^2 + 4\,\log(x) + 1}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*log(x)^2 + 4*log(x) + 1)/x^4
```

mupad [B] time = 0.41, size = 21, normalized size = 0.66

$$-\frac{\frac{\ln\left(\frac{1}{x}\right)^2}{4} - \frac{\ln\left(\frac{1}{x}\right)}{8} + \frac{1}{32}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/x)^2/x^5,x)
```

```
[Out] -(log(1/x)^2/4 - log(1/x)/8 + 1/32)/x^4
```

sympy [A] time = 0.13, size = 27, normalized size = 0.84

$$-\frac{\log\left(\frac{1}{x}\right)^2}{4x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{1}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x)**2/x**5,x)
```

```
[Out] -log(1/x)**2/(4*x**4) + log(1/x)/(8*x**4) - 1/(32*x**4)
```

3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$

<u>**Optimal**</u>. Leaf size=40

$$-\frac{\sqrt{\frac{\pi}{2}}\,x\,\mathrm{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

```
[Out] -1/2*x*erf(1/2*(-ln(a*x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)/(a*x^2)^(1/2)
```

<u>**Rubi**</u> **[A]** time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2300, 2180, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\,x\,\mathrm{Erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

<u>Rule 2180</u>

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

<u>Rule 2205</u>

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

<u>Rule 2300</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

<u>Rubi steps</u>

$$\begin{aligned}\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx &= \frac{x\,\mathrm{Subst}\left(\int \frac{e^{x/2}}{\sqrt{-x}}\,dx, x, \log\left(ax^2\right)\right)}{2\sqrt{ax^2}}\\ &= -\frac{x\,\mathrm{Subst}\left(\int e^{-\frac{x^2}{2}}\,dx, x, \sqrt{-\log\left(ax^2\right)}\right)}{\sqrt{ax^2}}\\ &= -\frac{\sqrt{\frac{\pi}{2}}\,x\,\mathrm{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.48

$$\frac{\sqrt{\frac{\pi}{2}}x\sqrt{\log(ax^2)}\,\text{erfi}\left(\frac{\sqrt{\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}\sqrt{-\log(ax^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^2]],x]
[Out] (Sqrt[Pi/2]*x*Erfi[Sqrt[Log[a*x^2]]/Sqrt[2]]*Sqrt[Log[a*x^2]])/(Sqrt[a*x^2]*Sqrt[-Log[a*x^2]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integrate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln(a\,x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^2))^(1/2),x)
[Out] int(1/(-ln(a*x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\ln(a\,x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a*x^2))^(1/2),x)
```

```
[Out] int(1/(-log(a*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**2)), x)
```

3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}} dx$

Optimal. Leaf size=39

$$\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\,\mathrm{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

```
[Out] 1/2*x*erfi(1/2*(-ln(a/x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)*(a/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2300, 2180, 2204}

$$\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\,\mathrm{Erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a/x^2]],x]
```

```
[Out] Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}} dx &= -\left(\frac{1}{2}\left(\sqrt{\frac{a}{x^2}}x\right)\mathrm{Subst}\left(\int \frac{e^{-x/2}}{\sqrt{-x}} dx, x, \log\left(\frac{a}{x^2}\right)\right)\right)\\
&= \left(\sqrt{\frac{a}{x^2}}x\right)\mathrm{Subst}\left(\int e^{\frac{x^2}{2}} dx, x, \sqrt{-\log\left(\frac{a}{x^2}\right)}\right)\\
&= \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}x\mathrm{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.54

$$-\frac{\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\sqrt{\log\left(\frac{a}{x^2}\right)}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a/x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erf[Sqrt[Log[a/x^2]]/Sqrt[2]]*Sqrt[Log[a/x^2]])
/Sqrt[-Log[a/x^2]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a/x^2))^(1/2),x)
```

```
[Out] int(1/(-ln(a/x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a/x^2))^(1/2),x)
```

```
[Out] int(1/(-log(a/x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a/x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a/x**2)), x)
```

3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}} dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{\pi}\, x\, (ax^n)^{-1/n} \operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] -x*erf((-ln(a*x^n))^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^(1/n))/n^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2300, 2180, 2205}

$$-\frac{\sqrt{\pi}\, x\, (ax^n)^{-1/n} \operatorname{Erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] -((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-\log(ax^n)}} dx &= \frac{\left(x (ax^n)^{-1/n}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{-x}} dx, x, \log(ax^n)\right)}{n} \\
&= -\frac{\left(2x (ax^n)^{-1/n}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{-\log(ax^n)}\right)}{n} \\
&= -\frac{\sqrt{\pi}\, x\, (ax^n)^{-1/n} \operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.44

$$\frac{\sqrt{\pi}\, x\, (ax^n)^{-1/n} \sqrt{\log(ax^n)} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} \sqrt{-\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]]*Sqrt[Log[a*x^n]])/(Sqrt[n]*(a*x^
n)^n^(-1)*Sqrt[-Log[a*x^n]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.20, size = 32, normalized size = 0.74

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{-n \log(x)-\log(a)}}{\sqrt{n}}\right)}{a^{\left(\frac{1}{n}\right)} \sqrt{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*erf(-sqrt(-n*log(x) - log(a))/sqrt(n))/(a^(1/n)*sqrt(n))
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln\left(a\,x^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^n))^(1/2),x)
```

```
[Out] int(1/(-ln(a*x^n))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(ax^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\ln\left(a\,x^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a*x^n))^(1/2),x)
```

```
[Out] int(1/(-log(a*x^n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**n)), x)
```

3.266 $\int \frac{\log(1+\sqrt{x}-x)}{x}\,dx$

Optimal. Leaf size=122

$$2\text{Li}_2\left(1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\text{Li}_2\left(\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right)$$

```
[Out] -2*ln(1/2+1/2*5^(1/2))*ln(1+5^(1/2)-2*x^(1/2))+ln(x)*ln(1-x+x^(1/2))-ln(x)*
ln(1-2*x^(1/2)/(-5^(1/2)+1))-2*polylog(2,2*x^(1/2)/(-5^(1/2)+1))+2*polylog(
2,1-2*x^(1/2)/(5^(1/2)+1))
```

Rubi [A] time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2530, 2524, 2357, 2317, 2391, 2316, 2315}

$$2\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-2\log\left(1-\frac{2}{1-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] - 2*Log[1 - (2*Sqrt[x]
)/(1 - Sqrt[5])]*Log[Sqrt[x]] + 2*Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + 2*Pol
yLog[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (2*Sqrt[x])/(1 - Sqrt
[5])]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2530

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rubi steps

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(1+x-x^2\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \frac{(1-2x)\log(x)}{1+x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \left(-\frac{2\log(x)}{1-\sqrt{5}-2x} - \frac{2\log(x)}{1+\sqrt{5}-2x}\right) dx, x, \sqrt{x}\right.$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) + 4\,\text{Subst}\left(\int \frac{\log(x)}{1-\sqrt{5}-2x}\,dx, x, \sqrt{x}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+}\right.$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\right.$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\right.$$

Mathematica [A] time = 0.08, size = 121, normalized size = 0.99

$$2\text{Li}_2\left(\frac{-2\sqrt{x}+\sqrt{5}+1}{1+\sqrt{5}}\right) - 2\text{Li}_2\left(-\frac{2\sqrt{x}}{-1+\sqrt{5}}\right) - 2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right) + \left(\log\left(\sqrt{5}-1\right) - \log\left(2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] + (Log[-1 + Sqrt[5]] -
 Log[-1 + Sqrt[5] + 2*Sqrt[x]])*Log[x] + Log[1 + Sqrt[x] - x]*Log[x] + 2*Po
lyLog[2, (1 + Sqrt[5] - 2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (-2*Sqrt[x
])/(-1 + Sqrt[5])]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(-x+\sqrt{x}+1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(log(-x + sqrt(x) + 1)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

maple [A] time = 0.07, size = 102, normalized size = 0.84

$$-\ln(x)\ln\left(\frac{2\sqrt{x}-1+\sqrt{5}}{\sqrt{5}-1}\right)-\ln(x)\ln\left(\frac{-2\sqrt{x}+1+\sqrt{5}}{\sqrt{5}+1}\right)+\ln(x)\ln\left(-x+\sqrt{x}+1\right)-2\,\mathrm{dilog}\left(\frac{2\sqrt{x}-1+\sqrt{5}}{\sqrt{5}-1}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-x+x^(1/2))/x,x)
```

```
[Out] ln(x)*ln(1-x+x^(1/2))-ln(x)*ln((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+1))-ln(x)*ln(
(-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))-2*dilog((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+
1))-2*dilog((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\sqrt{x}-x+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^(1/2) - x + 1)/x,x)
```

```
[Out] int(log(x^(1/2) - x + 1)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\sqrt{x}-x+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-x+x**(1/2))/x,x)
```

```
[Out] Integral(log(sqrt(x) - x + 1)/x, x)
```

3.267 $\int \frac{x\log(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=81

$$-\frac{a\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

```
[Out] -x/b+(d*x+c)*ln(d*x+c)/b/d-a*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2-a*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b^2
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{a\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\log(c+dx)}{a+bx}\,dx &= \int \left(\frac{\log(c+dx)}{b} - \frac{a\log(c+dx)}{b(a+bx)}\right) dx \\
&= \frac{\int \log(c+dx)\,dx}{b} - \frac{a\int \frac{\log(c+dx)}{a+bx}\,dx}{b} \\
&= -\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} + \frac{\operatorname{Subst}\left(\int \log(x)\,dx, x, c+dx\right)}{bd} + \frac{(ad)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b^2} \\
&= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} + \frac{a\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, \ldots\right)}{b^2} \\
&= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} - \frac{a\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.90

$$\frac{-ad\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right) + \log(c+dx)\left(-ad\log\left(\frac{d(a+bx)}{ad-bc}\right) + bc + bdx\right) - bdx}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] (-(b*d*x) + (b*c + b*d*x - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c + d
*x] - a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x\log(dx+c)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(d*x + c)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log(dx+c)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(d*x + c)/(b*x + a), x)
```

maple [A] time = 0.08, size = 114, normalized size = 1.41

$$-\frac{a\ln\left(\frac{ad-bc+(dx+c)b}{ad-bc}\right)\ln\left(dx+c\right)}{b^2}+\frac{x\ln\left(dx+c\right)}{b}-\frac{a\,\mathrm{dilog}\left(\frac{ad-bc+(dx+c)b}{ad-bc}\right)}{b^2}+\frac{c\ln\left(dx+c\right)}{bd}-\frac{x}{b}-\frac{c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*x+c)/(b*x+a),x)
```

```
[Out] 1/b*ln(d*x+c)*x+1/d/b*ln(d*x+c)*c-1/b*x-1/d/b*c-a/b^2*dilog((b*(d*x+c)+a*d-
b*c)/(a*d-b*c))-a/b^2*ln(d*x+c)*ln((b*(d*x+c)+a*d-b*c)/(a*d-b*c))
```

maxima [A] time = 0.68, size = 111, normalized size = 1.37

$$d\left(\frac{\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)a}{b^2d}-\frac{x}{bd}+\frac{c\log\left(dx+c\right)}{bd^2}\right)+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)\log\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*((log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*a/(b^2*d) - x/(b*d) + c*log(d*x + c)/(b*d^2)) + (x/b - a*log(
b*x + a)/b^2)*log(d*x + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(c+d\,x\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c + d*x))/(a + b*x),x)
```

```
[Out] int((x*log(c + d*x))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*log(c + d*x)/(a + b*x), x)
```

3.268 $\int \frac{\log(x)}{-1+x} \, dx$

Optimal. Leaf size=9

$$-\text{Li}_2(1-x)$$

```
[Out] -polylog(2,1-x)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2315}

$$-\text{PolyLog}(2, 1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(-1 + x),x]
[Out] -PolyLog[2, 1 - x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log(x)}{-1+x} \, dx = -\text{Li}_2(1-x)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\text{Li}_2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(-1 + x),x]
[Out] -PolyLog[2, 1 - x]
```

fricas [A] time = 0.42, size = 8, normalized size = 0.89

$$-\text{Li}_2(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="fricas")
[Out] -dilog(-x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="giac")
[Out] integrate(log(x)/(x - 1), x)
```

maple [A] time = 0.06, size = 5, normalized size = 0.56

$$-\text{dilog}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(x-1),x)
```

```
[Out] -dilog(x)
```

maxima [A] time = 0.64, size = 12, normalized size = 1.33

$$\log(x)\log(-x+1) + \mathrm{Li}_2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1) + dilog(x)
```

mupad [B] time = 0.02, size = 4, normalized size = 0.44

$$-\mathrm{Li}_2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x - 1),x)
```

```
[Out] -dilog(x)
```

sympy [C] time = 1.72, size = 10, normalized size = 1.11

$$-\mathrm{Li}_2\left((x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(-1+x),x)
```

```
[Out] -polylog(2, (x - 1)*exp_polar(I*pi))
```

3.269 $\int \frac{x \log(1-a-bx)}{a+bx} \, dx$

Optimal. Leaf size=43

$$\frac{a\mathrm{Li}_2(a+bx)}{b^2} - \frac{(-a-bx+1)\log(-a-bx+1)}{b^2} - \frac{x}{b}$$

```
[Out] -x/b-(-b*x-a+1)*ln(-b*x-a+1)/b^2+a*polylog(2,b*x+a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {43, 2416, 2389, 2295, 2393, 2391}

$$\frac{a\mathrm{PolyLog}(2,a+bx)}{b^2} - \frac{(-a-bx+1)\log(-a-bx+1)}{b^2} - \frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[1 - a - b*x])/(a + b*x),x]
```

```
[Out] -(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \int \left(\frac{\log(1-a-bx)}{b} - \frac{a\log(1-a-bx)}{b(a+bx)}\right) dx$$
$$= \frac{\int \log(1-a-bx)\,dx}{b} - \frac{a\int \frac{\log(1-a-bx)}{a+bx}\,dx}{b}$$
$$= -\frac{\text{Subst}(\int \log(x)\,dx, x, 1-a-bx)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, a+bx\right)}{b^2}$$
$$= -\frac{x}{b} - \frac{(1-a-bx)\log(1-a-bx)}{b^2} + \frac{a\text{Li}_2(a+bx)}{b^2}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.81

$$\frac{a\text{Li}_2(a+bx) + (a+bx-1)\log(-a-bx+1) - bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 - a - b*x])/(a + b*x),x]
```

```
[Out] (-(b*x) + (-1 + a + b*x)*Log[1 - a - b*x] + a*PolyLog[2, a + b*x])/b^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log(-bx-a+1)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(-b*x - a + 1)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log(-bx-a+1)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(-b*x - a + 1)/(b*x + a), x)
```

maple [A] time = 0.07, size = 77, normalized size = 1.79

$$\frac{x\ln(-bx-a+1)}{b} + \frac{a\,\text{dilog}(-bx-a+1)}{b^2} + \frac{a\ln(-bx-a+1)}{b^2} - \frac{x}{b} - \frac{a}{b^2} - \frac{\ln(-bx-a+1)}{b^2} + \frac{1}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-b*x-a+1)/(b*x+a),x)
```

```
[Out] 1/b*ln(-b*x-a+1)*x+1/b^2*dilog(-b*x-a+1)*a+1/b^2*ln(-b*x-a+1)*a-1/b*x-1/b^2
*ln(-b*x-a+1)-a/b^2+1/b^2
```

maxima [B] time = 0.64, size = 82, normalized size = 1.91

$$b\left(\frac{\left(\log(bx+a)\log(-bx-a+1) + \text{Li}_2(bx+a)\right)a}{b^3} - \frac{x}{b^2} + \frac{(a-1)\log(bx+a-1)}{b^3}\right) + \left(\frac{x}{b} - \frac{a\log(bx+a)}{b^2}\right)\log(-b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*((log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))*a/b^3 - x/b^2 + (a - 1
)*log(b*x + a - 1)/b^3) + (x/b - a*log(b*x + a)/b^2)*log(-b*x - a + 1)
```

mupad [B] time = 0.32, size = 59, normalized size = 1.37

$$-\frac{\ln\left(1-b\,x-a\right)+b\,\left(x-x\,\ln\left(1-b\,x-a\right)\right)-a\,\mathrm{Li}_2\left(1-b\,x-a\right)-a\,\ln\left(1-b\,x-a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(1 - b*x - a))/(a + b*x),x)
```

```
[Out] -(log(1 - b*x - a) + b*(x - x*log(1 - b*x - a)) - a*dilog(1 - b*x - a) - a*
log(1 - b*x - a))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(-a - bx + 1\right)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-b*x-a+1)/(b*x+a),x)
```

```
[Out] Integral(x*log(-a - b*x + 1)/(a + b*x), x)
```

3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$

Optimal. Leaf size=30

$$\text{Li}_2\left(-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

```
[Out] 1/2*ln(x)^2+ln(x)*ln(c*x/b+1)+polylog(2,-c*x/b)
```

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2357, 2301, 2317, 2391}

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

```
[Out] Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx &= \int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx\\
&= c\int \frac{\log(x)}{b+cx}\,dx+\int \frac{\log(x)}{x}\,dx\\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)-\int \frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx\\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)
\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 31, normalized size = 1.03

$$\mathrm{Li}_2\left(-\frac{cx}{b}\right)+\log(x)\log\left(\frac{b+cx}{b}\right)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

```
[Out] Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + PolyLog[2, -((c*x)/b)]
```

fricas [**F**] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(2\,cx+b)\log(x)}{cx^2+bx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="fricas")
```

```
[Out] integral((2*c*x + b)*log(x)/(c*x^2 + b*x), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2\,cx+b)\log(x)}{(cx+b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] integrate((2*c*x + b)*log(x)/((c*x + b)*x), x)
```

maple [**A**] time = 0.07, size = 31, normalized size = 1.03

$$\frac{\ln(x)^2}{2}+\ln(x)\ln\left(\frac{cx+b}{b}\right)+\mathrm{dilog}\left(\frac{cx+b}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] 1/2*ln(x)^2+ln(x)*ln((c*x+b)/b)+dilog((c*x+b)/b)
```

maxima [**A**] time = 0.55, size = 49, normalized size = 1.63

$$\left(\log\left(cx+b\right)+\log(x)\right)\log(x)-\log\left(cx+b\right)\log(x)+\log\left(\frac{cx}{b}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\mathrm{Li}_2\left(-\frac{cx}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="maxima")
```

```
[Out] (log(c*x + b) + log(x))*log(x) - log(c*x + b)*log(x) + log(c*x/b + 1)*log(x
) - 1/2*log(x)^2 + dilog(-c*x/b)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\ln(x)\,\left(b+2\,c\,x\right)}{x\,\left(b+c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*(b + 2*c*x))/(x*(b + c*x)),x)
```

```
[Out] int((log(x)*(b + 2*c*x))/(x*(b + c*x)), x)
```

sympy [C] time = 117.58, size = 192, normalized size = 6.40

$$b\left(\begin{cases} -\frac{1}{cx} & \text{for } b = 0 \\ \frac{\begin{cases} \log(c)\log(x) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } |x| < 1 \\ -\log(c)\log\left(\frac{1}{x}\right) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(c) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(c) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{1}{cx} \\ \frac{\log\left(\frac{b}{x}+c\right)}{b} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] b*Piecewise((-1/(c*x), Eq(b, 0)), (Piecewise((log(c)*log(x) + polylog(2, b*
exp_polar(I*pi)/(c*x)), Abs(x) < 1), (-log(c)*log(1/x) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*
log(c) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), True))/b, True)) - b*Piecewise((1/(c*x), Eq(b, 0)), (log
(b/x + c)/b, True))*log(x) - 2*c*Piecewise((x/b, Eq(c, 0)), (Piecewise((log
(b)*log(x) - polylog(2, c*x*exp_polar(I*pi)/b), Abs(x) < 1), (-log(b)*log(1
/x) - polylog(2, c*x*exp_polar(I*pi)/b), 1/Abs(x) < 1), (-meijerg(((), (1,
1)), ((0, 0), ()), x)*log(b) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(b
) - polylog(2, c*x*exp_polar(I*pi)/b), True))/c, True)) + 2*c*Piecewise((x/
b, Eq(c, 0)), (log(b + c*x)/c, True))*log(x)
```

3.271 $\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx$

Optimal. Leaf size=7

$$-\cos(x\log(x))$$

```
[Out] -cos(x*ln(x))
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4511}

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
[Out] -Cos[x*Log[x]]
```

Rule 4511

```
Int[Log[(b_.)*(x_)]*Sin[Log[(b_.)*(x_)]*(a_.)*(x_)], x_Symbol] :> -Simp[Cos
[a*x*Log[b*x]]/a, x] - Int[Sin[a*x*Log[b*x]], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx &= \int \sin(x\log(x))\,dx + \int \log(x)\sin(x\log(x))\,dx\\ &= -\cos(x\log(x))\end{aligned}$$

Mathematica [A] time = 0.11, size = 7, normalized size = 1.00

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
[Out] -Cos[x*Log[x]]
```

fricas [A] time = 0.43, size = 7, normalized size = 1.00

$$-\cos\left(x\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="fricas")
[Out] -cos(x*log(x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 9.46Not invertible Error: Bad Argument V
alue
```

maple [**A**] time = 0.08, size = 8, normalized size = 1.14

$$-\cos(x\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
[Out] -cos(x*ln(x))
```

maxima [**A**] time = 1.09, size = 7, normalized size = 1.00

$$-\cos\left(x\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="maxima")
[Out] -cos(x*log(x))
```

mupad [**B**] time = 0.42, size = 7, normalized size = 1.00

$$-\cos(x\ \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x*log(x)) + sin(x*log(x))*log(x),x)
[Out] -cos(x*log(x))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\log(x)+1\right)\sin\left(x\log(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
[Out] Integral((log(x) + 1)*sin(x*log(x)), x)
```

3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2} dx$

Optimal. Leaf size=68

$$-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}+\tan^{-1}(1-x)$$

```
[Out] -1/x-arctan(-1+x)-ln((1-(1-x)^2)/(1+(-1+x)^2))/x+1/2*ln(2-x)+1/2*ln(x)-1/2*
ln(x^2-2*x+2)
```

Rubi [A] time = 0.25, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2525, 12, 6728, 634, 617, 204, 628}

$$-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}+\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] - Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x + Log[2 -
 x]/2 + Log[x]/2 - Log[2 - 2*x + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx &= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\int \frac{4(1-x)}{(2-x)x^2\left(2-2x+x^2\right)}\,dx\\
&= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+4\int \frac{1-x}{(2-x)x^2\left(2-2x+x^2\right)}\,dx\\
&= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+4\int \left(\frac{1}{8(-2+x)}+\frac{1}{4x^2}+\frac{1}{8x}-\frac{x}{4\left(2-2x+x^2\right)}\right)dx\\
&= -\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\int \frac{x}{2-2x+x^2}\,dx\\
&= -\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx-\int \frac{1}{2-2x+x^2}\,dx\\
&= -\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\log\left(2-2x+x^2\right)-\text{Subst}\left(\int \frac{1}{-1-x^2}\right.\\
&= -\frac{1}{x}+\tan^{-1}(1-x)-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\log\left(2-2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.93

$$-\frac{\log\left(-\frac{(x-2)x}{x^2-2x+2}\right)}{x}-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}+\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] + Log[2 - x]/2 + Log[x]/2 - Log[-(((-2 + x)*x)/(2 -
 2*x + x^2))]/x - Log[2 - 2*x + x^2]/2
```

fricas [A] time = 0.42, size = 58, normalized size = 0.85

$$-\frac{2\,x\arctan\left(x-1\right)+x\log\left(x^2-2\,x+2\right)-x\log\left(x^2-2\,x\right)+2\,\log\left(-\frac{x^2-2\,x}{x^2-2\,x+2}\right)+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*arctan(x - 1) + x*log(x^2 - 2*x + 2) - x*log(x^2 - 2*x) + 2*log(-
(x^2 - 2*x)/(x^2 - 2*x + 2)) + 2)/x
```

giac [A] time = 0.22, size = 59, normalized size = 0.87

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(|x-2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="giac")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(abs(x - 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.08, size = 57, normalized size = 0.84

$$-\arctan\left(x-1\right)+\frac{\ln(x)}{2}+\frac{\ln\left(x-2\right)}{2}-\frac{\ln\left(x^2-2x+2\right)}{2}-\frac{\ln\left(\frac{(-x+2)x}{x^2-2x+2}\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((1-(x-1)^2)/(1+(x-1)^2))/x^2,x)
```

```
[Out] -1/x*ln(x*(-x+2)/(x^2-2*x+2))+1/2*ln(x-2)-1/2*ln(x^2-2*x+2)-arctan(x-1)-1/x
+1/2*ln(x)
```

maxima [A] time = 1.29, size = 57, normalized size = 0.84

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(x-2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="maxima")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(x - 2) + 1/2*log(x)
```

mupad [B] time = 0.46, size = 59, normalized size = 0.87

$$\frac{\ln\left(x\,\left(x-2\right)\right)}{2}-\mathrm{atan}\left(x-1\right)-\frac{\ln\left(x^2-2\,x+2\right)}{2}-\frac{\ln\left(2\,x-x^2\right)}{x}+\frac{\ln\left(x^2-2\,x+2\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x^2,x)
```

```
[Out] log(x*(x - 2))/2 - atan(x - 1) - log(x^2 - 2*x + 2)/2 - log(2*x - x^2)/x +
log(x^2 - 2*x + 2)/x - 1/x
```

sympy [A] time = 0.25, size = 46, normalized size = 0.68

$$\frac{\log\left(x^2-2x\right)}{2}-\frac{\log\left(x^2-2x+2\right)}{2}-\operatorname{atan}\left(x-1\right)-\frac{\log\left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((1-(-1+x)**2)/(1+(-1+x)**2))/x**2,x)
```

```
[Out] log(x**2 - 2*x)/2 - log(x**2 - 2*x + 2)/2 - atan(x - 1) - log((1 - (x - 1)*
*2)/((x - 1)**2 + 1))/x - 1/x
```

3.273 $\int \log\left(\sqrt{x}+x\right)\,dx$

Optimal. Leaf size=29

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

```
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2548, 376, 77}

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + x],x]
```

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\sqrt{x}+x\right)\,dx &= x\log\left(\sqrt{x}+x\right)-\int \frac{1+2\sqrt{x}}{2+2\sqrt{x}}\,dx\\
&= x\log\left(\sqrt{x}+x\right)-2\,\text{Subst}\left(\int \frac{x(1+2x)}{2+2x}\,dx,x,\sqrt{x}\right)\\
&= x\log\left(\sqrt{x}+x\right)-2\,\text{Subst}\left(\int \left(-\frac{1}{2}+x+\frac{1}{2(1+x)}\right)\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + x],x]
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

fricas [A] time = 0.43, size = 31, normalized size = 1.07

$$(x+1)\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*log(x + sqrt(x)) - x + sqrt(x) - 2*log(sqrt(x) + 1) - log(sqrt(x))
```

giac [A] time = 0.16, size = 23, normalized size = 0.79

$$x\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

maple [A] time = 0.06, size = 24, normalized size = 0.83

$$x\ln\left(x+\sqrt{x}\right)-x-\ln\left(\sqrt{x}+1\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+x^(1/2)),x)
[Out] -x-ln(x^(1/2)+1)+x*ln(x+x^(1/2))+x^(1/2)
```

maxima [A] time = 0.62, size = 23, normalized size = 0.79

$$x\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="maxima")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

mupad [B] time = 0.08, size = 23, normalized size = 0.79

$$\sqrt{x}-\ln\left(\sqrt{x}+1\right)-x+x\,\ln\left(x+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^(1/2)),x)
[Out] x^(1/2) - log(x^(1/2) + 1) - x + x*log(x + x^(1/2))
```

sympy [A] time = 7.63, size = 24, normalized size = 0.83

$$\sqrt{x}+x\log\left(\sqrt{x}+x\right)-x-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+x**(1/2)),x)
[Out] sqrt(x) + x*log(sqrt(x) + x) - x - log(sqrt(x) + 1)
```

3.274 $\int \log\left(-\frac{x}{1+x}\right) dx$

Optimal. Leaf size=18

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

[Out] x*ln(-x/(1+x))-ln(1+x)

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2486, 31}

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[Log[-(x/(1 + x))],x]

[Out] x*Log[-(x/(1 + x))] - Log[1 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(-\frac{x}{1+x}\right) dx &= x\log\left(-\frac{x}{1+x}\right)-\int \frac{1}{1+x}\, dx\\ &= x\log\left(-\frac{x}{1+x}\right)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[Log[-(x/(1 + x))],x]

[Out] x*Log[-(x/(1 + x))] - Log[1 + x]

fricas [A] time = 0.45, size = 18, normalized size = 1.00

$$x\log\left(-\frac{x}{x+1}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-x/(1+x)),x, algorithm="fricas")

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

giac [B] time = 0.16, size = 80, normalized size = 4.44

$$-\frac{\log\left(-\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)\left(\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)}-1\right)}\right)}{\frac{x}{x+1}-1}-\log\left(\frac{|x|}{|x+1|}\right)+\log\left(\left|-\frac{x}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="giac")
```

```
[Out] -log(-x/((x + 1)*(x/(x + 1) - 1)*(x/((x + 1)*(x/(x + 1) - 1)) - 1)))/(x/(x
+ 1) - 1) - log(abs(x)/abs(x + 1)) + log(abs(-x/(x + 1) + 1))
```

maple [A] time = 0.20, size = 28, normalized size = 1.56

$$\ln\left(\frac{1}{x+1}\right)-\left(-1+\frac{1}{x+1}\right)(x+1)\ln\left(-1+\frac{1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1/(x+1)*x),x)
```

```
[Out] ln(1/(x+1))-ln(-1+1/(x+1))*(-1+1/(x+1))*(x+1)
```

maxima [A] time = 0.64, size = 18, normalized size = 1.00

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="maxima")
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

mupad [B] time = 0.36, size = 18, normalized size = 1.00

$$x\,\ln\left(-\frac{x}{x+1}\right)-\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-x/(x + 1)),x)
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.78

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x/(1+x)),x)
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

3.275 $\int \log\left(\frac{-1+x}{1+x}\right) dx$

Optimal. Leaf size=27

$$-\left((1-x)\log\left(-\frac{1-x}{x+1}\right)\right)-2\log(x+1)$$

[Out] -(1-x)*ln((-1+x)/(1+x))-2*ln(1+x)

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2486, 31}

$$-(1-x)\log\left(-\frac{1-x}{x+1}\right)-2\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[Log[(-1 + x)/(1 + x)],x]

[Out] -((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{-1+x}{1+x}\right) dx &= -(1-x)\log\left(-\frac{1-x}{1+x}\right)-2\int \frac{1}{1+x}\,dx\\ &= -(1-x)\log\left(-\frac{1-x}{1+x}\right)-2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.78

$$(x-1)\log\left(\frac{x-1}{x+1}\right)-2\log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[Log[(-1 + x)/(1 + x)],x]

[Out] (-1 + x)*Log[(-1 + x)/(1 + x)] - 2*Log[1 + x]

fricas [A] time = 0.42, size = 21, normalized size = 0.78

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="fricas")
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

giac [B] time = 0.23, size = 103, normalized size = 3.81

$$-\frac{2\log\left(\frac{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}+1}{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}-1}\right)}{\frac{x-1}{x+1}-1}-2\log\left(\frac{|x-1|}{|x+1|}\right)+2\log\left(\left|\frac{x-1}{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="giac")
```

```
[Out] -2*log((((x - 1)/(x + 1) + 1)/((x - 1)/(x + 1) - 1) + 1)/(((x - 1)/(x + 1)
+ 1)/((x - 1)/(x + 1) - 1) - 1))/((x - 1)/(x + 1) - 1) - 2*log(abs(x - 1)/a
bs(x + 1)) + 2*log(abs((x - 1)/(x + 1) - 1))
```

maple [A] time = 0.17, size = 35, normalized size = 1.30

$$2\ln\left(-\frac{2}{x+1}\right)+\left(1-\frac{2}{x+1}\right)(x+1)\ln\left(1-\frac{2}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x-1)/(x+1)),x)
```

```
[Out] 2*ln(-2/(x+1))+ln(1-2/(x+1))*(1-2/(x+1))*(x+1)
```

maxima [A] time = 0.58, size = 25, normalized size = 0.93

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="maxima")
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x + 1) - log(x - 1)
```

mupad [B] time = 0.08, size = 21, normalized size = 0.78

$$x\ \ln\left(\frac{x-1}{x+1}\right)-\ln\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x - 1)/(x + 1)),x)
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

sympy [A] time = 0.12, size = 15, normalized size = 0.56

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-1+x)/(1+x)),x)
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x**2 - 1)
```

3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} dx$

Optimal. Leaf size=57

$$\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x+1}-\tan^{-1}(x)$$

```
[Out] -1/(1+x)-arctan(x)+1/2*ln(-x^2+1)-ln((-x^2+1)/(x^2+1))/(1+x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2525, 12, 2074, 260, 635, 203}

$$\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x+1}-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) - ArcTan[x] + Log[1 - x^2]/2 - Log[(1 - x^2)/(1 + x^2)]/(1 +
x) - Log[1 + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx &= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + \int \frac{4x}{-1-x+x^4+x^5}\,dx \\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + 4\int \frac{x}{-1-x+x^4+x^5}\,dx \\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + 4\int \left(\frac{1}{4(1+x)^2} + \frac{x}{4\left(-1+x^2\right)} + \frac{-1-x}{4\left(1+x^2\right)}\right)dx \\
&= -\frac{1}{1+x} - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + \int \frac{x}{-1+x^2}\,dx + \int \frac{-1-x}{1+x^2}\,dx \\
&= -\frac{1}{1+x} + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx \\
&= -\frac{1}{1+x} - \tan^{-1}(x) + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [C] time = 0.05, size = 60, normalized size = 1.05

$$\frac{1}{2}\left(\log\left(1-x^2\right) - \frac{2\left(\log\left(\frac{1-x^2}{x^2+1}\right)+1\right)}{x+1} + (-1+i)\log(-x+i) - (1+i)\log(x+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] ((-1 + I)*Log[I - x] - (1 + I)*Log[I + x] + Log[1 - x^2] - (2*(1 + Log[(1 -
 x^2)/(1 + x^2)]))/(1 + x))/2
```

fricas [A] time = 0.42, size = 54, normalized size = 0.95

$$-\frac{2\,(x+1)\arctan(x) + (x+1)\log\left(x^2+1\right) - (x+1)\log\left(x^2-1\right) + 2\,\log\left(-\frac{x^2-1}{x^2+1}\right) + 2}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(x) + (x + 1)*log(x^2 + 1) - (x + 1)*log(x^2 - 1) + 2
*log(-(x^2 - 1)/(x^2 + 1)) + 2)/(x + 1)
```

giac [A] time = 0.21, size = 56, normalized size = 0.98

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1} - \frac{1}{x+1} - \arctan(x) - \frac{1}{2}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(|x+1|\right) + \frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="giac")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

maple [C] time = 0.28, size = 112, normalized size = 1.96

$$-\frac{\ln\left(\frac{-x^2+1}{x^2+1}\right)}{x+1}+\frac{ix\ln(x-i)-x\ln(x-i)-ix\ln(x+i)-x\ln(x+i)+x\ln\left(x^2-1\right)+i\ln(x-i)-\ln(x-i)-i\ln(x}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-x^2+1)/(x^2+1))/(x+1)^2,x)
```

```
[Out] -ln((-x^2+1)/(x^2+1))/(x+1)+1/2*(I*ln(x-I)*x-I*ln(x+I)*x+I*ln(x-I)-ln(x-I)*
x-I*ln(x+I)-ln(x+I)*x+ln(x^2-1)*x-ln(x-I)-ln(x+I)+ln(x^2-1)-2)/(x+1)
```

maxima [A] time = 1.45, size = 54, normalized size = 0.95

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(x + 1) + 1/2*log(x - 1)
```

mupad [B] time = 0.42, size = 55, normalized size = 0.96

$$\frac{\ln\left(x^2-1\right)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\mathrm{atan}(x)-\frac{1}{x+1}+\frac{\ln\left(x^2+1\right)}{x+1}-\frac{\ln\left(1-x^2\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-(x^2 - 1)/(x^2 + 1))/(x + 1)^2,x)
```

```
[Out] log(x^2 - 1)/2 - log(x^2 + 1)/2 - atan(x) - 1/(x + 1) + log(x^2 + 1)/(x + 1
) - log(1 - x^2)/(x + 1)
```

sympy [A] time = 0.20, size = 41, normalized size = 0.72

$$\frac{\log\left(x^2-1\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\mathrm{atan}(x)-\frac{4}{4x+4}-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-x**2+1)/(x**2+1))/(1+x)**2,x)
```

```
[Out] log(x**2 - 1)/2 - log(x**2 + 1)/2 - atan(x) - 4/(4*x + 4) - log((1 - x**2)/
(x**2 + 1))/(x + 1)
```

3.277 $\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2} dx$

Optimal. Leaf size=60

$$\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\text{Li}_2\left(1-\frac{2}{ix+1}\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] I*n*arctan(x)^2+2*n*arctan(x)*ln(2/(1+I*x))+arctan(x)*ln(c*(x^2+1)^n)+I*n*p
olylog(2,1-2/(1+I*x))
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {203, 2470, 4920, 4854, 2402, 2315}

$$in\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[2/(1 + I*x)] + ArcTan[x]*Log[c*(1 + x^2
)^n] + I*n*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx = \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{x\tan^{-1}(x)}{1+x^2}\,dx$$
$$= in\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2n)\int \frac{\tan^{-1}(x)}{i-x}\,dx$$
$$= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}$$
$$= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2in)\,\text{Subst}\left(\int\right.$$
$$= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+in\text{Li}_2\left(1-\frac{2}{1+ix}\right.$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.03

$$\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\text{Li}_2\left(\frac{x+i}{x-i}\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2i}{-x+i}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[(2*I)/(I - x)] + ArcTan[x]*Log[c*(1 + x
^2)^n] + I*n*PolyLog[2, (I + x)/(-I + x)]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

maple [C] time = 0.62, size = 249, normalized size = 4.15

$$-\frac{i\pi\arctan(x)\text{csgn}(ic)\,\text{csgn}\left(i\left(x^2+1\right)^n\right)\text{csgn}\left(ic\left(x^2+1\right)^n\right)}{2}+\frac{i\pi\arctan(x)\text{csgn}(ic)\,\text{csgn}\left(ic\left(x^2+1\right)^n\right)^2}{2}+\frac{i\pi\,\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(x^2+1)^n)/(x^2+1),x)
```

```
[Out] arctan(x)*ln((x^2+1)^n)-n*ln(x^2+1)*arctan(x)-1/2*I*n*ln(x-I)*ln(x^2+1)+1/2
*I*n*dilog(-1/2*I*(x+I))+1/2*I*n*ln(x-I)*ln(-1/2*I*(x+I))+1/4*I*n*ln(x-I)^2
+1/2*I*n*ln(x+I)*ln(x^2+1)-1/2*I*n*dilog(1/2*I*(x-I))-1/2*I*n*ln(x+I)*ln(1/
2*I*(x-I))-1/4*I*n*ln(x+I)^2+1/2*I*arctan(x)*Pi*csgn(I*(x^2+1)^n)*csgn(I*c*
(x^2+1)^n)^2-1/2*I*arctan(x)*Pi*csgn(I*(x^2+1)^n)*csgn(I*c*(x^2+1)^n)*csgn(
I*c)-1/2*I*arctan(x)*Pi*csgn(I*c*(x^2+1)^n)^3+1/2*I*arctan(x)*Pi*csgn(I*c*(
x^2+1)^n)^2*csgn(I*c)+arctan(x)*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(x^2 + 1)^n)/(x^2 + 1),x)
```

```
[Out] int(log(c*(x^2 + 1)^n)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(x**2+1)**n)/(x**2+1),x)
```

```
[Out] Integral(log(c*(x**2 + 1)**n)/(x**2 + 1), x)
```

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} dx$

Optimal. Leaf size=61

$$i\text{Li}_2\left(\frac{2}{1-ix}-1\right)+\log\left(\frac{x^2}{x^2+1}\right)\tan^{-1}(x)+i\tan^{-1}(x)^2-2\log\left(2-\frac{2}{1-ix}\right)\tan^{-1}(x)$$

```
[Out] I*arctan(x)^2-2*arctan(x)*ln(2-2/(1-I*x))+arctan(x)*ln(x^2/(x^2+1))+I*polyl
og(2,-1+2/(1-I*x))
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {203, 2526, 12, 4924, 4868, 2447}

$$i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)+\log\left(\frac{x^2}{x^2+1}\right)\tan^{-1}(x)+i\tan^{-1}(x)^2-2\log\left(2-\frac{2}{1-ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] I*ArcTan[x]^2 - 2*ArcTan[x]*Log[2 - 2/(1 - I*x)] + ArcTan[x]*Log[x^2/(1 + x
^2)] + I*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2526

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[(u*D[RFx, x])/RFx, x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-\int \frac{2\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\ &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2\int \frac{\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\ &= i\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2i\int \frac{\tan^{-1}(x)}{x(i+x)}\,dx\\ &= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+2\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx\\ &= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+i\text{Li}_2\left(-1+\frac{2}{1-ix}\right)\end{aligned}$$

Mathematica [B] time = 0.05, size = 239, normalized size = 3.92

$$-\frac{1}{2}i\text{Li}_2\left(-\frac{1}{2}i(i-x)\right)+i\text{Li}_2(-i(i-x))+\frac{1}{2}i\text{Li}_2\left(-\frac{1}{2}i(x+i)\right)-i\text{Li}_2(-i(x+i))-\frac{1}{2}i\log\left(\frac{x^2}{x^2+1}\right)\log(-x+i)+\frac{1}{2}i\log(x+i$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] (-1/4*I)*Log[I - x]^2 + I*Log[I - x]*Log[(-I)*x] - (I/2)*Log[I - x]*Log[(-1
/2*I)*(I + x)] + (I/2)*Log[(-1/2*I)*(I - x)]*Log[I + x] - I*Log[I*x]*Log[I
+ x] + (I/4)*Log[I + x]^2 - (I/2)*Log[I - x]*Log[x^2/(1 + x^2)] + (I/2)*Log
[I + x]*Log[x^2/(1 + x^2)] - (I/2)*PolyLog[2, (-1/2*I)*(I - x)] + I*PolyLog
[2, (-I)*(I - x)] + (I/2)*PolyLog[2, (-1/2*I)*(I + x)] - I*PolyLog[2, (-I)*
(I + x)]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

maple [B] time = 0.19, size = 158, normalized size = 2.59

$$i\ln(-ix)\ln(x-i)-i\ln(ix)\ln(x+i)-\frac{i\ln\left(-\frac{i(x+i)}{2}\right)\ln(x-i)}{2}+\frac{i\ln\left(\frac{i(x-i)}{2}\right)\ln(x+i)}{2}-\frac{i\ln\left(\frac{x^2}{x^2+1}\right)\ln(x-i)}{2}+\frac{i\ln\left(\frac{x^2}{x^2+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/(x^2+1)*x^2)/(x^2+1),x)
```

```
[Out] -1/2*I*ln(x-I)*ln(1/(x^2+1)*x^2)-1/2*I*dilog(-1/2*I*(x+I))-1/2*I*ln(x-I)*ln
(-1/2*I*(x+I))+I*dilog(-I*x)+I*ln(x-I)*ln(-I*x)-1/4*I*ln(x-I)^2+1/2*I*ln(x+
I)*ln(1/(x^2+1)*x^2)+1/2*I*dilog(1/2*I*(x-I))+1/2*I*ln(x+I)*ln(1/2*I*(x-I))
-I*dilog(I*x)-I*ln(x+I)*ln(I*x)+1/4*I*ln(x+I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2/(x^2 + 1))/(x^2 + 1),x)
```

```
[Out] int(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2/(x**2+1))/(x**2+1),x)
```

```
[Out] Integral(log(x**2/(x**2 + 1))/(x**2 + 1), x)
```

3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}+\frac{i\text{Li}_2\left(\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}-1\right)}{\sqrt{a}\sqrt{b}}+\frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}-\frac{2\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] I*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*ln(
c*x^2/(b*x^2+a))/a^(1/2)/b^(1/2)-2*arctan(x*b^(1/2)/a^(1/2))*ln(2-2*a^(1/2)
/(a^(1/2)-I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*polylog(2,-1+2*a^(1/2)/(a^(1/2)-I
*x*b^(1/2)))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {205, 2526, 12, 4924, 4868, 2447}

$$\frac{i\text{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}+\frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}-\frac{2\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (I*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (I*PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2526

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[(u*D[RFx, x])/RFx, x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
```

```
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \int \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,x\left(a+bx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{x\left(a+bx^2\right)}\,dx}{\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{(2i)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{x\left(i+\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}\,dx}{\sqrt{a}\,\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{2\int \frac{\log\left(2-\frac{}{1-}\right)}{1+\frac{bx}{a}}}{a}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{i\mathrm{Li}_2\left(-1+\right.}{\sqrt{a}\,\sqrt{}}$$

Mathematica [B] time = 0.20, size = 373, normalized size = 2.26

$$\frac{2\log\left(\sqrt{-a}-\sqrt{b}\,x\right)\log\left(\frac{cx^2}{a+bx^2}\right)-2\log\left(\sqrt{-a}+\sqrt{b}\,x\right)\log\left(\frac{cx^2}{a+bx^2}\right)+4\mathrm{Li}_2\left(\frac{\sqrt{b}\,x}{\sqrt{-a}}+1\right)-2\mathrm{Li}_2\left(\frac{a-\sqrt{-a}\,\sqrt{b}\,x}{2a}\right)+2\mathrm{Li}_2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (-4*Log[(Sqrt[b]*x)/Sqrt[-a]]*Log[Sqrt[-a] - Sqrt[b]*x] + Log[Sqrt[-a] - Sq
rt[b]*x]^2 + 4*Log[(a*Sqrt[b]*x)/(-a)^(3/2)]*Log[Sqrt[-a] + Sqrt[b]*x] - Lo
g[Sqrt[-a] + Sqrt[b]*x]^2 + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(a - Sqrt[-a]*S
qrt[b]*x)/(2*a)] - 2*Log[Sqrt[-a] + Sqrt[b]*x]*Log[(a + Sqrt[-a]*Sqrt[b]*x)
/(2*a)] + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] - 2*Log[Sqrt
[-a] + Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] + 4*PolyLog[2, 1 + (Sqrt[b]*x)/S
qrt[-a]] - 2*PolyLog[2, (a - Sqrt[-a]*Sqrt[b]*x)/(2*a)] + 2*PolyLog[2, (a +
 Sqrt[-a]*Sqrt[b]*x)/(2*a)] - 4*PolyLog[2, 1 + (a*Sqrt[b]*x)/(-a)^(3/2)])/(
4*Sqrt[-a]*Sqrt[b])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="fricas")
[Out] integral(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="giac")
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)
[Out] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2),x)
[Out] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(b*x**2+a))/(b*x**2+a),x)
```

```
[Out] Integral(log(c*x**2/(a + b*x**2))/(a + b*x**2), x)
```

3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2518}

$$\frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] time = 0.65, size = 134, normalized size = 4.62

$$\frac{\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-2\left(-\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)+\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(e^{-2\tanh^{-1}(ax)}+1\right)-\log\left(\right.\right.\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] - Log[1 -
I/E^ArcTanh[a*x]] + Log[1 + I/E^ArcTanh[a*x]]) - PolyLog[2, (-I)/E^ArcTanh[
a*x]] + PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

fricas [A] time = 0.44, size = 37, normalized size = 1.28

$$\frac{\text{Li}_2\left(-\frac{ax-\sqrt{ax+1}\,\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x - sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\log\left(ax+1\right)-\log\left(-ax+1\right)\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(-ax+1\right)^2-4}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) + I*sqrt(-a*x + 1)))/a - integrate(-sqrt(a*x + 1)*(log
(a*x + 1) - log(-a*x + 1))/(2*(a^2*x^2 - 1)*sqrt(a*x + 1) + (2*I*a^2*x^2 -
2*I)*sqrt(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\ln\left(1+\frac{\sqrt{1-a\,x}\,1{}\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1),x)
```

```
[Out] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2518}

$$\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] time = 0.57, size = 134, normalized size = 4.62

$$\frac{\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-2\left(\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(e^{-2\tanh^{-1}(ax)}+1\right)+\log\left(1\right.\right.\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] + Log[1 -
I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

fricas [A] time = 0.44, size = 36, normalized size = 1.24

$$\frac{\text{Li}_2\left(-\frac{ax+\sqrt{ax+1}\,\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(-I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\log\left(ax+1\right)-\log\left(-ax+1\right)\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(-ax+1\right)^2-4}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) - I*sqrt(-a*x + 1)))/a + integrate(sqrt(a*x + 1)*(log(
a*x + 1) - log(-a*x + 1))/(2*(a^2*x^2 - 1)*sqrt(a*x + 1) - (2*I*a^2*x^2 - 2
*I)*sqrt(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\ln\left(1-\frac{\sqrt{1-a\,x}\,1{}\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

3.282 $\int \log\left(e^{a+bx}\right) dx$

Optimal. Leaf size=17

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2157, 30}

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^{a+bx}\right) dx &= \frac{\operatorname{Subst}\left(\int x\,dx, x, \log\left(e^{a+bx}\right)\right)}{b}\\ &= \frac{\log^2\left(e^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

fricas [A] time = 0.39, size = 10, normalized size = 0.59

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.17, size = 10, normalized size = 0.59

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.06, size = 15, normalized size = 0.88

$$\frac{\ln\left(e^{bx+a}\right)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(b*x+a)),x)
[Out] 1/2*ln(exp(b*x+a))^2/b
```

maxima [A] time = 0.62, size = 10, normalized size = 0.59

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x
```

mupad [B] time = 0.07, size = 17, normalized size = 1.00

$$x\ln\left(e^{bx}\,e^{a}\right) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(a + b*x)),x)
[Out] x*log(exp(b*x)*exp(a)) - (b*x^2)/2
```

sympy [A] time = 0.08, size = 8, normalized size = 0.47

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(b*x+a)),x)
[Out] a*x + b*x**2/2
```

3.283 $\int \log \left(e^{a+bx^n}\right) dx$

Optimal. Leaf size=27

$$x\log\left(e^{a+bx^n}\right)-\frac{bnx^{n+1}}{n+1}$$

```
[Out] -b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2548, 12, 30}

$$x\log\left(e^{a+bx^n}\right)-\frac{bnx^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x^n)],x]
```

```
[Out] -((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^{a+bx^n}\right) dx &= x\log\left(e^{a+bx^n}\right)-\int bnx^n\,dx\\ &= x\log\left(e^{a+bx^n}\right)-(bn)\int x^n\,dx\\ &= -\frac{bnx^{1+n}}{1+n}+x\log\left(e^{a+bx^n}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.93

$$x\left(\log\left(e^{a+bx^n}\right)-\frac{bnx^n}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x^n)],x]
```

```
[Out] x*(-((b*n*x^n)/(1 + n)) + Log[E^(a + b*x^n)])
```

fricas [A] time = 0.46, size = 20, normalized size = 0.74

$$\frac{bxx^n+(an+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

giac [A] time = 0.23, size = 16, normalized size = 0.59

$$ax + \frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="giac")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

maple [A] time = 0.16, size = 27, normalized size = 1.00

$$-\frac{bn\,x^{n+1}}{n+1} + x\ln\left(\mathrm{e}^{b\,x^{n}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(b*x^n+a)),x)
```

```
[Out] -b*n*x^(n+1)/(n+1)+x*ln(exp(b*x^n+a))
```

maxima [A] time = 0.53, size = 16, normalized size = 0.59

$$ax + \frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="maxima")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

mupad [B] time = 0.60, size = 49, normalized size = 1.81

$$\begin{cases} x\,\ln\left(\mathrm{e}^{a+\frac{b}{x}}\right) + b\,\ln(x) & \text{if } n = -1 \\ x\,\ln\left(\mathrm{e}^{a+b\,x^{n}}\right) - \frac{b\,n\,x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(a + b*x^n)),x)
```

```
[Out] piecewise(n == -1, x*log(exp(a + b/x)) + b*log(x), n ~= -1, x*log(exp(a + b
*x^n)) - (b*n*x^(n + 1))/(n + 1))
```

sympy [A] time = 2.55, size = 34, normalized size = 1.26

$$\begin{cases} \frac{anx}{n+1} + \frac{ax}{n+1} + \frac{bxx^{n}}{n+1} & \text{for } n \neq -1 \\ ax + b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(a+b*x**n)),x)
```

```
[Out] Piecewise((a*n*x/(n + 1) + a*x/(n + 1) + b*x*x**n/(n + 1), Ne(n, -1)), (a*x
 + b*log(x), True))
```

3.284 $\int e^x \log(a + be^x)\,dx$

Optimal. Leaf size=25

$$\frac{(a + be^x)\log(a + be^x)}{b} - e^x$$

```
[Out] -exp(x)+(a+b*exp(x))*ln(a+b*exp(x))/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2194, 2554, 12, 2248, 43}

$$e^x \log(a + be^x) + \frac{a\log(a + be^x)}{b} - e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + (a*Log[a + b*E^x])/b + E^x*Log[a + b*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x \log(a+be^x)\,dx &= e^x\log(a+be^x) - \int \frac{be^{2x}}{a+be^x}\,dx\\ &= e^x\log(a+be^x) - b\int \frac{e^{2x}}{a+be^x}\,dx\\ &= e^x\log(a+be^x) - b\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, e^x\right)\\ &= e^x\log(a+be^x) - b\,\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)dx, x, e^x\right)\\ &= -e^x + \frac{a\log(a+be^x)}{b} + e^x\log(a+be^x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{(a+be^x)\log(a+be^x)}{b} - e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[a + b*E^x],x]
[Out] -E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

fricas [A] time = 0.40, size = 25, normalized size = 1.00

$$-\frac{be^x - (be^x + a)\log(be^x + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="fricas")
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a))/b
```

giac [A] time = 0.16, size = 26, normalized size = 1.04

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="giac")
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

maple [A] time = 0.06, size = 34, normalized size = 1.36

$$e^x\ln(b\,e^x + a) + \frac{a\ln(b\,e^x + a)}{b} - e^x - \frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(b*exp(x)+a),x)
[Out] exp(x)*ln(b*exp(x)+a)-exp(x)+a*ln(b*exp(x)+a)/b-a/b
```

maxima [A] time = 0.74, size = 26, normalized size = 1.04

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

mupad [B] time = 0.66, size = 27, normalized size = 1.08

$$e^x \ln(a + b e^x) - e^x + \frac{a \ln(a + b e^x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*log(a + b*exp(x)),x)
```

```
[Out] exp(x)*log(a + b*exp(x)) - exp(x) + (a*log(a + b*exp(x)))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(a+b*exp(x)),x)
```

```
[Out] Timed out
```

3.285 $\int e^{a+bx}\log(x)\,dx$

Optimal. Leaf size=26

$$\frac{\log(x)e^{a+bx}}{b}-\frac{e^a\mathrm{Ei}(bx)}{b}$$

```
[Out] -exp(a)*Ei(b*x)/b+exp(b*x+a)*ln(x)/b
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2194, 2554, 12, 2178}

$$\frac{\log(x)e^{a+bx}}{b}-\frac{e^a\mathrm{Ei}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Log[x],x]
```

```
[Out] -((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\log(x)\,dx&=\frac{e^{a+bx}\log(x)}{b}-\int\frac{e^{a+bx}}{bx}\,dx\\&=\frac{e^{a+bx}\log(x)}{b}-\frac{\int\frac{e^{a+bx}}{x}\,dx}{b}\\&=-\frac{e^a\mathrm{Ei}(bx)}{b}+\frac{e^{a+bx}\log(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.85

$$\frac{e^a\left(e^{bx}\log(x)-\mathrm{Ei}(bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Log[x],x]
[Out] (E^a*(-ExpIntegralEi[b*x] + E^(b*x)*Log[x]))/b
```

fricas [A] time = 0.42, size = 23, normalized size = 0.88

$$-\frac{\operatorname{Ei}(bx)\,e^{a}-e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="fricas")
[Out] -(Ei(b*x)*e^a - e^(b*x + a)*log(x))/b
```

giac [A] time = 0.17, size = 24, normalized size = 0.92

$$-\frac{\operatorname{Ei}(bx)\,e^{a}}{b}+\frac{e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="giac")
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

maple [A] time = 0.62, size = 26, normalized size = 1.00

$$\frac{\operatorname{Ei}(1,-bx)\,\mathrm{e}^{a}}{b}+\frac{\mathrm{e}^{bx+a}\ln(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*ln(x),x)
[Out] exp(b*x+a)*ln(x)/b+1/b*exp(a)*Ei(1,-b*x)
```

maxima [A] time = 0.86, size = 24, normalized size = 0.92

$$-\frac{\operatorname{Ei}(bx)\,e^{a}}{b}+\frac{e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="maxima")
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

mupad [B] time = 0.37, size = 20, normalized size = 0.77

$$-\frac{\mathrm{e}^{a}\left(\operatorname{ei}(b\,x)-\mathrm{e}^{b\,x}\ln(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*log(x),x)
[Out] -(exp(a)*(ei(b*x) - exp(b*x)*log(x)))/b
```

sympy [A] time = 7.38, size = 26, normalized size = 1.00

$$\left(\begin{cases} x & \text{for}\ b=0 \\ \frac{e^{bx}}{b} & \text{otherwise} \end{cases}\right)e^{a}\log(x)-\left(\begin{cases} x & \text{for}\ b=0 \\ \frac{\operatorname{Ei}(bx)}{b} & \text{otherwise} \end{cases}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*ln(x),x)
```

```
[Out] Piecewise((x, Eq(b, 0)), (exp(b*x)/b, True))*exp(a)*log(x) - Piecewise((x,
Eq(b, 0)), (Ei(b*x)/b, True))*exp(a)
```

3.286 $\int \frac{x^2}{x+\log(x)}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{x^2}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x^2/(x+ln(x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(x + Log[x]),x]
[Out] Defer[Int][x^2/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log(x)}\,dx$$

Mathematica [A] time = 7.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(x + Log[x]),x]
[Out] Integrate[x^2/(x + Log[x]), x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{x+\log(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(x^2/(x + log(x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(x^2/(x + log(x)), x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+ln(x)),x)
```

```
[Out] int(x^2/(x+ln(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x + log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^2}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + log(x)),x)
```

```
[Out] int(x^2/(x + log(x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\log{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x+ln(x)),x)
```

```
[Out] Integral(x**2/(x + log(x)), x)
```

3.287 $\int \frac{x}{x+\log(x)}\,dx$

Optimal. Leaf size=11

$$\mathrm{Int}\left(\frac{x}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x/(x+ln(x)),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(x + Log[x]),x]
[Out] Defer[Int][x/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log(x)}\,dx$$

Mathematica [A] time = 5.72, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(x + Log[x]),x]
[Out] Integrate[x/(x + Log[x]), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{x+\log(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="fricas")
[Out] integral(x/(x + log(x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="giac")
[Out] integrate(x/(x + log(x)), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+ln(x)),x)
```

```
[Out] int(x/(x+ln(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x + log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + log(x)),x)
```

```
[Out] int(x/(x + log(x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log{\left(x \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+ln(x)),x)
```

```
[Out] Integral(x/(x + log(x)), x)
```

3.288 $\int \frac{1}{x+\log(x)}\,dx$

Optimal. Leaf size=9

$$\mathrm{Int}\left(\frac{1}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(1/(x+ln(x)),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x + Log[x])^(-1),x]
[Out] Defer[Int][(x + Log[x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log(x)}\,dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x + Log[x])^(-1),x]
[Out] Integrate[(x + Log[x])^(-1), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x+\log(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x + log(x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/(x + log(x)), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+ln(x)),x)
```

```
[Out] int(1/(x+ln(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + log(x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{x + \ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + log(x)),x)
```

```
[Out] int(1/(x + log(x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \log{\left(x \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+ln(x)),x)
```

```
[Out] Integral(1/(x + log(x)), x)
```

3.289 $\int \frac{1}{x(x+\log(x))}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{1}{x(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate(1/x/(x+ln(x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(x+\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(x + Log[x])),x]
[Out] Defer[Int][1/(x*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{x(x+\log(x))}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x+\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(x + Log[x])),x]
[Out] Integrate[1/(x*(x + Log[x])), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x^2+x\log(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^2 + x*log(x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\log(x)\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+\ln(x))\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+ln(x)),x)
```

```
[Out] int(1/x/(x+ln(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\log(x)\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + log(x))*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x\,(x+\ln(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + log(x))),x)
```

```
[Out] int(1/(x*(x + log(x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x+\log\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+ln(x)),x)
```

```
[Out] Integral(1/(x*(x + log(x))), x)
```

3.290 $\int \frac{1}{x^2(x+\log(x))} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate(1/x^2/(x+ln(x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(x+\log(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(x + Log[x])),x]
[Out] Defer[Int][1/(x^2*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x^2(x+\log(x))} \, dx = \int \frac{1}{x^2(x+\log(x))} \, dx$$

Mathematica [A] time = 6.66, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(x+\log(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(x + Log[x])),x]
[Out] Integrate[1/(x^2*(x + Log[x])), x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^3+x^2\log(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^3 + x^2*log(x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+\log(x))x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x^2), x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+\ln(x))\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+ln(x)),x)
```

```
[Out] int(1/x^2/(x+ln(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\log(x)\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + log(x))*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2\ (x+\ln(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + log(x))),x)
```

```
[Out] int(1/(x^2*(x + log(x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x+\log\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x+ln(x)),x)
```

```
[Out] Integral(1/(x**2*(x + log(x))), x)
```

3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$

Optimal. Leaf size=13

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203, 260}

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x + 4*x*Log[x]^2),x]
[Out] Log[1 + 4*Log[x]^2]/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x+4x\log^2(x)}\,dx &= \text{Subst}\left(\int \frac{x}{1+4x^2}\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\log\left(1+4\log^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x + 4*x*Log[x]^2),x]
[Out] Log[1 + 4*Log[x]^2]/8
```

fricas [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{1}{8}\log\left(4\log(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

giac [A] time = 0.16, size = 11, normalized size = 0.85

$$\frac{1}{8} \log\left(4 \log(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

maple [A] time = 0.07, size = 12, normalized size = 0.92

$$\frac{\ln\left(4 \ln(x)^2 + 1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(x+4*x*ln(x)^2),x)
```

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

maxima [A] time = 0.79, size = 9, normalized size = 0.69

$$\frac{1}{8} \log\left(\log(x)^2 + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*log(log(x)^2 + 1/4)
```

mupad [B] time = 0.46, size = 11, normalized size = 0.85

$$\frac{\ln\left(4 \ln(x)^2 + 1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x + 4*x*log(x)^2),x)
```

```
[Out] log(4*log(x)^2 + 1)/8
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\log\left(\log(x)^2 + \frac{1}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(x+4*x*ln(x)**2),x)
```

```
[Out] log(log(x)**2 + 1/4)/8
```

3.292 $\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$

Optimal. Leaf size=9

$$\log\left(\frac{\log(x)}{x}+1\right)$$

```
[Out] ln(1+ln(x)/x)
```

Rubi [A] time = 0.07, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6712, 31}

$$\log\left(\frac{\log(x)}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] Log[1 + Log[x]/x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6712

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[c*q, Subst[Int
[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a,
b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integer
Q[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\log(x)}{x(x+\log(x))}\,dx &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\log(x)}{x}\right)\\ &= \log\left(1+\frac{\log(x)}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 10, normalized size = 1.11

$$\log(x+\log(x))-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] -Log[x] + Log[x + Log[x]]
```

fricas [A] time = 0.42, size = 10, normalized size = 1.11

$$\log\left(x+\log(x)\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] log(x + log(x)) - log(x)
```

giac [A] time = 0.17, size = 14, normalized size = 1.56

$$-\log(x) + \log\left(-x - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="giac")
[Out] -log(x) + log(-x - log(x))
```

maple [A] time = 0.07, size = 11, normalized size = 1.22

$$-\ln(x) + \ln\left(x + \ln(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x/(x+ln(x)),x)
[Out] -ln(x)+ln(x+ln(x))
```

maxima [A] time = 0.90, size = 10, normalized size = 1.11

$$\log\left(x + \log(x)\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="maxima")
[Out] log(x + log(x)) - log(x)
```

mupad [B] time = 0.37, size = 10, normalized size = 1.11

$$\ln\left(x + \ln(x)\right) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 1)/(x*(x + log(x))),x)
[Out] log(x + log(x)) - log(x)
```

sympy [A] time = 0.14, size = 8, normalized size = 0.89

$$-\log\left(x\right) + \log\left(x + \log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x/(x+ln(x)),x)
[Out] -log(x) + log(x + log(x))
```

3.293 $\int \frac{1+x}{\log(x)(x+\log(x))}\,dx$

Optimal. Leaf size=13

$$\mathrm{li}(x) + \log(\log(x)) - \log(x + \log(x))$$

```
[Out] Li(x)+ln(ln(x))-ln(x+ln(x))
```

Rubi [A] time = 0.14, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 2353, 2298, 2302, 29, 6684}

$$\mathrm{li}(x) + \log(\log(x)) - \log(x + \log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(Log[x]*(x + Log[x])),x]
```

```
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{\log(x)(x+\log(x))}\,dx &= \int \left(\frac{1+x}{x\log(x)}+\frac{-1-x}{x(x+\log(x))}\right)dx\\
&= \int \frac{1+x}{x\log(x)}\,dx+\int \frac{-1-x}{x(x+\log(x))}\,dx\\
&= -\log(x+\log(x))+\int \left(\frac{1}{\log(x)}+\frac{1}{x\log(x)}\right)dx\\
&= -\log(x+\log(x))+\int \frac{1}{\log(x)}\,dx+\int \frac{1}{x\log(x)}\,dx\\
&= -\log(x+\log(x))+\text{li}(x)+\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log(x)\right)\\
&= \log(\log(x))-\log(x+\log(x))+\text{li}(x)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$\text{li}(x)+\log(\log(x))-\log(x+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(Log[x]*(x + Log[x])),x]
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

fricas [A] time = 0.40, size = 13, normalized size = 1.00

$$-\log\left(x+\log(x)\right)+\log\left(\log(x)\right)+\text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="fricas")
[Out] -log(x + log(x)) + log(log(x)) + log_integral(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x+\log(x)\right)\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="giac")
[Out] integrate((x + 1)/((x + log(x))*log(x)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x+\ln(x))\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/ln(x)/(x+ln(x)),x)
[Out] int((x+1)/ln(x)/(x+ln(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{x\log(x)}\,dx-\log\left(x+\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="maxima")
[Out] integrate((x + 1)/(x*log(x)), x) - log(x + log(x))
```

mupad [B] time = 0.39, size = 13, normalized size = 1.00

$$\ln(\ln(x)) - \ln(x + \ln(x)) + \mathrm{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(log(x)*(x + log(x))),x)
[Out] log(log(x)) - log(x + log(x)) + logint(x)
```

sympy [A] time = 4.30, size = 15, normalized size = 1.15

$$-\log\left(x + \log(x)\right) + \log\left(\log(x)\right) + \mathrm{Ei}\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/ln(x)/(x+ln(x)),x)
[Out] -log(x + log(x)) + log(log(x)) + Ei(log(x))
```

3.294 $\int \log\left(2+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=67

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

```
[Out] -1/6*ln(1-(1+1/x)^(1/2))+1/2*ln(1+(1+1/x)^(1/2))-1/3*ln(2+(1+1/x)^(1/2))+x*
ln(2+((1+x)/x)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2548, 12, 2058}

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -Log[1 - Sqrt[1 + x^(-1)]]/6 + Log[1 + Sqrt[1 + x^(-1)]]/2 - Log[2 + Sqrt[1
 + x^(-1)]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(2+\sqrt{\frac{1+x}{x}}\right) dx &= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{2\left(-1-x-2x\sqrt{\frac{1+x}{x}}\right)}\,dx\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\int \frac{1}{-1-x-2x\sqrt{\frac{1+x}{x}}}\,dx\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)+\text{Subst}\left(\int \frac{1}{2+x-2x^2-x^3}\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)+\text{Subst}\left(\int \left(-\frac{1}{6(-1+x)}+\frac{1}{2(1+x)}-\frac{1}{3(2+x)}\right) dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= -\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right)+x\log\left(2+\sqrt{\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.79

$$x\log\left(\sqrt{\frac{1}{x}+1}+2\right)+\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(2\sqrt{\frac{1}{x}+1}+1\right)\right)-\tanh^{-1}\left(2\sqrt{\frac{1}{x}+1}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ArcTanh[(1 + 2*Sqrt[1 + x^(-1)])/3]/3 - ArcTanh[3 + 2*Sqrt[1 + x^(-1)]] + x
*Log[2 + Sqrt[1 + x^(-1)]]
```

fricas [A] time = 0.45, size = 48, normalized size = 0.72

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) + 2) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/6
*log(sqrt((x + 1)/x) - 1)
```

giac [A] time = 0.25, size = 88, normalized size = 1.31

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log\left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6\,\mathrm{sgn}(x)}-\frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{3\,\mathrm{sgn}(x)}+\frac{\log\left(\left|-3x+3\sqrt{x^2+x}-1\right|\right)}{6\,\mathrm{sgn}(x)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) -
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) + 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

maple [B] time = 0.14, size = 108, normalized size = 1.61

$$x\ln\left(2+\sqrt{\frac{x+1}{x}}\right)+\frac{-3\sqrt{\frac{x+1}{x}}\,x\ln(-3x+1)-\sqrt{9}\,\sqrt{(x+1)x}\,\ln\left(\frac{15x+4\sqrt{9}\,\sqrt{x^2+x}+3}{9x-3}\right)+6\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\right.}{18\sqrt{\frac{x+1}{x}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+((x+1)/x)^(1/2)),x)
```

```
[Out] x*ln(2+((x+1)/x)^(1/2))+1/18/((x+1)/x)^(1/2)/x*(-9^(1/2)*ln(1/3*(4*9^(1/2)*
(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(x+1))^(1/2)-3*((x+1)/x)^(1/2)*x*ln(1-3*x
)+6*ln(x+1/2+(x^2+x)^(1/2))*(x*(x+1))^(1/2))
```

maxima [A] time = 0.56, size = 67, normalized size = 1.00

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+2\right)}{\frac{x+1}{x}-1}-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 2)/((x + 1)/x - 1) - 1/3*log(sqrt((x + 1)/x) + 2) + 1
/2*log(sqrt((x + 1)/x) + 1) - 1/6*log(sqrt((x + 1)/x) - 1)
```

mupad [B] time = 0.64, size = 63, normalized size = 0.94

$$\frac{\ln\left(-5\sqrt{\frac{x+1}{x}}-5\right)}{2}-\frac{\ln\left(\frac{\sqrt{\frac{x+1}{x}}}{9}-\frac{1}{9}\right)}{6}-\frac{\ln\left(-\frac{5\sqrt{\frac{x+1}{x}}}{9}-\frac{10}{9}\right)}{3}+x\ln\left(\sqrt{\frac{x+1}{x}}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) + 2),x)
```

```
[Out] log(- 5*((x + 1)/x)^(1/2) - 5)/2 - log(((x + 1)/x)^(1/2)/9 - 1/9)/6 - log(-
 (5*((x + 1)/x)^(1/2))/9 - 10/9)/3 + x*log(((x + 1)/x)^(1/2) + 2)
```

sympy [A] time = 40.98, size = 53, normalized size = 0.79

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{6}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - log(sqrt(1 + 1/x) - 1)/6 + log(sqrt(1 + 1/x) +
 1)/2 - log(sqrt(1 + 1/x) + 2)/3
```

3.295 $\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

```
[Out] 1/2*arctanh(((1+x)/x)^(1/2))+x*ln(1+((1+x)/x)^(1/2))-1/2/(1+(1+1/x)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2548, 12, 44, 207}

$$-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/(2*(1 + Sqrt[1 + x^(-1)])) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{2\left(-1-x-x\sqrt{\frac{1+x}{x}}\right)}dx$$
$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\int \frac{1}{-1-x-x\sqrt{\frac{1+x}{x}}}dx$$
$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)^2}dx,x,\sqrt{\frac{1+x}{x}}\right)$$
$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(-1+x^2\right)}\right)dx,x,\sqrt{\frac{1+x}{x}}\right)$$
$$= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}dx,x,\sqrt{\frac{1+x}{x}}\right)$$
$$= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1+x}{x}}\right)+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.06

$$\frac{1}{4}\left(-2\sqrt{\frac{1}{x}+1}\,x+2x+4x\log\left(\sqrt{\frac{1}{x}+1}+1\right)+\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (2*x - 2*Sqrt[1 + x^(-1)]*x + 4*x*Log[1 + Sqrt[1 + x^(-1)]] + Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x])/4
```

fricas [A] time = 0.45, size = 49, normalized size = 0.98

$$\frac{1}{4}(4x+1)\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}x\sqrt{\frac{x+1}{x}}+\frac{1}{2}x-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) + 1) - 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) - 1)
```

giac [A] time = 0.23, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}x-\frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{4\,\text{sgn}(x)}-\frac{\sqrt{x^2+x}}{2\,\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 1) + 1/2*x - 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) - 1/2*sqrt(x^2 + x)/sgn(x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \ln\left(1+\sqrt{\frac{x+1}{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+((x+1)/x)^(1/2)),x)
```

```
[Out] int(ln(1+((x+1)/x)^(1/2)),x)
```

maxima [A] time = 0.59, size = 68, normalized size = 1.36

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+1\right)}{\frac{x+1}{x}-1}-\frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)}+\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 1)/((x + 1)/x - 1) - 1/2/(sqrt((x + 1)/x) + 1) + 1/4*
log(sqrt((x + 1)/x) + 1) - 1/4*log(sqrt((x + 1)/x) - 1)
```

mupad [B] time = 0.45, size = 38, normalized size = 0.76

$$\frac{x}{2}+\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\ln\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{x\sqrt{\frac{1}{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) + 1),x)
```

```
[Out] x/2 + atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) + 1) - (x*(1/x + 1
)^(1/2))/2
```

sympy [A] time = 45.79, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}-\frac{1}{2\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 1) - log(sqrt(1 + 1/x) - 1)/4 + log(sqrt(1 + 1/x) +
 1)/4 - 1/(2*(sqrt(1 + 1/x) + 1))
```

3.296 $\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=21

$$x\log\left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log(x+1)$$

[Out] 1/2*ln(1+x)+1/2*x*ln(1+1/x)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2453, 2448, 263, 31}

$$x\log\left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[Log[Sqrt[(1 + x)/x]],x]

[Out] x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx &= \int \log\left(\sqrt{1+\frac{1}{x}}\right) dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)x} dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{1+x} dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{1}{2}\left(\log(x) + (x+1)\log\left(\frac{x+1}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[(1 + x)/x]],x]
[Out] (Log[x] + (1 + x)*Log[(1 + x)/x])/2
```

fricas [A] time = 0.40, size = 18, normalized size = 0.86

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right) + \frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="fricas")
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

giac [B] time = 0.16, size = 47, normalized size = 2.24

$$\frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x} - 1\right)} + \frac{1}{2}\,\log\left(\frac{|x+1|}{|x|}\right) - \frac{1}{2}\,\log\left(\left|\frac{x+1}{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="giac")
[Out] 1/2*log((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(abs(x + 1)/abs(x)) - 1/2*log(a
bs((x + 1)/x - 1))
```

maple [A] time = 0.17, size = 22, normalized size = 1.05

$$\frac{\left(\frac{1}{x}+1\right)x\ln\left(\frac{1}{x}+1\right)}{2} - \frac{\ln\left(\frac{1}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln((x+1)/x),x)
[Out] -1/2*ln(1/x)+1/2*ln(1/x+1)*(1/x+1)*x
```

maxima [A] time = 0.59, size = 18, normalized size = 0.86

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right) + \frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="maxima")
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

mupad [B] time = 0.06, size = 18, normalized size = 0.86

$$\frac{\ln\left(x+1\right)}{2} + \frac{x\,\ln\left(\frac{x+1}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x + 1)/x)/2,x)
```

```
[Out] log(x + 1)/2 + (x*log((x + 1)/x))/2
```

sympy [**A**] time = 0.11, size = 17, normalized size = 0.81

$$\frac{x \log\left(\frac{x+1}{x}\right)}{2} + \frac{\log(2x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln((1+x)/x),x)
```

```
[Out] x*log((x + 1)/x)/2 + log(2*x + 2)/2
```

3.297 $\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)}+x\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

```
[Out] -1/2*arctanh((1+1/x)^(1/2))+x*ln(-1+((1+x)/x)^(1/2))-1/2/(1-(1+1/x)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2548, 44, 207}

$$-\frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)}+x\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/(2*(1 - Sqrt[1 + x^(-1)])) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + Sq
rt[(1 + x)/x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \int \frac{1}{-2+\left(-2+2\sqrt{1+\frac{1}{x}}\right)x}dx$$
$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)^2(1+x)}dx, x, \sqrt{1+\frac{1}{x}}\right)$$
$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2} - \frac{1}{2\left(-1+x^2\right)}\right)dx, x, \sqrt{1+\frac{1}{x}}\right)$$
$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)} + x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}dx, x, \sqrt{1+\frac{1}{x}}\right)$$
$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right) + x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.06

$$\frac{1}{2}\left(\sqrt{\frac{1}{x}+1}+1\right)x + x\log\left(\sqrt{\frac{1}{x}+1}-1\right) - \frac{1}{4}\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ((1 + Sqrt[1 + x^(-1)])*x)/2 + x*Log[-1 + Sqrt[1 + x^(-1)]] - Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x]/4
```

fricas [A] time = 0.43, size = 49, normalized size = 0.98

$$\frac{1}{4}(4x+1)\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{1}{2}x\sqrt{\frac{x+1}{x}} + \frac{1}{2}x - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) - 1) + 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) + 1)
```

giac [A] time = 0.23, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{1}{2}x + \frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{4\,\text{sgn}(x)} + \frac{\sqrt{x^2+x}}{2\,\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 1) + 1/2*x + 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) + 1/2*sqrt(x^2 + x)/sgn(x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \ln\left(-1+\sqrt{\frac{x+1}{x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+((x+1)/x)^(1/2)),x)
```

```
[Out] int(ln(-1+((x+1)/x)^(1/2)),x)
```

maxima [A] time = 0.63, size = 68, normalized size = 1.36

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-1\right)}{\frac{x+1}{x}-1}+\frac{1}{2\left(\sqrt{\frac{x+1}{x}}-1\right)}-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 1)/((x + 1)/x - 1) + 1/2/(sqrt((x + 1)/x) - 1) - 1/4*
log(sqrt((x + 1)/x) + 1) + 1/4*log(sqrt((x + 1)/x) - 1)
```

mupad [B] time = 0.43, size = 38, normalized size = 0.76

$$\frac{x}{2}-\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\ln\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{x\sqrt{\frac{1}{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) - 1),x)
```

```
[Out] x/2 - atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) - 1) + (x*(1/x + 1
)^(1/2))/2
```

sympy [A] time = 46.26, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}+\frac{1}{2\left(\sqrt{1+\frac{1}{x}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 1) + log(sqrt(1 + 1/x) - 1)/4 - log(sqrt(1 + 1/x) +
 1)/4 + 1/(2*(sqrt(1 + 1/x) - 1))
```

3.298 $\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=69

$$\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

```
[Out] 1/2*ln(1-(1+1/x)^(1/2))-1/3*ln(2-(1+1/x)^(1/2))-1/6*ln(1+(1+1/x)^(1/2))+x*l
n(-2+((1+x)/x)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2548, 706, 31, 633}

$$\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
+ x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{-2+\left(-2+4\sqrt{1+\frac{1}{x}}\right)x}dx$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-2+x)\left(-1+x^2\right)}dx,x,\sqrt{1+\frac{1}{x}}\right)$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x}dx,x,\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{-2-}{-1+}\right.$$

$$= -\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}dx,x,\sqrt{1+\right.$$

$$= \frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\right.$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.93

$$\frac{1}{6}\left(\log\left(2-\sqrt{\frac{1}{x}+1}\right)+6x\log\left(\sqrt{\frac{1}{x}+1}-2\right)-\log\left(\sqrt{\frac{1}{x}+1}+1\right)-6\tanh^{-1}\left(3-2\sqrt{\frac{1}{x}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (-6*ArcTanh[3 - 2*Sqrt[1 + x^(-1)]] + Log[2 - Sqrt[1 + x^(-1)]] + 6*x*Log[-
2 + Sqrt[1 + x^(-1)]] - Log[1 + Sqrt[1 + x^(-1)]])/6
```

fricas [A] time = 0.43, size = 48, normalized size = 0.70

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) - 2) - 1/6*log(sqrt((x + 1)/x) + 1) + 1/2
*log(sqrt((x + 1)/x) - 1)
```

giac [A] time = 0.27, size = 88, normalized size = 1.28

$$x\log\left(\sqrt{\frac{x+1}{x}}-2\right)+\frac{\log\left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6\,\mathrm{sgn}(x)}+\frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{3\,\mathrm{sgn}(x)}-\frac{\log\left(\left|-3x+3\sqrt{x^2+x}-1\right|\right)}{6\,\mathrm{sgn}(x)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) + 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) +
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

maple [A] time = 0.14, size = 107, normalized size = 1.55

$$x\ln\left(-2+\sqrt{\frac{x+1}{x}}\right)+\frac{-3\sqrt{\frac{x+1}{x}}\,x\ln(-3x+1)+\sqrt{9}\,\sqrt{(x+1)x}\,\ln\left(\frac{15x+4\sqrt{9}\,\sqrt{x^2+x}+3}{9x-3}\right)-6\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}\right.}{18\sqrt{\frac{x+1}{x}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2+((x+1)/x)^(1/2)),x)
```

```
[Out] x*ln(-2+((x+1)/x)^(1/2))+1/18/((x+1)/x)^(1/2)/x*(9^(1/2)*((x+1)*x)^(1/2)*ln
(1/3*(15*x+4*9^(1/2)*(x^2+x)^(1/2)+3)/(3*x-1))-3*((x+1)/x)^(1/2)*x*ln(-3*x+
1)-6*((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2)))
```

maxima [A] time = 0.61, size = 67, normalized size = 0.97

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-2\right)}{\frac{x+1}{x}-1}-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 2)/((x + 1)/x - 1) - 1/6*log(sqrt((x + 1)/x) + 1) + 1
/2*log(sqrt((x + 1)/x) - 1) - 1/3*log(sqrt((x + 1)/x) - 2)
```

mupad [B] time = 0.32, size = 63, normalized size = 0.91

$$\frac{\ln\left(5-5\sqrt{\frac{x+1}{x}}\right)}{2}-\frac{\ln\left(\frac{\sqrt{\frac{x+1}{x}}}{9}+\frac{1}{9}\right)}{6}-\frac{\ln\left(\frac{10}{9}-\frac{5\sqrt{\frac{x+1}{x}}}{9}\right)}{3}+x\ln\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) - 2),x)
```

```
[Out] log(5 - 5*((x + 1)/x)^(1/2))/2 - log(((x + 1)/x)^(1/2)/9 + 1/9)/6 - log(10/
9 - (5*((x + 1)/x)^(1/2))/9)/3 + x*log(((x + 1)/x)^(1/2) - 2)
```

sympy [A] time = 41.30, size = 53, normalized size = 0.77

$$x\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-2\right)}{3}+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) - log(sqrt(1 + 1/x) - 2)/3 + log(sqrt(1 + 1/x) -
 1)/2 - log(sqrt(1 + 1/x) + 1)/6
```

3.299 $\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx$

Optimal. Leaf size=9

$$\frac{x^{ax}}{a}$$

```
[Out] x^(a*x)/a
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2553}

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(a*x) + x^(a*x)*Log[x],x]
[Out] x^(a*x)/a
```

Rule 2553

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx &= \int x^{ax}\,dx+\int x^{ax}\log(x)\,dx\\ &= \frac{x^{ax}}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(a*x) + x^(a*x)*Log[x],x]
[Out] x^(a*x)/a
```

fricas [A] time = 0.44, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="fricas")
[Out] x^(a*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{ax}\log(x)+x^{ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^(a*x)*log(x) + x^(a*x), x)
```

maple [A] time = 0.07, size = 11, normalized size = 1.22

$$\frac{e^{ax\ln(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(a*x)+x^(a*x)*ln(x),x)
```

```
[Out] 1/a*exp(a*x*ln(x))
```

maxima [A] time = 0.82, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="maxima")
```

```
[Out] x^(a*x)/a
```

mupad [B] time = 0.37, size = 9, normalized size = 1.00

$$\frac{x^{a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(a*x) + x^(a*x)*log(x),x)
```

```
[Out] x^(a*x)/a
```

sympy [A] time = 0.26, size = 10, normalized size = 1.11

$$\begin{cases} \frac{x^{ax}}{a} & \text{for}\: a \neq 0 \\ x \log{\left(x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(a*x)+x**(a*x)*ln(x),x)
```

```
[Out] Piecewise((x**(a*x)/a, Ne(a, 0)), (x*log(x), True))
```

3.300 $\int \log^m(x)^p \, dx$

Optimal. Leaf size=26

$$(-\log(x))^{-mp}\log^m(x)^p\Gamma(mp+1,-\log(x))$$

```
[Out] GAMMA(m*p+1,-ln(x))*(ln(x)^m)^p/((-ln(x))^(m*p))
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6720, 2299, 2181}

$$(-\log(x))^{-mp}\log^m(x)^p\text{Gamma}(mp+1,-\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^m)^p,x]
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \log^m(x)^p\,dx &= \left(\log^{-mp}(x)\log^m(x)^p\right)\int \log^{mp}(x)\,dx\\ &= \left(\log^{-mp}(x)\log^m(x)^p\right)\text{Subst}\left(\int e^x x^{mp}\,dx, x, \log(x)\right)\\ &= \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$(-\log(x))^{-mp}\log^m(x)^p\Gamma(mp+1,-\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^m)^p,x]
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

fricas [A] time = 0.46, size = 16, normalized size = 0.62

$$\cos(\pi mp)\Gamma(mp+1, -\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="fricas")
[Out] cos(pi*m*p)*gamma(m*p + 1, -log(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\log(x)^m)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="giac")
[Out] integrate((log(x)^m)^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (\ln(x)^m)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)^m)^p,x)
[Out] int((ln(x)^m)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\log(x)^m)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="maxima")
[Out] integrate((log(x)^m)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int (\ln(x)^m)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^m)^p,x)
[Out] int((log(x)^m)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\log(x)^m)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(x)**m)**p,x)
[Out] Integral((log(x)**m)**p, x)
```

3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}\,(2a+b)e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*(2*a+b)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b)+x*(a
+b*ln(x))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2294, 2299, 2180, 2204}

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}\,(2a+b)e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] -((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2)*E^(a/b))
+ (x*Sqrt[a + b*Log[x]])/b
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{x\sqrt{a+b\log(x)}}{b} + \frac{(-2a-b)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}$$
$$= \frac{x\sqrt{a+b\log(x)}}{b} + \frac{(-2a-b)\,\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx, x, \log(x)\right)}{2b}$$
$$= \frac{x\sqrt{a+b\log(x)}}{b} - \frac{(2a+b)\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\log(x)}\right)}{b^2}$$
$$= -\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{x\sqrt{a+b\log(x)}}{b}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 1.20

$$\frac{2x(a+b\log(x)) - (2a+b)e^{-\frac{a}{b}}\sqrt{-\frac{a+b\log(x)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{a+b\log(x)}{b}\right)}{2b\sqrt{a+b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*x*(a + b*Log[x]) - ((2*a + b)*Gamma[1/2, -((a + b*Log[x])/b)]*Sqrt[-((a
+ b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.28, size = 89, normalized size = 1.48

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\,\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}} + \frac{\sqrt{\pi}\,a\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\,\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}\,b} + \frac{\sqrt{b\log(x)+a}\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi)*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(p
i)*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log
(x) + a)*x/b
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{\sqrt{b\ln(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

```
[Out] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

maxima [B] time = 0.70, size = 108, normalized size = 1.80

$$-\frac{\frac{2\sqrt{\pi}a\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}+\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) +
 sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqrt
(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{\sqrt{a+b\,\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a + b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a + b*log(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a + b*log(x)), x)
```

3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\pi}\,(2a-b)e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*a-b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)-x*(a-
b*ln(x))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2294, 2299, 2180, 2205}

$$-\frac{\sqrt{\pi}\,(2a-b)e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] -((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(2*b^(3/2)) -
 (x*Sqrt[a - b*Log[x]])/b
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(-2a+b)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}$$
$$= -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(-2a+b)\,\text{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx, x, \log(x)\right)}{2b}$$
$$= -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(2a-b)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\log(x)}\right)}{b^2}$$
$$= -\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{x\sqrt{a-b\log(x)}}{b}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 1.11

$$\frac{-2x(a-b\log(x)) - \left((b-2a)e^{a/b}\sqrt{\frac{a}{b}-\log(x)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\right)}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] (-((-2*a + b)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]) - 2*x*(a
 - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.34, size = 74, normalized size = 1.16

$$\frac{\sqrt{\pi}\,a\,\text{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} - \frac{\sqrt{\pi}\,\text{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log(x)+a}\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) - 1/2*sqrt(pi)
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*x/
b
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{\sqrt{-b\ln(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

```
[Out] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

maxima [A] time = 0.65, size = 94, normalized size = 1.47

$$-\frac{2\sqrt{\pi}\,a\sqrt{b}\,\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - \sqrt{\pi}\,b^{\frac{3}{2}}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} + 2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - sqrt(
pi)*b^(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) + 2*sqrt(-b*log(x) + a
)*b*e^((b*log(x) - a)/b + a/b))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{\sqrt{a-b\,\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a - b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a - b*log(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{(x)}}{\sqrt{a-b\log{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a - b*log(x)), x)
```

3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}(2Ab-B(2a+b))\operatorname{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

```
[Out] 1/2*(2*A*b-(2*a+b)*B)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(
a/b)+B*x*(a+b*ln(x))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2294, 2299, 2180, 2204}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}(2Ab-B(2a+b))\operatorname{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] ((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2
)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{Bx\sqrt{a+b\log(x)}}{b} + \frac{(2Ab-(2a+b)B)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}$$
$$= \frac{Bx\sqrt{a+b\log(x)}}{b} + \frac{(2Ab-(2a+b)B)\,\text{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx, x, \log(x)\right)}{2b}$$
$$= \frac{Bx\sqrt{a+b\log(x)}}{b} + \frac{(2Ab-(2a+b)B)\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\log(x)}\right)}{b^2}$$
$$= \frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{Bx\sqrt{a+b\log(x)}}{b}$$

Mathematica [A] time = 0.15, size = 80, normalized size = 1.16

$$\frac{e^{-\frac{a}{b}}(2Ab-B(2a+b))\sqrt{-\frac{a+b\log(x)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)+2Bx(a+b\log(x))}{2b\sqrt{a+b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*B*x*(a + b*Log[x]) + ((2*A*b - (2*a + b)*B)*Gamma[1/2, -((a + b*Log[x])/
b)]*Sqrt[-((a + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [B] time = 0.31, size = 129, normalized size = 1.87

$$-\frac{\sqrt{\pi}\,A\,\text{erf}\left(-\frac{\sqrt{b\log(x)+a}\,\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}}+\frac{\sqrt{\pi}\,B\,\text{erf}\left(-\frac{\sqrt{b\log(x)+a}\,\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}}+\frac{\sqrt{\pi}\,Ba\,\text{erf}\left(-\frac{\sqrt{b\log(x)+a}\,\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}\,b}+\sqrt{b\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(pi)*A*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + 1/2*sqr
t(pi)*B*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(pi)*B*
a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log(x)
 + a)*B*x/b
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{B\ln(x)+A}{\sqrt{b\ln(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(x))/(b*ln(x)+a)^(1/2),x)
```

```
[Out] int((A+B*ln(x))/(b*ln(x)+a)^(1/2),x)
```

maxima [B] time = 0.74, size = 156, normalized size = 2.26

$$\frac{\frac{2\sqrt{\pi}A\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{b\sqrt{-\frac{1}{b}}}-\frac{\left(\frac{\sqrt{\pi}b\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(pi)*A*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) -
2*sqrt(pi)*B*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/(b*sqrt(-1/b)) -
 (sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqr
t(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))*B/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln(x)}{\sqrt{a+b\,\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(x))/(a + b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a + b*log(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\log{(x)}}{\sqrt{a+b\log{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a + b*log(x)), x)
```

3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\pi}\,e^{a/b}(2aB+2Ab-bB)\operatorname{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*A*b+2*B*a-B*b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(
3/2)-B*x*(a-b*ln(x))^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2294, 2299, 2180, 2205}

$$-\frac{\sqrt{\pi}\,e^{a/b}(2aB+2Ab-bB)\operatorname{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] -((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(
2*b^(3/2)) - (B*x*Sqrt[a - b*Log[x]])/b
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}$$
$$= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\,\text{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx, x, \log(x)\right)}{2b}$$
$$= -\frac{Bx\sqrt{a-b\log(x)}}{b}-\frac{(2Ab+2aB-bB)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\log(x)}\right)}{b^2}$$
$$= -\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 1.11

$$\frac{e^{a/b}(2aB+2Ab-bB)\sqrt{\frac{a}{b}-\log(x)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)-2Bx(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]
- 2*B*x*(a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.31, size = 106, normalized size = 1.49

$$\frac{\sqrt{\pi}Ba\,\text{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}}+\frac{\sqrt{\pi}A\,\text{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}-\frac{\sqrt{\pi}B\,\text{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}}-\frac{\sqrt{-b\log(x)+a}\,Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*B*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) + sqrt(pi)*A
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - 1/2*sqrt(pi)*B*erf(-sq
rt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*B*x/b
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{B\ln(x)+A}{\sqrt{-b\ln(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(x)+A)/(-b*ln(x)+a)^(1/2),x)
```

```
[Out] int((B*ln(x)+A)/(-b*ln(x)+a)^(1/2),x)
```

maxima [B] time = 0.66, size = 130, normalized size = 1.83

$$-\frac{\frac{2\sqrt{\pi}Ba\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}+2\sqrt{\pi}A\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-\frac{\left(\sqrt{\pi}b^{\frac{3}{2}}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*B*a*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) + 2*s
qrt(pi)*A*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - (sqrt(pi)*b^(3
/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - 2*sqrt(-b*log(x) + a)*b*e^((
b*log(x) - a)/b + a/b))*B/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln(x)}{\sqrt{a-b\,\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(x))/(a - b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a - b*log(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a - b*log(x)), x)
```

3.305 $\int x^2 \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=98

$$-\frac{1}{3}\text{Ei}(3\log(x))+\frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right)-\frac{1}{2}x\text{Li}_3\left(e^{2ix}\right)-\frac{1}{4}i\text{Li}_4\left(e^{2ix}\right)+\frac{ix^4}{12}-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log(\log(x)\sin(x))$$

```
[Out] 1/12*I*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(ln(x)*sin(x)
)+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x))-1/4*I*polylog
(4,exp(2*I*x))
```

Rubi [A] time = 0.29, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {30, 2555, 12, 6688, 14, 3717, 2190, 2531, 6609, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\frac{ix^4}{12}-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])/3 + (x^
3*Log[Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (x*PolyLog[3,
 E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int x^2 \log(\log(x)\sin(x))\,dx = \frac{1}{3}x^3\log(\log(x)\sin(x)) - \int \frac{x^2(1+x\cot(x)\log(x))}{3\log(x)}\,dx$$
$$= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \frac{x^2(1+x\cot(x)\log(x))}{\log(x)}\,dx$$
$$= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^2\left(x\cot(x)+\frac{1}{\log(x)}\right)dx$$
$$= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \left(x^3\cot(x)+\frac{x^2}{\log(x)}\right)dx$$
$$= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^3\cot(x)\,dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\,dx$$
$$= \frac{ix^4}{12} + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{2}{3}i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx - \frac{1}{3}\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(x)\right)$$
$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \int x^2\log\left(1\right.$$
$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right.$$
$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right.$$
$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right.$$
$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right.$$

Mathematica [A] time = 0.06, size = 95, normalized size = 0.97

$$\frac{1}{192}i\left(64i\text{Ei}(3\log(x)) - 96x^2\text{Li}_2\left(e^{-2ix}\right) + 96ix\text{Li}_3\left(e^{-2ix}\right) + 48\text{Li}_4\left(e^{-2ix}\right) - 16x^4 + 64ix^3\log\left(1-e^{-2ix}\right) - 64ix^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - 16*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x^3*Log[1
- E^((-2*I)*x)] - (64*I)*x^3*Log[Log[x]*Sin[x]] - 96*x^2*PolyLog[2, E^((-2*
I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)])
```

fricas [C] time = 0.51, size = 234, normalized size = 2.39

$$\frac{1}{3}x^3\log\left(\log(x)\sin(x)\right) - \frac{1}{6}x^3\log\left(\cos(x)+i\sin(x)+1\right) - \frac{1}{6}x^3\log\left(\cos(x)-i\sin(x)+1\right) - \frac{1}{6}x^3\log\left(-\cos(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin(x)) - 1/2*I*x^2
*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin(x)) + 1/2*I*x^2
*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)) - x*polylog(3,
 cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*polylog(3, -cos(
x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x) + I*sin(x)) +
I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*sin(x)) - I*poly
log(4, -cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(\log(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x^2*log(log(x)*sin(x)), x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(\ln(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(ln(x)*sin(x)),x)
[Out] int(x^2*ln(ln(x)*sin(x)),x)
```

maxima [A] time = 2.06, size = 94, normalized size = 0.96

$$\frac{1}{12}\left(2i\pi - 4\log(2)\right)x^3 - \frac{1}{4}ix^4 + \frac{1}{3}x^3\log\left(\log(x)\right) + ix^2\mathrm{Li}_2\left(-e^{(ix)}\right) + ix^2\mathrm{Li}_2\left(e^{(ix)}\right) - 2x\mathrm{Li}_3(-e^{(ix)}) - 2x\mathrm{Li}_3(e^{(ix)}) - \frac{1}{3}\,\mathrm{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="maxima")
[Out] 1/12*(2*I*pi - 4*log(2))*x^3 - 1/4*I*x^4 + 1/3*x^3*log(log(x)) + I*x^2*dilo
g(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*polylog
(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*polylog(4,
 e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(\ln(x)\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(log(x)*sin(x)),x)
[Out] int(x^2*log(log(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \log{\left(\log{\left(x \right)} \sin{\left(x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(ln(x)*sin(x)),x)
[Out] Integral(x**2*log(log(x)*sin(x)), x)
```

3.306 $\int x \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=80

$$-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right)-\frac{1}{4}\text{Li}_3\left(e^{2ix}\right)+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

```
[Out] 1/6*I*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(ln(x)*sin(x))
+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A] time = 0.18, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {30, 2555, 12, 6688, 14, 3717, 2190, 2531, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\text{Ei}(2\log(x))+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2 + (x^2
*Log[Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2
*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int x\log(\log(x)\sin(x))\,dx &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \int \frac{x(1+x\cot(x)\log(x))}{2\log(x)}\,dx \\ &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \frac{x(1+x\cot(x)\log(x))}{\log(x)}\,dx \\ &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x\left(x\cot(x)+\frac{1}{\log(x)}\right)dx \\ &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \left(x^2\cot(x)+\frac{x}{\log(x)}\right)dx \\ &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x^2\cot(x)\,dx - \frac{1}{2}\int \frac{x}{\log(x)}\,dx \\ &= \frac{ix^3}{6} + \frac{1}{2}x^2\log(\log(x)\sin(x)) + i\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(x)\right) \\ &= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \int x\log\left(1-\right. \\ &= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) \\ &= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) \\ &= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.99

$$\frac{1}{48}\left(-24\text{Ei}(2\log(x)) - 24ix\text{Li}_2\left(e^{-2ix}\right) - 12\text{Li}_3\left(e^{-2ix}\right) - 8ix^3 - 24x^2\log\left(1-e^{-2ix}\right) + 24x^2\log(\log(x)\sin(x)) +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-2*I)
*x)] + 24*x^2*Log[Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)] - 12*P
olyLog[3, E^((-2*I)*x)])/48
```

fricas [C] time = 0.47, size = 174, normalized size = 2.18

$$\frac{1}{2}x^2\log\left(\log(x)\sin(x)\right) - \frac{1}{4}x^2\log\left(\cos(x)+i\sin(x)+1\right) - \frac{1}{4}x^2\log\left(\cos(x)-i\sin(x)+1\right) - \frac{1}{4}x^2\log\left(-\cos(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)) - 1/2*I*x*dil
og(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) + 1/2*I*x*dilog(-
cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3, cos(x) + I*sin(
x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -cos(x) + I*sin(x)
) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(\log(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x*log(log(x)*sin(x)), x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int x \ln(\ln(x)\sin(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(ln(x)*sin(x)),x)
[Out] int(x*ln(ln(x)*sin(x)),x)
```

maxima [A] time = 1.92, size = 70, normalized size = 0.88

$$\frac{1}{12}\left(3i\pi - 6\log(2)\right)x^2 - \frac{1}{3}ix^3 + \frac{1}{2}x^2\log\left(\log(x)\right) + ix\mathrm{Li}_2\left(-e^{(ix)}\right) + ix\mathrm{Li}_2\left(e^{(ix)}\right) - \frac{1}{2}\mathrm{Ei}\left(2\log(x)\right) - \mathrm{Li}_3(-e^{(ix)}) - \mathrm{Li}_3(e^{($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="maxima")
[Out] 1/12*(3*I*pi - 6*log(2))*x^2 - 1/3*I*x^3 + 1/2*x^2*log(log(x)) + I*x*dilog(
-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I*x)) -
polylog(3, e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\, \ln(\ln(x)\, \sin(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(log(x)*sin(x)),x)
[Out] int(x*log(log(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log\left(\log(x)\sin(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(ln(x)*sin(x)),x)
[Out] Integral(x*log(log(x)*sin(x)), x)
```

3.307 $\int \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=52

$$-\mathrm{li}(x)+\frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)+x\log(\log(x)\sin(x))$$

```
[Out] 1/2*I*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(ln(x)*sin(x))+1/2*I*polylog(2,exp(2
*I*x))
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2549, 3717, 2190, 2279, 2391, 2298}

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,e^{2ix}\right)-\mathrm{li}(x)+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)+x\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
+ (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log(\log(x)\sin(x))\,dx = x\log(\log(x)\sin(x)) - \int \left(x\cot(x) + \frac{1}{\log(x)}\right)dx$$
$$= x\log(\log(x)\sin(x)) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx$$
$$= \frac{ix^2}{2} + x\log(\log(x)\sin(x)) - \text{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) + \int \log\left(1-e^{2ix}\right)dx$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)$$
$$= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.90

$$-\text{li}(x) + \frac{1}{2}i\left(x^2 + \text{Li}_2\left(e^{2ix}\right)\right) - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]*Sin[x]],x]
```

```
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[Log[x]*Sin[x]] - LogIntegral[x] + (I/2)*(
x^2 + PolyLog[2, E^((2*I)*x)])
```

fricas [B] time = 0.46, size = 109, normalized size = 2.10

$$x\log\left(\log(x)\sin(x)\right) - \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right) - \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right) - \frac{1}{2}x\log\left(-\cos(x) + i\sin(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x)
- I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I
*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(
x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x)) - l
og_integral(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\log(x)\sin(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(log(x)*sin(x)), x)
```

maple [C] time = 0.84, size = 368, normalized size = 7.08

$$-\frac{i\pi x\,\text{csgn}\left(i\left(e^{2ix}-1\right)\right)\text{csgn}\left(i\ln(x)\right)\text{csgn}\left(i\left(e^{2ix}-1\right)\ln(x)\right)}{2} + \frac{i\pi x\,\text{csgn}\left(i\left(e^{2ix}-1\right)\right)\text{csgn}\left(i\left(e^{2ix}-1\right)\ln(x)\right)^2}{2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x)),x)
```

```
[Out] -x*ln(exp(I*x))-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*
(exp(2*I*x)-1))*x-1/2*I*Pi*x-I*dilog(exp(I*x))+1/2*I*Pi*csgn(ln(x)*sin(x))^
3*x+I*ln(exp(I*x))*ln(exp(I*x)+1)-1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*
sin(x))^2*x-ln(2)*x-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x-I*ln(exp(I*x)
)*ln(exp(2*I*x)-1)+1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))
^2*x+1/2*I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(
I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*x^2+1/2*I*Pi*csgn(
I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+1/2*I*Pi*csg
n(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(I*ln(x)*sin(x))^2*x-1/2
*I*Pi*csgn(I*ln(x)*sin(x))^3*x+1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin
(x))*x+I*dilog(exp(I*x)+1)+x*ln(ln(x))+Ei(1,-ln(x))
```

maxima [A] time = 1.76, size = 43, normalized size = 0.83

$$\frac{1}{2}\left(i\pi - 2\log(2)\right)x - \frac{1}{2}ix^2 + x\log\left(\log(x)\right) - \mathrm{Ei}\left(\log(x)\right) + i\,\mathrm{Li}_2\left(-e^{(ix)}\right) + i\,\mathrm{Li}_2\left(e^{(ix)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - 1/2*I*x^2 + x*log(log(x)) - Ei(log(x)) + I*dilog(
-e^(I*x)) + I*dilog(e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\ln(x)\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x)),x)
```

```
[Out] int(log(log(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\log(x)\sin(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(log(log(x)*sin(x)), x)
```

3.308 $\int \frac{\log(\log(x)\sin(x))}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\log(\log(x)\sin(x))}{x},x\right)$$

```
[Out] CannotIntegrate(ln(ln(x)*sin(x))/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x,x]
[Out] Defer[Int][Log[Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx=\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Mathematica [A] time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x,x]
[Out] Integrate[Log[Log[x]*Sin[x]]/x, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\log(x)\sin(x)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="fricas")
[Out] integral(log(log(x)*sin(x))/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log(x)\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="giac")
[Out] integrate(log(log(x)*sin(x))/x, x)
```

maple [A] time = 3.18, size = 0, normalized size = 0.00

$$\int \frac{\ln(\ln(x)\sin(x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(ln(x)*sin(x))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\log(2)+1\right)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - integrat
e(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\ln\left(\ln(x)\,\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x))/x,x)
```

```
[Out] int(log(log(x)*sin(x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log(x)\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x,x)
```

```
[Out] Integral(log(log(x)*sin(x))/x, x)
```

3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\cot(x)}{x},x\right)+\text{Ei}(-\log(x))-\frac{\log(\log(x)\sin(x))}{x}$$

```
[Out] Ei(-ln(x))-ln(ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x^2,x]
[Out] ExpIntegralEi[-Log[x]] - Log[Log[x]*Sin[x]]/x + Defer[Int][Cot[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx &= -\frac{\log(\log(x)\sin(x))}{x}-\int \frac{-1-x\cot(x)\log(x)}{x^2\log(x)}\,dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}-\int \left(-\frac{\cot(x)}{x}-\frac{1}{x^2\log(x)}\right)dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx+\int \frac{1}{x^2\log(x)}\,dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx+\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx,x,\log(x)\right)\\
&= \text{Ei}(-\log(x))-\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x^2,x]
[Out] Integrate[Log[Log[x]*Sin[x]]/x^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\log(x)\sin(x)\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="fricas")
[Out] integral(log(log(x)*sin(x))/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="giac")
[Out] integrate(log(log(x)*sin(x))/x^2, x)
```

maple [A] time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\ln(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x^2,x)
[Out] int(ln(ln(x)*sin(x))/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x\left(\mathrm{Ei}\left(-\log(x)\right)+\overline{\mathrm{Ei}\left(-\log(x)\right)}\right)-2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)x}\,dx+2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)x}\,dx+2\,\mathrm{lo}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="maxima")
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*log(2) - log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*log(log(x)))/x
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(\ln(x)\,\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x))/x^2,x)
[Out] int(log(log(x)*sin(x))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x**2,x)
[Out] Integral(log(log(x)*sin(x))/x**2, x)
```

3.310 $\int x^2 \log\left(e^x \log(x)\sin(x)\right) dx$

Optimal. Leaf size=103

$$-\frac{1}{3}\text{Ei}(3\log(x))+\frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right)-\frac{1}{2}x\text{Li}_3\left(e^{2ix}\right)-\frac{1}{4}i\text{Li}_4\left(e^{2ix}\right)+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\right.$$

```
[Out] (-1/12+1/12*I)*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(exp(
x)*ln(x)*sin(x))+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x)
)-1/4*I*polylog(4,exp(2*I*x))
```

Rubi [A] time = 0.20, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {30, 2555, 12, 14, 3717, 2190, 2531, 6609, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}x^3\log\left(1-e^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/12 + I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])
/3 + (x^3*Log[E^x*Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (
x*PolyLog[3, E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(e^x\log(x)\sin(x)\right)dx &= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\int\frac{1}{3}x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&=\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&=\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int\left(x^3(1+\cot(x))+\frac{x^2}{\log(x)}\right)dx\\
&=\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int x^3(1+\cot(x))\,dx-\frac{1}{3}\int\frac{x^2}{\log(x)}dx\\
&=\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int\left(x^3+x^3\cot(x)\right)dx-\frac{1}{3}\text{Subst}\left(\int\frac{e^{3x}}{x}dx,x,\text{lo}\right.\\
&=-\frac{x^4}{12}-\frac{1}{3}\text{Ei}(3\log(x))+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{3}\int x^3\cot(x)\,dx\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)+\frac{2}{3}i\int\frac{e^{2ix}x^3}{1-e^{2ix}}dx\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\\
&=\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 0.97

$$\frac{1}{192}i\left(64i\text{Ei}(3\log(x))-96x^2\text{Li}_2\left(e^{-2ix}\right)+96ix\text{Li}_3\left(e^{-2ix}\right)+48\text{Li}_4\left(e^{-2ix}\right)+(-16+16i)x^4+64ix^3\log\left(1-e^{-2ix}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - (16 - 16*I)*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x
^3*Log[1 - E^((-2*I)*x)] - (64*I)*x^3*Log[E^x*Log[x]*Sin[x]] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])
```

fricas [C] time = 0.49, size = 241, normalized size = 2.34

$$-\frac{1}{12}x^4+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{6}x^3\log\left(\cos(x)+i\sin(x)+1\right)-\frac{1}{6}x^3\log\left(\cos(x)-i\sin(x)+1\right)-\frac{1}{6}x^3\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/12*x^4 + 1/3*x^3*log(e^x*log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin
(x)) - 1/2*I*x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin
(x)) + 1/2*I*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)
) - x*polylog(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*
```

```
polylog(3, -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x
) + I*sin(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*
sin(x)) - I*polylog(4, -cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(e^x \log(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e^x*log(x)*sin(x)), x)
```

maple [F] time = 2.47, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(\mathrm{e}^x \ln(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] int(x^2*ln(exp(x)*ln(x)*sin(x)),x)
```

maxima [A] time = 2.42, size = 94, normalized size = 0.91

$$\frac{1}{12}\left(2i\pi - 4\log(2)\right)x^3 - \left(\frac{1}{4}i - \frac{1}{4}\right)x^4 + \frac{1}{3}x^3\log\left(\log(x)\right) + i x^2 \mathrm{Li}_2\left(-e^{(ix)}\right) + i x^2 \mathrm{Li}_2\left(e^{(ix)}\right) - 2x\mathrm{Li}_3(-e^{(ix)}) - 2x\mathrm{Li}_3(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(2*I*pi - 4*log(2))*x^3 - (1/4*I - 1/4)*x^4 + 1/3*x^3*log(log(x)) + I*
x^2*dilog(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x
*polylog(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*po
lylog(4, e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(\mathrm{e}^x\,\ln(x)\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(exp(x)*log(x)*sin(x)),x)
```

```
[Out] int(x^2*log(exp(x)*log(x)*sin(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Timed out
```

3.311 $\int x \log \left(e^x \log(x) \sin(x)\right) dx$

Optimal. Leaf size=85

$$-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right)-\frac{1}{4}\text{Li}_3\left(e^{2ix}\right)+\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)$$

```
[Out] (-1/6+1/6*I)*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(exp(x)
*ln(x)*sin(x))+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {30, 2555, 12, 14, 3717, 2190, 2531, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\text{Ei}(2\log(x))+\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/6 + I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2
 + (x^2*Log[E^x*Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyL
og[3, E^((2*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(e^x\log(x)\sin(x)\right)dx &= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\int\frac{1}{2}x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2(1+\cot(x))+\frac{x}{\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2(1+\cot(x))\,dx-\frac{1}{2}\int\frac{x}{\log(x)}dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2+x^2\cot(x)\right)dx-\frac{1}{2}\text{Subst}\left(\int\frac{e^{2x}}{x}dx,x,\log\right.\\
&= -\frac{x^3}{6}-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\cot(x)\,dx\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+i\int\frac{e^{2ix}x^2}{1-e^{2ix}}dx\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+
\end{aligned}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.96

$$\frac{1}{48}\left(-24\text{Ei}(2\log(x))-24ix\text{Li}_2\left(e^{-2ix}\right)-12\text{Li}_3\left(e^{-2ix}\right)+(-8-8i)x^3-24x^2\log\left(1-e^{-2ix}\right)+24x^2\log\left(e^x\log(x)\text{si}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8 + 8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-
2*I)*x)] + 24*x^2*Log[E^x*Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)
] - 12*PolyLog[3, E^((-2*I)*x)])/48
```

fricas [C] time = 0.51, size = 181, normalized size = 2.13

$$-\frac{1}{6}x^3+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{4}x^2\log\left(\cos(x)+i\sin(x)+1\right)-\frac{1}{4}x^2\log\left(\cos(x)-i\sin(x)+1\right)-\frac{1}{4}x^2\log\left(-\text{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/6*x^3 + 1/2*x^2*log(e^x*log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)
) - 1/2*I*x*dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) +
1/2*I*x*dilog(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3,
cos(x) + I*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -co
s(x) + I*sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(e^x\log(x)\sin(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(e^x*log(x)*sin(x)), x)
```

maple [F] time = 2.35, size = 0, normalized size = 0.00

$$\int x \ln \left(\mathrm{e}^{x} \ln(x) \sin(x)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] int(x*ln(exp(x)*ln(x)*sin(x)),x)
```

maxima [A] time = 1.43, size = 70, normalized size = 0.82

$$\frac{1}{12}\left(3i\pi - 6\log(2)\right)x^2 - \left(\frac{1}{3}i - \frac{1}{3}\right)x^3 + \frac{1}{2}x^2\log\left(\log(x)\right) + i\,x\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,x\mathrm{Li}_2\left(e^{(i\,x)}\right) - \frac{1}{2}\mathrm{Ei}\left(2\log(x)\right) - \mathrm{Li}_3(-e^{(i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*pi - 6*log(2))*x^2 - (1/3*I - 1/3)*x^3 + 1/2*x^2*log(log(x)) + I*
x*dilog(-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(
I*x)) - polylog(3, e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(\mathrm{e}^{x}\,\ln(x)\,\sin(x)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(exp(x)*log(x)*sin(x)),x)
```

```
[Out] int(x*log(exp(x)*log(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log \left(e^{x} \log (x) \sin (x)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(x*log(exp(x)*log(x)*sin(x)), x)
```

3.312 $\int \log\left(e^x \log(x)\sin(x)\right)\,dx$

Optimal. Leaf size=57

$$-\text{li}(x)+\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)+\left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)$$

```
[Out] (-1/2+1/2*I)*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(exp(x)*ln(x)*sin(x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2549, 3717, 2190, 2279, 2391, 2298}

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\text{li}(x)+\left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - LogI
ntegral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \log\left(e^x \log(x)\sin(x)\right)\,dx = x\log\left(e^x\log(x)\sin(x)\right) - \int \left(x + x\cot(x) + \frac{1}{\log(x)}\right)dx$$
$$= -\frac{x^2}{2} + x\log\left(e^x\log(x)\sin(x)\right) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx$$
$$= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx$$
$$= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + \int \log\left(1-e^{2i\right.$$
$$= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{\right.$$
$$= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.98

$$\frac{1}{2}\left(-2\text{li}(x) + i\text{Li}_2\left(e^{2ix}\right) + (-1+i)x^2 - 2x\log\left(1-e^{2ix}\right) + 2x\log\left(e^x\log(x)\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] ((-1 + I)*x^2 - 2*x*Log[1 - E^((2*I)*x)] + 2*x*Log[E^x*Log[x]*Sin[x]] - 2*L
ogIntegral[x] + I*PolyLog[2, E^((2*I)*x)])/2
```

fricas [B] time = 0.49, size = 116, normalized size = 2.04

$$-\frac{1}{2}x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right) - \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right) - \frac{1}{2}x\log\left(-\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(e^x*log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/
2*x*log(cos(x) - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*
log(-cos(x) - I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(
cos(x) - I*sin(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x)
- I*sin(x)) - log_integral(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(e^x\log(x)\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(e^x*log(x)*sin(x)), x)
```

maple [C] time = 1.04, size = 583, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] 1/2*ln(exp(x))^2-ln(2)*x+I*dilog(exp(I*x)+1)+Ei(1,-ln(x))-I*ln(exp(2*I*x)-1
)*ln(exp(I*x))-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^3*x+1/2*I
*x^2+1/2*I*Pi*x*csgn(ln(x)*sin(x))^3-1/2*I*Pi*x+I*ln(exp(I*x)+1)*ln(exp(I*x
))+1/2*I*Pi*x*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))*csgn(I*(exp(2*I*x)-1)*ln
(x))+1/2*I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2*x-1/2*I*Pi*csgn((ex
p((1+I)*x)-exp((1-I)*x))*ln(x))^3*x-1/2*I*Pi*x*csgn(I*(exp(2*I*x)-1)*ln(x))
^3+1/2*I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-ex
p((1-I)*x)))*x+1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp
((1+I)*x)-exp((1-I)*x))*ln(x))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(ex
p((1+I)*x)-exp((1-I)*x)))^2*x-x*ln(exp(I*x))-I*dilog(exp(I*x))+1/2*I*Pi*x*c
sgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))^2+1/2*I*Pi*x*csgn(I*exp(-I*x))*cs
gn(ln(x)*sin(x))^2+1/2*I*Pi*x*csgn(ln(x)*sin(x))^2*csgn(I*(exp(2*I*x)-1)*ln
(x))-1/2*I*Pi*x*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*
ln(x))-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x
)-exp((1-I)*x))*ln(x))*x+x*ln(ln(x))-1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(
x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x+1/2*I*Pi*x*csgn(I*(exp(2*I*x)-1))*csgn(
I*(exp(2*I*x)-1)*ln(x))^2
```

maxima [A] time = 1.29, size = 43, normalized size = 0.75

$$\frac{1}{2}\left(i\,\pi - 2\,\log(2)\right)x - \left(\frac{1}{2}i - \frac{1}{2}\right)x^2 + x\log\left(\log(x)\right) - \mathrm{Ei}\left(\log(x)\right) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - (1/2*I - 1/2)*x^2 + x*log(log(x)) - Ei(log(x)) +
I*dilog(-e^(I*x)) + I*dilog(e^(I*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(e^x\,\ln(x)\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x)),x)
```

```
[Out] int(log(exp(x)*log(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(e^x \log(x) \sin(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x)), x)
```

3.313 $\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx$

Optimal. Leaf size=16

$$\text{Int}\left(\frac{\log\left(e^x \log(x)\sin(x)\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x,x]
[Out] Defer[Int][Log[E^x*Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx = \int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx$$

Mathematica [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x,x]
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e^x \log(x)\sin(x)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="fricas")
[Out] integral(log(e^x*log(x)*sin(x))/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="giac")
[Out] integrate(log(e^x*log(x)*sin(x))/x, x)
```

maple [A] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e^{x}\ln(x)\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\log(2)+1\right)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
x + integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - inte
grate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\ln\left(e^{x}\,\ln(x)\,\sin(x)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x))/x,x)
```

```
[Out] int(log(exp(x)*log(x)*sin(x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x))/x, x)
```

3.314 $\int \frac{\log(e^x \log(x)\sin(x))}{x^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\cot(x)}{x},x\right)+\text{Ei}(-\log(x))+\log(x)-\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}$$

```
[Out] Ei(-ln(x))+ln(x)-ln(exp(x)*ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e^x\log(x)\sin(x)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] + Log[x] - Log[E^x*Log[x]*Sin[x]]/x + Defer[Int][Cot
[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e^x\log(x)\sin(x)\right)}{x^2}\,dx &= -\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}+\int \frac{1+\cot(x)+\frac{1}{x\log(x)}}{x}\,dx\\
&= -\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}+\int \left(\frac{1+\cot(x)}{x}+\frac{1}{x^2\log(x)}\right)dx\\
&= -\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}+\int \frac{1+\cot(x)}{x}\,dx+\int \frac{1}{x^2\log(x)}\,dx\\
&= -\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}+\int \left(\frac{1}{x}+\frac{\cot(x)}{x}\right)dx+\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx,x,\log(x)\right)\\
&= \text{Ei}(-\log(x))+\log(x)-\frac{\log\left(e^x\log(x)\sin(x)\right)}{x}+\int \frac{\cot(x)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 2.46, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^x\log(x)\sin(x)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e^x\log(x)\sin(x)\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(log(e^x*log(x)*sin(x))/x^2, x)
```

giac [**A**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(log(e^x*log(x)*sin(x))/x^2, x)
```

maple [**A**]   time = 4.36, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e^x \ln(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

maxima [**A**]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{x\left(\mathrm{Ei}\left(-\log(x)\right) + \overline{\mathrm{Ei}\left(-\log(x)\right)}\right) - 2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)x}\,dx + 2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)x}\,dx + 2\,x\log}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*x*log(x) + 2*log(2) - log(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*lo
g(log(x)))/x
```

mupad [**A**]   time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(e^x\ \ln(x)\ \sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x))/x^2,x)
```

```
[Out] int(log(exp(x)*log(x)*sin(x))/x^2, x)
```

sympy [**A**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^x \log(x)\sin(x)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x**2,x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x))/x**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```